Compton amplitude and the nucleon structure functions

from lattice QCD

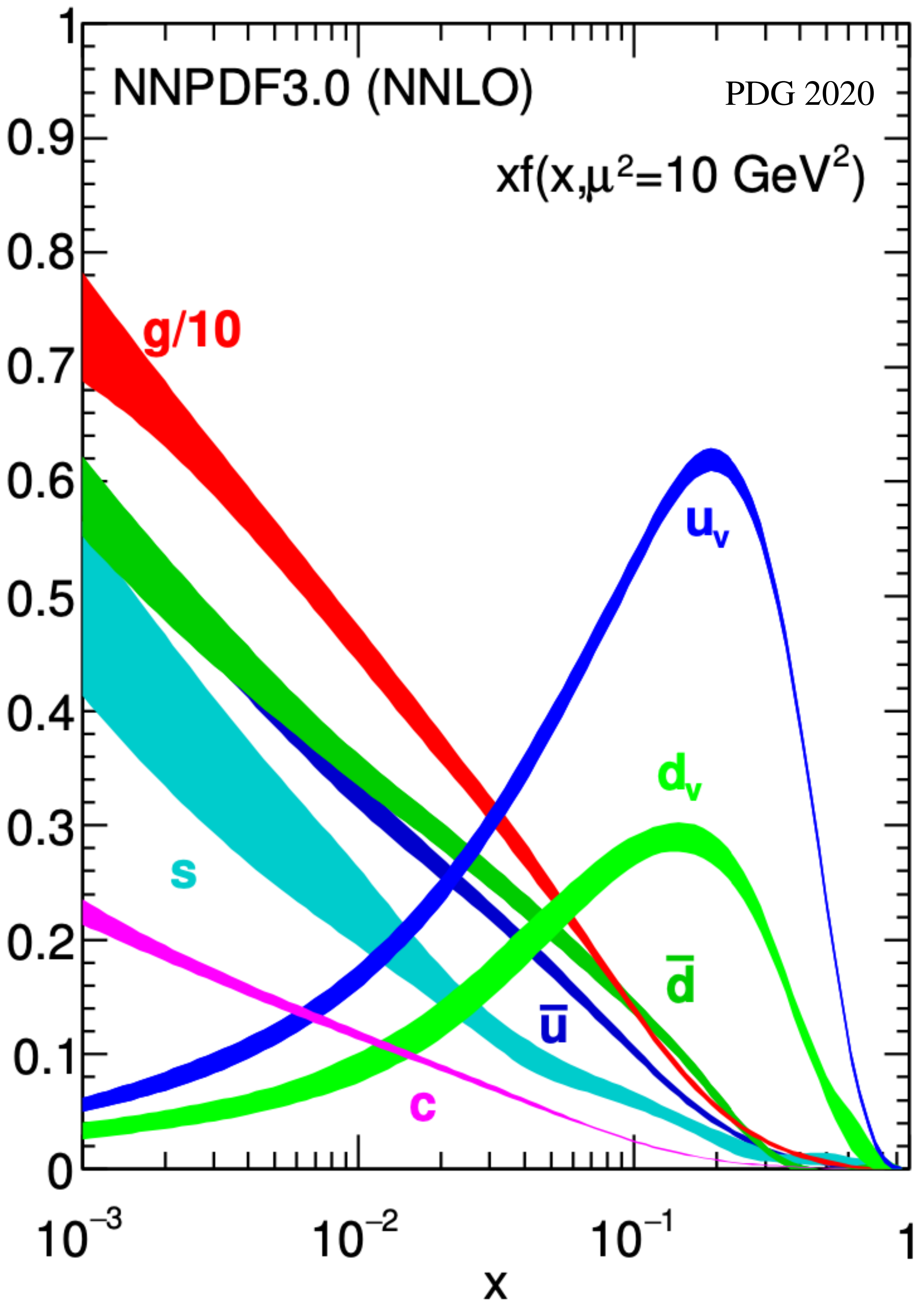
K. Utku Can
CSSM, The University of Adelaide

in collaboration with QCDSF:
R. D. Young, J. M. Zanotti (Adelaide),
R. Horsley (Edinburgh), P.E.L. Rakow (Liverpool), H.Perlt (Leipzig),
G. Schierholz (DESY), H. Stüben (Hamburg), Y. Nakamura (RIKEN, Kobe)

PDFLattice 2024, JLab

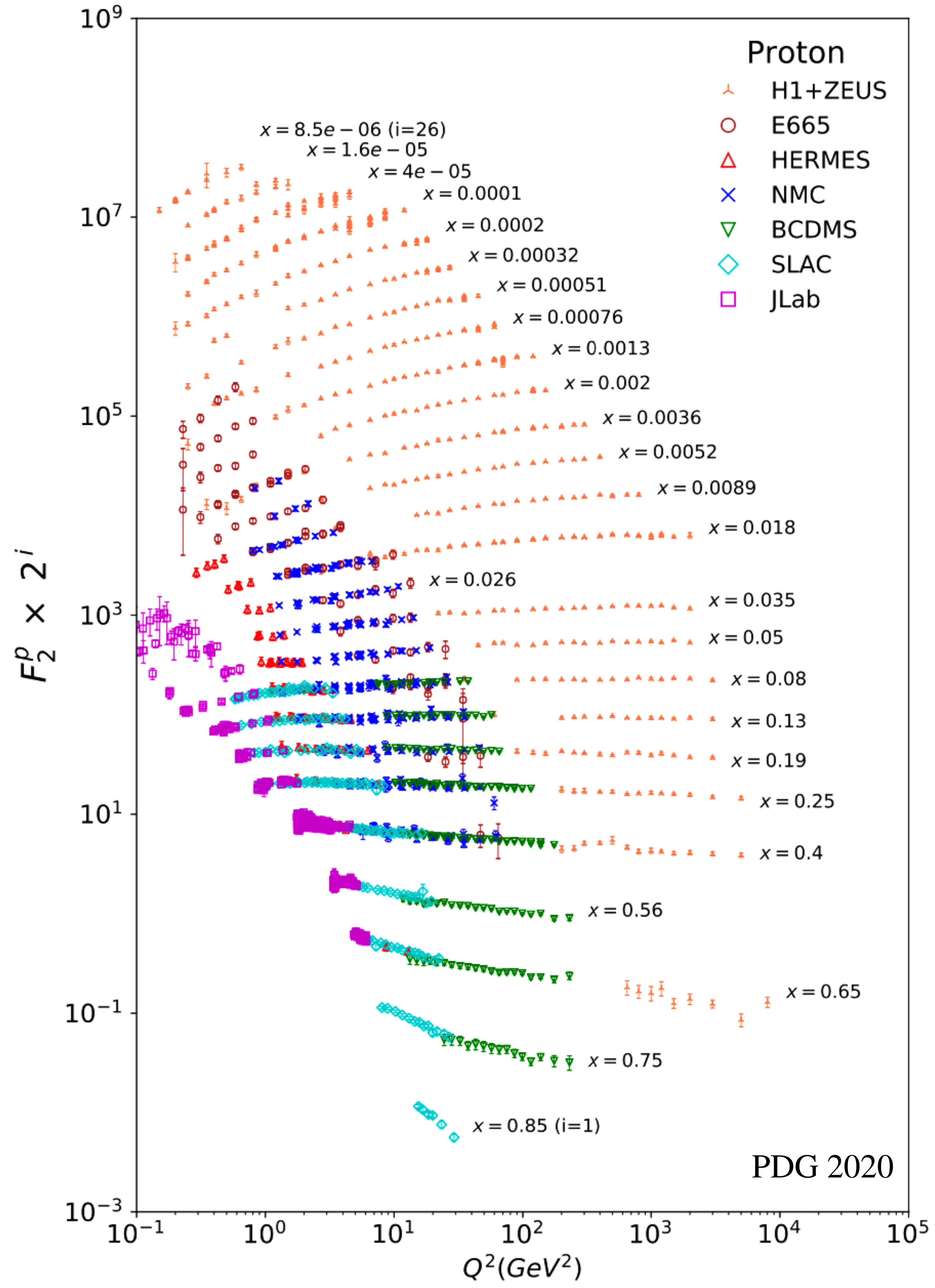
18 – 20 November 2024

Motivation

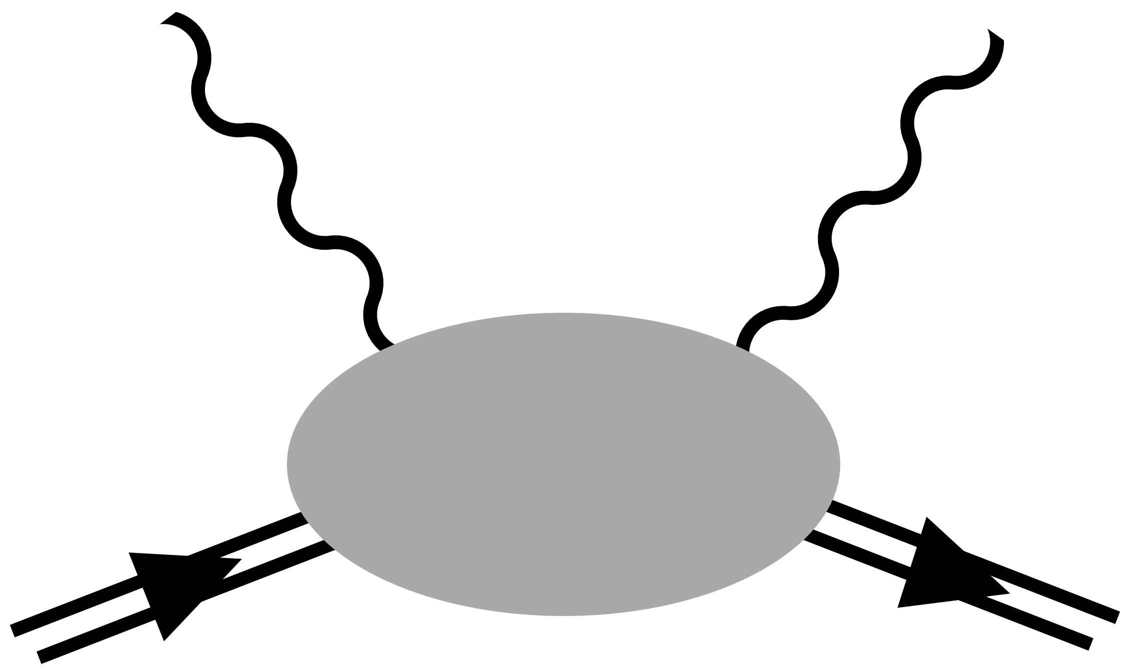


- Nucleon structure (leading twist)
  - Parton distributions from first principles
  - Constraining the high- and low-x regions better

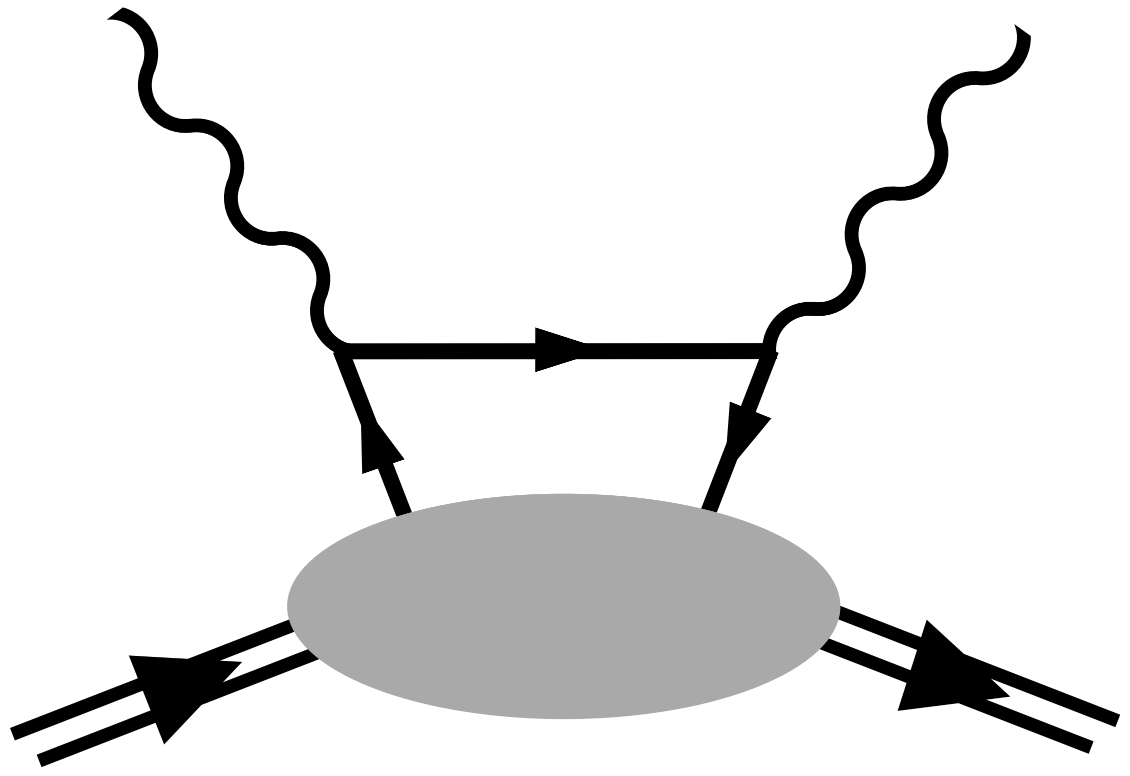
- Scaling and Power corrections
  - Q^2 cuts of global QCD analyses
  - Large-x, low-Q^2:
    - Higher-twist contributions
    - Target mass corrections



