



wakis:

3D Electromagnetic Time- Domain Wake and Impedance Solver

ABP-CEI Meeting, 25th April 2024

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😊 **Special thanks to:** Chiara Antuono, Giovanni Rumolo, Benoit Salvant, Leonardo Sito, Jean Luc Vay (Berkeley Lab) and yet again to Lorenzo Giacomel

Outline

1. Introduction to Beam-Coupling Impedance simulations & Motivation

..... Slides 4 - 11

2. The Finite Integration Technique (FIT) step-by-step:

..... Slides 13 - 38

2.1. Numerical Algorithm in free-space

2.2. Geometry definition: STL importer

2.3. Adding material tensors: ε and μ

2.4. Particle beam injection & Absorbing boundaries

2.5. Adding conductivity σ

2.6. Low- β simulations

3. Conclusions, Challenges & Future work

..... Slides 40 - 42

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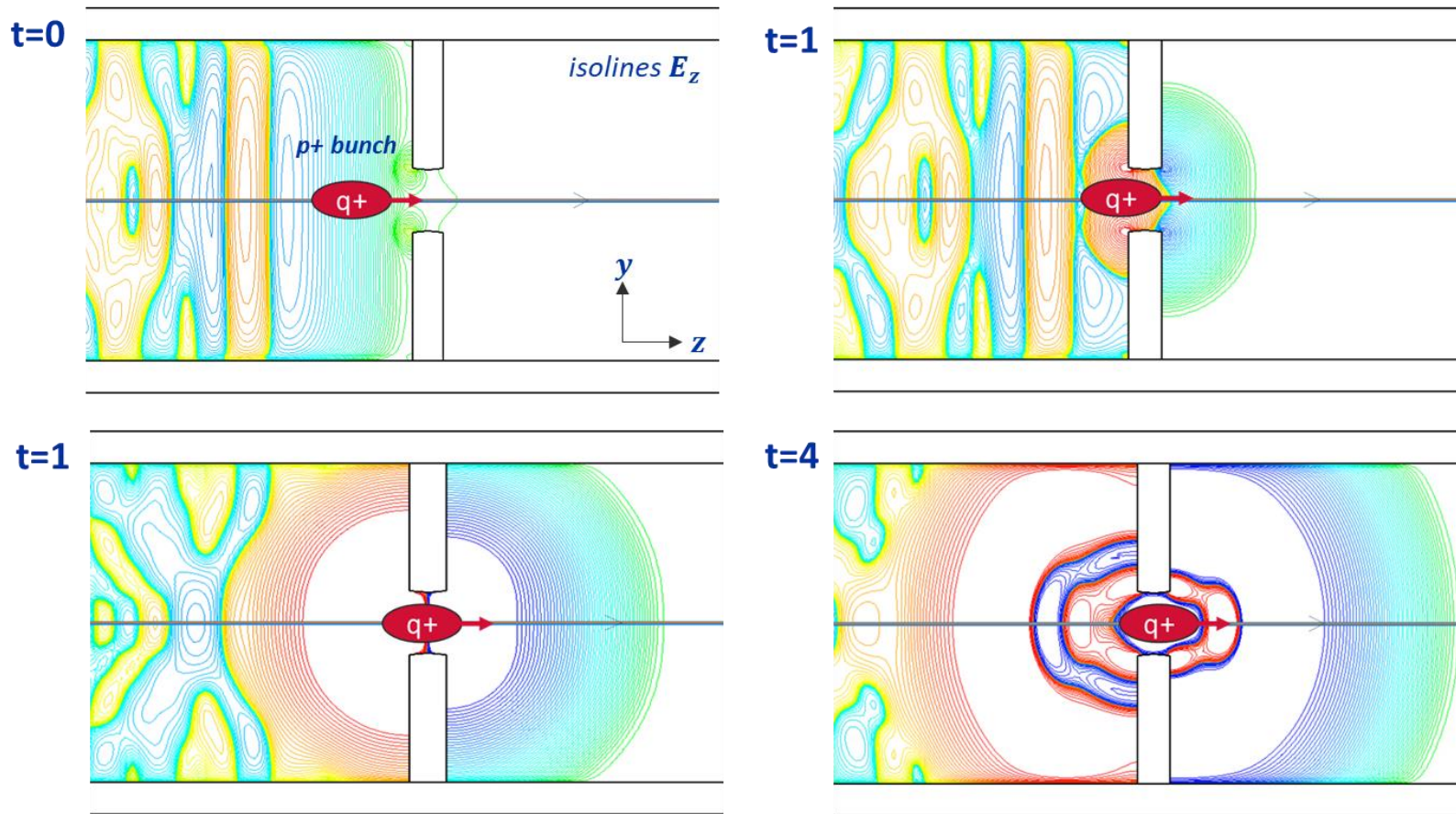
2.5. Adding conductivity σ

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Beam Coupling Impedance $Z(f)$



Wakefields are generated as the particle beam traverses the different accelerator devices and ‘perceives’ **discontinuities**:

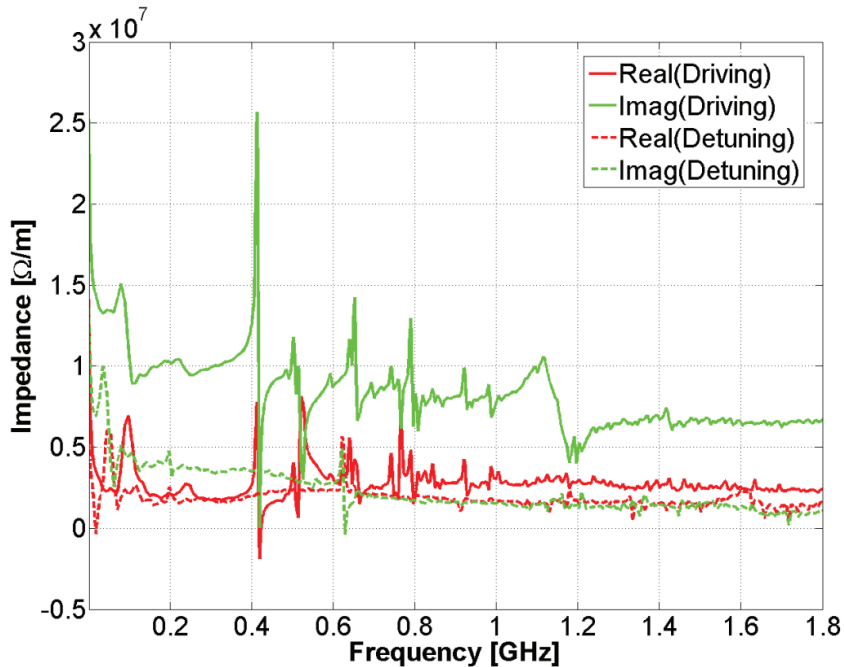
- in the **geometry**
- or the **electromagnetic properties** ($\epsilon, \mu, \sigma \dots$)

These wakefields will affect the trailing particles/bunches. The **beam-coupling impedance** is the **frequency-dependent property of each accelerator device**, used to quantify the wakefields’ effects.

Fig: 3D Time domain simulation of a smooth pipe with obstacle with a passing proton beam (1-4) with CST®

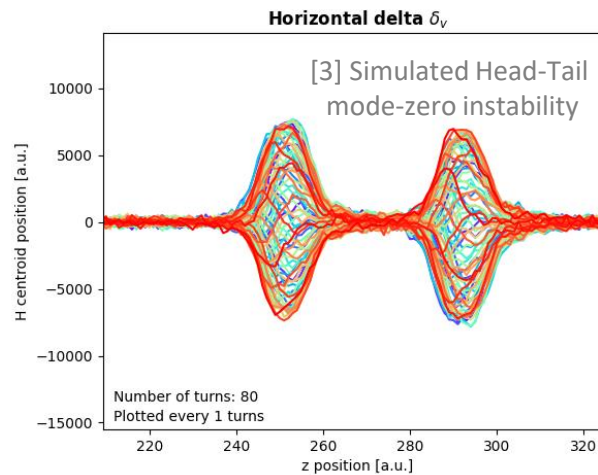
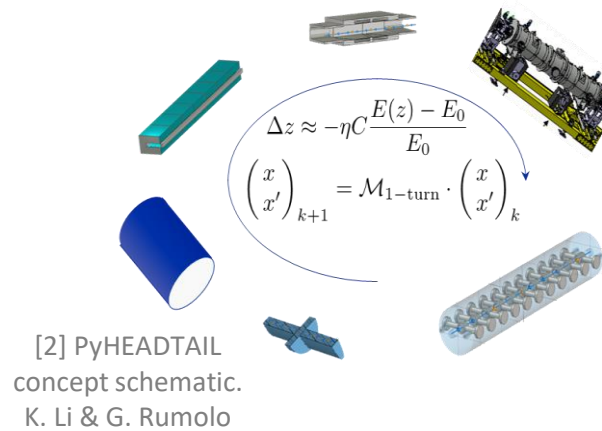
Beam Coupling Impedance $Z(f)$ (II)

- The impedance of all relevant accelerator components is gathered in each accelerator's **impedance model**



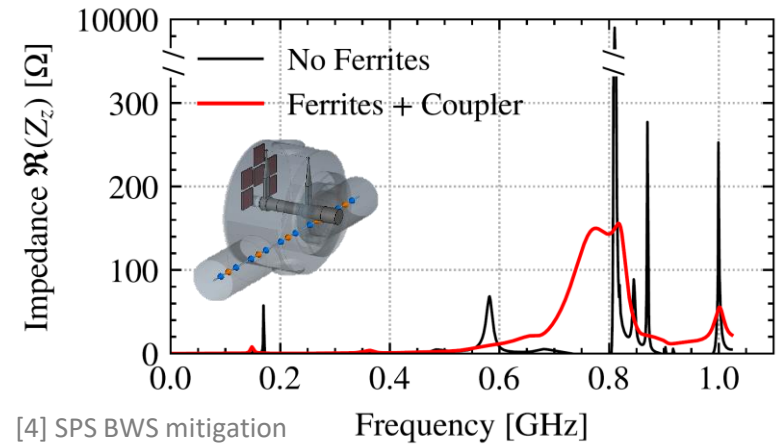
[1] SPS Transverse vertical impedance model.
C. Zannini

- And used in beam-dynamics codes (pyHeadTail, Xsuite) to **predict beam behaviour**

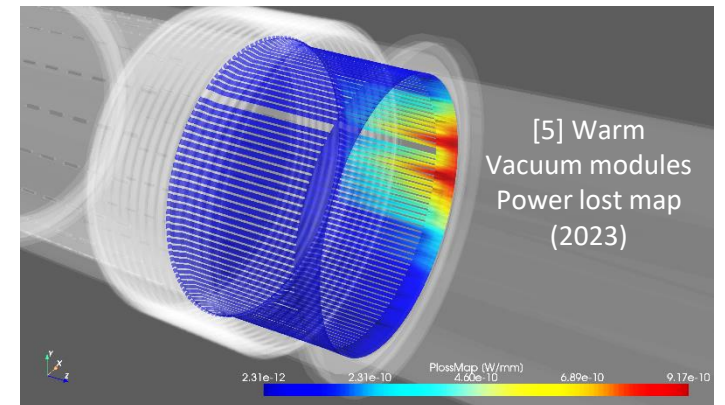


[3] Simulated Head-Tail mode-zero instability

- Or assess **beam-induced heating** of individual accelerator components and propose mitigation solutions



[4] SPS BWS mitigation solution (2023)



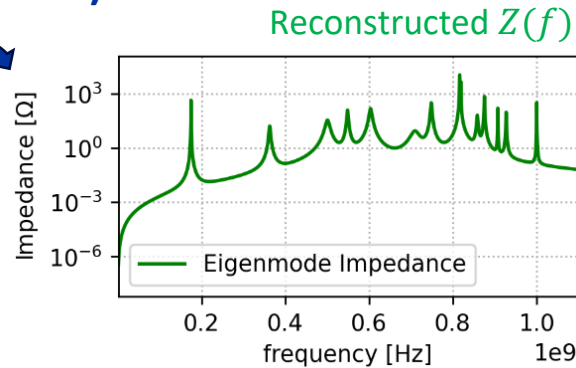
How to obtain $Z(f)$?

Directly in frequency domain (FD)

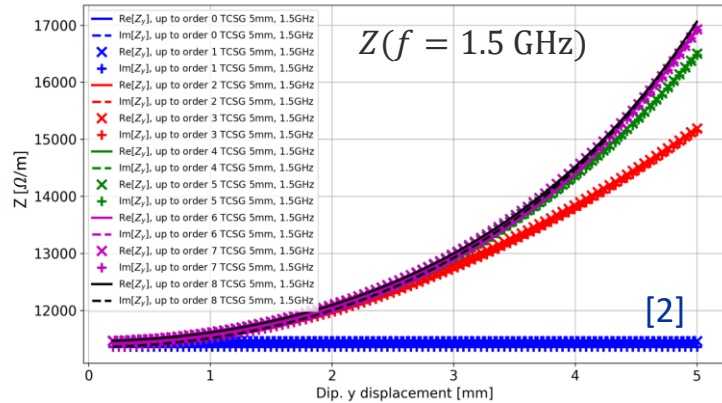
○ Eigenmode solver (CST®):

Mode	Frequency
1	8.02724981695
2	10.1781072579
3	10.3554337227
4	12.5109337728
5	14.7562186034

f, R_s, Q



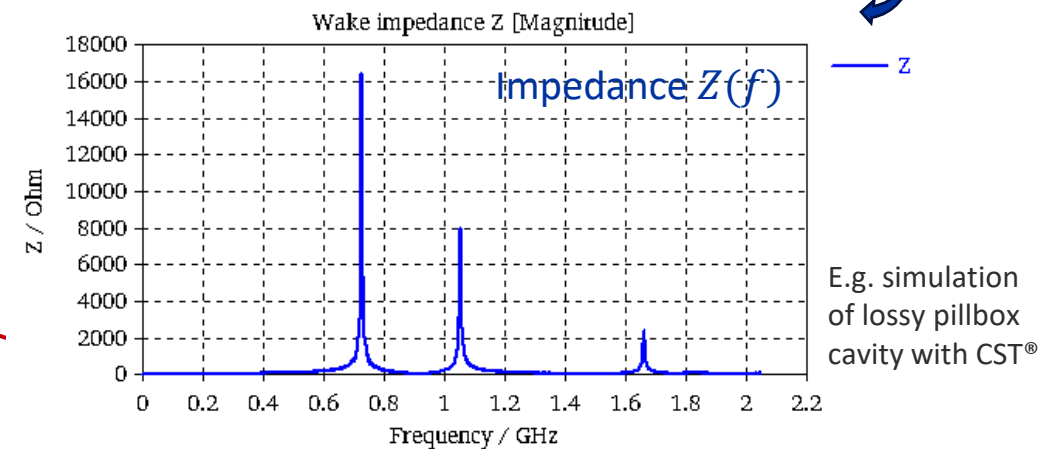
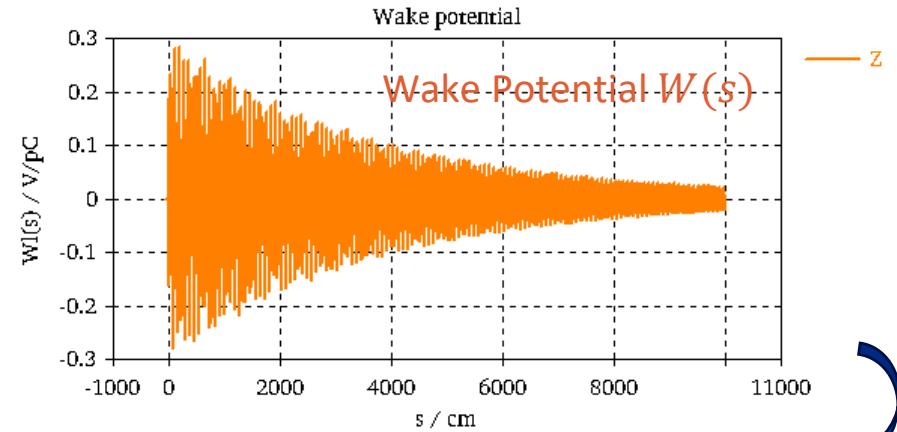
○ Resistive wall impedance: (IW2D) [1]



E.g. vertical TCSG CFC collimator vertical dipolar impedance at $f = 1.5 \text{ GHz}$ vs. vertical source offset

From the time domain (TD)

○ Wakefield solver (CST®):



E.g. simulation of lossy pillbox cavity with CST®

[1] N. Mounet. The LHC Transverse Coupled-Bunch Instability, PhD thesis 5305 (EPFL, 2012)

[2] N. Mounet. ImpedanceWake2D BE-ABP-CEI 22/04/2021

How to obtain $Z(f)$?

Commercial /
not open-source codes

Directly in frequency domain (FD)

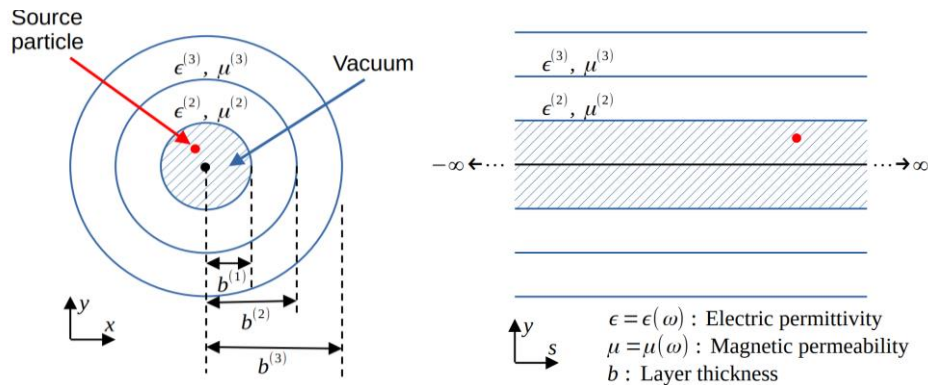
○ Eigenmode solver (CST®):

$$\left. \begin{aligned} \text{rot } \vec{E} &= j\omega\mu\vec{H} \\ \text{rot } \vec{H} &= j\omega\left(\epsilon + \frac{\sigma}{j\omega}\right)\vec{E} \end{aligned} \right\} \text{Eigenvalue problem}$$

$$\text{rot } \frac{1}{\underline{\underline{\mu}}} \text{rot } \vec{E} = \underline{\underline{\epsilon}}\omega^2\vec{E}$$

Fundamental EM modes of 3D loss-less resonant structures
without excitation + Q-factor postprocessing

○ Resistive wall impedance: (IW2D@CERN) [1]



From the time domain (TD)

○ Wakefield solver (CST®):

3D time domain Maxwell equations (integral form)

$$\oint_{\partial A} \vec{E} \cdot d\vec{s} = - \iint_A \frac{\partial \vec{B}}{\partial t} \cdot d\vec{A}$$

$$\oint_{\partial A} \vec{H} \cdot d\vec{s} = - \iint_A \left(\frac{\partial \vec{D}}{\partial t} + \vec{J} \right) \cdot d\vec{A}$$

$$\oiint_{\partial V} \vec{B} \cdot d\vec{A} = 0$$

$$\oiint_{\partial V} \vec{D} \cdot d\vec{A} = \iiint_V \rho dV$$

with beam
excitation
 $\vec{J} = v_z \lambda(z) \vec{e}_z$

+ wake potential and impedance computation:

$$W(r_1, r_2, s) = \frac{1}{q_1} \int_{-\infty}^{\infty} dz \left[\vec{E}(r_1, r_2, z, t) + c\vec{e}_z \times \vec{B}(r_1, r_2, z, t) \right]_{t=\frac{s+z}{c}}$$

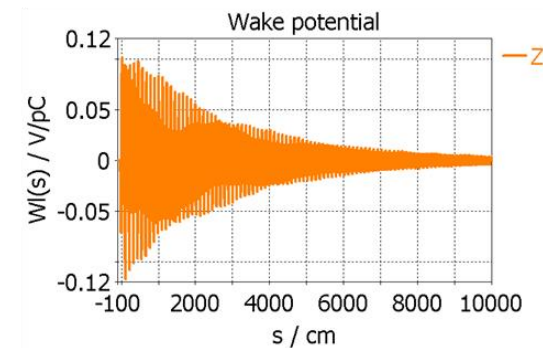
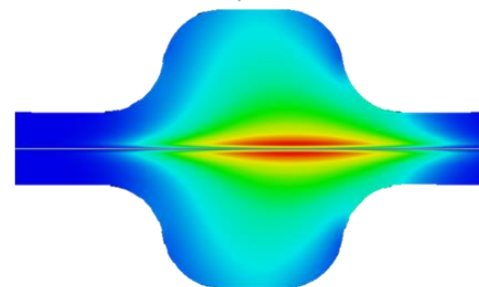
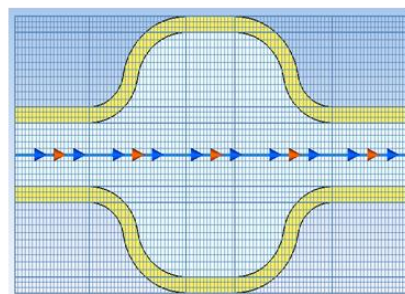
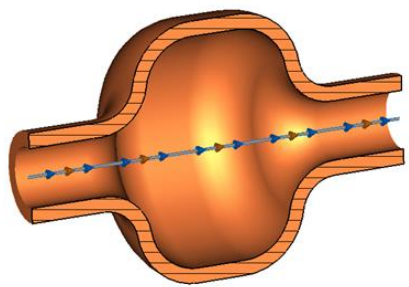
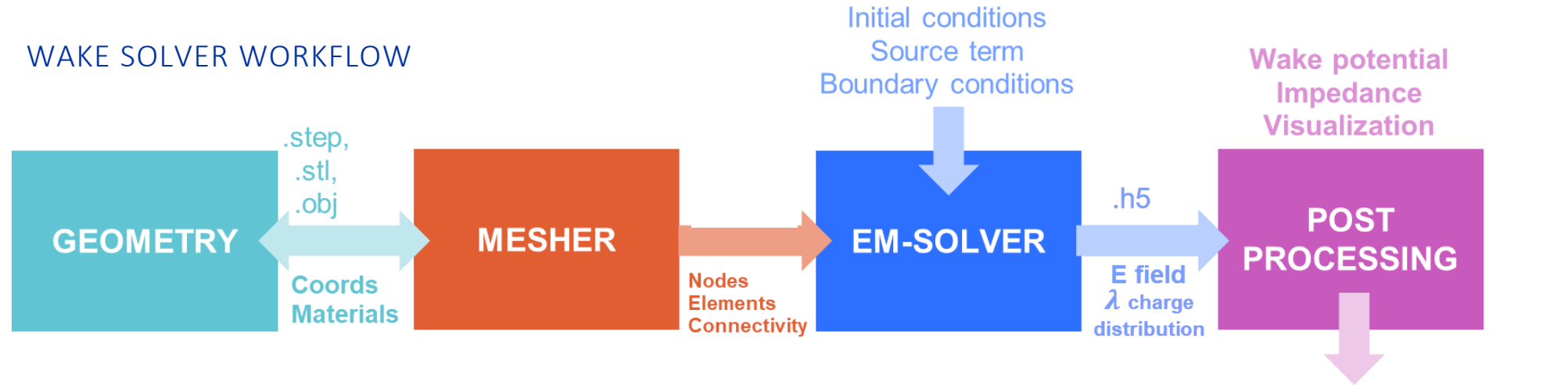
$$Z_{||}(\omega) = - \frac{\int_{-\infty}^{\infty} W(s) e^{-i\omega s} ds}{\int_{-\infty}^{\infty} c\lambda(s) e^{-i\omega s} ds} \text{ (FT + deconvolution)}$$

[1] N. Mounet. The LHC Transverse Coupled-Bunch Instability, PhD thesis 5305 (EPFL, 2012)

Project objective

Develop an **open-source Wakefield Solver** (i.e., 3D electromagnetic time-domain) at **CERN**

WAKE SOLVER WORKFLOW

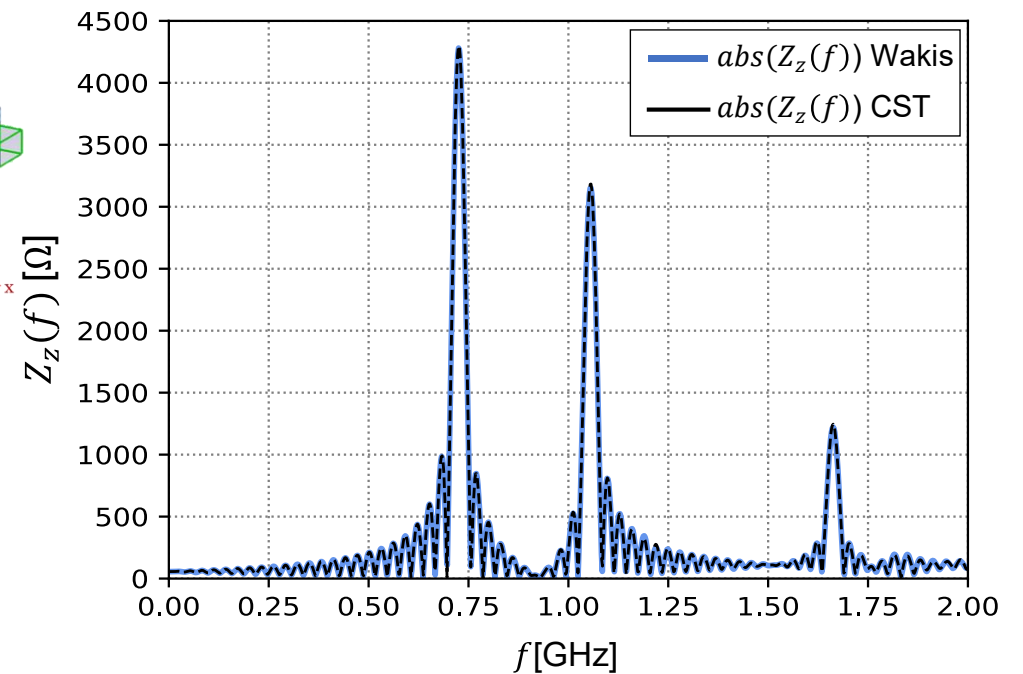
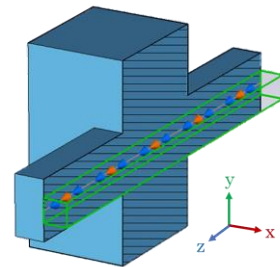


