

Synergies between QCD global analysis and LQCD JAM examples

Nobuo Sato



Parton Distributions and Lattice
Calculations (PDFLattice 2024)
Jefferson Lab, Nov 18, 2024

Jefferson Lab

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Outline

1. Gluon helicity PDF
2. Nucleon transversity/tensorcharge
3. Pion PDF

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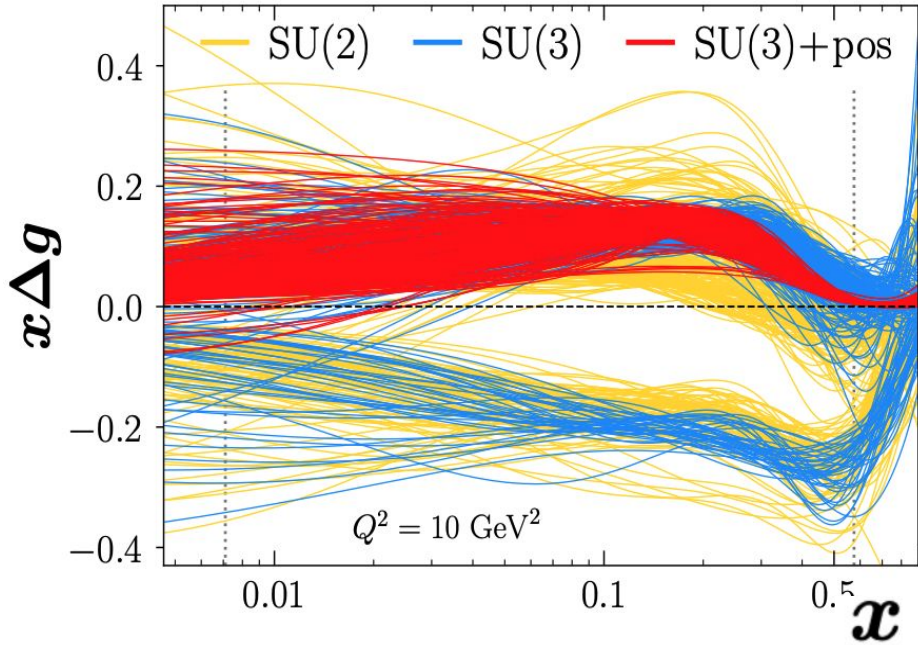


Zhou, NS, Melnitchouk '22

Karpie, Whitehill, Melnitchouk, Monahan, Orginos, Qiu, Richards, NS, Zafeiropoulos '23

Hunt-Smith, Cocuzza, Melnitchouk, NS, Thomas, White '24

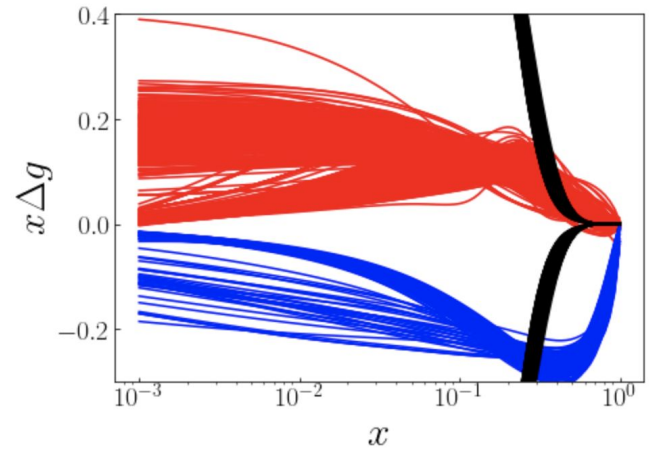
Zhou, Melnitchouk, NS '21

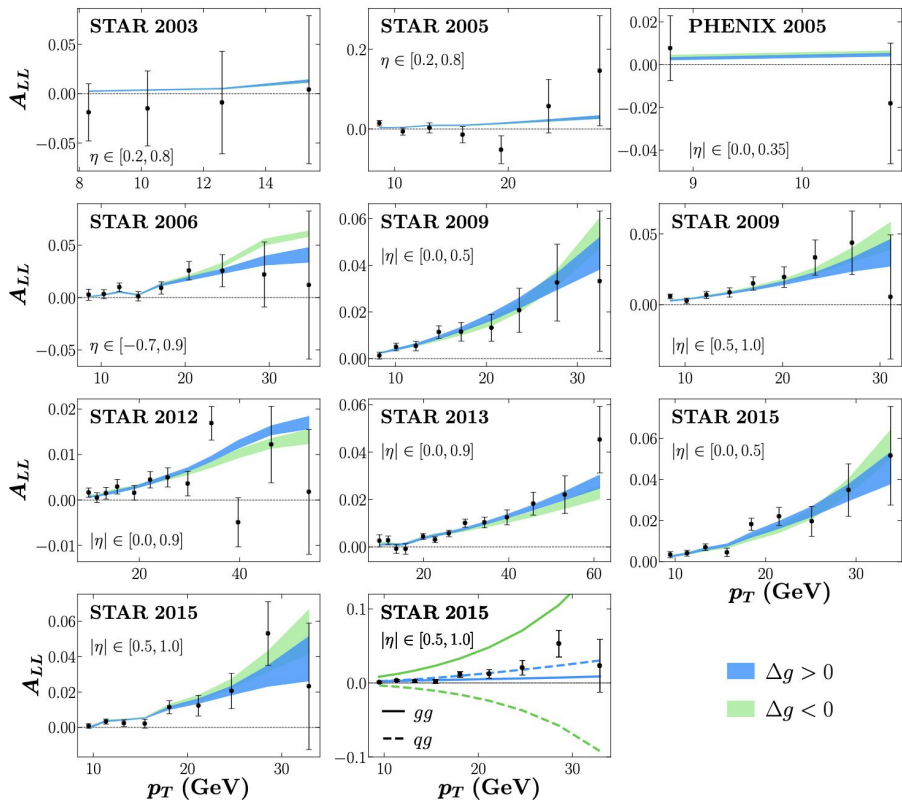


$$|\Delta g| < g$$

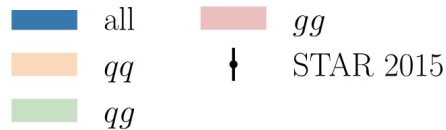
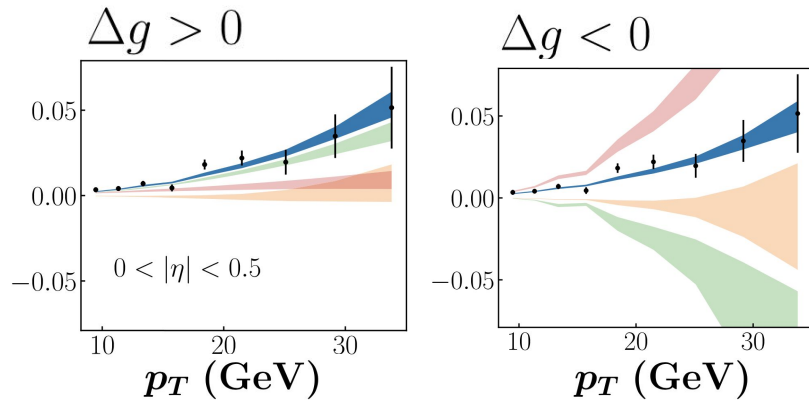
PDF positivity constraint

- Sign of gluon-hpdf is not uniquely determined by existing experimental data (DIS $W^2 > 10 \text{ GeV}^2$)
- PDF positivity constraints + data strongly disfavors negative g-hpdf
- Negative g-hpdf violates significantly pdf positivity constraint
- PDF positivity is not a strict requirement in QCD





$$|A_{LL}| < 1 \quad \checkmark$$



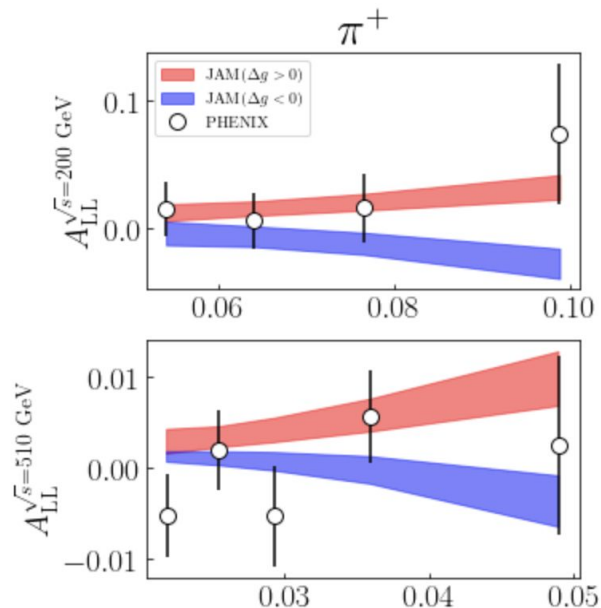
$$\begin{aligned}
 A_{LL}^{\text{jet}}(p_T, y) \propto & a_{gg}[\Delta g \otimes \Delta g] + \sum_q a_{qq}[\Delta q \otimes \Delta g] \\
 & + \sum_{q, q'} a_{qq'}[\Delta q \otimes \Delta q'] + \mathcal{O}(\alpha_s),
 \end{aligned}$$

g-hpdf enters quadratically, and different subchannels contribute with different signs and strengths

Measurement of charged pion double spin asymmetries at midrapidity in longitudinally polarized $p + p$ collisions at $\sqrt{s} = 510$ GeV

PHENIX Collaboration • U.A. Acharya (Georgia State U.) et al. (Apr 6, 2020)

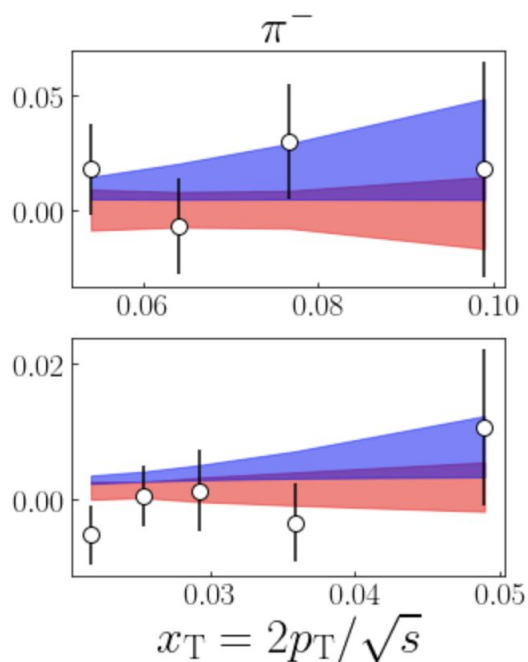
Published in: *Phys.Rev.D* 102 (2020) 3, 032001 • e-Print: [2004.02681](https://arxiv.org/abs/2004.02681) [hep-ex]