Forward Compton Amplitude



$$=$$



$$+\ \mathcal{O}\left(\frac{M_N^2}{Q^2}, \frac{\mu^2}{Q^2}\right)$$

- We calculate the LHS: the physical amplitude
- also known as OPE without OPE [Chambers et al. PRL118]

- Leading twist:
  - local matrix elements (ME) or quasi/pseudo-distributions

- Power corrections relevant at low-Q^2
- Difficult to calculate from local ME

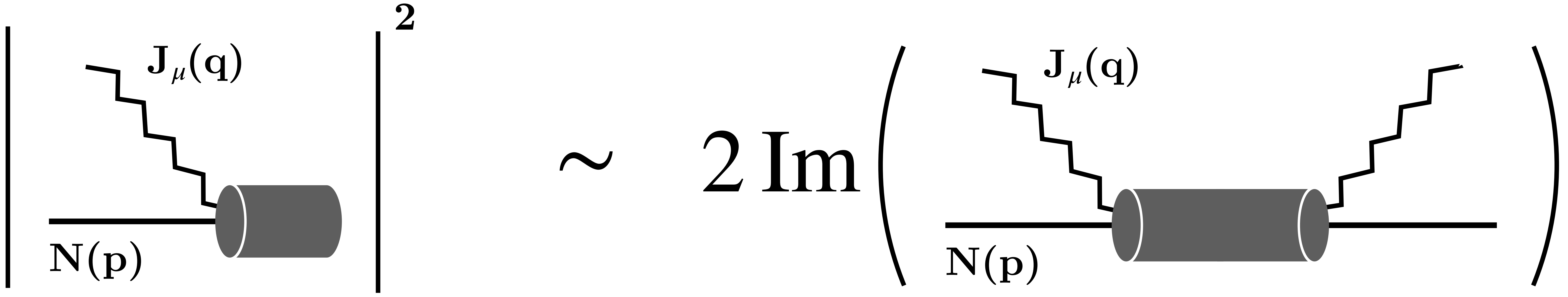
Forward Compton Amplitude

$$T_{\mu\nu}(p,q) = i\int d^4z\, e^{iq\cdot z}\rho_{ss'}\langle p,s'|\mathscr{T}\{J_\mu(z)J_\nu(0)\}|p,s\rangle$$, spin avg. $\rho_{ss'} = \frac{1}{2}\delta_{ss'}$ $\omega = \frac{2p\cdot q}{Q^2} = x^{-1}$

Same Lorentz decomposition as the Hadronic Tensor

$$= \left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2}\right)\mathscr{F}_1(\omega,Q^2) + \left(p_\mu - \frac{p\cdot q}{q^2}q_\mu\right)\left(p_\nu - \frac{p\cdot q}{q^2}q_\nu\right)\frac{\mathscr{F}_2(\omega,Q^2)}{p\cdot q}$$

Compton Structure Functions (SF)



DIS Cross Section ~ Hadronic Tensor

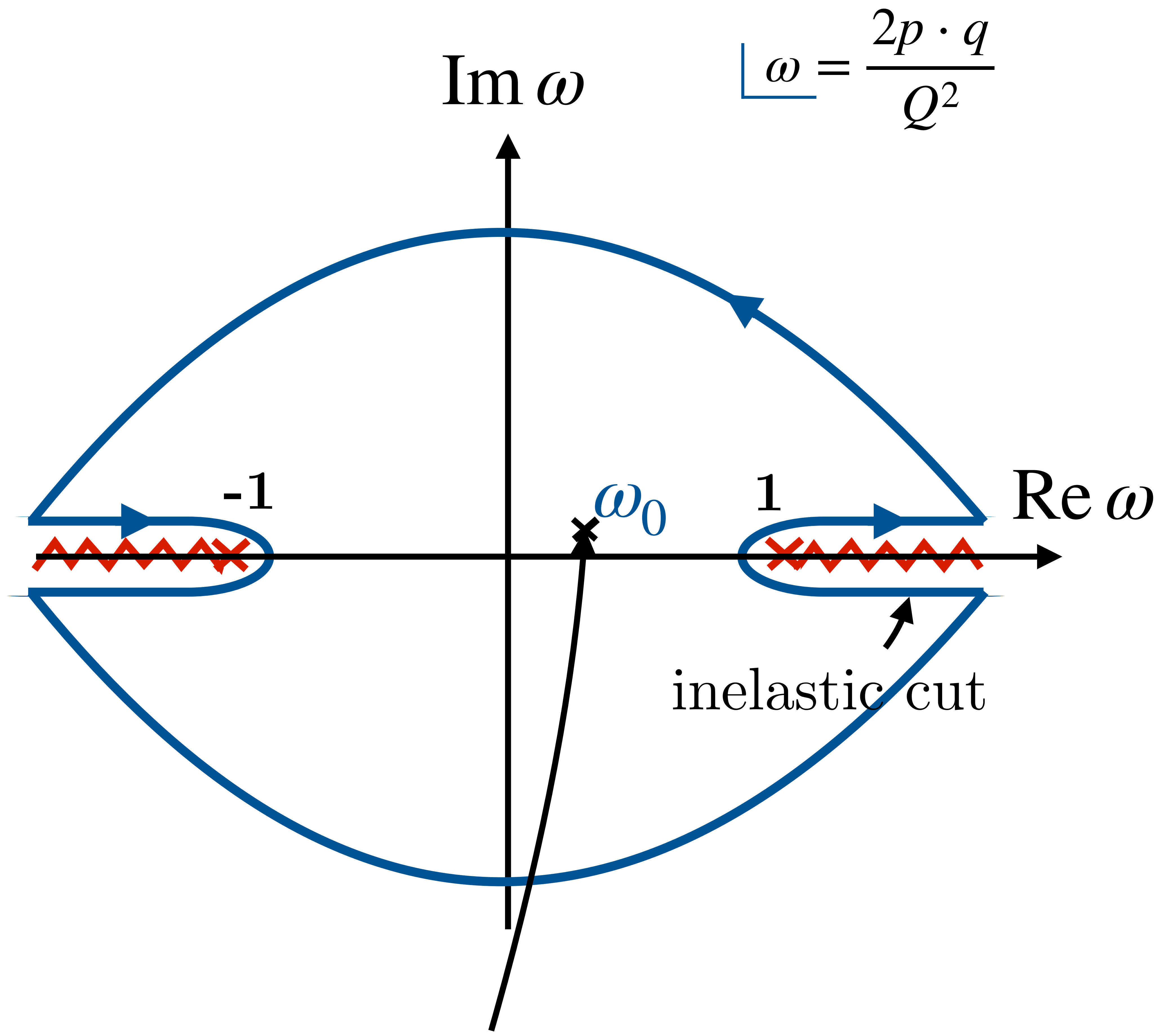
Forward Compton Amplitude ~ Compton Tensor

Nucleon Structure Functions

- **Dispersion relations connect Compton SFs to DIS SFs:**

$$\overbrace{\overline{\mathcal{F}}_1(\omega,Q^2)}^{\mathcal{F}_1(\omega,Q^2)-\mathcal{F}_1(0,Q^2)} = 2\omega^2\int_0^1 dx\frac{2x\,F_1(x,Q^2)}{1-x^2\omega^2-i\varepsilon}$$

$$\mathcal{F}_{2,3}(\omega,Q^2) = 4\omega\int_0^1 dx\frac{F_{2,3}(x,Q^2)}{1-x^2\omega^2-i\varepsilon}$$



Minkowski and Euclidean formulations are equivalent (no need for $i\varepsilon$ prescription) for the Compton Amplitude in the unphysical region

Nucleon Structure Functions

- using the Taylor expansion, $\frac{1}{1-(x\omega)^2} = \sum_{n=1}^{\infty} (x\omega)^{2n-2}$

$\omega = \frac{2p\cdot q}{Q^2}$

$$\overline{\mathcal{F}}_1(\omega, Q^2) = \sum_{n=1}^{\infty} 2\omega^{2n}\, 2\int_0^1 dx\, x^{2n-1} F_1(x, Q^2) = \sum_{n=1}^{\infty} 2\omega^{2n} M_{2n}^{(1)}(Q^2)$$

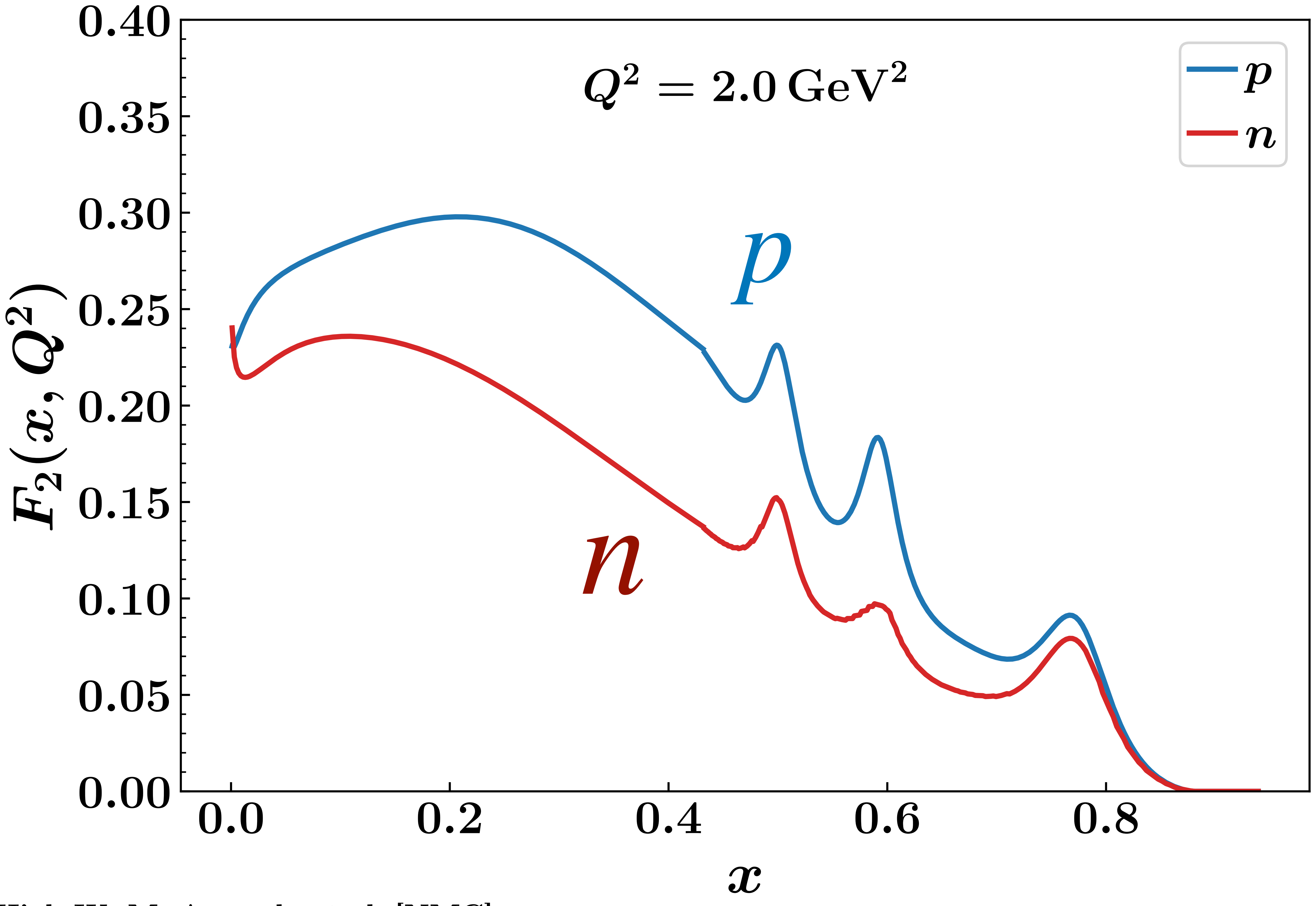
$$\mathcal{F}_2(\omega, Q^2) = \sum_{n=1}^{\infty} 4\omega^{2n-1} \int_0^1 dx\, x^{2n-2} F_2(x, Q^2) = \sum_{n=1}^{\infty} 4\omega^{2n-1} M_{2n}^{(2)}(Q^2)$$

$$\mathcal{F}_3(\omega, Q^2) = \sum_{n=1}^{\infty} 4\omega^{2n-2} \int_0^1 dx\, x^{2n-2} F_3(x, Q^2) = \sum_{n=1}^{\infty} 4\omega^{2n-2} M_{2n-1}^{(3)}(Q^2)$$