PhD motivation



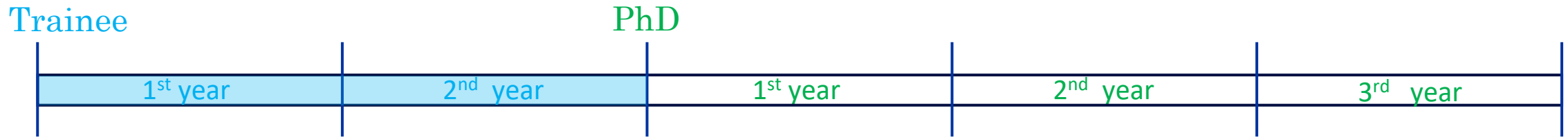
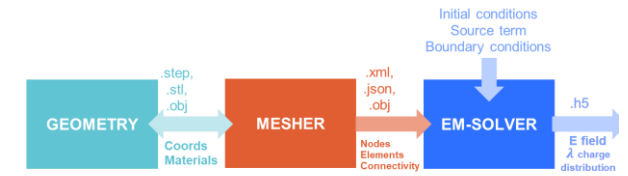
(v0.1) Developed wakis package:

- Computation of wake potential and impedance from pre-computed fields
- Longitudinal $Z_{||}$ and transverse Z_{\perp} (dipolar & quadrupolar)
- Benchmarked with CST[®] Wakefield fields



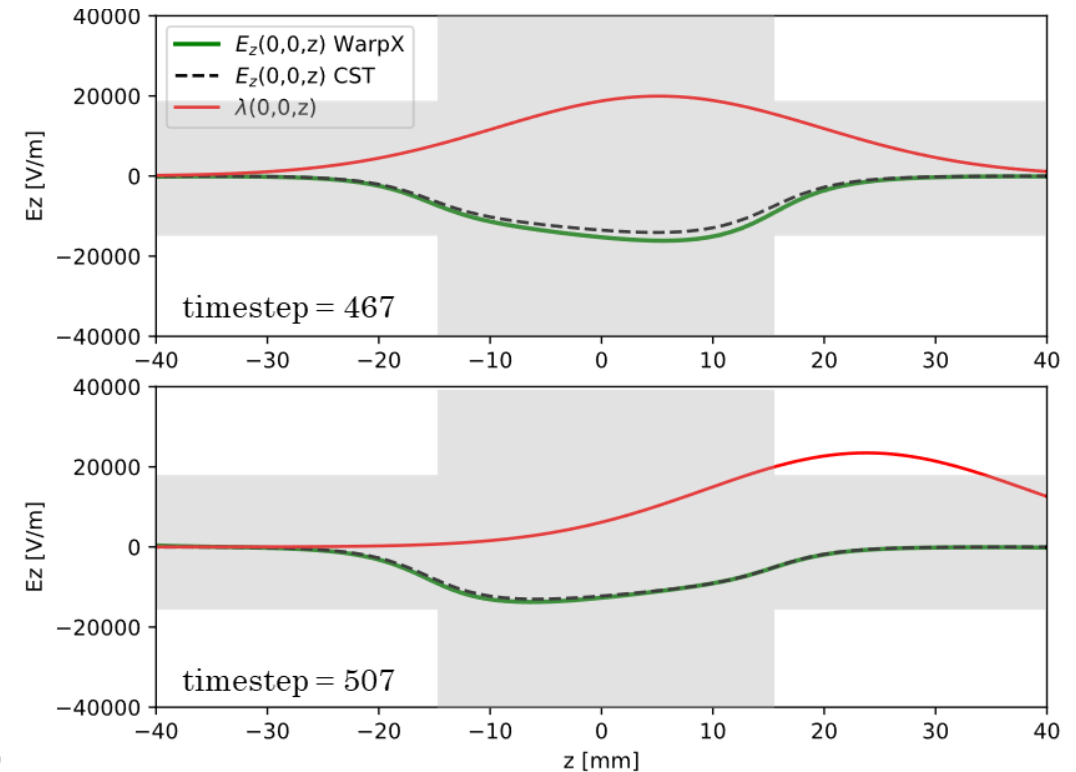
[1] Progress and challenges of an in-house wake/impedance solver.
ABP information meeting, 28th April 2022.
<https://indico.cern.ch/event/1154158/>

PhD motivation (II)



(v0.2) Explored open-source EM solvers to couple with *wakis*:

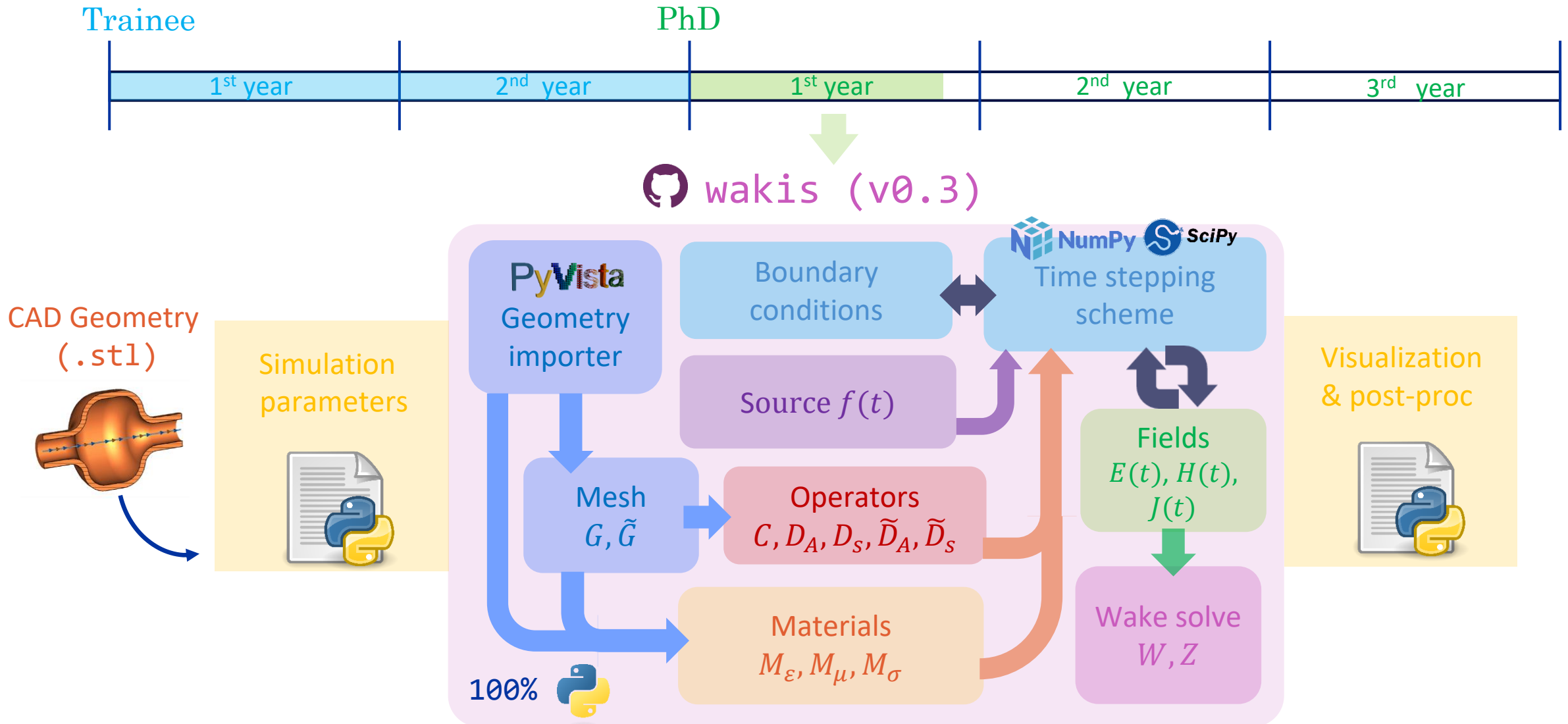
- [WarpX](#) was the most promising option: PIC+FDTD, mesh refinement (AMR), PML boundaries, python API
- Found **very good agreement for pillbox cavity**, AMReX developed importer for more complex geometries
- WarpX at the time, **did not include material tensors** other than vacuum/PEC. Found limitations in beam injection & PML behaviour too, **C++ core, complexity...**



[2] Progress on in-house wake solver “wakis”. ABP-CEI meeting, 3rd August 202
<https://indico.cern.ch/event/1283485/>

PhD status

(v.0.3) Decided to explore CST®'s numerical method, the **Finite Integration Technique (FIT)** in a python mockup 😊



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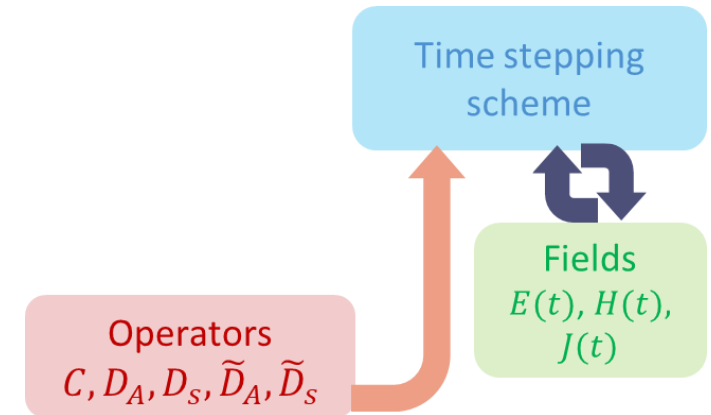
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Finite Integration Technique

Maxwell Equations (Integral form)

$$\oint_{\partial A} \mathbf{E} \cdot d\mathbf{s} = - \iint_A \frac{\partial \mathbf{B}}{\partial t} \cdot d\mathbf{A}$$

$$\oint_{\partial A} \mathbf{H} \cdot d\mathbf{s} = - \iint_A \left(\frac{\partial \mathbf{D}}{\partial t} + \mathbf{J} \right) \cdot d\mathbf{A}$$

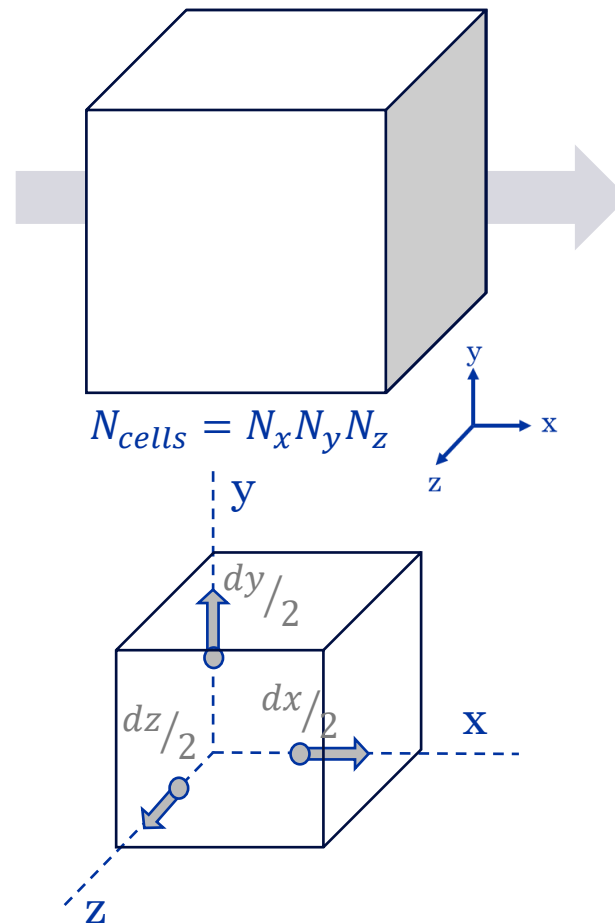
$$\oiint_{\partial V} \mathbf{B} \cdot d\mathbf{A} = 0$$

$$\oiint_{\partial V} \mathbf{D} \cdot d\mathbf{A} = \iiint_V \rho dV$$

$$\mathbf{D} = \underline{\underline{\epsilon}} \mathbf{E}, \quad \mathbf{B} = \underline{\underline{\mu}} \mathbf{H}, \quad \mathbf{J} = \underline{\underline{\sigma}} \mathbf{E} + \rho \mathbf{v}$$

*Formulation by T. Weiland: Wakefields and Impedances, 1991

1st approximation
Domain discretization
 dx, dy, dz



Maxwell Grid Equations*

$$\mathbf{C} \mathbf{D}_s \mathbf{e} = - \mathbf{D}_A \frac{\partial \mathbf{b}}{\partial t}$$

$$\tilde{\mathbf{C}} \tilde{\mathbf{D}}_s \mathbf{h} = \tilde{\mathbf{D}}_A \left(\frac{\partial \mathbf{d}}{\partial t} + \mathbf{j} \right)$$

$$\mathbf{S} \mathbf{D}_A \mathbf{b} = 0$$

$$\tilde{\mathbf{S}} \tilde{\mathbf{D}}_A \left(\frac{\partial \mathbf{d}}{\partial t} + \mathbf{j} \right) = 0$$

$$\mathbf{d} = \tilde{\mathbf{D}}_\epsilon \mathbf{e}, \quad \mathbf{b} = \mathbf{D}_\mu \mathbf{h}, \quad \mathbf{j} = \tilde{\mathbf{D}}_\sigma \mathbf{e} + \mathbf{j}_{src}$$

- Operators
- Grid areas and lengths
- Materials

Finite Integration Technique (II)

Maxwell Grid Equations

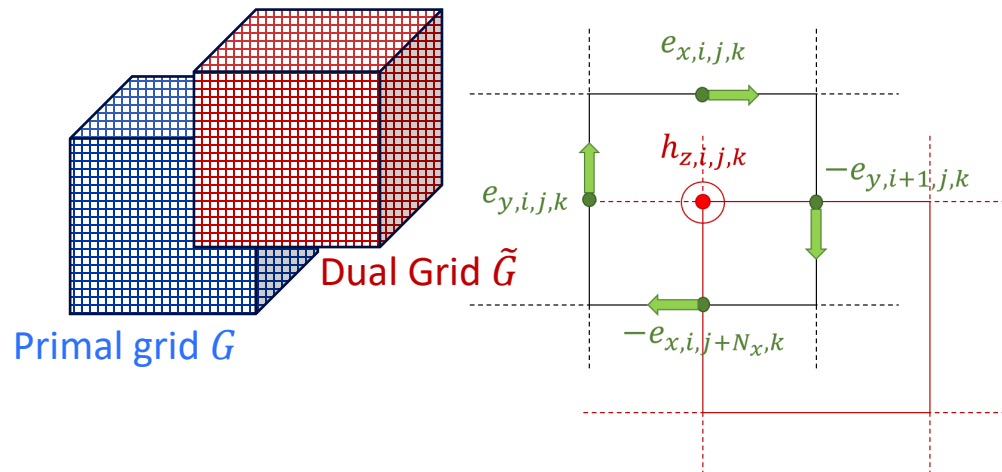
$$\begin{aligned} \mathbf{C}\mathbf{D}_s\mathbf{e} &= -\mathbf{D}_A\frac{\partial\mathbf{b}}{\partial t} \\ \tilde{\mathbf{C}}\tilde{\mathbf{D}}_s\mathbf{h} &= \tilde{\mathbf{D}}_A\left(\frac{\partial\mathbf{d}}{\partial t} + \mathbf{j}\right) \\ \mathbf{d} &= \tilde{\mathbf{D}}_\epsilon\mathbf{e}, \quad \mathbf{b} = \mathbf{D}_\mu\mathbf{h}, \\ \mathbf{j} &= \tilde{\mathbf{D}}_\sigma\mathbf{e} + \mathbf{j}_{src} \end{aligned}$$

$$\begin{aligned} \mathbf{S}\mathbf{D}_A\mathbf{b} &= \mathbf{0} \\ \tilde{\mathbf{S}}\tilde{\mathbf{D}}_A\left(\frac{\partial\mathbf{d}}{\partial t} + \mathbf{j}\right) &= \mathbf{0} \end{aligned}$$

2nd approximation
Evolution in timestep Δt

$$\frac{\partial a}{\partial t} = \frac{a^{n+1} - a^n}{\Delta t}$$

Domain discretization
With Yee grid



Time-stepping scheme (leapfrog)

$$\begin{aligned} \mathbf{h}^{n+1} &= \mathbf{h}^n - \Delta t \tilde{\mathbf{D}}_s \mathbf{D}_\mu^{-1} \mathbf{D}_A^{-1} \mathbf{C} \mathbf{e}^{n+0.5} \\ \mathbf{e}^{n+1.5} &= \mathbf{e}^{n+0.5} + \Delta t \mathbf{D}_s \tilde{\mathbf{D}}_\epsilon^{-1} \tilde{\mathbf{D}}_A^{-1} \tilde{\mathbf{C}} \mathbf{h}^n - \tilde{\mathbf{D}}_\epsilon^{-1} \mathbf{j}_{src}^n \end{aligned}$$

✓ 2nd order convergence in time

✓ Can pre-compute most of the quantities → speed 🚀

✓ No matrix inversion needed during time loop

Gauss Laws (for E and H) considered satisfied* $\forall t$ for no charges \mathbf{j}_{ext}

FIT (III): Operators C , \tilde{C}

Time-stepping scheme in vacuum

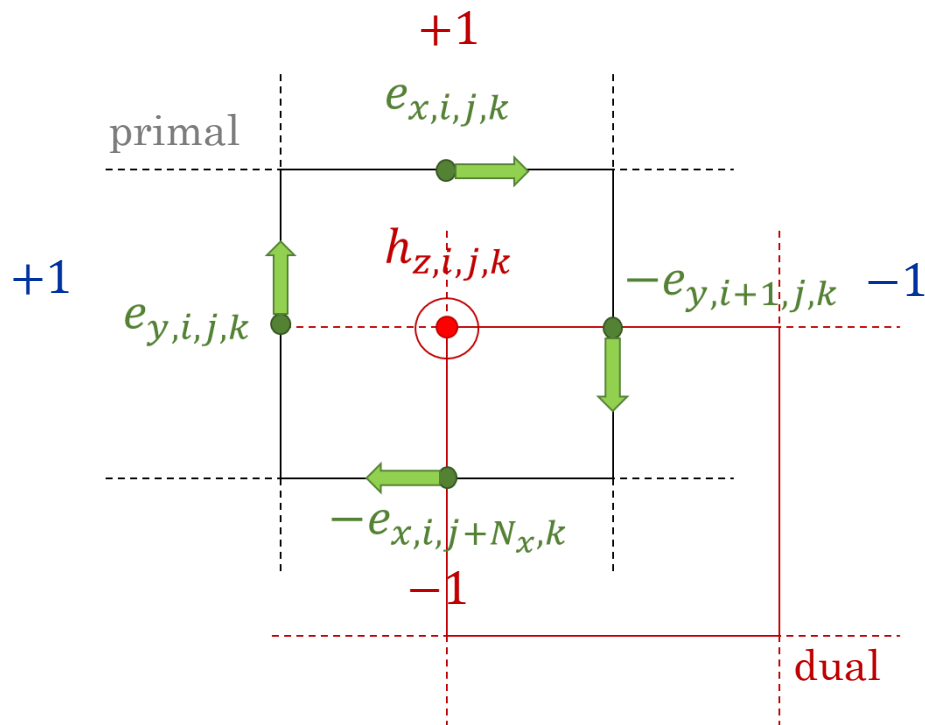
$$\mathbf{h}^{n+1} = \mathbf{h}^n - \Delta t \mu_0^{-1} \tilde{D}_s D_A^{-1} \mathbf{C} \mathbf{e}^{n+0.5}$$

$$\mathbf{e}^{n+1.5} = \mathbf{e}^{n+0.5} + \Delta t \varepsilon_0^{-1} D_s \tilde{D}_A^{-1} \tilde{\mathbf{C}} \mathbf{h}^n - \varepsilon_0^{-1} \mathbf{j}^n$$

The **curl operator C** on primal grid and **\tilde{C}** on the dual grid, with $\tilde{C} = C^t$ is a $3N_{cells} \times 3N_{cells}$ sparse matrix made of bands of **+1** and **-1**

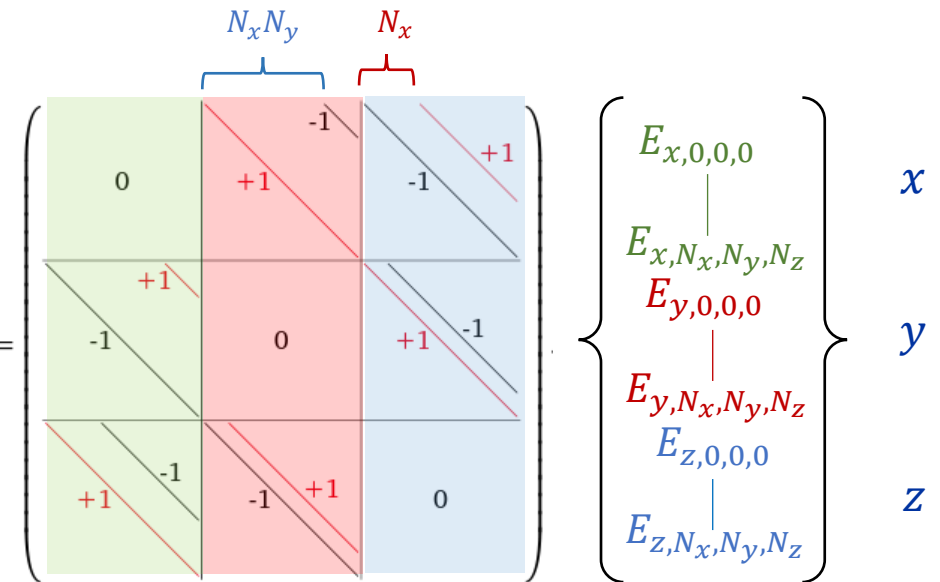
By modifying the columns or rows of C , one can add **boundary conditions**:

- **Perfect Electric Conductor (PEC)**: columns to zero at ijk of boundary cells
- **Perfect Magnetic Conductor (PMC)**: rows to zero at ijk of boundary cells



Sparcity of C

$$C = \begin{pmatrix} 0 & -P_z & P_y \\ P_z & 0 & -P_x \\ -P_y & P_x & 0 \end{pmatrix} =$$



Size: $3N_{cells} \times 3N_{cells}$

Lexicographic indexation

FIT (IV): Diagonal matrices $D_A, D_S, \tilde{D}_A, \tilde{D}_S$

Time-stepping scheme vacuum

$$\mathbf{h}^{n+1} = \mathbf{h}^n - \Delta t \mu_0^{-1} \tilde{\mathbf{D}}_S \mathbf{D}_A^{-1} \mathbf{C} \mathbf{e}^{n+0.5}$$

$$\mathbf{e}^{n+1.5} = \mathbf{e}^{n+0.5} + \Delta t \varepsilon_0^{-1} \mathbf{D}_S \tilde{\mathbf{D}}_A^{-1} \tilde{\mathbf{C}} \mathbf{h}^n - \varepsilon_0^{-1} \mathbf{j}^n$$

The diagonal matrices contain the **grid** information:

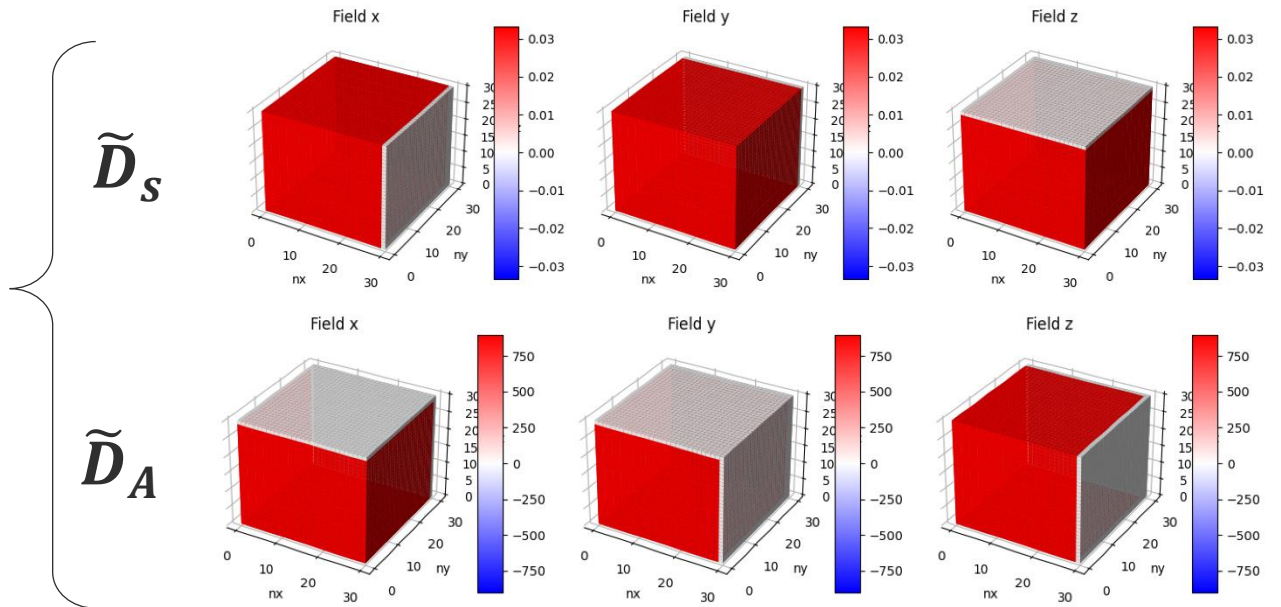
- $D_A^{-1} = \text{diag} \{ A_{x,0,0,0}^{-1}, \dots, A_{x,N_x,N_y,N_z}^{-1}, A_{y,0,0,0}^{-1}, \dots, A_{y,N_x,N_y,N_z}^{-1}, A_{z,0,0,0}^{-1}, \dots, A_{z,N_x,N_y,N_z}^{-1} \}$
- $D_S = \text{diag} \{ l_{x,0,0,0}, \dots, l_{x,N_x,N_y,N_z}, l_{y,0,0,0}, \dots, l_{y,N_x,N_y,N_z}, l_{z,0,0,0}, \dots, l_{z,N_x,N_y,N_z} \}$

Where (i.e. x-direction):

- $l_{x,i,j,k} = x_{i+1,j,k} - x_{i,j,k}$
- $A_{x,i,j,k} = l_{y,i,j,k} \times l_{z,i,j,k}$

- $\tilde{\mathbf{D}}_A$ and $\tilde{\mathbf{D}}_S$ are analogous for the **dual grid**. Due to the Yee staggering, some of the **components are zero**:

- If not set to zero, but equal to D_A and D_S at the opposite end → **Periodic boundary conditions**



