Combination of aN^{3}LO PDFs and implications for Higgs production

Giacomo Magni
PDF4LHC 2024, 2 December 2024

Based on:
[arxiv.org:2411.05373]

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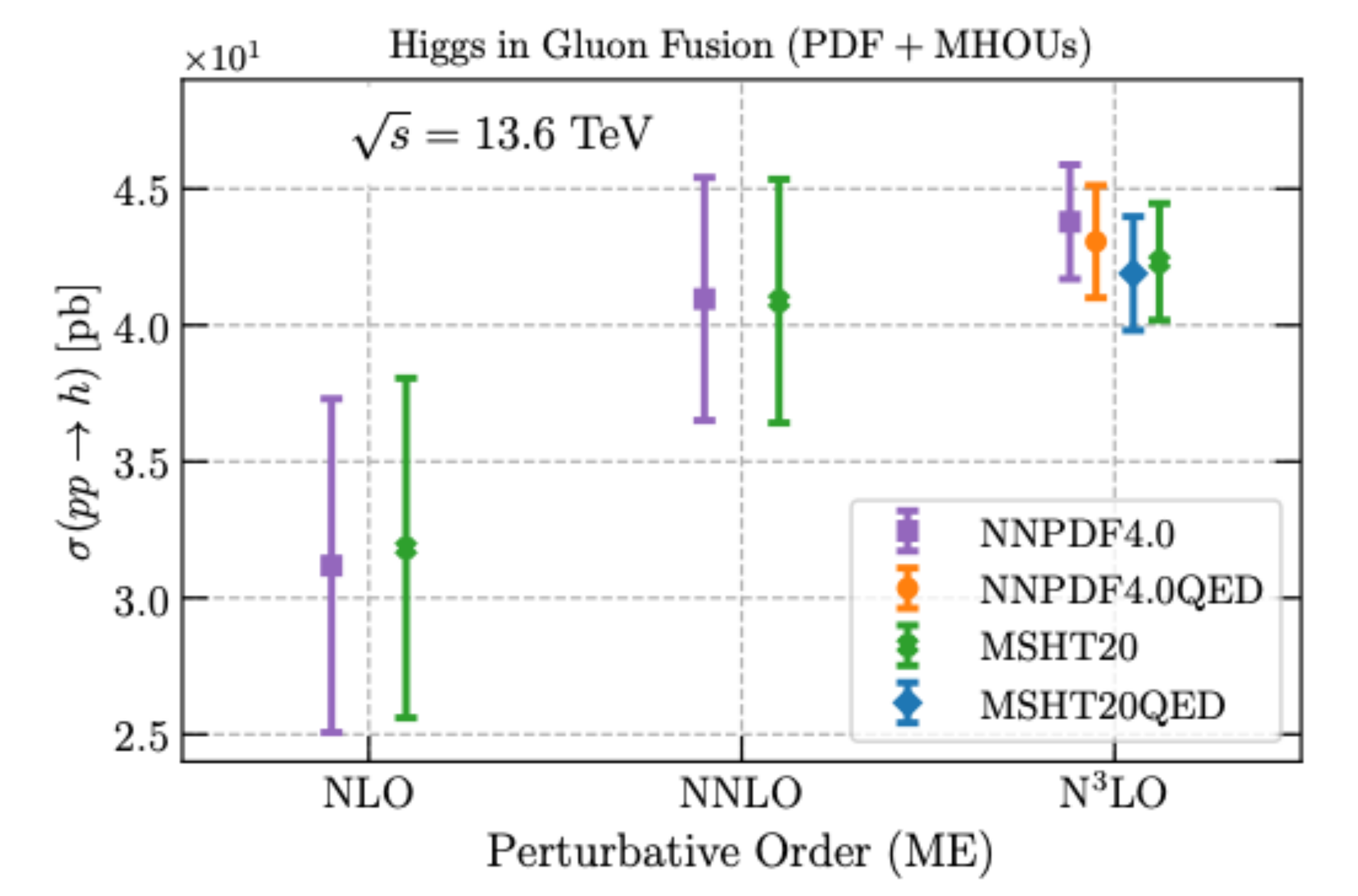
Going beyond NNLO PDFs

➡ **PDFs accuracy** and their uncertainties are nowadays a **limitation** for many **precisions LHC analysis**.

➡ Effects **beyond NNLO QCD** corrections, **QED** corrections, and possibly **theoretical uncertainties** should be taken into account during PDF fits.

- With these goals in mind MSHT and NNPDF have published **approximate N^3LO PDFs**, where besides the available information we introduce an **estimate of PDF theory uncertainties**.
- In both cases **variants including photon PDF**, computed with LuxQED formalism, are also available.
- These studies suggest that the impact of aN^3LO PDFs lead to sizeable changes in standard LHC cross-sections (e.g. Higgs).



OUTLINE

1. Which **N^{3}LO corrections** (relevant for PDF fits) **are known?**
   Which corrections are included in MSHT20 and NNPDF40 aN^{3}LO?

