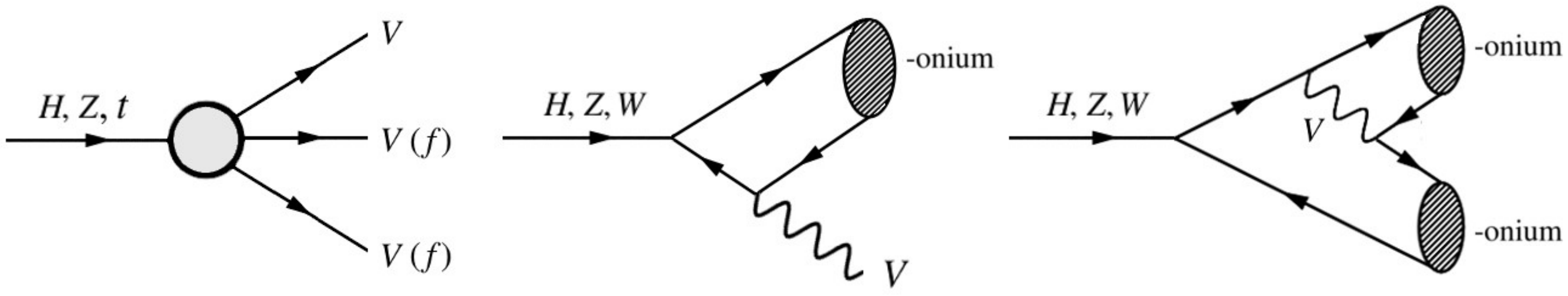
Rare and exclusive few-body decays of the Higgs boson

LHC Higgs WG1 meeting
CERN, 3rd Sept. 2024



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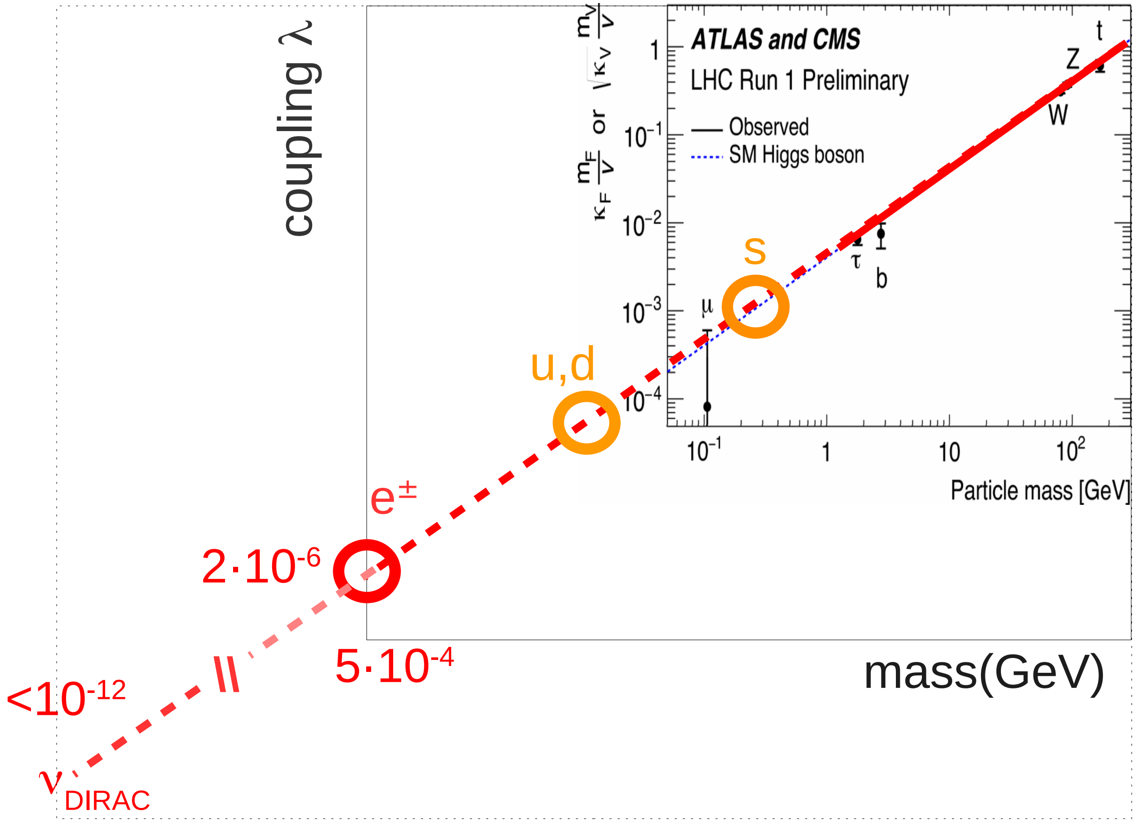
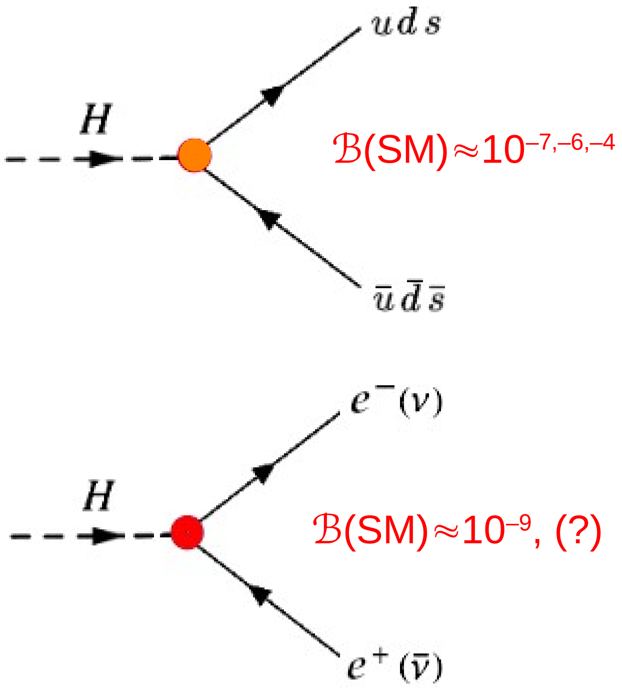
Details in: arxiv:2312.11211, to appear in JPG

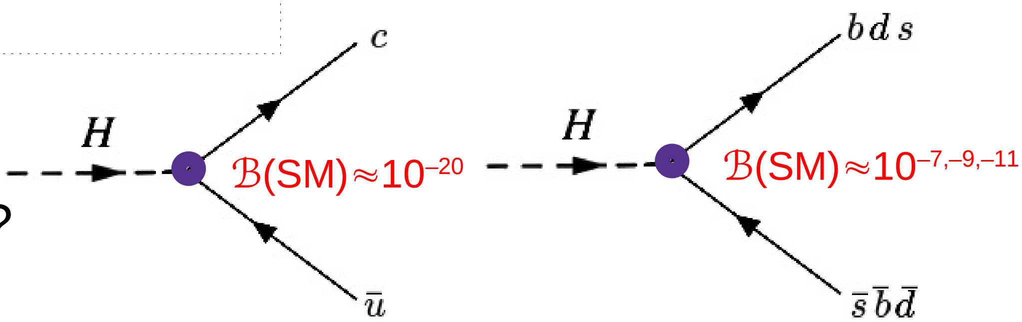
Many Higgs boson open questions...

- Do the lightest fermions (u,d,s,e,ν) that form the stable visible matter in universe, acquire their masses through their Higgs (Yukawa) couplings?

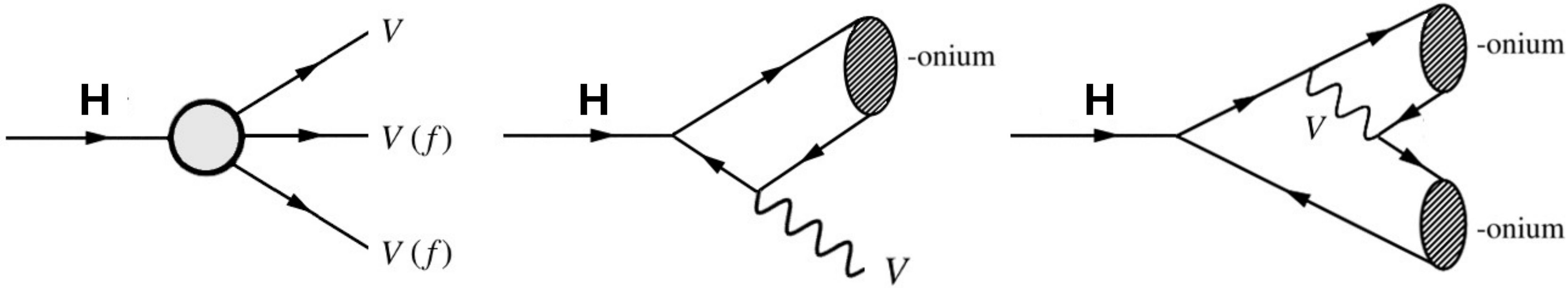




- Are there FCNCs mediated by Higgs boson H→qq' at tree level?



Rare & exclusive few-body Higgs boson decays



- Physics motivations:
  - Search for suppressed/forbidden SM decays.
  - Probe quark/lepton Yukawa couplings ($H \to cc, qq, \ell\ell$) via exclusive final states with charm mesons, light mesons, or leptonia.
  - Search for FCNC Higgs decays.
  - Perform precision tests of the QCD factorization formalism & constraint nonperturbative hadronic bound-state parameters.

- Work performed:
  - Revisit theor. & exp. status of 2-,3-,4-body rare/exclusive decays (branching fractions: $\mathcal{B} < 10^{-5}$) of the SM Higgs boson.
  - Collect all $\mathcal{B}$'s computed (41) and/or with exp. upper limits (20) to date. Update a few old TH $\mathcal{B}$'s. Identify missing decays & compute their $\mathcal{B}$'s (22).
  - Provide ($\mathcal{L}_{int}$-based) projections for HL-LHC, FCC-ee, FCC-hh searches: Help guide and prioritize future experimental searches.

