Recent Advances in PDFs From Lattice QCD

Martha Constantinou

Temple University

Parton Distributions and Lattice Calculations (PDFLattice 2024)

JLab, November 18-20, 2024

OUTLINE

A. Fundamentals

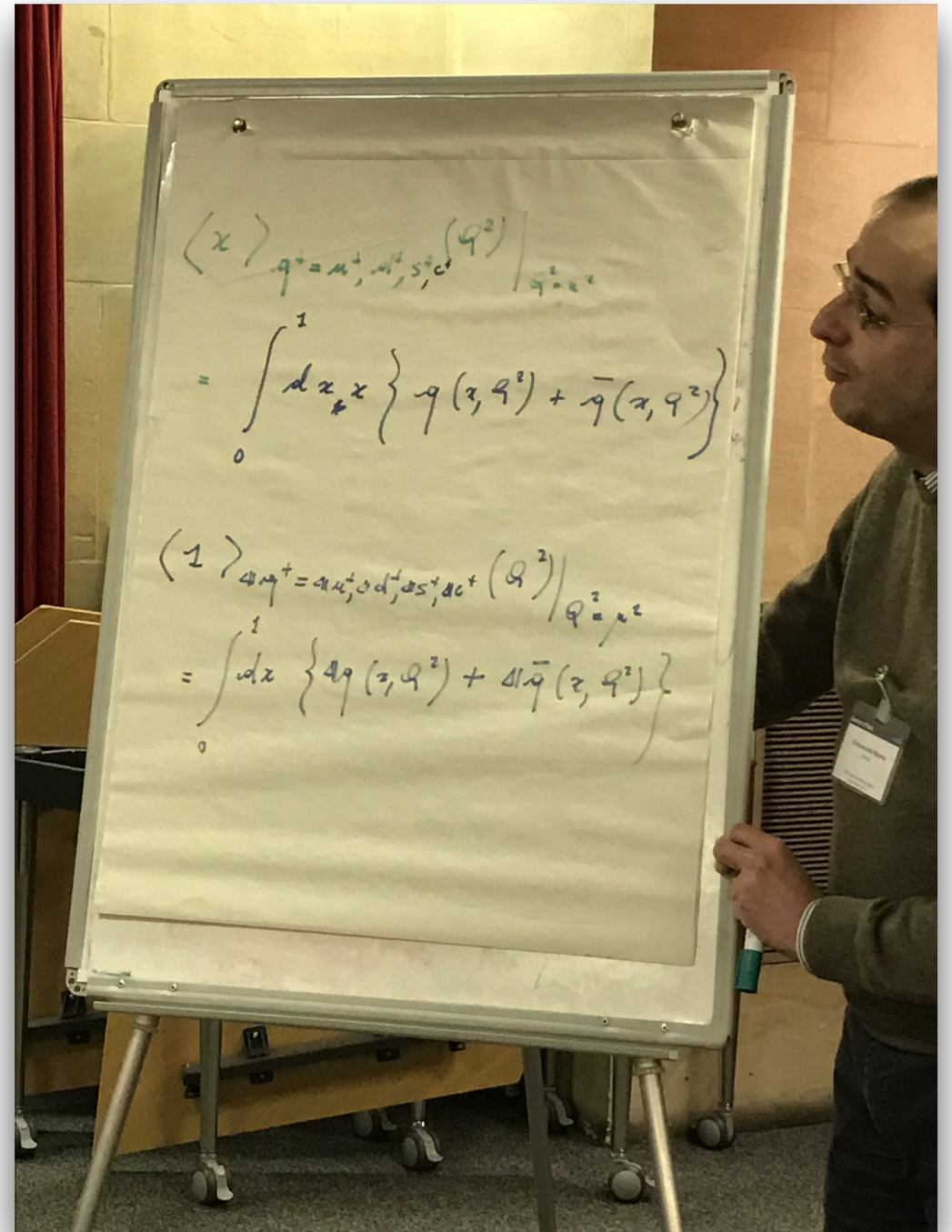
B. Updates since the PDFLattice 2020 whitepaper
[Constantinou et al., Prog. Part. Nucl. Phys. 121 (2021) 103908]

C. New lattice QCD results:
- quark PDFs (unpolarized, helicity, transversity)
- flavor decomposition of PDFs (light and strange quark)
- gluon PDFs (unpolarized, polarized)
- twist-3 PDFs

D. Synergy with phenomenology

E. Concluding remarks

We have come long way since PDFLattice 2017…



Key Questions

PDFLattice 2024– Uncertainty Quantification
Workshop Goals: Key Questions to Answer

Aurore Courtoy[1], Cynthia Keppel[1], Andreas Kronfeld[1], Huey-Wen Lin[1], Emanuele R. Nocera[1], Fredrick Olness[1], and Jianwei Qiu[1]

[1]Organizers

November 2024

We encourage speakers to address the following points throughout their presentations. The use of blackboard explanations is welcome to ensure clarity and foster interactive discussions. The presentations will serve as a baseline for the White Paper. By the end of this workshop, we aim to (partially) answer the following grouped key questions:

1. Accessing PDFs: lattice and pheno approaches

- How does lattice QCD assess moments or functions related to PDFs through their respective OPEs?
- How does a phenomenological fit (global analysis) assess PDFs using a data-driven methodology grounded in the QCD factorization formalism?
- What are the current efforts, directions, and challenges in both lattice and pheno/global analyses? How can we foster synergy by establishing a common language between them?

2. Momentum and validity considerations

- How do we define the regimes of validity and equivalence between zP_z and $\xi^- P^+$?
- What role do large-momentum corrections play?
- How should we treat small-momentum results when lattice QCD's x-dependent methods require large momentum?

3. Inverse problem and objective functions

- What inverse problem techniques are used to access x-dependent PDFs?
- How do neural networks (NNs), choice of parametrization, and other methods contribute to solving these problems?
- What criteria should guide the choice of objective functions, model averaging, and classification of uncertainties for both lattice and pheno approaches?

4. Combining lattice and experimental knowledge about PDFs

- What are the efforts to combine lattice and experimental data in pheno analyses?
- How do integral constraints, Lagrange multipliers, and "apple-to-apple" data comparisons factor into this process?

5. Uncertainty Quantification (UQ) and bias/variance tradeoff

- What interpretation (frequentist, Bayesian, ...) applies to the provided PDF uncertainties?
- How do we propagate uncertainties using methods such as bootstrap, importance sampling, and the Hessian formalism?
- How do we combine uncertainties from aleatoric and epistemic sources in lattice and pheno approaches?
- How can we manage the bias/variance tradeoff and evaluate the generalization power in both pheno and lattice analyses?

6. Benchmarking and Definitions

- How can we benchmark moment extractions for a given set of corrections and systematic uncertainties, including large-momentum, higher-twist, and truncation effects?
- How do we discuss the definitions and matching of quasi/pseudo-distributions, and what challenges arise in handling these distributions?
- Is it possible to develop a combined benchmark for lattice and pheno analyses?

Accessing PDFs/GPDs from lattice QCD

Novel Approaches

★	Hadronic tensor	[K.F. Liu, S.J. Dong, PRL 72 (1994) 1790, K.F. Liu, PoS(LATTICE 2015) 115]
	Auxiliary scalar quark	[U. Aglietti et al., Phys. Lett. B441, 371 (1998), arXiv:hep-ph/9806277]
	Fictitious heavy quark	[W. Detmold, C. J. D, Lin, Phys. Rev. D73, 014501 (2006)]
	Auxiliary scalar quark	[V. Braun & D. Mueller, Eur. Phys. J. C55, 349 (2008), arXiv:0709.1348]
	Higher moments	[Z. Davoudi, M. Savage, Phys. Rev. D86, 054505 (2012)]
	Quasi-distributions (LaMET)	[X. Ji, PRL 110 (2013) 262002, arXiv:1305.1539; Sci. China PPMA. 57, 1407 (2014)]
	Compton amplitude and OPE	[A. Chambers et al. (QCDSF), PRL 118, 242001 (2017), arXiv:1703.01153]
	Pseudo-distributions	[A. Radyushkin, Phys. Rev. D 96, 034025 (2017), arXiv:1705.01488]
	Good lattice cross sections	[Y-Q Ma & J. Qiu, Phys. Rev. Lett. 120, 022003 (2018), arXiv:1709.03018]
	PDFs without Wilson line	[Y. Zhao Phys.Rev.D 109 (2024) 9, 094506, arXiv:2306.14960]
	Moments of PDFs of any order	[A. Shindler, Phys.Rev.D 110 (2024) 5, L051503, arXiv:2311.18704]

Novel Approaches

★	Hadronic tensor	[K.F. Liu, S.J. Dong, PRL 72 (1994) 1790, K.F. Liu, PoS(LATTICE 2015) 115]
	Auxiliary scalar quark	[U. Aglietti et al., Phys. Lett. B441, 371 (1998), arXiv:hep-ph/9806277]
	Fictitious heavy quark	[W. Detmold, C. J. D, Lin, Phys. Rev. D73, 014501 (2006)]
	Auxiliary scalar quark	[V. Braun & D. Mueller, Eur. Phys. J. C55, 349 (2008), arXiv:0709.1348]
	Higher moments	[Z. Davoudi, M. Savage, Phys. Rev. D86, 054505 (2012)]
	Quasi-distributions (LaMET)	[X. Ji, PRL 110 (2013) 262002, arXiv:1305.1539; Sci. China PPMA. 57, 1407 (2014)]
	Compton amplitude and OPE	[A. Chambers et al. (QCDSF), PRL 118, 242001 (2017), arXiv:1703.01153]
	Pseudo-distributions	[A. Radyushkin, Phys. Rev. D 96, 034025 (2017), arXiv:1705.01488]
	Good lattice cross sections	[Y-Q Ma & J. Qiu, Phys. Rev. Lett. 120, 022003 (2018), arXiv:1709.03018]
	PDFs without Wilson line	[Y. Zhao Phys.Rev.D 109 (2024) 9, 094506, arXiv:2306.14960]
	Moments of PDFs of any order	[A. Shindler, Phys.Rev.D 110 (2024) 5, L051503, arXiv:2311.18704]

★ Reviews of methods and applications

- ***A guide to light-cone PDFs from Lattice QCD: an overview of approaches, techniques and results***
  K. Cichy & M. Constantinou (invited review) Advances in HEP 2019, 3036904, arXiv:1811.07248

- ***Large Momentum Effective Theory***
  X. Ji, Y.-S. Liu, Y. Liu, J.-H. Zhang, and Y. Zhao (2020), 2004.03543

- ***The x-dependence of hadronic parton distributions: A review on the progress of lattice QCD***
  M. Constantinou (invited review) Eur. Phys. J. A 57 (2021) 2, 77, arXiv:2010.02445

Novel Approaches

quasi-PDFs

[X. Ji, PRL 110 (2013) 262002]

pseudo-PDFs

[A. Radyushkin, PRD 96, 034025 (2017)]

Good lattice cross sections

[Y-Q Ma & J. Qiu, PRL 120, 022003 (2018)]

x-dependent distribution functions

current-current correlators

[Y-Q Ma & J. Qiu, PRL 120, 022003 (2018)]

Other methods

[Y. Zhao, PRD 109 (2024) 9, 094506]

[A. Shindler, PRD 110 (2024) 5, L051503]

...

Several talks and poster in PDFLattice 2024

Novel Approaches

quasi-PDFs

[X. Ji, PRL 110 (2013) 262002]

Same raw lattice data

pseudo-PDFs

[A. Radyushkin, PRD 96, 034025 (2017)]

Good lattice cross sections

[Y-Q Ma & J. Qiu, PRL 120, 022003 (2018)]

x-dependent distribution functions

current-current correlators

[Y-Q Ma & J. Qiu, PRL 120, 022003 (2018)]

Other methods

[Y. Zhao, PRD 109 (2024) 9, 094506]

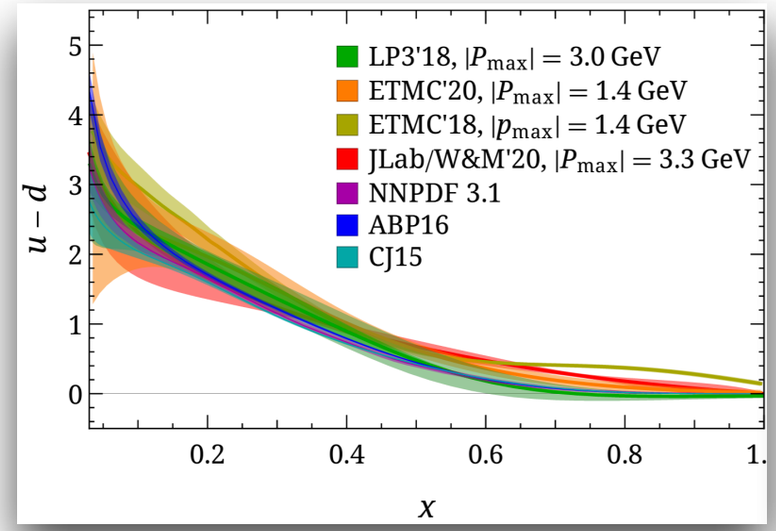
[A. Shindler, PRD 110 (2024) 5, L051503]

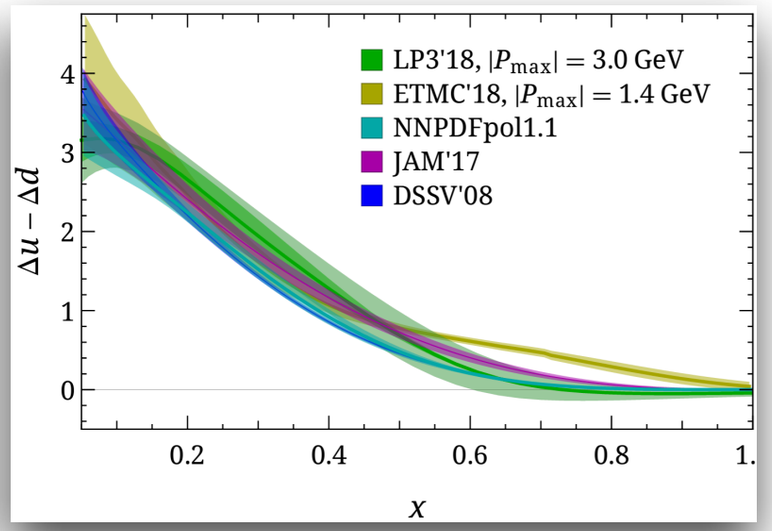
...

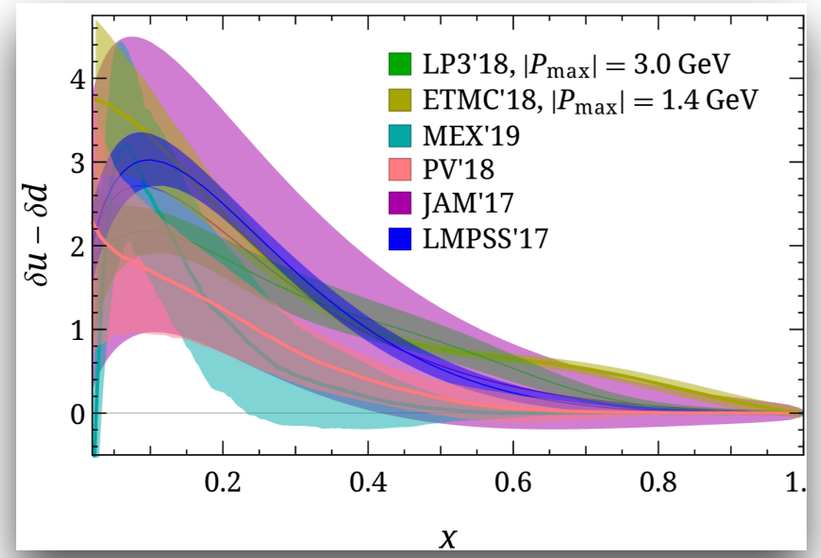
Several talks and poster in PDFLattice 2024

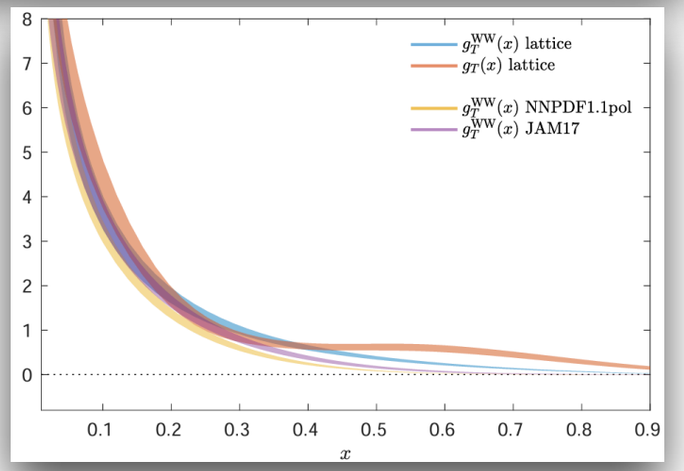
Collection of results

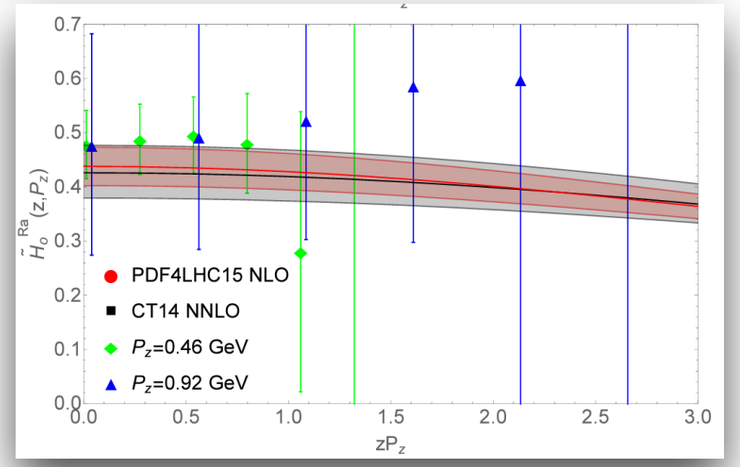
[2020 PDFLattice Report, Prog.Part.Nucl.Phys. 121 (2021) 103908]