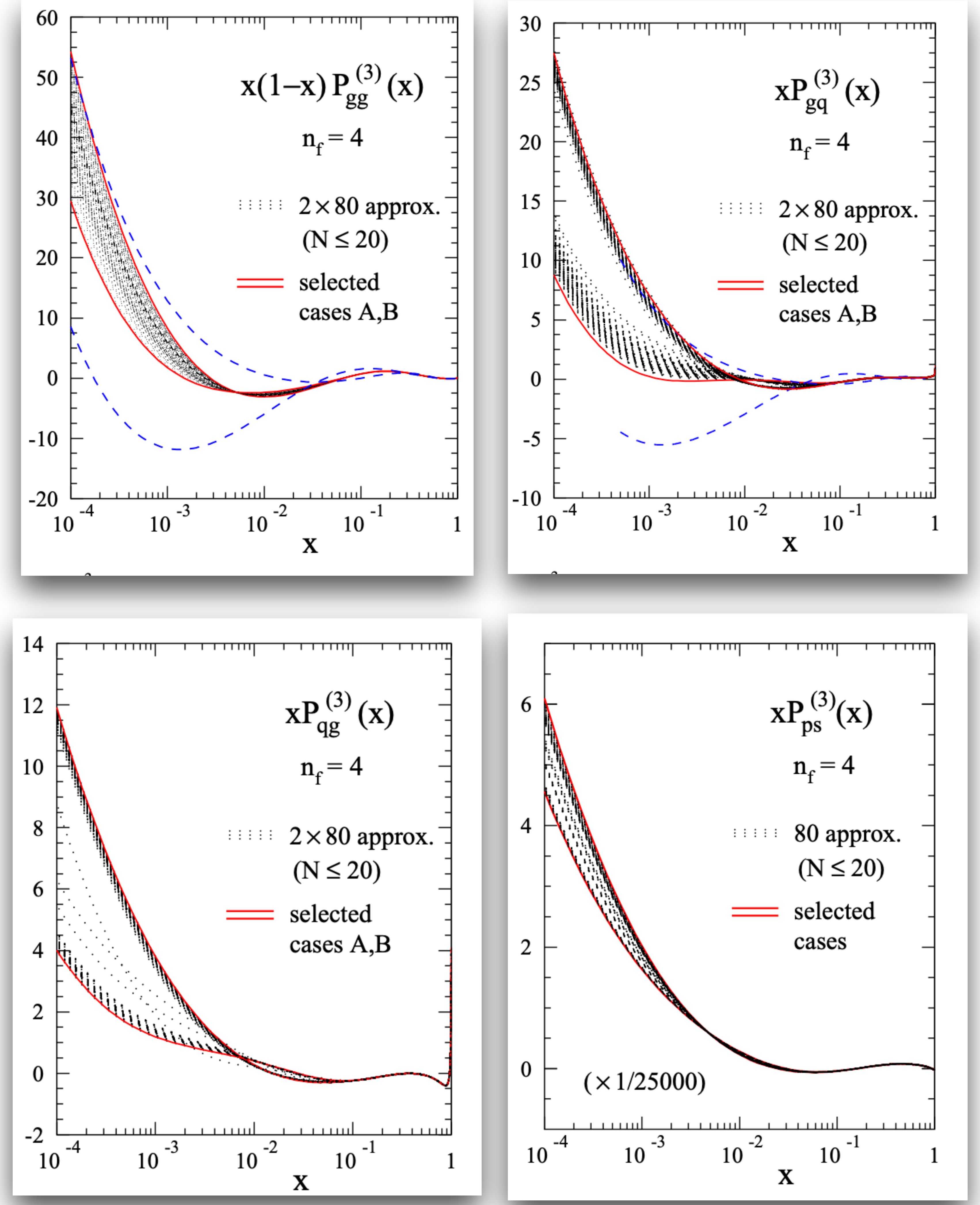
2. The combined **MSHT20xNNPDF40_an3lo** and **MSHT20xNNPDF40_an3lo_qed** PDF sets.

3. What is the **effect of more recents DGLAP N^{3}LO** corrections to our PDF fits?
   Are aN^{3}LO PDFs stable?

N^3LO corrections and PDF fits

See J. Blümlein talk

- **DGLAP evolution**: accurate numerical approximations N^3LO splitting functions (10 lowest moments, large-x and small-x limits).
- **Matching conditions**: all the relevant terms are now known (all exact, $a_{Hg}^{(3)}$ parametric).
- **DIS coefficients**: massless coefficients (both NC and CC) are known. Massive NC can be approximated.
- **Hadronic coefficients:** some DY coefficients are known, but not yet available in a format suitable for PDFs fits (besides the k-factor). Corrections to Jets and $t\bar{t}$ processes are still unknown.