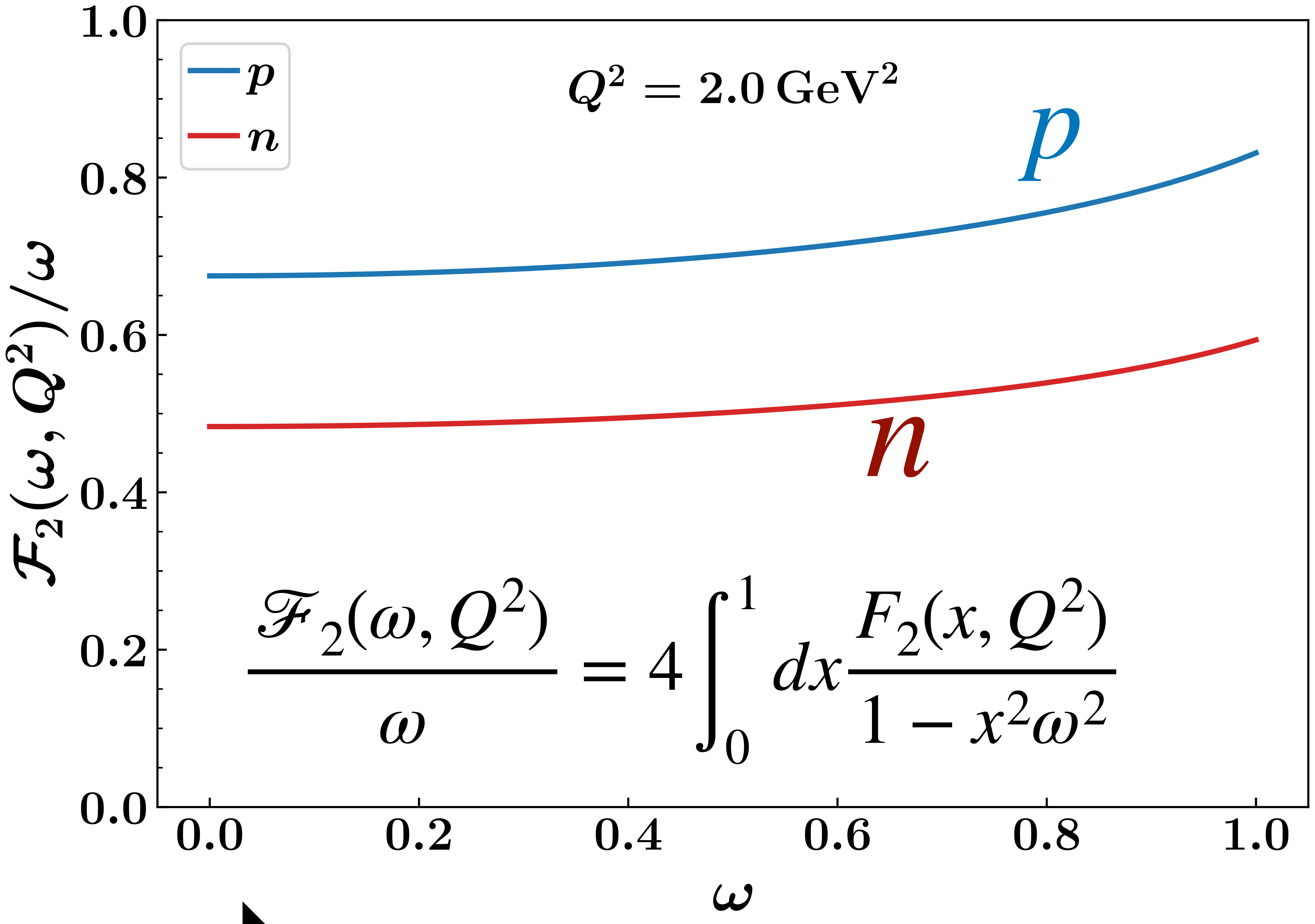
- Compton structure functions are given in terms of towers of physical Mellin moments

Shape of the Compton Amplitude

Structure functions



Compton Amplitudes



High-W: M. Arneodo et al. [NMC], PLB364, 107-115 (1995), [hep-ph/9509406]

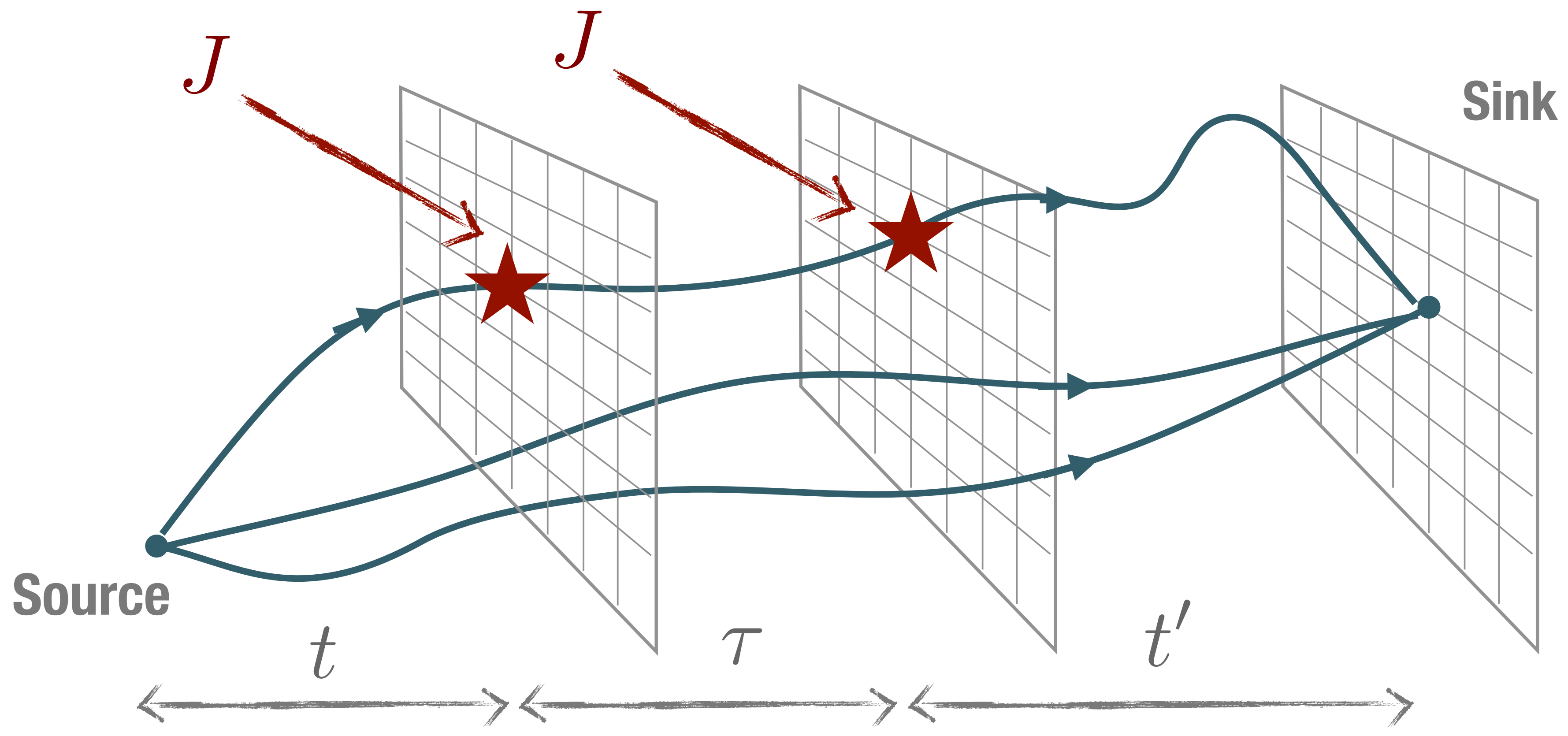
Low-W: M.E. Christy and P.E. Bosted, PRC81, 055213 (2010), [0712.3731]

dispersion relation

Feynman–Hellmann

Modify the Euclidean QCD action, extract energy shifts:

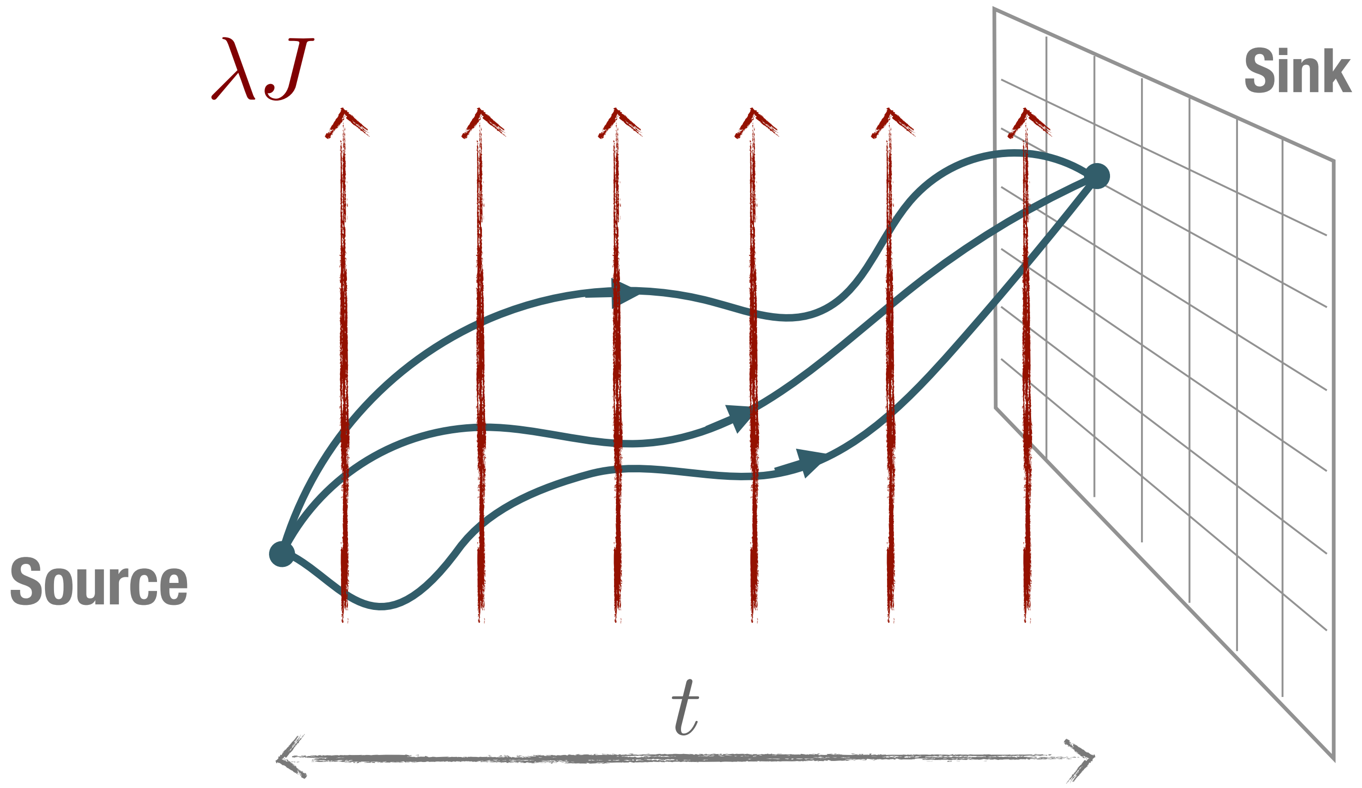
$$S \to S(\lambda) = S + \lambda \int d^4x \cos(q \cdot x)\, \bar{\psi}(x)\, \Gamma_\mu \psi(x) \qquad \Gamma_\mu \in \{\mathbf{1}, \gamma_\mu, \gamma_5\gamma_\mu, \dots\}$$