Charged-pion cross sections and double-helicity asymmetries in polarized p+p collisions at $\sqrt{s}=200$ GeV

PHENIX Collaboration • A. Adare (Colorado U.) et al. (Sep 5, 2014)

Published in: *Phys.Rev.D* 91 (2015) 3, 032001 • e-Print: [1409.1907](https://arxiv.org/abs/1409.1907) [hep-ex]

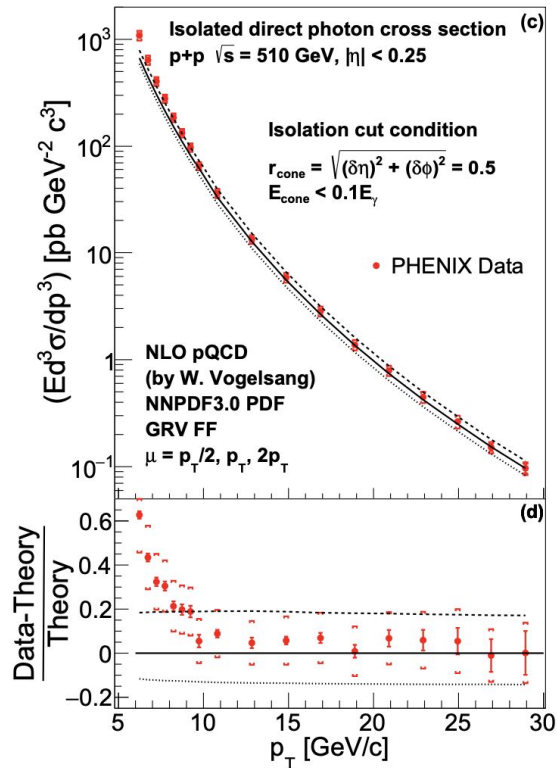
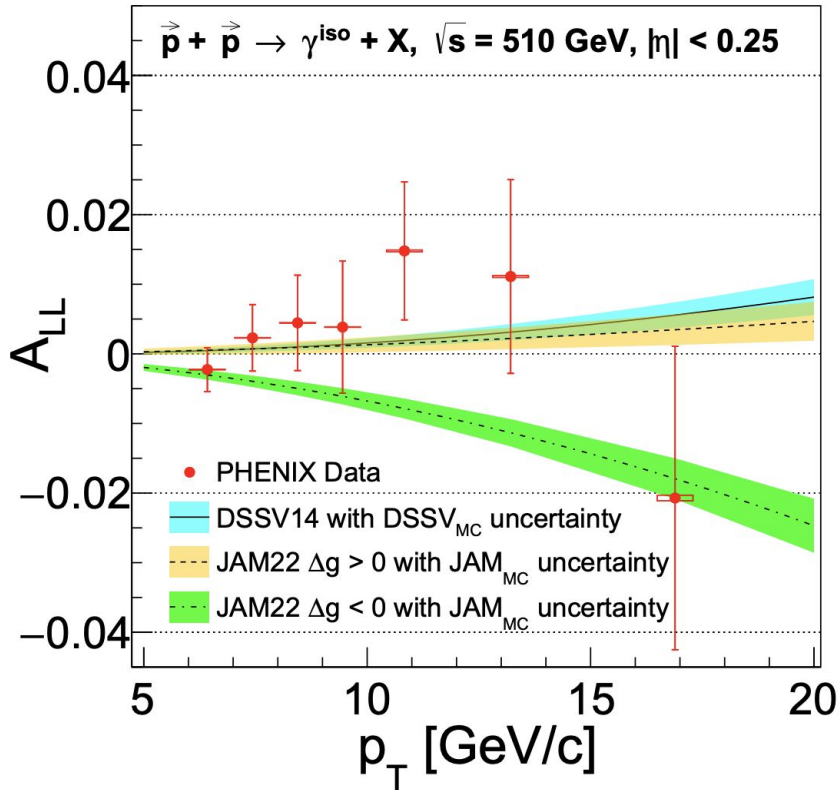


- PHENIX collaboration stated that the gluon spin contribution is positive
- The two solutions for g-hpdf found by JAM describe the data equally well

Measurement of Direct-Photon Cross Section and Double-Helicity Asymmetry at $\sqrt{s} = 510$ GeV in $\vec{p} + \vec{p}$ Collisions

PHENIX Collaboration • U. Acharya (Georgia State U., Atlanta) et al. (Feb 16, 2022)

e-Print: [2202.08158](https://arxiv.org/abs/2202.08158) [hep-ex]

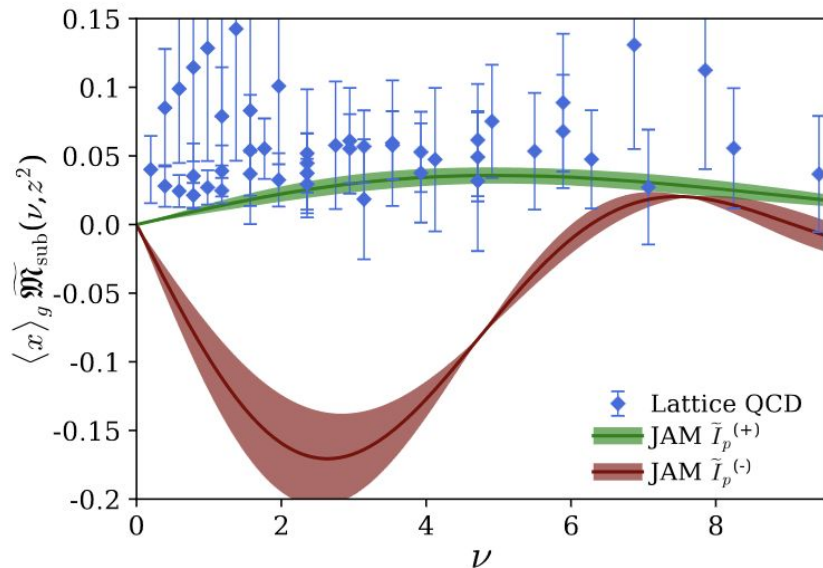


- PHENIX collaboration stated that negative g-hpdf is disfavored by more than 2.8σ
- However, only last 3 high- p_T A_{LL} points are well described in pQCD (see denominator of A_{LL})

Toward the determination of the gluon helicity distribution in the nucleon from lattice quantum chromodynamics

HadStruc Collaboration • Colin Egerer (Jefferson Lab) et al. (Jul 18, 2022)

Published in: *Phys.Rev.D* 106 (2022) 9, 094511 • e-Print: [2207.08733](https://arxiv.org/abs/2207.08733) [hep-lat]

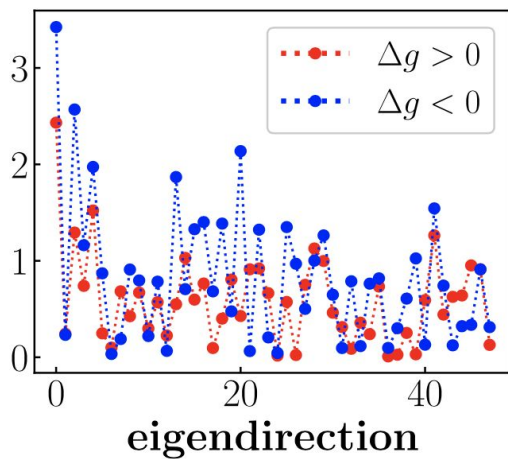
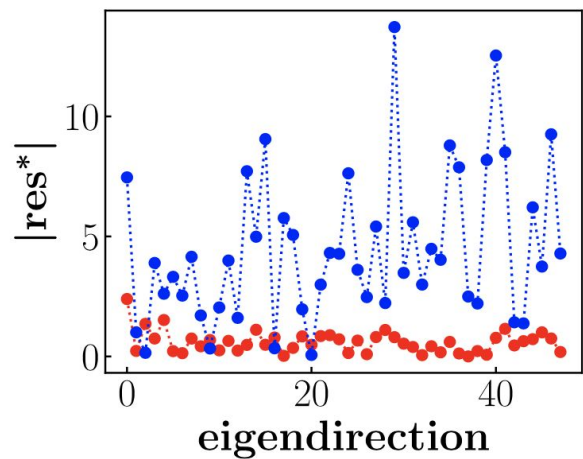
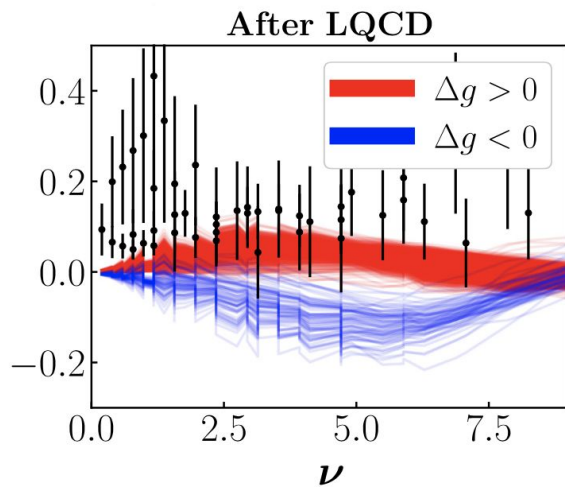
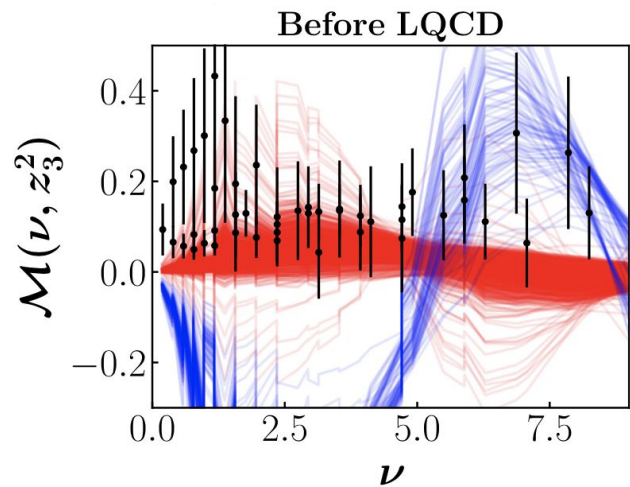


$$\widetilde{M}^{\mu\nu;\alpha\beta}(p, z) = \langle p | F^{\mu\nu}(0) W(0; z) \widetilde{F}^{\alpha\beta}(z) | p \rangle$$

$$\widetilde{\mathfrak{M}}(\nu, z^2) = \frac{\widetilde{M}_{00}(p, z)/p_0 p_3 Z_L(z_3/a)}{M_{00}(p=0, z)/m^2}$$

$$\begin{aligned} \widetilde{\mathfrak{M}}(\nu, z^2) \langle x_g \rangle_{\mu^2} = & \tilde{\mathcal{I}}_p(\nu, \mu^2) - \frac{\alpha_s N_c}{2\pi} \int_0^1 du \tilde{\mathcal{I}}_p(u\nu, \mu^2) \left\{ \ln \left(z^2 \mu^2 \frac{e^{2\gamma_E}}{4} \right) \right. \\ & \left(\left[\frac{2u^2}{\bar{u}} + 4u\bar{u} \right]_+ - \left(\frac{1}{2} + \frac{4}{3} \frac{\langle x_S \rangle_{\mu^2}}{\langle x_g \rangle_{\mu^2}} \right) \delta(\bar{u}) \right) \\ & + 4 \left[\frac{u + \ln(1-u)}{\bar{u}} \right]_+ - \left(\frac{1}{\bar{u}} - \bar{u} \right)_+ - \frac{1}{2} \delta(\bar{u}) + 2\bar{u}u \left. \right\} \\ & - \frac{\alpha_s C_F}{2\pi} \int_0^1 du \tilde{\mathcal{I}}_S(u\nu, \mu^2) \left\{ \ln \left(z^2 \mu^2 \frac{e^{2\gamma_E}}{4} \right) \tilde{\mathcal{B}}_{gq}(u) + 2\bar{u}u \right\} + \mathcal{O}(\Lambda_{\text{QCD}}^2 z^2), \end{aligned}$$