Theoretical methods

- For rare few-body decays into elementary particles (γ,ν): MG5@NLO
- For exclusive hadronic channels: 3 models based on pQCD factorization to describe decay of heavy object $\mathcal{O}$(100 GeV) into much lighter quarks/mesons:

Decay width = hard ME (perturbative) ⊗ quark with mom. *x* (nonpQCD)

1. Light cone (LC): nonperturbative mesons described by LCDAs. Employed for light (uds) mesons. Decay amplitude ∝ f_M decay constant

$$A(\mathrm{X} \to \mathrm{M} + \gamma) \sim f_\mathrm{M} \int_0^1 \mathrm{d}x\, H_\mathrm{M}(x,\mu)\phi_\mathrm{M}(x,\mu)$$

$$f_\mathrm{M}^2 = 4N_\mathrm{c}\frac{|\phi_\mathrm{M}(0)|^2}{m_\mathrm{M}}$$ (from meson leptonic width)

$\Phi_M(x) \approx x\delta(x)$ LCDA: Amplitude of q at mom. *x*

2. Soft-Collinear Effective Theory (SCET): EFT multiple scale resum. + LCDAs. Mostly used for light (uds) mesons.

$$A = -if_\mathrm{M}E\int_0^1 \mathrm{d}x H_\mathrm{M}(x,\mu)\phi_\mathrm{M}(x,\mu) + \text{ power corr}$$

$$\phi_\mathrm{M}(x,\mu) = 6x(1-x)\left[1 + \sum_{n=1}^{\infty} a_n^\mathrm{M}(\mu)\, C_n^{3/2}(2x-1)\right]$$

(a_n moments from latt.-QCD or sum rules)

(Heavy-Quark Effective Theory, HQET: for light+heavy-quark mesons)

3. Non-Relativistic QCD (NRQCD): LDMEs for decays into charm/bottom mesons:

$$\mathrm{d}\Gamma[\mathrm{X} \to \mathrm{M}(n) + \mathrm{Y}] = \sum_n \mathrm{d}\hat{\Gamma}[\mathrm{X} \to (\mathrm{Q\overline{Q}})_n + \mathrm{Y}]\,\langle \mathcal{O}^\mathrm{M}(n)\rangle$$

$$\textstyle\sum_n \mathrm{d}\hat{\Gamma}[\mathrm{X} \to (\mathrm{Q\overline{Q}})_n + \mathrm{Y}] \propto \frac{1}{2m_\mathrm{X}}|\mathcal{A}|^2 d\Phi_3$$

$$\langle \mathcal{O}^{\eta_c}(^1S_0^{[1]})\rangle = \frac{N_\mathrm{c}}{2\pi}|R^{\eta_c}(0)|^2, \quad \langle \mathcal{O}^{\mathrm{J}/\psi}(^3S_1^{[1]})\rangle = \frac{3N_\mathrm{c}}{2\pi}|R^{\mathrm{J}/\psi}(0)|^2$$

Theoretical parameters

- Numerical parameters used for estimations of rare/exclusive Higgs decay widths (MG5@NLO, LC, SCET/HQET frameworks):

masses and widths		couplings and CKM elements		meson masses and form-factor parameters (in GeV)			
m_{ee}	1.02991 MeV	$\alpha(0)$	1/137.036	$m_{\pi^\pm}$	0.13957	f_π	0.1304 [45]
$m_{\mu\mu}$	0.21126 GeV	$\alpha(m_Z)$	1/128.943	$m_{K^\pm}$	0.493677	f_K	0.1562 [45]
$m_{\tau\tau}$	3.5537 GeV	$\alpha_s(m_Z)$	0.1180			λ_K	0.255 [69]
m_c	1.50 GeV	$\sin^2\theta_w$	0.2351	$m_{\rho^\pm}$	0.77526	f_ρ	0.212 [45]
m_b	4.18 GeV	G_F	$1.1664 \cdot 10^{-5}$ GeV^{-2}	$m_{K^{*\pm}}$	0.89166	f_{K^*}	0.203 [45]
m_t	172.69 GeV	$\lvert V_{ud,cs}\rvert$	0.974	m_ϕ	1.0194	f_ϕ	0.223 [14]
m_W	80.377 GeV	$\lvert V_{us,cd}\rvert$	0.225	$m_{D^\pm}$	1.86966	f_D	0.212 [70]
m_Z	91.188 GeV	$\lvert V_{cb,ts}\rvert$	0.041			λ_D	0.354 [71]
m_H	125.25 GeV	$\lvert V_{ub}\rvert$	0.00382	$m_{D_s^\pm}$	1.96835	f_{D_s}	0.2499 [70]
Γ_W	2.085 GeV	$\lvert V_{tb}\rvert$	0.999	$m_{D^{*0}}$	2.007	$f_{D^{*0}}$	1.28 f_D [72]
$\Gamma_{W\to\mu\nu}$	0.1063 Γ_W	$\lvert V_{td}\rvert$	0.0086	$m_{D^{*\pm}}$	2.01027	f_{D^*}	1.28 f_D [72]
Γ_Z	2.4955 GeV			$m_{D_s^{*\pm}}$	2.1123	$f_{D_s^*}$	1.26 f_{D_s} [72]
$\Gamma_{Z\to\ell\ell}$	0.0337 Γ_Z			$m_{B^\pm}$	5.27934	f_B	0.190 [70]
Γ_H	4.1 MeV [73]					λ_B	0.338 [74]
$\Gamma_{H\to\gamma\gamma}$	$2.5 \cdot 10^{-3}$ Γ_H			$m_{B^{*\pm,0}}$	5.32471	f_{B^*}	0.941 f_B [75]
Γ_t	1.331 GeV [76]					f_{B_s}	0.2303 [70]
						λ_{B_s}	0.438 [77]
				$m_{B_s^{*0}}$	5.415	$f_{B_s^*}$	0.953 f_{B_s} [75]
				$m_{B_c^\pm}$	6.27447	f_{B_c}	0.427 [75]
				$m_{B_c^{*\pm}}$	6.32877 [75]	$f_{B_c^*}$	0.988 f_{B_c} [75]

Experimental limits projections

- Number of Higgs bosons at different colliders:
  N(LHC,13 TeV) ≈ 10e6 → N(HL-LHC) ≈ 350e6 → N(FCC-hh) ≈ 2.8e10
  → N(FCC-ee) ≈ 2e6 (with ~0 backgds.)

Collider	$W^\pm$ bosons		Z bosons		H bosons		top quarks	
	σ(W)	N(W)	σ(Z)	N(Z)	σ(H)	N(H)	$\sigma(t\bar{t})$	N(top)
LEP	4.0 pb	0.8×10^5	59 nb	2×10^7	~2, 1 fb	~5	–	–
FCC-ee	4.0 pb	5×10^8	59 nb	6×10^{12}	200, 30 fb	1.9×10^6	0.5 pb	3.8×10^6
Increase factor LEP ↦ FCC-ee	1	6250	1	300,000	70, 30	400,000	–	–
Tevatron (1.96 TeV, 10 fb^{-1})	25.3 nb	2.5×10^8	7.6 nb	7.6×10^7	1.1 pb	1.1×10^4	7.1 pb	1.4×10^5
HL-LHC (14 TeV, 2×3 ab^{-1})	200 nb	1.2×10^{12}	62.5 nb	3.8×10^{11}	58 pb	3.5×10^8	1 nb	1.2×10^{10}
FCC-hh (100 TeV, 30 ab^{-1})	1300 nb	4.1×10^{13}	415 nb	1.2×10^{13}	0.93 nb	2.8×10^{10}	35 nb	2.1×10^{12}
Increase factor Tevatron ↦ HL-LHC	8	4800	8.2	5000	52.7	31 800	141	86 000
Increase factor HL-LHC ↦ FCC-hh	6.5	34	6.7	32	16	80	35	175

- Extrapolation of present (13 TeV) $\mathcal{B}$ limits for HL-LHC via:

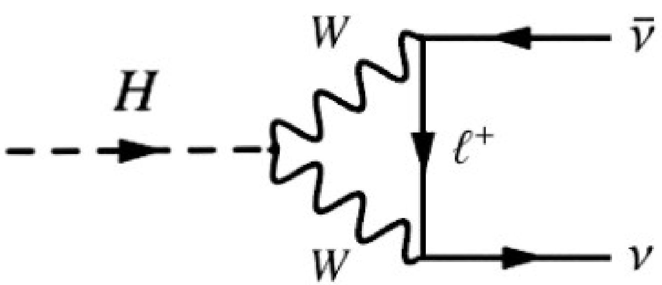
– Existing dedicated CMS/ATLAS studies for a few channels.
– Stat.-based projection of current bounds: $\sqrt{2\times3\ \mathrm{ab}^{-1}/\mathcal{L}_{\mathrm{int}}(13\,\mathrm{TeV})}$ ~6.5× improvement

- Indicate whether H decay will be producible at FCC-ee/FCC-hh by simply checking the relation $[BR(X)\times N(X)]>1$?

Very suppressed SM Higgs decays

Rare 2-, 3-, 4-body Higgs boson decays

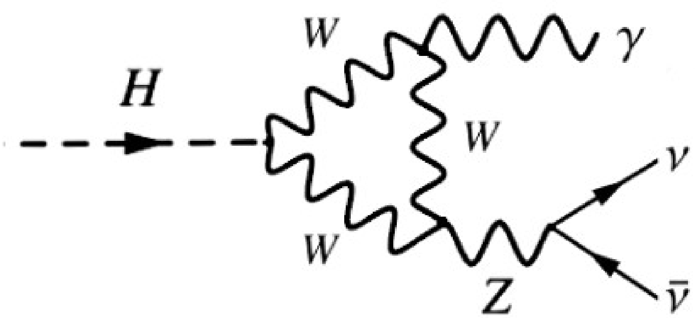
- Ultra-rare H → 2ν invisible decay: Loop-induced for massless ν's in the SM



$\mathcal{B}$ = 7e-36(!): Loops + double-chirality flip. Negligible compared to std. H → ZZ* → 4ν invisible decay ($\mathcal{B} \approx$ 0.1%).

But very dependent on actual nu-mass-generation mechanism realized in nature.

- Rare H→γ+2ν (monophoton Higgs) decay:



$\mathcal{B}$ = 3.7e-4 (~20% larger than naive

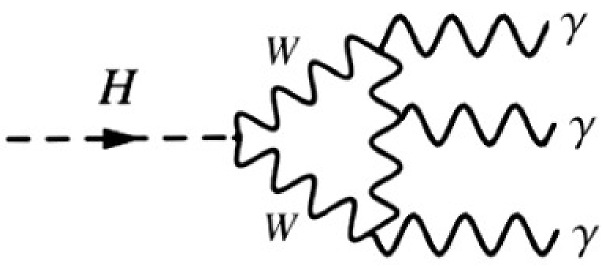
$\mathcal{B}$(H → γZ(2ν))=7.7e-3×0.2, due to extra

W-induced channels). It should be visible at HL-LHC & FCC.

	Branching fraction	Framework	Exp. limits 2023	Exp. limits HL-LHC	Producible at FCC-ee	Producible at FCC-hh
$H \to \nu + \bar{\nu}$	7.2×10^{-36}	NLO SM, MG5_aMC (this work)	–	–	✘	✘
$H \to \gamma + \nu + \bar{\nu}$	$(3.74 \pm 0.01) \times 10^{-4}$	NLO SM, MG5_aMC (this work)	–	–	✓	✓
	3.4×10^{-4}	NLO SM [82]				

Rare 2-, 3-, 4-body Higgs boson decays

- Ultra-rare H → 3γ (triphoton Higgs) decay: Test of fundamental symmetries:

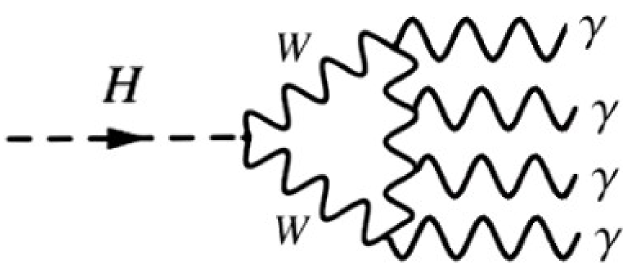


Forbidden in QED (C-violation):

$C(H) = 0 \neq C(3\gamma) = (-1)^3 = -1$

but not in full EW theory. However, final state must be composed of 3 spatially symmetric γ's with zero total ang. momentum: $\mathcal{B}$=1e-40(!)