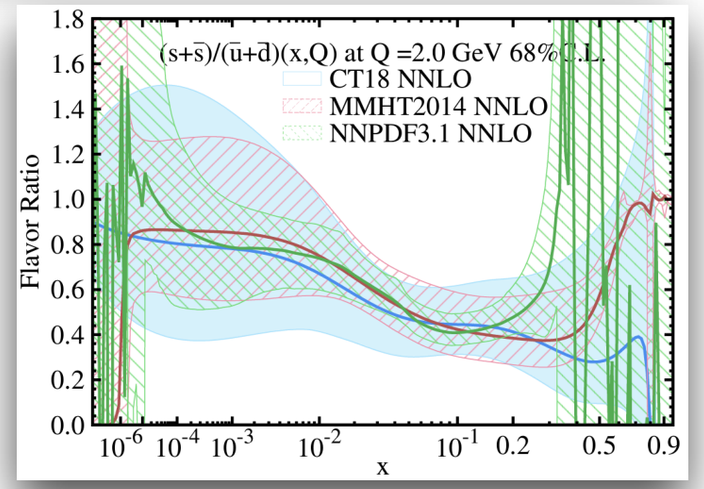






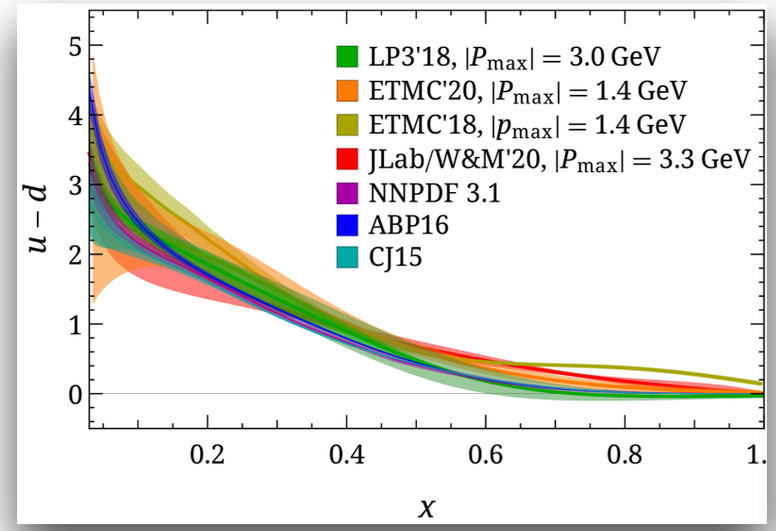


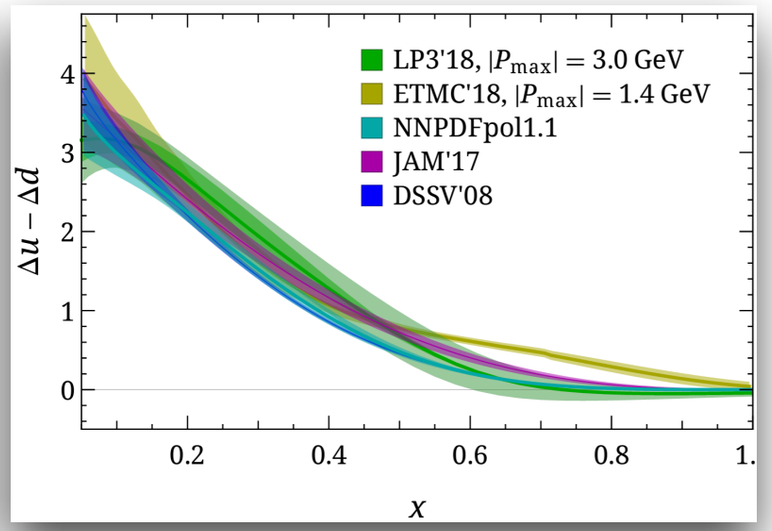


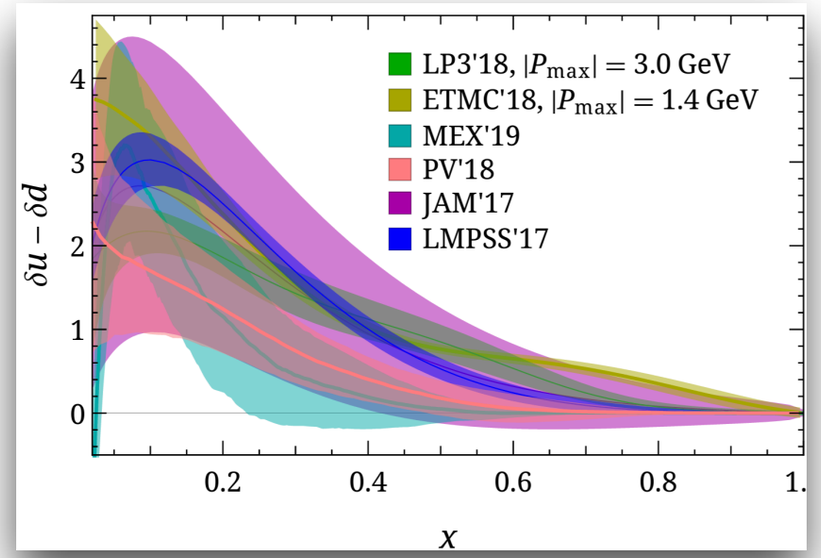


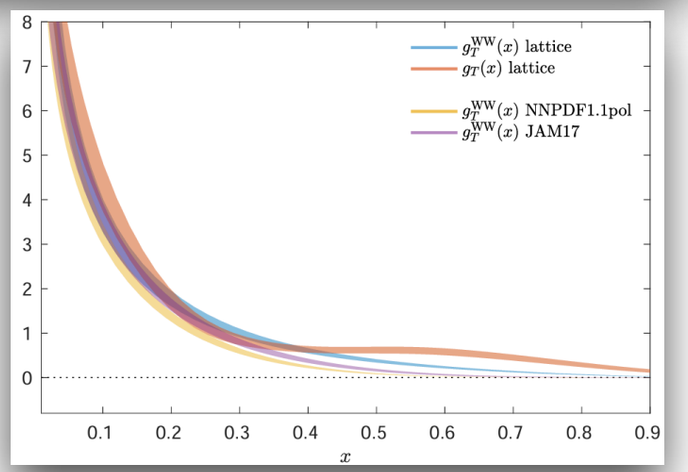
Collection of results

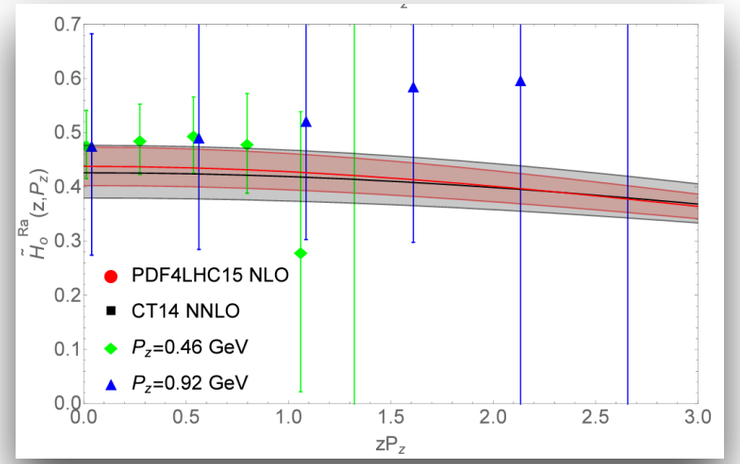
[2020 PDFLattice Report, Prog.Part.Nucl.Phys. 121 (2021) 103908]

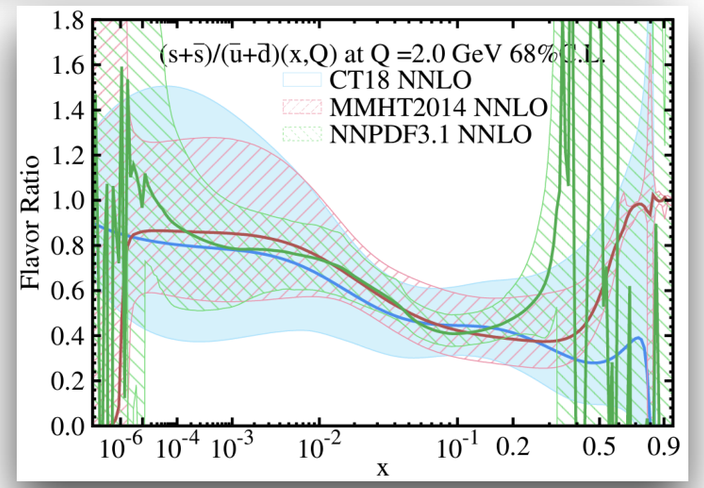












★ Several improvements:

- More calculations at physical quark masses
- Ensembles at various lattice spacings
- Addressing systematic uncertainties due to methodologies

Chris Monahan @ Mon 1:30 pm

Quark PDFs:

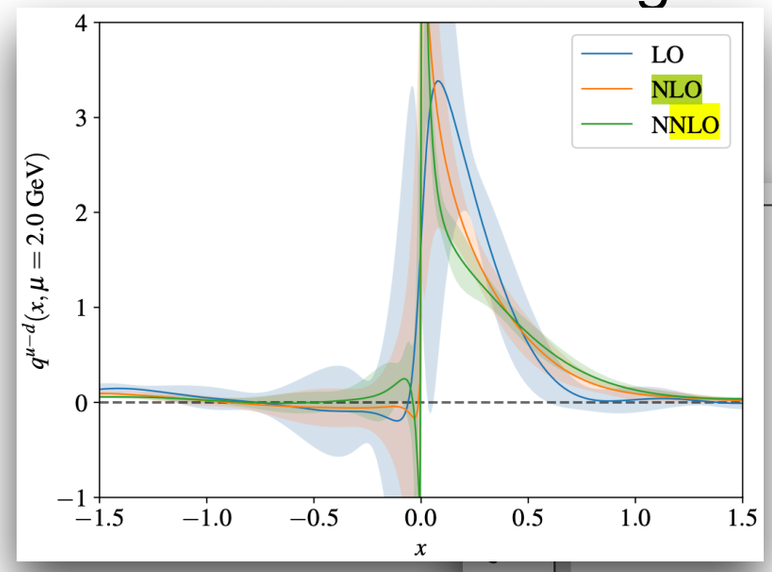
The unpolarized case

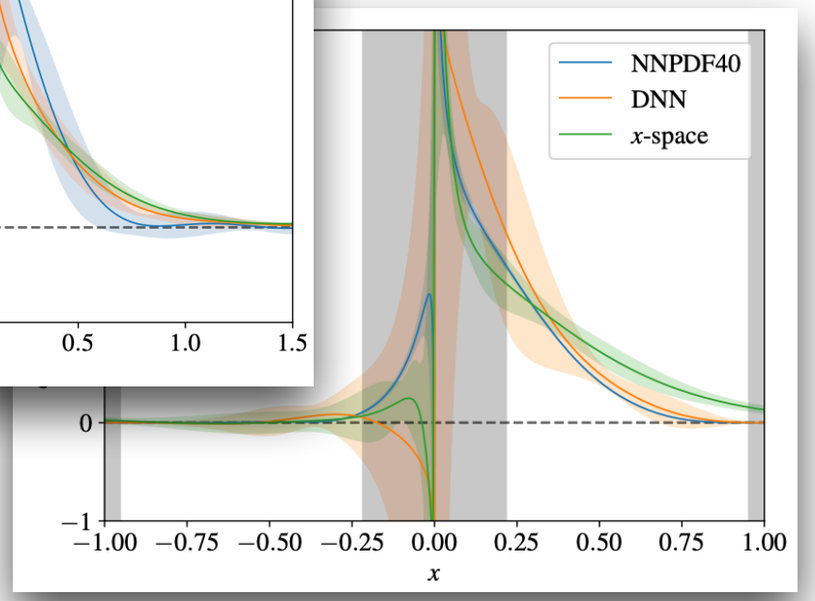
Refining the unpolarized proton PDF (u-d)

★ x-dependence reconstruction

[Gao et al., PRD 107 (2023) 7, 074509]

- HISQ fermions, $m_\pi = 140$ MeV a=0.076 fm
- quasi-PDFs (hybrid renorm) and pseudo-PDFs method: $P \leq 1.5$ GeV
- Deep Neural Network for inverse problem
- NNLO for matching





- NNLO effect small but non-negligible
- DNN improves agreement with global fits (large errors)

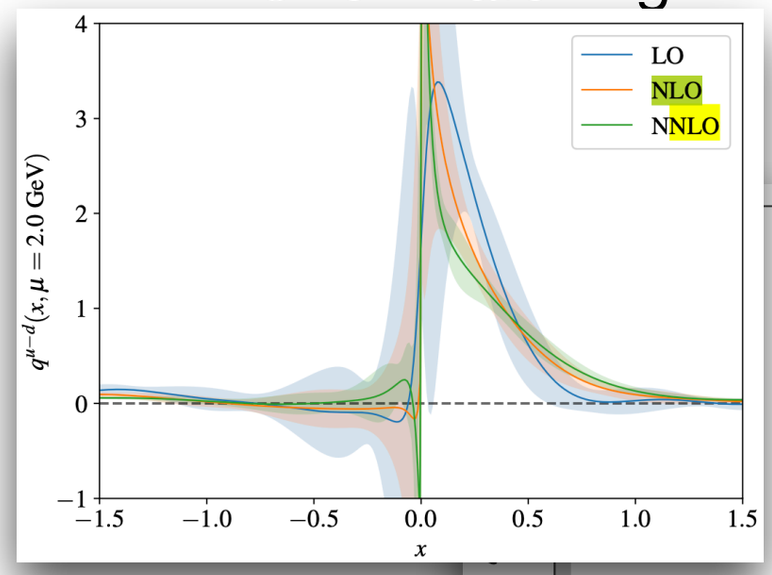
Accessing small and large x challenging

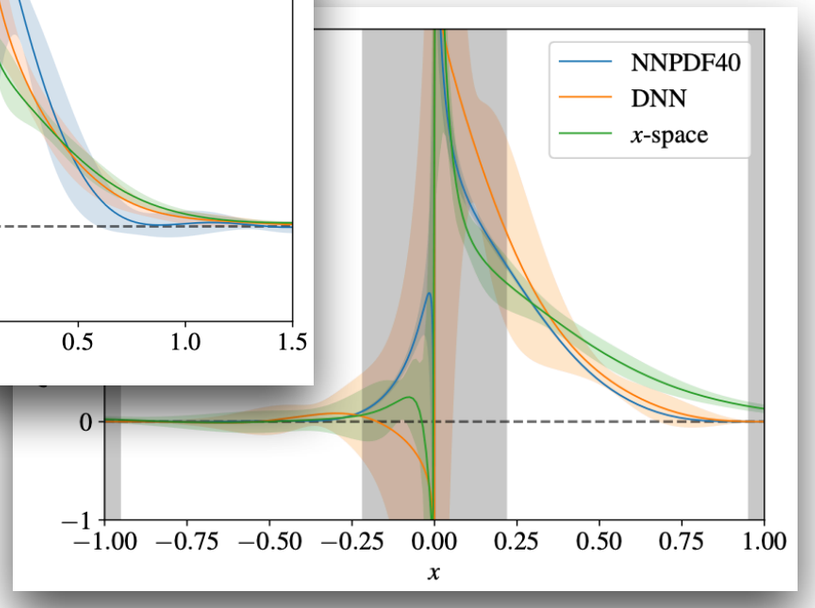
Refining the unpolarized proton PDF (u-d)

★ x-dependence reconstruction

[Gao et al., PRD 107 (2023) 7, 074509]

- HISQ fermions, $m_\pi = 140$ MeV a=0.076 fm
- quasi-PDFs (hybrid renorm) and pseudo-PDFs method: $P \leq 1.5$ GeV
- Deep Neural Network for inverse problem
- NNLO for matching



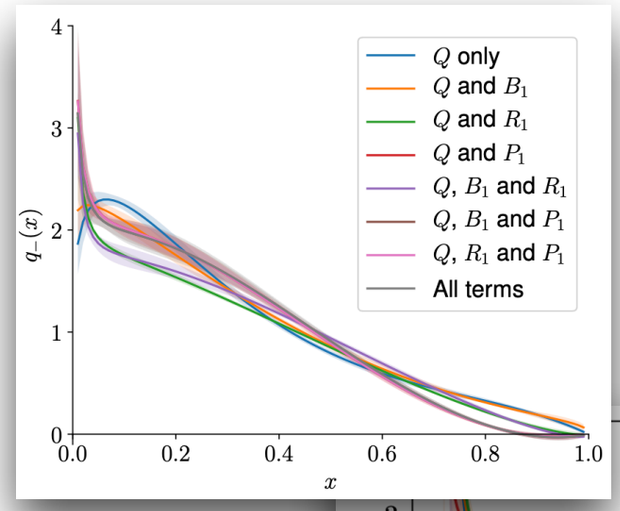


- NNLO effect small but non-negligible
- DNN improves agreement with global fits (large errors)
- Accessing small and large x challenging