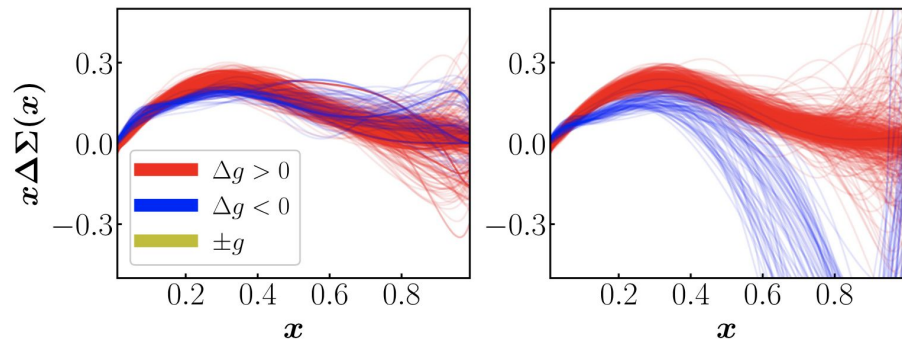
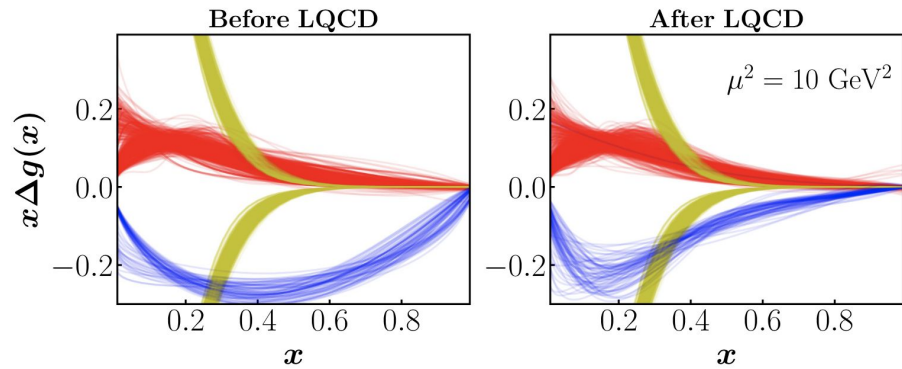
$$\tilde{\mathcal{I}}_p(\nu) = \frac{i}{2} \int_{-1}^1 dx e^{-ix\nu} x \Delta g(x).$$



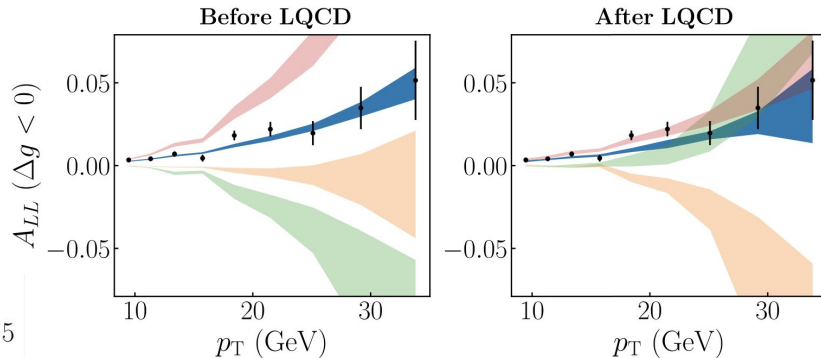
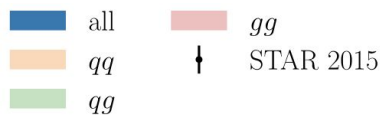
$$\begin{aligned}\chi^2 &= (\mathbf{d} - \mathbf{t})^T \boldsymbol{\Sigma}^{-1} (\mathbf{d} - \mathbf{t}) \\ &= (\mathbf{d} - \mathbf{t})^T \mathbf{U} \mathbf{D}^{-1} \mathbf{U}^T (\mathbf{d} - \mathbf{t}) \\ &= \sum_i \text{res}_i^{*2}.\end{aligned}$$



- PCA projections of residuals reveal strong correlations between LQCD data points
- The correlations prevent determination of g-hpdf sign

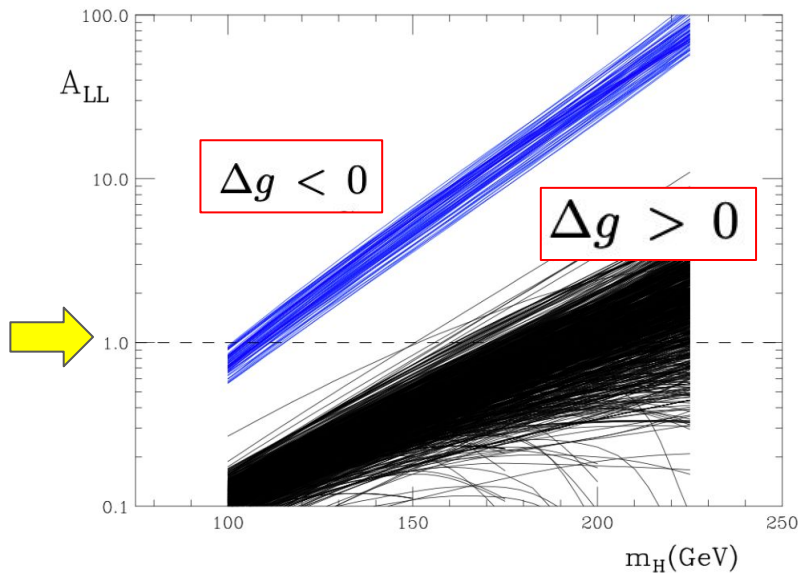


- LQCD distorts significantly the negative g-hpdf at higher $x > 0.3$
- Note that both solutions violate pdf positivity bounds for $x > 0.3$
- Before inclusion of LQCD data, singlet-hpdf were stable for both solutions
- Inclusion of LQCD data forces the quark singlet-hpdf to become negative at $x > 0.4$ for the negative g-hpdf



Higgs production at RHIC and the positivity of the gluon helicity distribution

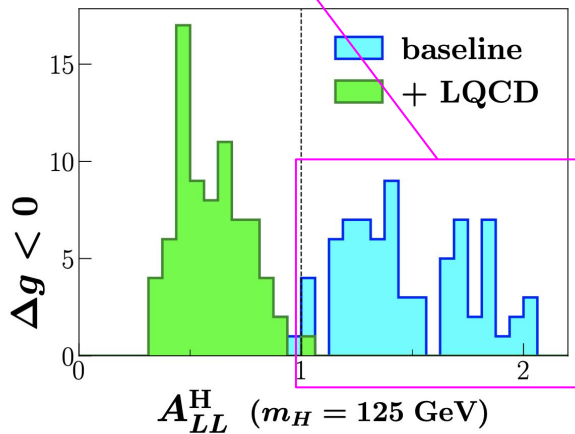
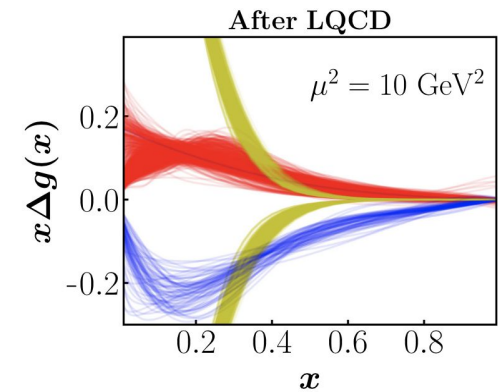
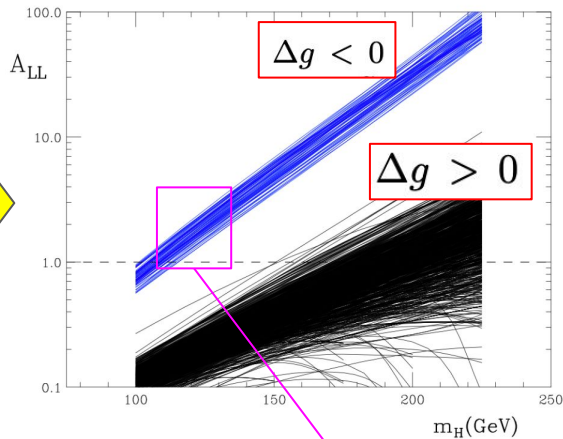
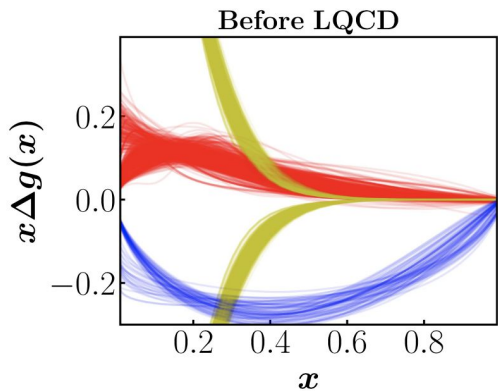
Daniel de Florian^a, Stefano Forte^b, Werner Vogelsang^c



- Higgs A_{LL} is directly sensitive to g-hpdf squared at LO
- Calculations of $A_{LL}(H)$ with negative g-hpdf can lead to unphysical results

$$A_{LL}^H(\tau) = \frac{[\Delta g \otimes \Delta g]}{[g \otimes g]} + \mathcal{O}(\alpha_s),$$

Can Higgs A_{LL} fully discriminate negative g -hpdf?