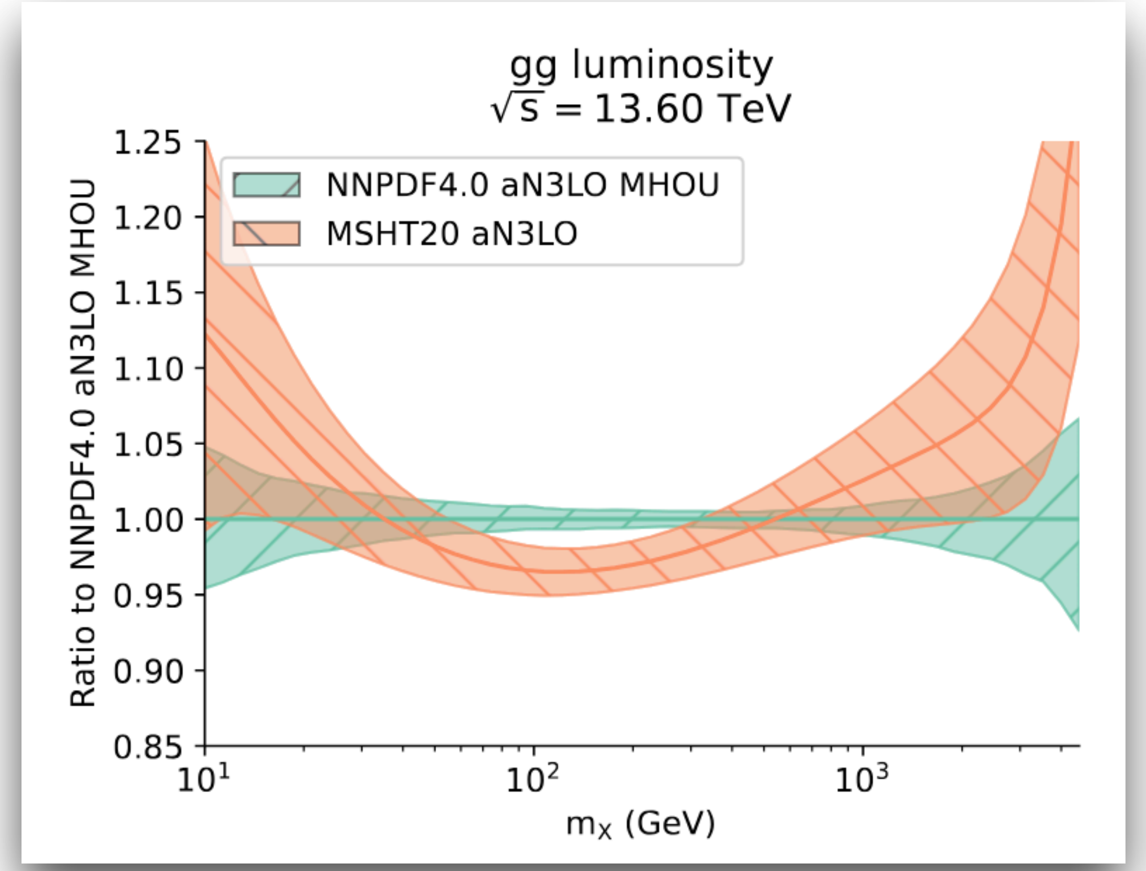
From: [arxiv:2302.07593],[arxiv:2307.04158] [arxiv:2404.09701] [arxiv:2410.08089].

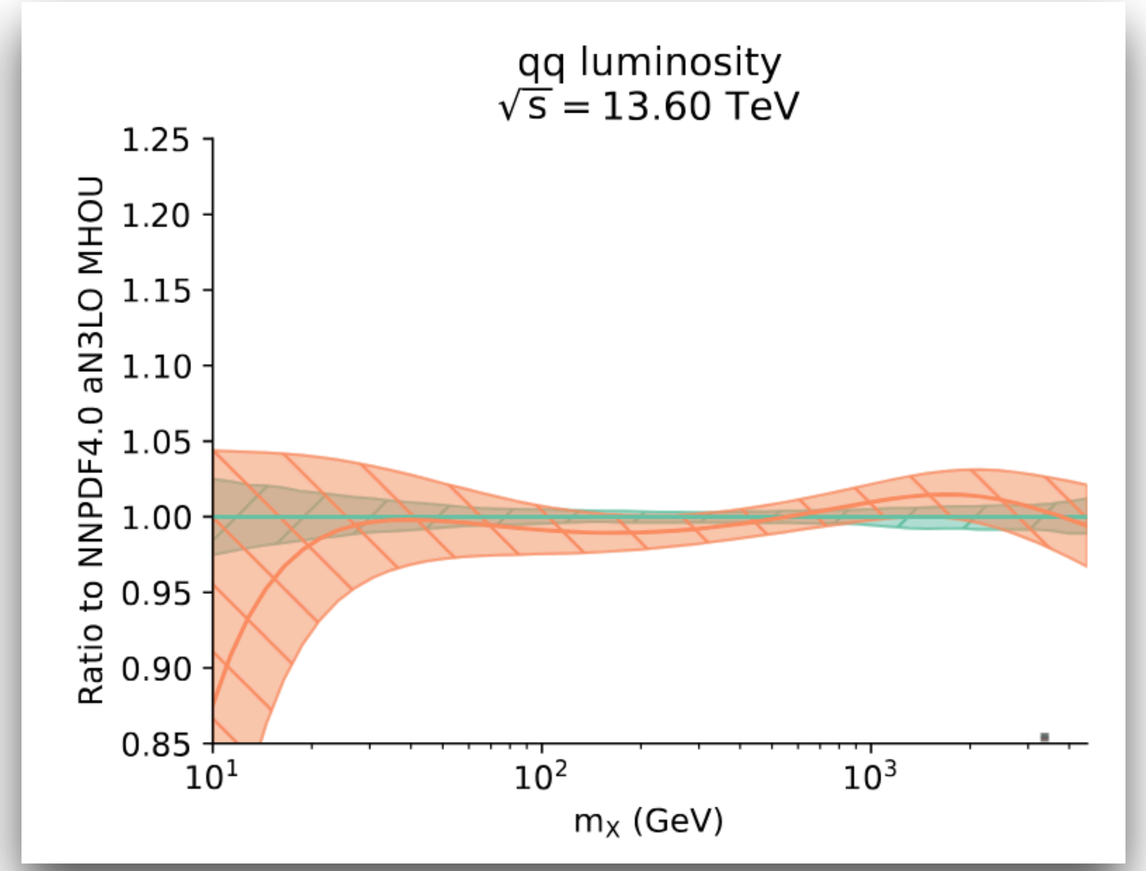
- **Great progress has been done on DGLAP evolution!**
- **Approximated terms have theoretical uncertainty which can be propagated into a PDF fit.**