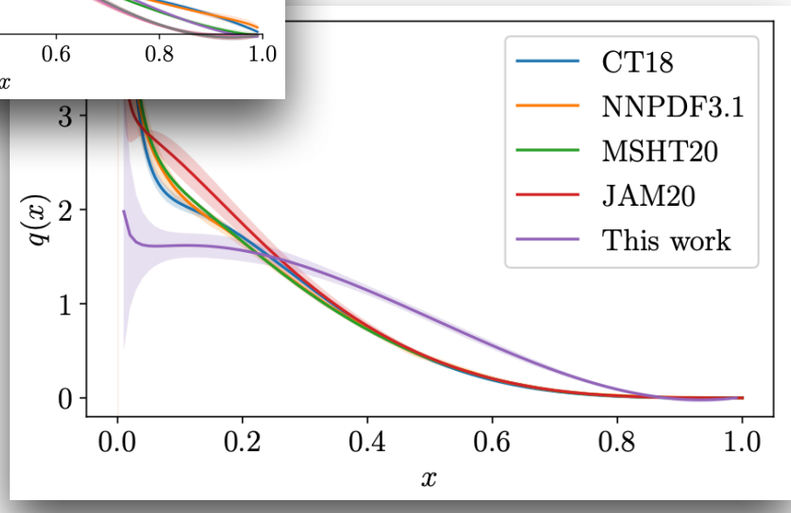
★ Cont. limit - higher twist effects

[Karpie et al., JHEP 11 (2021) 024]

- Clover, a=0.075, 0.065, 0.048 fm
- $m_\pi = 440$ MeV
- Excited states: sGEVP
- Jacobi polynomials for inverse problem to model finite-a & higher twist effects



$P_1 : a/z$
$P_1 : ap$
B_1 : higher twist



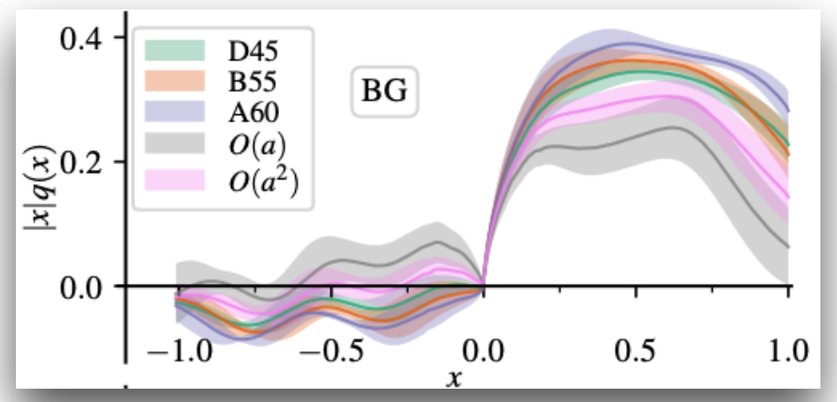
- Isolated effects from each term not clear

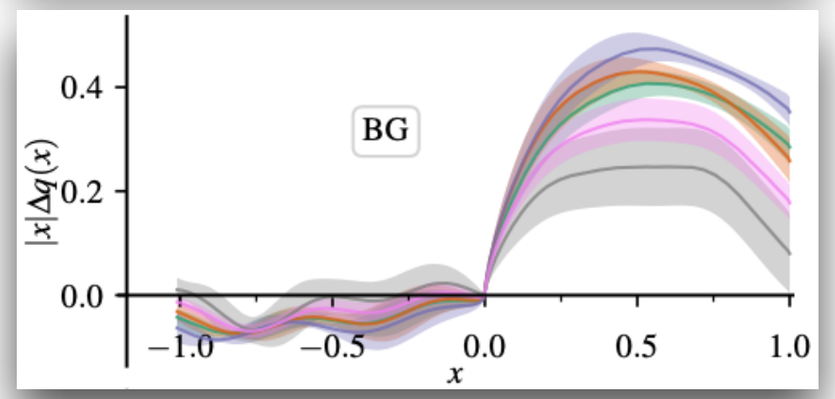
Refining the unpolarized proton PDF (u-d)

★ Continuum limit

[Alexandrou et al., PRD 103 (2021) 094512]

- No $\mathcal{O}(a)$ improvement for nonlocal operators [Green et al., PRL 121, 022004 (2018)]
- TM fermions:
  $m_\pi = 370$ MeV (a =0.093, 0.082, 0.064 fm)
- quasi-PDFs method: P ~ 1.7 - 1.8 GeV
- $a \to 0$ at fixed x





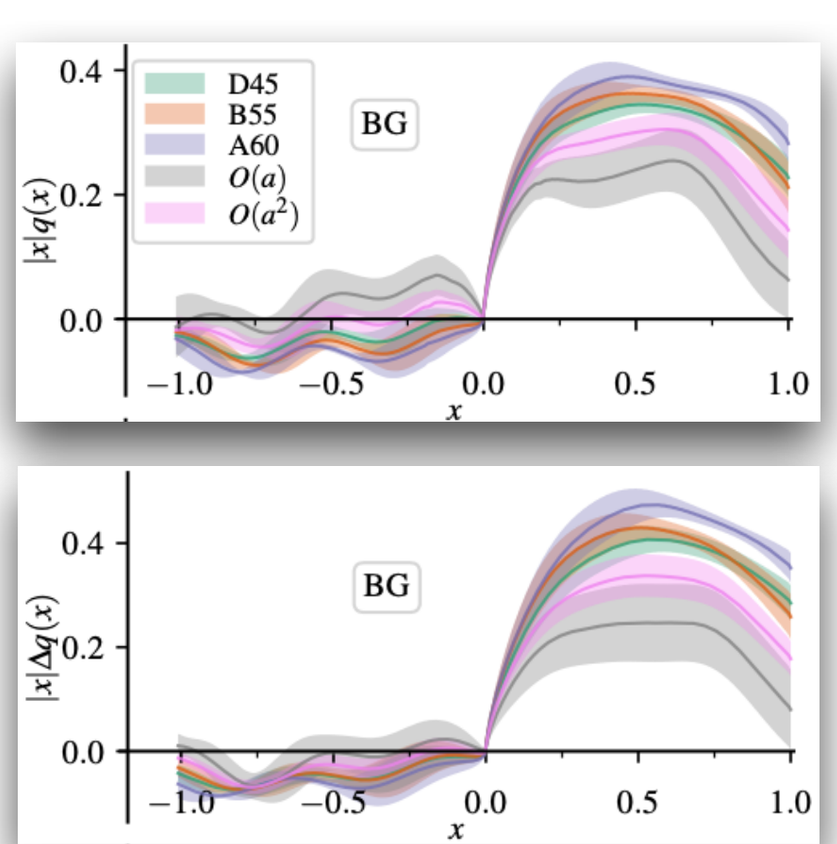
- Data do not favor $\mathcal{O}(a)$ or $\mathcal{O}(a^2)$ extrapolations

Refining the unpolarized proton PDF (u-d)

★ Continuum limit

[Alexandrou et al., PRD 103 (2021) 094512]

- No $\mathcal{O}(a)$ improvement for nonlocal operators [Green et al., PRL 121, 022004 (2018)]
- TM fermions: $m_\pi = 370$ MeV (a =0.093, 0.082, 0.064 fm)
- quasi-PDFs method: P ~ 1.7 - 1.8 GeV
- $a \to 0$ at fixed x

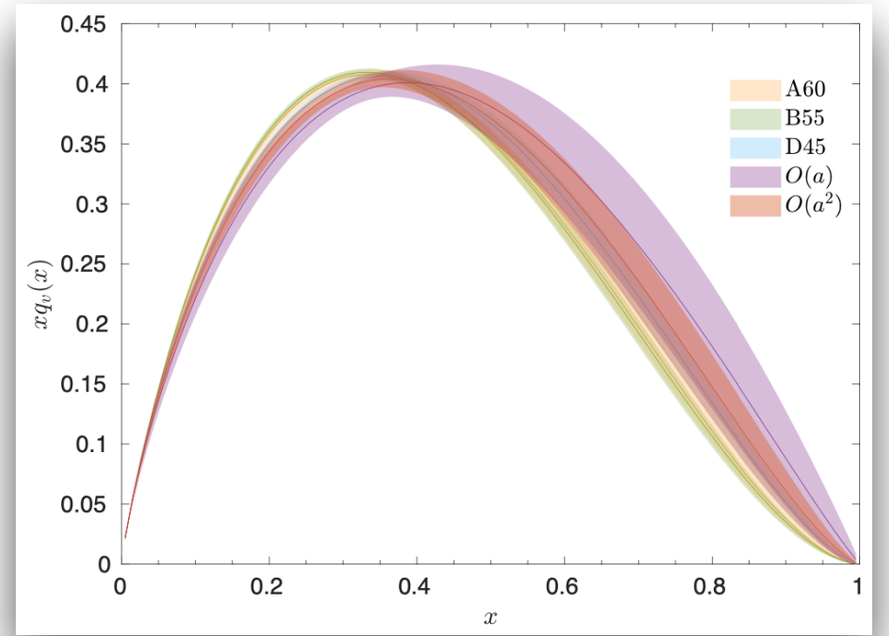


- Data do not favor $\mathcal{O}(a)$ or $\mathcal{O}(a^2)$ extrapolations

[Bhat et al., PRD 106 (2022) 5, 054504]

- Analysis of same raw data within pseudo-PDFs method
- Important to assess the influence of methodology on systematic uncertainties

- NNLO for matching (small effect)



- Extrapolation inflates the statistical uncertainties
- Conclusions remain the same as quasi-PDF method

Quark PDFs:

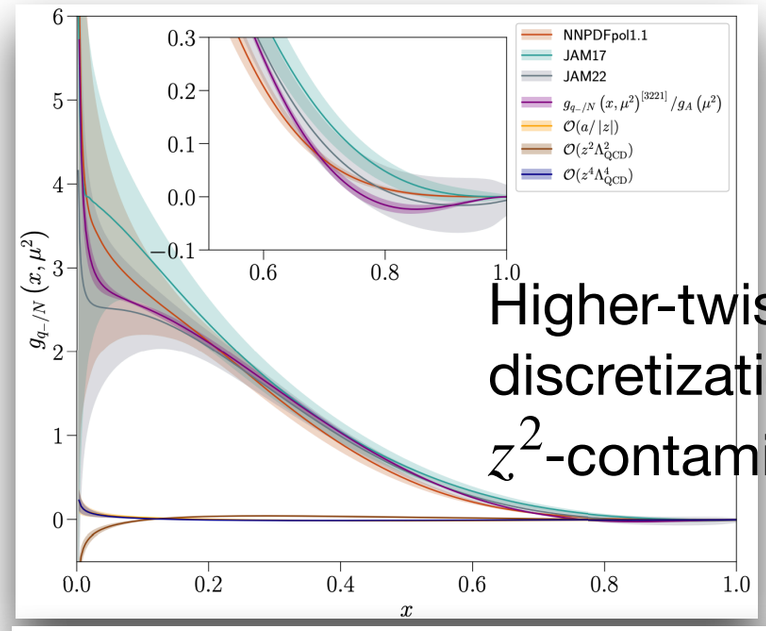
helicity and transversity

Helicity proton PDF (u-d)

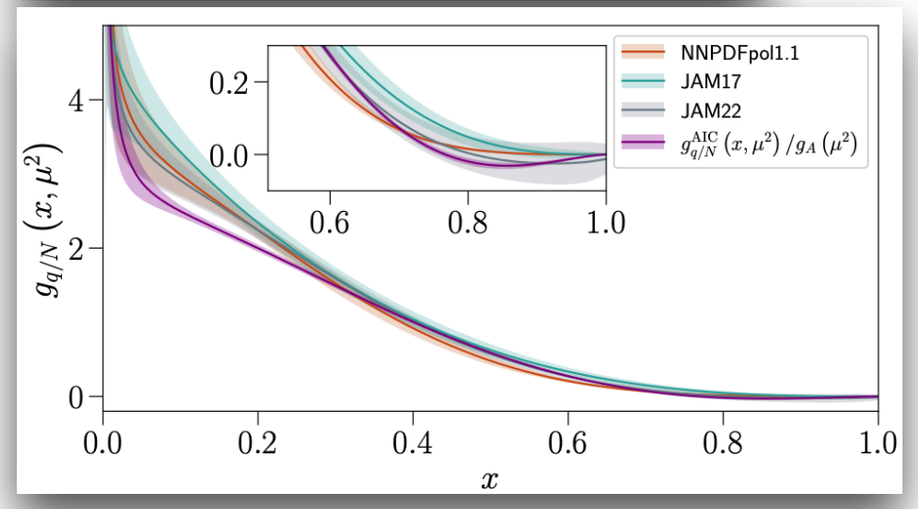
★ Addressing systematic effects

[Edwards et al., JHEP 03 (2023) 086]

- Clover, a=0.094 fm
- $m_\pi = 358$ MeV
- Distillation method



Higher-twist effects, discretization effects, z^2-contaminating amplitude



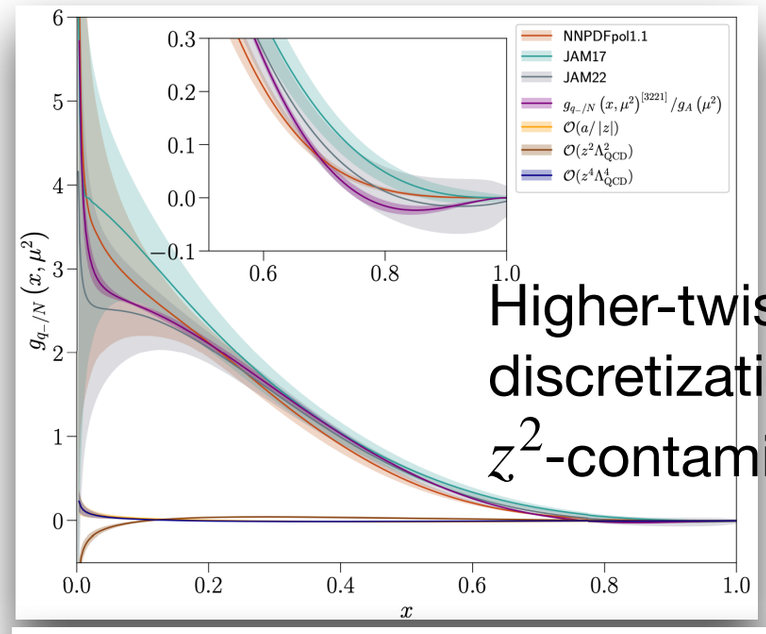
Implementation of corrected Akaike Information Criterion

Helicity proton PDF (u-d)

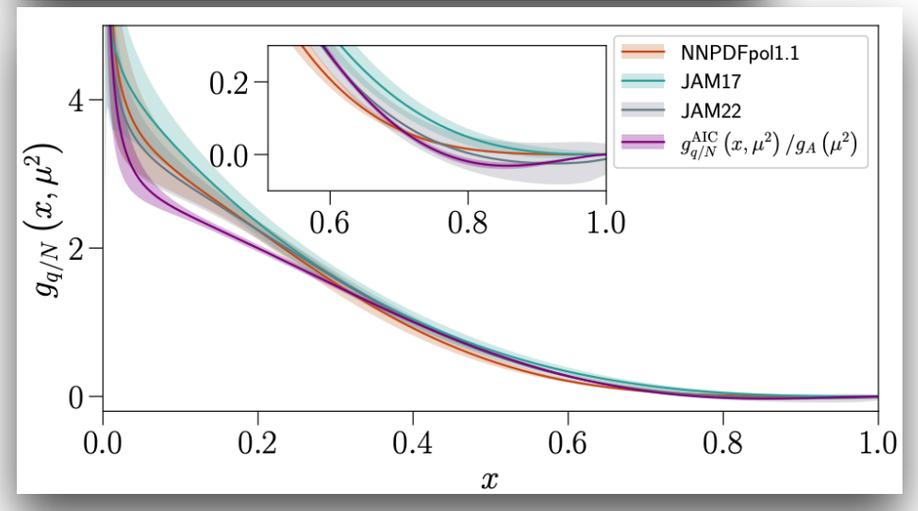
★ Addressing systematic effects

[Edwards et al., JHEP 03 (2023) 086]