Negative g -hpdf with LQCD constraints still admits a physical Higgs A_{LL}

New Data-Driven Constraints on the Sign of Gluon Polarization in the Proton

[N. T. Hunt-Smith](#)¹, [C. Cocuzza](#)^{1,2}, [W. Melnitchouk](#)³, [N. Sato](#)³, [A. W. Thomas](#)¹, and [M. J. White](#)¹ (JAM Collaboration-Spin PDF Analysis Group)

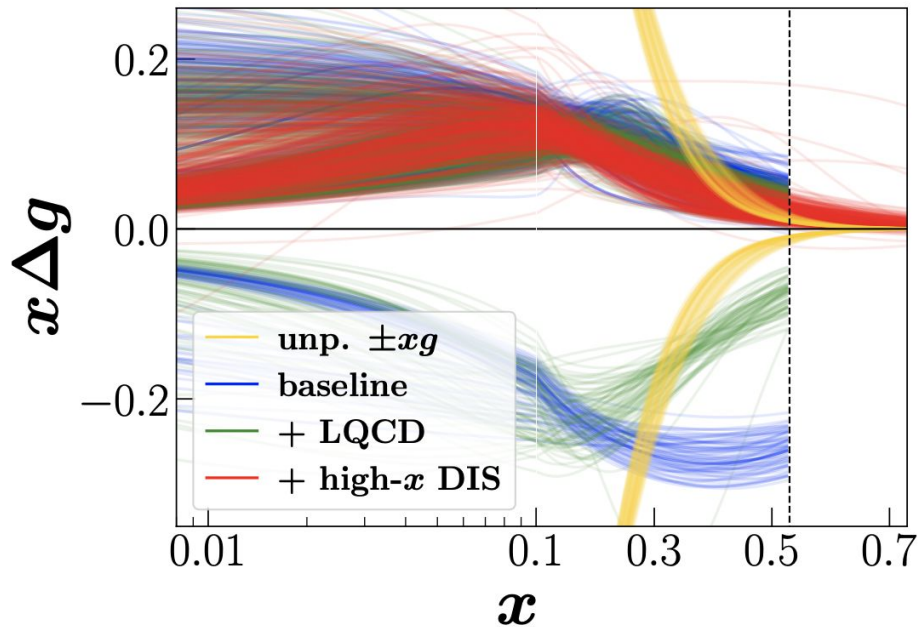
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1370 additional data points for pol DIS (+ high-x DIS)

Reaction	$\chi_{\text{red}}^2(\Delta g > 0)$			$\chi_{\text{red}}^2(\Delta g < 0)$			N
	baseline	+ LQCD	+ high- x DIS	baseline	+ LQCD	+ high- x DIS	
<i>Polarized</i>							
Inclusive DIS	0.95	0.96	1.21	0.98	1.12	1.25	1735*
SIDIS	0.85	0.84	1.08	0.84	0.96	1.11	231
Inclusive jets	0.84	0.89	0.90	0.88	1.10	1.44	83
Inclusive W^\pm/Z	0.60	0.60	0.99	0.83	0.84	1.32	18
<i>Total</i>	0.89	0.90	1.18	0.92	1.06	1.24	2067
<i>Unpolarized</i>							
Inclusive DIS	1.17	1.17	1.17	1.18	1.18	1.19	3908
SIDIS	0.99	0.99	1.04	0.99	0.99	1.02	1490
Inclusive jets	1.28	1.28	1.30	1.29	1.29	1.30	198
Drell-Yan	1.21	1.21	1.21	1.24	1.24	1.24	205
Inclusive W^\pm/Z	1.01	1.01	1.01	1.03	1.03	1.04	153
<i>Total</i>	1.14	1.14	1.14	1.15	1.15	1.15	5954
SIA	0.86	0.86	0.89	0.90	0.90	0.92	564
LQCD	—	0.57	0.58	—	1.18	3.92	48
<i>Total</i>	1.08	1.10	1.13	1.10	1.12	1.17	8633

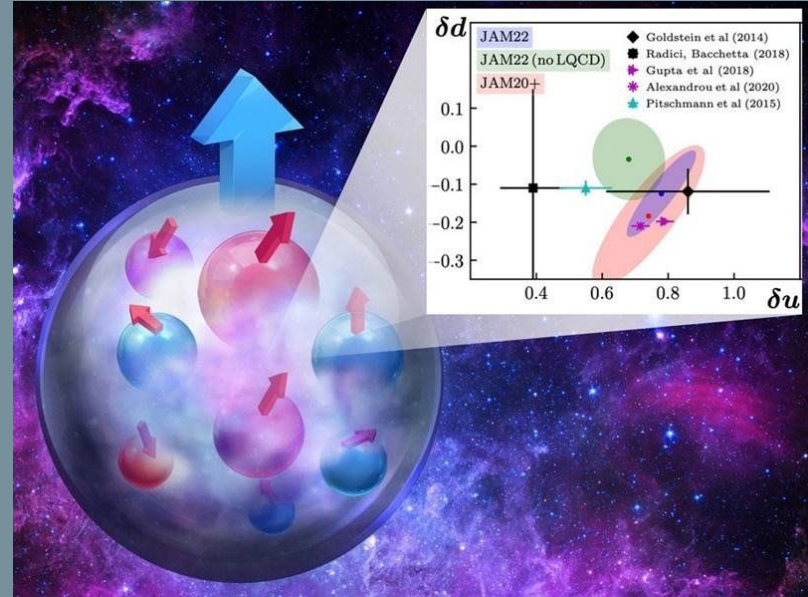
Takeaways

- For the first time, we were able to discriminate the sign of g-hpdf using data-driven approach
- Constraints from LQCD along with DSAs from jets and DIS at large-x were crucial to achieve the resolution of g-hpdf sign
- Inclusion of LQCD is becoming increasingly important in global analysis
- Experimental constraints at large x on gluon - hpdf are still scarce, and more data needed to reach precision similar to unpolarized gluon density (EIC - small x, JLab12/22 - high x)

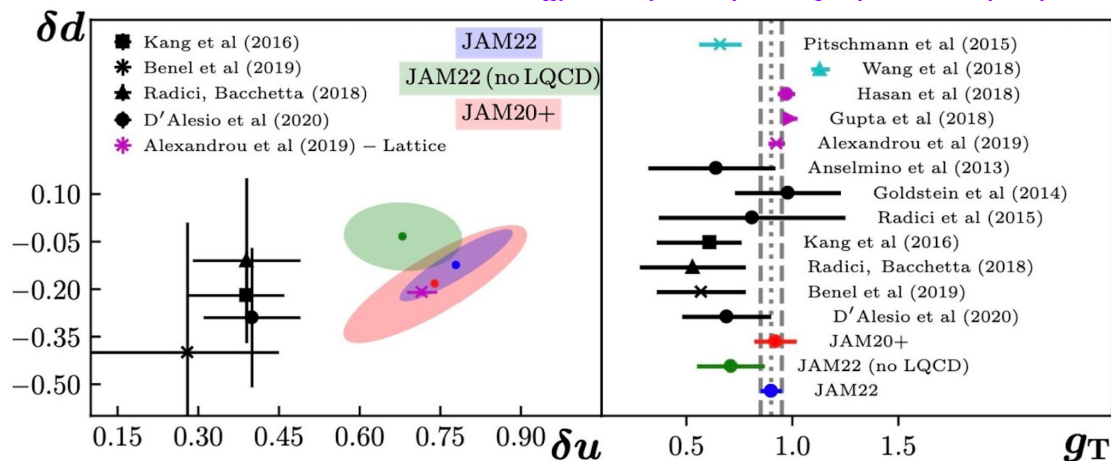


Outline

1. Gluon helicity PDF
2. **Nucleon transversity/tensorcharge**
3. Pion PDF



Pitonyak, Cocuzza, Metz, Prokudin, NS, '24 (PRL)
Cocuzza, Metz, Pitonyak, Prokudin, NS, Seidl '24 (PRL)
Cocuzza, Metz, Pitonyak, Prokudin, NS, Seidl '24 (PRD)



- TMD+CT3 pheno in tension with other analyzes (δu)
- Radici, Bacchetta, and Benel, Courtoy, Ferro-Hernandez used collinear di-hadron observables to extract tensor charges
- **New fresh look at collinear di-hadron pheno**

Observable	Reactions	Non-Perturbative Function(s)	χ^2/npts	Exp. Refs.
$A_{UT}^{\sin(\phi_h - \phi_S)}$	$e + (p, d)^\uparrow \rightarrow e + (\pi^+, \pi^-, \pi^0) + X$	$f_{1T}^\perp(x, \vec{k}_T^2)$	182.9/166 = 1.10	[22, 24, 27]
$A_{UT}^{\sin(\phi_h + \phi_S)}$	$e + (p, d)^\uparrow \rightarrow e + (\pi^+, \pi^-, \pi^0) + X$	$h_1(x, \vec{k}_T^2), H_1^\perp(z, z^2 \vec{p}_T^2)$	181.0/166 = 1.09	[22, 24, 27]
$*A_{UT}^{\sin \phi_S}$	$e + p^\uparrow \rightarrow e + (\pi^+, \pi^-, \pi^0) + X$	$h_1(x), \tilde{H}(z)$	18.6/36 = 0.52	[22, 24, 27]
$A_{UC/UL}$	$e^+ + e^- \rightarrow \pi^+ \pi^- (UC, UL) + X$	$H_1^\perp(z, z^2 \vec{p}_T^2)$	154.9/176 = 0.88	[29–32]
$A_{T, \mu^+ \mu^-}^{\sin \phi_S}$	$\pi^- + p^\uparrow \rightarrow \mu^+ \mu^- + X$	$f_{1T}^\perp(x, \vec{k}_T^2)$	6.92/12 = 0.58	[34]
$A_N^{W/Z}$	$p^\uparrow + p \rightarrow (W^+, W^-, Z) + X$	$f_{1T}^\perp(x, \vec{k}_T^2)$	30.8/17 = 1.81	[35]
A_N^π	$p^\uparrow + p \rightarrow (\pi^+, \pi^-, \pi^0) + X$	$h_1(x), F_{FT}(x, x) = \frac{1}{\pi} f_{1T}^{\perp(1)}(x), H_1^{\perp(1)}(z), \tilde{H}(z)$	70.4/60 = 1.17	[7, 9, 10, 13]
Lattice g_T	—	$h_1(x)$	1.82/1 = 1.82	[89]