- **4-pt functions**

$$t, t' \gg \frac{1}{\Delta E}, \quad \frac{\langle C_4(t,\tau,t')\rangle}{\langle C_2(t)\rangle\langle C_2(t')\rangle} \propto \langle N|J(\tau_E)J|N\rangle$$

$$\int_0^\infty d\tau_E \to \langle N|JJ|N\rangle$$



- **Feynman—Hellmann**

$$t \gg \frac{1}{\Delta E}, \qquad \left.\frac{\partial^2 E}{\partial \lambda^2}\right|_{\lambda \to 0} \propto \langle N|JJ|N\rangle$$

Energy shifts

Isolate the 2nd-order energy shift

$$G^{(2)}_{\lambda}(\mathbf{p};t) \sim A_\lambda(\mathbf{p}) e^{-E_{N_\lambda}(\mathbf{p})t}$$

$$E_{N_\lambda}(\mathbf{p}) = E_N(\mathbf{p}) + \lambda \left.\frac{\partial E_{N_\lambda}(\mathbf{p})}{\partial \lambda}\right|_{\lambda=0} + \frac{\lambda^2}{2!}\left.\frac{\partial^2 E_{N_\lambda}(\mathbf{p})}{\partial^2 \lambda}\right|_{\lambda=0} + \mathcal{O}(\lambda^3)$$

$$= E_N(\mathbf{p}) + \Delta E^o_N(\mathbf{p}) + \Delta E^e_N(\mathbf{p})$$

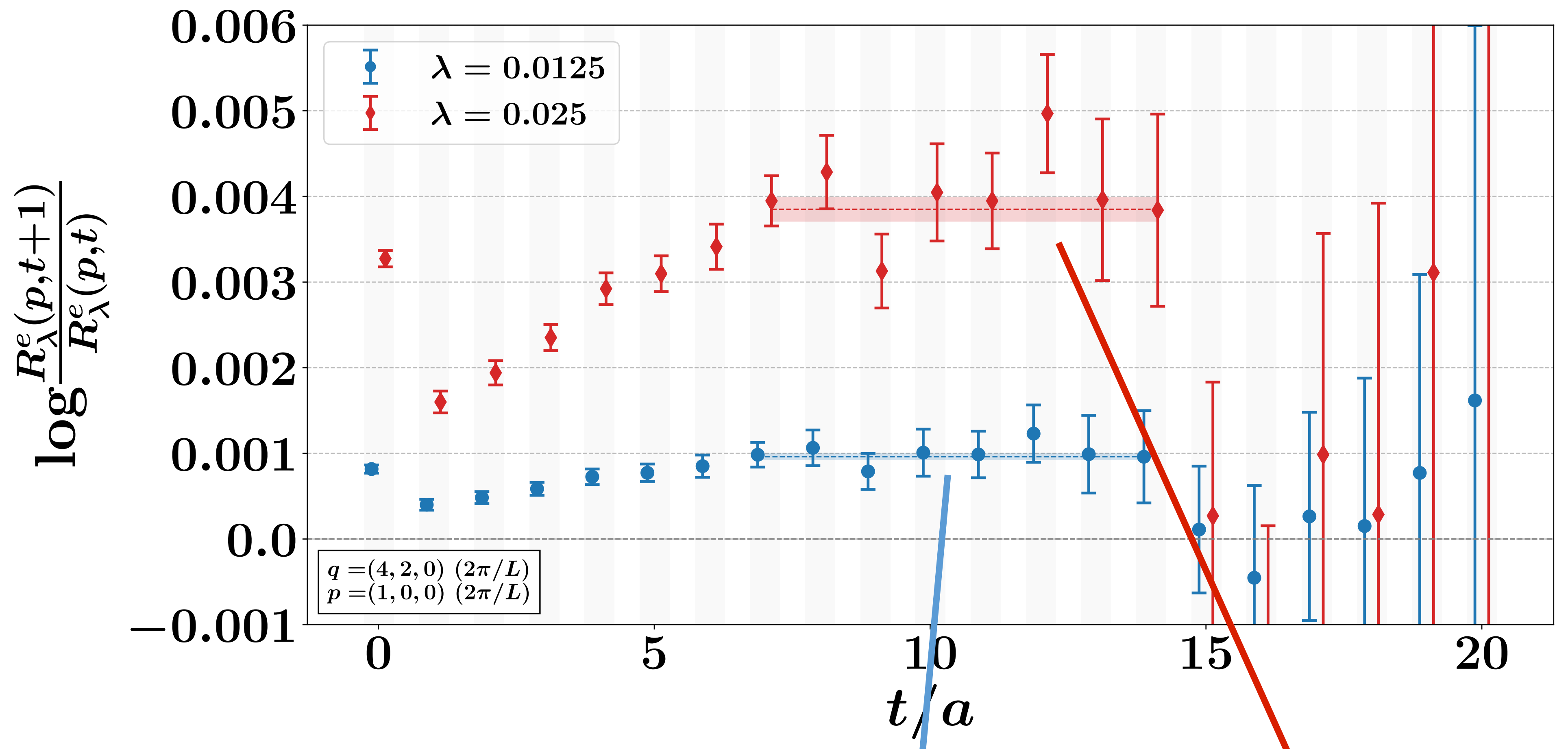
Ratio of perturbed to unperturbed Euclidean 2-pt correlation functions

$$R^e_\lambda(\mathbf{p},t) \equiv \frac{\langle G^{(2)}_{+\lambda}(\mathbf{p},t)\rangle \langle G^{(2)}_{-\lambda}(\mathbf{p},t)\rangle}{\left(\langle G^{(2)}(\mathbf{p},t)\rangle\right)^2}$$

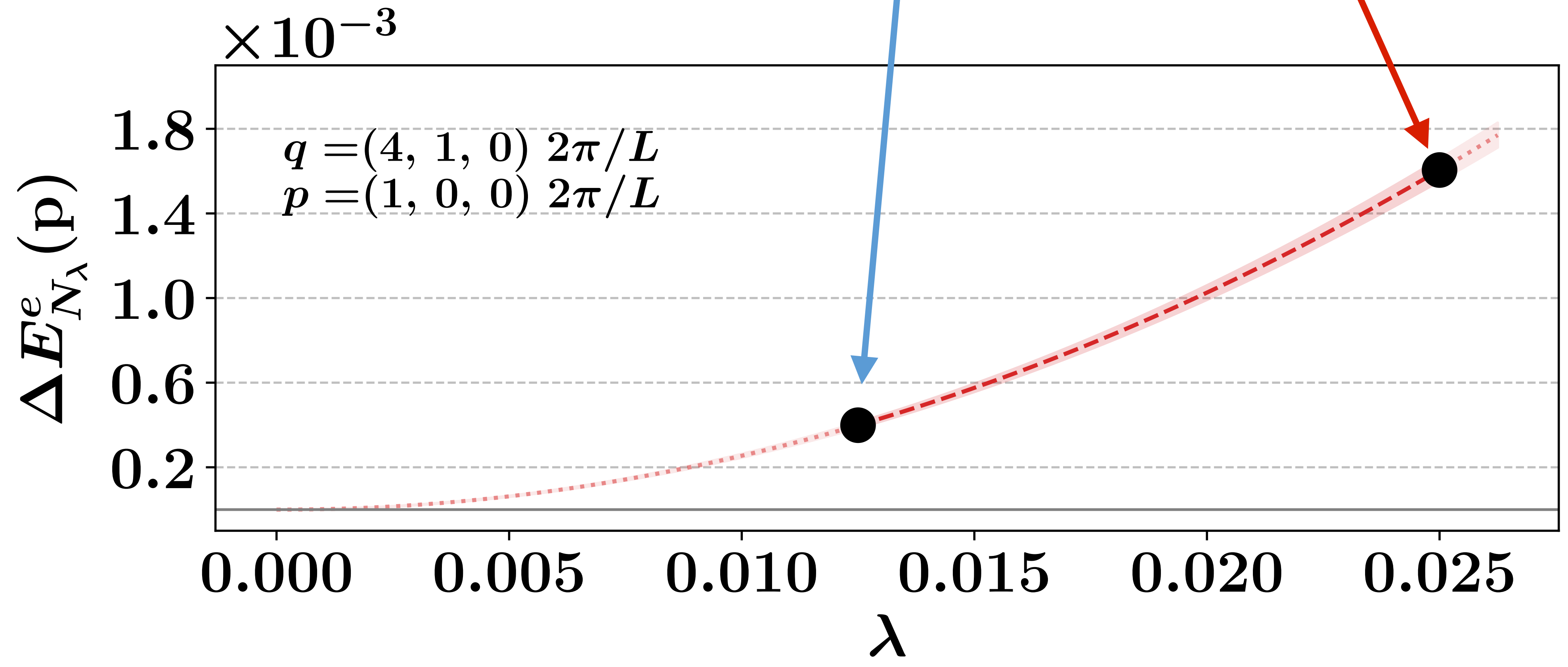
$$\xrightarrow{t\gg 0} A_\lambda(\mathbf{p}) e^{-2\Delta E^e_{N_\lambda}(\mathbf{p})t}$$

$\langle\cdots\rangle \equiv$ ensemble average, statistical uncertainty from a bootstrap procedure