- Ultra-rare H → 4γ (4-photon Higgs) decay:



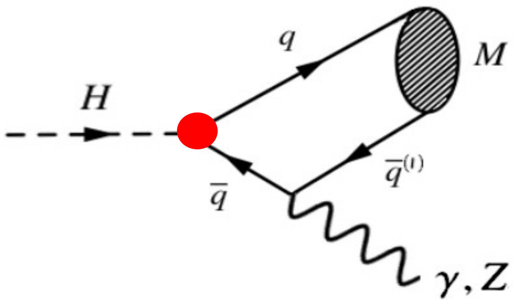
$\mathcal{B}$ = 4.6e-12 (suppressed by heavy loops and $\alpha^4 \approx$ 3e-9). Not observable at LHC/FCC (unless BSM), but limits could be set from existing H→a(γγ) a(γγ) searches.

			Exp. limits		Producible at	
	Branching fraction	Framework	2023	HL-LHC	FCC-ee	FCC-hh
$H \rightarrow \gamma+\gamma+\gamma$	1.0×10^{-40}	NLO SM, MG5_aMC (this work)	–	–	✘	✘
$H \rightarrow \gamma+\gamma+\gamma+\gamma$	$(4.56 \pm 0.01) \times 10^{-12}$	NLO SM, MG5_aMC (this work)	–	–	✘	✘

Higgs quark Yukawa couplings

Higgs quark Yukawas via exclusive H→VM+V decays

- Higgs decays into a meson + EW boson offer the possibility to constrain (very) suppressed Yukawa couplings to valence quarks (H→cc,qq):



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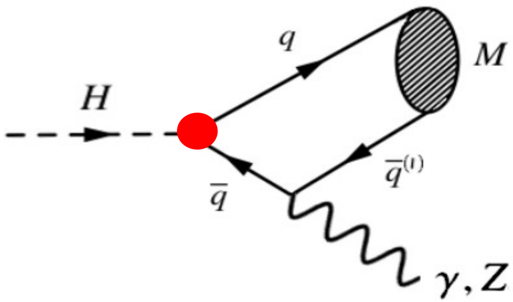
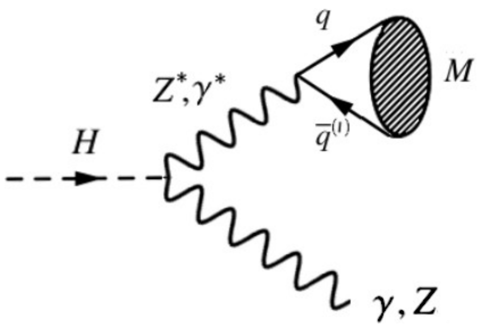




G. T. Bodwin et al. arXiv:1306.5770 [hep-ph]
G. Isidori et al., arXiv:1305.0663 [hep-ph]
A. L. Kagan et al. arXiv:1406.1722 [hep-ph]
M. König, M. Neubert, arXiv:1505.03870 [hep-ph]
G. Perez et al., arXiv:1505.06689 [hep-ph]

- However, direct (Yukawa-sensitive) contribution is much smaller (for SM g_{qH}) than the indirect H→Z,γ followed by Z,γ→VM conversion.

- Destructive interference of dir./indir. amplitudes:

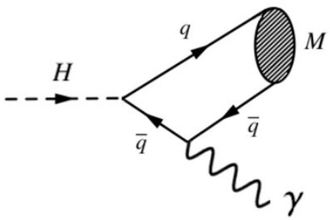
$$\Gamma(H \to V\gamma) = \frac{1}{8\pi}\frac{m_H^2 - m_V^2}{m_H^2}\left|\mathcal{A}_{\rm direct} + \mathcal{A}_{\rm indirect}\right|^2$$

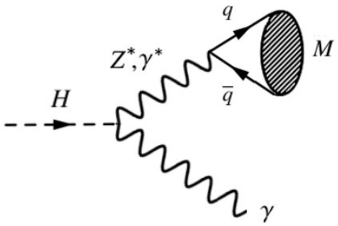
$A_{\rm direct} \propto g^2_{qH}$: For SM Yukawas, the dir. contribution is very small (except for c,b-quark), but limits on BSM-enhanced light-q Yukawas can still be set.

Process	$\frac{\mathcal{A}_{\rm direct}}{\mathcal{A}_{\rm indirect}}$
$H \to \gamma + \rho^0$	
$H \to \gamma + \omega$	-0.002
$H \to \gamma + \phi$	
$H \to \gamma + J/\psi$	-0.06
$H \to \gamma + \psi(2S)$	
$H \to \gamma + \Upsilon(1S)$	
$H \to \gamma + \Upsilon(2S)$	-0.8
$H \to \gamma + \Upsilon(3S)$	

Exclusive Higgs decays into vector-meson + γ

- Exclusive $H \to VM + \gamma$ decays computed by multiple groups in SCET (q), NRQCD (Q):



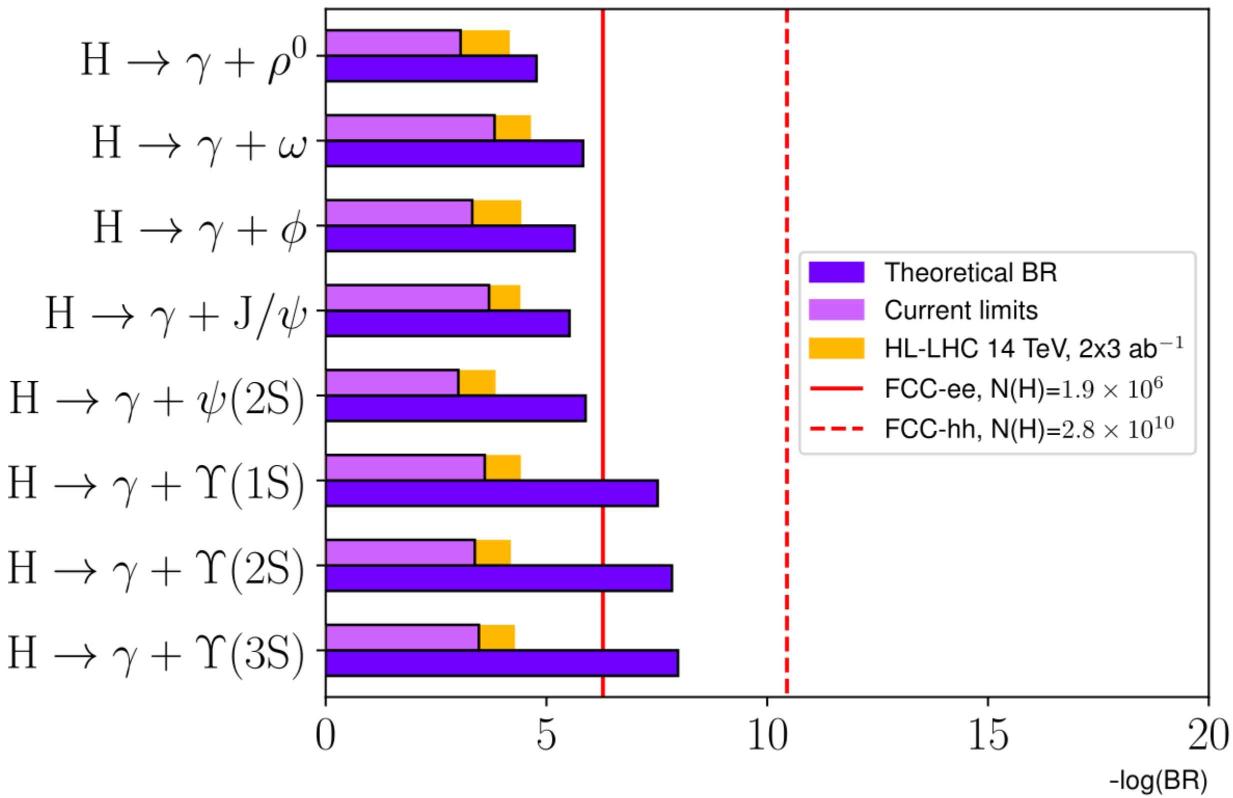


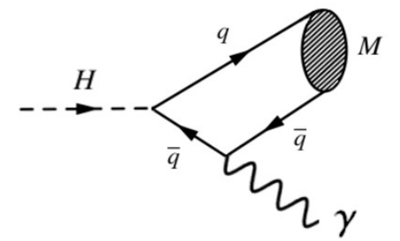
$H \to \gamma$ +	M	Branching fraction	Framework	Exp. limits 2023	Exp. limits HL-LHC	Producible at FCC-ee	Producible at FCC-hh
$H \to \gamma$ +	ρ^0	$(1.68 \pm 0.08) \times 10^{-5}$	SCET+LCDA [14]	$< 8.8\times 10^{-4}$ [88]	$\lesssim 6.8 \times 10^{-5}$	✓	✓
	ω	$(1.48 \pm 0.08) \times 10^{-6}$	SCET+LCDA [14]	$< 1.5\times 10^{-4}$ [91]	$\lesssim 2.2 \times 10^{-5}$	✓	✓
	ϕ	$(2.31 \pm 0.11) \times 10^{-6}$	SCET+LCDA [14]	$< 4.8\times 10^{-4}$ [88]	$\lesssim 3.7 \times 10^{-5}$	✓	✓
	J/ψ	$(2.95 \pm 0.17) \times 10^{-6}$	SCET+LCDA [14]	$< 2.0\times 10^{-4}$ [93]	$\lesssim 3.9 \times 10^{-5}$ [62]	✓	✓
		$(3.01 \pm 0.15) \times 10^{-6}$	NRQCD (NLL)+LDME [94]				
		$\left(2.99^{+0.16}_{-0.15}\right) \times 10^{-6}$	NRQCD+LCDA [95]				
	ψ(2S)	$(1.3 \pm 0.0) \times 10^{-6}$	SCET+LCDA [14]	$< 9.9\times 10^{-4}$ [92, 93]	$\lesssim 1.4 \times 10^{-4}$	✓	✓
	Υ(1S)	$\left(4.61^{+1.76}_{-1.23}\right) \times 10^{-9}$	SCET+LCDA [14]	$< 2.5\times 10^{-4}$ [93]	$\lesssim 3.8 \times 10^{-5}$	✘	✓
		$\left(9.97^{+4.04}_{-3.03}\right) \times 10^{-9}$	NRQCD (NLL)+LDME [94]				
		3.0×10^{-8}	NRQCD (NLO)+LDME [96]				
		$\left(5.22^{+2.02}_{-1.70}\right) \times 10^{-9}$	NRQCD+LCDA [95]				
	Υ(2S)	$\left(2.34^{+0.76}_{-1.00}\right) \times 10^{-9}$	SCET+LCDA [14]	$< 4.2\times 10^{-4}$ [93]	$\lesssim 6.4 \times 10^{-5}$	✘	✓
		$\left(2.62^{+1.39}_{-0.91}\right) \times 10^{-9}$	NRQCD (NLL)+LDME [94]				
		1.4×10^{-8}	NRQCD (NLO)+LDME [96]				
		$\left(1.42^{+0.72}_{-0.57}\right) \times 10^{-9}$	NRQCD+LCDA [95]				
	Υ(3S)	$\left(2.13^{+0.76}_{-1.12}\right) \times 10^{-9}$	SCET+LCDA [14]	$< 3.4\times 10^{-4}$ [93]	$\lesssim 5.2 \times 10^{-5}$	✘	✓
		$\left(1.87^{+1.05}_{-0.69}\right) \times 10^{-9}$	NRQCD (NLL)+LDME [94]				
		1.1×10^{-8}	NRQCD (NLO)+LDME [96]				
		$\left(9.1^{+4.8}_{-3.8}\right) \times 10^{-10}$	NRQCD+LCDA [95]				