Collaboration	References	Observable	Process	Nonperturbative function(s)
Belle	[64]	$d\sigma/dz dM_h$	$e^+e^- \rightarrow (\pi^+\pi^-)X$	D_1
Belle	[112]	$A^{e^+e^-}$	$e^+e^- \rightarrow (\pi^+\pi^-)(\pi^+\pi^-)X$	D_1, H_1^{\triangleleft}
HERMES	[118]	A_{UT}^{SIDIS}	$ep^\uparrow \rightarrow e'(\pi^+\pi^-)X$	$D_1, H_1^{\triangleleft}, h_1$
COMPASS	[117]	A_{UT}^{SIDIS}	$\mu\{p, D\}^\uparrow \rightarrow \mu'(\pi^+\pi^-)X$	$D_1, H_1^{\triangleleft}, h_1$
STAR	[97,121]	A_{UT}^{pp}	$p^\uparrow p \rightarrow (\pi^+\pi^-)X$	$D_1, H_1^{\triangleleft}, h_1$
ETMC	[77]	$\delta u, \delta d$	LQCD	h_1
PNDME	[71]	$\delta u, \delta d$	LQCD	h_1

$$\frac{d\sigma}{dz dM_h} = \frac{4\pi N_c \alpha_{\text{em}}^2}{3s} \sum_q \bar{e}_q^2 D_1^q(z, M_h)$$

 $h_1(x; \mu^2)$ Transversity (TPDF)

$$A^{e^+e^-}(z, M_h, \bar{z}, \bar{M}_h) = \frac{\sin^2 \theta \sum_q e_q^2 H_1^{\triangleleft, q}(z, M_h) H_1^{\triangleleft, \bar{q}}(\bar{z}, \bar{M}_h)}{(1 + \cos^2 \theta) \sum_q e_q^2 D_1^q(z, M_h) D_1^{\bar{q}}(\bar{z}, \bar{M}_h)}$$

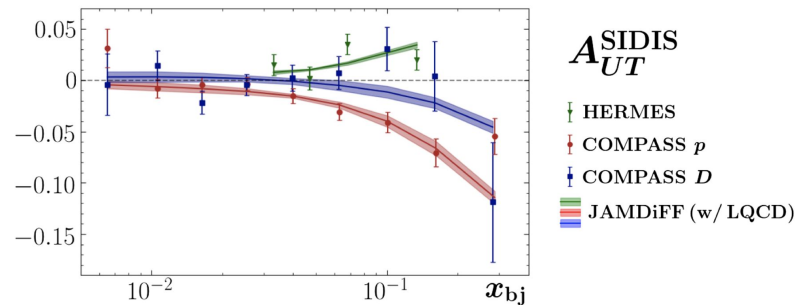
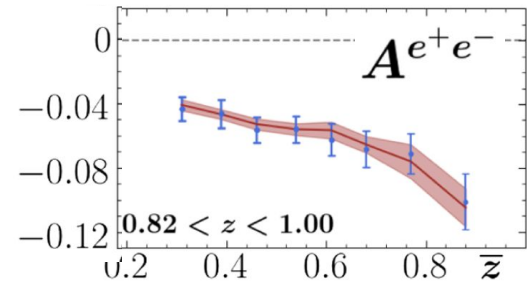
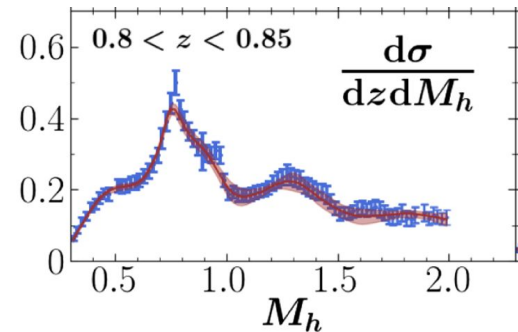
$H_1^{\triangleleft}(z, M_h; \mu^2)$ Interference FF (IFF)

$D_1(z, M_h; \mu^2)$ Dihadron FF (DiFFs)

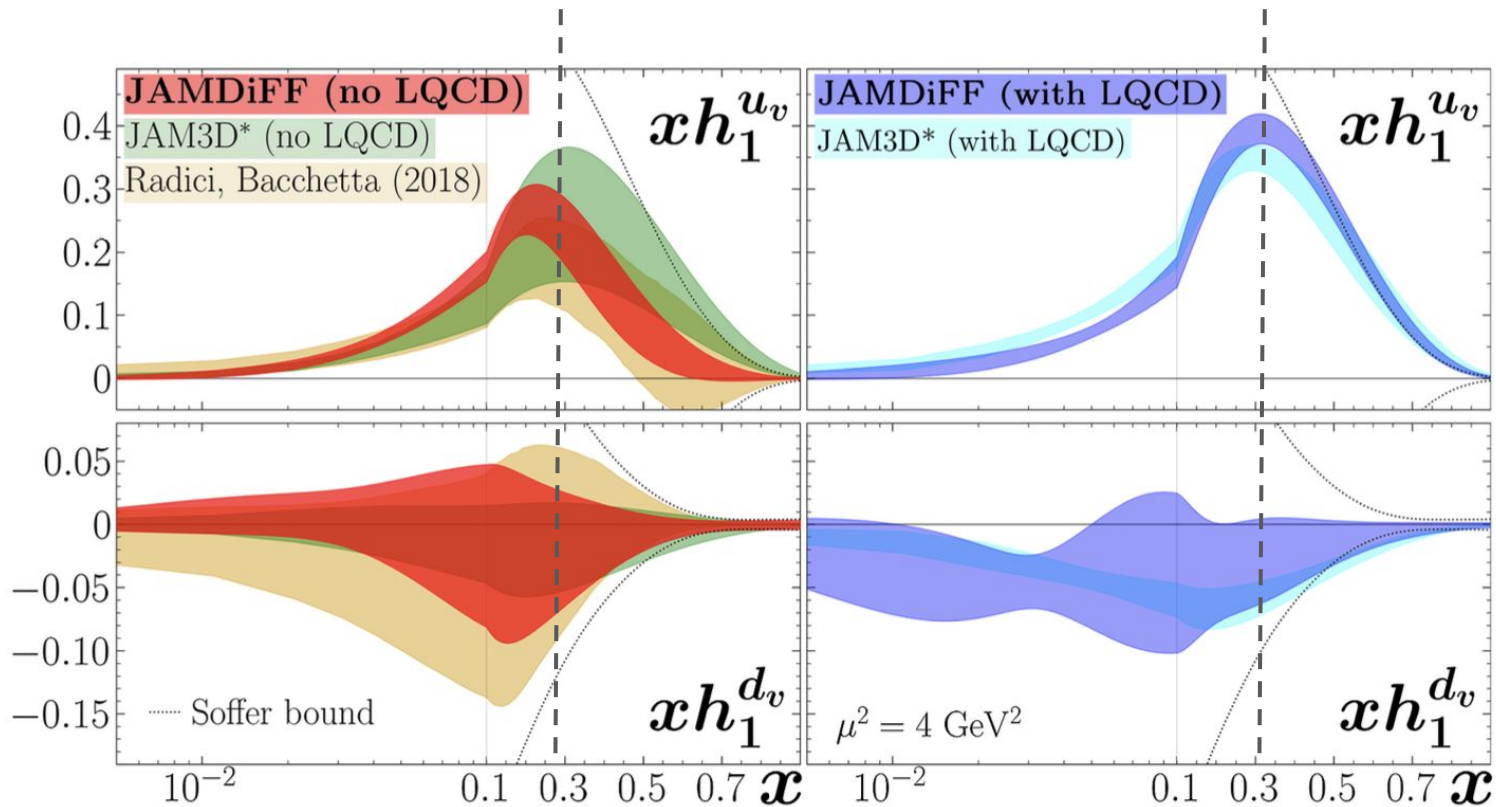
$$A_{UT}^{\text{SIDIS}} = c(y) \frac{\sum_q e_q^2 h_1^q(x) H_1^{\triangleleft, q}(z, M_h)}{\sum_q e_q^2 f_1^q(x) D_1^q(z, M_h)}$$

$$A_{UT}^{pp} = \frac{2P_{hT} \sum_i \sum_{a,b,c,d} \int_{x_a^{\min}}^1 dx_a \int_{x_b^{\min}}^1 \frac{dx_b}{z} h_1^a(x_a) f_1^b(x_b) \frac{d\Delta\hat{\sigma}_{a^\uparrow b \rightarrow c^\uparrow d}}{d\hat{t}} H_1^{\triangleleft, c}(z, M_h)}{2P_{hT} \sum_i \sum_{a,b,c,d} \int_{x_a^{\min}}^1 dx_a \int_{x_b^{\min}}^1 \frac{dx_b}{z} f_1^a(x_a) f_1^b(x_b) \frac{d\hat{\sigma}_{ab \rightarrow cd}}{d\hat{t}} D_1^c(z, M_h)}$$

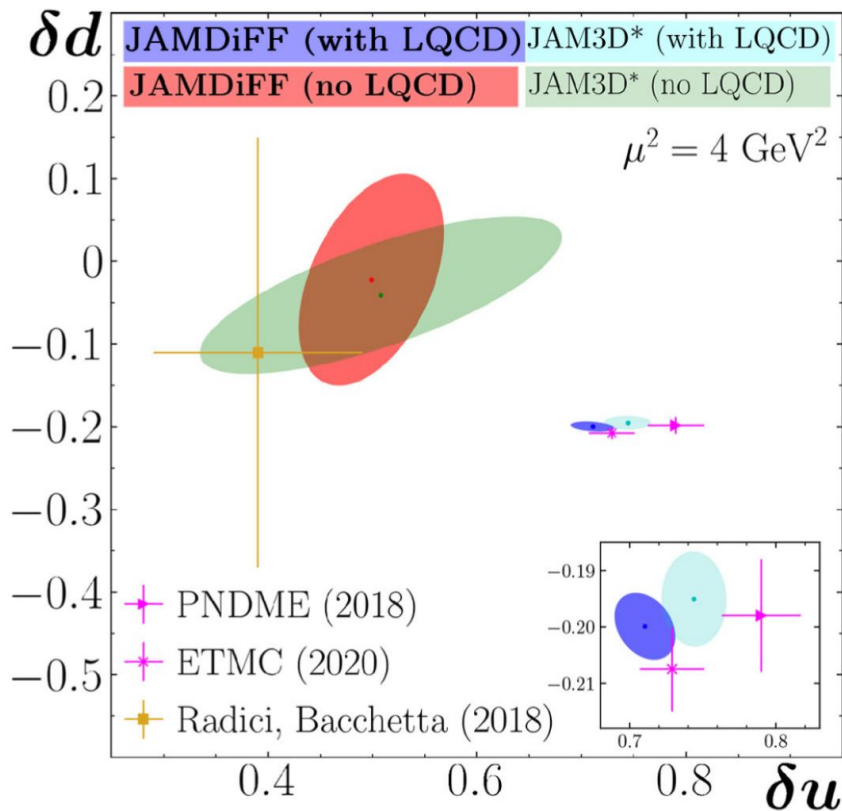
Experiment	Binning	N_{dat}	χ^2_{red}		
			JAMDiFF		
			(w/ LQCD)	(no LQCD)	(SIDIS only)
Belle (cross section) [64]	z, M_h	1094	1.01	1.01	1.01
	z, M_h	55	1.27	1.24	1.28
Belle (Artru-Collins) [112]	M_h, \bar{M}_h	64	0.60	0.60	0.60
	z, \bar{z}	64	0.42	0.42	0.41
HERMES [118]	x_{bj}	4	1.77	1.70	1.67
	M_h	4	0.41	0.42	0.47
	z	4	1.20	1.17	1.13
COMPASS (p) [117]	x_{bj}	9	1.98	0.65	0.59
	M_h	10	0.92	0.94	0.93
	z	7	0.77	0.60	0.63
COMPASS (D) [117]	x_{bj}	9	1.37	1.42	1.22
	M_h	10	0.45	0.37	0.38
	z	7	0.50	0.46	0.46
STAR [121] $\sqrt{s} = 200$ GeV $R < 0.3$	$M_h, \eta < 0$	5	2.57	2.56	
	$M_h, \eta > 0$	5	1.34	1.55	
	$P_{hT}, \eta < 0$	5	0.98	1.00	
	$P_{hT}, \eta > 0$	5	1.73	1.74	
	η	4	0.52	1.46	
STAR [97] $\sqrt{s} = 500$ GeV $R < 0.7$	$M_h, \eta < 0$	32	1.30	1.10	
	$M_h, \eta > 0$	32	0.81	0.78	
	$P_{hT}, \eta > 0$	35	1.09	1.07	
	η	7	2.97	1.83	
ETMC δu [77]		1	0.71		
ETMC δd [77]		1	1.02		
PNDME δu [71]		1	8.68		
PNDME δd [71]		1	0.04		
Total $\chi^2_{\text{red}} (N_{\text{dat}})$			1.01 (1475)	0.98 (1471)	0.96 (1341)