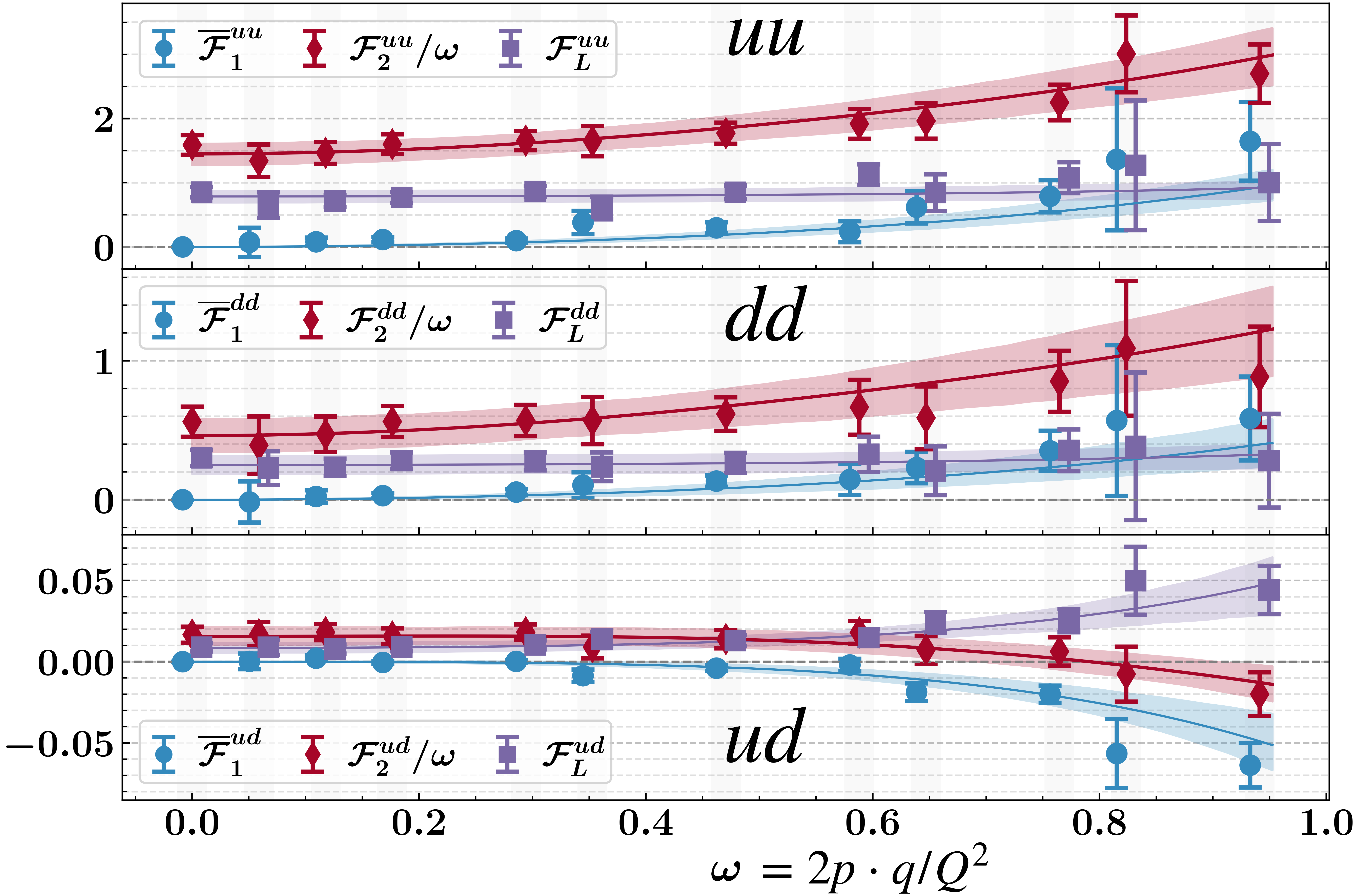
- Extract energy shifts for each λ



- Get the 2nd order derivative



Compton Structure Functions

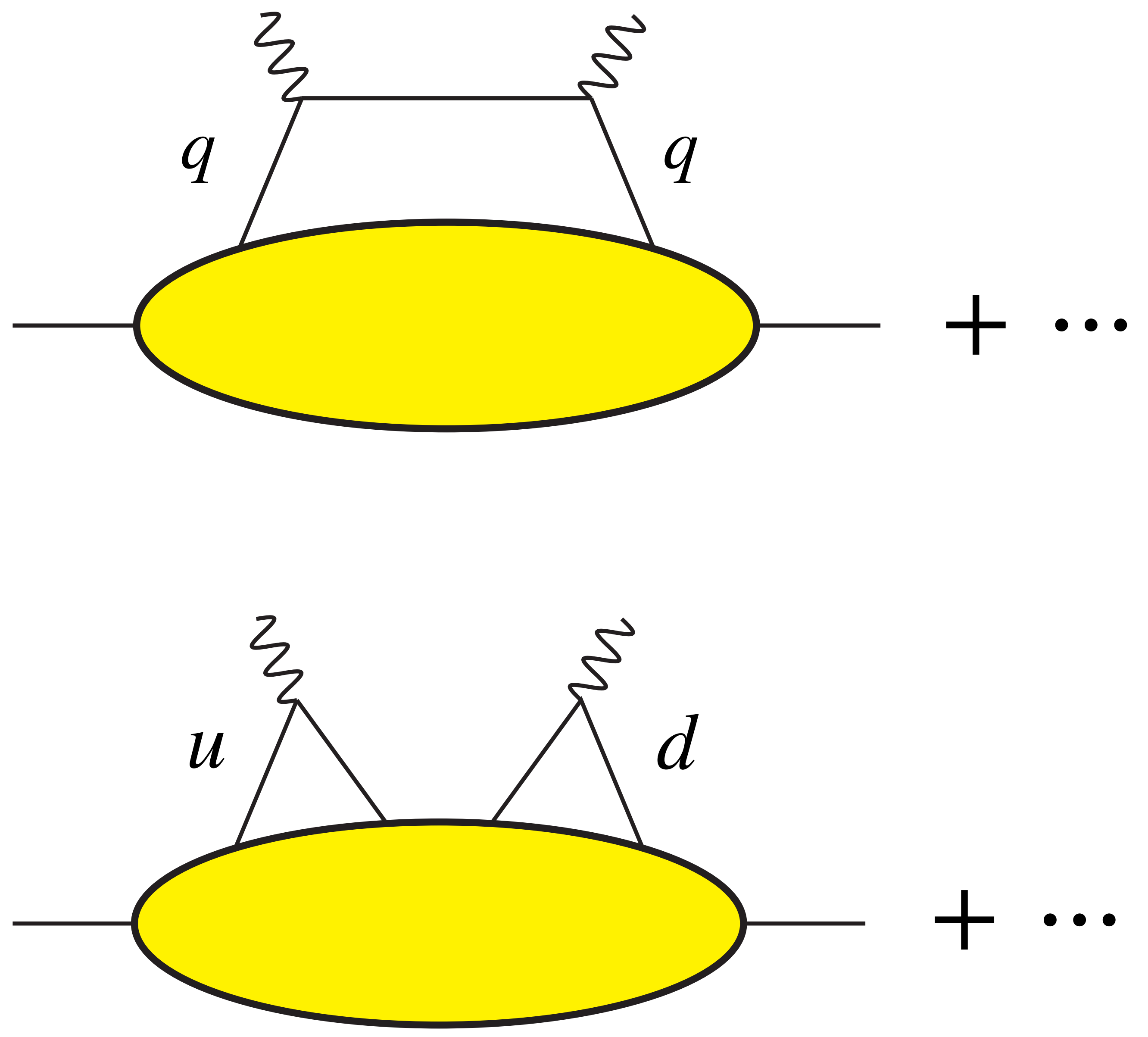


48^3x96, 2+1 flavour

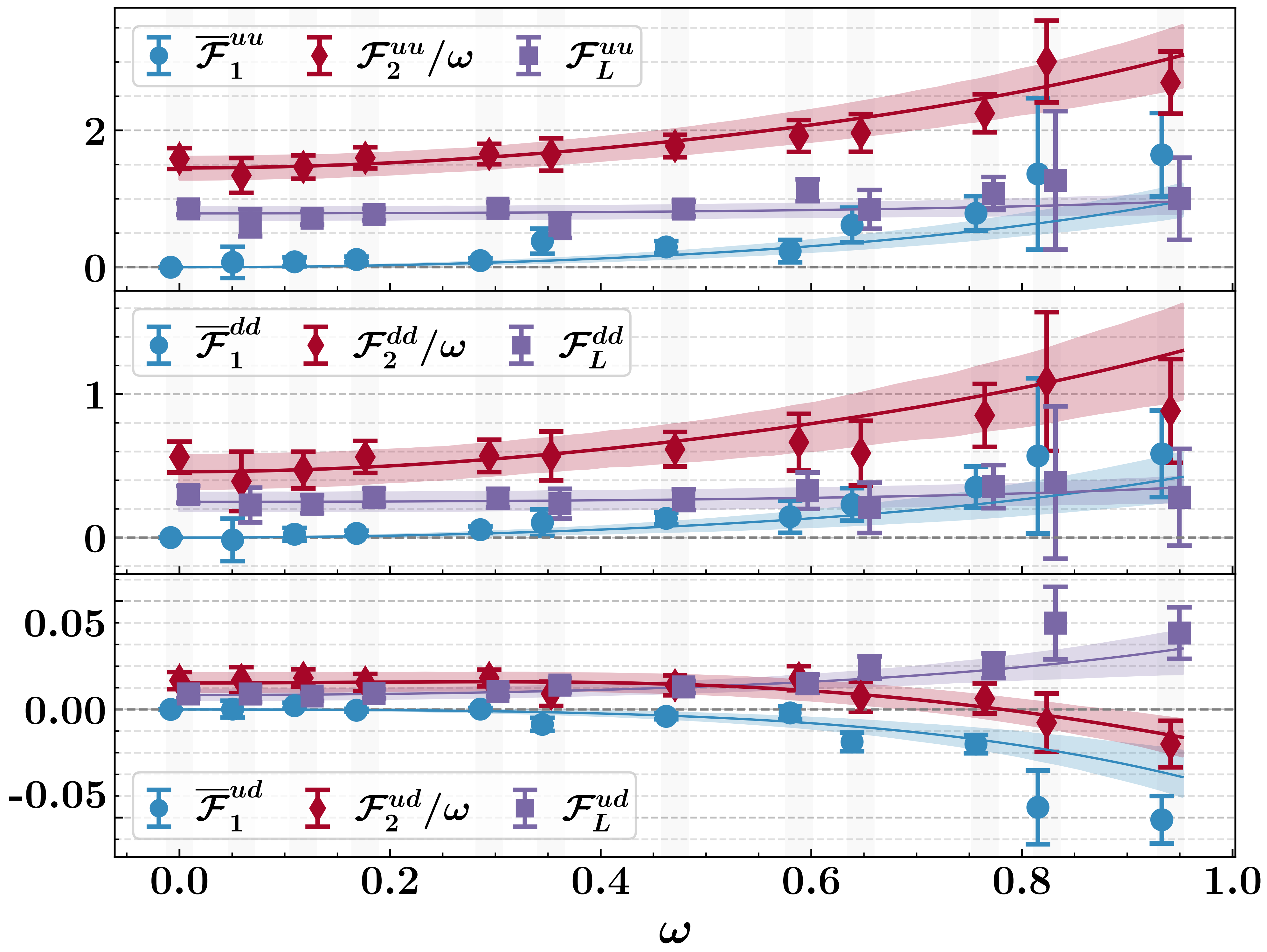
$a = 0.068$ fm

$m_\pi \sim 420$ MeV

$Q^2 = 4.9\ GeV^2$



Moments | Fit details



- **Fit to an expansion of Mellin moments, e.g.**

$$\overline{\mathcal{F}}_1^{qq}(\omega, Q^2) = \sum_{n=1}^{\infty} 2\omega^{2n} M_{2n}^{(1)}(Q^2)$$

- **Enforce monotonic decreasing of moments** **for u and d only, not necessarily true for $u-d$**

$$M_2^{(1)}(Q^2) \geq M_4^{(1)}(Q^2) \geq \cdots \geq M_{2n}^{(1)}(Q^2) \geq \cdots \geq 0$$

We truncate at $n = 4$ [$\mathcal{O}(\omega^8)$], inclusive
No dependence to truncation order for $3 \leq n \leq 10$

- **Bayesian approach by MCMC method**

Sample the moments from Uniform priors
individually for u- and d-quark

$$M_2(Q^2) \sim \mathcal{U}(0, 1)$$
$$M_{2n}(Q^2) \sim \mathcal{U}\left(0, M_{2n-2}(Q^2)\right)$$

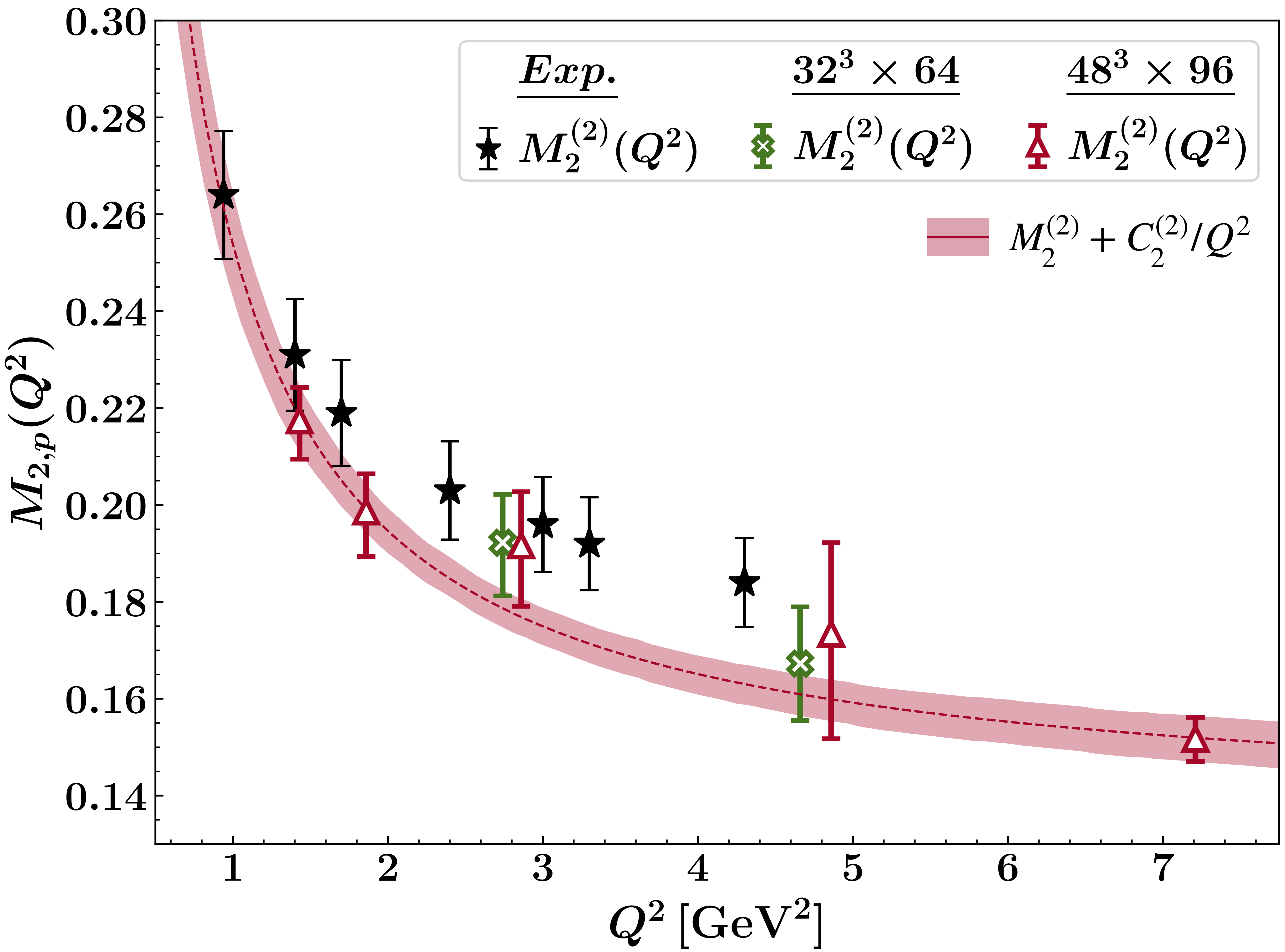
Normal Likelihood function, $exp(-\chi^2/2)$