- Clover, a=0.094 fm
- $m_\pi = 358$ MeV
- Distillation method



Higher-twist effects, discretization effects, z^2-contaminating amplitude

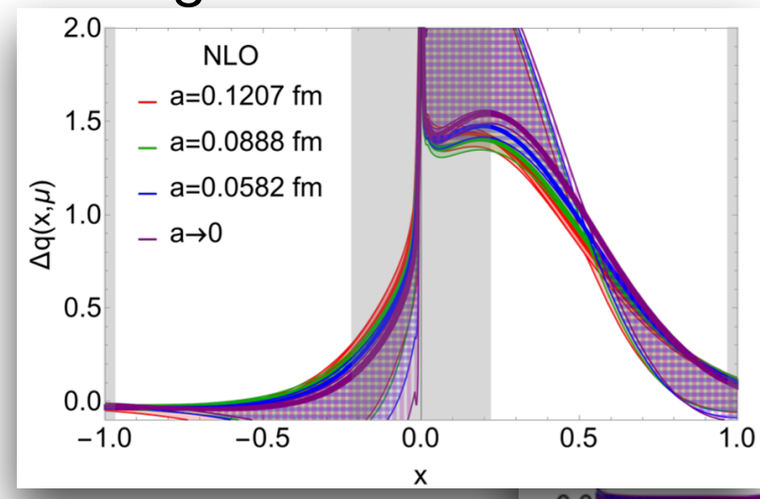


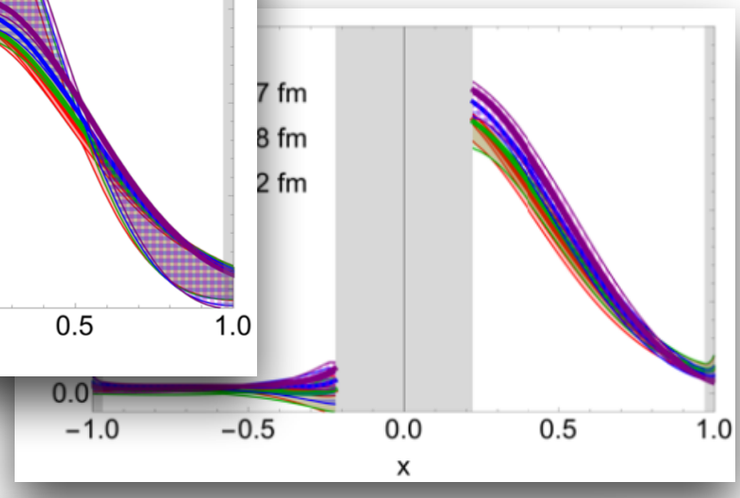
Implementation of corrected Akaike Information Criterion

★ Continuum limit - renormalization

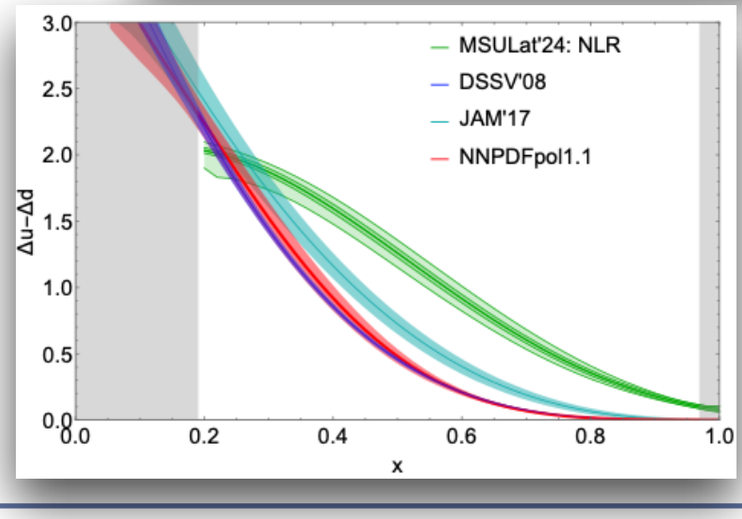
[Holligan et al., PLB 854 (2024) 138731]

- HISQ/clover fermions:
  $m_\pi = 315$ MeV (a =0.121, 0.089, 0.058 fm)
- quasi-PDFs method: P ~ 1.75 GeV
- renormalization-group resummation
- leading-renormalon resummation in matching





- RGR & LRR: reduced errors

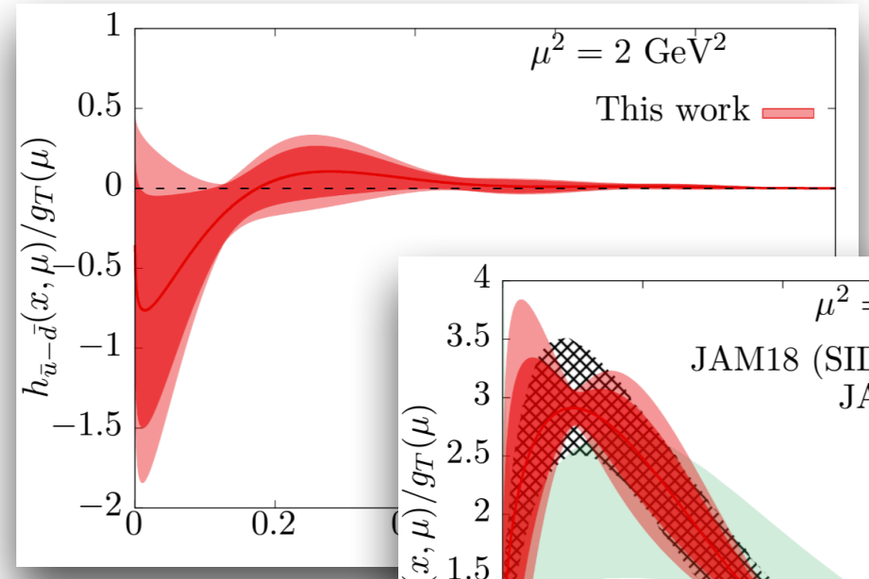


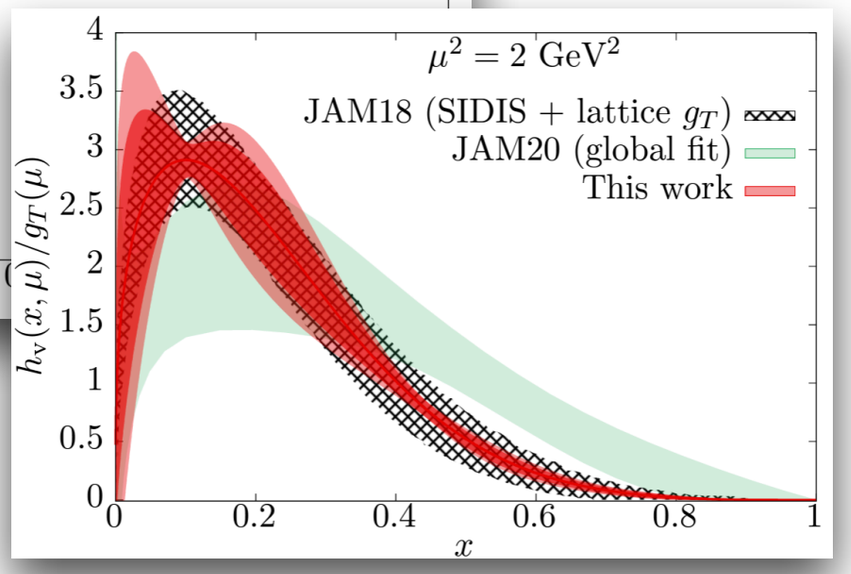
Transversity proton PDF (u-d)

★ x-dependence reconstruction

[Egerer et al., PRD 105 (2022) 3, 034507]

- Clover, a=0.094 fm
- $m_\pi = 358$ MeV
- Excited states: sGEVP
- Jacobi polynomials for inverse problem to model finite-a & higher twist effects





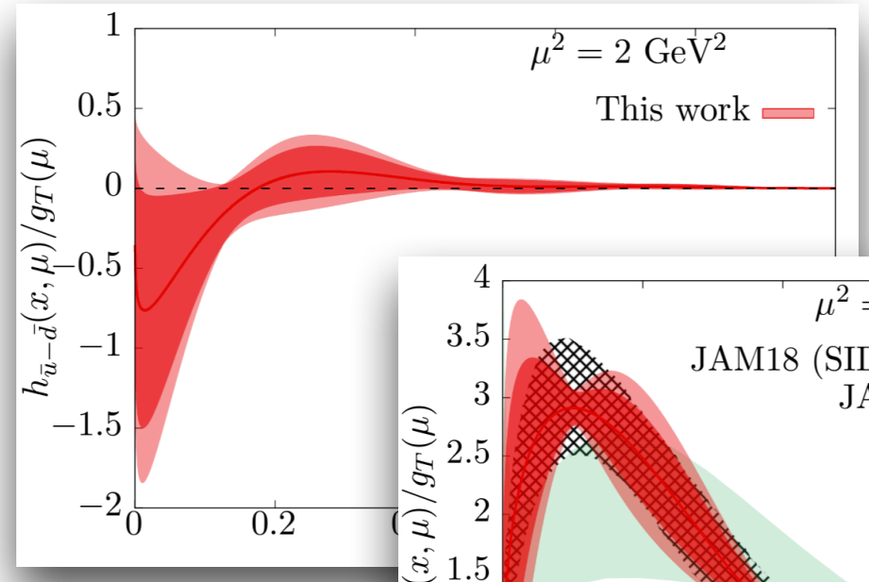
- Quark region compatible with JAM18
- Anti-quark region compatible with zero

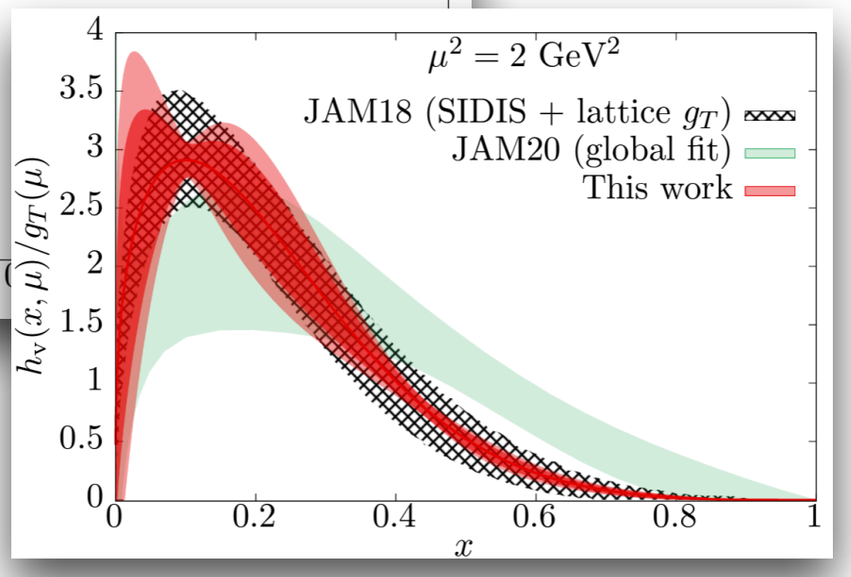
Transversity proton PDF (u-d)

★ x-dependence reconstruction

[Egerer et al., PRD 105 (2022) 3, 034507]

- Clover, a=0.094 fm
- $m_\pi = 358$ MeV
- Excited states: sGEVP
- Jacobi polynomials for inverse problem to model finite-a & higher twist effects



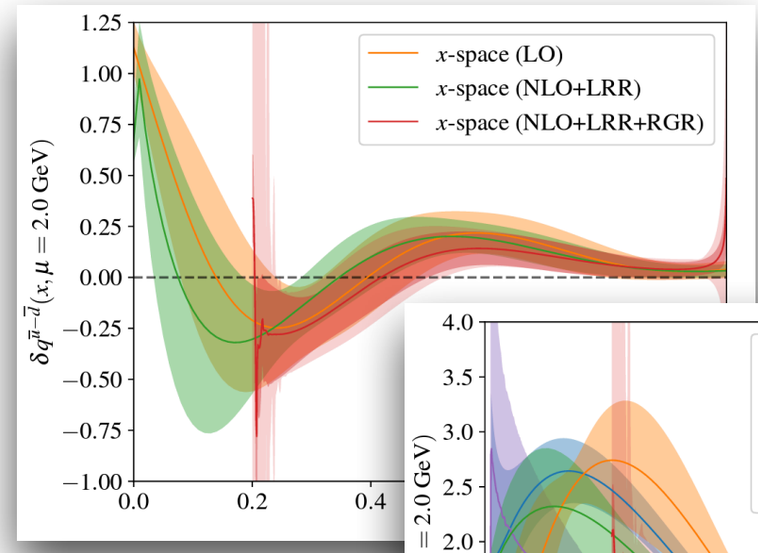


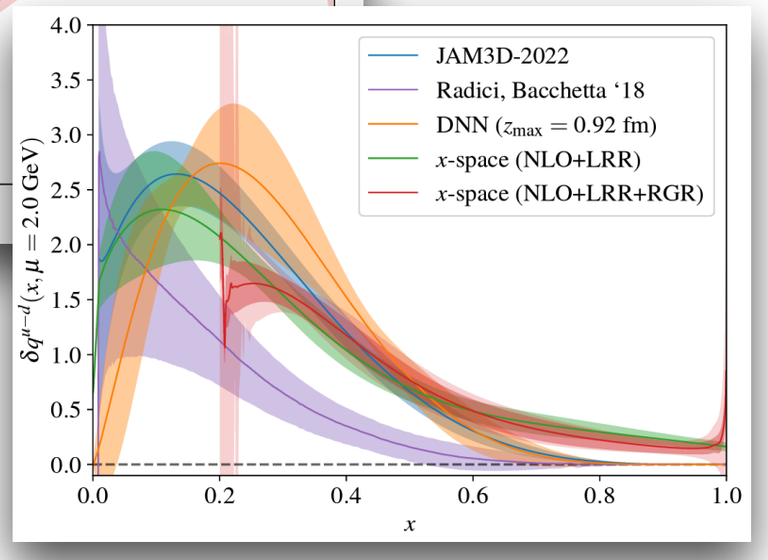
- Quark region compatible with JAM18
- Anti-quark region compatible with zero

★ Physical pion mass

[Gao et al., PRD 109 (2024) 5, 054506]

- HISQ fermions, $m_\pi = 140$MeV, a=0.076fm
- quasi-PDFs (hybrid renorm) and pseudo-PDFs method: P : 0.5, 0.9, 1.2, 1.7 GeV
- Hybrid renormalization, LRR (NLO)
- Deep Neural Network for inverse problem





- x>0.6 affected by approach
- Anti-quark region almost zero

Quark PDFs:

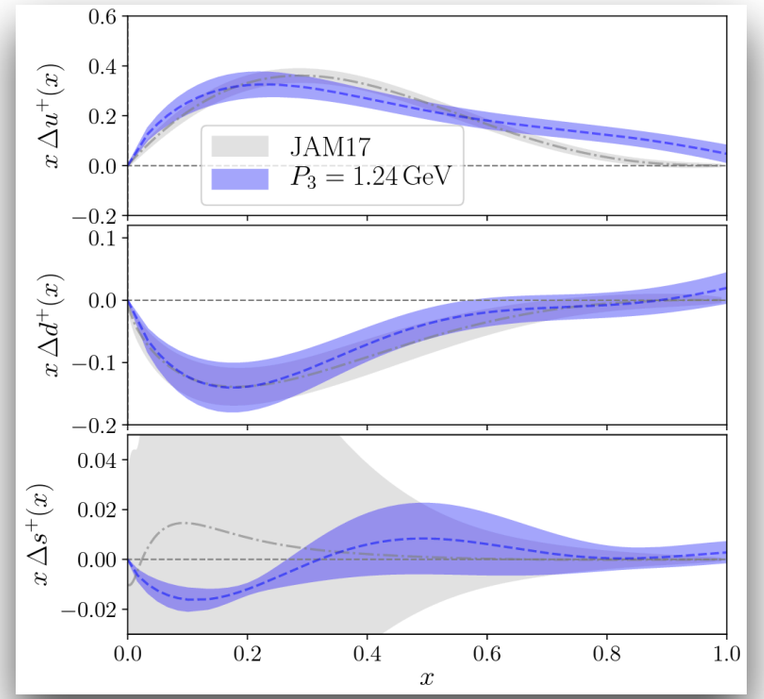
flavor decomposition

Flavor decomposition of PDFs

★ Helicity

[Alexandrou et al., PRL 126 (2021) 10, 102003]

- TM fermions: $m_\pi = 260$ MeV (a =0.093 fm)
- quasi-PDFs method: $P \leq 1.24$ GeV
- Inclusion of both connected and disconnected diagrams for light quarks



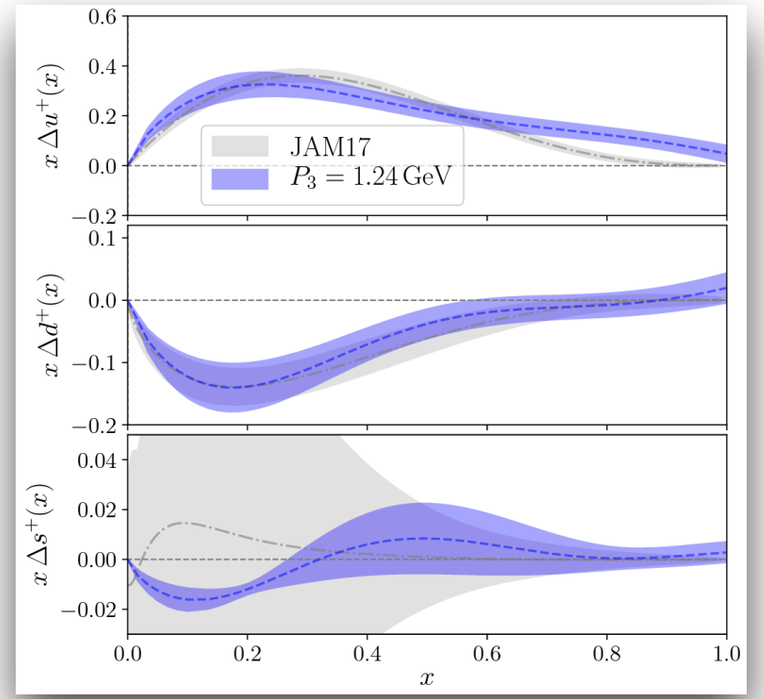
- Light-quark disconnected most sizable for helicity
- Control of statistical uncertainties