*generated with wakis built-in 3D plotting

FIT (V): Fields E, H, J

Time-stepping scheme vacuum

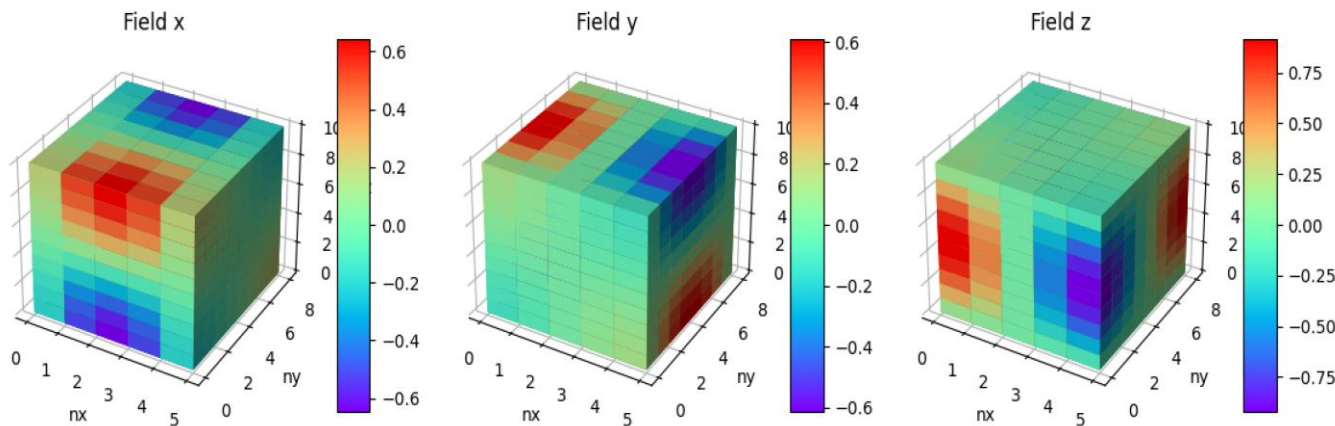
$$h^{n+1} = h^n - \Delta t \mu_0^{-1} \tilde{D}_s D_A^{-1} C e^{n+0.5}$$

$$e^{n+1.5} = e^{n+0.5} + \Delta t \varepsilon_0^{-1} D_s \tilde{D}_A^{-1} \tilde{C} h^n - \varepsilon_0^{-1} j^n$$

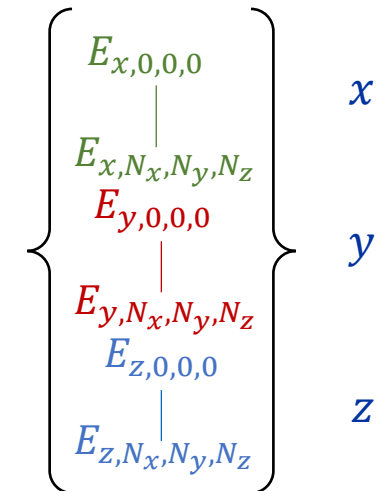
The fields are stored in memory as 3 (x, y, z) 3d numpy matrices `np.zeros(N_x, N_y, N_z)`. In the time-stepping, they must be converted to 1d arrays with lexicographic indexing:

$$n = 1 + (i - 1) + (j - 1)N_x + (k - 1)N_xN_y$$

Class `Field` has custom **magic methods** (`__getitem__`, `__setitem__`, `__mul__`, ...) to handle 3d to collapsed conversion, operations and built-in **visualization methods** (`inspect()`, `inspect3d()`)



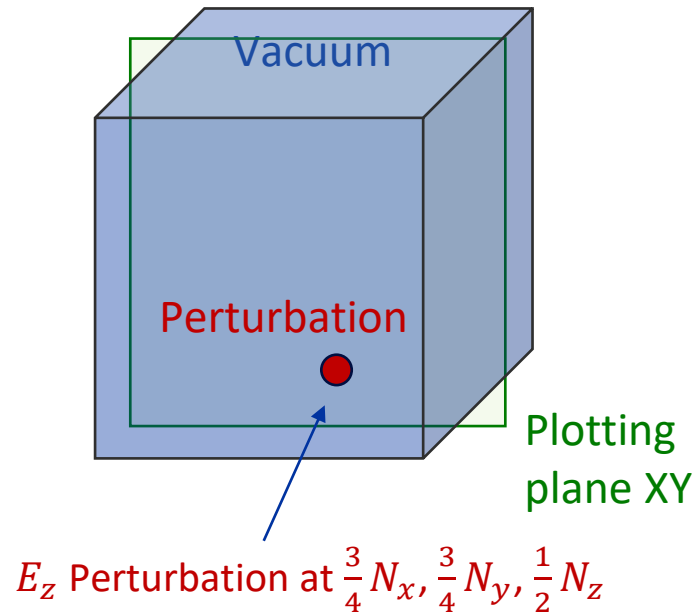
field.py
`class Field`



Lexicographic
indexation

*E.g., E field in a coarse-meshed cube resonator generated with `wakis` built-in 3D plotting solver. `E.inspect3d()`

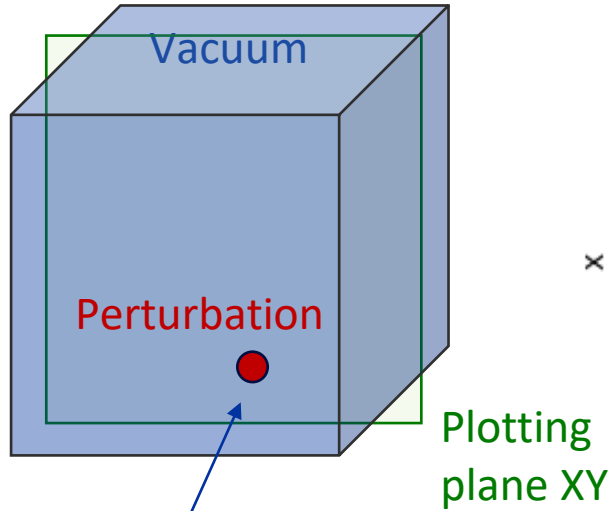
Example: Perturbation in free-space with BC



[examples/script_noeb_fit.py](#)

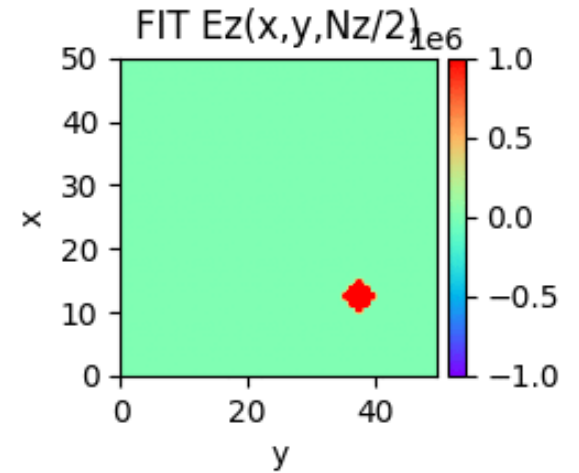
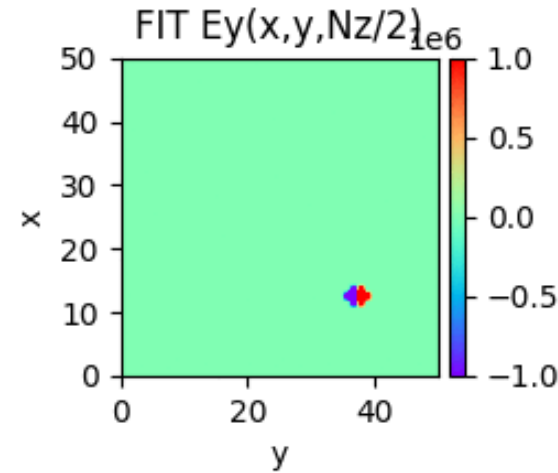
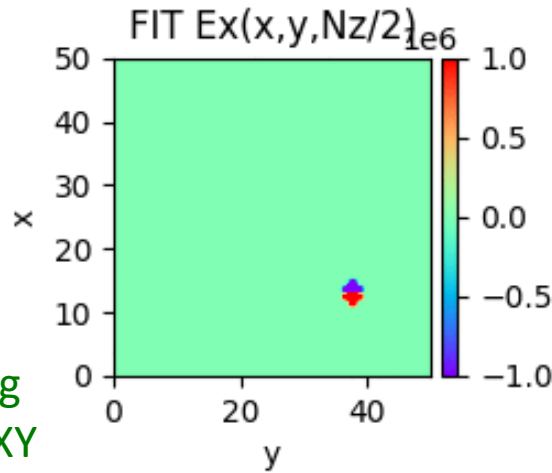
A static field perturbation in E_z generates a spheric wavefront that is reflected for PEC BCs or re-enters the domain for Periodic BCs

Example: Perturbation in free-space with BC



E_z Perturbation at $\frac{3}{4}N_x, \frac{3}{4}N_y, \frac{1}{2}N_z$

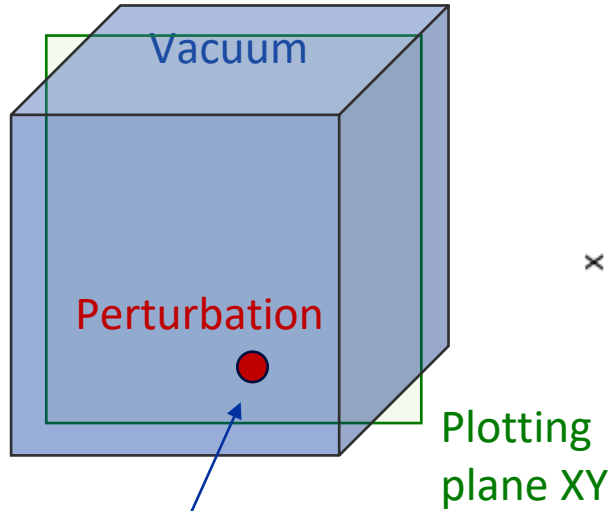
Boundary conditions (BCs): All Periodic



[examples/script_noeb_fit.py](#)

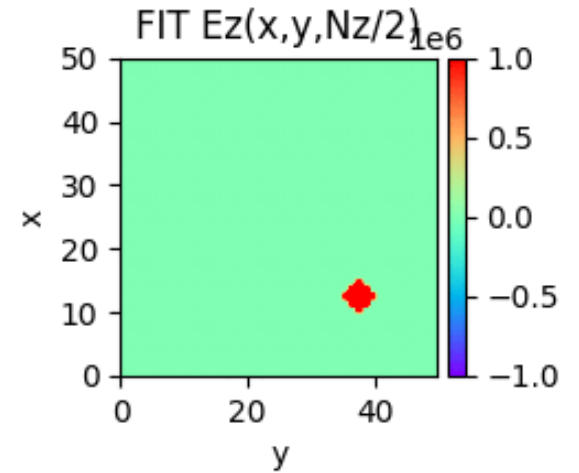
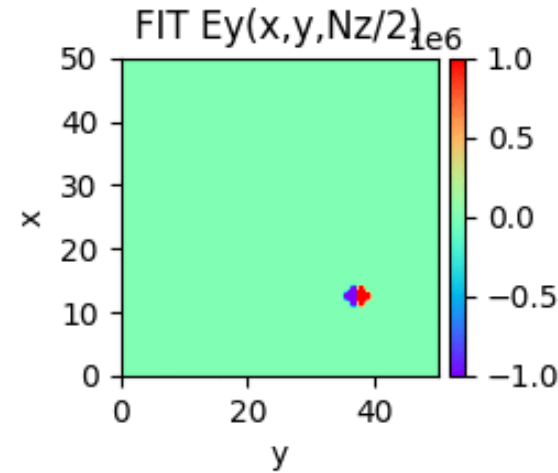
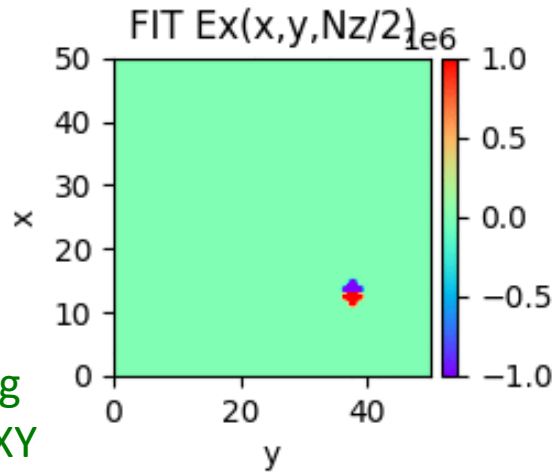
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Example: Perturbation in free-space with BC

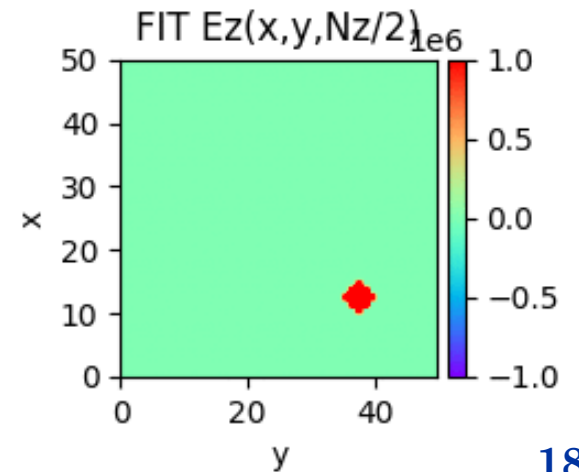
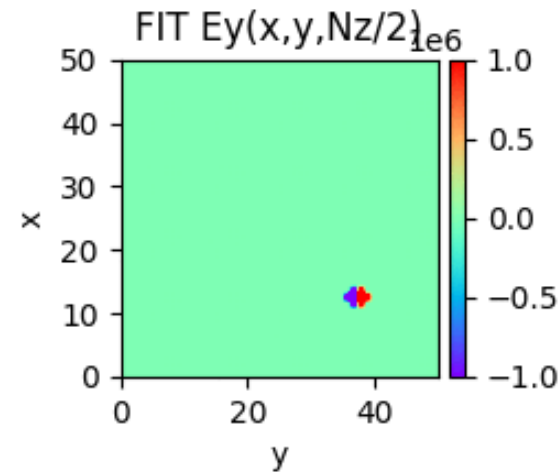
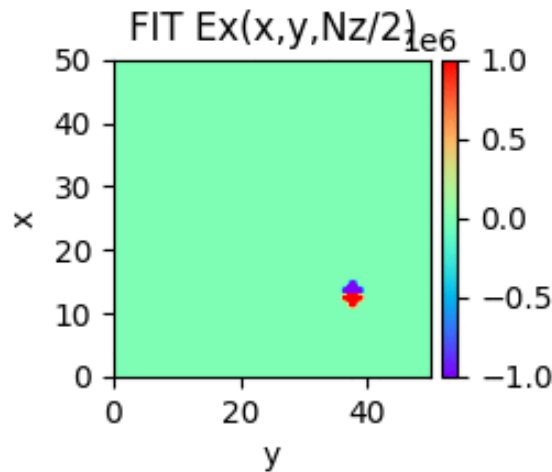


E_z Perturbation at $\frac{3}{4}N_x, \frac{3}{4}N_y, \frac{1}{2}N_z$

Boundary conditions (BCs): All Periodic



Boundary conditions (BCs): all PEC



 [examples/script_noeb_fit.py](https://github.com/IFN-GV/examples/script_noeb_fit.py)

A static field perturbation in E_z generates a spheric wavefront that is reflected for PEC BCs or re-enters the domain for Periodic BCs

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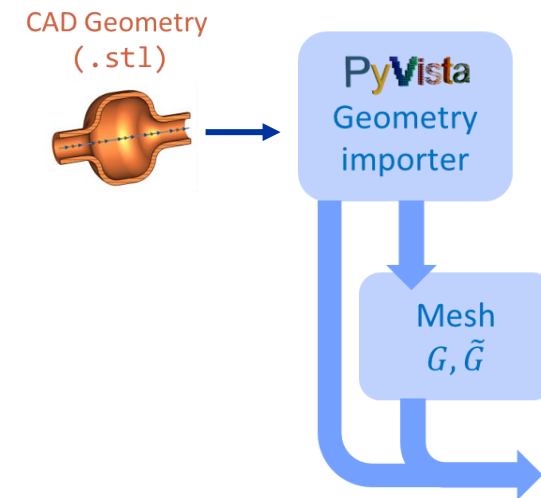
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2.6. Low- β simulations

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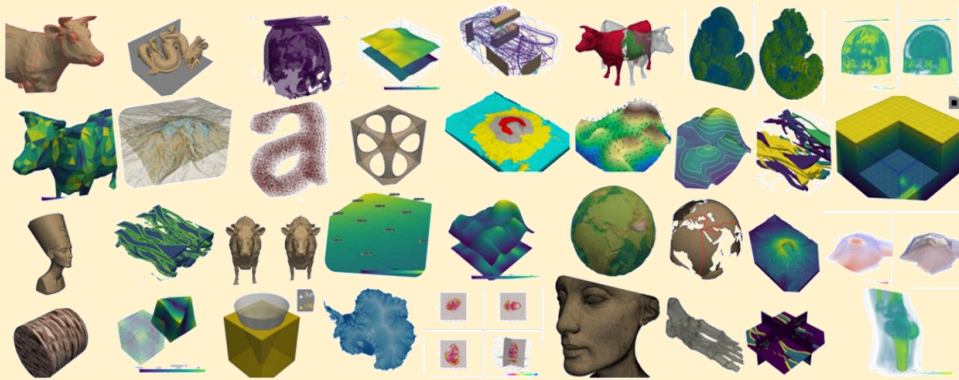
..... Slides 40 - 42



Importing geometry with PyVista

PyVista

3D plotting and mesh analysis through a streamlined interface for the Visualization Toolkit (VTK)



<https://github.com/pyvista/pyvista>

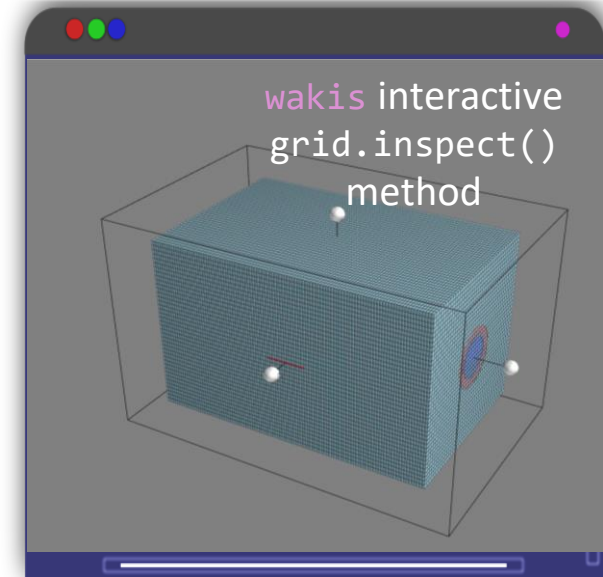
<https://docs.pyvista.org/version/stable/>

*Pyvista `extract cells inside surface filter`

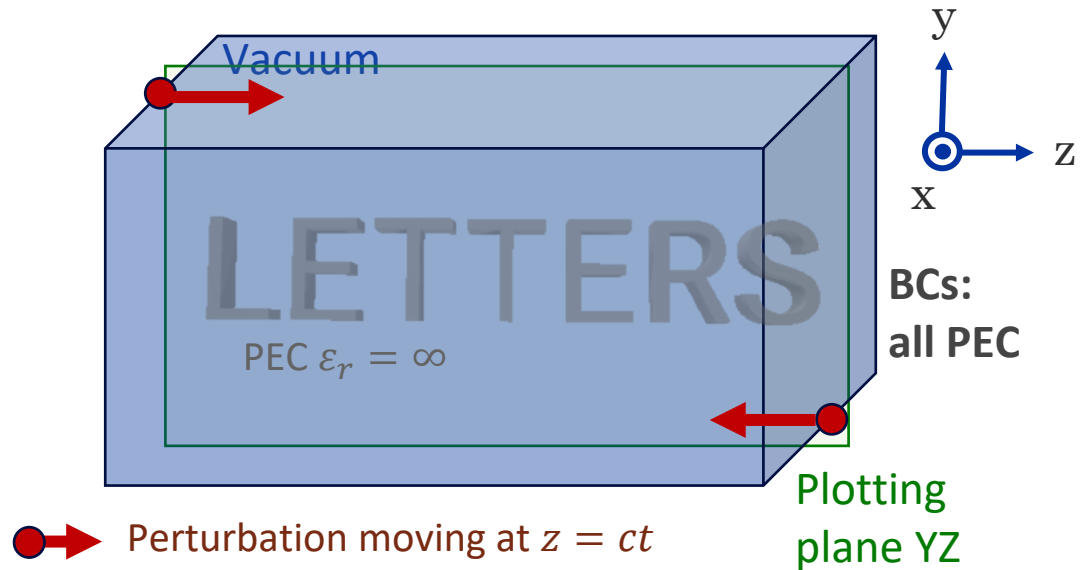
** Simplified goniometer geometry by C. Antuono

Using PyVista capabilities, `wakis` can:

- Import CAD geometry: `stl = pv.read(stl_file)`
- Generate a grid: `grid = pv.StructuredGrid(X, Y, Z)`
- Find the cells that are inside the geometry with advanced **collision filters*** → **mask for material properties**
- State-of-the-art **interactive 3d plotting**



Example: Perturbation with imported STL

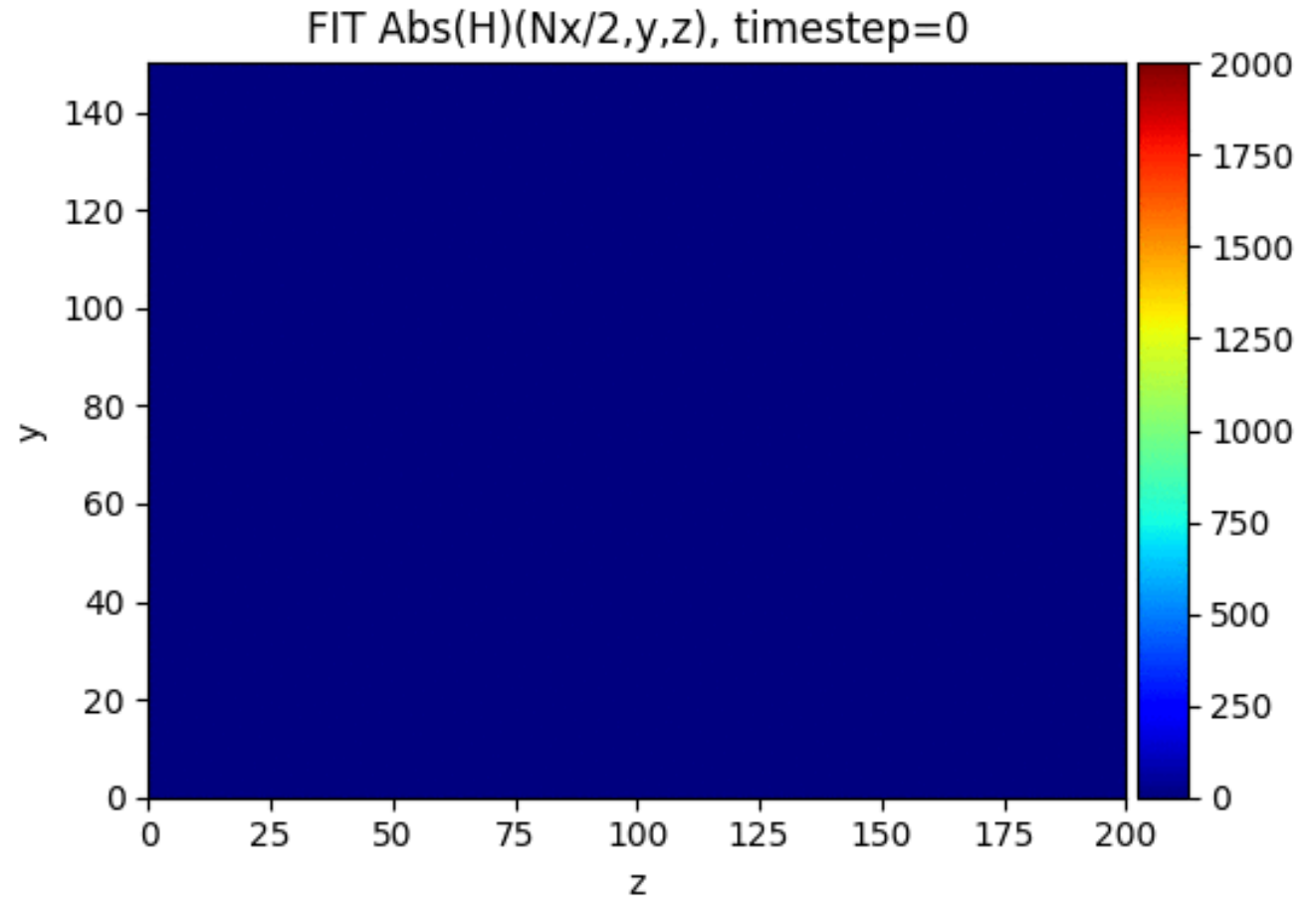
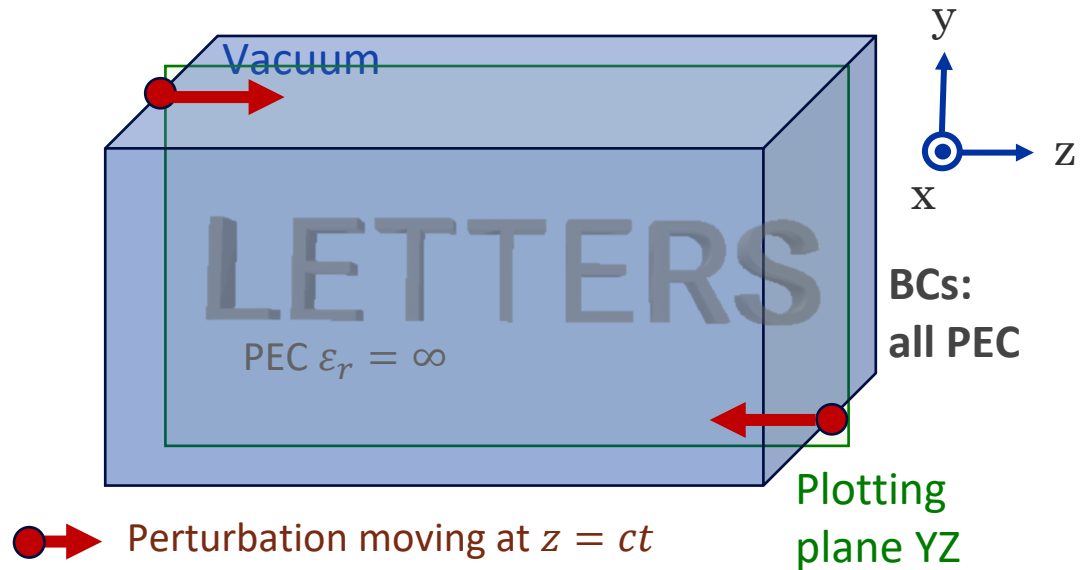