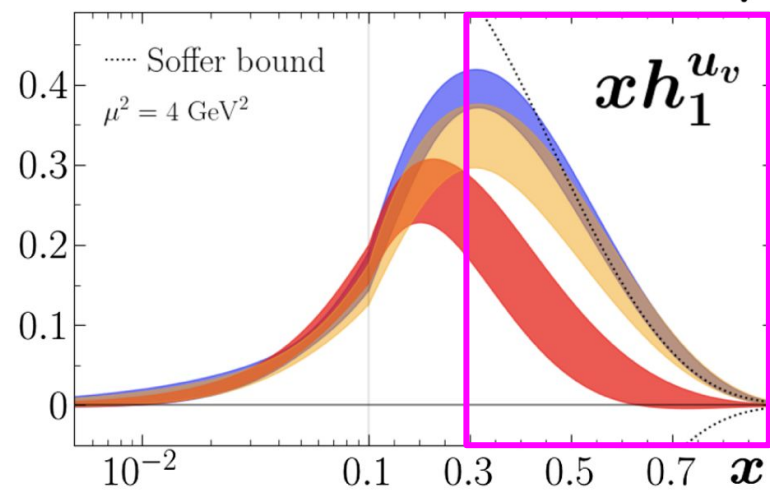
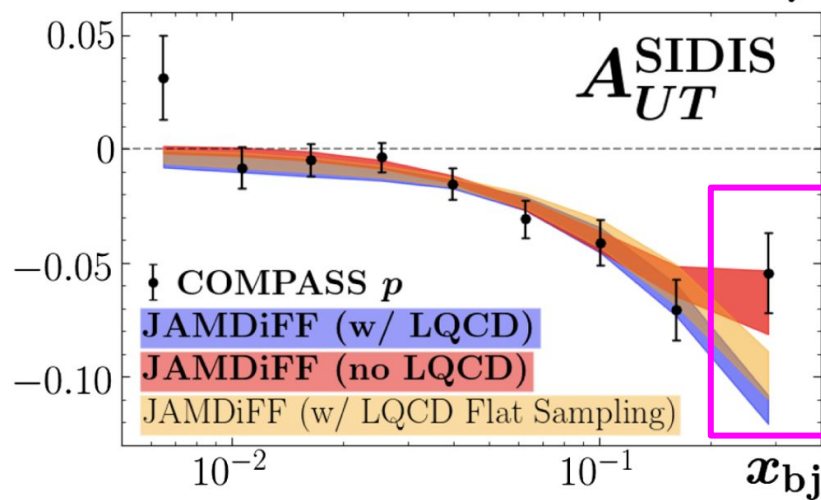
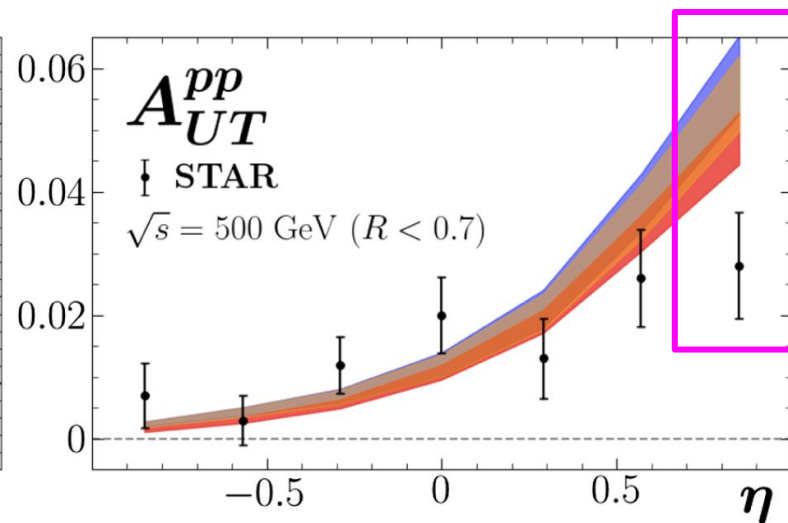
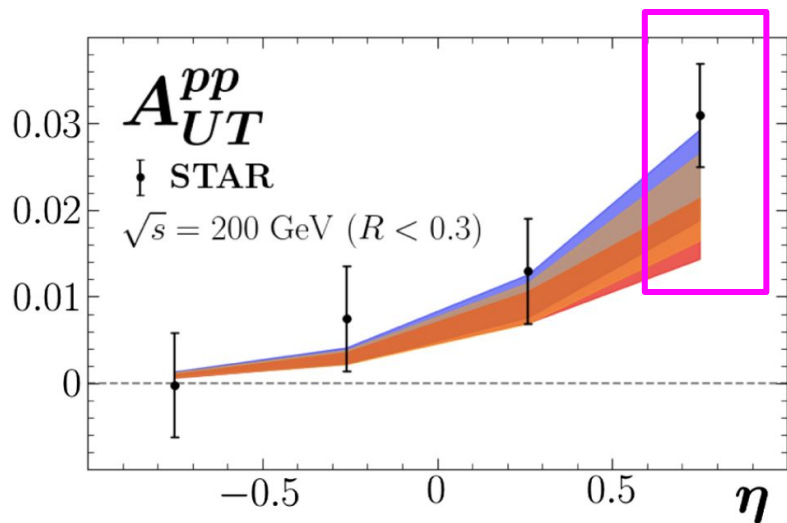
Reconstructed TPDF



Reconstructed TPDF

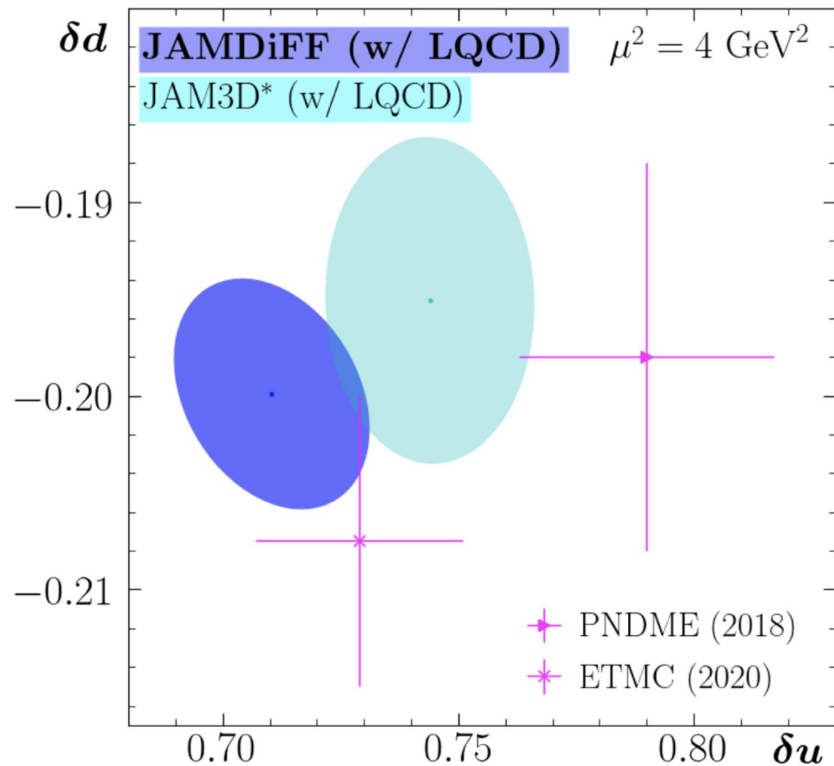


Experiment	χ^2_{red}	
	JAMDiFF (w/ LQCD)	JAMDiFF (no LQCD)
Belle (cross section) [64]	1.01	1.01
Belle (Artru-Collins) [112]	1.27	1.24
	0.60	0.60
HERMES [118]	0.42	0.42
	1.77	1.70
COMPASS (p) [117]	0.41	0.42
	1.20	1.17
COMPASS (D) [117]	1.98	0.65
	0.92	0.94
STAR [121]	0.77	0.60
	1.37	1.42
COMPASS (D) [117]	0.45	0.37
	0.50	0.46
STAR [121]	2.57	2.56
	1.34	1.55
$\sqrt{s} = 200 \text{ GeV}$ $R < 0.3$	0.98	1.00
	1.73	1.74
STAR [97]	0.52	1.46
	1.30	1.10
$\sqrt{s} = 500 \text{ GeV}$ $R < 0.7$	0.81	0.78
	1.09	1.07
ETMC δu [77]	2.97	1.83
ETMC δd [77]	0.71	
PNDME δu [71]	1.02	
PNDME δd [71]	8.68	
PNDME δd [71]	0.04	
Total $\chi^2_{\text{red}} (N_{\text{dat}})$	1.01 (1475)	0.98 (1471)