Flavor decomposition of PDFs

★ Helicity

[Alexandrou et al., PRL 126 (2021) 10, 102003]

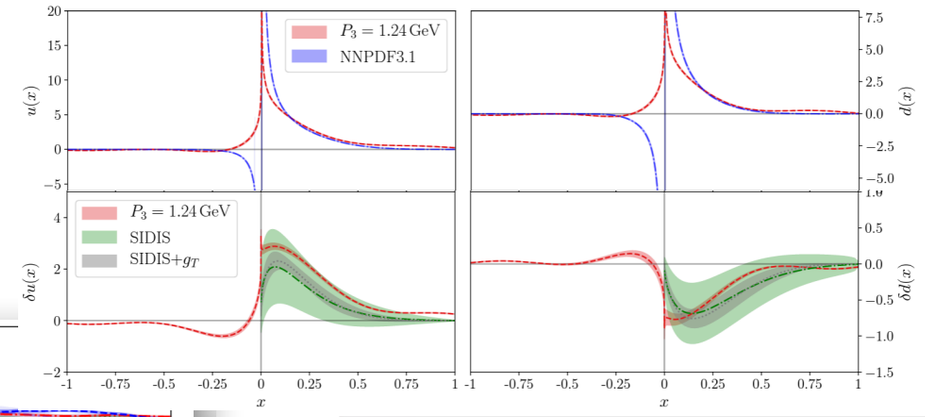
- TM fermions: $m_\pi = 260$ MeV (a =0.093 fm)
- quasi-PDFs method: $P \leq 1.24$ GeV
- Inclusion of both connected and disconnected diagrams for light quarks

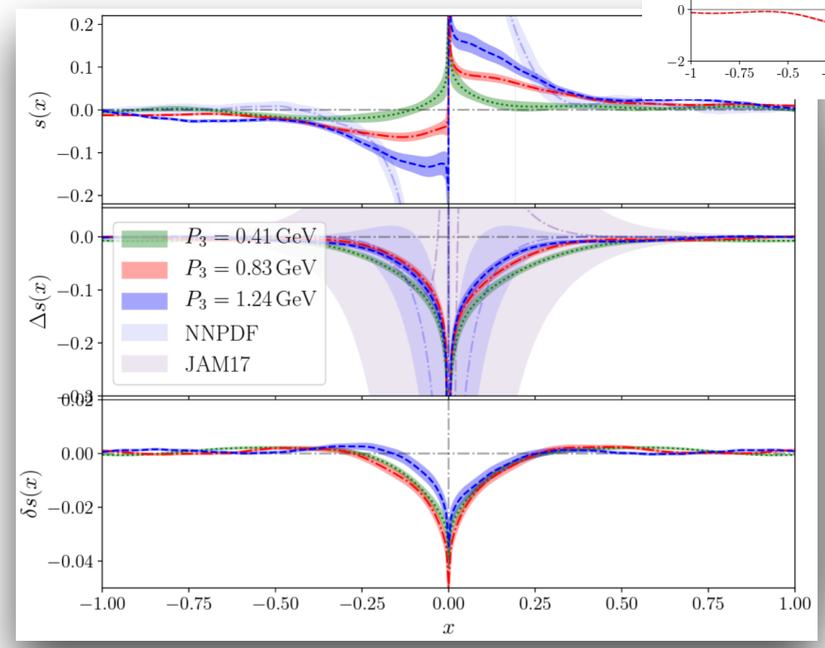


- Light-quark disconnected most sizable for helicity
- Control of statistical uncertainties

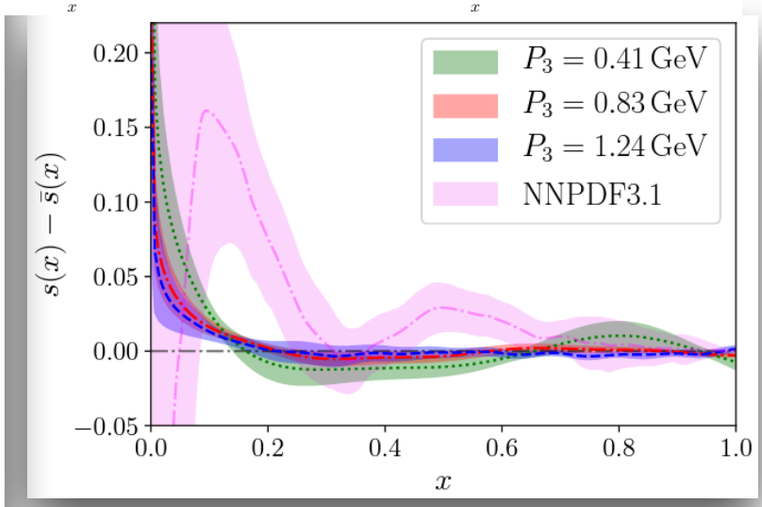
★ Unpolarized, helicity, transversity

[Alexandrou et al., PRD 104 (2021) 5, 054503]





- s-quark << light-quark PDFs



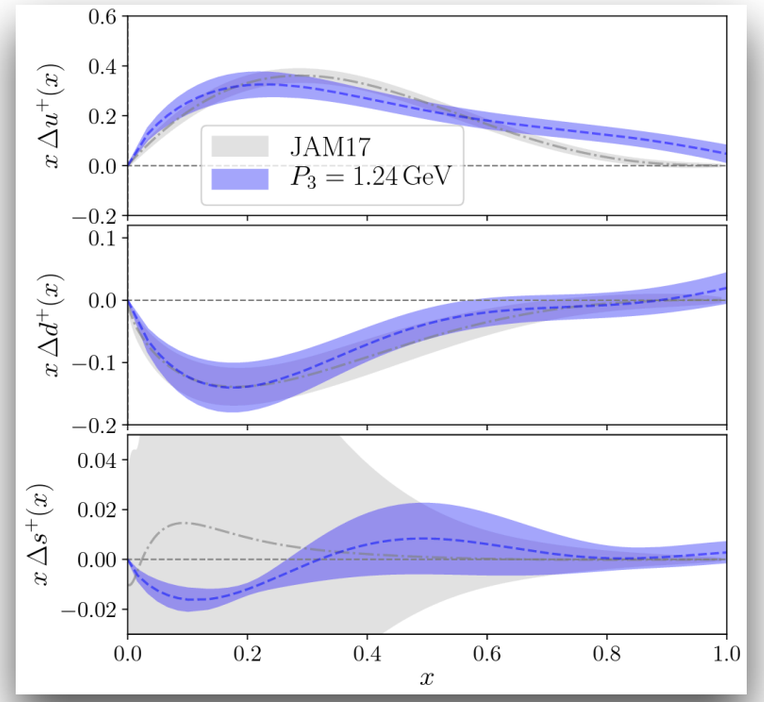
- Asymmetry~0 at $x \geq 0.2$

Flavor decomposition of PDFs

★ Helicity

[Alexandrou et al., PRL 126 (2021) 10, 102003]

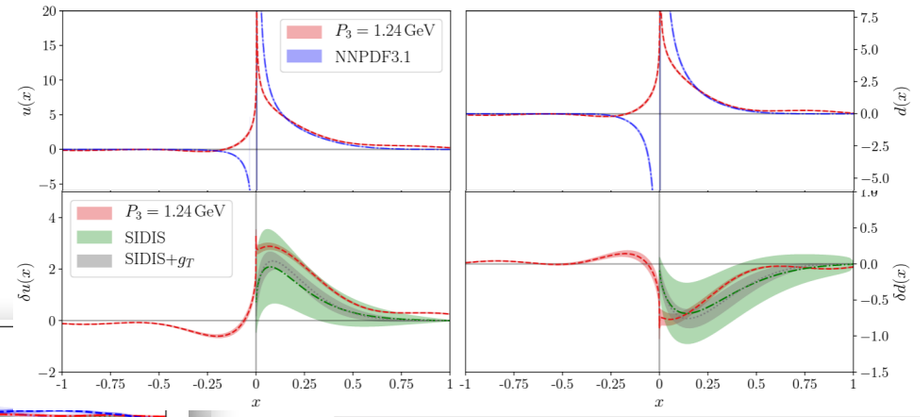
- TM fermions: $m_\pi = 260$ MeV (a =0.093 fm)
- quasi-PDFs method: $P \leq 1.24$ GeV
- Inclusion of both connected and disconnected diagrams for light quarks

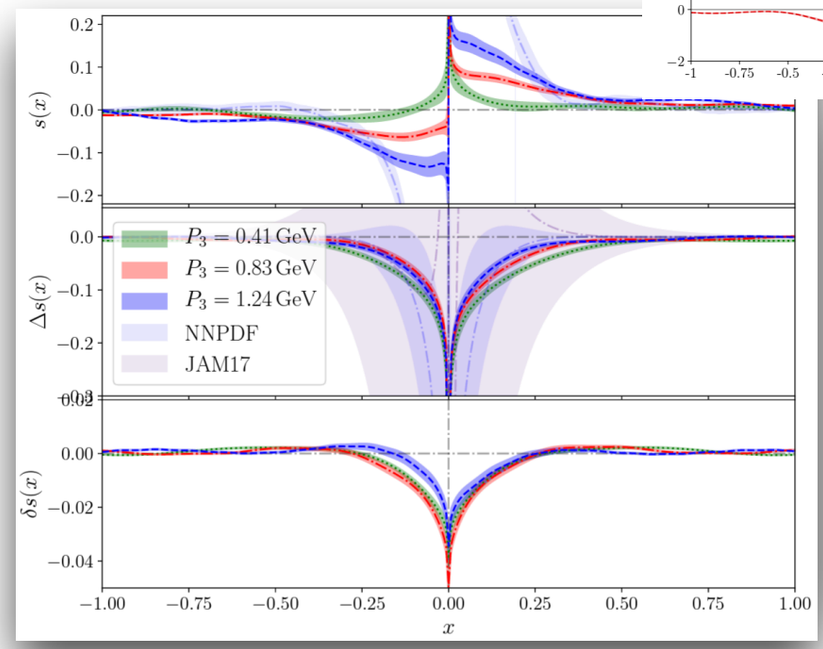


- Light-quark disconnected most sizable for helicity
- Control of statistical uncertainties

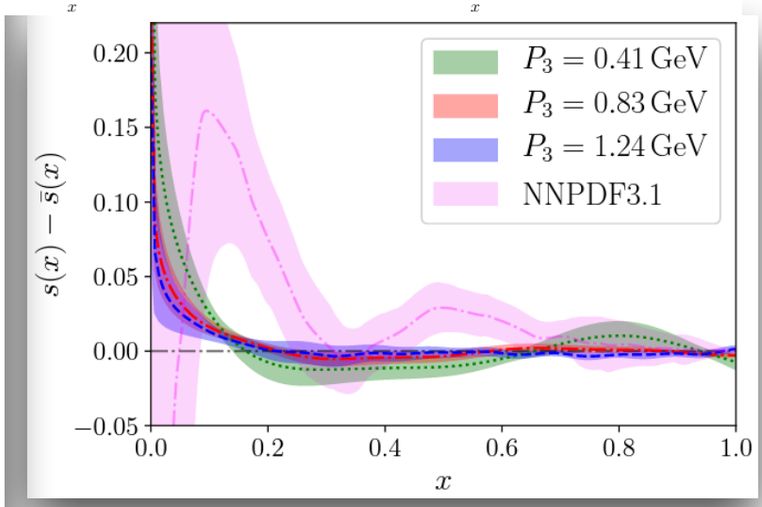
★ Unpolarized, helicity, transversity

[Alexandrou et al., PRD 104 (2021) 5, 054503]





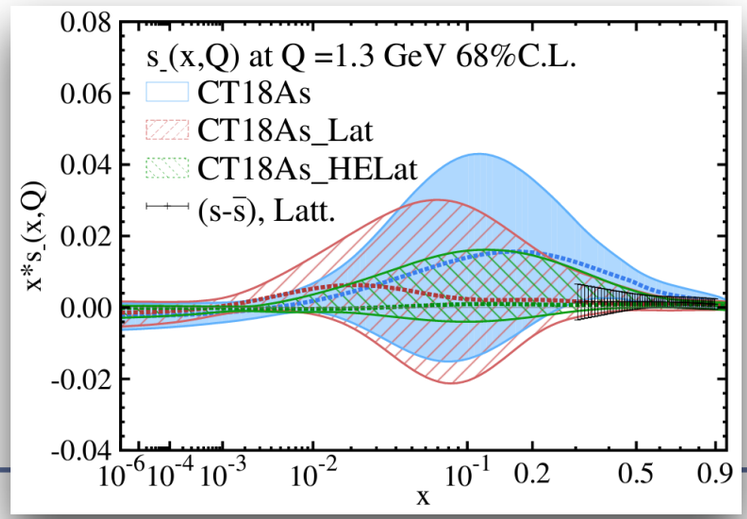
- s-quark << light-quark PDFs



- Asymmetry~0 at $x \geq 0.2$

LQCD in CTEQ-TEA framework

[Hou et al., PRD 107 (2023) 7, 076018]
[Zhang et al., PRD 104 (2021) 9, 094511]



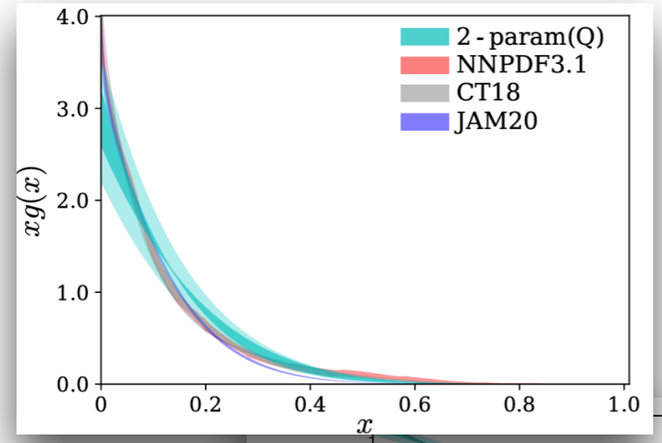
Gluonic Sector

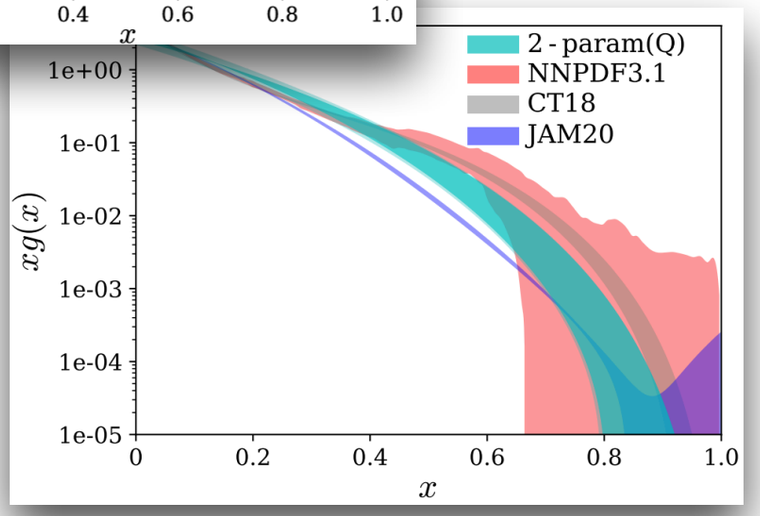
Gluon unpolarized PDF

★ New techniques

[Khan et al., JHEP 11, 148 (2021)]

- Clover, a=0.094 fm
- $m_\pi = 358$ MeV
- pseudo-ITD method: $P \leq 2.5$ GeV
- Excited states: sGEVP





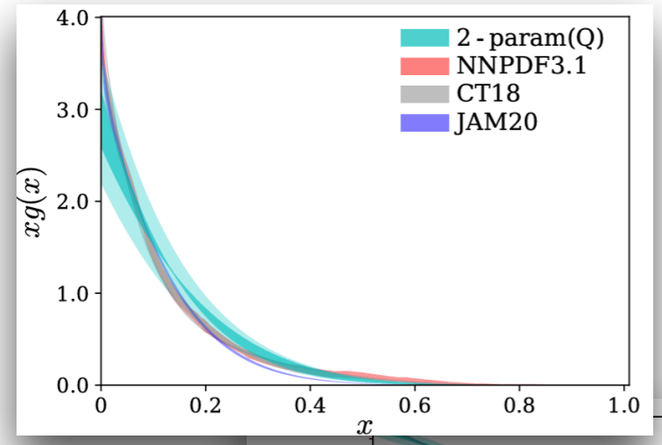
- Methodology allows higher P

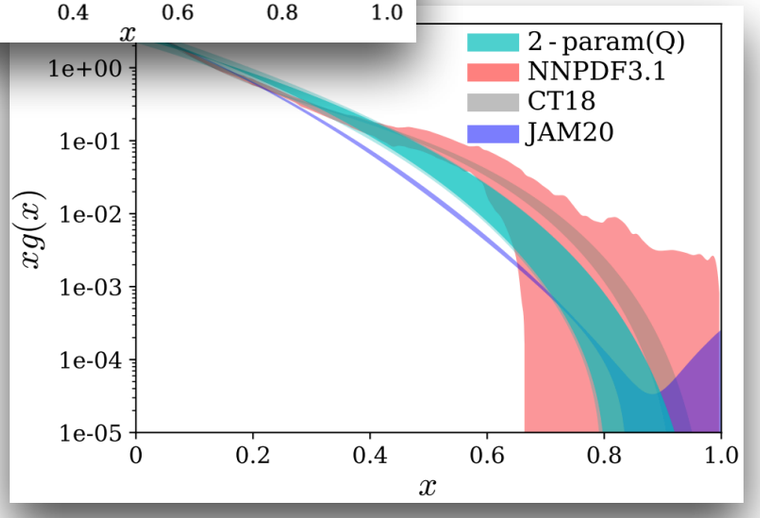
Gluon unpolarized PDF

★ New techniques

[Khan et al., JHEP 11, 148 (2021)]