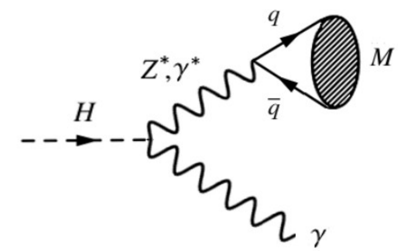
- Theory $\mathcal{B}$'s: $\mathcal{O}(10^{-5}$–$10^{-10})$. Factors 2–3 differences among SCET-NRQCD for Υ(nS)
- Experiment: 8 channels studied with limits: $\mathcal{B} < \mathcal{O}(10^{-3}$–$10^{-4})$

Exclusive Higgs decays into vector-meson + γ

- Most recent & future limits on exclusive $H \to VM+\gamma$ decays:



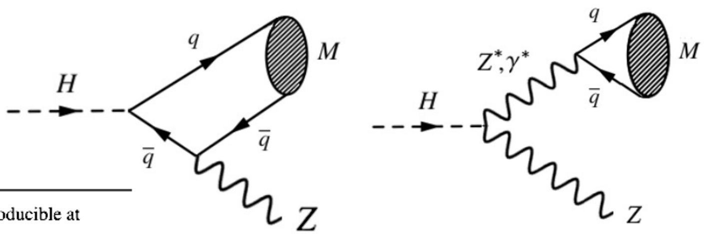




- Theory $\mathcal{B}$'s: $\mathcal{O}(10^{-5}\text{–}10^{-10})$. Factors 2–3 differences among SCET-NRQCD for Υ(nS)
- Experiment: 8 channels studied with limits $\mathcal{B}<\mathcal{O}(10^{-3}\text{–}10^{-4})$
- HL-LHC: $H\to\rho+\gamma$ may be observed (possible upper bound for u,d Yukawas?)
- 5 (all 8) producible channels at FCC-ee (FCC-hh).

Exclusive Higgs decays into vector-meson + Z

- Exclusive H → VM + Z decays computed by multiple groups in EFT (q), NRQCD (Q):



H → Z +	M	Branching fraction	Framework	Exp. limits 2023	Exp. limits HL-LHC	Producible at FCC-ee	Producible at FCC-hh
H → Z +	π^0	$(2.3 \pm 0.1) \times 10^{-6}$	EFT+NRQM [10]	–	–	✓	✓
		$(2.3 \pm 0.1) \times 10^{-6}$	EFT+LCDA [102]				
	η	$(8.3 \pm 0.9) \times 10^{-7}$	EFT+LCDA [102]	–	–	✓	✓
	ρ^0	$(1.4 \pm 0.1) \times 10^{-5}$	EFT+NRQM [10]	$< 1.2\times 10^{-2}$ [103]	$\lesssim 1.8\times 10^{-3}$	✓	✓
		$(7.19 \pm 0.29) \times 10^{-6}$	EFT+LCDA [102]				
	ω	$(1.6 \pm 0.1) \times 10^{-6}$	EFT+NRQM [10]	–	–	✓	✓
		$(5.6 \pm 0.2) \times 10^{-7}$	EFT+LCDA [102]				
	η'	$(1.24 \pm 0.13) \times 10^{-6}$	EFT+LCDA [102]	–	–	✓	✓
	ϕ	$(4.2 \pm 0.3) \times 10^{-6}$	EFT+NRQM [10]	$< 3.6\times 10^{-3}$ [103]	$\lesssim 5.4\times 10^{-4}$	✓	✓
		$(2.42 \pm 0.10) \times 10^{-6}$	EFT+LCDA [102]				
	η_c	$(1.0 \pm 0.1) \times 10^{-5}$	EFT+NRQM [10]	–	–	✓	✓
		$(1.0 \pm 0.0) \times 10^{-5}$	EFT+LCDA [104]				
	J/ψ	3.4×10^{-6}	NRQCD (NLO)+LMDE [105]	$< 1.9\times 10^{-3}$ [106]	$\lesssim 2.1\times 10^{-4}$ [64]	✓	✓
		3.2×10^{-6}	EFT+NRQM [107]				
		$(2.3 \pm 0.1) \times 10^{-6}$	EFT+LCDA [102]				
	$\psi(2S)$	1.5×10^{-6}	EFT+NRQM [107]	$< 6.6\times 10^{-3}$ [106]	$\lesssim 1.0\times 10^{-3}$	✓	✓
	η_b	$(2.69 \pm 0.05) \times 10^{-5}$	EFT+LCDA [104]	–	–	✓	✓
		$\left(4.739^{+0.276}_{-0.244}\right) \times 10^{-5}$	EFT (NLO)+LCDA [108]				
	$\Upsilon(1S)$	1.7×10^{-5}	NRQCD (NLO)+LMDE [105]	–	–	✓	✓
		1.7×10^{-5}	EFT+NRQM [107]				
		$(1.54 \pm 0.06) \times 10^{-5}$	EFT+LCDA [102]				
	$\Upsilon(2S)$	8.9×10^{-6}	EFT+NRQM [107]	–	–	✓	✓
		$(7.5 \pm 0.3) \times 10^{-6}$	EFT+LCDA [102]				
	$\Upsilon(3S)$	6.7×10^{-6}	EFT+NRQM [107]	–	–	✓	✓
		$(5.63 \pm 0.24) \times 10^{-6}$	EFT+LCDA [102]				

Less sensitivity to g_{qH}, but probe CP-even,CP-odd HγZ couplings:

$$\frac{A_{\rm direct}}{A_{\rm indirect}}$$

<0.0001: $H \to Z + \pi^0$, $H \to Z + \eta$, $H \to Z + \rho^0$, $H \to Z + \omega$, $H \to Z + \eta'$, $H \to Z + \phi$

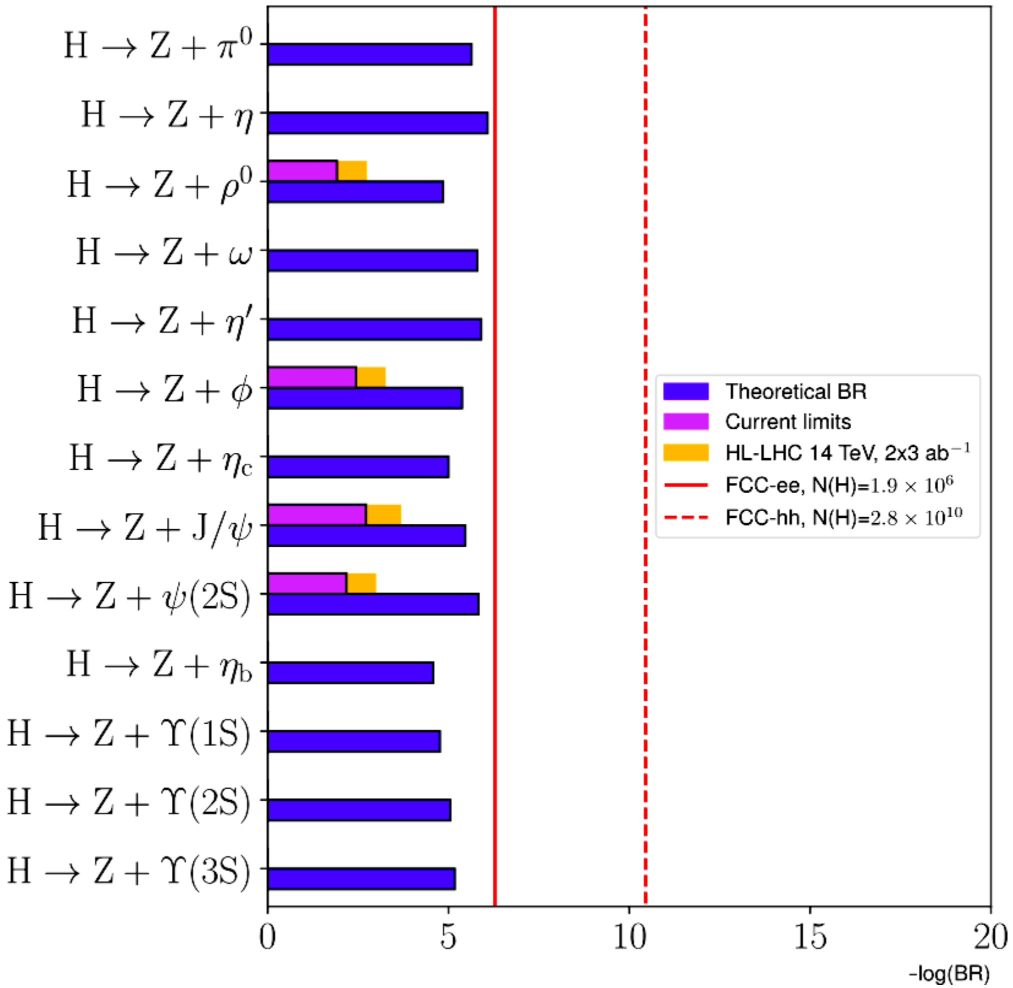
-0.014: $H \to Z + \eta_c$, $H \to Z + J/\psi$, $H \to Z + \psi(2S)$

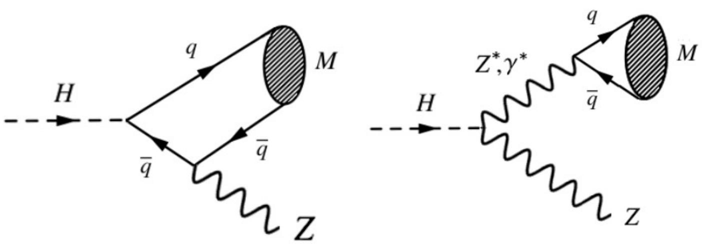
+0.05: $H \to Z + \eta_b$, $H \to Z + \Upsilon(1S)$, $H \to Z + \Upsilon(2S)$, $H \to Z + \Upsilon(3S)$