Takeaways

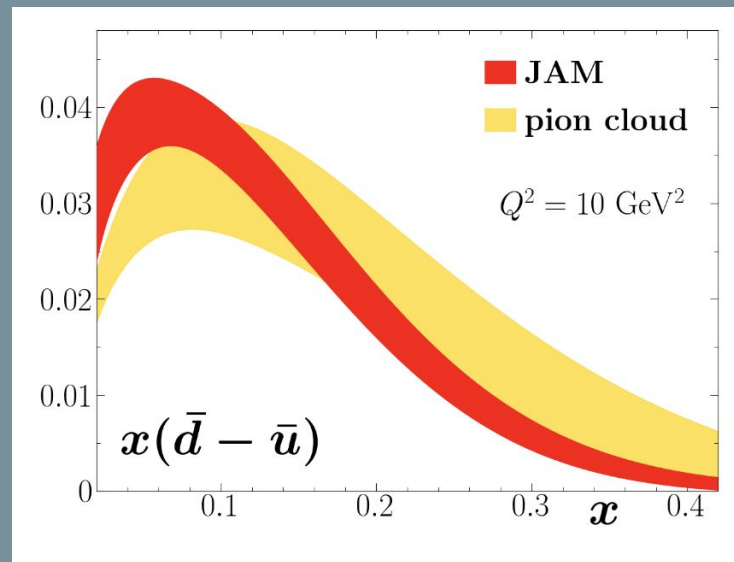
- At present there is no significant tension between LQCD and experimental reconstruction of nucleon tensor charges
- Different reconstructions of tensor charges are mostly driven by large x data
- More high x data is needed to reach accurate reconstruction of TPDF above $x > 0.3$
- Inclusion of LQCD calculations as priors are very informative/useful in QCD phenomenology
- **The JAMDiFF results and JAM3D* results are very similar and one can perform a combined analysis (TMD+CT3 & DiFF) -> indicates possible universal nature of all SSAs and nucleon tensor charges**



Outline

1. Gluon helicity PDF
2. Nucleon transversity/tensorcharge
3. **Pion PDF**

Cocuzza, Melnitchouk, Metz, NS '21

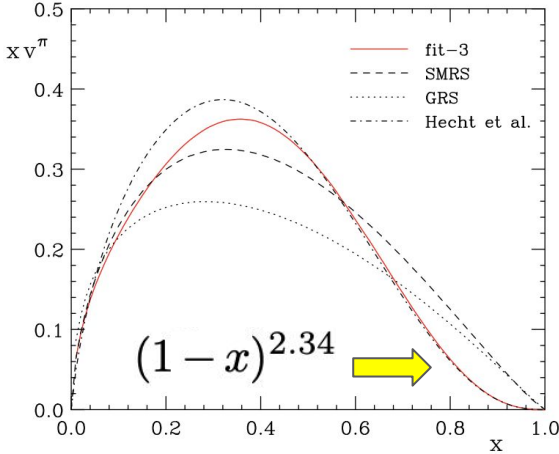


$$(\bar{d} - \bar{u})(x) = [(f_{n\pi^+} + f_{\Delta^0\pi^+} - f_{\Delta^{++}\pi^-}) \otimes \bar{q}_v^\pi](x),$$

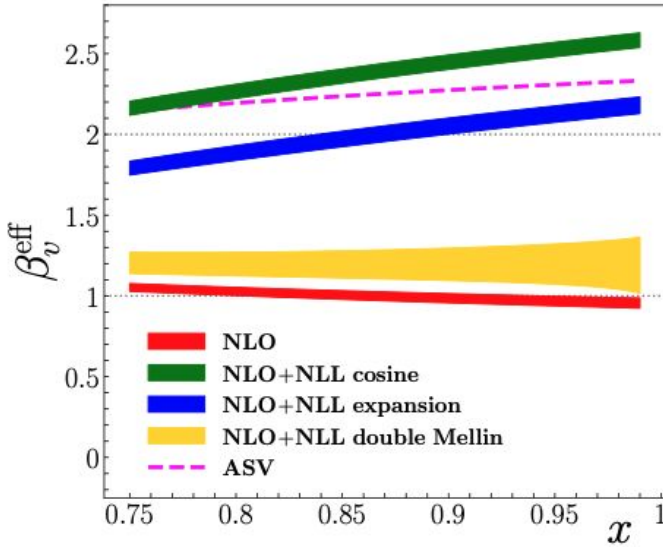
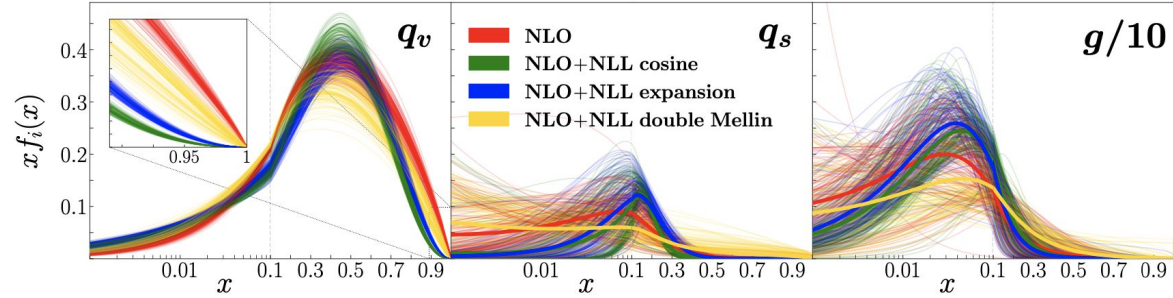
Large x valence pion PDFs

Barry, Ji, Melnitchouk, NS '21

Aicher, Schafer, Vogelsang '10



- Aicher et al, showed that large x asymptotics qV is sensitive to threshold corrections.
- Their analysis found similar asymptotic behavior as in DSE expectations.

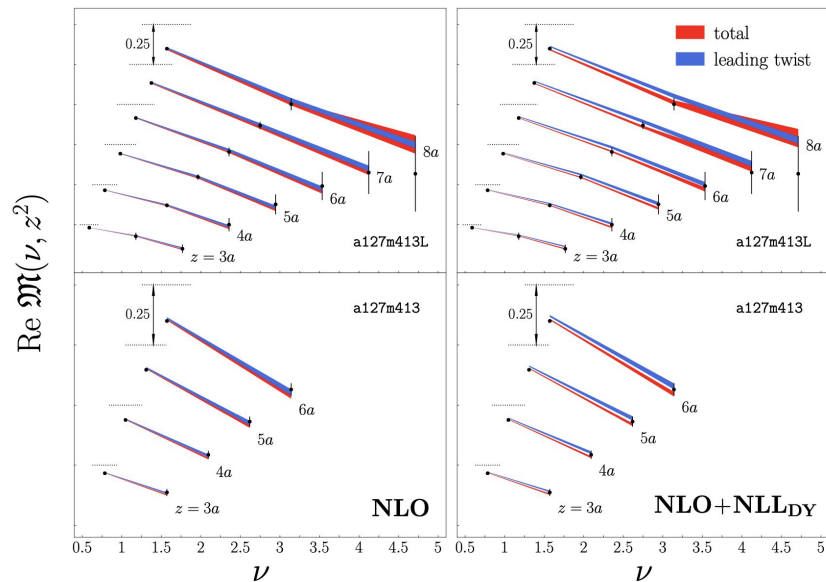


$$\beta_v^{\text{eff}}(x, \mu) = \frac{\partial \log |q_v(x, \mu)|}{\partial \log(1-x)}$$

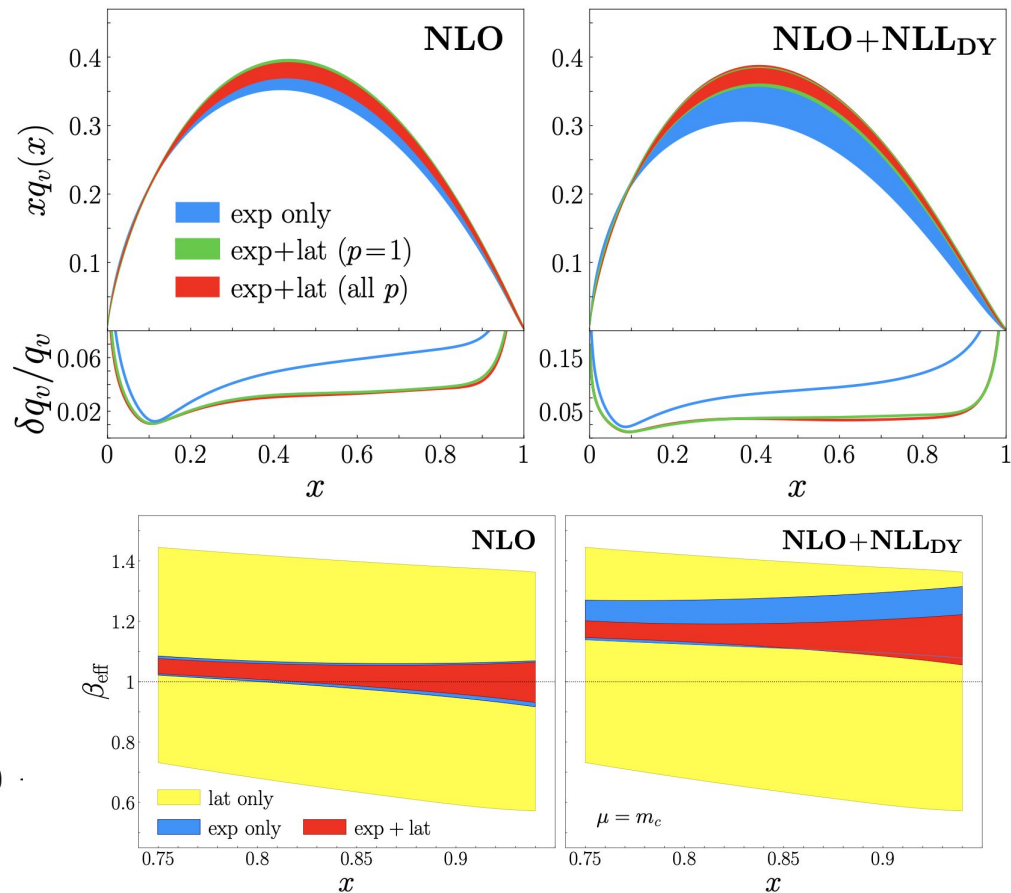
- Improved resummation setup showed consistency of fixed order NLO results.
- DSE vs pQCD back to a disagreement

Inclusion of LQCD **JAM+HadStruc**

Barry et al. '22



$$\text{Re } \mathfrak{M}(\nu, z^2) = \int_0^1 dx q_v(x, \mu_{\text{lat}}) \mathcal{C}^{\text{Rp-ITD}}(x\nu, z^2, \mu_{\text{lat}}) + z^2 B_1(\nu) + \frac{a}{|z|} P_1(\nu) + e^{-m_\pi(L-z)} F_1(\nu).$$



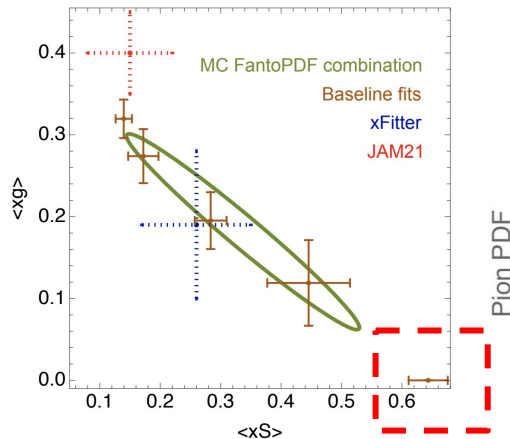
An analysis of parton distributions in a pion with Bézier parametrizations