- Clover, a=0.094 fm
- $m_\pi = 358$ MeV
- pseudo-ITD method: $P \leq 2.5$ GeV
- Excited states: sGEVP



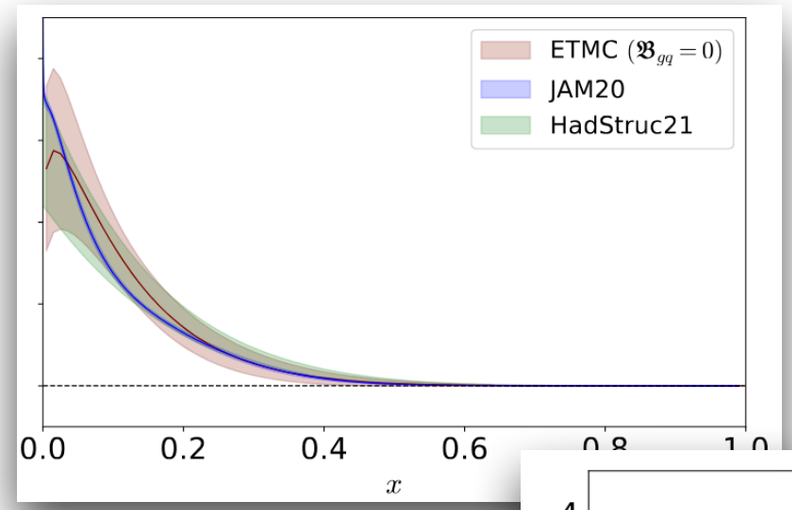


- Methodology allows higher P

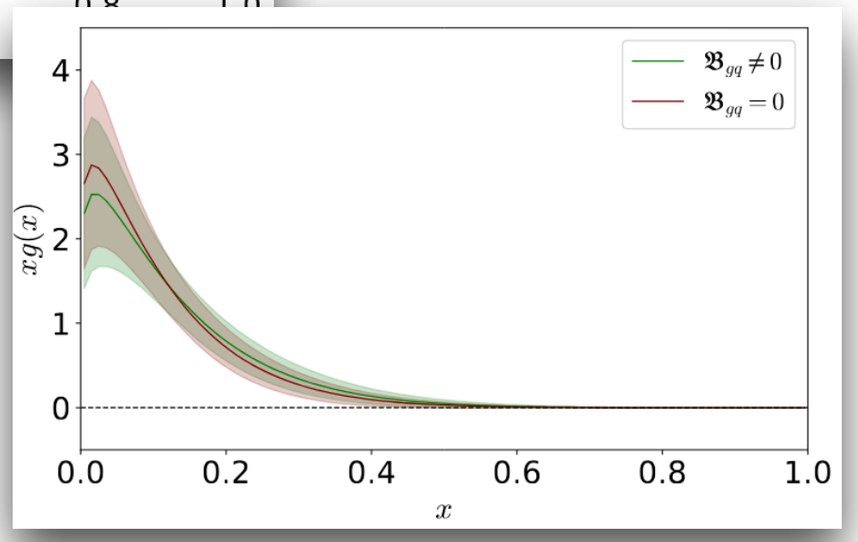
★ Mixing elimination

[Delmar et al., PRD 108 (2023) 9, 094515]

- TM fermions: $m_\pi = 260$ MeV (a =0.093 fm)
- pseudo-ITD method: P ~ 1.7 GeV



- Full agreement despite difference in techniques



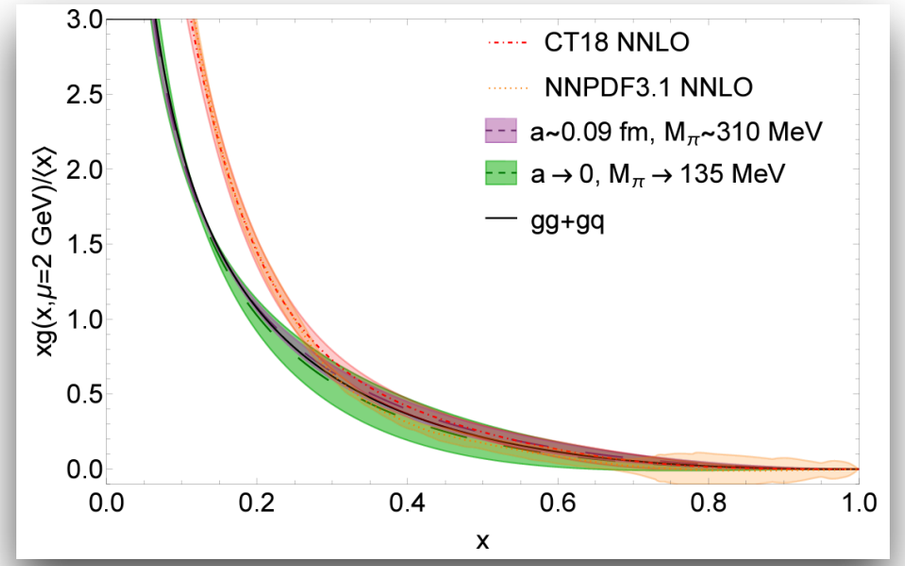
- Effect of mixing with singlet PDF ([PRD 104 (2021) 5, 054503]) within uncertainties

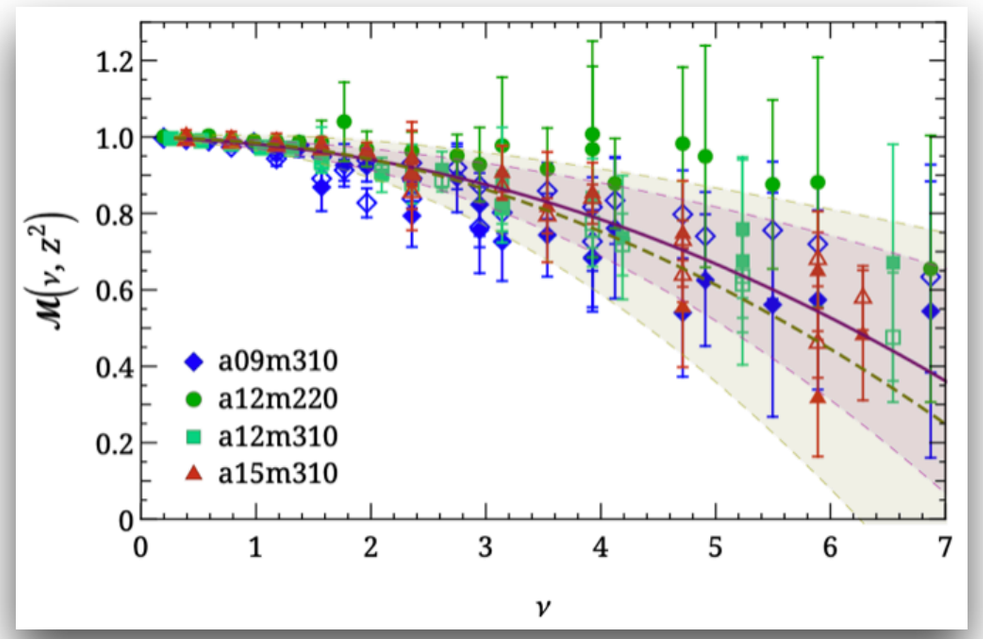
Gluon unpolarized PDF

★ Chiral extrapolation, Continuum limit

[Fan et al., PRD 108, 014508 (2023)]

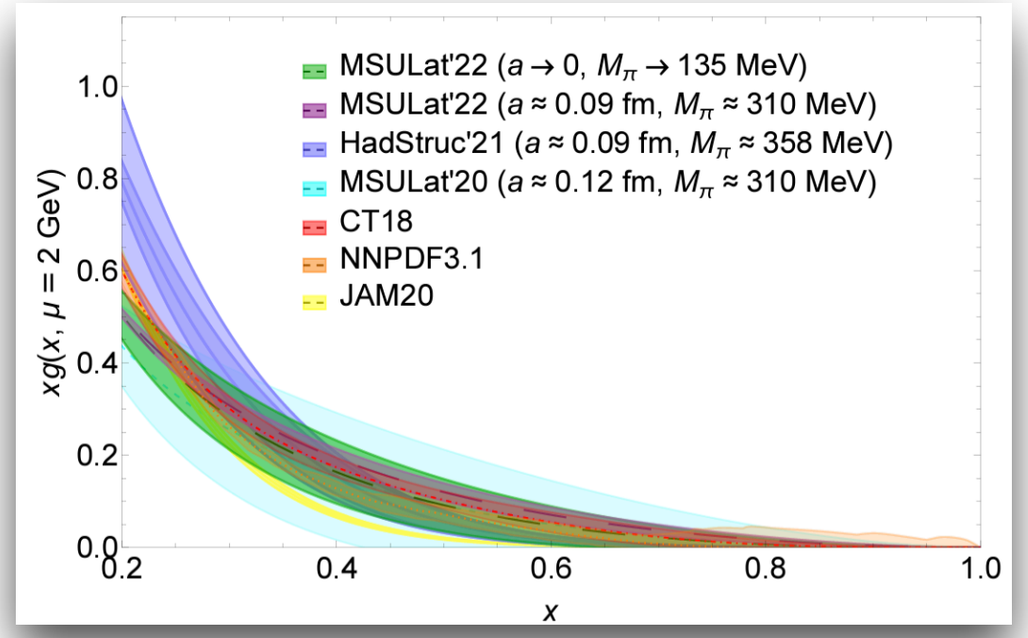
- HISQ/clover fermions:
  $m_\pi^s = 315$ MeV, $m_\pi^v = 220, 310, 690$ MeV
  ($a = 0.09, 0.12, 0.15$ fm)
- pseudo-ITP method: $P \sim 3$ GeV





- Control of statistical uncertainties challenging for low pion masses, as well as for high Ioffe times

- Continuum extrapolation increases errors
- Tension of ratio between LQCD and global analysis in x<0.35 region



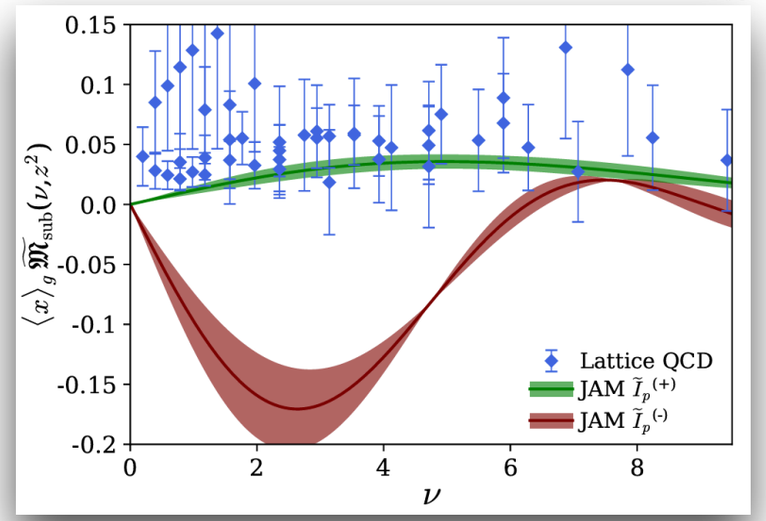
- Better agreement in PDF

Exploration of Gluon Helicity PDF

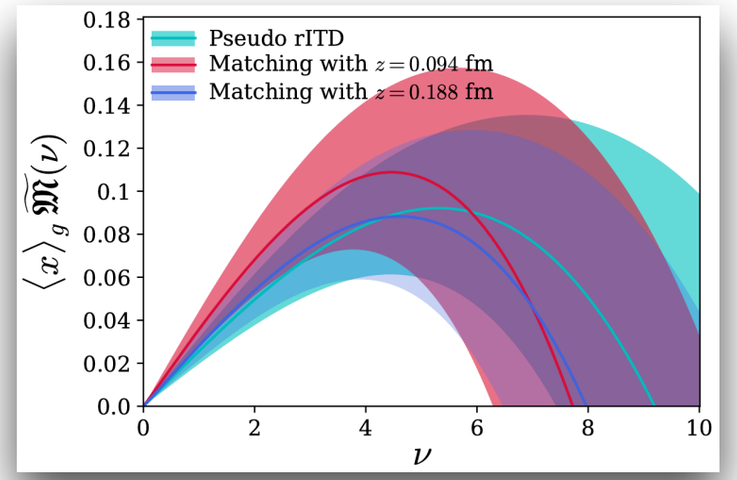
★ Positivity Constraints

[Egerer et al., PRD 106 (2022) 9, 094511]

- Clover, a=0.094 fm, m_π=358 MeV



- Hint for nonzero gluon spin
- LQCD can distinguish between
  + and - solutions of global fits



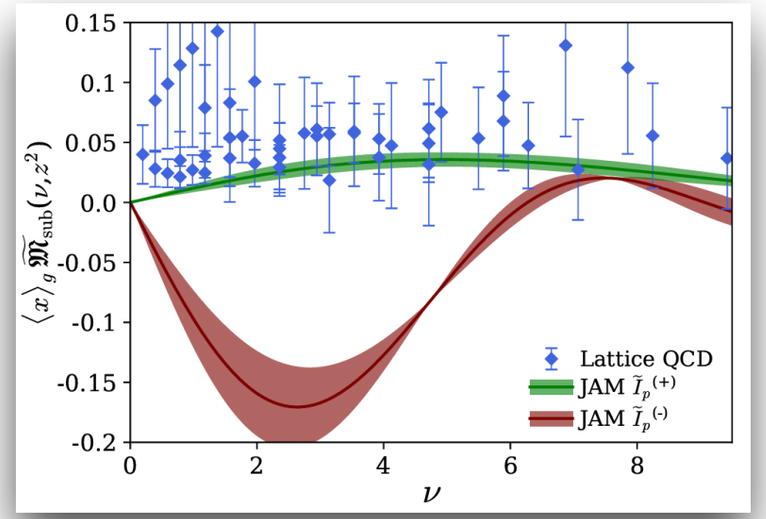
- Grand challenge: statistical uncertainties
- Higher momenta needed

Exploration of Gluon Helicity PDF

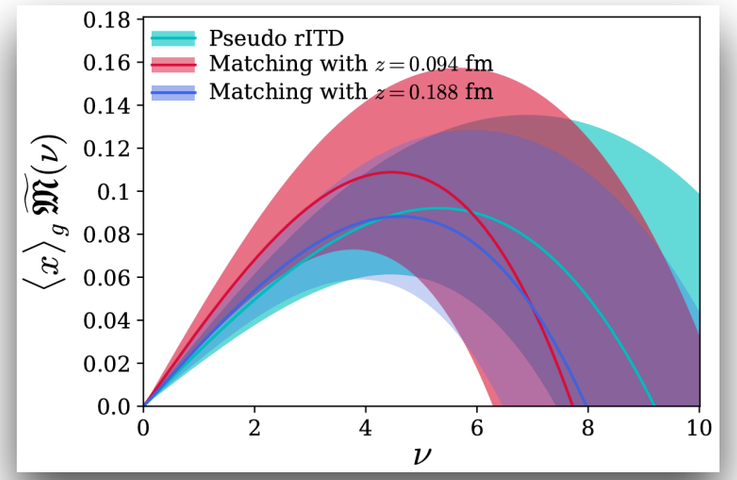
★ Positivity Constraints

[Egerer et al., PRD 106 (2022) 9, 094511]

- Clover, a=0.094 fm, m_π=358 MeV

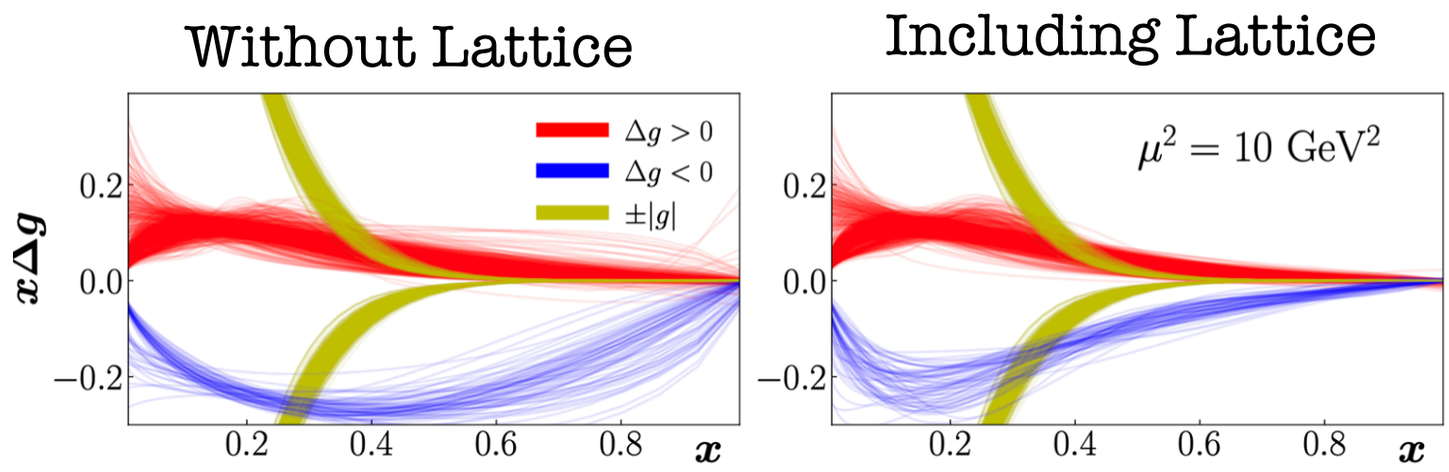


- Hint for nonzero gluon spin
- LQCD can distinguish between + and - solutions of global fits