- Theory $\mathcal{B}$'s: $\mathcal{O}(10^{-5}$–$10^{-7})$. Factors ×2 max. differences among predictions
- Experiment: 4 channels studied with limits: $\mathcal{B} < \mathcal{O}(10^{-2}$–$10^{-3})$

Exclusive Higgs decays into vector-meson + Z

- Most recent & future limits on exclusive $H \to VM + Z$ decays:



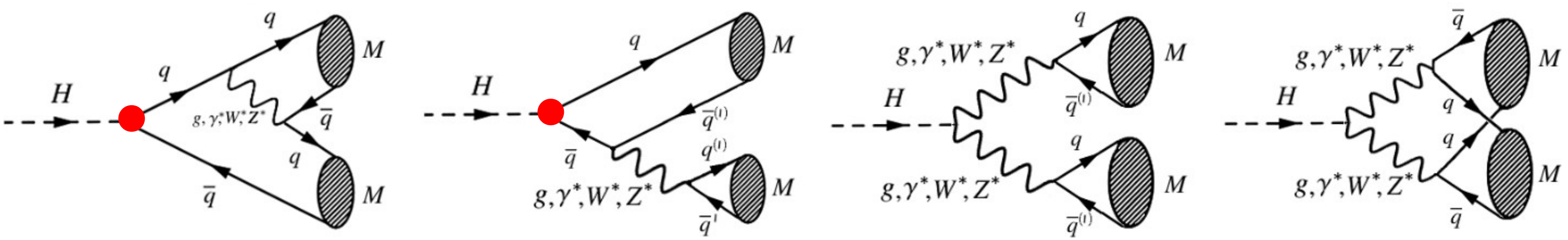


- Botomonia have largest $\mathcal{B}$'s, but no exp. bound set so far.
- No observable channel at HL-LHC
- All channels producible at FCC-ee

- Theory $\mathcal{B}$'s: $\mathcal{O}(10^{-5}$–$10^{-7})$. Factors ×2 max. differences among predictions
- Experiment: 4 channels studied with limits: $\mathcal{B} < \mathcal{O}(10^{-2}$–$10^{-3})$

Exclusive Higgs decays into two mesons

- Can exclusive double-meson Higgs decay constrain Yukawa couplings ?



- Challenges:
  – Multitude of hard channels contribute, not well-controlled theoretically (not all predictions include all diagrams plotted above...).
  – Exclusive pair meson formation: Doubly suppressed → very small rates.
  – Calculations carried out in multiple frameworks (LC+LCDA, NRQCD/ NRCSM, NRQCD+LDME,...) predict very small rates: $\mathcal{O}(10^{-9}-10^{-11})$

Exclusive Higgs decays into two mesons

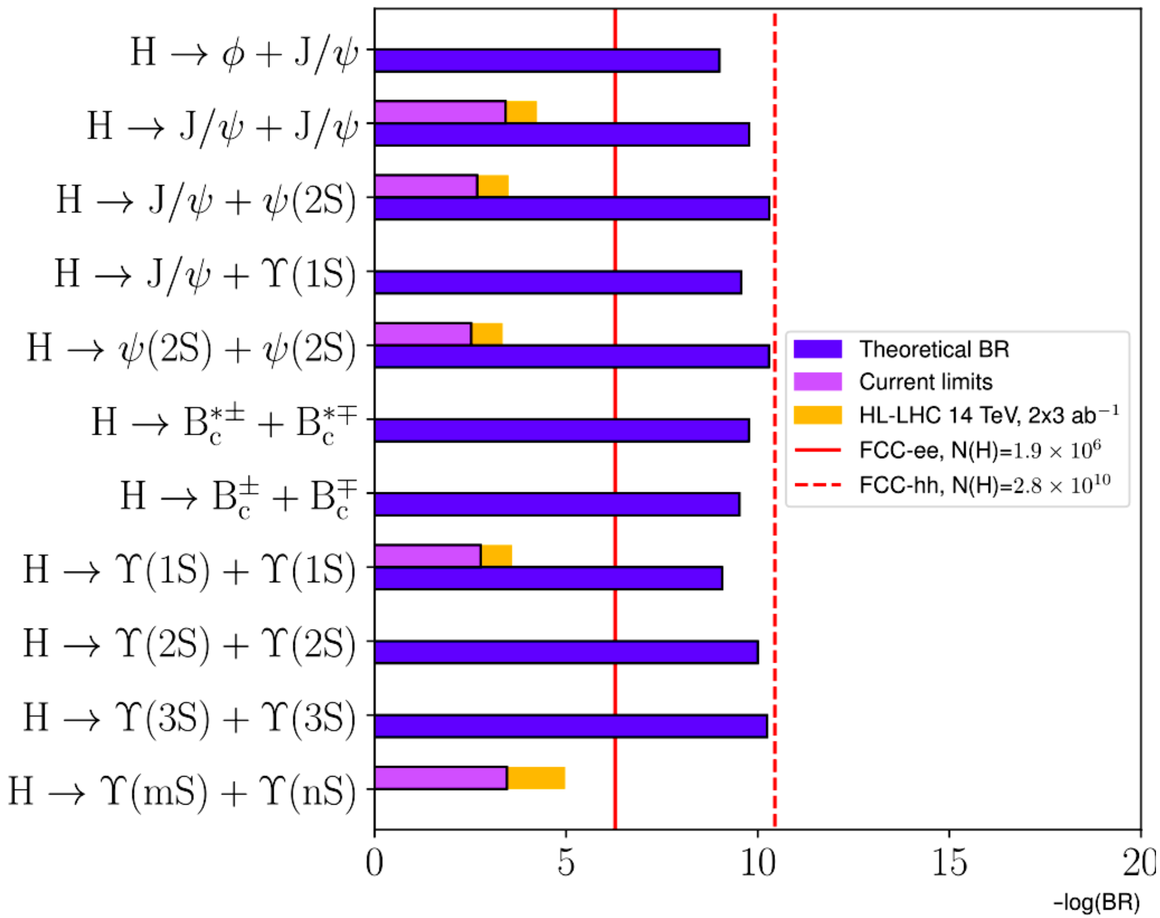
■ Exclusive $H \to VM + VM$ decay branching fractions:

H →	M	+	M	Branching fraction	Framework	Exp. limits 2023	Exp. limits HL-LHC	Producible at FCC-ee	Producible at FCC-hh
H →	ϕ	+	J/ψ	1.0×10^{-9}	LC+LCDA [120]	–	–	✘	✓
				$(5.8-6.0) \times 10^{-9}$	NRQCD+LDME [125]				
				1.7×10^{-10}	RQM [122]				
	J/ψ	+	J/ψ	2.1×10^{-10}	RQM [124]	$< 4.7\times 10^{-4}$ [109]	$\lesssim 7.1\times 10^{-5}$	✘	✓
				$(5.9 \pm 2.3) \times 10^{-10}$	NRQCD/NRCSM [123]				
				1.5×10^{-10}	LC+LCDA [120]				
	$\psi(2S)$	+	J/ψ	$\mathcal{O}(5) \times 10^{-11}$	–	$< 2.6\times 10^{-3}$ [109]	$\lesssim 3.9\times 10^{-4}$	✘	✓
			$\psi(2S)$	$(5.1 \pm 2.0) \times 10^{-11}$	NRQCD/NRCSM [123]	$< 3.6\times 10^{-3}$ [109]	$\lesssim 5.5\times 10^{-4}$	✘	✓
	$B_c^{*\mp}$	+	$B_c^{*\pm}$	$(1.4-1.7) \times 10^{-10}$	RQM [121]	–	–	✘	✓
	$B_c^{\mp}$	+	$B_c^{\pm}$	$(2.0-3.0) \times 10^{-10}$	RQM [121]	–	–	✘	✓
	$\Upsilon(1S)$	+	J/ψ	$(2.7-3.6) \times 10^{-10}$	NRQCD+LDME [125]	–	–	✘	✓
				1.6×10^{-11}	LC+LCDA [120]				
				$(8.5-9.2) \times 10^{-10}$	NRQCD+LDME [125]				
				1.8×10^{-10}	RQM [122]				
			$\Upsilon(1S)$	2.3×10^{-9}	RQM [124]	$< 2.0\times 10^{-3}$ [109]	$\lesssim 3.0\times 10^{-4}$	✘	✓
				$(4.3 \pm 0.9) \times 10^{-10}$	NRQCD/NRCSM [123]				
				2.3×10^{-9}	LC+LCDA [120]				
	$\Upsilon(2S)$	+	$\Upsilon(2S)$	$(1.0 \pm 0.2) \times 10^{-10}$	NRQCD/NRCSM [123]	–	–	✘	✓
	$\Upsilon(3S)$	+	$\Upsilon(3S)$	$(5.7 \pm 1.2) \times 10^{-11}$	NRQCD/NRCSM [123]	–	–	✘	✓
	$\Upsilon(mS)$	+	$\Upsilon(nS)$		–	$< 4.3\times 10^{-4}$ [109]	$\lesssim 1.1\times 10^{-5}$ [64]	✘	✘

- Theory BRs: $\mathcal{O}(10^{-9}$–$10^{-11})$. Predictions within factor 10 (depending on diags. considered)
- Experiment: 5 channels searched-for with limits: $\mathcal{O}(10^{-3}$–$10^{-4})$.
- No (All) producible channels at FCC-ee (FCC-hh)

Exclusive Higgs decays into two mesons

■ Most recent & future limits on $H \to VM + VM$ decays:

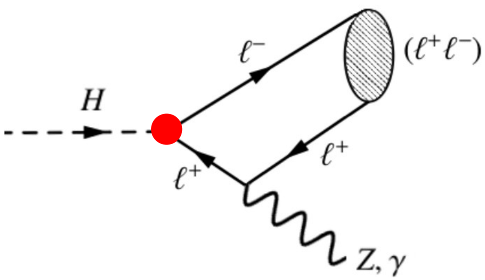


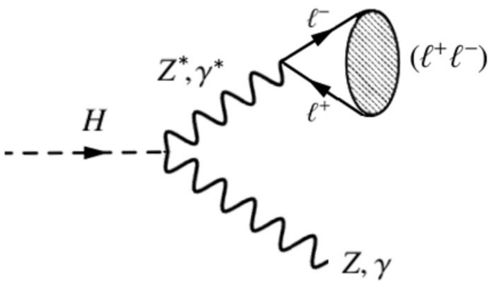
- Theory BRs: $\mathcal{O}(10^{-9}$–$10^{-11})$. Predictions within factor 10 (depending on diags. considered)
- Experiment: 5 channels searched-for with limits: $\mathcal{O}(10^{-3}$–$10^{-4})$.
- No (All) producible channels at FCC-ee (FCC-hh)

Higgs lepton Yukawa couplings

Exclusive Higgs decays into leptonium + γ, Z

- Can Higgs decays into leptonium + EW boson constrain the e, μ (and τ) Yukawas? Contributions from direct+indirect mechanisms. Never computed to date. Expected decay width much smaller than for quarkonium counterparts:





$$f_M^2 = 4N_c \frac{|\phi_M(0)|^2}{m_M}$$

Leptonium wavefunction at origin:

$$|\phi_{n,(\ell\ell)}(r=0)|^2 = \frac{(m_\ell \alpha(0))^3}{8\pi n^3}$$

$$[\alpha(0) m_{\ell\ell} / (\alpha_s(m_{q\bar{q}}) m_{q\bar{q}})]^3 \ll 1$$

Ortholeptonium: VM = $(\ell\ell)_1$ is the only state possible (CP conservation) with γ.