Lucas Kotz ¹, Aurore Courtoy ^{2,*}, Pavel Nadolsky ^{1,†}, Fredrick Olness ¹ and Maximiliano Ponce-Chavez ²

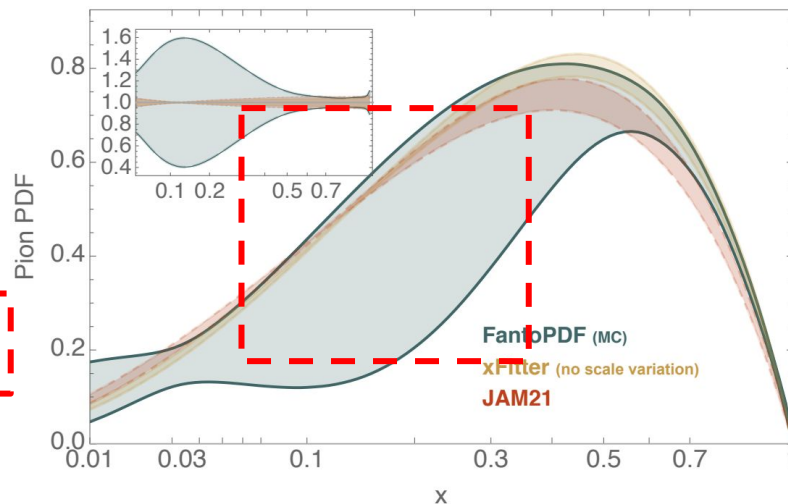
¹Department of Physics, Southern Methodist University, Dallas, TX 75275-0175, USA

²Instituto de Física, Universidad Nacional Autónoma de México, Apartado Postal 20-364, 01000 Ciudad de México, Mexico

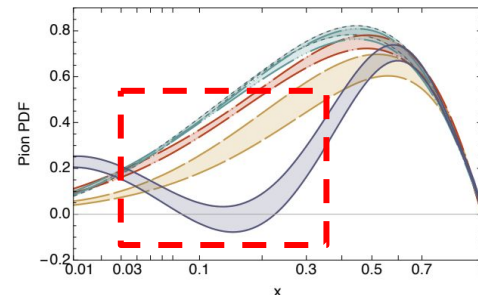
(Dated: April 17, 2024)



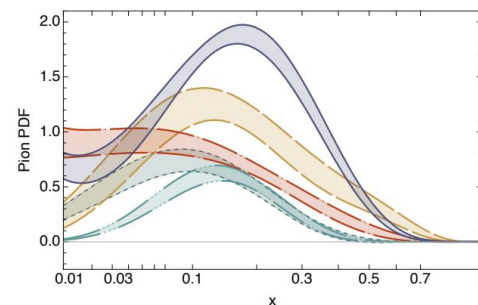
$xV(x,Q)$ at $Q=1.4$ GeV, 68% c.l. (band)



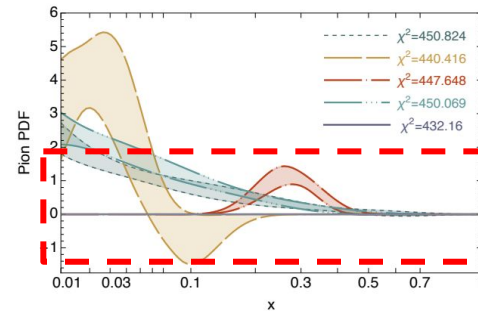
$xV(x,Q)$ at $Q=1.4$ GeV, 68% c.l. (band)



$xS(x,Q)$ at $Q=1.4$ GeV, 68% c.l. (band)



$xg(x,Q)$ at $Q=1.4$ GeV, 68% c.l. (band)



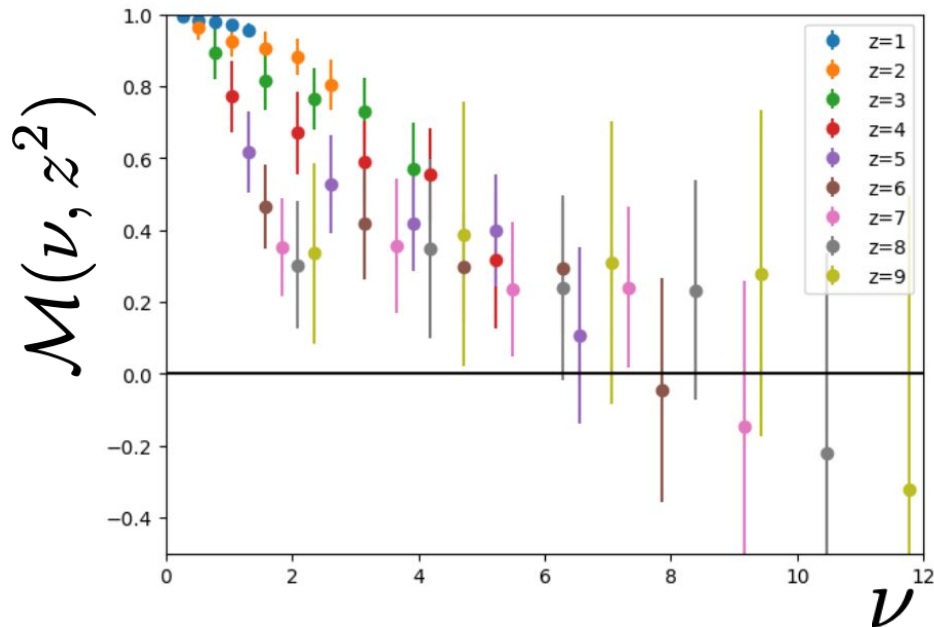
- No gluons inside pion gives best χ^2
- Sea quark pdf larger than valence quark $x \sim 0.1$

Toward the First Gluon Parton Distribution from the LaMET

William Good (Michigan State U. and Michigan State U., East Lansing (main)), Kinza

Hasan (Michigan State U.), Huey-Wen Lin (Michigan State U.)

Sep 4, 2024



ensemble	a12m310 (310 MeV)
a (fm)	0.1207(11)
$L^3 \times T$	$24^3 \times 64$
M_π^{val} (MeV)	309.0(11)
P_z (GeV)	[0, 1.71]
N_{cfg}	1013
N_{meas}	~ 1.2 M
t_{sep}	[5, 9]

Reduced pseudo-Ioffe Time Distribution from one ensemble

$$\mathcal{M}(v, z^2) = \frac{\mathcal{M}(zP_z, z^2)/\mathcal{M}(0 \cdot P_z, 0)}{\mathcal{M}(z \cdot 0, z^2)/\mathcal{M}(0 \cdot 0, 0)}$$

Lattice + JAM



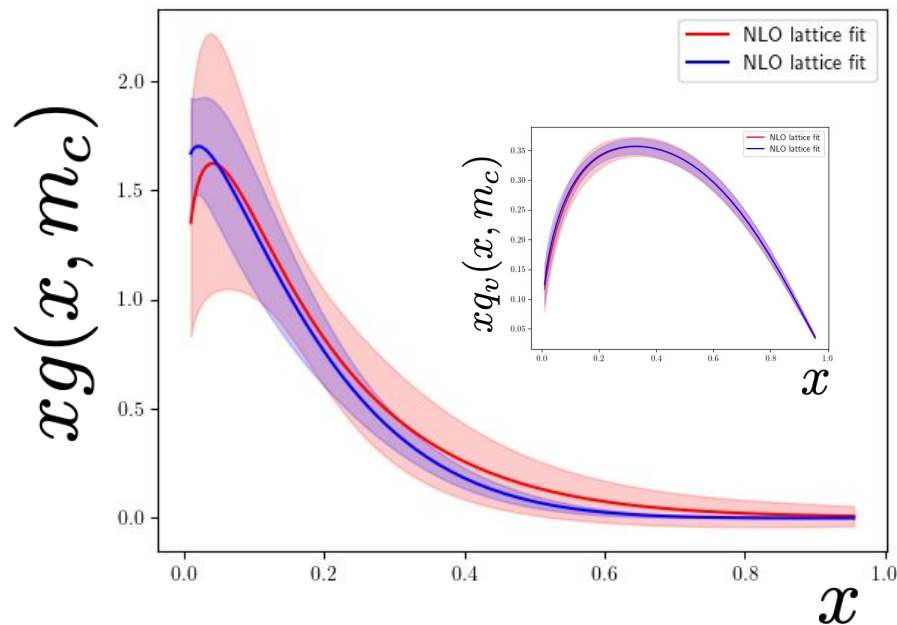
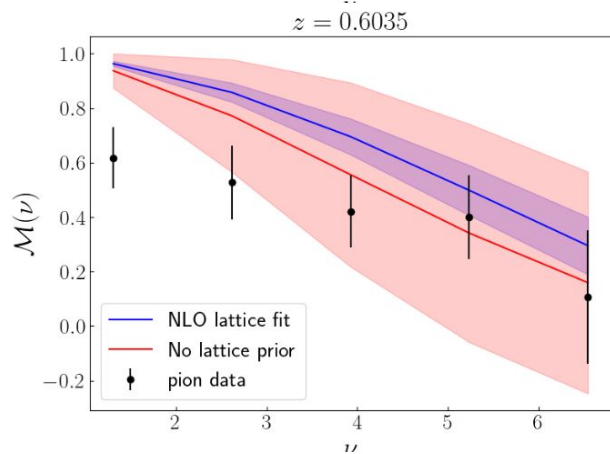
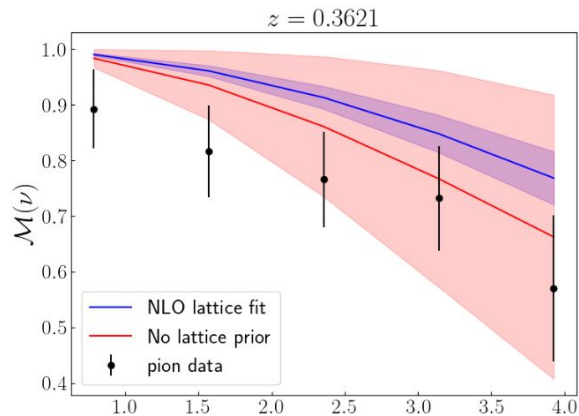
Alexis
NieMiera

William
Good

Patrick
Barry

In collab with
Huey-Wen & Wally

Preliminary Results



- No gluons are disfavored by LQCD
- LQCD data has a strong impact relative to exp. observables

chi ² /N_pts	Before Lattice	After Lattice
Expt	0.88(5)	0.84(4)
Theory	4.3(85)	1.86(7)

Holistic approach to QCD global analysis