- Grand challenge: statistical uncertainties
- Higher momenta needed

[Karpie et al., PRD 109 (2024) 3, 036031]



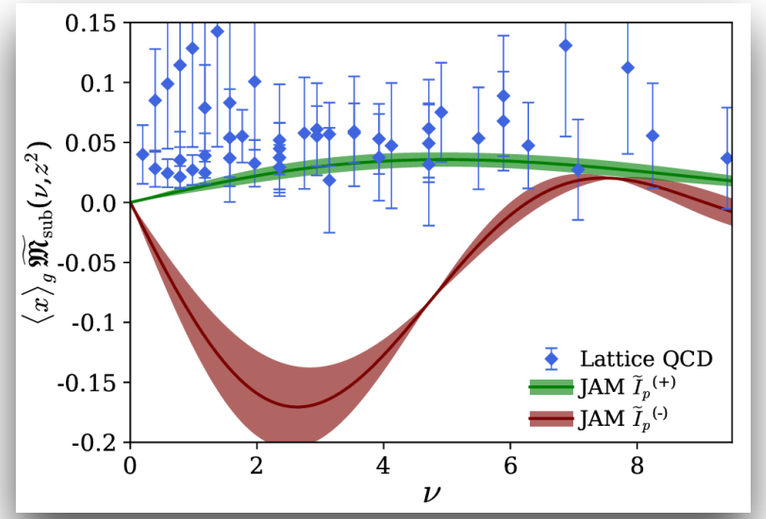
JAM analysis: No positivity constraint ($\Delta g > |g|$ for some regions of x)

Exploration of Gluon Helicity PDF

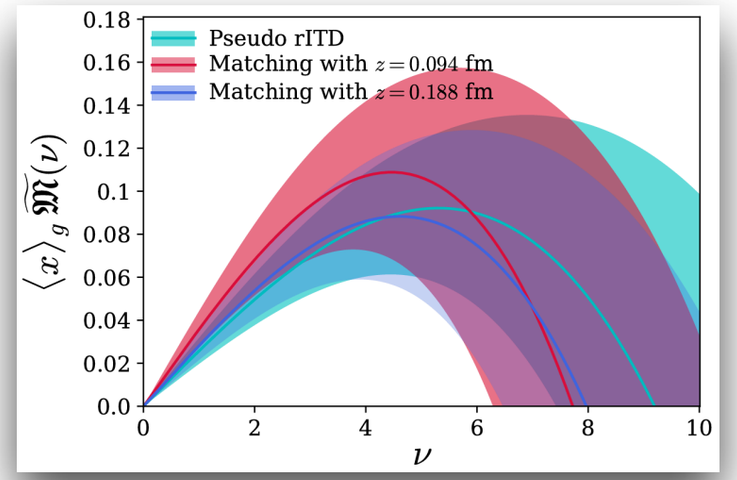
★ Positivity Constraints

[Egerer et al., PRD 106 (2022) 9, 094511]

- Clover, a=0.094 fm, m_π=358 MeV

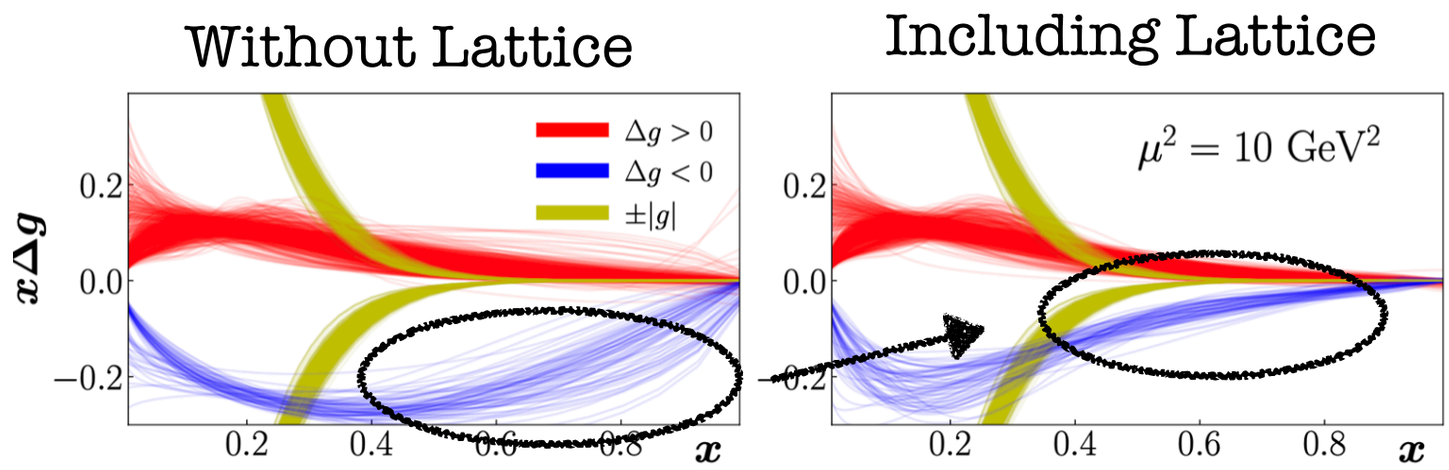


- Hint for nonzero gluon spin
- LQCD can distinguish between + and - solutions of global fits



- Grand challenge: statistical uncertainties
- Higher momenta needed

[Karpie et al., PRD 109 (2024) 3, 036031]



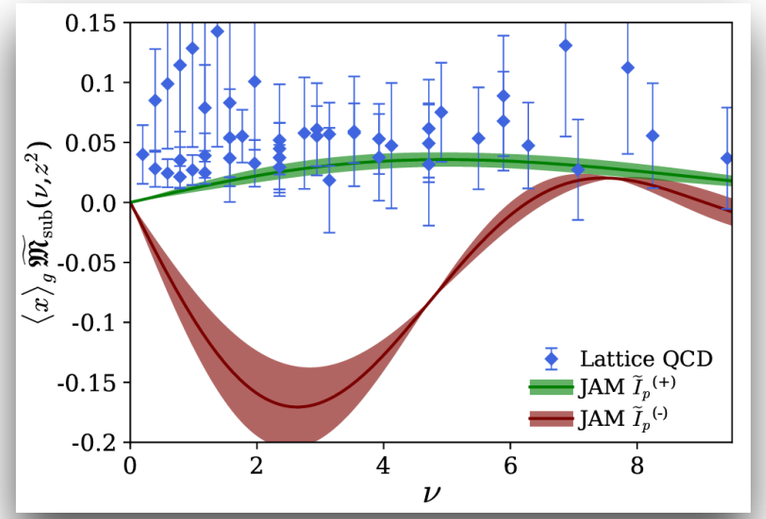
JAM analysis: No positivity constraint ($\Delta g > |g|$ for some regions of x)

Exploration of Gluon Helicity PDF

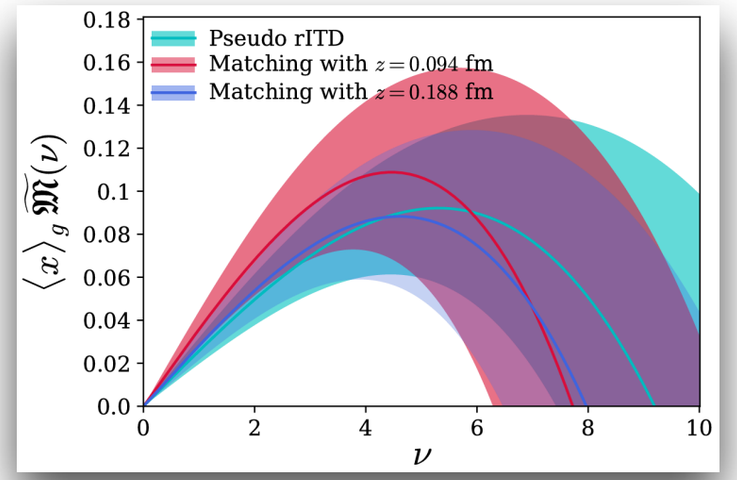
★ Positivity Constraints

[Egerer et al., PRD 106 (2022) 9, 094511]

- Clover, a=0.094 fm, m_π=358 MeV

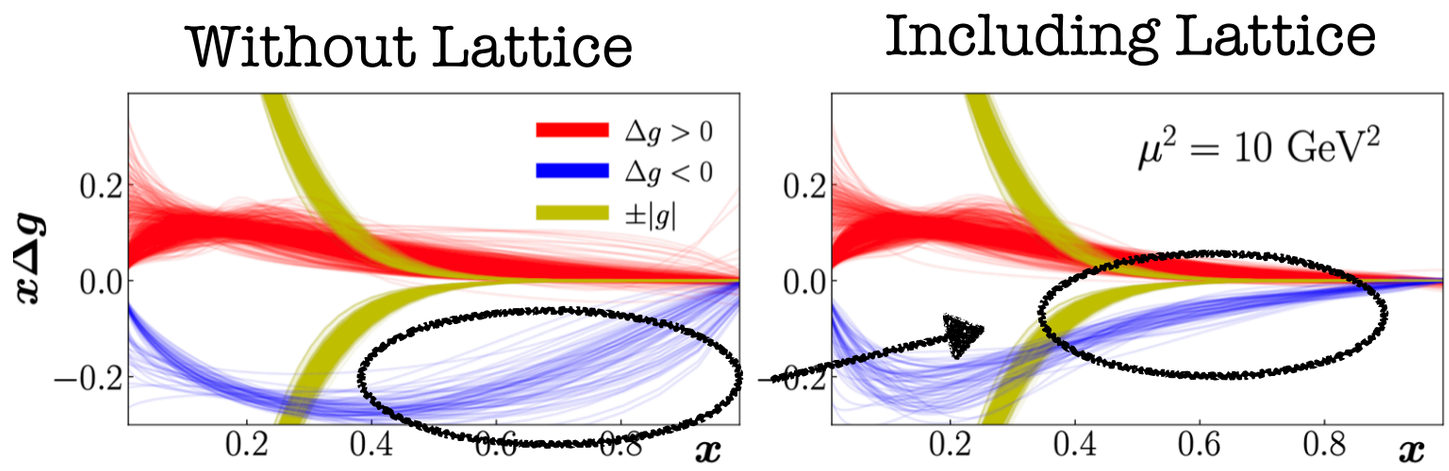


- Hint for nonzero gluon spin
- LQCD can distinguish between + and - solutions of global fits



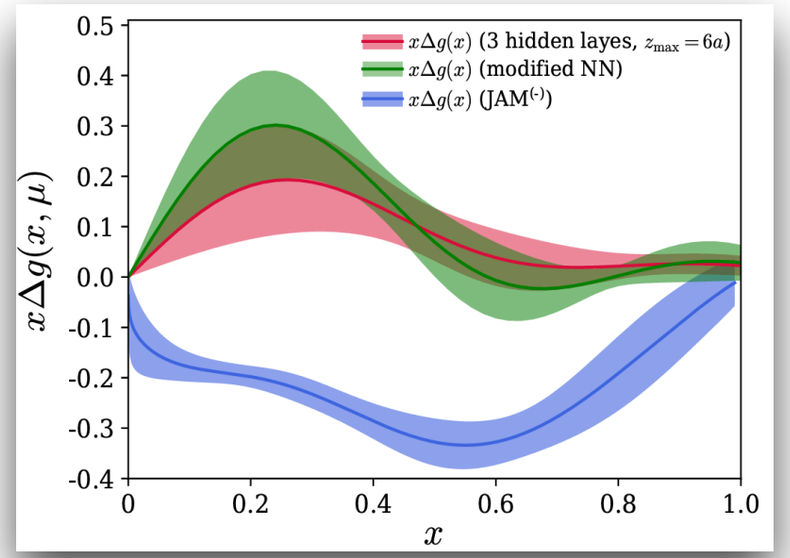
- Grand challenge: statistical uncertainties
- Higher momenta needed

[Karpie et al., PRD 109 (2024) 3, 036031]



JAM analysis: No positivity constraint ($\Delta g > |g|$ for some regions of x)

[Khan et al., PRD 108, 074502]



- Neural network analysis disfavors negative gluon polarizability

Twist-3 PDFs

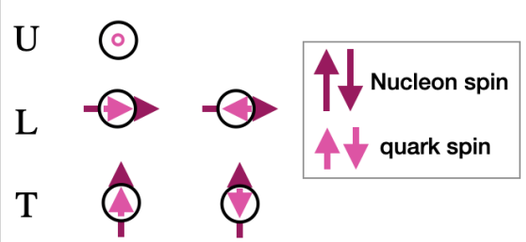
Twist-classification of PDFs, GPDs, TMDs

★ Twist: The order in Q^{-1} entering factorization

$$f_i = f_i^{(0)} + \frac{f_i^{(1)}}{Q} + \frac{f_i^{(2)}}{Q^2} \cdots$$

Twist-2 $(f_i^{(0)})$

Nucleon \ Quark	U (γ^+)	L ($\gamma^+\gamma^5$)	T (σ^{+j})
U	$H(x,\xi,t)$ $E(x,\xi,t)$ unpolarized		
L		$\widetilde{H}(x,\xi,t)$ $\widetilde{E}(x,\xi,t)$ helicity	
T			H_T, E_T $\widetilde{H}_T, \widetilde{E}_T$ transversity



(Selected) Twist-3 $(f_i^{(1)})$

Nucleon \ $\mathcal{O}$	γ^j	$\gamma^j\gamma^5$	σ^{jk}
U	G_1, G_2 G_3, G_4		
L		$\widetilde{G}_1, \widetilde{G}_2$ $\widetilde{G}_3, \widetilde{G}_4$	
T			$H_2'(x,\xi,t)$ $E_2'(x,\xi,t)$

★ Twist-2: probabilistic densities - a wealth of information exists (mostly on PDFs)

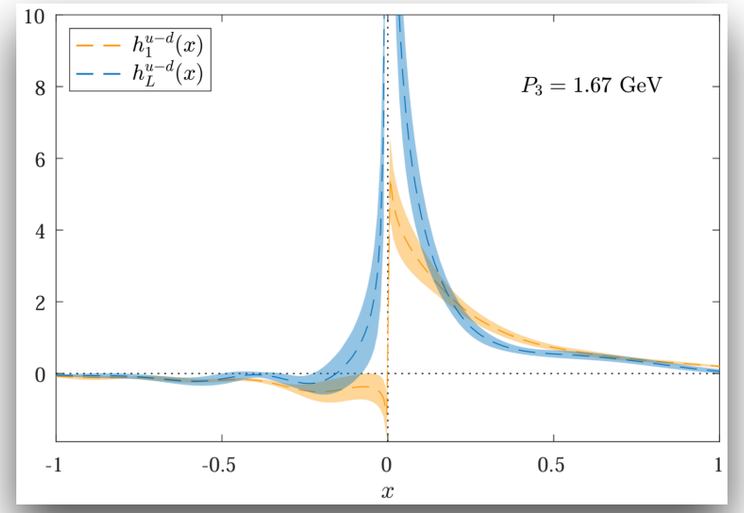
★ Twist-3: poorly known, but very important and have physical interpretation:
- as sizable as twist-2
- contain information about quark-gluon correlations inside hadrons
- appear in QCD factorization theorems for various observables (e.g. g_2)

The extraction of twist-3 is very challenges both experimentally and theoretically

Twist-3 $h_L(x)$ PDF

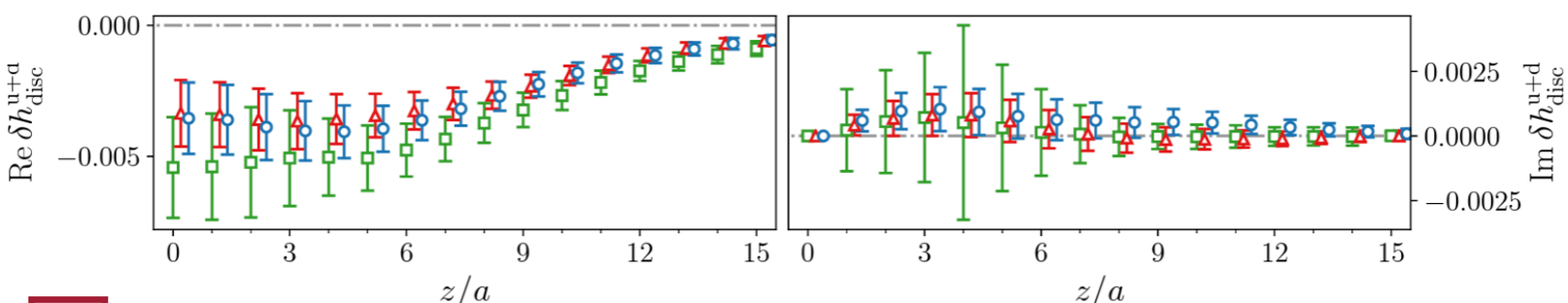
[Bhattacharya et al., PRD 104 (2021) 11, 114510]