[examples/script_letters_fit.py](#)

A field perturbation in E_z moving at $z = ct$ excites the field in the vacuum domain revealing the PEC imported geometry

*STL files generated with online tool: <https://text2stl.mestres.fr/>

Example: Perturbation with imported STL



[examples/script_letters_fit.py](https://github.com/examples/script_letters_fit.py)

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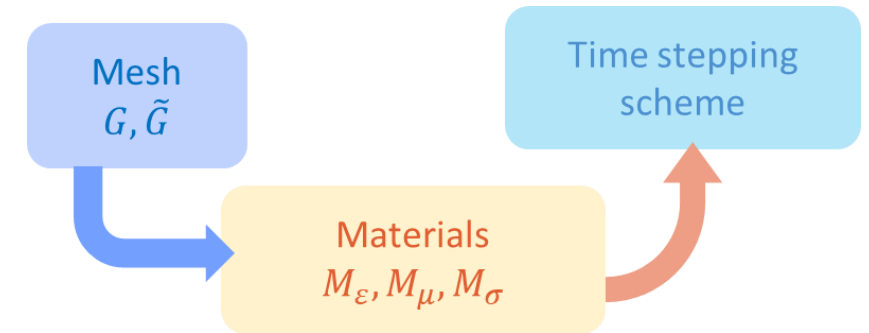
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Material tensors for ϵ, μ

Time-stepping scheme with ϵ, μ

$$\mathbf{h}^{n+1} = \mathbf{h}^n - \Delta t \tilde{\mathbf{D}}_s \mathbf{D}_\mu^{-1} \mathbf{D}_A^{-1} \mathbf{C} \mathbf{e}^{n+0.5}$$

$$\mathbf{e}^{n+1.5} = \mathbf{e}^{n+0.5} + \Delta t \mathbf{D}_s \tilde{\mathbf{D}}_\epsilon^{-1} \tilde{\mathbf{D}}_A^{-1} \tilde{\mathbf{C}} \mathbf{h}^n - \tilde{\mathbf{D}}_\epsilon^{-1} \mathbf{j}^n$$

The material matrices are considered **diagonal* tensors**:

- User can specify **relative permittivity/permeability**: $\epsilon_r = \epsilon / \epsilon_0$
- The matrix will be built with a **background material** $\epsilon_{r,bg}, \mu_{r,bg}$
- The **material associated with each imported stl geometry** will overwrite the background material in the tensor (numpy + pyvista)
- User can also specify **anisotropic materials** by defining $\epsilon_{r,x}, \epsilon_{r,y}, \epsilon_{r,z}, \dots$

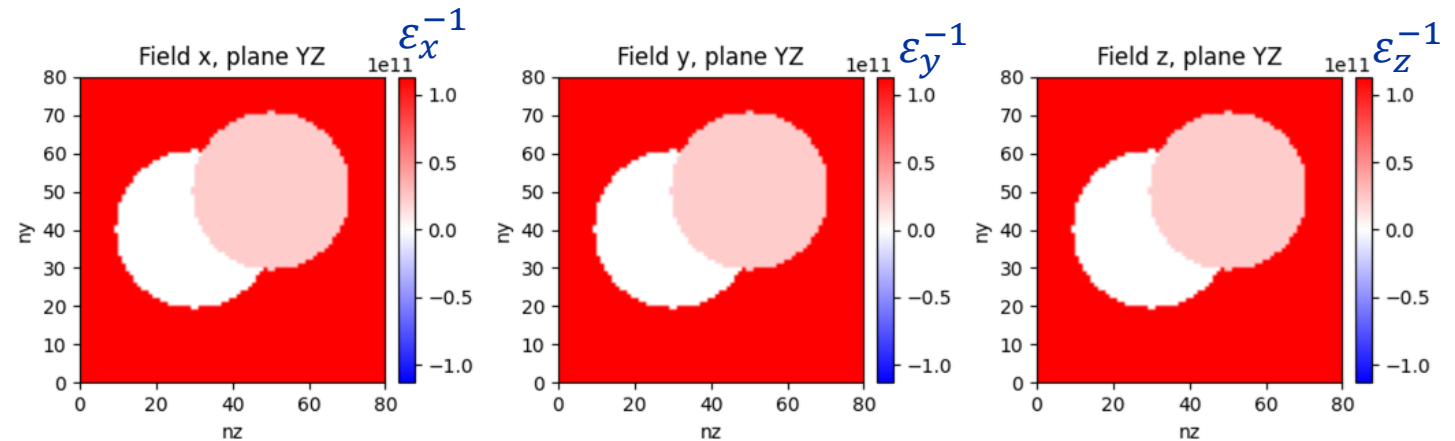
$$\mathbf{D}_\epsilon^{-1} = \text{diag} \left\{ \epsilon_{x,0.0,0}^{-1}, \dots, \epsilon_{x,N_x,N_y,N_z}^{-1}, \epsilon_{y,0.0,0}^{-1}, \dots, \epsilon_{y,N_x,N_y,N_z}^{-1}, \epsilon_{z,0.0,0}^{-1}, \dots, \epsilon_{z,N_x,N_y,N_z}^{-1} \right\} 3 \times N_{\text{cells}}$$

$$\mathbf{D}_\mu^{-1} = \text{diag} \left\{ \mu_{x,0.0,0}^{-1}, \dots, \mu_{x,N_x,N_y,N_z}^{-1}, \mu_{y,0.0,0}^{-1}, \dots, \mu_{y,N_x,N_y,N_z}^{-1}, \mu_{z,0.0,0}^{-1}, \dots, \mu_{z,N_x,N_y,N_z}^{-1} \right\} 3 \times N_{\text{cells}}$$

 [examples/test/test_materials.py](https://github.com/IFN-GV/examples/test/test_materials.py)

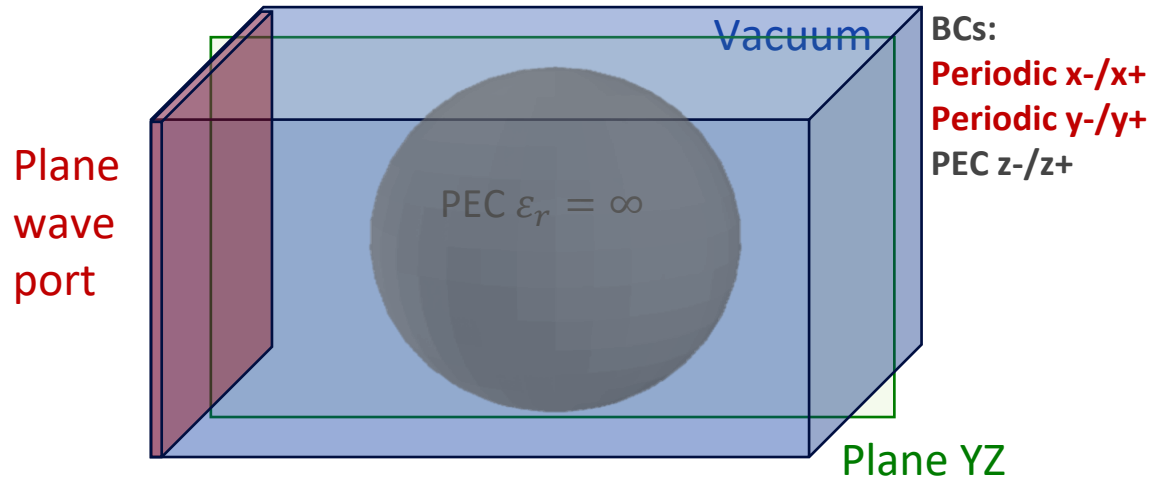
A PEC sphere $\epsilon^{-1} = 0$ and a Dielectric sphere $\epsilon^{-1} = (5\epsilon_0)^{-1}$ imported in a vacuum background $\epsilon^{-1} = \epsilon_0^{-1}$.

User can inspect the material tensors by:
`solver.ieps.inspect(plane='YZ')`

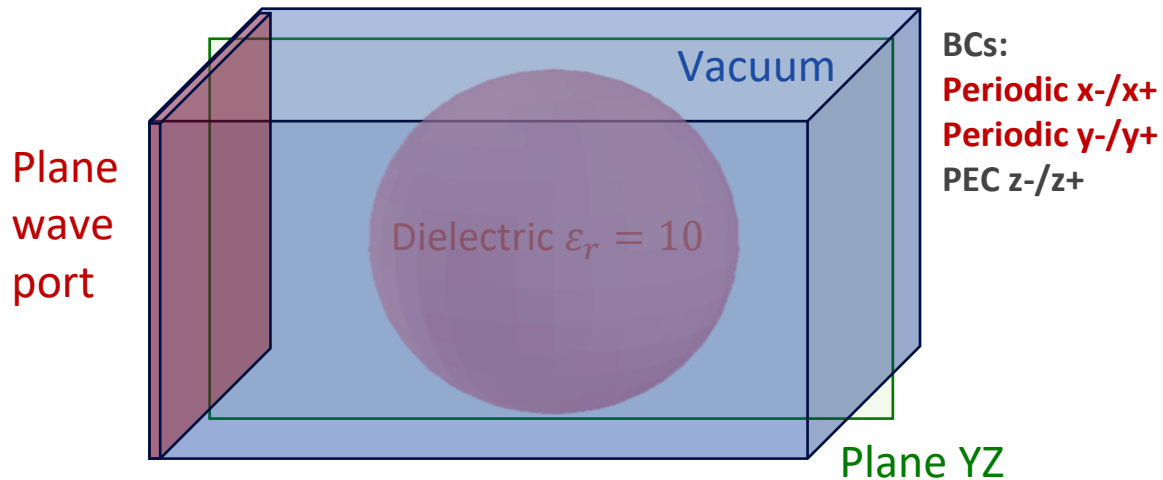


*This approximation is only not valid for Gyrotropic materials

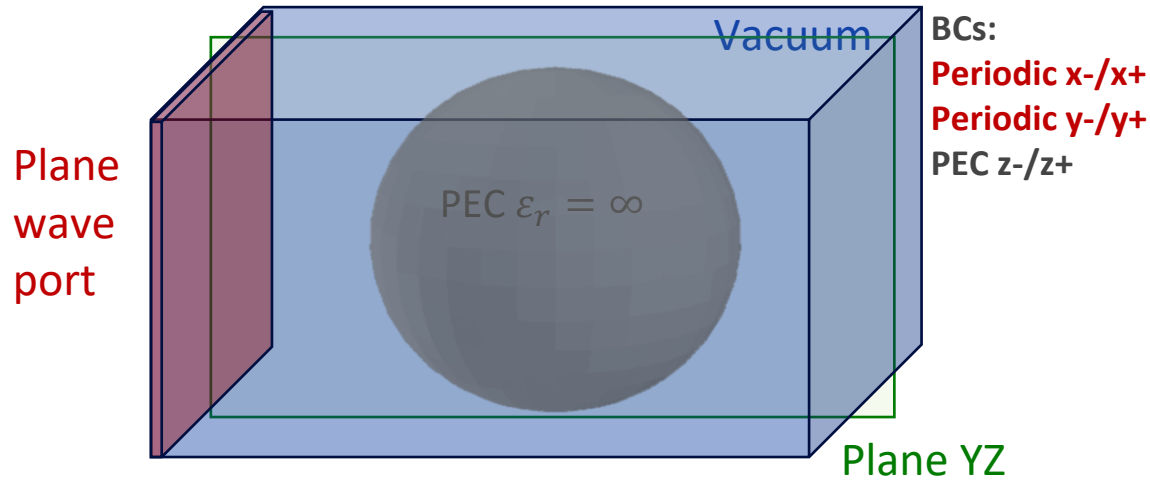
Example: Planewave interacting with sphere



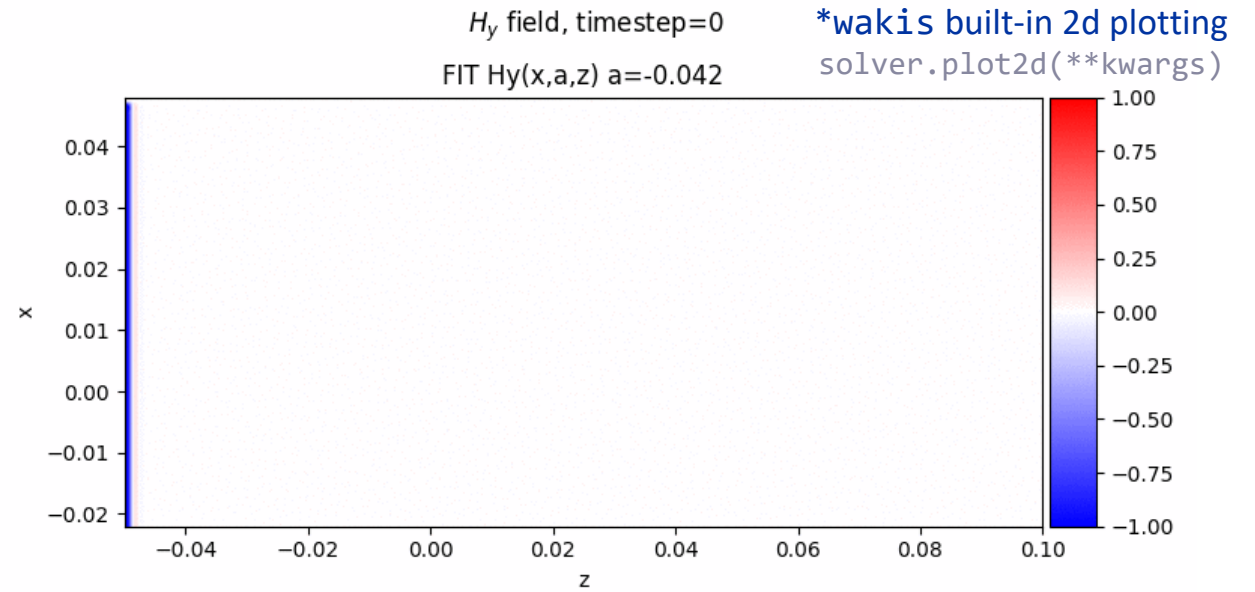
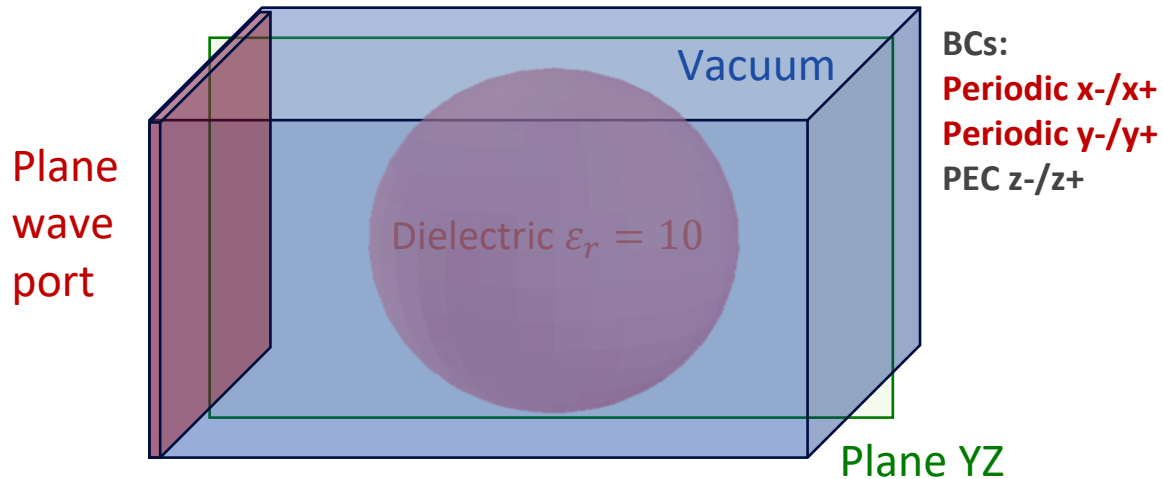
Special thanks to Prof. M. Cotelo (UPM) for the help with the planewave port setup and validation



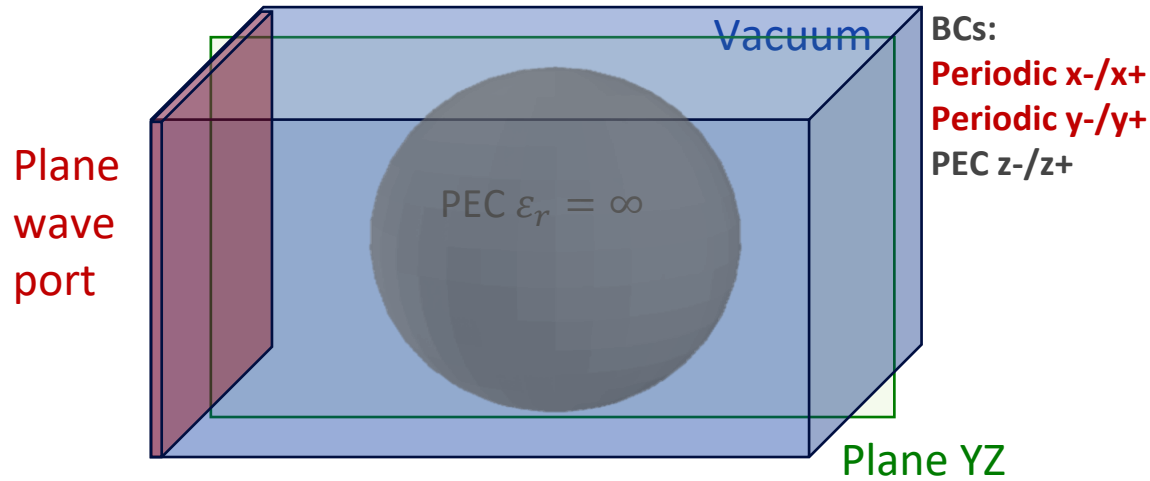
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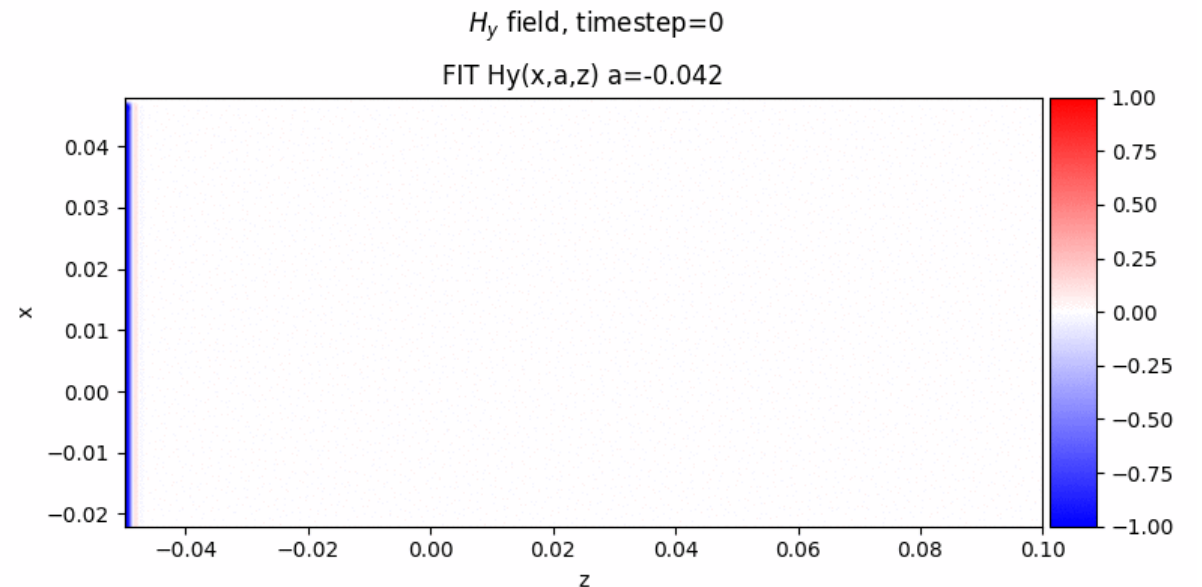
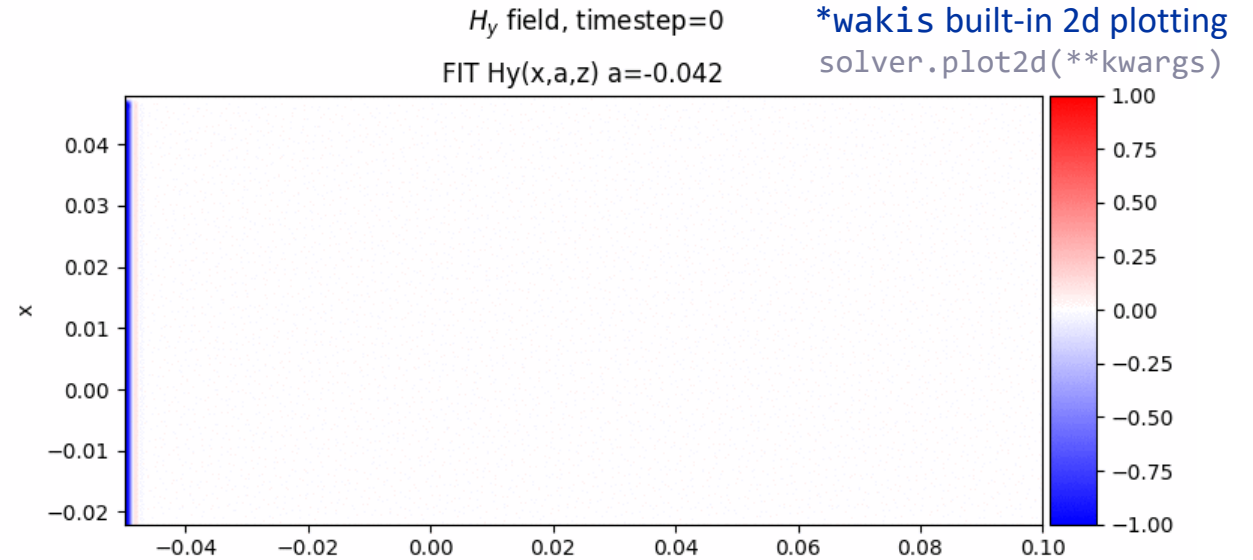
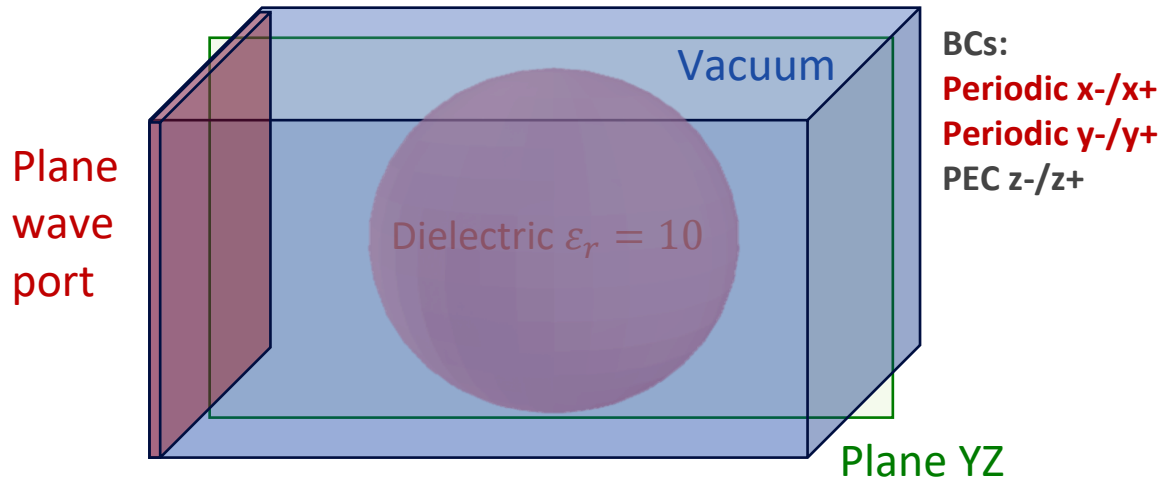
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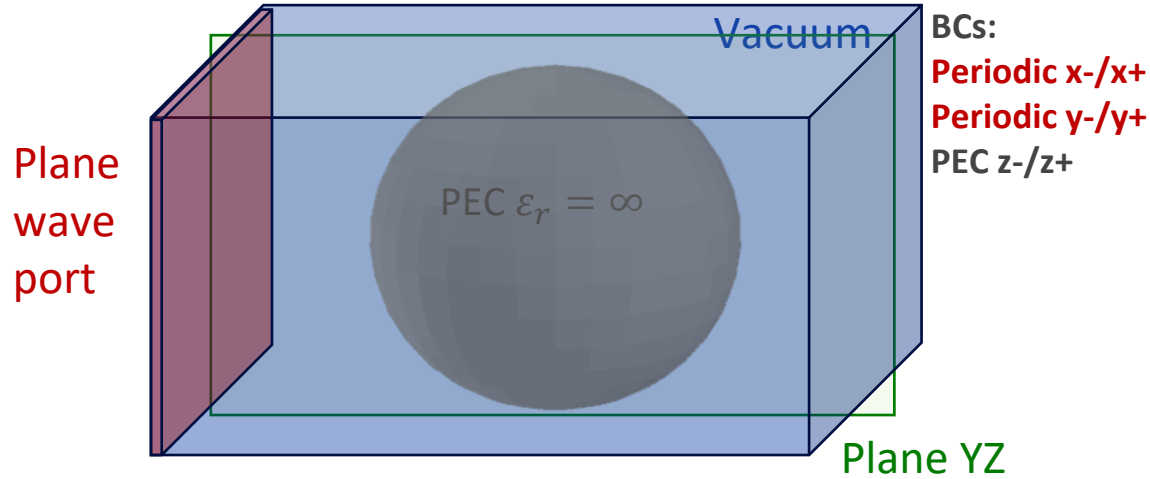
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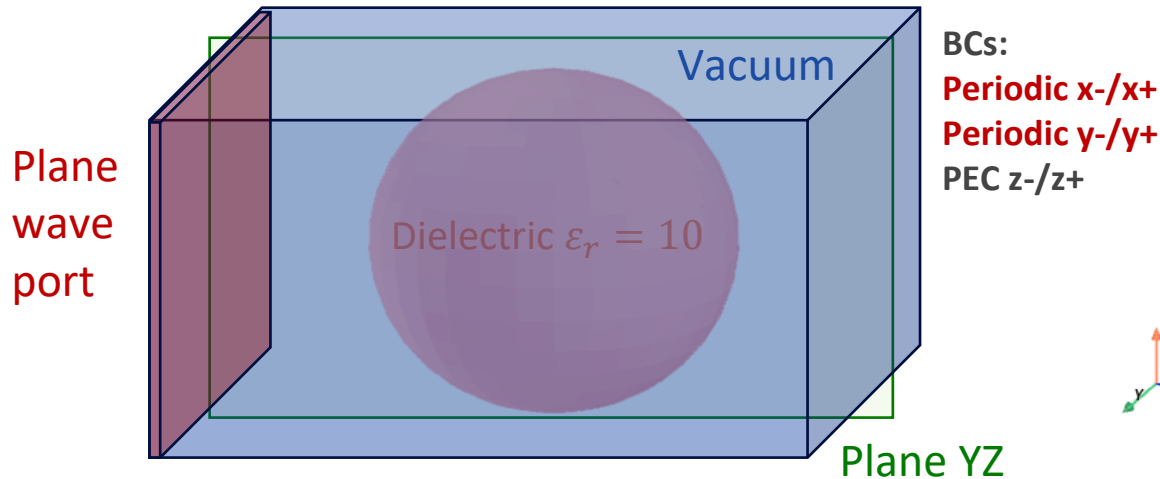
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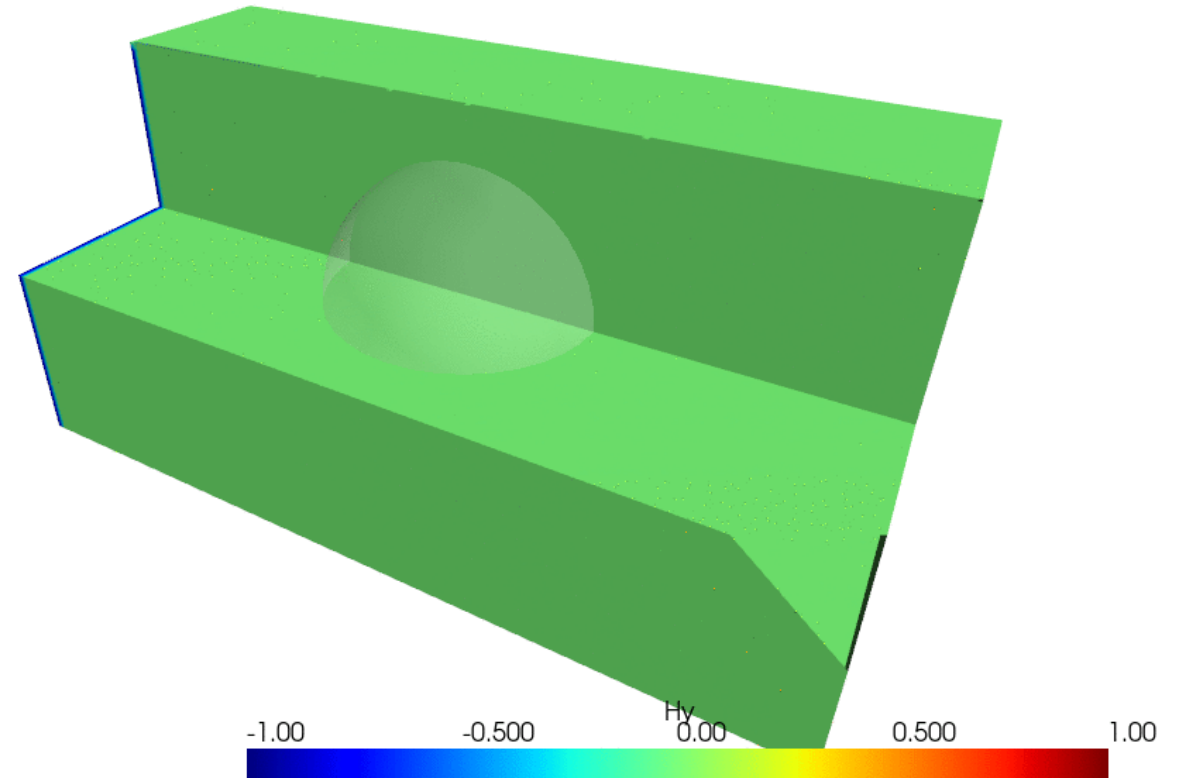