Paraleptonium: PS= $(\ell\ell)_0$ also possible (but suppressed) with Z (long. polarized)

- H→$(\ell\ell)_1$+γ decay widths computed using similar approach as for H→(qq)+γ:

$$\mathcal{B}(\mathrm{H} \to \gamma + (\ell^+\ell^-)_1) = \frac{1}{8\pi} \frac{\mathrm{m_H^2} - \mathrm{m_{\ell\ell}^2}}{\mathrm{m_H^2}\,\Gamma_\mathrm{H}} |\mathcal{A}_\mathrm{dir} + \mathcal{A}_\mathrm{ind}|^2$$

(destructive interference)

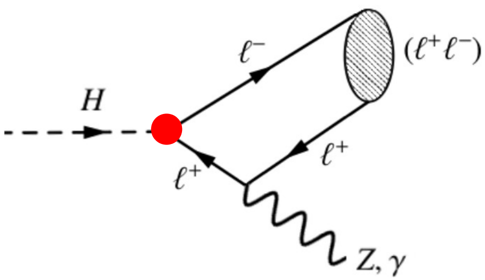
$$\mathcal{A}_\mathrm{dir} = 2Q_\ell \sqrt{4\pi\alpha(0)} \left(\sqrt{2}\mathrm{G_F}\, m_{\ell\ell}\right)^{1/2} \frac{m_\mathrm{H}^2 - m_{\ell\ell}^2}{\sqrt{m_\mathrm{H}}(m_\mathrm{H}^2 - m_{\ell\ell}^2/2 - 2m_\ell^2)} \phi_{\mathrm{n},(\ell\ell)}(0)$$

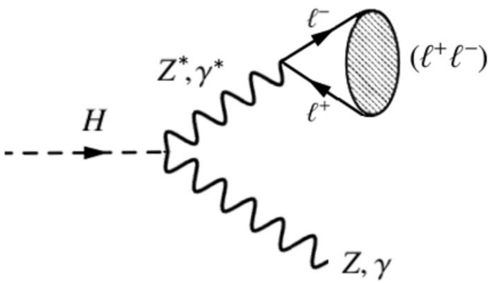
$$\mathcal{A}_\mathrm{ind} = \frac{Q_\ell \sqrt{4\pi\alpha(0)}\, f_{\ell\ell}}{m_{\ell\ell}} \left(16\pi\Gamma_{\mathrm{H}\to\gamma\gamma}\right)^{1/2} \frac{m_\mathrm{H}^2 - m_{\ell\ell}^2}{m_\mathrm{H}^2} = -\frac{2Q_\ell\sqrt{4\pi\alpha(0)}}{m_{\ell\ell}^{3/2}} \left(16\pi\Gamma_{\mathrm{H}\to\gamma\gamma}\right)^{1/2} \frac{m_\mathrm{H}^2 - m_{\ell\ell}^2}{m_\mathrm{H}^2} \phi_{\mathrm{n},(\ell\ell)}(0)$$

$\mathcal{A}_\mathrm{dir}/\mathcal{A}_\mathrm{indir}$	
~10^{-8}	$\mathrm{H} \to \gamma + (\mathrm{ee})_1$
~10^{-3}	$\mathrm{H} \to \gamma + (\mu\mu)_1$
~0.2	$\mathrm{H} \to \gamma + (\tau\tau)_1$

Exclusive Higgs decays into leptonium + Z

- Can Higgs decays into leptonium + EW boson constrain the e, μ (and τ) Yukawas? Contributions from direct+indirect mechanisms. Never computed to date. Expected decay width much smaller than for quarkonium counterparts:





$$f_M^2 = 4N_c \frac{|\phi_M(0)|^2}{m_M}$$

Leptonium wavefunction at origin:

$$|\phi_{n,(\ell\ell)}(r=0)|^2 = \frac{(m_\ell \alpha(0))^3}{8\pi n^3}$$

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Paraleptonium: PS= $(\ell\ell)_0$ also possible (but suppressed) with Z (long. polarized)

- $H \to (\ell\ell)_1 + Z$ decay widths computed using similar approach as for $H \to (qq) + Z$:

$$\Gamma(H \to Z + (\ell^+\ell^-)_1) = \Gamma_1 + \Gamma_2 + \Gamma_3,$$

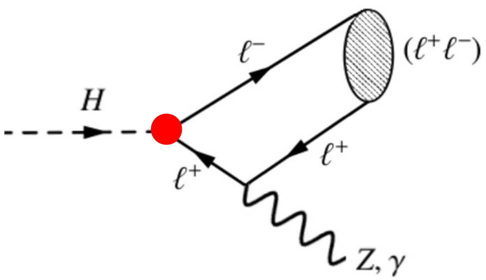
$$\Gamma_1 = \frac{m_H^3 (g_{\ell\ell} f_{\ell\ell})^2}{16\pi v^4} \frac{\lambda^{1/2}(1, r_Z, r_{\ell\ell})}{(1 - r_{\ell\ell}/r_Z)^2} [(1 - r_Z - r_{\ell\ell})^2 + 8 r_Z r_{\ell\ell}],$$

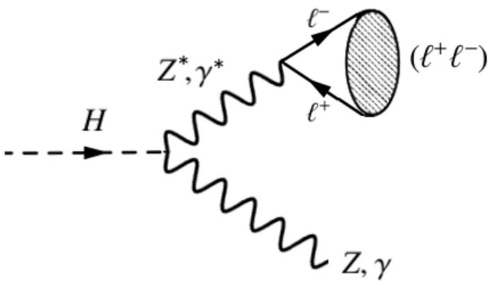
$$\Gamma_2 = \frac{\alpha(0)^3 f_{\ell\ell}^2 Q_\ell^2 m_H^3}{32\pi^2 v^2 \sin^2\theta_W} \frac{C_{Z\gamma}^2}{m_{\ell\ell}^2} \lambda^{1/2}(1, r_Z, r_{\ell\ell}) [(1 - r_Z - r_{\ell\ell})^2 + 2 r_Z r_{\ell\ell}],$$

$$\Gamma_3 = \frac{3\alpha(0)^2 f_{\ell\ell}^2 g_{\ell\ell} Q_\ell m_H C_{Z\gamma}}{8\pi \cos\theta_W \sin^2\theta_W v^2} \frac{\lambda^{1/2}(1, r_Z, r_{\ell\ell})}{1 - r_{\ell\ell}/r_Z} (1 - r_Z - r_{\ell\ell}),$$

Exclusive Higgs decays into leptonium + Z

- Can Higgs decays into leptonium + EW boson constrain the e, μ (and τ) Yukawas? Contributions from direct+indirect mechanisms. Never computed to date. Expected decay width much smaller than for quarkonium counterparts:





$$f_M^2 = 4N_c \frac{|\phi_M(0)|^2}{m_M}$$

Leptonium wavefunction at origin:

$$|\phi_{n,(\ell\ell)}(r=0)|^2 = \frac{(m_\ell \alpha(0))^3}{8\pi n^3}$$

$$[\alpha(0) m_{\ell\ell} / (\alpha_s(m_{q\bar{q}}) m_{q\bar{q}})]^3 \ll 1$$

Ortholeptonium: VM = $(\ell\ell)_1$ is the only state possible (CP conservation) with γ.

Paraleptonium: PS= $(\ell\ell)_0$ also possible (but suppressed) with Z (long. polarized)

- H→$(\ell\ell)_0$ + Z decay widths computed using similar approach as for H→(qq) + Z:

$$\Gamma(\mathrm{H} \to \mathrm{Z} + (\ell^+\ell^-)_0) = \frac{m_\mathrm{H}^3}{4\pi v^4} \lambda^{3/2}(1, r_\mathrm{Z}, r_{\ell\ell}) \left| F^{\mathrm{Z}+\ell\ell_0} \right|^2 \quad \text{with } F^{\mathrm{Z}+\ell\ell_0} = F_\mathrm{dir}^{\mathrm{Z}+\ell\ell_0} + F_\mathrm{ind}^{\mathrm{Z}+\ell\ell_0}$$

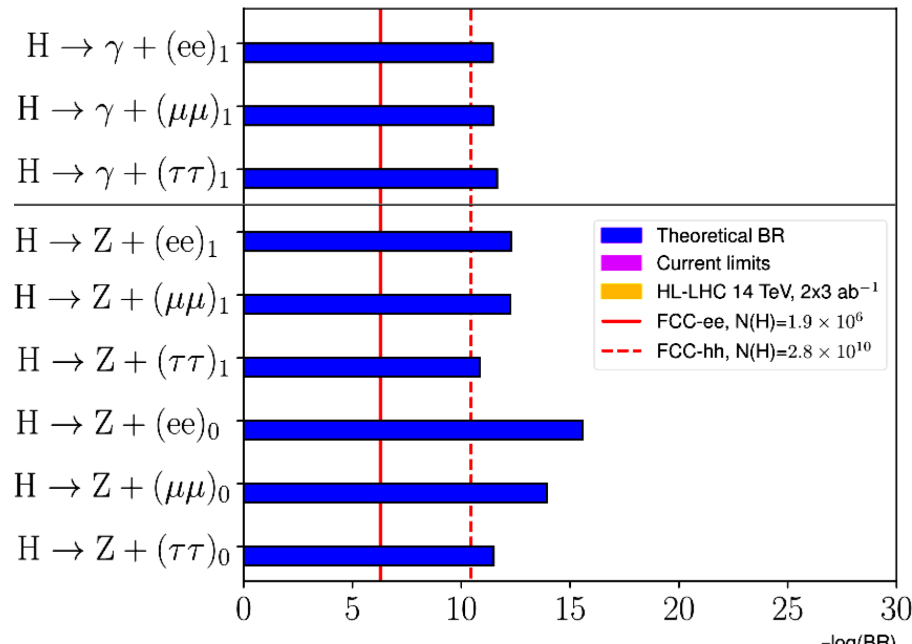
$$F_\mathrm{dir}^{\mathrm{Z}+\ell\ell_0} = -f_{\ell\ell}\, a_\ell \frac{m_\ell^2}{m_\mathrm{H}^2} \frac{1 - r_\mathrm{Z}^2 + 2 r_\mathrm{Z} \ln r_\mathrm{Z}}{(1 - r_\mathrm{Z})^3}, \qquad F_\mathrm{ind}^{\mathrm{Z}+\ell\ell_0} = f_{\ell\ell}\, a_\ell .$$