- TM fermions: $m_\pi = 260$ MeV (a =0.093 fm)
- quasi-PDFs method: $P \sim 1.7$ GeV



- h_L as sizable as h_1

- Flavor decomposition:
  - similar operator to transversity PDF
  - no mixing (gluon transversity)
  - quark singlet tansversity PDF has negligible disconn. contributions



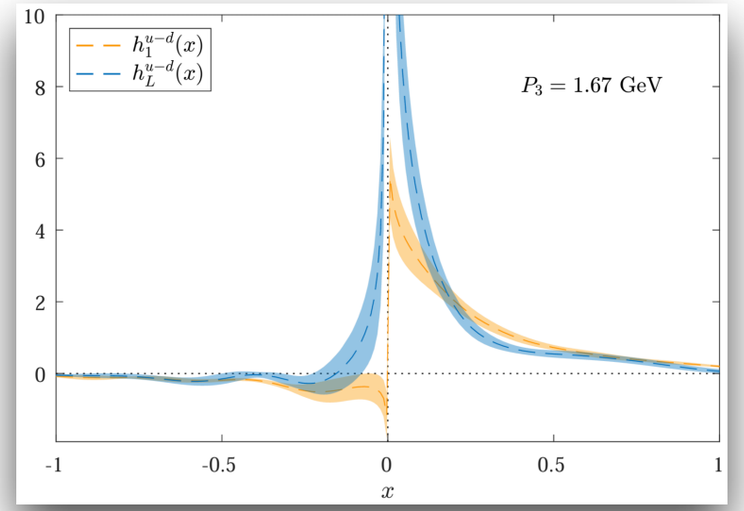
[Alexandrou et al., PRD 104 (2021) 5, 054503]

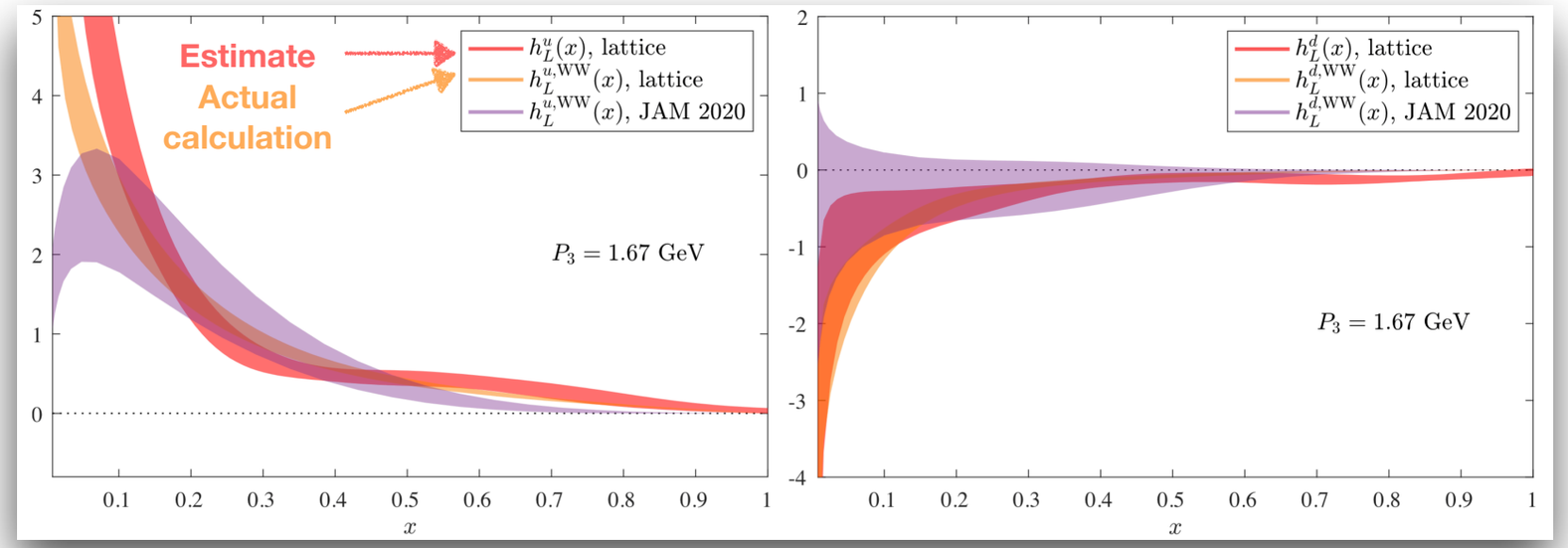
Twist-3 $h_L(x)$ PDF

[Bhattacharya et al., PRD 104 (2021) 11, 114510]

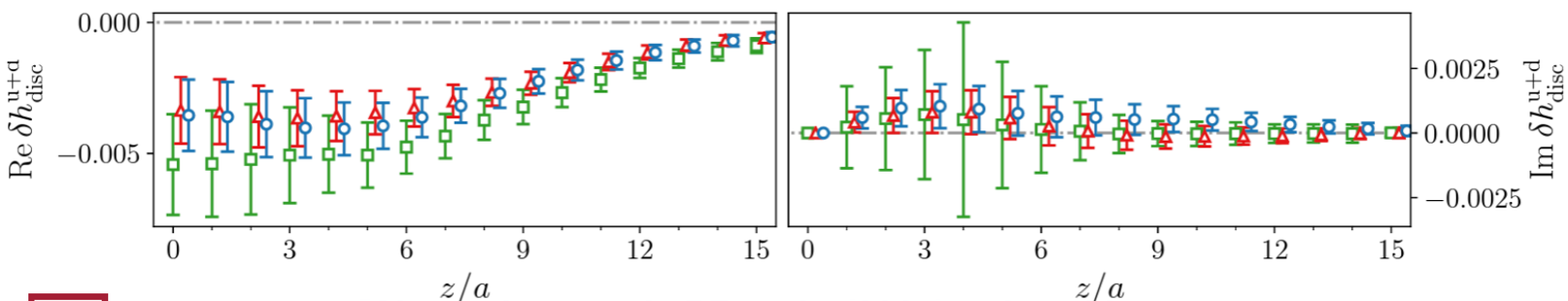
- TM fermions: $m_\pi = 260$ MeV (a =0.093 fm)
- quasi-PDFs method: $P \sim 1.7$ GeV
- Wandzura - Wilczek approximation

$$h_L^{\mathrm{WW}}(x) = 2x \int_x^1 dy \, \frac{h_1(y)}{y^2}$$





- h_L as sizable as h_1
- Flavor decomposition:
  - similar operator to transversity PDF
  - no mixing (gluon transversity)
  - quark singlet tansversity PDF has negligible disconn. contributions
- h_L^u dominant - tension between h_L & h_L^{WW}
- h_L^d <0 and decays faster then h_L^u
- Lattice data suggest that twist-3 $h_L(x)$ determined from twist-2 counterpart within uncertainties



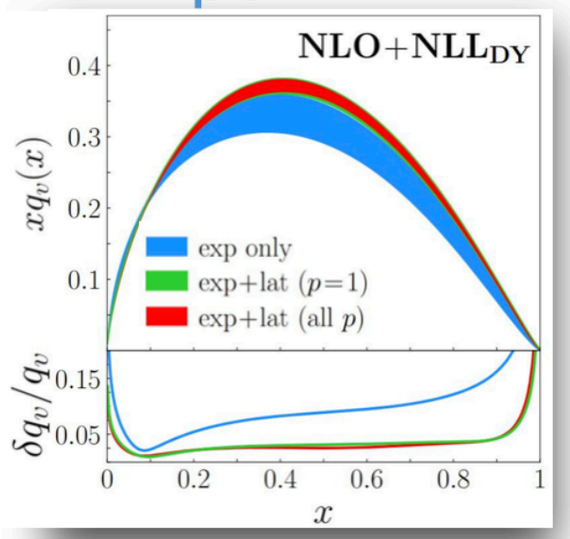
[Alexandrou et al., PRD 104 (2021) 5, 054503]

Synergy/Complementarity of lattice and phenomenology

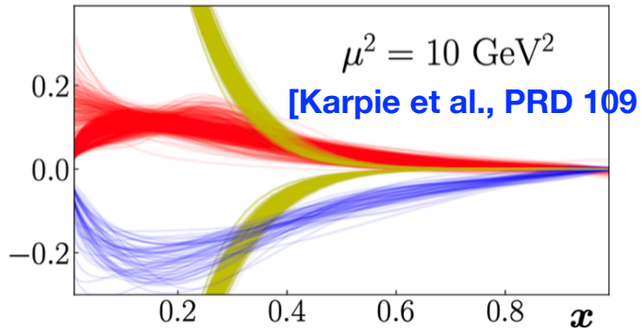
Beyond the scope of this talk

Synergies: constraints & predictive power of lattice QCD

pion PDF

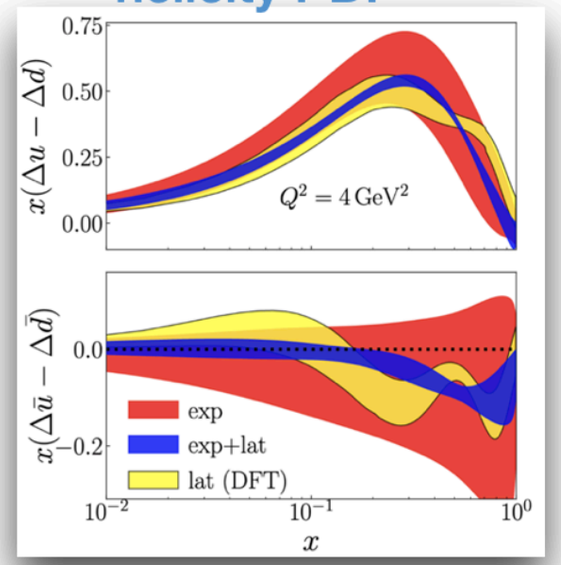


[JAM/HadStruc, PRD105 (2022) 114051]



[Karpie et al., PRD 109 (2024) 3, 036031]

helicity PDF



[JAM & ETMC, PRD 103 (2021) 016003]

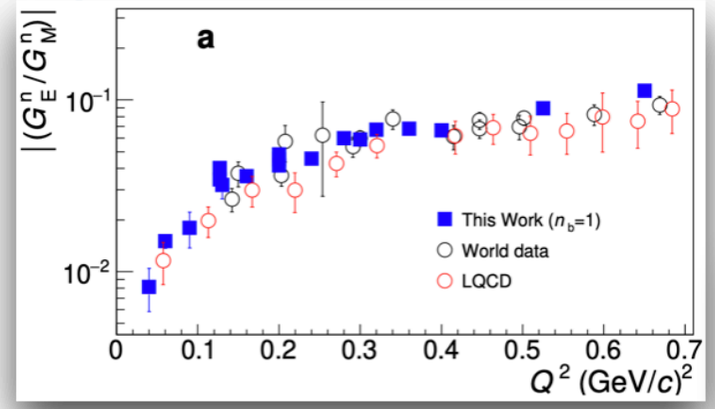
Lattice

Theory, pheno-menology

QCD

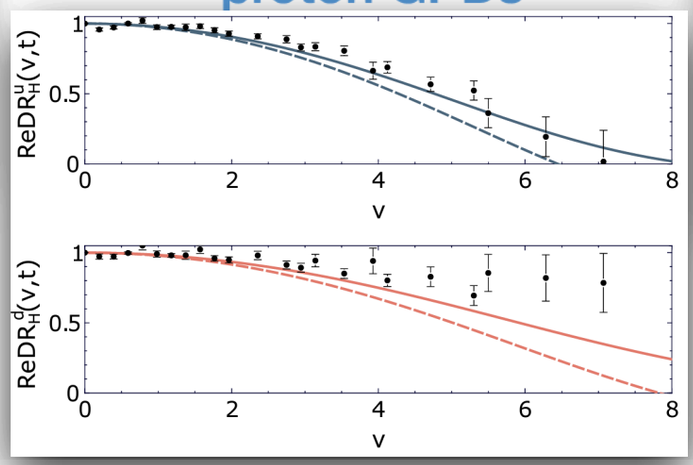
Experiments, global analysis

proton & neutron radius



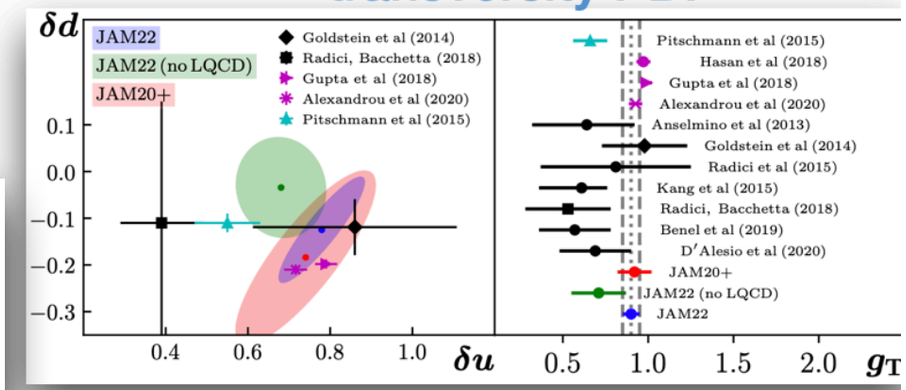
[Atac et al., Nature Comm. 12, 1759 (2021)]

proton GPDs

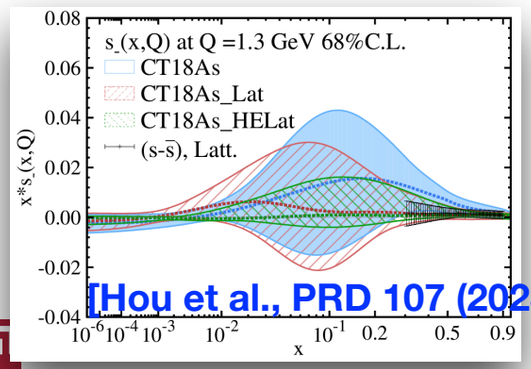


[Cichy et al., PRD (2024), arXiv:2409.17955]

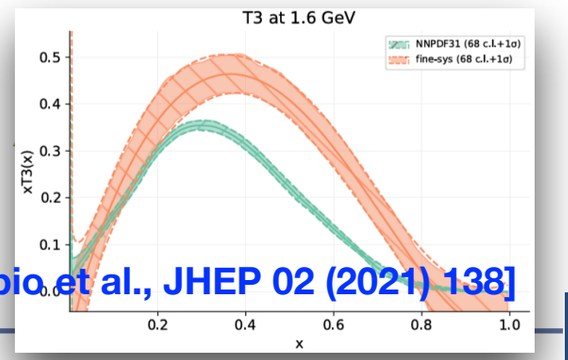
transversity PDF



[JAM, PRD 106 (2022) 3, 034014]



[Hou et al., PRD 107 (2023) 7, 076018]



[Del Debbio et al., JHEP 02 (2021) 138]

Synergy of lattice, theory, phenomenology in the horizon

Synergy of lattice, theory, phenomenology in the horizon

QUARK-GLUON TOMOGRAPHY COLLABORATION

U.S. DEPARTMENT OF ENERGY | Office of Science

Award Number: DE-SC0023646

1. **Theoretical studies** of high-momentum transfer processes using perturbative QCD methods and study of GPDs properties
2. **Lattice QCD** calculations of GPDs and related structures
3. **Global analysis** of GPDs based on experimental data using modern data analysis techniques for inference and uncertainty quantification

Synergy of lattice, theory, phenomenology in the horizon

QUARK-GLUON TOMOGRAPHY COLLABORATION

U.S. DEPARTMENT OF ENERGY | Office of Science

Award Number: DE-SC0023646

1. **Theoretical studies** of high-momentum transfer processes using perturbative QCD methods and study of GPDs properties
2. **Lattice QCD** calculations of GPDs and related structures
3. **Global analysis** of GPDs based on experimental data using modern data analysis techniques for inference and uncertainty quantification

Other GPD global analysis efforts:

- Gepard [https://gepard.phy.hr/]
- PARTONS [https://partons.cea.fr]
- EXCLAIM [https://exclaimcollab.github.io/web.github.io/#/]

Concluding remarks

Concluding Remarks

- ★ Impressive progress in the extraction of PDFs from Lattice QCD
- ★ New Developments in several promising directions:
  DA, GPDs, TMDs
- ★ Several sources of systematic uncertainties are being investigated but there are many more
- ★ Synergy with phenomenology has the potential to enhance the impact of lattice QCD data and complement data sets

Concluding Remarks

- ★ Impressive progress in the extraction of PDFs from Lattice QCD
- ★ New Developments in several promising directions:
  DA, GPDs, TMDs
- ★ Several sources of systematic uncertainties are being investigated but there are many more
- ★ Synergy with phenomenology has the potential to enhance the impact of lattice QCD data and complement data sets

Thank you

U.S. DEPARTMENT OF ENERGY | Office of Science

DOE Early Career Award (NP)
Grant No. DE-SC0020405
& Grant No. DE-SC0025218

U.S. DEPARTMENT OF ENERGY | Office of Science
QUARK-GLUON TOMOGRAPHY COLLABORATION

Award Number:
DE-SC0023646