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Hy field, timestep=0

Generated with wakis built-in 3d plotting solver.plot3d(**kwargs) based on pyVista



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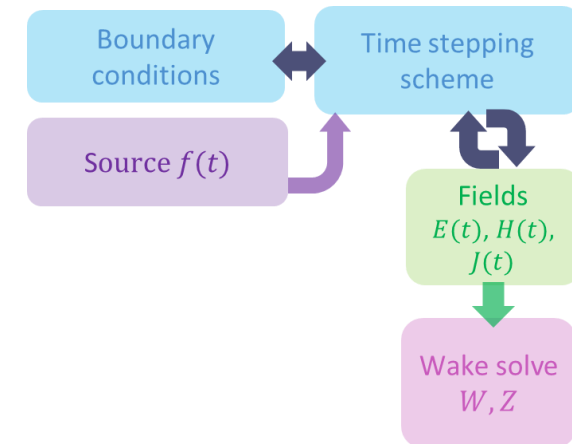
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Beam injection as current J_z

- **Sources (E, H, J)** are managed in a dedicated class inside `sources.py`

Each source should have a `source.update(t)` method that will be called every timestep dt in the simulation loop.

- For the case of a **particle beam**, the source is a **current in J_z** , applied at $x_s, y_s \forall z$ with a gaussian time profile.

- q is the charge in [C], typically $\approx 10^{-9}$ C
- σ_z is the beam size [m]
- β is the ratio of the beam speed v to the speed of light c

$$J_z(x_s, y_s, z) = \frac{q\beta c}{\sqrt{2\pi\sigma_z}} e^{-\frac{(s-s_0)^2}{2\sigma_z^2}}$$

$$s = z - \beta ct; \quad s_0 = z_{min} - \beta ct$$

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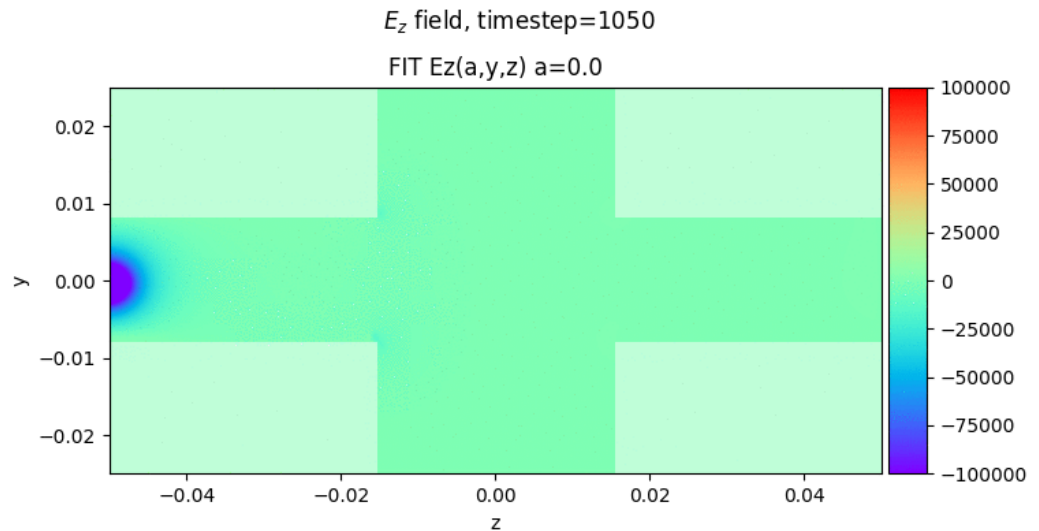
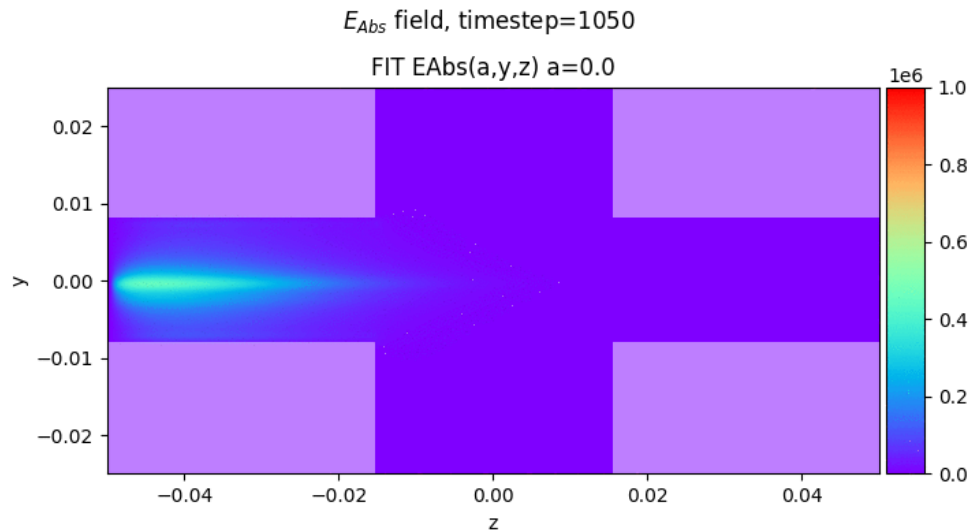
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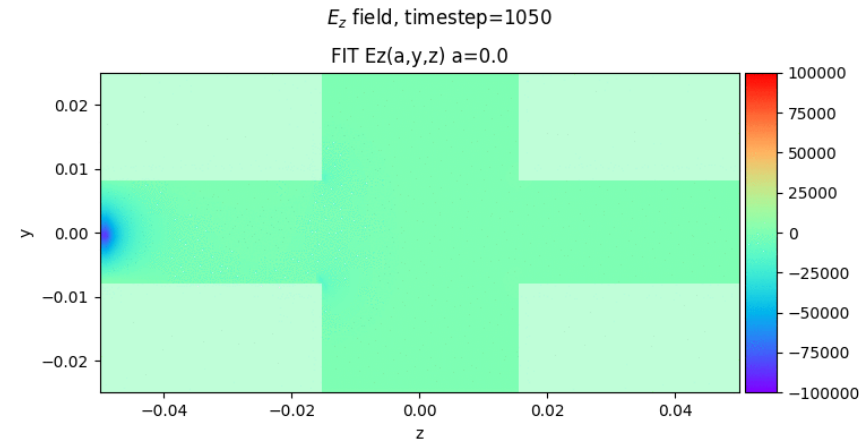
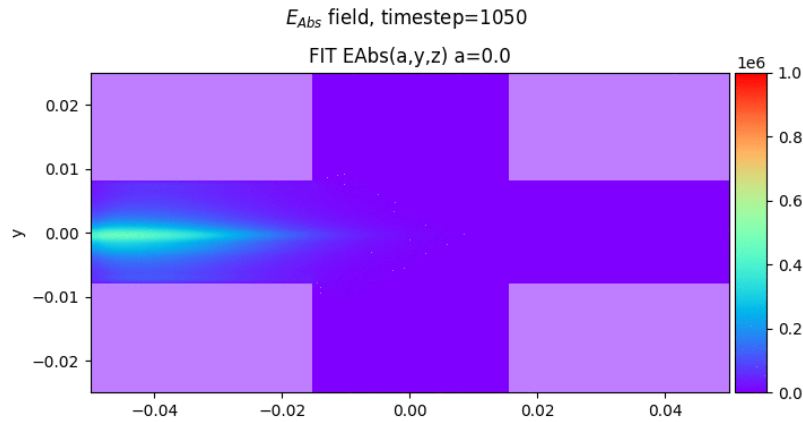
Minimizing injection perturbation: ABC

- The **perturbation** appears due to the violation of the continuity law $\tilde{S}\tilde{D}_A \left(\frac{\partial d}{\partial t} + j \right) = 0$
- We can **mitigate it with boundary conditions**: 1st attempt was the FOEXTRAP* absorbing boundary condition (ABC)

Minimizing injection perturbation: ABC

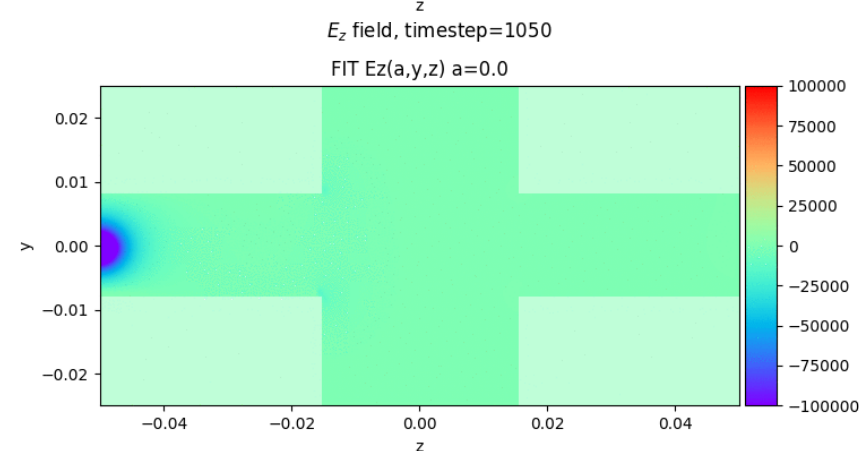
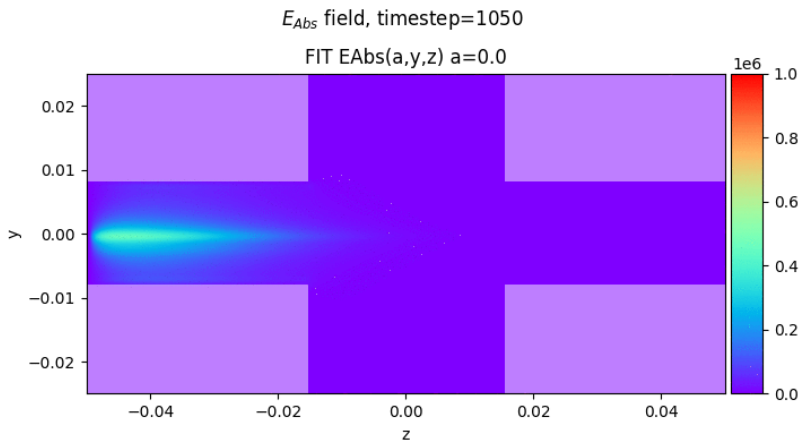
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ABC

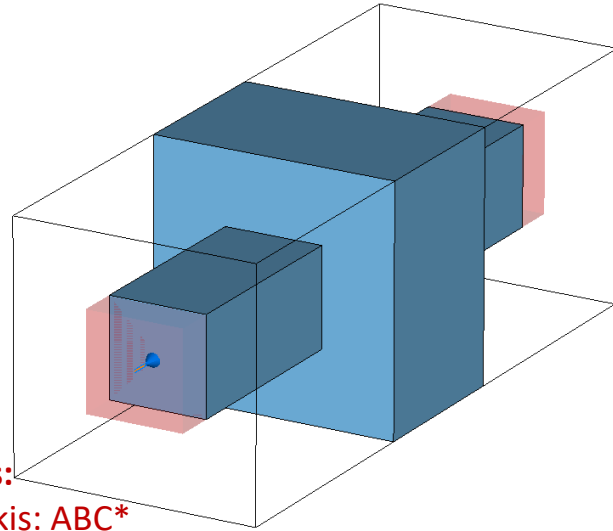


Thanks to Prof. M. Cotelo (UPM) for the ABC idea

PEC



Example: PEC cubic cavity



Geometry:
 $L_{cav} = 30$ mm
 $h_{cav} = 50$ mm
 $w_{cav} = 50$ mm
 $L_{pipe} = 100$ mm
 $h_{pipe} = 15$ mm
 $w_{pipe} = 15$ mm

Beam:
 $\sigma_z = 18.5$ mm (5 GHz)
 $q = 1e-9$ C

Mesh:
nx, ny, nz = 50, 50, 150
total: 375000 cells

Cutoff frequency: ~10 GHz

BCs:
wakis: ABC*
WarpX: PML*
CST: CPML

Remarks

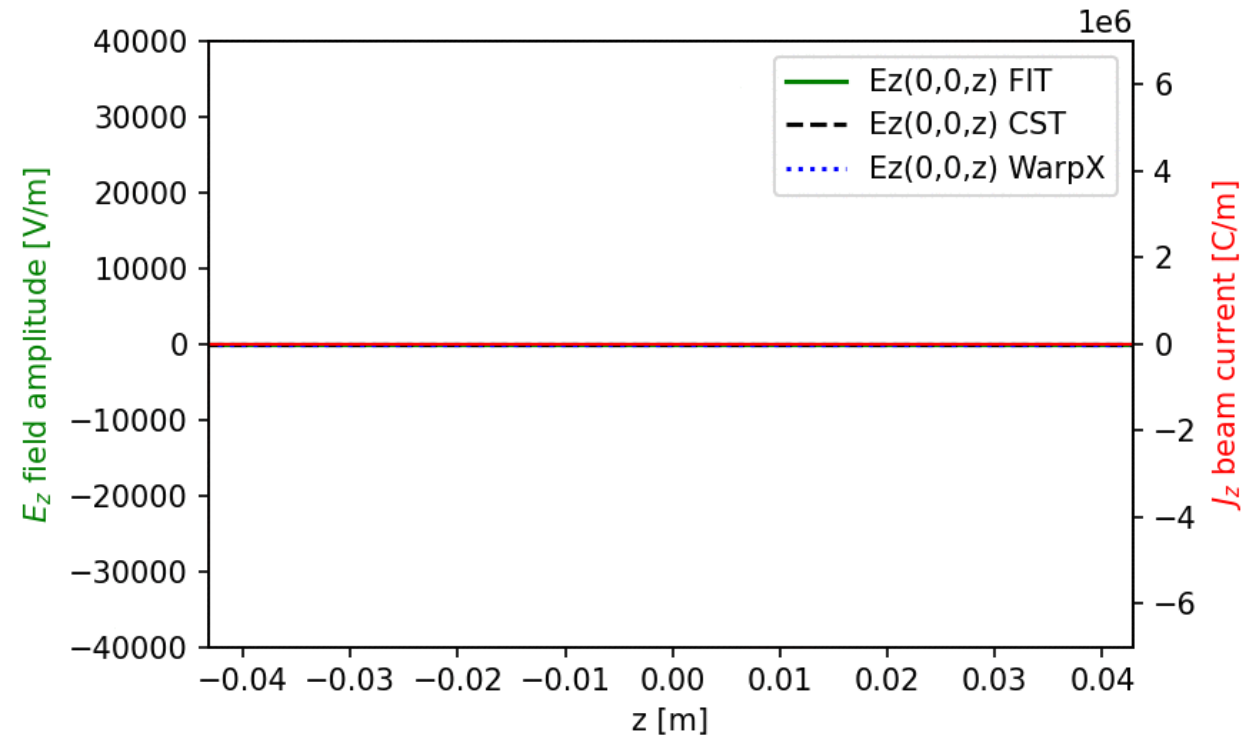
- ✓ Very good agreement
- WarpX shows a **non-vanishing injection perturbation** while in wakis it disappears
- Some phase error comes from the **CST field monitor extraction** (very close to simulation's timestep)



Simulation time: for 1m wavelength (8760 timesteps):

- **CST:** 42s, 16 threads, in `abpimp60g01`
- **Wakis:** 3m40s, single core in `abpimp60g01`
- **WarpX:** 12m, single core, #MP 1e6, in `abpimp60g01`

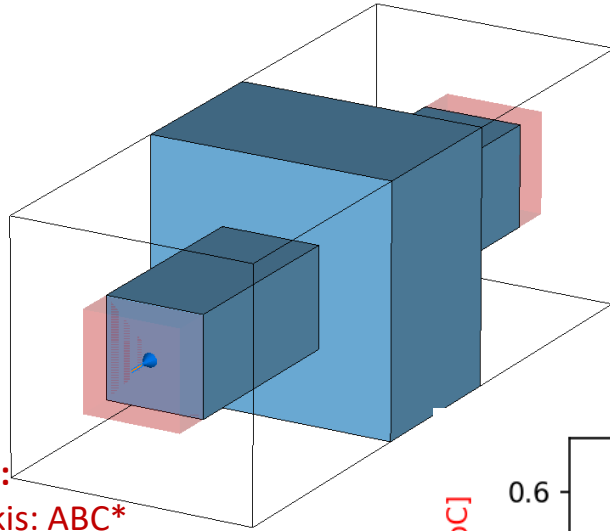
timestep=0



Example: PEC cubic cavity



benchmarks/cubcavitymm/



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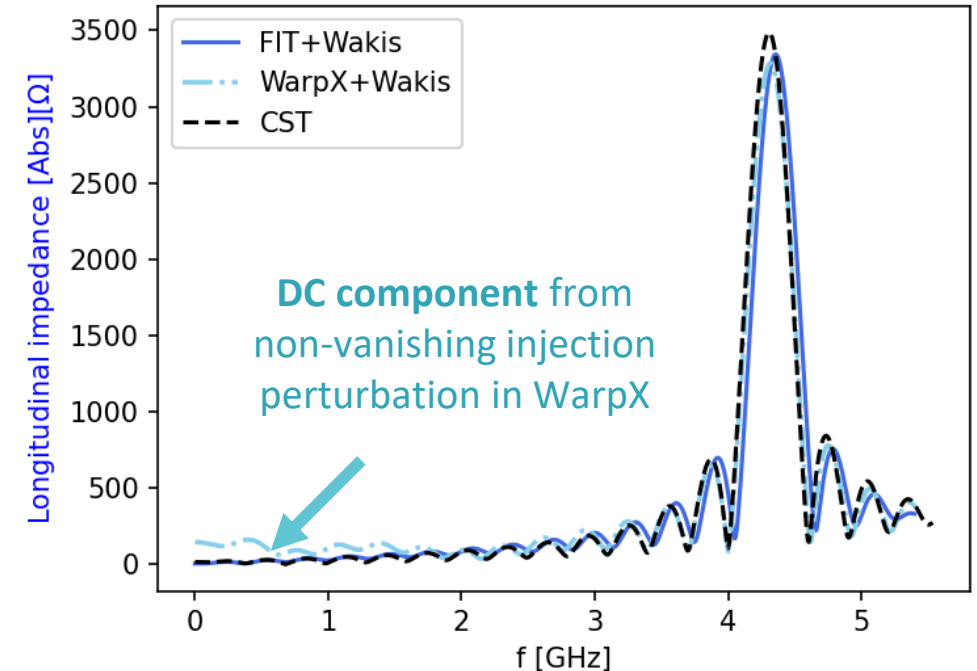
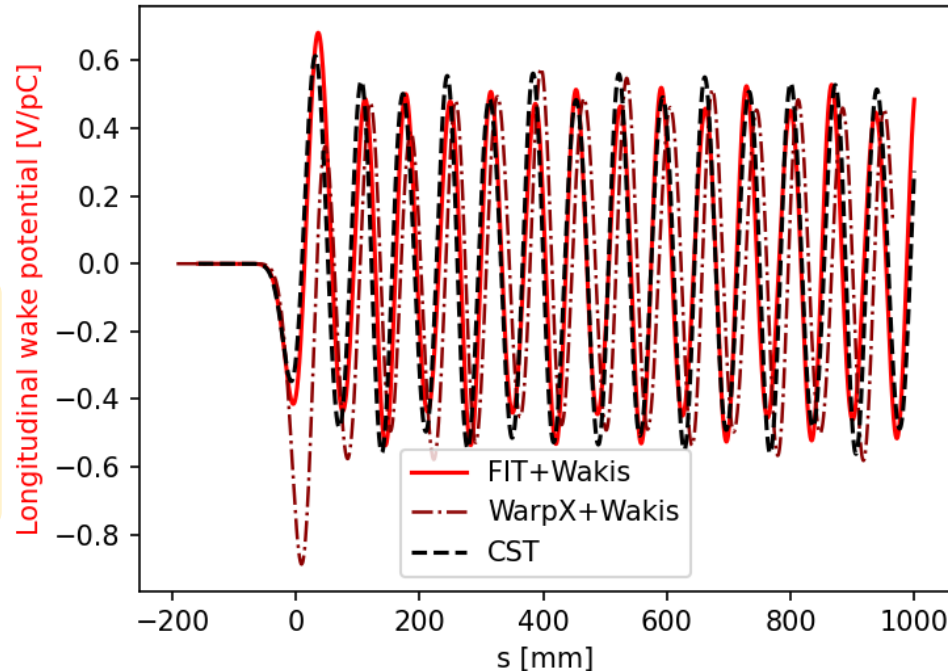
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BCs:
wakis: ABC*
WarpX: PML*
CST: CPML

*last 10 cells in z- and z+ were removed for the wake calculation to mitigate injection perturbation

Simulation wakis vs WarpX vs CST: (keeping same mesh)



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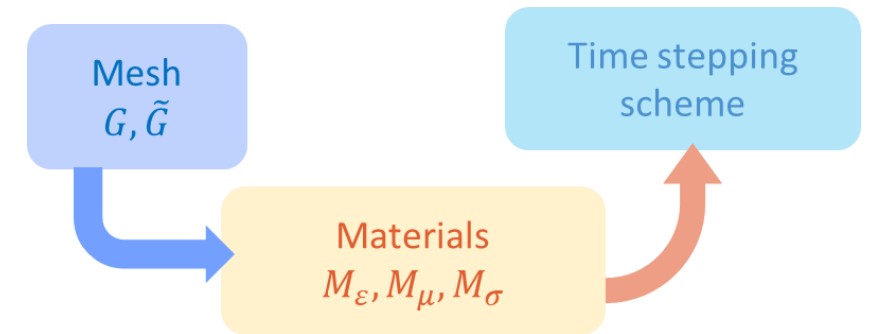
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Conductivity in time domain

Relative permittivity is often a **frequency-dependent, complex quantity**:

$$\varepsilon_r(\omega) = \varepsilon'_r(\omega) - j\varepsilon_r''(\omega)$$

The **simple model** for a **metal with broadband conductivity σ** is:

$$\varepsilon_r(\omega) = \varepsilon_r - j \frac{\sigma}{\omega \varepsilon_0}$$

Time-stepping scheme:

$$\mathbf{h}^{n+1} = \mathbf{h}^n - \Delta t \tilde{\mathbf{D}}_s \mathbf{D}_\mu^{-1} \mathbf{D}_A^{-1} \mathbf{C} \mathbf{e}^{n+0.5}$$

$$\mathbf{e}^{n+1.5} = \mathbf{e}^{n+0.5} + \Delta t \mathbf{D}_s \tilde{\mathbf{D}}_\varepsilon^{-1} \tilde{\mathbf{D}}_A^{-1} \tilde{\mathbf{C}} \mathbf{h}^n - \tilde{\mathbf{D}}_\varepsilon^{-1} \mathbf{j}_{beam}^n$$



How to implement this in a **time domain (TD)** simulations?