Exclusive Higgs decays into leptonium + γ, Z

- Exclusive $H \to (\ell\ell)_{0,1} + \gamma, Z$ decay branching fractions:

					Exp. limits		Producible at	
$H \to V$	+	$(\ell\ell)$	Branching fraction	Framework	2023	HL-LHC	FCC-ee	FCC-hh
$H \to \gamma$	+	$(ee)_1$	3.5×10^{-12}	(this work)	–	–	✘	✘
		$(\mu\mu)_1$	3.5×10^{-12}	(this work)	–	–	✘	✘
		$(\tau\tau)_1$	2.2×10^{-12}	(this work)	–	–	✘	✘
$H \to Z$	+	$(ee)_1$	5.2×10^{-13}	(this work)	–	–	✘	✘
		$(\mu\mu)_1$	5.7×10^{-13}	(this work)	–	–	✘	✘
		$(\tau\tau)_1$	1.4×10^{-11}	(this work)	–	–	✘	✘
		$(ee)_0$	2.7×10^{-16}	(this work)	–	–	✘	✘
		$(\mu\mu)_0$	1.1×10^{-14}	(this work)	–	–	✘	✘
		$(\tau\tau)_0$	3.2×10^{-12}	(this work)	–	–	✘	✘

- Tiny branching fractions: $\mathcal{O}(10^{-12}$–$10^{-16})$
- No channel searched-for to date. Leptonia are long-lived = clear LLP signature (displaced 3γ, e^+e^-, $\mu^+\mu^-$ vertices)
- Unless BSM enhances lepton Yukawas, no SM channels producible at FCC-ee. $H \to (\tau\tau) + Z$ may be reachable at FCC-hh



Higgs FCNC decays

FCNC Higgs decays

- Flavour-changing neutral currents (via H boson) are a powerful probe of many BSM scenarios. Only possible via suppressed W loops in the SM:
- Exclusive FCNC H → qq' decay:
  - Very suppressed SM FCNC vertex (negligible SM background).
  - Well-controlled elementary vertex (small theoretical uncertainty)
- SM H → qq' branching fractions:

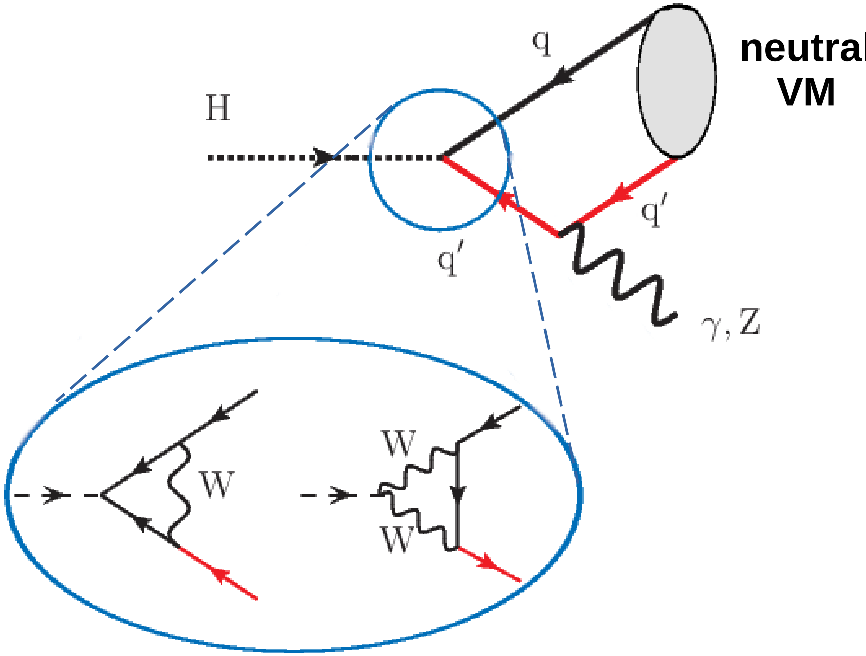
$$\mathcal{B}(\mathrm{H} \to \mathrm{qq'}) \equiv \mathcal{B}(\mathrm{H} \to \mathrm{q\bar{q}'} + \mathrm{q'\bar{q}})$$

$$\mathcal{B}(\mathrm{H} \to \mathrm{sb}) = (8.9 \pm 1.5) \cdot 10^{-8}$$

$$\mathcal{B}(\mathrm{H} \to \mathrm{db}) = (3.8 \pm 0.6) \cdot 10^{-9}$$

$$\mathcal{B}(\mathrm{H} \to \mathrm{ds}) = 1.19 \cdot 10^{-11}$$

$$\mathcal{B}(\mathrm{H} \to \mathrm{uc}) = (2.7 \pm 0.5) \cdot 10^{-20}$$



[J.F.Kamenik et al., arXiv:2306.17520 [hep-ph].]
[I.Aranda et al., arXiv:2009.07166 [hep-ph].

- H → VM^0 + γ,Z with exclusive meson formation from LCDA-based expression:

$$\mathcal{B}(\mathrm{H} \to \gamma + \mathrm{VM(qq')}) = \frac{\alpha(0)}{2\, m_\mathrm{H}} \left(\frac{f_\mathrm{VM} m_\mathrm{VM}}{2\, \lambda_\mathrm{VM}(\mu)} Q_\mathrm{q} \right)^2 \frac{|\kappa_{\mathrm{qq'}}|^2 + |\kappa_{\mathrm{q'q}}|^2}{\Gamma_\mathrm{H}},$$

$$\mathcal{B}(\mathrm{H} \to \mathrm{Z} + \mathrm{VM(qq')}) = \frac{9 m_\mathrm{H} [f^{\perp}_\mathrm{VM}(\mu_\mathrm{HZ})]^2}{8\pi v^2} v_\mathrm{q}^2 \frac{|\kappa_{\mathrm{qq'}}|^2 + |\kappa_{\mathrm{q'q}}|^2}{2\, \Gamma_\mathrm{H}} \frac{r_\mathrm{Z}}{(1 - r_\mathrm{Z})^3} (1 - r_\mathrm{Z}^2 + 2 r_\mathrm{Z} \ln r_\mathrm{Z})^2.$$

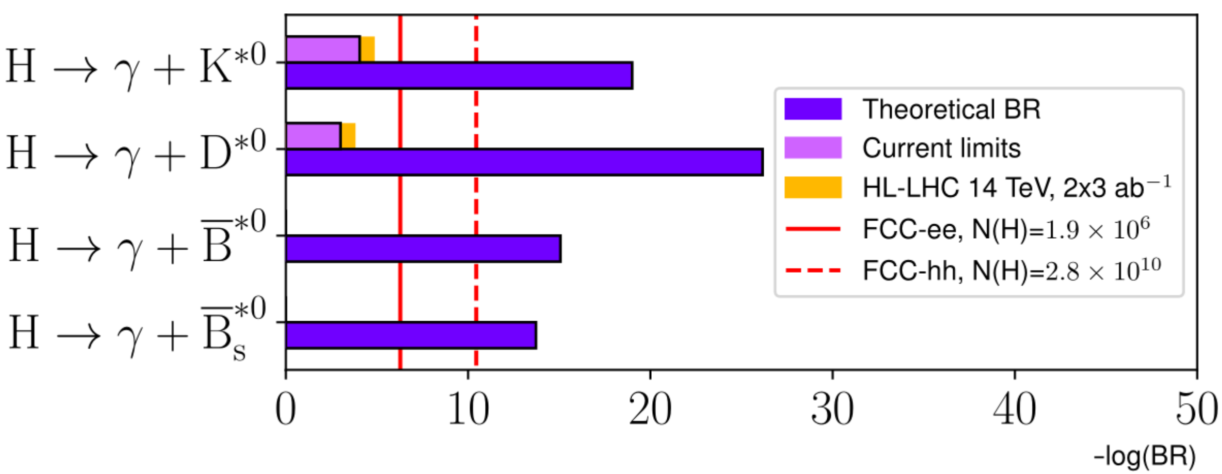
With known theoretical ingredients: f_M, HQET λ_M,..

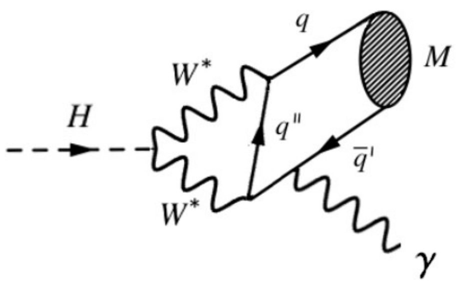
Exclusive Higgs decays into γ + flavoured-meson

- Exclusive $H \to VM^0 + \gamma$ decays computed here within EFT+LCDA approach:

$H \to X$ + M		Branching fraction	Framework	Exp. limits 2023	Exp. limits HL-LHC	Producible at FCC-ee	Producible at FCC-hh
$H \to \gamma$ +	K^{*0}	1.6×10^{-19}	EFT+LCDA (this work)	$< 1.7 \times 10^{-4}$ [91, 93]	$\lesssim 2.6 \times 10^{-5}$	✘	✘
	D^{*0}	6.7×10^{-27}	EFT+LCDA (this work)	$< 1.0 \times 10^{-3}$ [104]	$\lesssim 1.5 \times 10^{-4}$	✘	✘
	B^{*0}	8.2×10^{-16}	EFT+LCDA (this work)	–	–	✘	✘
	B_s^{*0}	1.8×10^{-14}	EFT+LCDA (this work)	–	–	✘	✘

- Current & future experimental limits:





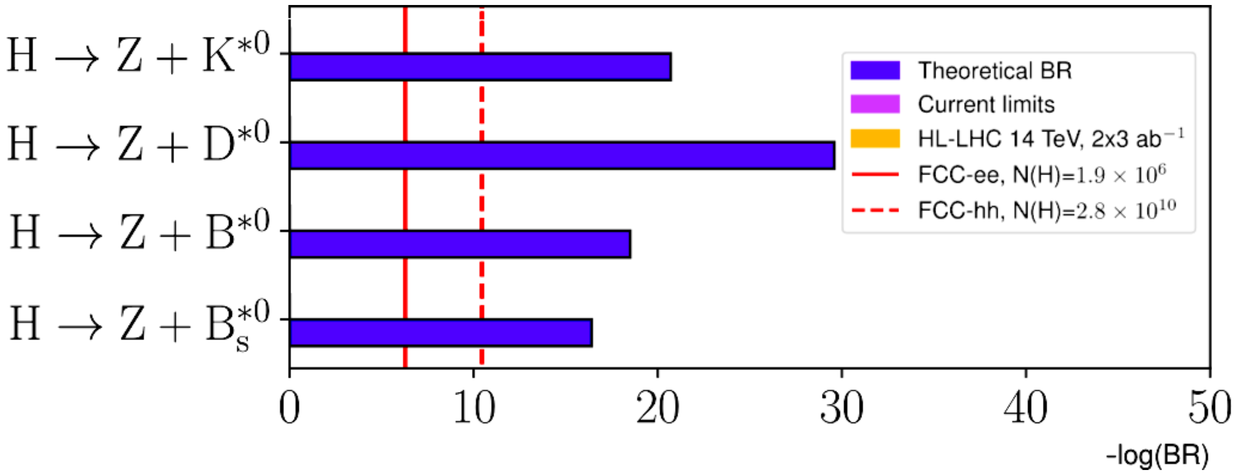
- Tiny theoretical SM branching fractions: $\mathcal{O}(10^{-14}–10^{-27})$. Any visible signal is BSM!
- Experiment: 2 channels searched-for with limits: $\mathcal{O}(10^{-3} – 10^{-4})$
- No channels producible at FCC-ee (FCC-hh) either: Very strong BSM FCNC limits.