See slide 17 for complete list of references.

MSHT20 and NNPDF40 aN^{3}LO

Both studies include all the available N^{3}LO informations by the time of publication.





MSHT20 aN3LO [arxiv:2207.04739]

- **Splittings functions**: **in-house approximation**. Singlet sector: 4 moments. Non-singlet: 8 moments + large-x and small-x limits.
- **Matching conditions**: $a^{PS(3)}_{Hq}$, $a^{(3)}_{gq,H}$ exact, $a^{NS,(3)}_{qq,H}$, $a^{(3)}_{gg,H}$, $a^{(3)}_{Hg}$ approximated.
- **DIS**: approximated massive NC corrections. Extended TR scheme.
- **Theory uncertainties**: incomplete N^{3}LO unc. and missing higher orders unc. computed with **nuisance parameters.**
- **Hadronic processes**: **approximated K-factors**.

NNPDF40 aN3LO [arxiv:2402.1863]

- **Splittings functions**: **in-house approximation**, based on all the current known terms, but $P^{(3)}_{gq}, P^{(3)}_{gg}$ only with 5 moments.
- **Matching conditions**: all up to date, but superseded $a^{(3)}_{Hg}$.
- **DIS**: approximated massive NC corrections. Extended FONLL scheme.
- **Theory uncertainties**: computed with a **covariance matrix**, incomplete N^{3}LO unc. from parametric variations, missing higher orders unc. from scale variations.
- **Hadronic processes**: **NNLO scale variations** as proxy for incomplete N^{3}LO.

MSHT20 and NNPDF40 aN^{3}LO

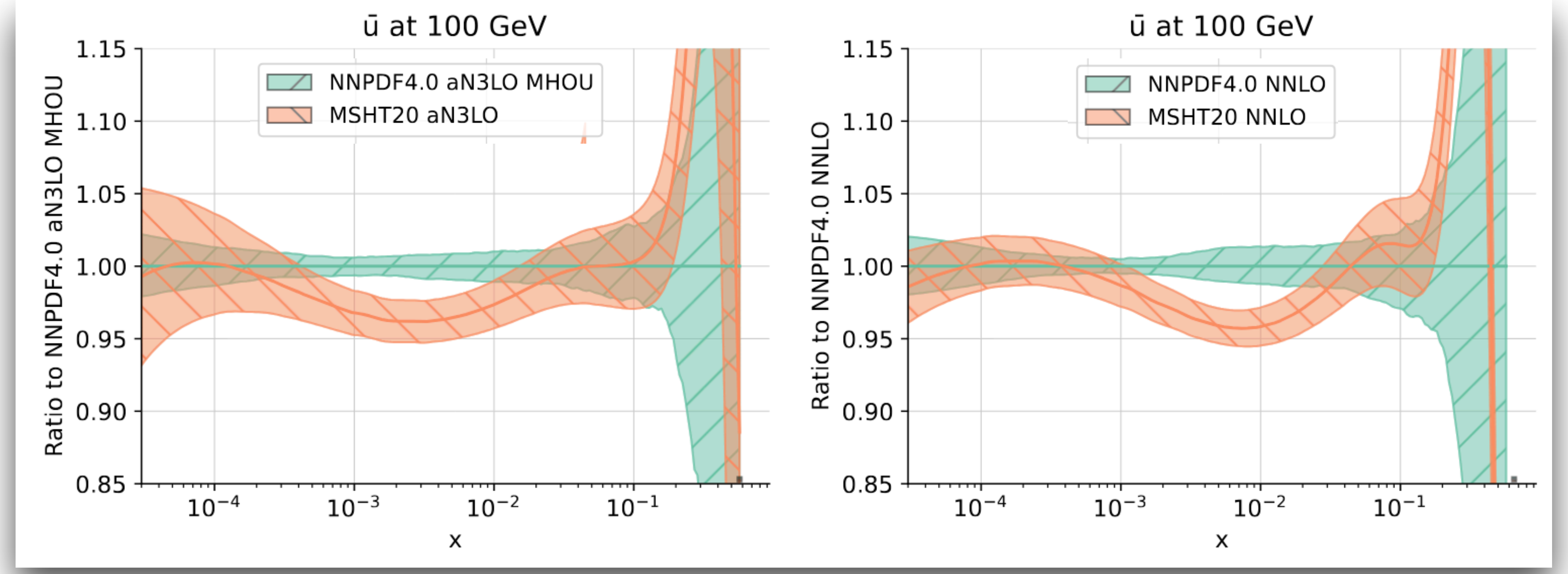
MSHT20 and NNPDF4.0 aN3LO relies on **different methodologies**: kinematic coverage, fitting methodology, input parameters, fitted charm...