- In TD, fields are purely real $\Re \rightarrow$ all matrix elements should be real
- Introducing frequency dependence involves a deconvolution... (computationally expensive 💰 💰)

Conductivity in time domain

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$$\varepsilon_r(\omega) = \varepsilon'_r(\omega) - j\varepsilon''_r(\omega)$$

The **simple model** for a **metal with broadband conductivity σ** is:

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From the **Frequency domain Maxwell equations**:

Ampere's law: $\nabla \times H = J_{beam} + j\omega D = J_{beam} + j\omega \varepsilon E$;

$$\nabla \times H(\omega) = J_{beam} + j\omega \varepsilon_0 \left(\varepsilon_r + \frac{\sigma}{j\omega \varepsilon_0} \right) E(\omega);$$

$$\nabla \times H(\omega) = J_{beam} + j\omega \varepsilon' E(\omega) + \sigma E(\omega)$$

To convert to TD: $j\omega \leftrightarrow d/dt$, $A(\omega) \leftrightarrow A(t)^*$

$$\nabla \times H(t) = J_{beam} + \frac{d(\varepsilon' E(t))}{dt} + \sigma E(t)$$

Time-stepping scheme:

$$h^{n+1} = h^n - \Delta t \tilde{D}_s D_\mu^{-1} D_A^{-1} C e^{n+0.5}$$

$$e^{n+1.5} = e^{n+0.5} + \Delta t D_s \tilde{D}_\varepsilon^{-1} \tilde{D}_A^{-1} \tilde{C} h^n - \tilde{D}_\varepsilon^{-1} j_{beam}^n$$



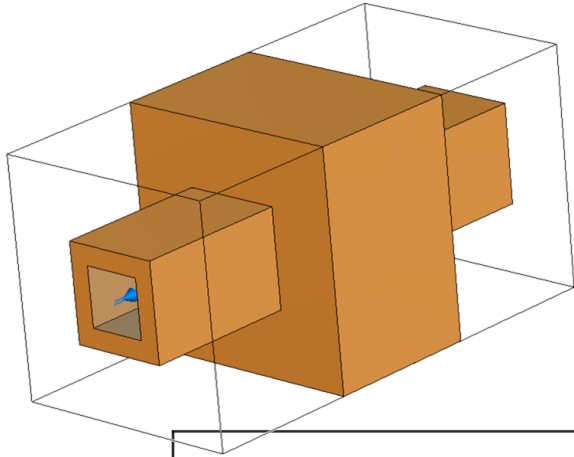
Time-stepping scheme **with σ** :

$$h^{n+1} = h^n - \Delta t \tilde{D}_s D_\mu^{-1} D_A^{-1} C e^{n+0.5}$$

$$e^{n+1.5} = e^{n+0.5} + \Delta t D_s \tilde{D}_\varepsilon^{-1} \tilde{D}_A^{-1} \tilde{C} h^n - \tilde{D}_\varepsilon^{-1} j_{beam}^n - \tilde{D}_\varepsilon^{-1} \tilde{D}_\sigma e^{n+0.5}$$

+ constitutive relation for J
 $J(t) = \sigma E(t) + J_{beam}(t)$

Example: lossy cubic cavity



Conductivity: 10 S/m

Skin-Depth: $\delta = \sqrt{\frac{1}{\pi f \mu \sigma}} \approx 2 \text{ mm}$

N_{cells} : 542754

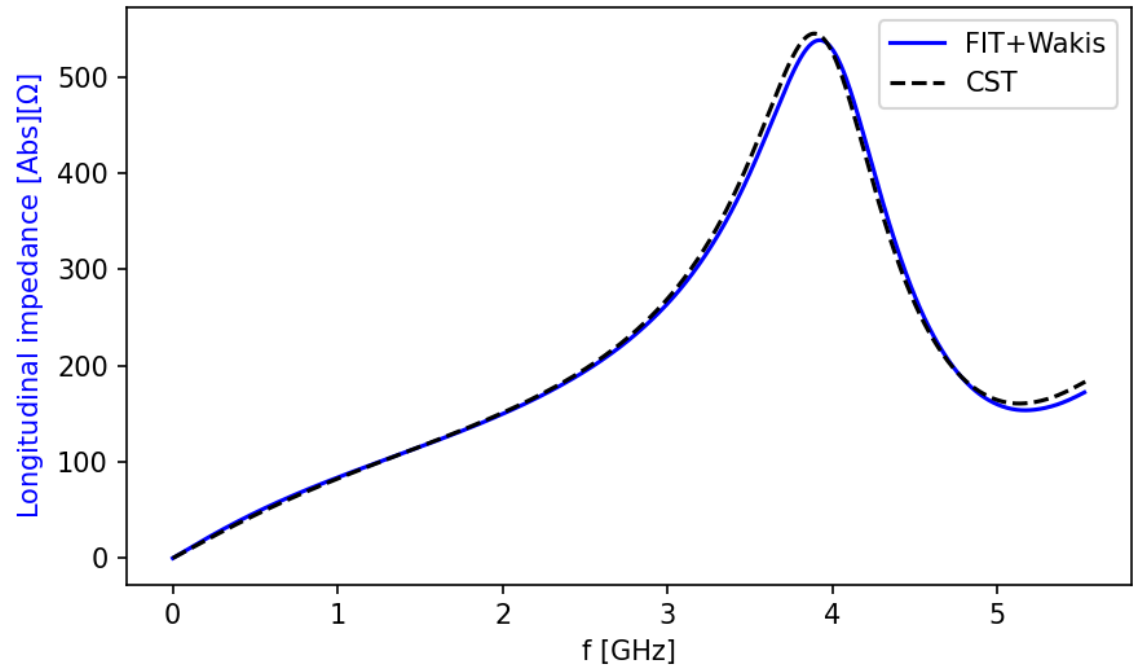
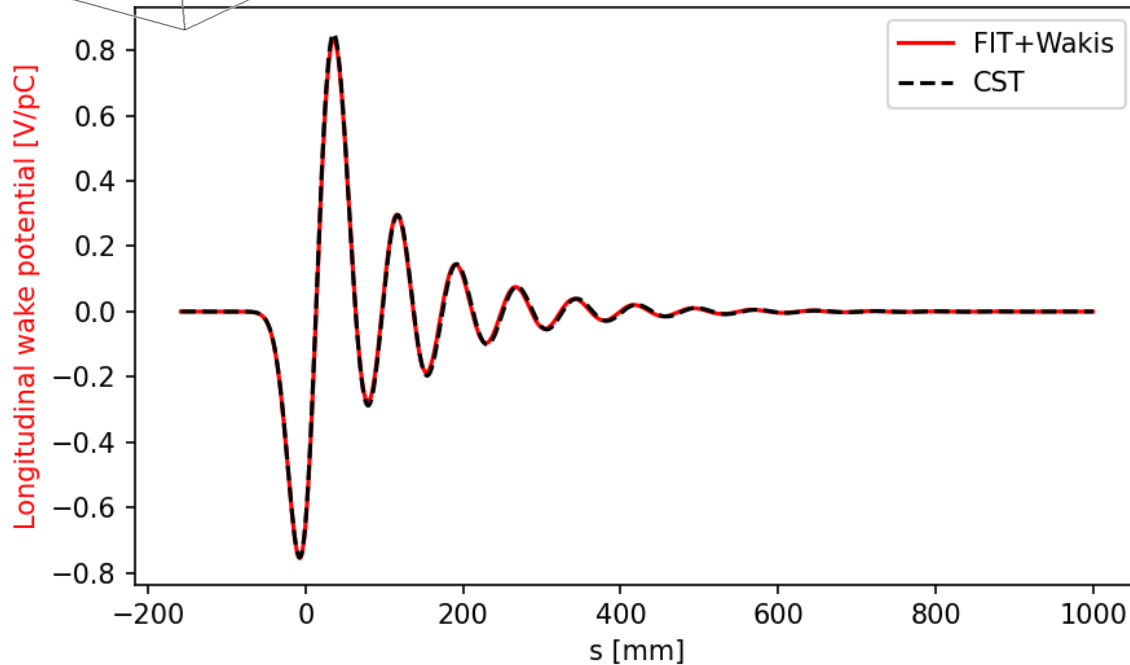
Runtime: 4', 30'', single core in `abpimp60g01`

Δ_{cell} : 0.5 mm > 3δ

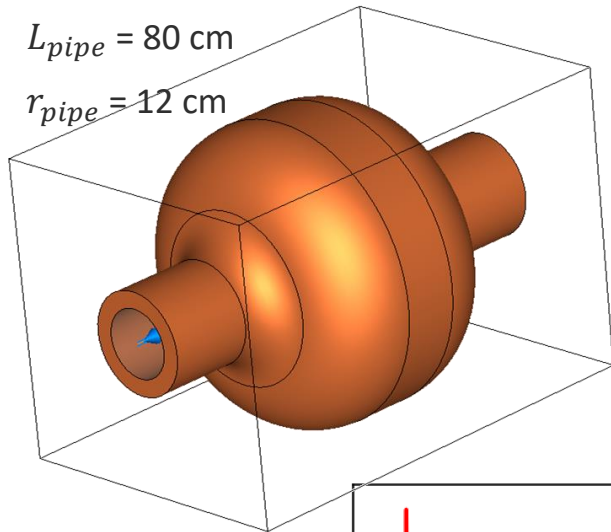
Remarks:

- To match CST results, we had to remove 10 cells z-/z+ and increase the mesh by 20%
- Low conductivity was chosen to have a fast decaying wake

Benchmark with CST Wakefield Solver



Example: lossy fancy-shaped* cavity



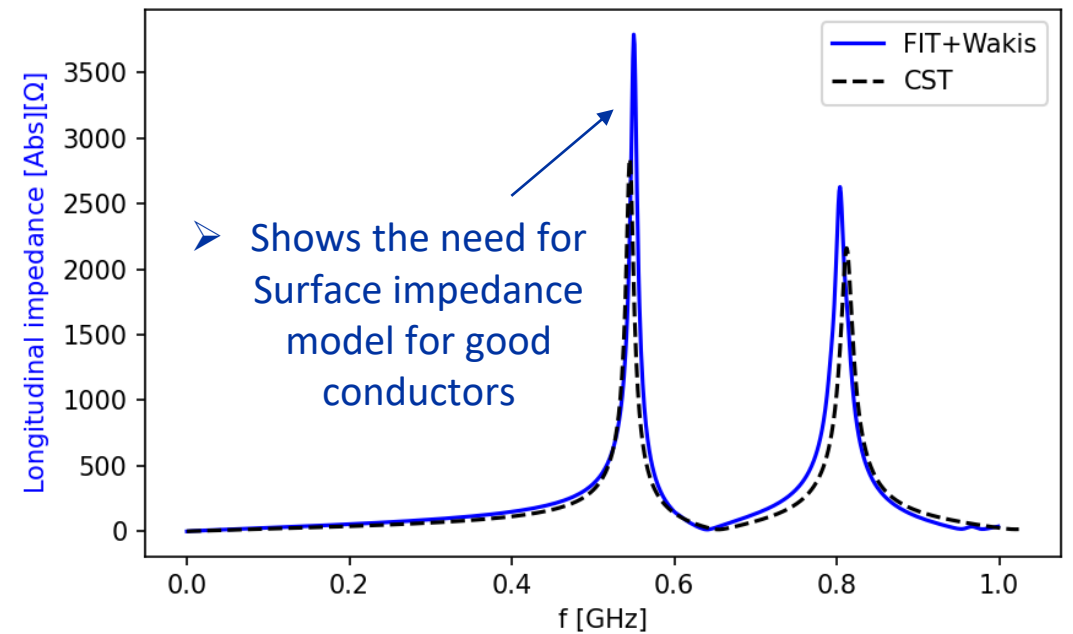
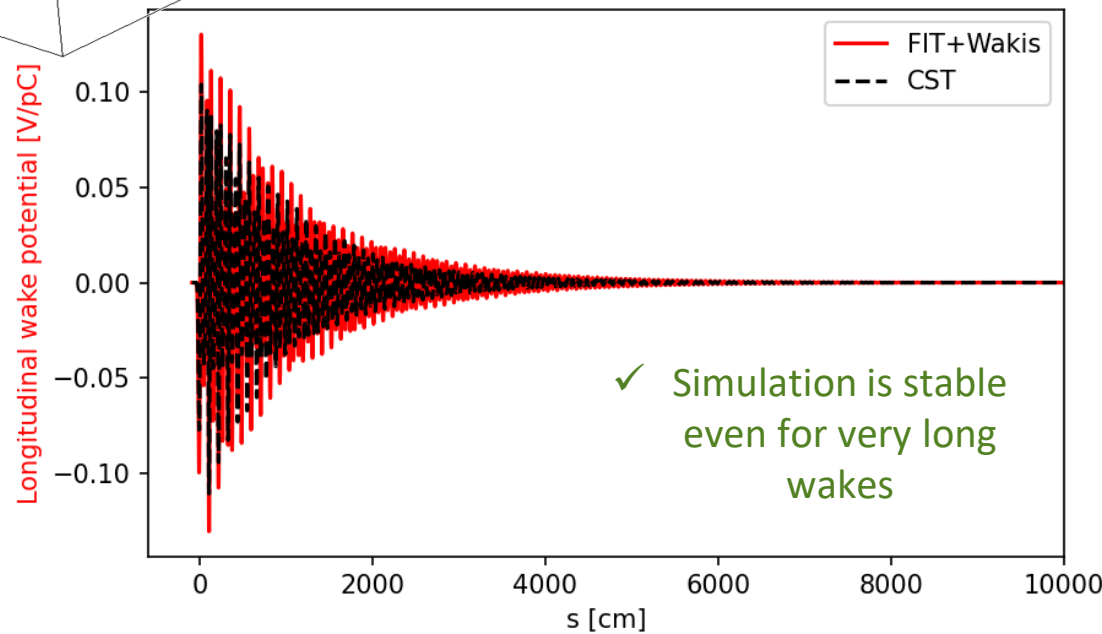
Conductivity: 1000 S/m
Skin-Depth: $\delta = \sqrt{\frac{1}{\pi f \mu \sigma}} \approx 0.5 \text{ mm}$
 N_{cells} : 271600,
Wavelength: 100 m
 Δ_{cell} : 1.5 mm $< 3\delta$
Runtime: 20' 40" single core in `abpimp60g01`

Remarks:

- Staircased grid can induce the frequency shift at 800 MHz
- Skin Depth** is not well modelled → difference in shunt impedance

**for $\sigma > 100 \text{ S/m}$, CST uses surface impedance model (Leontovich)

Benchmark with CST Wakefield Solver



Outline

1. Introduction to Beam-Coupling Impedance simulations & Motivation

..... Slides 4 - 11

2. The Finite Integration Technique (FIT) step-by-step:

2.1. Numerical Algorithm in free-space

2.2. Geometry definition: STL importer

2.3. Adding material tensors: ϵ and μ

2.4. Particle beam injection & Absorbing boundaries

2.5. Adding conductivity σ

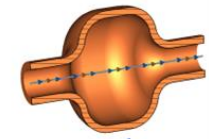
2.6. Low- β simulations

..... Slides 13 - 38

3. Conclusions, Challenges & Future work

..... Slides 40 - 42

CAD Geometry
(.stl)



Simulation
script



Simulation script example



```
from wakis import GridFIT3D, SolverFIT3D, WakeSolver
import pyvista as pv

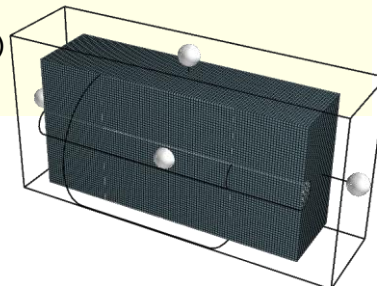
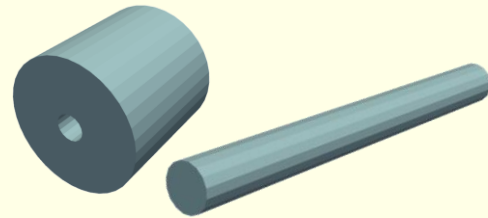
# ----- Domain and Grid setup -----
# Number of mesh cells
Nx = 57
Ny = 57
Nz = 109
#dt = 5.707829241e-12

# Geometry Import
stl_cavity = 'cavity.stl'
stl_pipe = 'beampipe.stl'
stl_solids = {'cavity': stl_cavity, 'pipe': stl_pipe}

# Materials
stl_materials = {'cavity': 'vacuum', 'pipe': 'vacuum'}
background = [1.0, 1.0, 100] # lossy metal [ $\epsilon_r$ ,  $\mu_r$ ,  $\sigma$ ]

# Domain bounds (from stl)
surf = pv.read(stl_cavity) + pv.read(stl_pipe)
xmin, xmax, ymin, ymax, zmin, zmax = surf.bounds

# Set grid and geometry
grid = GridFIT3D(xmin, xmax, ymin, ymax, zmin, zmax, Nx, Ny, Nz,
                stl_solids=stl_solids,
                stl_materials=stl_materials)
#grid.inspect()
```



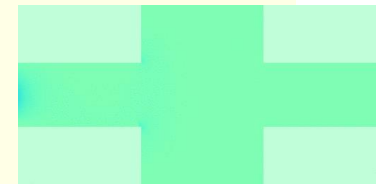
```
# ----- Beam source -----
# Beam parameters and wake obj.
beta = 0.8 # beam relativistic beta
sigmaz = beta*6e-2 # [m] -> multiplied by beta to have f_max cte
q = 1e-9 # [C]
xs = 0. # x source position [m]
ys = 0. # y source position [m]
xt = 0. # x test position [m]
yt = 0. # y test position [m]
# tinj = 8.53*sigmaz/(beta*c) # injection time offset [s]

wake = WakeSolver(q=q, sigmaz=sigmaz, beta=beta,
                 xsource=xs, ysource=ys, xtest=xt, ytest=yt,
                 save=True, logfile=True)

# ----- Solver & Simulation -----
# boundary conditions and solver obj.
bc_low=['pec', 'pec', 'pec']
bc_high=['pec', 'pec', 'pec']
solver = SolverFIT3D(grid, wake,
                    bc_low=bc_low, bc_high=bc_high,
                    use_stl=True, bg=background)

# Run wakefield time-domain simulation
wakelength = 5. #[m]
add_space = 10 # no. cells to remove for the wake calculation

solver.wakesolve(wakelength=wakelength, add_space=add_space,
                plot=True, plot_every=30, save_J=True,
                **plotkw)
```

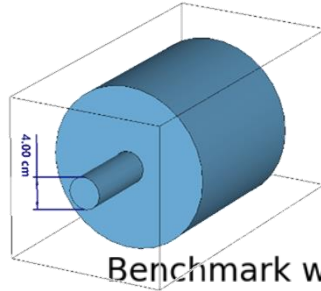


Example: results for different β

 [benchmarks/betacavity/](https://github.com/benchmarks/betacavity/)

Simulation of a cylindrical pillbox* below cut-off for different relativistic β values

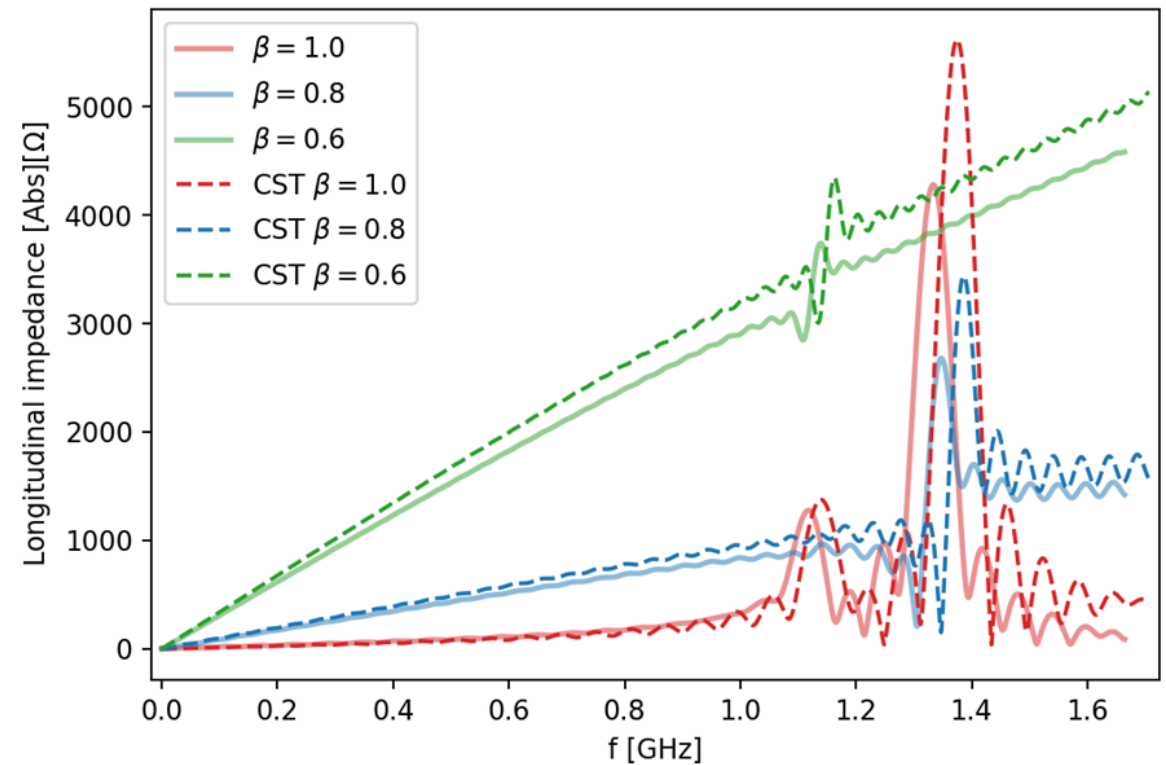
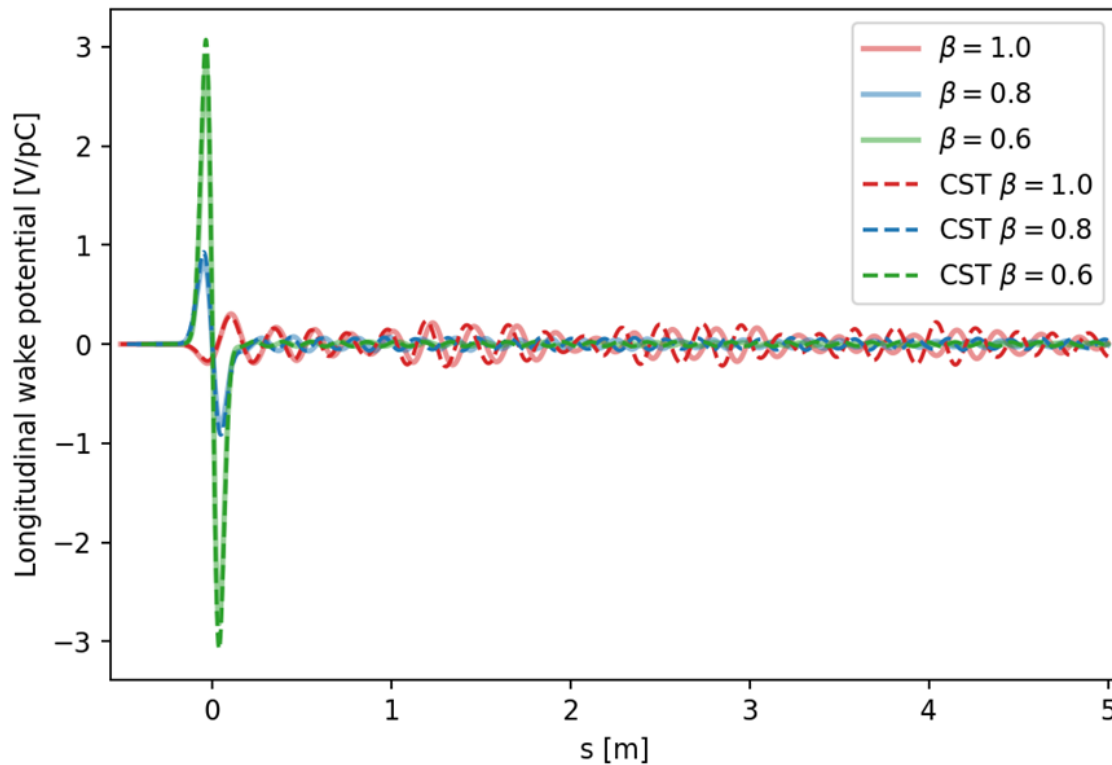
*Model by Elena Macchia



Benchmark with CST Wakefield Solver

Work in progress!