$$\chi^2 = \sum_i \frac{\left(\overline{\mathcal{F}}_i - \overline{\mathcal{F}}^{obs}(\omega_i)\right)^2}{\sigma_i^2}$$

stat. uncertainty
via bootstrap analysis

Moments | Power Corrections

- **Unique ability to study the Q^2 dependence of moments!**



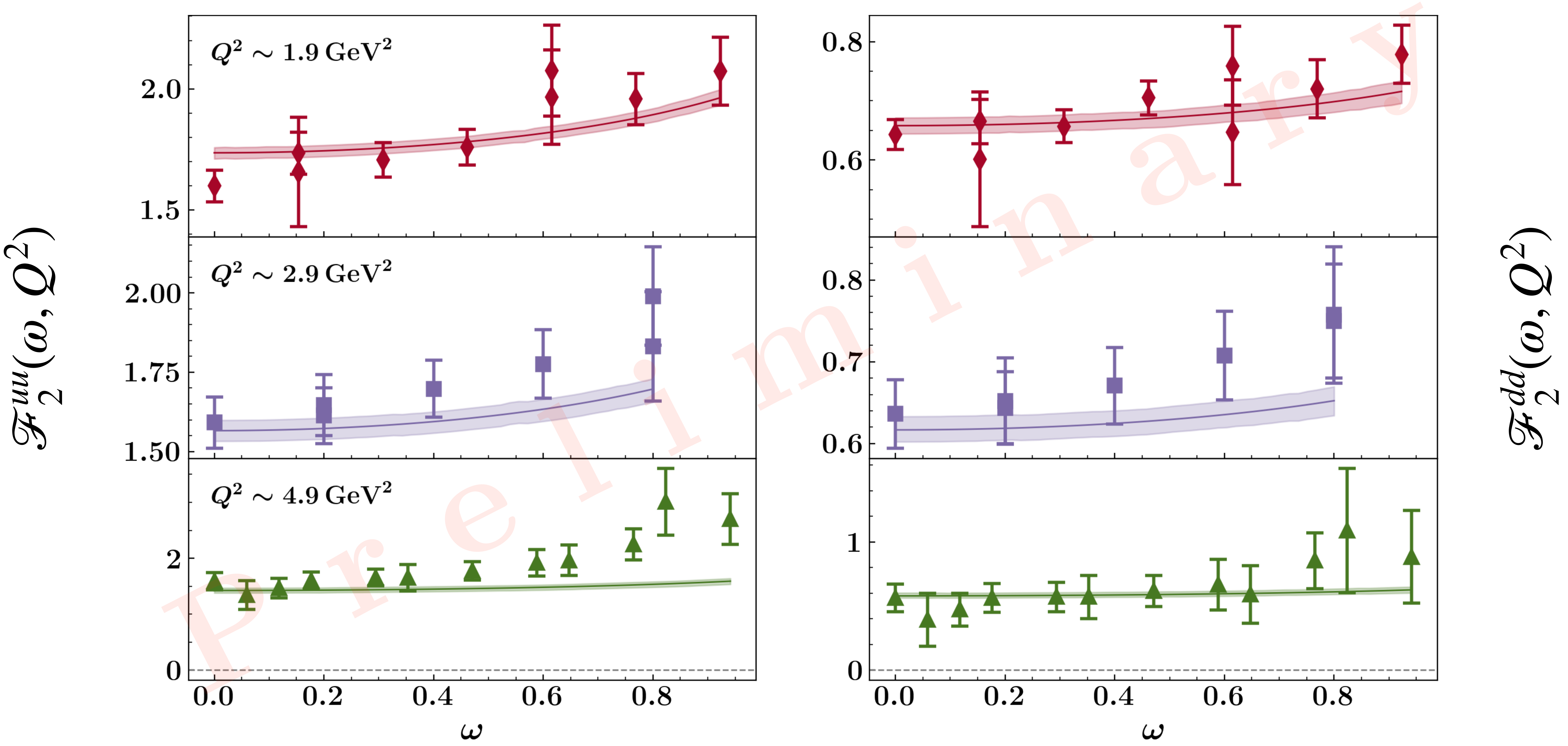
- Power corrections below $\sim 3\, GeV^2$?
  - Naive modelling via:
  - $M_2^{(2)}(Q^2) = M_2^{(2)} + C_2^{(2)}/Q^2$
  - $C_2^{(2)}$ is a catch all correction term

  - Can we distinguish
    - Target mass corrections,
    - Elastic ($x = 1$),
    - $\ln Q^2$ scaling, and
    - genuine higher twist contributions?

★ Exp $M_2^{(2)}$: C. S. Armstrong, R. Ent, C. E. Keppel, S. Liuti, G. Niculescu, and I. Niculescu, Phys. Rev. D 63, 094008 (2001), arXiv:hep-ph/0104055.

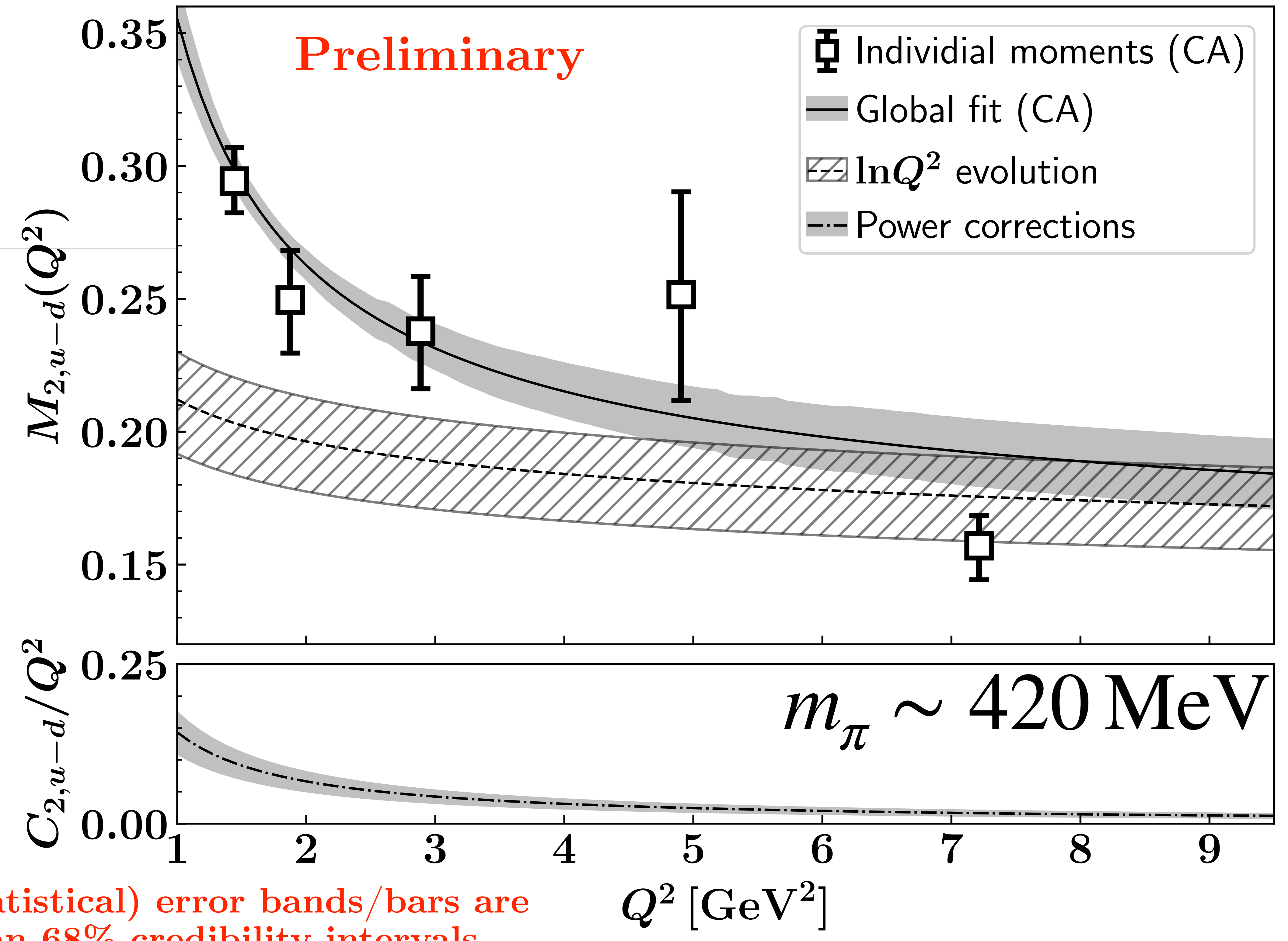
$\mathcal{F}_2$ | Global fit

- **Can we perform a global fit in Q^2 to all our Compton amplitude results?**



$\mathscr{F}_2$ | Global fit

- fit to u_V and d_V quarks; get the non-singlet $u-d$
- Assume a parametric form for the SFs w/HT
  - $f_q(x,Q^2) = a_q x^{b_q}(1-x)^{c_q}\left(1 + \frac{d_q x^{e_q}(1-x)^{f_q}}{Q^2}\right)$
- $a_q, b_q, c_q, d_q, e_q, f_q$ are free fit parameters, $q = [u,d]$
- a_q is normalised to lowest even moment, $M_{2,q}$



All (statistical) error bands/bars are Bayesian 68% credibility intervals

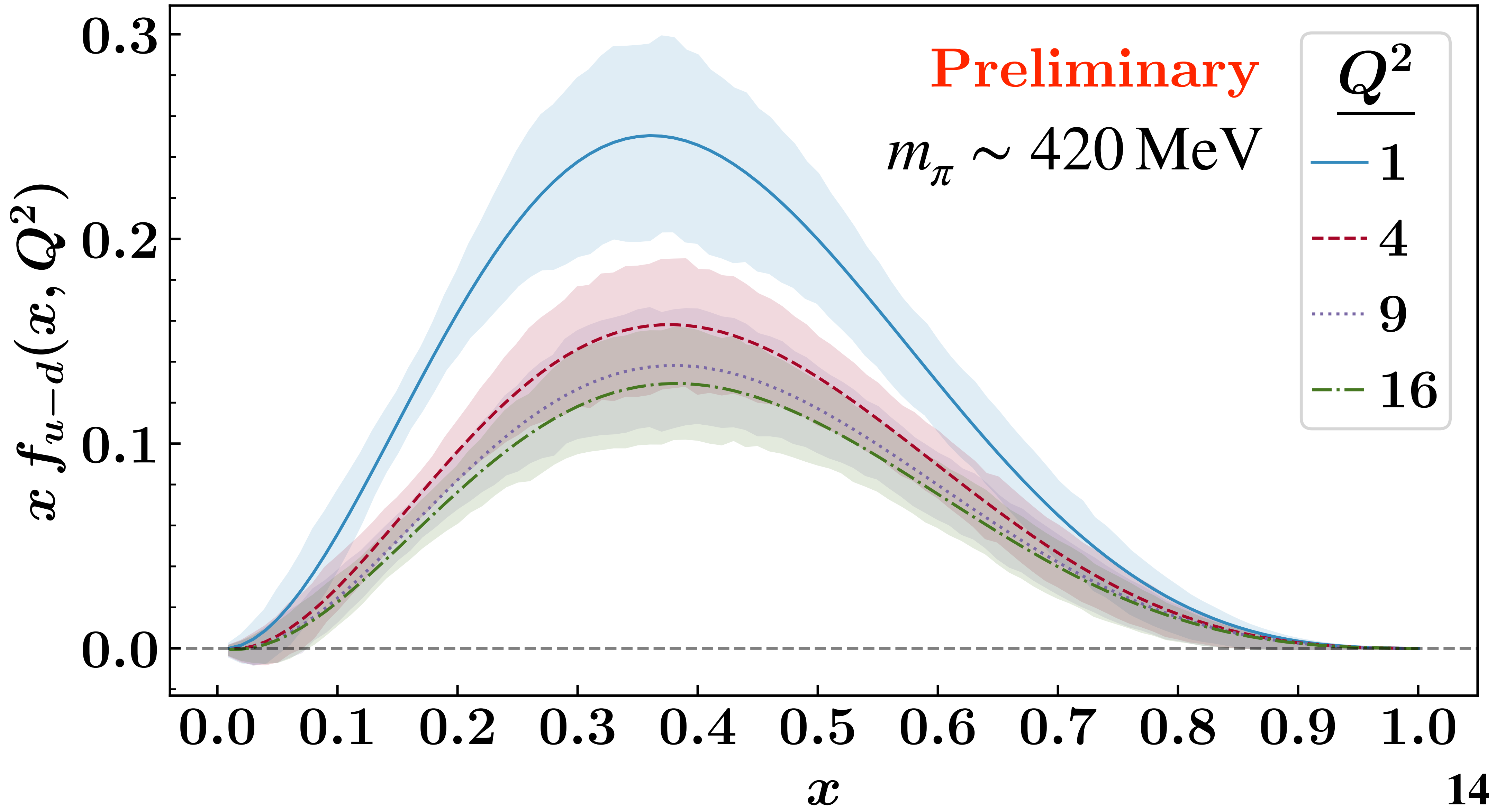
- Input $f_q(x,Q^2)$ to the dispersion relation:

Compton structure function is described by a generalised hypergeometric series,

- $$\frac{\mathscr{F}_2(\omega,Q^2)}{4\omega} = M_2(Q^2)\sum_{n=1}^{N}\left(A_{2n}(b,c,Q^2) + \frac{C_{2n}(b,c,d,e,f,Q^2)}{Q^2}\right)\omega^{2n-2}$$

$Q_0^2 = 4$ GeV^2

A_{2n}, C_{2n} are known functions
non-singlet $\ln Q^2$ evolution at LO embedded in A_{2n}, C_{2n}



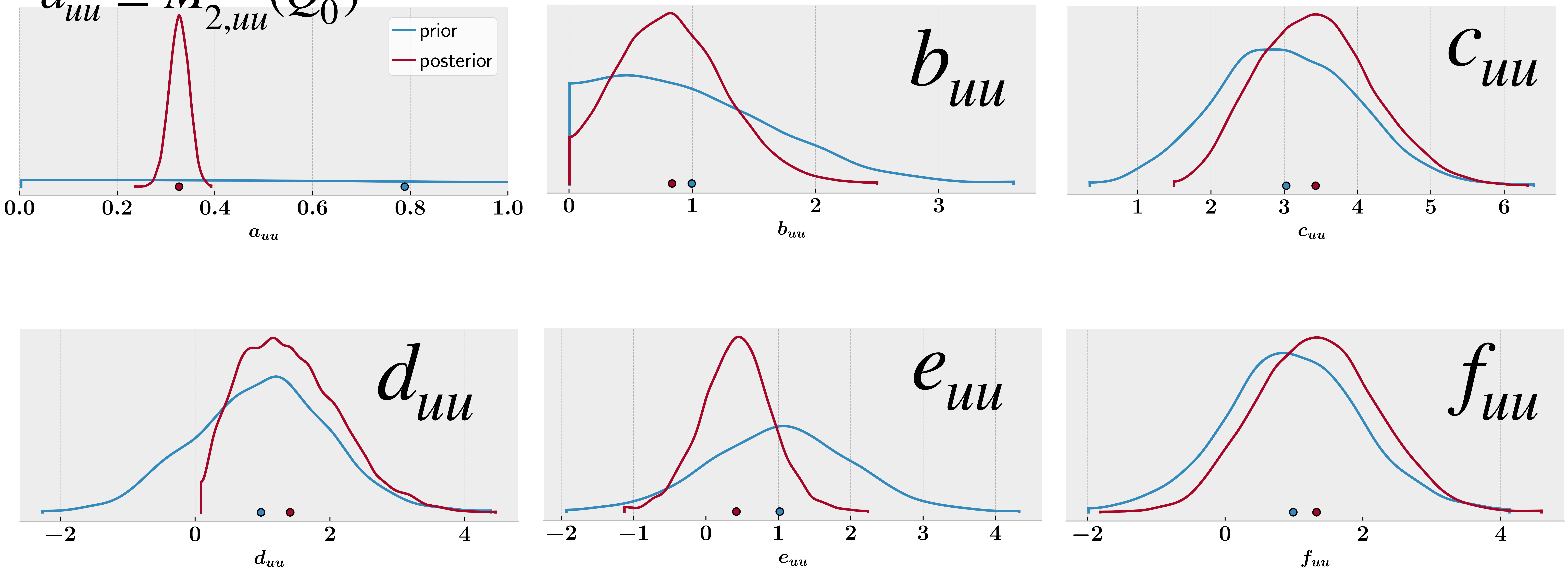
$\mathcal{F}_2$ | Bayesian Priors / Posteriors

- Constrain the parameter space via priors
- Posteriors (except for a_{uu}) are not unique

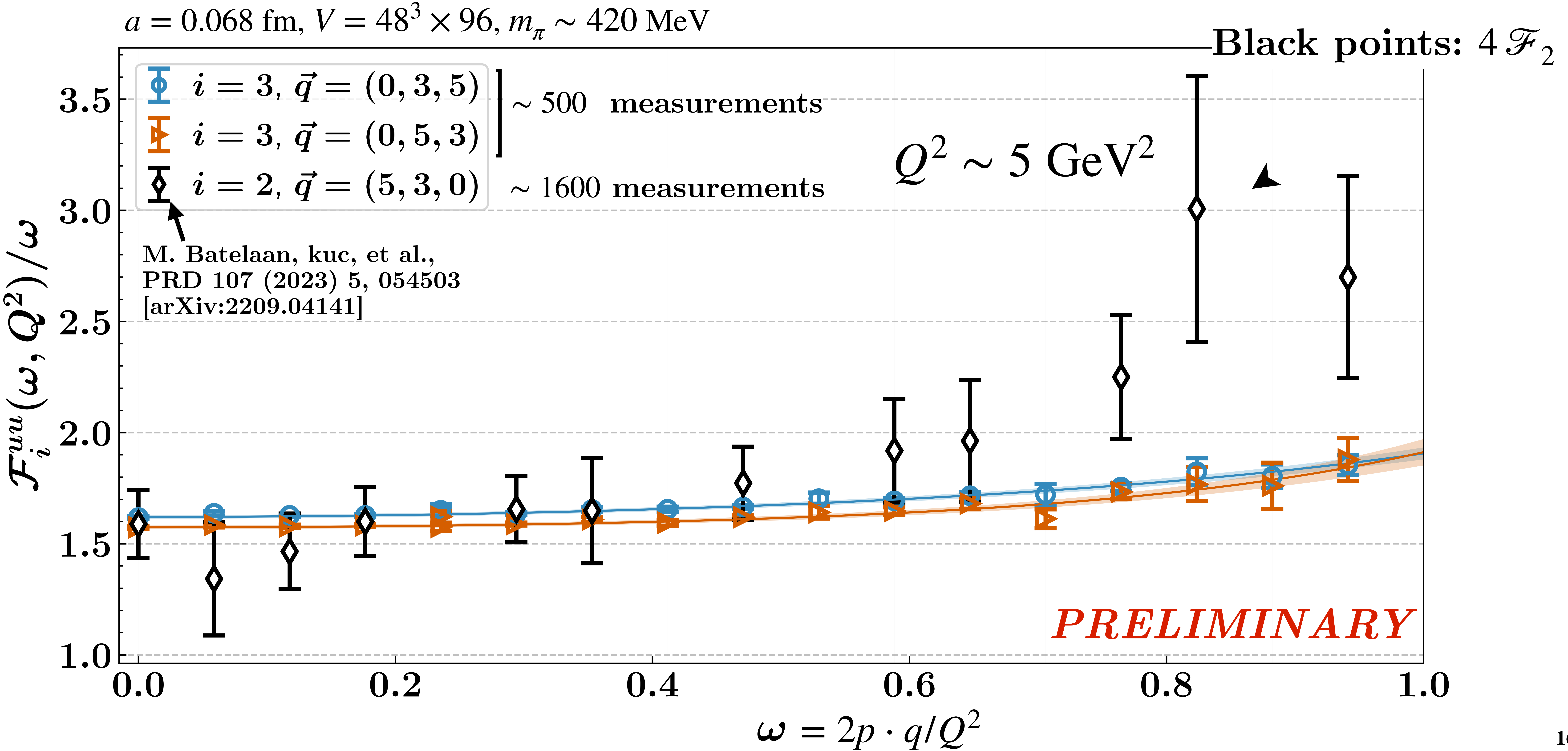
parametrisation

$$f_q(x, Q^2) = a_q x^{b_q}(1-x)^{c_q}\left(1 + \frac{d_q x^{e_q}(1-x)^{f_q}}{Q^2}\right)$$

$a_{uu} \equiv M^{(2)}_{2,uu}(Q_0^2)$

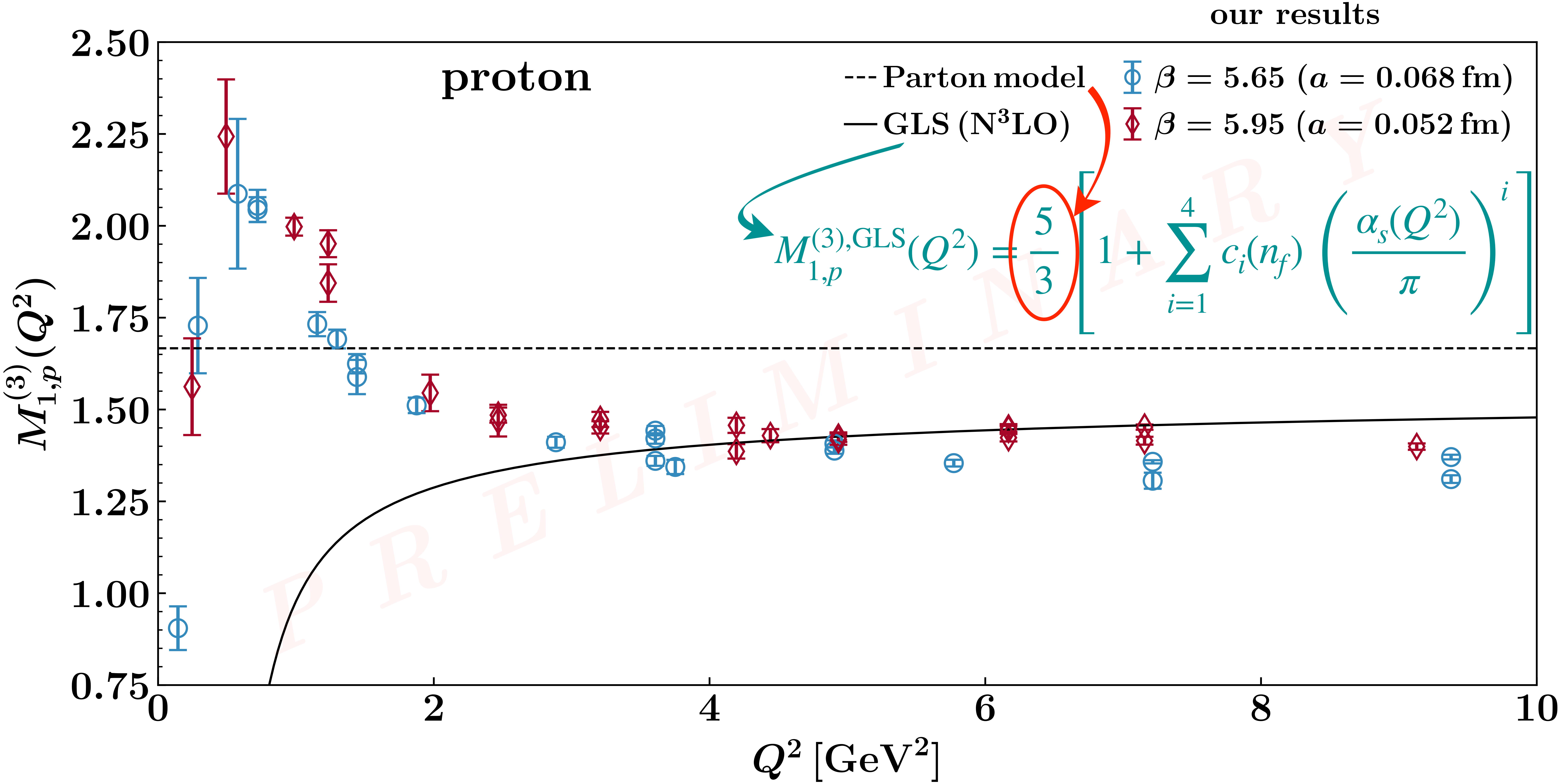


$\mathscr{F}_3^{\gamma Z}$ | Parity-odd structure function



$\mathscr{F}_3^{\gamma Z}$ | First moment

$a = 0.068, 0.052\,\mathrm{fm}$
$m_\pi \sim 410\,\mathrm{MeV}$
$48^3\mathrm{x}96$, 2+1 flavour