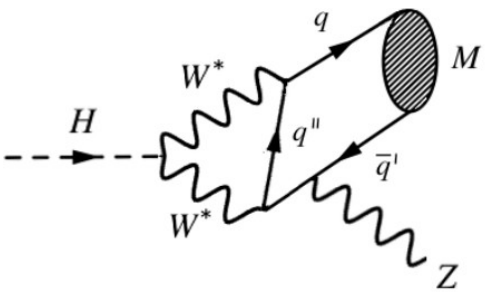
Exclusive Higgs decays into Z + flavoured-meson

- Exclusive $H \to VM^0 + Z$ decays computed here within EFT+LCDA approach:

$H \to X$ + M		Branching fraction	Framework	Exp. limits 2023	Exp. limits HL-LHC	Producible at FCC-ee	Producible at FCC-hh
$H \to Z$ +	K^{*0}	1.4×10^{-21}	EFT+LCDA (this work)	–	–	✘	✘
	D^{*0}	1.8×10^{-30}	EFT+LCDA (this work)	–	–	✘	✘
	B^{*0}	2.4×10^{-19}	EFT+LCDA (this work)	–	–	✘	✘
	B_s^{*0}	2.9×10^{-17}	EFT+LCDA (this work)	–	–	✘	✘

- Future experimental limits:

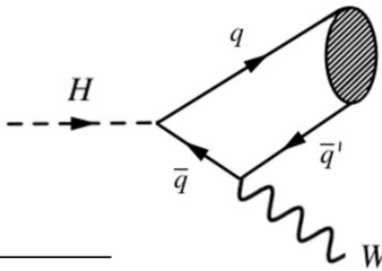




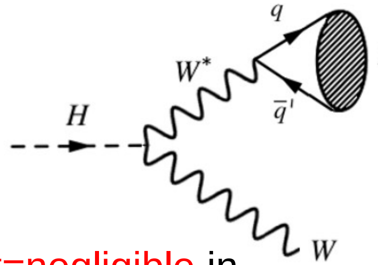
- Tiny theoretical SM branching fractions: $\mathcal{O}(10^{-17}$–$10^{-30})$. Any visible signal is BSM!
- Experiment: No channels searched-for to date
- No channels producible at FCC-ee (FCC-hh) either: Very strong BSM FCNC limits.

Exclusive Higgs decays into W + charged meson

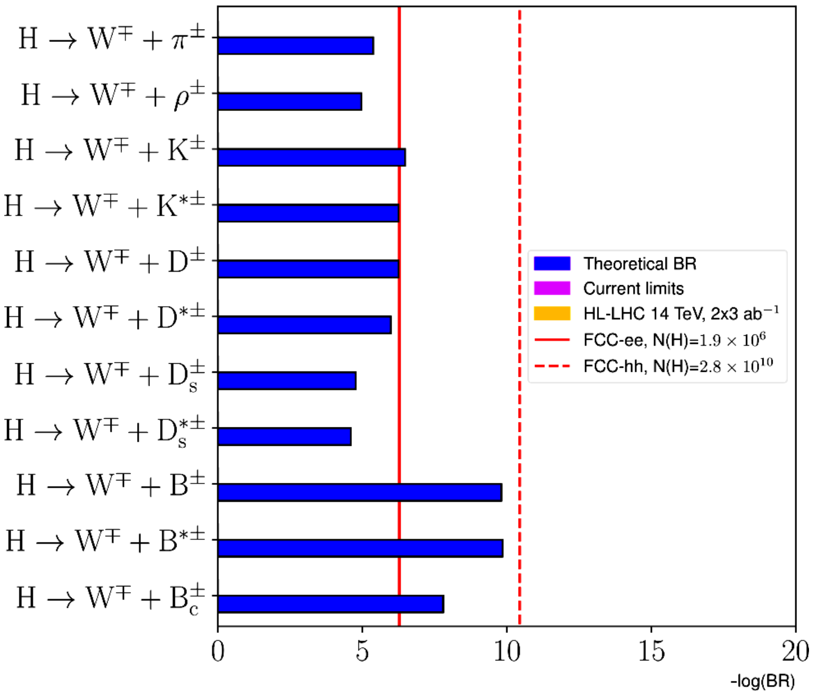
- Exclusive $H \to M^{\pm} + W^{\mp}$ decays computed in EFT+LCDA/NRQM models:



Direct=negligible in SM (unless Higgs FCNC)



H → W	+	M	Branching fraction	Framework	Exp. limits 2023	Exp. limits HL-LHC	Producible at FCC-ee	Producible at FCC-hh
$H \to W^{\mp}$	+	$\pi^{\pm}$	$(4.2 \pm 0.2) \times 10^{-6}$	EFT+NRQM [10]	–	–	✓	✓
			$(4.3 \pm 0.2) \times 10^{-6}$	EFT+LCDA [102]				
		$\rho^{\pm}$	$(1.5 \pm 0.1) \times 10^{-5}$	EFT+NRQM [10]	–	–	✓	✓
			$(1.09 \pm 0.05) \times 10^{-5}$	EFT+LCDA [102]				
		$K^{\pm}$	$(3.3 \pm 0.1) \times 10^{-7}$	EFT+NRQM [10]	–	–	✗	✓
			$(3.3 \pm 0.1) \times 10^{-7}$	EFT+LCDA [102]				
		$K^{*\pm}$	$(4.3 \pm 0.2) \times 10^{-7}$	EFT+NRQM [10]	–	–	✗	✓
			$(5.6 \pm 0.4) \times 10^{-7}$	EFT+LCDA [102]				
		$D^{\pm}$	$(5.8 \pm 0.6) \times 10^{-7}$	EFT+NRQM [10]	–	–	✓	✓
			$(5.6 \pm 0.5) \times 10^{-7}$	EFT+LCDA [102]				
		$D^{*\pm}$	$(1.3 \pm 0.1) \times 10^{-6}$	EFT+NRQM [10]	–	–	✓	✓
			$(1.04 \pm 0.14) \times 10^{-6}$	EFT+LCDA [102]				
		$D_s^{\pm}$	$(1.6 \pm 0.1) \times 10^{-5}$	EFT+NRQM [10]	–	–	✓	✓
			$(1.71 \pm 0.11) \times 10^{-5}$	EFT+LCDA [102]				
		$D_s^{*\pm}$	$(3.5 \pm 0.2) \times 10^{-5}$	EFT+NRQM [10]	–	–	✓	✓
			$(2.51 \pm 0.19) \times 10^{-5}$	EFT+LCDA [102]				
		$B^{\pm}$	$(1.6 \pm 0.4) \times 10^{-10}$	EFT+NRQM [10]	–	–	✗	✓
			$(1.54 \pm 0.40) \times 10^{-10}$	EFT+LCDA [102]				
		$B^{*\pm}$	$(1.3 \pm 0.2) \times 10^{-5}$	EFT+NRQM [10]	–	–	✓	✓
			$(1.41 \pm 0.36) \times 10^{-10}$	EFT+LCDA [102]				
		$B_c^{\pm}$	$(1.6 \pm 0.2) \times 10^{-8}$	EFT+NRQM [10]	–	–	✗	✓
			$(8.21 \pm 0.83) \times 10^{-8}$	EFT+LCDA [102]				



- Theoretical branching fractions: $\mathcal{O}(10^{-5} - 10^{-10})$, enhanced if FCNC-Higgs couplings
- Experiment: No searches performed to date.
- Future: 7 (all 11) channels producible at FCC-ee (FCC-hh)

Summary: Rare & exclusive Higgs decays

- Revisited theory & exp. status of 2-,3-,4-body rare/exclusive decays of the SM Higgs boson (branching fractions: $\mathcal{B} < 10^{-5}$):
  $\mathcal{B}$'s predictions for 41 channels, and exp. upper limits for 20 channels.
- Physics motivations:
  – Probe light-quark & lepton Yukawa couplings.
  – Search for FCNC Higgs decays. Search for suppressed/forbidden SM decays.
- Explicitly computed $\mathcal{B}$'s for 22 new channels:
  – Neutrinos, multi-photons: $H \to 2\nu$, $\gamma+2\nu$, 3γ, 4γ with SM rates $\mathcal{O}(10^{-4}$–$10^{-40})$
  – Exclusive leptonium: $H \to (\ell\ell)_{0,1} + \gamma,Z$ with SM rates $\mathcal{O}(10^{-12}$–$10^{-16})$
  – FCNC decays: $H \to VM^0 + \gamma,Z$ with SM rates $\mathcal{O}(10^{-14}$–$10^{-30})$
- Provided ($\mathcal{L}_{int}$-based) projections for HL-LHC, FCC-ee, FCC-hh searches. "Promising" decays at HL-LHC ("easy" new limits, or observation at reach):

			Branching fraction	Exp. limits 2023	Exp. limits HL-LHC	$\mathcal{B}$(th)/$\mathcal{B}$(exp)
H →	$\gamma\gamma\gamma\gamma$		$(4.56 \pm 0.01) \times 10^{-12}$	–	–	–
	γ +	ρ^0	$(1.68 \pm 0.08) \times 10^{-5}$	$< 8.8 \times 10^{-4}$ [88]	$\lesssim 6.8 \times 10^{-5}$	$\sim 1/4$
		J/ψ	$(2.95 \pm 0.17) \times 10^{-6}$	$< 2.6 \times 10^{-4}$ [90, 92]	$\lesssim 3.9 \times 10^{-5}$ [62]	$\sim 1/10$
	$W^{\mp}$ +	$\rho^{\pm}$	$(1.5 \pm 0.1) \times 10^{-5}$	–	–	–
		$D_s^{*\pm}$	$(3.5 \pm 0.2) \times 10^{-5}$	–	–	–
	Z +	ρ^0	$(1.4 \pm 0.1) \times 10^{-5}$	$< 1.2 \times 10^{-2}$ [106]	$\lesssim 1.8 \times 10^{-3}$	$\sim 1/100$
		$\Upsilon(1S)$	1.7×10^{-5}	–	–	–

Back-up slides