- ✓ 1st attempt shows some agreement, but mesh needs to be optimized
- ✓ The behaviour with β is consistent



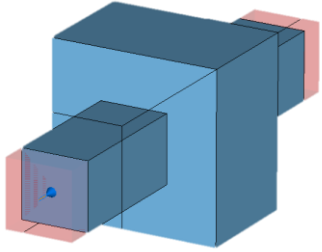
Outline

1. Introduction to Beam-Coupling Impedance simulations & Motivation Slides 4 - 11
2. The Finite Integration Technique (FIT) step-by-step: Slides 13 - 38
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 - 2.5. Adding conductivity σ
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3. Conclusions, Challenges & Future work Slides 40 - 42

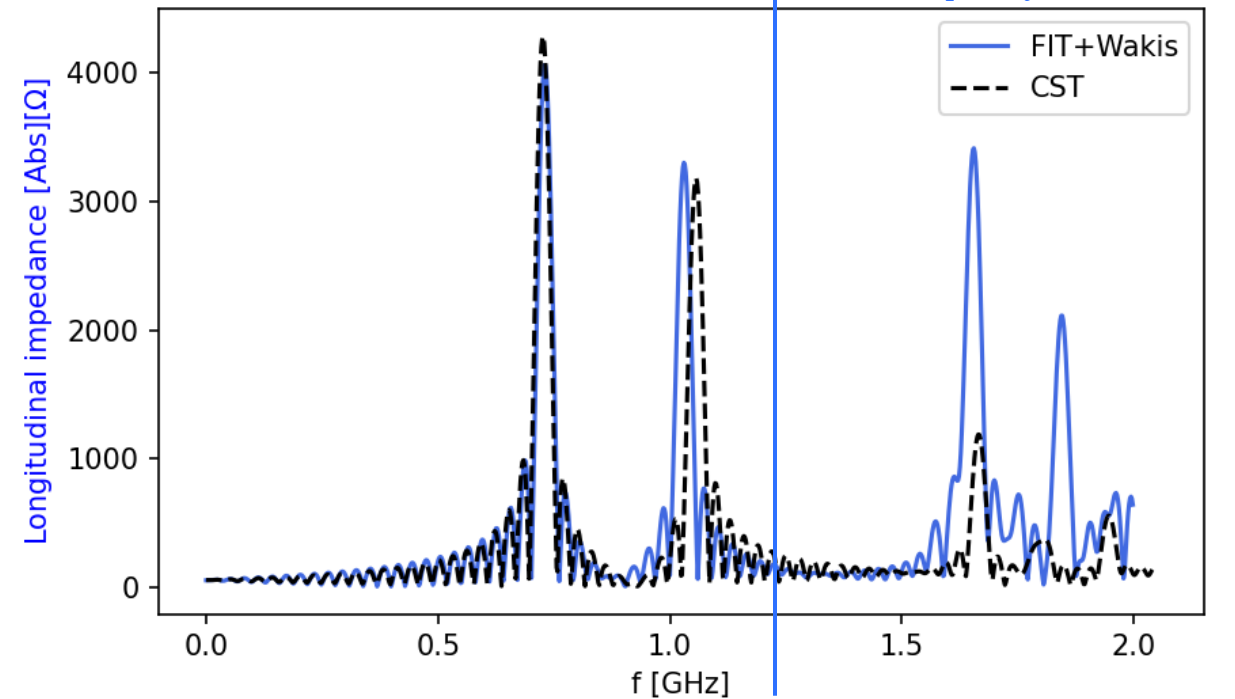
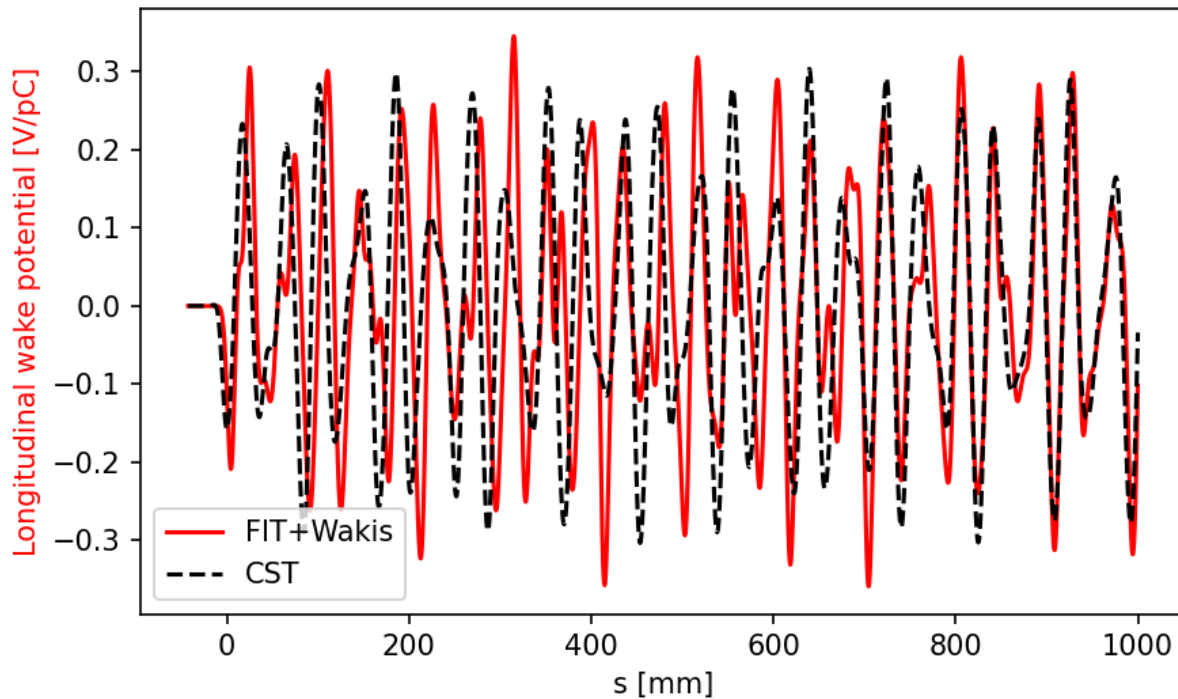
Challenges: Simulations above cut-off f

When the bunches are short, they excite to higher frequencies: $f_{max} = \frac{\beta c}{3\sigma_z}$

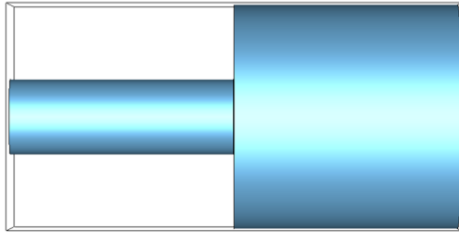
When $f_{max} > f_{cutoff}$ of the pipe, some modes are in propagation and reach the boundaries



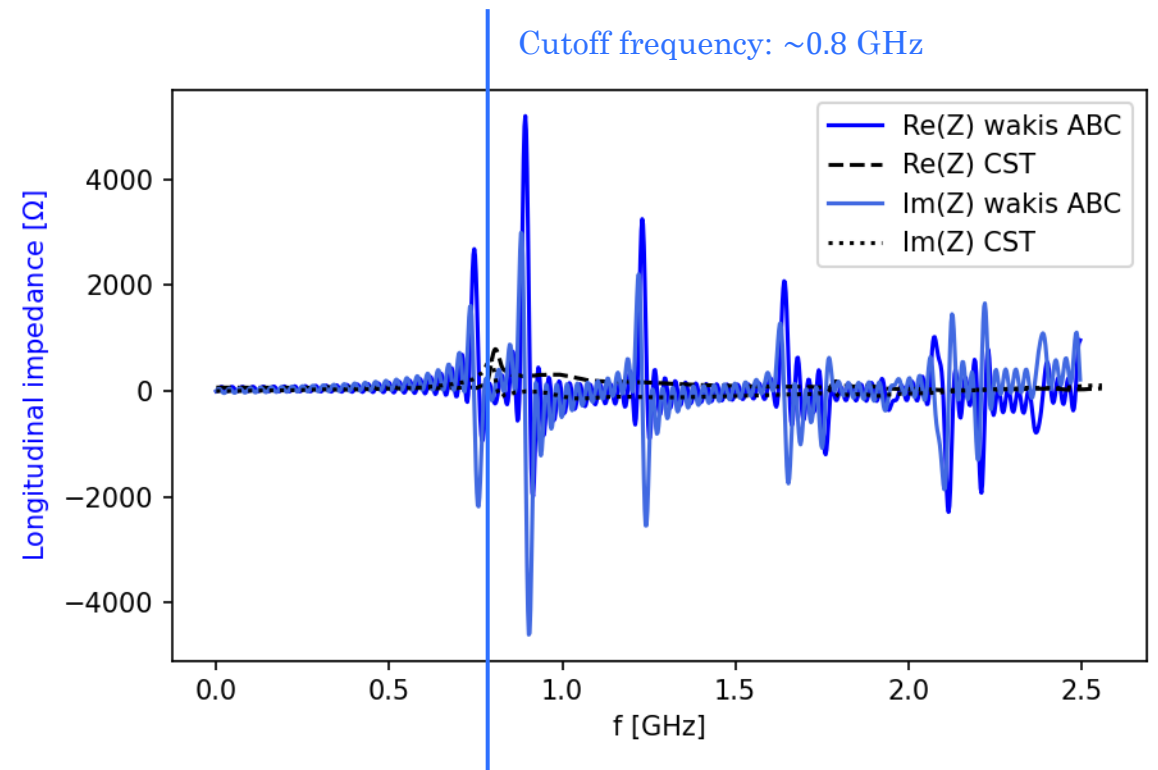
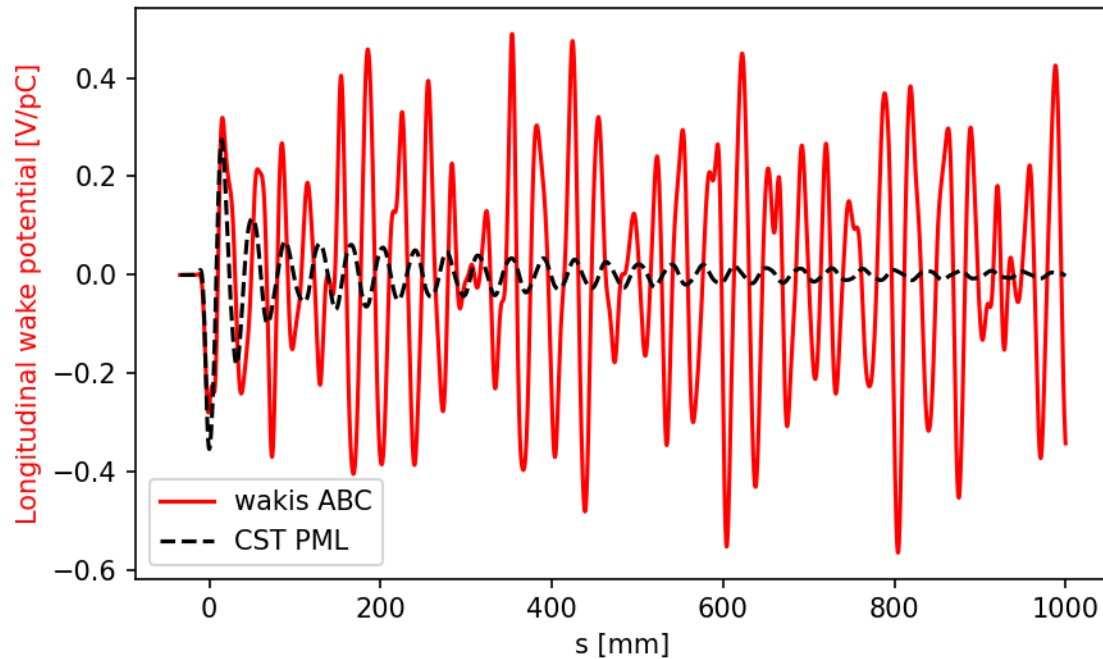
wakis does not show good agreement above cut-off (yet)
Perfect matching layers (PML) BCs are needed to simulate propagating modes



Challenges: Simulations above cut-off f

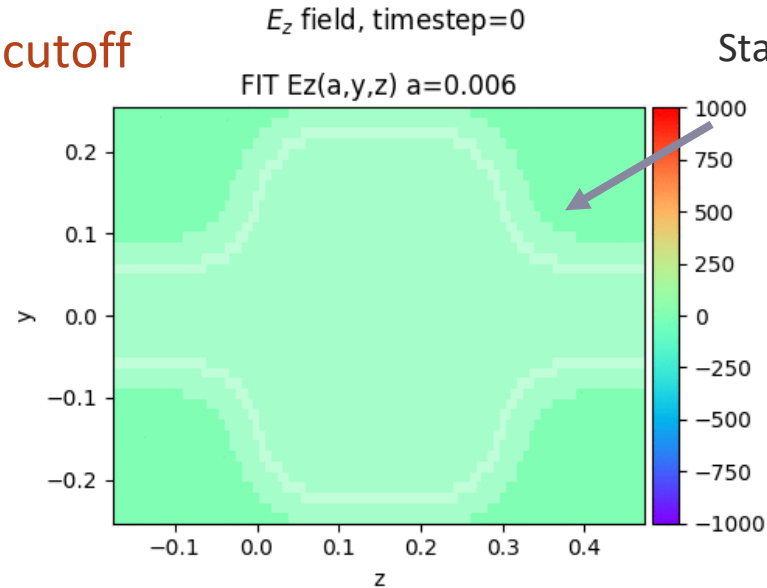


This becomes more evident in a non-resonant structure, like a transition (i.e., Step-out)
All modes should propagate, but in **wakis** they are reflected back. To simulate propagating modes, we need the **PML** boundary conditions

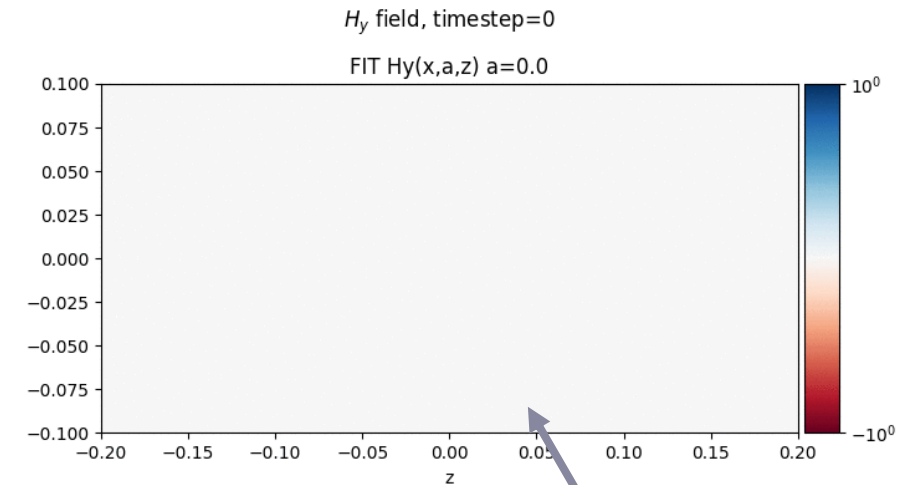


More challenges for the near future

- I. PML & Simulations above cutoff
- II. Staircased grid
- III. Numerical dispersion
- IV. GPU acceleration
- V. Surface impedance
- VI. Wake extrapolation (genetic algorithm)
- VII. Frequency dependent (dispersive) materials
- VIII. Frequency domain monitors



Staircased grid with curved geometry generates artifacts and difficulties convergence



CuPy

Acceleration should be possible with cupy since it supports `scipy.sparse`

For some optic applications, the code shows numerical dispersion

 [examples/script_wavepacket_fit.py](https://github.com/abp-cei/examples/blob/main/script_wavepacket_fit.py)

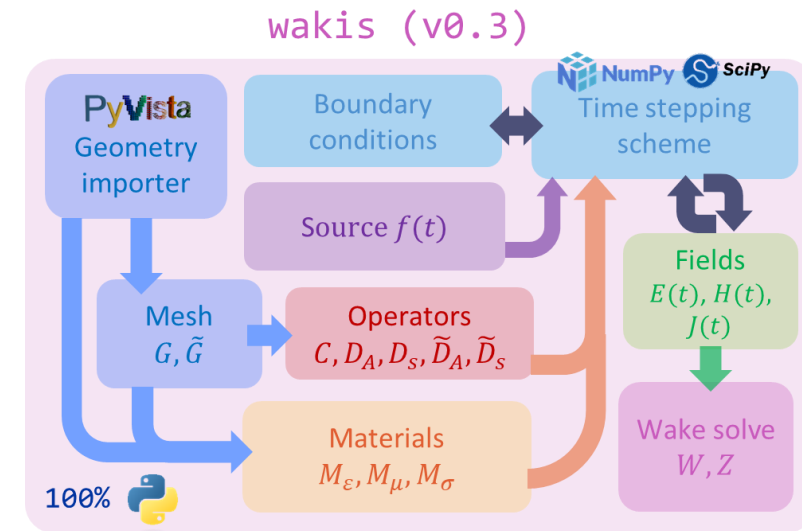
Conclusions

- Developed a **3D Electromagnetic and Wake Solver in time domain**, 100% in python: **wakis**
 - i. Uses the **Finite Integration Technique** and **scipy.sparse** matrices, allowing for fast computations, **extendable to GPU**
 - ii. Different **boundary conditions**: PEC, PMC, Periodic, ABC
 - iii. **Importing CAD geometry** with **pyvista**
 - iv. **Simulate materials with ϵ, μ, σ** with diagonal tensors. Possibility of **anisotropic properties**.
 - v. Simulate **several sources**: Planewave, gaussian wave packet, and the **particle beam for different β**
 - vi. Several **on the fly built-in plotting** capabilities in 1d, 2d, 3d
 - vii. **Interactive visualization** methods to inspect the grid and the geometry
 - viii. +10 examples and benchmark simulations (⌚ ~2' to 20') already available on GitHub.

○ **wakis** solver has been **benchmarked with CST Wakefield Solver®** with pillbox cavities of different materials and geometries, below cut-off.

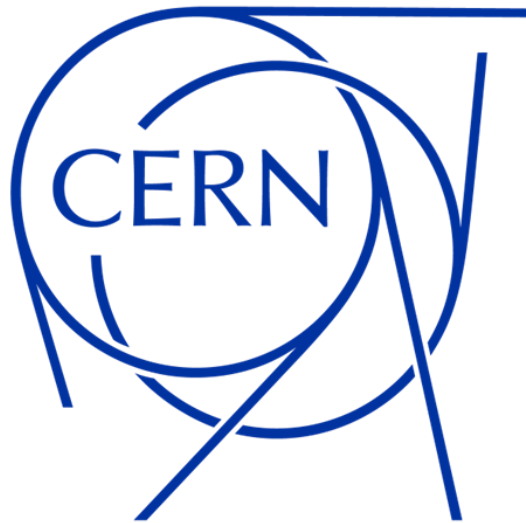
○ It is built also as a **general-purpose 3D TD EM code**, capable of simulating optical lenses, diffraction gratings and much more.

➤ However, **many challenges need to be investigated**: simulations above cutoff with PML boundaries, advanced mesh generation, GPU acceleration, numerical dispersion correction, surface impedance condition... That **will be addressed during the PhD!**



<https://github.com/ImpedanCEI/FITwakis>

Thank you 😊 !!!



wakis:

3D Electromagnetic Time- Domain
Wake and Impedance Solver

Elena de la Fuente García (BE-ABP-CEI)

Bibliography used

1990 Joint US-CERN Accelerator Course
Hilton Head, So. Carolina

Wake Fields and Impedances

T. Weiland, R. Wanzenberg

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1

- i. [Wakefields and Impedances \[T. Weiland, 1991\]](#)
- ii. [TE/TM FIT for accelerators \[I. Zarg, 2005\]](#)
- iii. [Open boundaries for FIT \[MC. Balk, 2005\]](#)
- iv. [2D expansion \[I. Zarg, 2015\]](#)
- v. [2D freq. domain \[R. Schumann, 2000\]](#)
- vi. [Frequency domain FIT and FEM Uwe Niedermayer PhD thesis \[2015\]](#)
- vii. [Eigenmode + MPI + Gyrotropic materials Klaus Klopfer PhD thesis \[2014\]](#)
- viii. [EMcLAW: An unsplit Godunov method for Maxwell 's Equations. \(UPM\) Moreno, José A. PhD Thesis \[2020\]](#)

Absorbing boundary condition (ABC)

A first attempt to **reduce the perturbation of the E_z field** when the beam current enters/exits it to use **absorbing boundary conditions (ABC)**

- Since PML formulation is complex, the simplest ABC, the **FOEXTRAP**, was tested first. This is a first order extrapolation that mimics a continuous field at the boundary cells

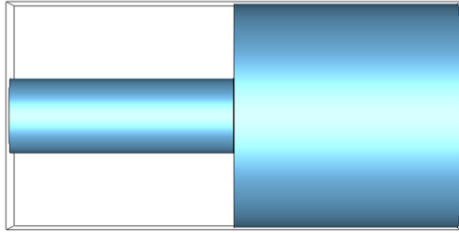
```
...120  def one_step(self):
121
122      if self.step_0:
123          self.set_ghosts_to_0()
124          self.step_0 = False
125
126          #if self.use_conductors:
127              #self.set_field_in_conductors_to_0()
128
129      self.H.fromarray(self.H.toarray() -
130                      self.dt*self.tDsiDmuiDaC*self.E.toarray()
131                      )
132
133      self.E.fromarray(self.E.toarray() +
134                      self.dt*(self.itDaiDepsDstC * self.H.toarray() - self.iDeps*self.J.to
135                      )
136
137      #update ABC
138      if self.activate_abc:
139          self.update_abc()
```

It has to be updated every timestep

```
401      def apply_bc_to_C(self):
505
506          # Absorbing boundary conditions ABC
507          if any(True for x in self.bc_low if x.lower() == 'abc'):
508              self.activate_abc = True
509
510  def update_abc(self):
511      '''
512      Apply ABC algo to the selected BC,
513      to be applied after each timestep
514      '''
515
516      if self.bc_low[0].lower() == 'abc':
517          for d in ['x', 'y', 'z']:
518              self.E[0, :, :, d] = self.E[1, :, :, d]
519              self.H[0, :, :, d] = self.H[1, :, :, d]
```

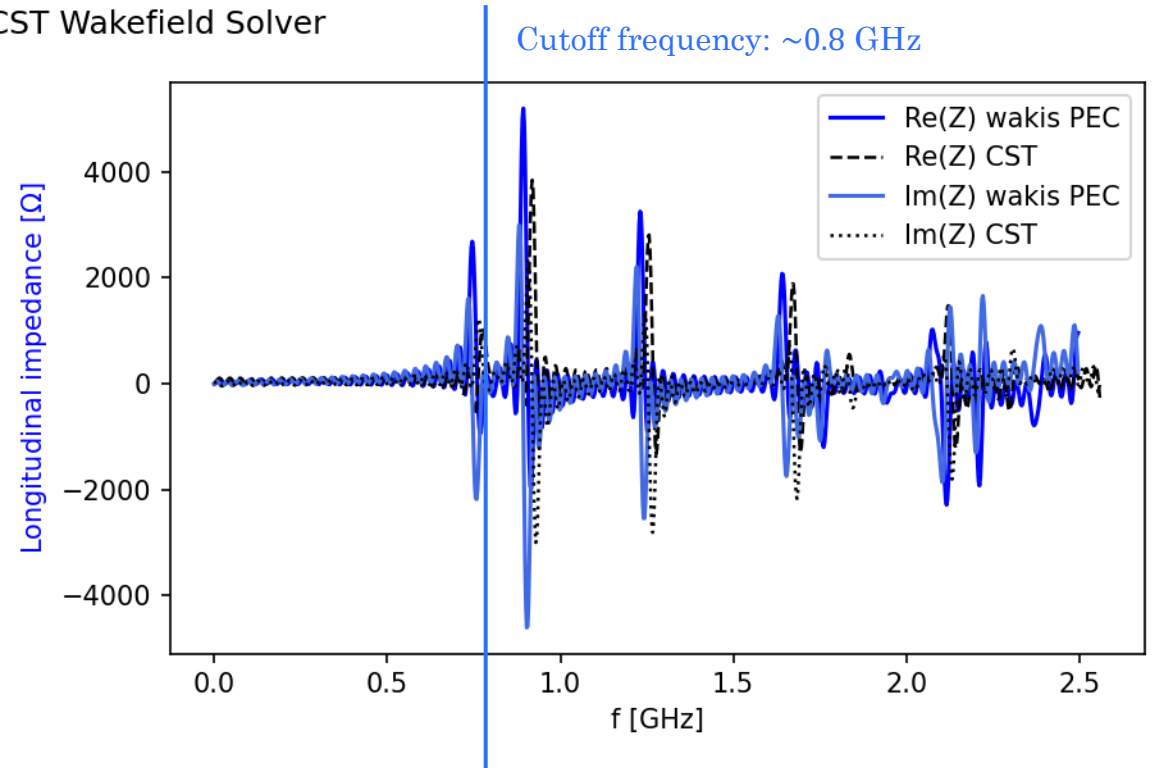
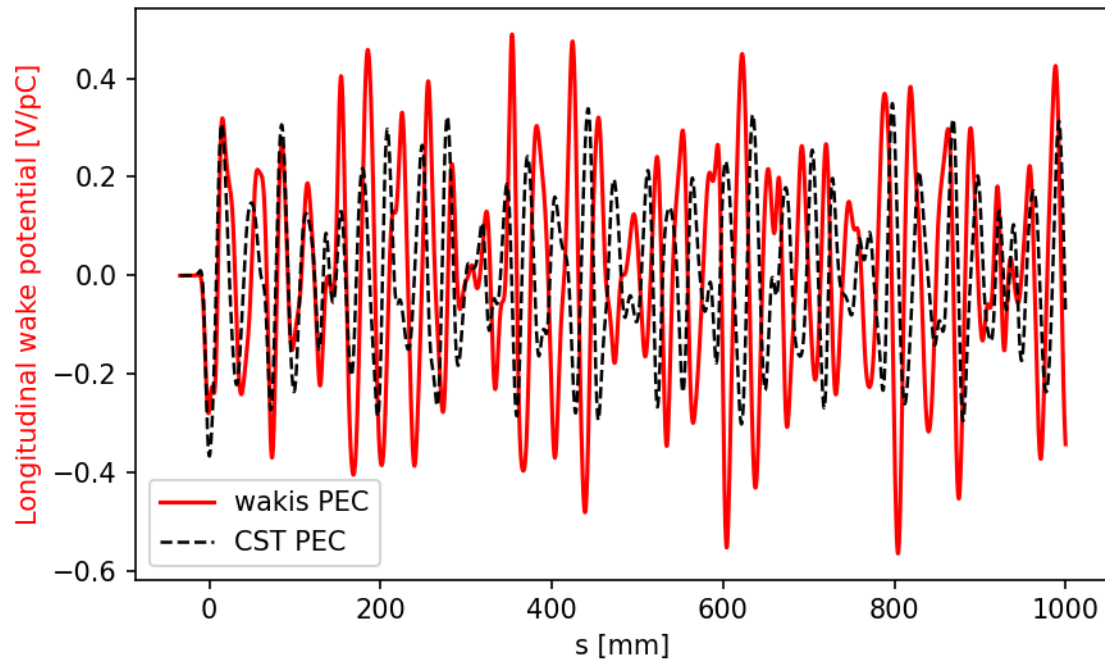
Same for all 6 boundaries (low and high, x, y, z)