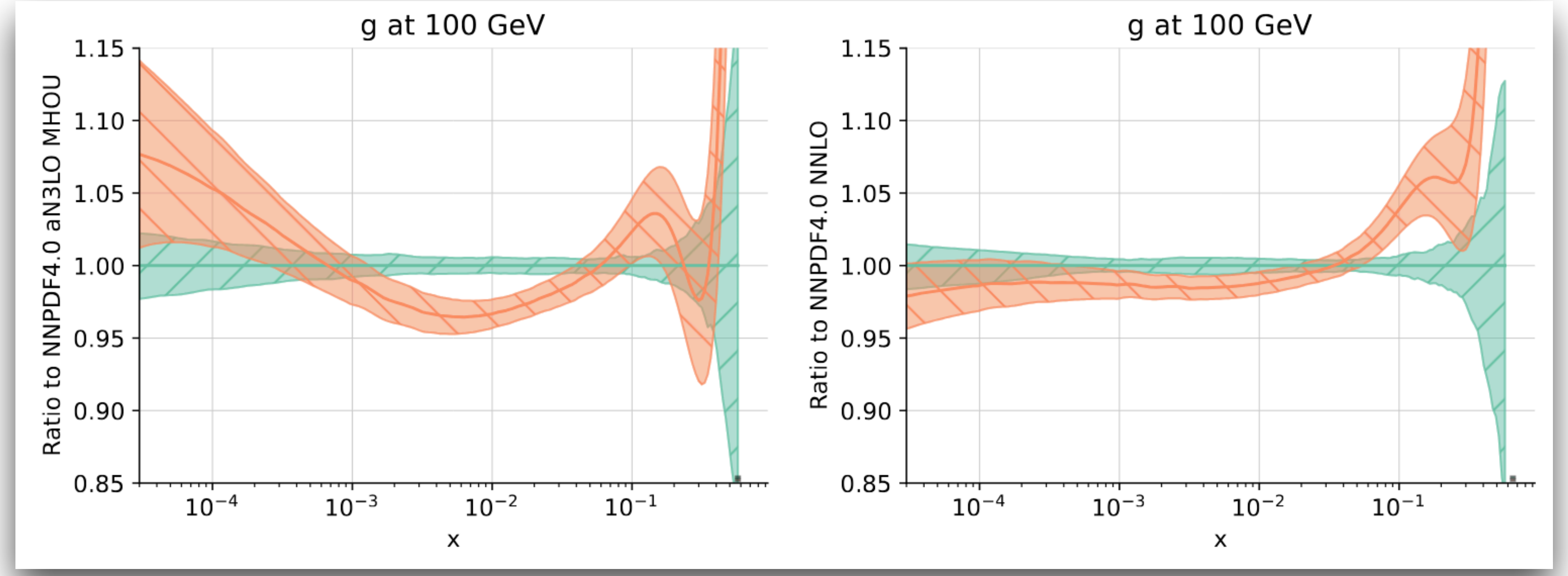
- Most of the differences are already present at NNLO.
- The largest aN^{3}LO effects are visible in the gluon, where MSHT favors a larger suppression.

However, the shape of **aN^{3}LO / NNLO ratio** is similar.