$\mathcal{F}_3^{\gamma Z}$

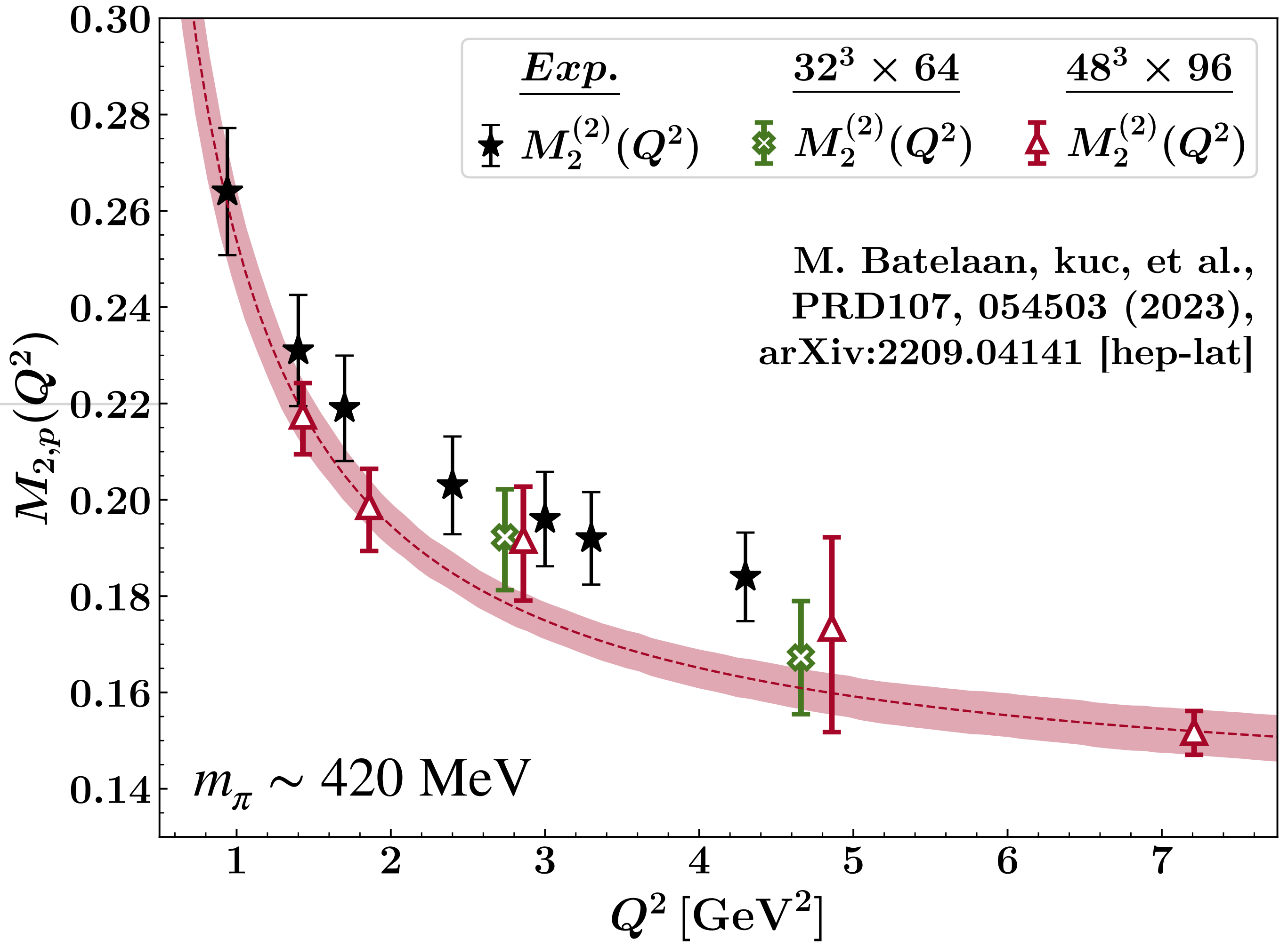
- Statistical precision is much improved w.r.t $\mathcal{F}_2$
- $\mathcal{F}_3$ is purely non-singlet
- Our current results have a good low/mid-Q^2 coverage, $0.1 \lesssim Q^2 \lesssim 10$ GeV^2
    - additional lattice spacings, m_π on the way
- Allows for a direct test of Gross-Llewellyn-Smith sum rule (benchmark quantity?)

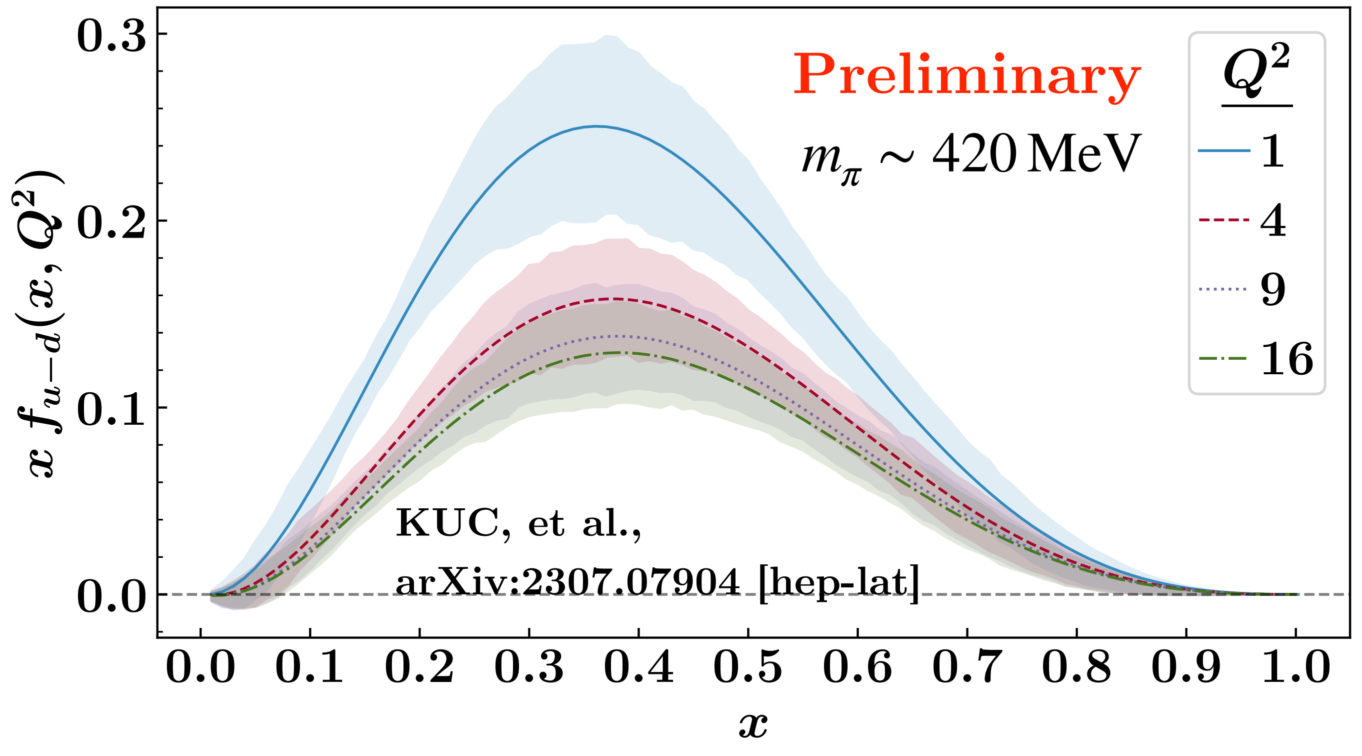
- There are systematic uncertainties to control
- A global fit similar to $\mathcal{F}_2$ can be performed
- $\mathcal{F}_3$ is a good candidate to incorporate to phenomenological global fits

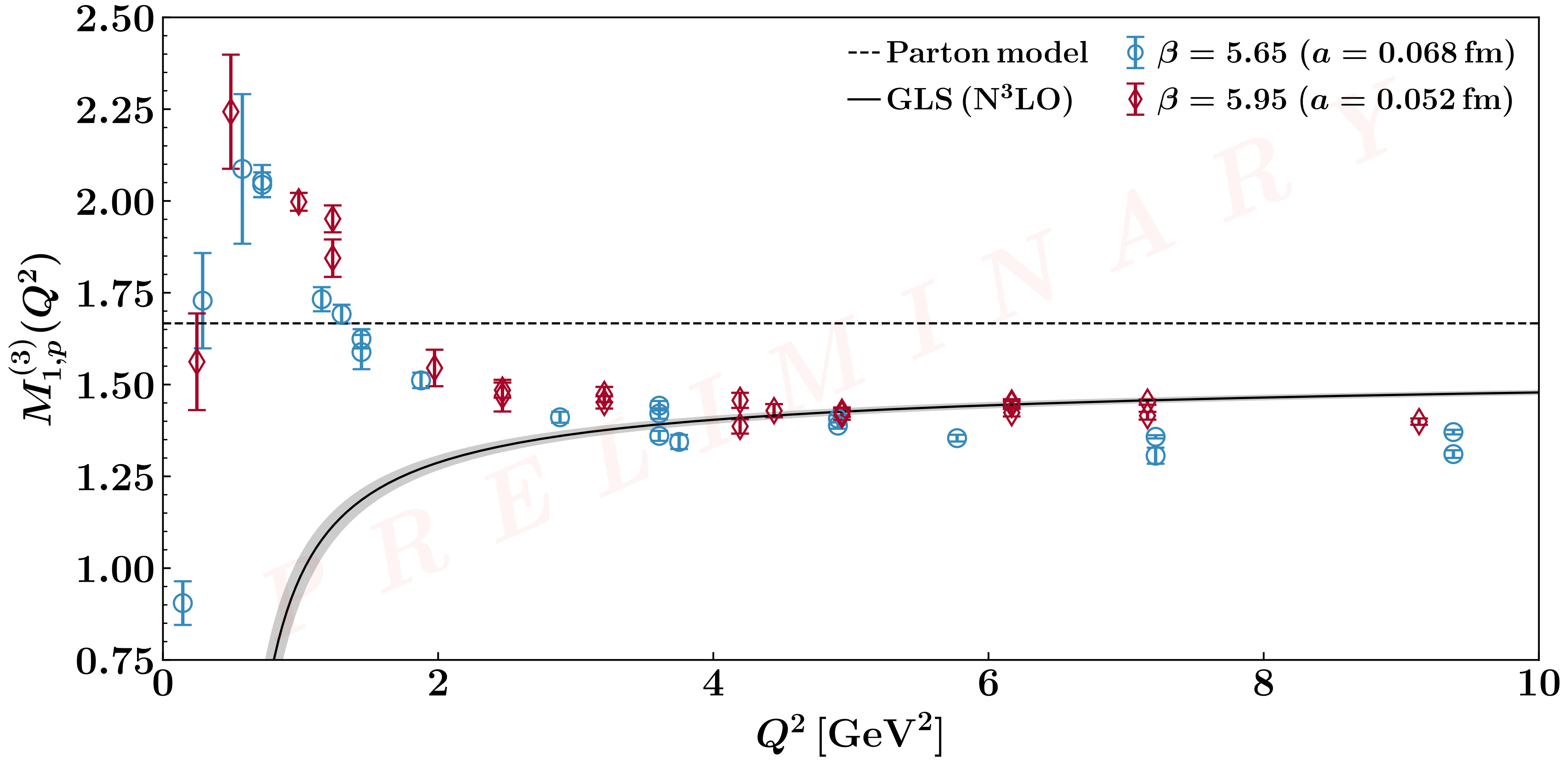
Summary & Outlook

- ■ Physical Compton amplitude, can be matched to OPE
- ☑ Can extract moments of DIS structure functions
- ☑ Systematic investigation of power corrections, higher-twist effects and scaling is within reach
- ☑ Exploratory investigation of x-dependence w/scaling and HT
- ☑ First moment of F_3 (GLS sum rule) can serve as a benchmark quantity
- ☐ Plans to incorporate Compton amplitude results to the JAM framework
- ☐ Exploring synergies with quasi/pseudo methods would be beneficial

- ■ Our approach can be extended to:
  - ☑ GPDs: A. Hannaford-Gunn et al. *Phys.Rev.D* **105**, 014502 [arXiv:2110.11532], and *Phys.Rev.D* **110** (2024) 1, 014509 [2405.06256]
  - ☐ spin-dependent structure functions, g_1, g_2







Acknowledgements

- The numerical configuration generation (using the BQCD lattice QCD program)) and data analysis (using the Chroma software library) was carried on the
  - DiRAC Blue Gene Q and Extreme Scaling (EPCC, Edinburgh, UK) and Data Intensive (Cambridge, UK) services,
  - the GCS supercomputers JUQUEEN and JUWELS (NIC, Jülich, Germany) and
  - resources provided by HLRN (The North-German Supercomputer Alliance),
  - the NCI National Facility in Canberra, Australia (supported by the Australian Commonwealth Government) and
  - the Phoenix HPC service (University of Adelaide).
- RH is supported by STFC through grant ST/P000630/1.
- PELR is supported in part by the STFC under contract ST/G00062X/1.
- KUC, RDY and JMZ are supported by the Australian Research Council grants DP190100297 and DP220103098.