Challenges: Simulations above cut-off f

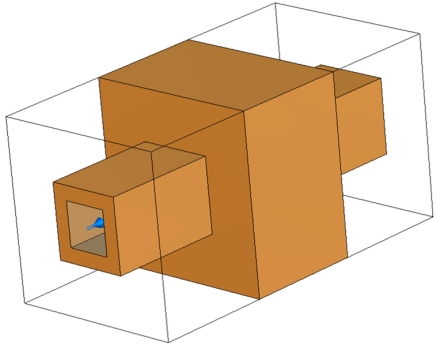


The agreement is okay when we use PEC boundaries In both wakis and CST®

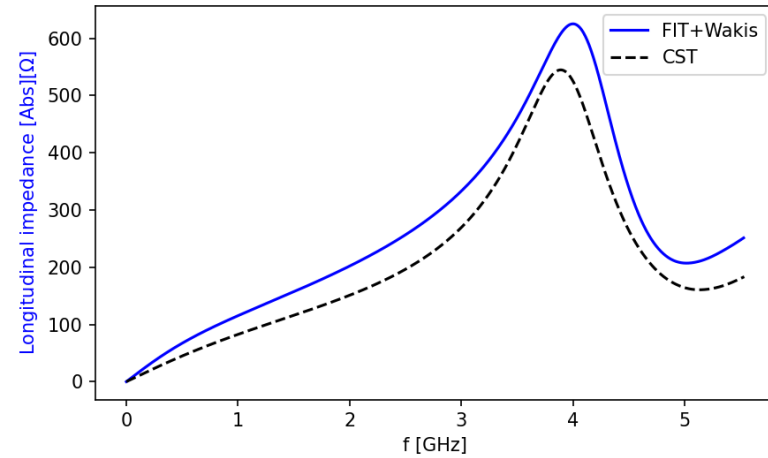
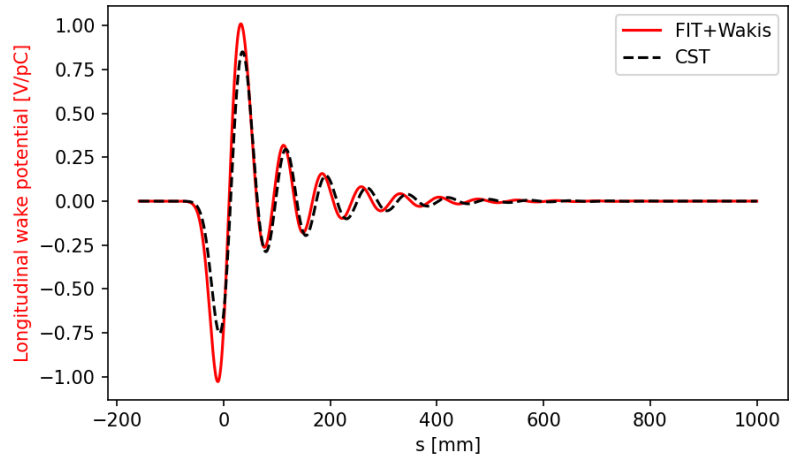
Benchmark with CST Wakefield Solver



Optimizing results with wakis



Benchmark with CST Wakefield Solver



If we remove the last 8 cells in z-/z+ with the add_space parameter



Conductivity: 10 S/m

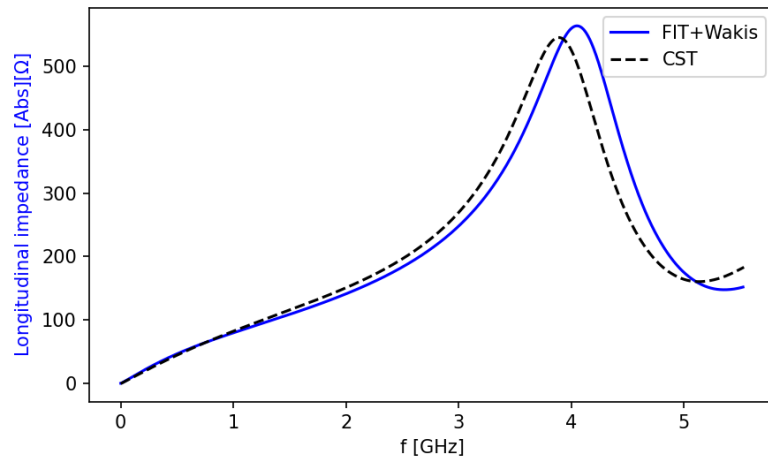
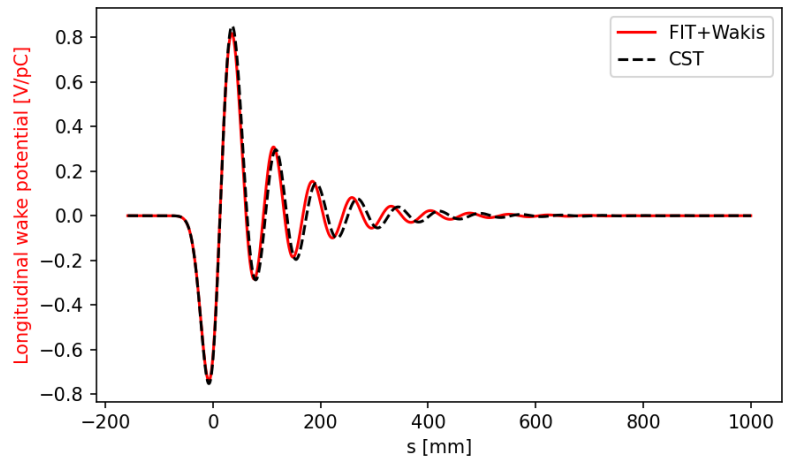
Skin-Depth: $\delta = \sqrt{\frac{1}{\pi f \mu \sigma}} \approx 2 \text{ mm}$

N_{cells} : 542754

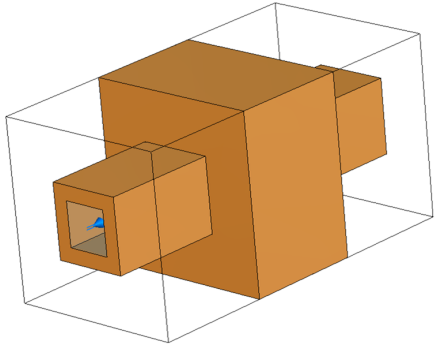
Runtime: 4', 30'',
single core in `abpimp60g01`

Δ_{cell} : 0.5 mm > 3 δ

Benchmark with CST Wakefield Solver



Optimizing results with wakis



Conductivity: 10 S/m

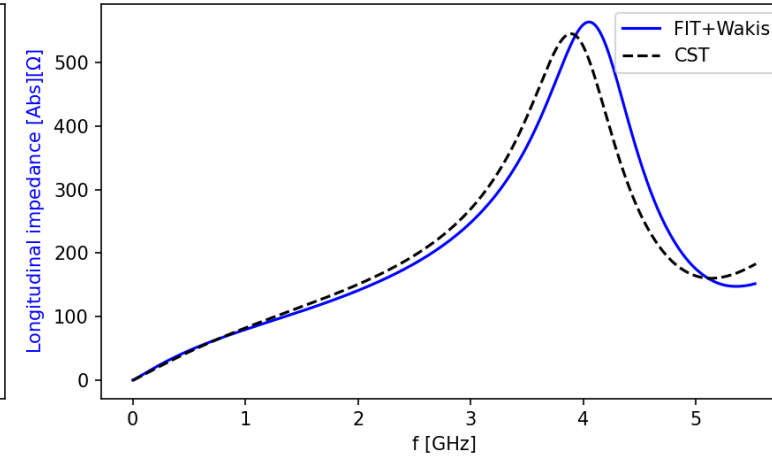
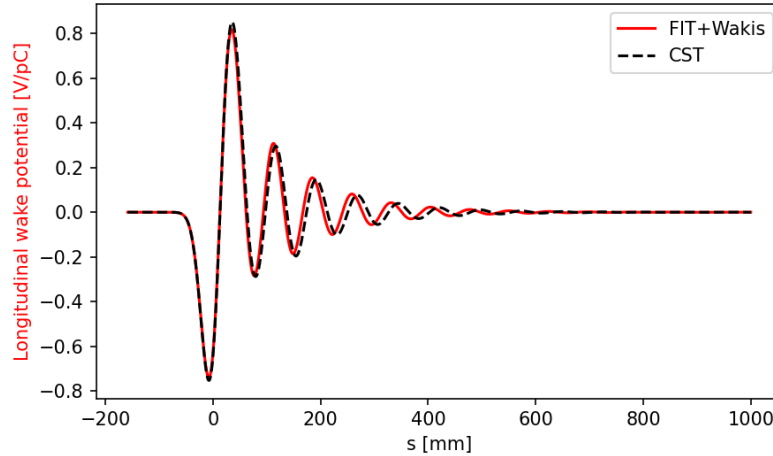
Skin-Depth: $\delta = \sqrt{\frac{1}{\pi f \mu \sigma}} \approx 2 \text{ mm}$

N_{cells} : 542754

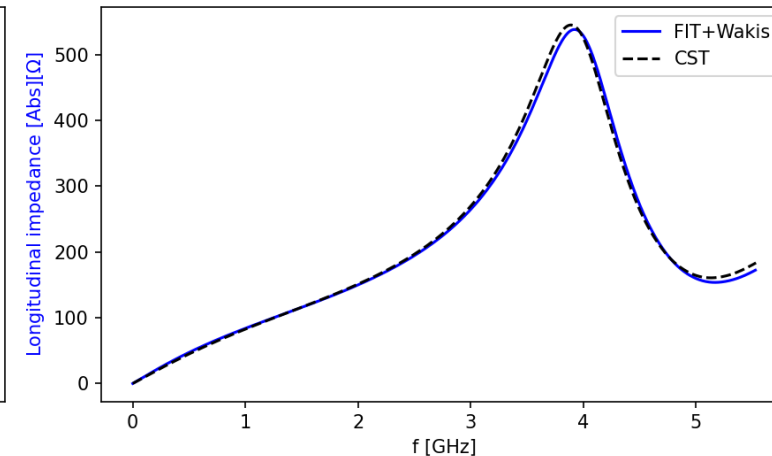
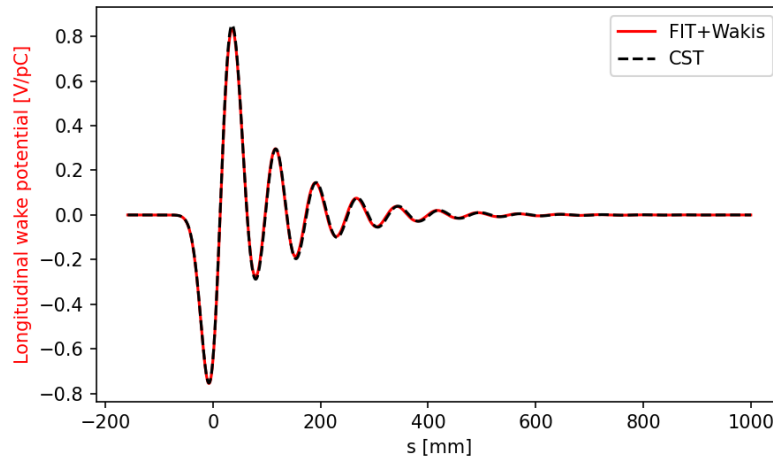
Runtime: 4', 30'',
single core in `abpimp60g01`

Δ_{cell} : 0.5 mm > 3 δ

Benchmark with CST Wakefield Solver



Benchmark with CST Wakefield Solver



Then, if we increase the mesh resolution by 15%



FITwakis GitHub overview



elenafluengar new benchmark with a bigger cavity above cutoff		8c6a436 · 14 hours ago	🕒 182 Commits
📁 benchmarks	new benchmark with a bigger cavity above cutoff		14 hours ago
📁 examples	gaussian wave packet example		last week
📄 .gitignore	include cst		last week
📄 conductors.py	Added implicit function conductor		3 years ago
📄 conductors3d.py	fixing a bug in sphere conductor		4 years ago
📄 field.py	updated __add__ to sum two Field objects		4 months ago
📄 grid2D.py	fixing a small bug		3 years ago
📄 grid3D.py	add conductors functions to fit		4 months ago
📄 gridFIT3D.py	small bug fix for stl_scale		2 months ago
📄 materials.py	typo		3 months ago
📄 pmlBlock2D.py	fixing a small bug		4 years ago
📄 pmlBlock3D.py	3D PMLs now working		4 years ago
📄 solver2D.py	Modified 2d em soolver		3 years ago
📄 solver3D.py	change CFL to default 0.5		last month
📄 solverFIT3D.py	add dt as parameter		14 hours ago
📄 wakeSolver.py	add wakelength to init		2 days ago

FDTD EM solver
by Lorenzo

FIT EM Solver

Wake Solver

← Benchmarks vs CST, WarpX ...

← Examples & university tests:

- Plane wave propagation
- Gaussian wavepacket propagation
- Cubic Resonator

← Field class to manage matrix formulation
 E, H, J, ϵ, μ are instances of this class

← GridFIT3D class in charge of STL importer and grid definition

← Pre-defined materials library (vacuum, dielectric, PEC)

← SolverFIT3D class that solves Maxwell equations

← Wakis(v0.2) code is refactored into class WakeSolver

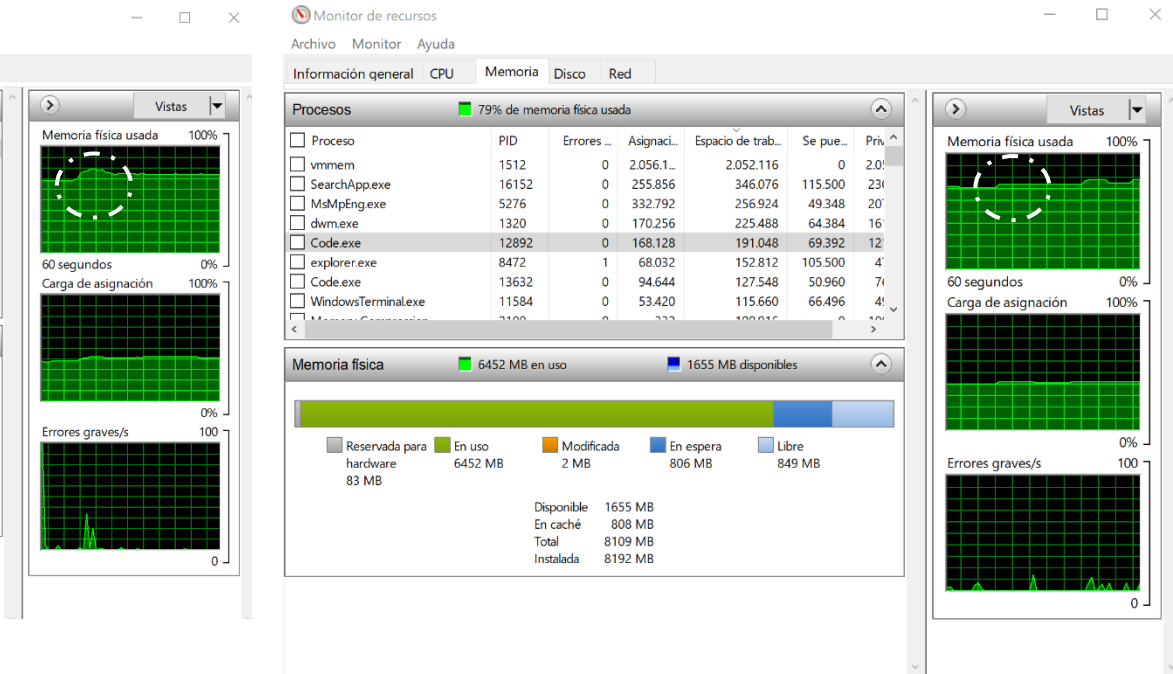
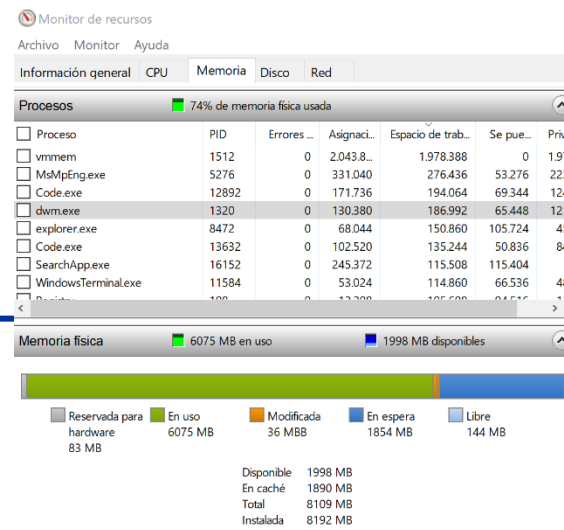
SolverFIT3D development (I): memory optimization

```
class SolverFIT3D
  func __init__
  func one_step
  func emsolve
  func wakesolve
  func beam
  func apply_bc_to_C
  func update_abc
  func set_ghosts_to_0
  func apply_conductors
  func set_field_in_conductors_to_0
  func apply_stl
  func attrcleanup ←
  func plot3D
  func plot2D
  func plot1D
```

```
632 def attrcleanup(self):
633
634     # Fields
635     del self.L, self.tl, self.iA, self.itA
636     if hasattr(self, 'BC'):
637         del self.BC
638         del self.Dbc
639
640     # Matrices
641     del self.Px, self.Py, self.Pz
642     del self.Ds, self.iDa, self.tDs, self.itDa
643     del self.C
```

Deletes from memory the matrices that will not be used for the timestepping routine:

- Improves memory allocation by 60%
- Increases speed performance by 5%



SolverFIT3D development (II): 1D, 2D, 3D plotting

```
class SolverFIT3D
  func __init__
  func one_step
  func emsolve
  func wakesolve
  func beam
  func apply_bc_to_C
  func update_abc
  func set_ghosts_to_0
  func apply_conductors
  func set_field_in_conductors_to_0
  func apply_stl
  func attrcleanup
  func plot3D
  func plot2D
  func plot1D
```

```
def plot3D(self, field='E', component='z', clim=None, hide_solids=None,
           show_solids=None, add_stl=None, stl_opacity=0.1, stl_colors='white',
           title=None, cmap='jet', clip_volume=True, clip_normal='-y',
           clip_box=False, clip_bounds=None, off_screen=False, zoom=0.5,
           nan_opacity=1.0, n=None):
    """
    Built-in 3D plotting using PyVista

    Parameters:
    -----
    field: str, default 'E'
           3D field magnitude ('E', 'H', or 'J') to plot
           To plot a component 'Ex', 'Hy' is also accepted
    component: str, default 'z'
              3D field component ('x', 'y', 'z', 'Abs') to plot. It will be overridden
              if a component is defined in field
```

Using [PyVista](#) (vtk based) functions.

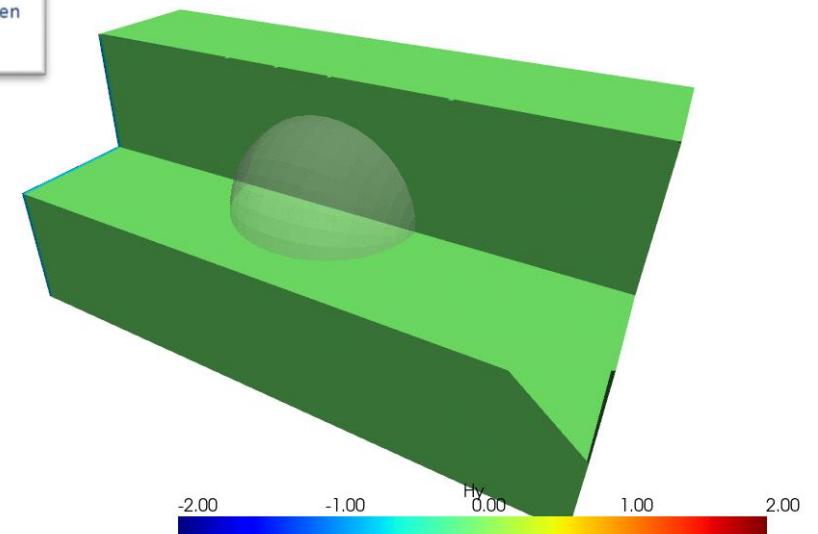
Plots can also be interactive:

- `clip_volume` or `clip_normal` flags when `off_screen = True`

Plot3D example:

[examples/script_planewave_fit.py](#)

A planewave interacting with a dielectric sphere (*University test*)



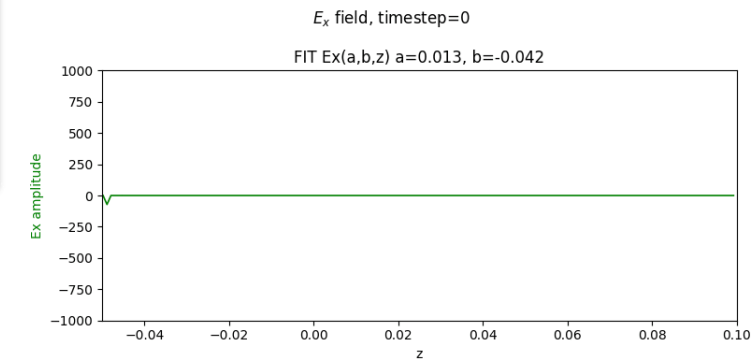
SolverFIT3D development (II): 1D, 2D, 3D plotting

```
class SolverFIT3D
  func __init__
  func one_step
  func emsolve
  func wakesolve
  func beam
  func apply_bc_to_C
  func update_abc
  func set_ghosts_to_0
  func apply_conductors
  func set_field_in_conductors_to_0
  func apply_stl
  func attrcleanup
  func plot3D
  func plot2D
  func plot1D
```

```
def plot2D(self, field='E', component='z', plane='ZY', pos=0.5, norm=None,
           vmin=None, vmax=None, figsize=[8,4], cmap='jet', patch_alpha=0.1,
           patch_reverse=False, add_patch=False, title=None, off_screen=False,
           n=None, interpolation='antialiased'):
    '''
    Built-in 2D plotting of a field slice using matplotlib
```

```
def plot1D(self, field='E', component='z', line=None, pos=0.5,
           xscale='linear', yscale='linear', xlim=None, ylim=None,
           figsize=[8,4], title=None, off_screen=False, n=None, **kwargs):
    '''
    Built-in 1D plotting of a field line using matplotlib
```

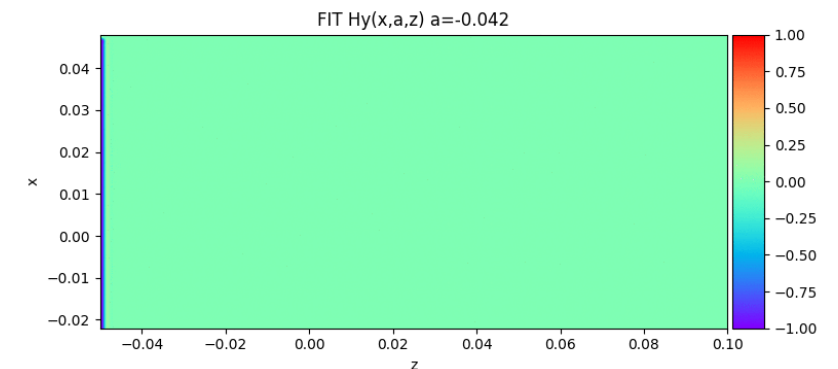
Matplotlib based coutourf (2D) and line plot (1D)



Plot2D and 1D example:

[examples/script_planewave_fit.py](#)

A planewave propagating through vacuum being reflected at the PEC boundary



SolverFIT3D development (IV): EM solve

```
class SolverFIT3D
  func __init__
  func one_step
  func emsolve
  func wakesolve
  func beam
  func apply_bc_to_C
  func update_abc
  func set_ghosts_to_0
  func apply_conductors
  func set_field_in_conductors_to_0
  func apply_stl
  func attrcleanup
  func plot3D
  func plot2D
  func plot1D
```

```
def emsolve(self, Nt, source=None, save=False, fields=['E'], components=['Abs'],
            every=1, subdomain=None, plot=False, plot_every=1, **kwargs):
    ...
    Run the simulation and save the selected field components in HDF5 files
    for every timestep. Each field will be saved in a separate HDF5 file 'Xy.h5'
    where X is the field and y the component.

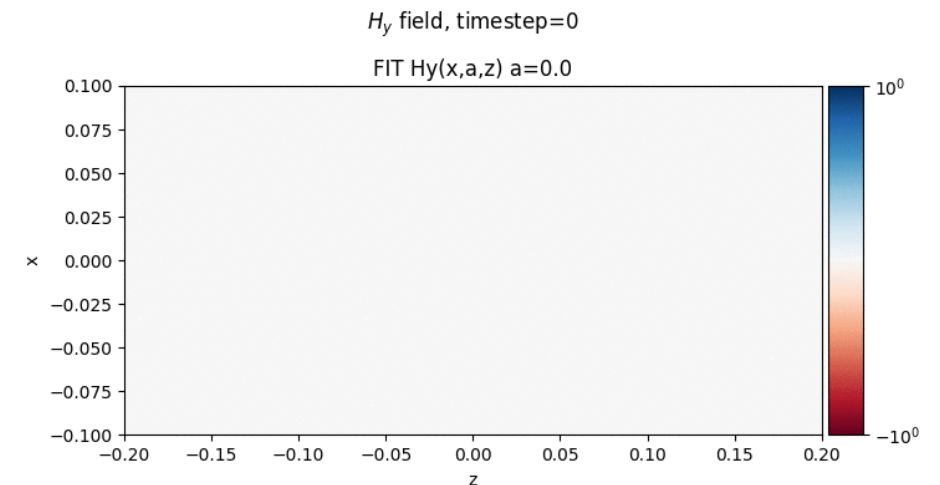
    Parameters:
    -----
    Nt: int
        Number of timesteps to run
    source: func
        Function defining the time-dependent source.
        It should be in the form `func(solver, t)`
```

Runs Electromagnetic
time domain simulation
given an initial condition
or source

EM solve example:

[examples/script_wavpacket_fit.py](#)

A gaussian wavepacket
propagating through vacuum
domain (*University test*)



SolverFIT3D development (V): Wake solve

```

class SolverFIT3D
  func __init__
  func one_step
  func emsolve
  func wakesolve
    func beam
  func apply_bc_to_C
  func update_abc
  func set_ghosts_to_0
  func apply_conductors
  func set_field_in_conductors_to_0
  func apply_stl
  func attrcleanup
  func plot3D
  func plot2D
  func plot1D
  
```

```

247 def wakesolve(self, wavelength, wake=None,
248               save_J=False, add_space=None,
249               plot=False, plot_every=1, **kwargs):
250     ...
251     Run the EM simulation and compute the longitudinal (z) and transverse (x,y)
252     wake potential WP(s) and impedance Z(s).
253
254     The `Ez` field is saved every timestep in a subdomain (xtest, ytest, z) around
255     the beam trajectory in HDF5 format file `Ez.h5`.
256
257     The computed results are available as Solver class attributes:
258     - wake potential: WP (longitudinal), WPx, WPy (transverse) [V/pC]
259     - impedance: Z (longitudinal), Zx, Zy (transverse) [Ohm]
260     - beam charge distribution: lambdas (distance) [C/m] lambda_f (spectrum) [C]
261
262     Parameters:
263     -----
264     wavelength: float
265         Desired length of the wake in [m] to be computed
  
```

- Runs Wakefield time domain simulation given a wavelength.
- Beam source injected is defined by WakeSolver object.
- Computes wake potential and impedance by saving Ez field every timestep and using the routines in WakeSolver class (needs h5py)

Wake solve example:

[examples/script_cubcavity_fit.py](#)

A gaussian beam traversing a PEC cubic pillbox cavity