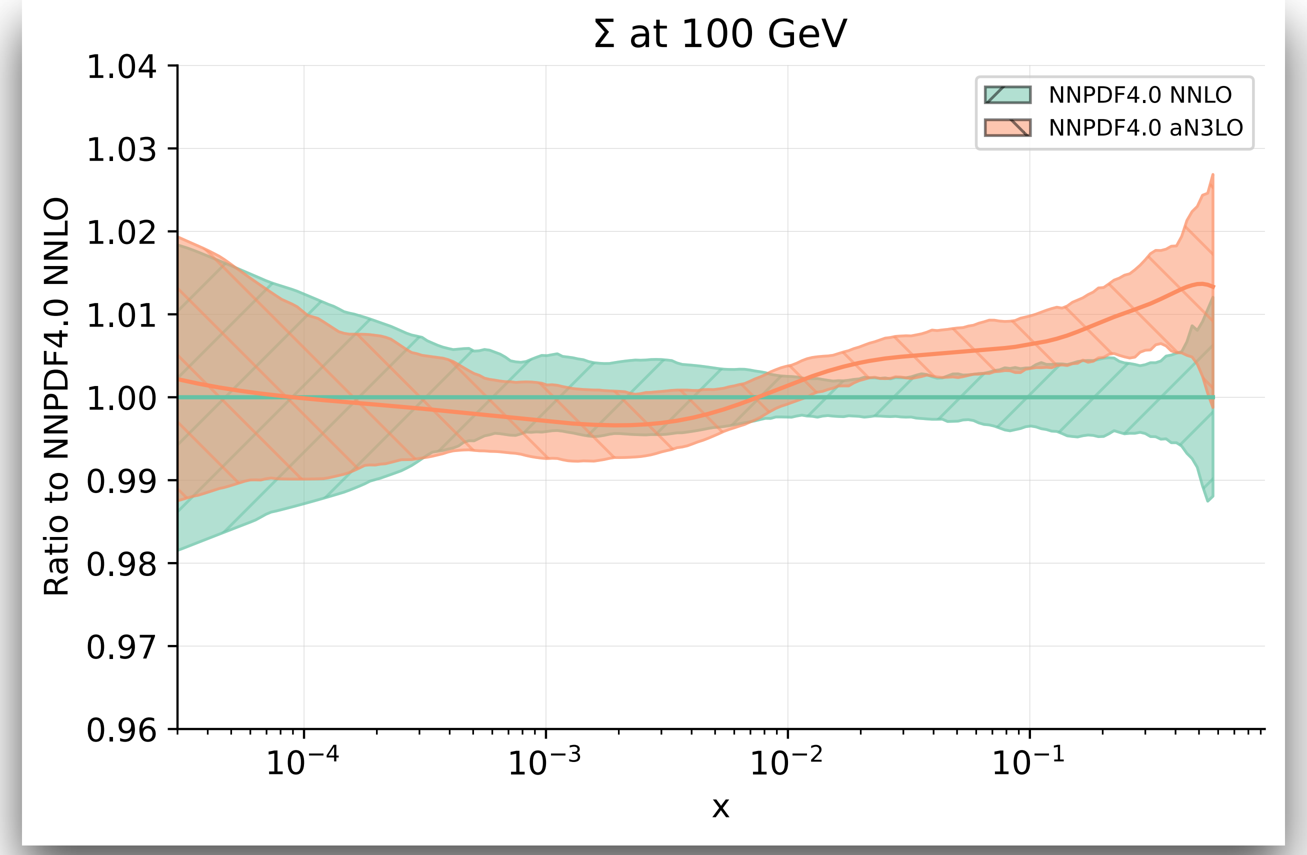
MSHT20 / NNPDF40

aN^{3}LO | **NNLO**

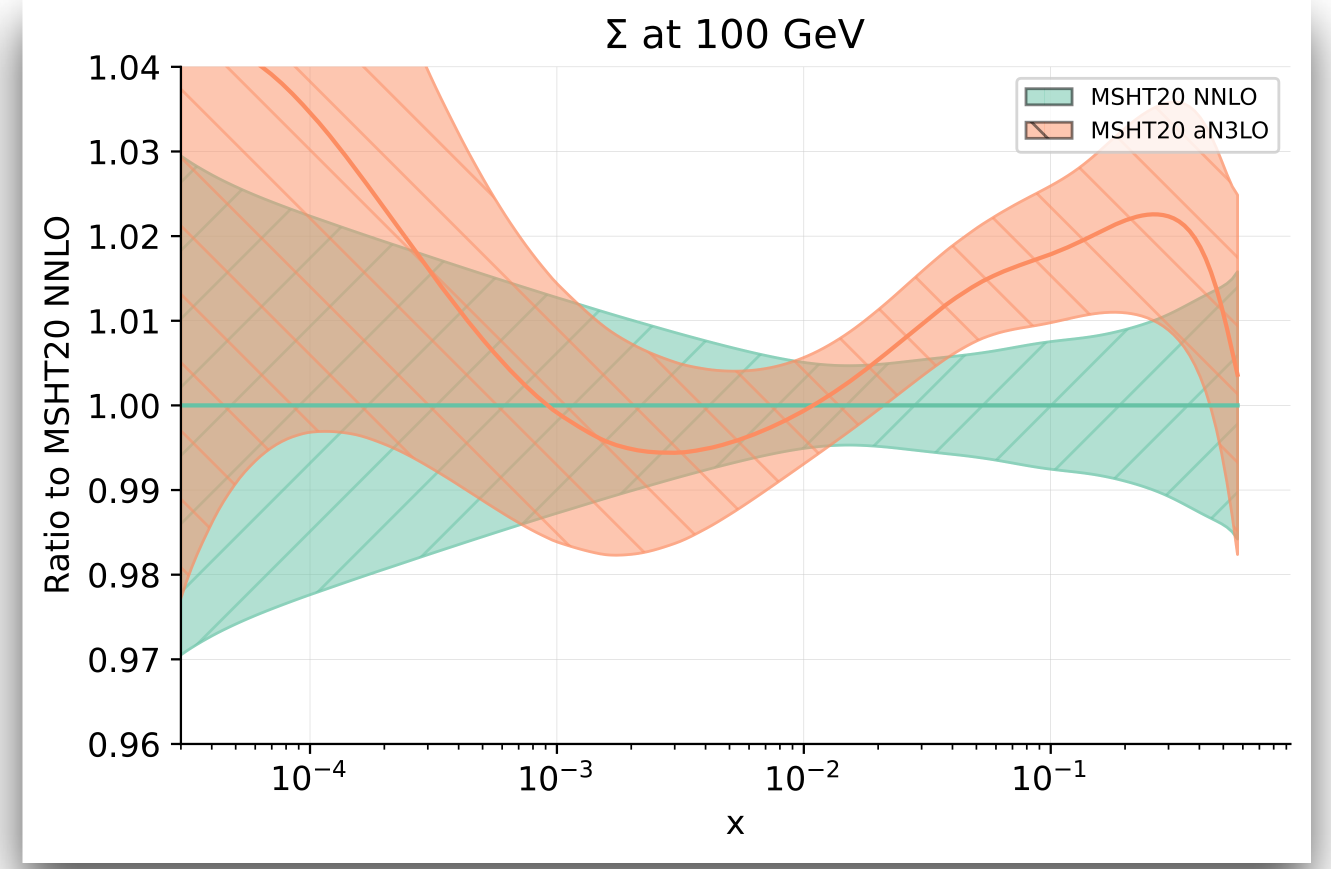




NNPDF4.0 aN^{3}LO / NNLO



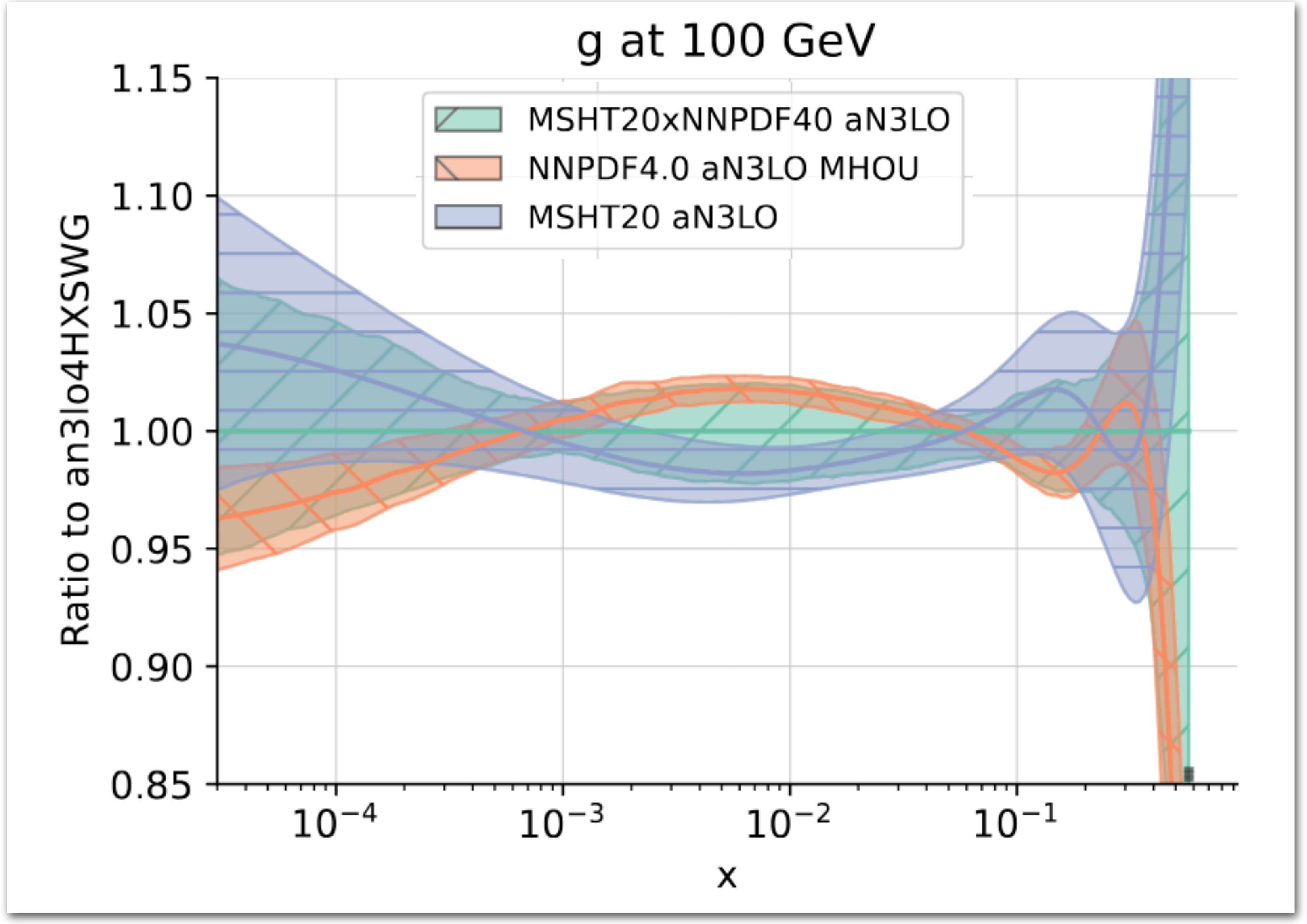
MSHT20 aN^{3}LO / NNLO



The combined aN^{3}LO set [arxiv.org:2411.05373]

As differences persist, in order to provide a **conservative estimate of PDFu and gain in accuracy,** we can construct an unweighted combination the 2 PDFs set:

MSHT20xNNPDF40_an3lo

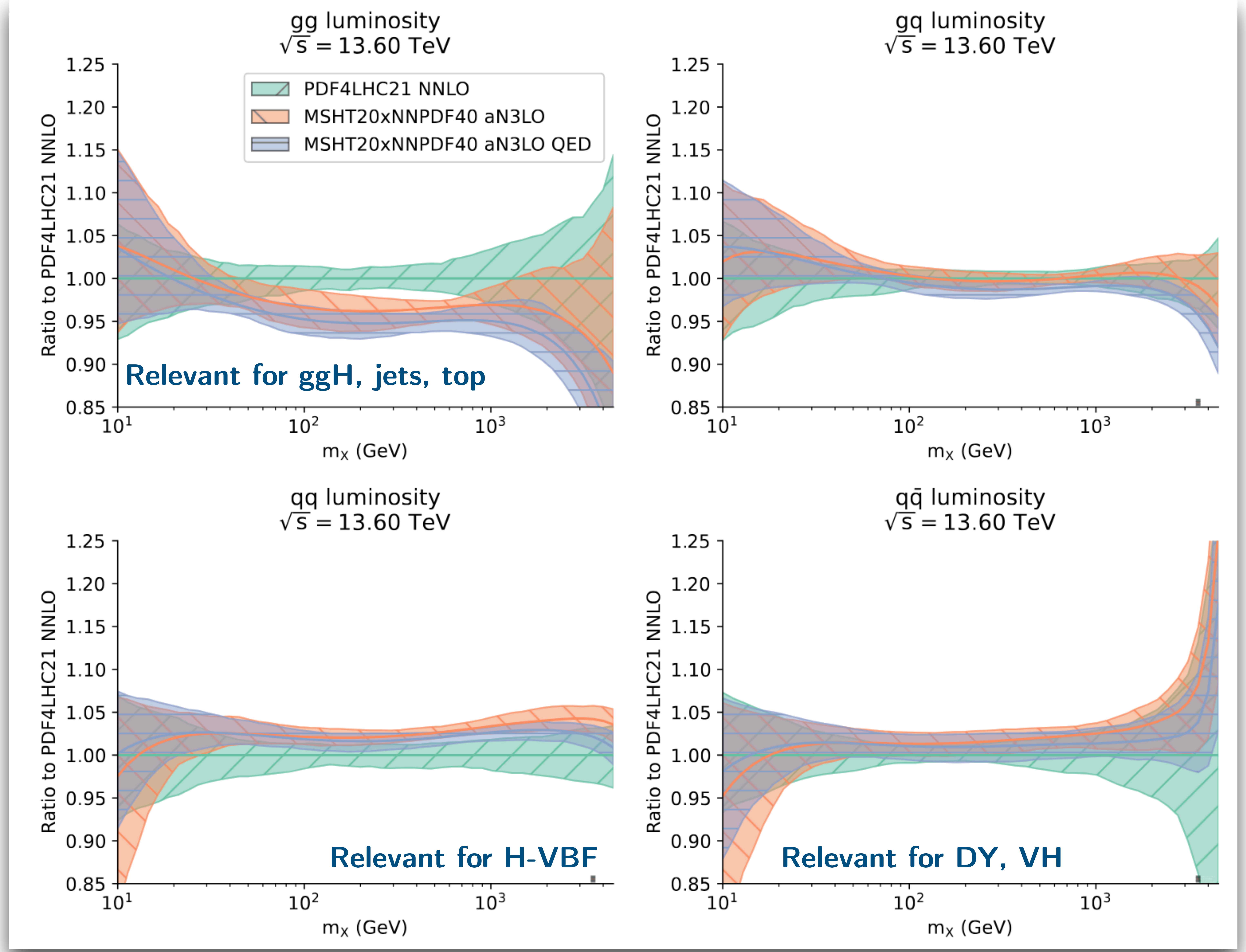


- **Same approach ad for PDF4LHC** combinations: merge $N_{\text{rep}} = 100$ replicas from the the 2 original sets.
- **Variants with QED effects** (and NNLO) are also available.
- Can be easily extended to other aN3LO PDF determinations.
- Caveat: no attempt to minimize input differences. Combination set must be used only for sufficiently high Q^2 scales.

	PDF set	pert. order (PDF)
Reference →	PDF4LHC21_mc	NNLO_{QCD}
Benchmark sets →	MSHT20xNNPDF40_nnlo	NNLO_{QCD}
	MSHT20xNNPDF40_nnlo_qed	$\text{NNLO}_{\text{QCD}} \otimes \text{NLO}_{\text{QED}}$
New aN^{3}LO sets →	MSHT20xNNPDF40_an3lo	$\text{aN}^3\text{LO}_{\text{QCD}}$
	MSHT20xNNPDF40_an3lo_qed	$\text{aN}^3\text{LO}_{\text{QCD}} \otimes \text{NLO}_{\text{QED}}$

The combined aN^{3}LO set: luminosities

- Larger effect w.r.t PDF4LHC21 is visible in the gluon-gluon luminosity, suppression of $\approx 3\,\%$.
- At high invariant mass QED effects are also sizeable for the gluon.
- QED corrections are similar in magnitude to aN^{3}LO effects especially for in the quark sector.
- Differences between NNLO PDF combination and PDF4LHC21 is smaller than the the aN^{3}LO - NNLO PDF shift.



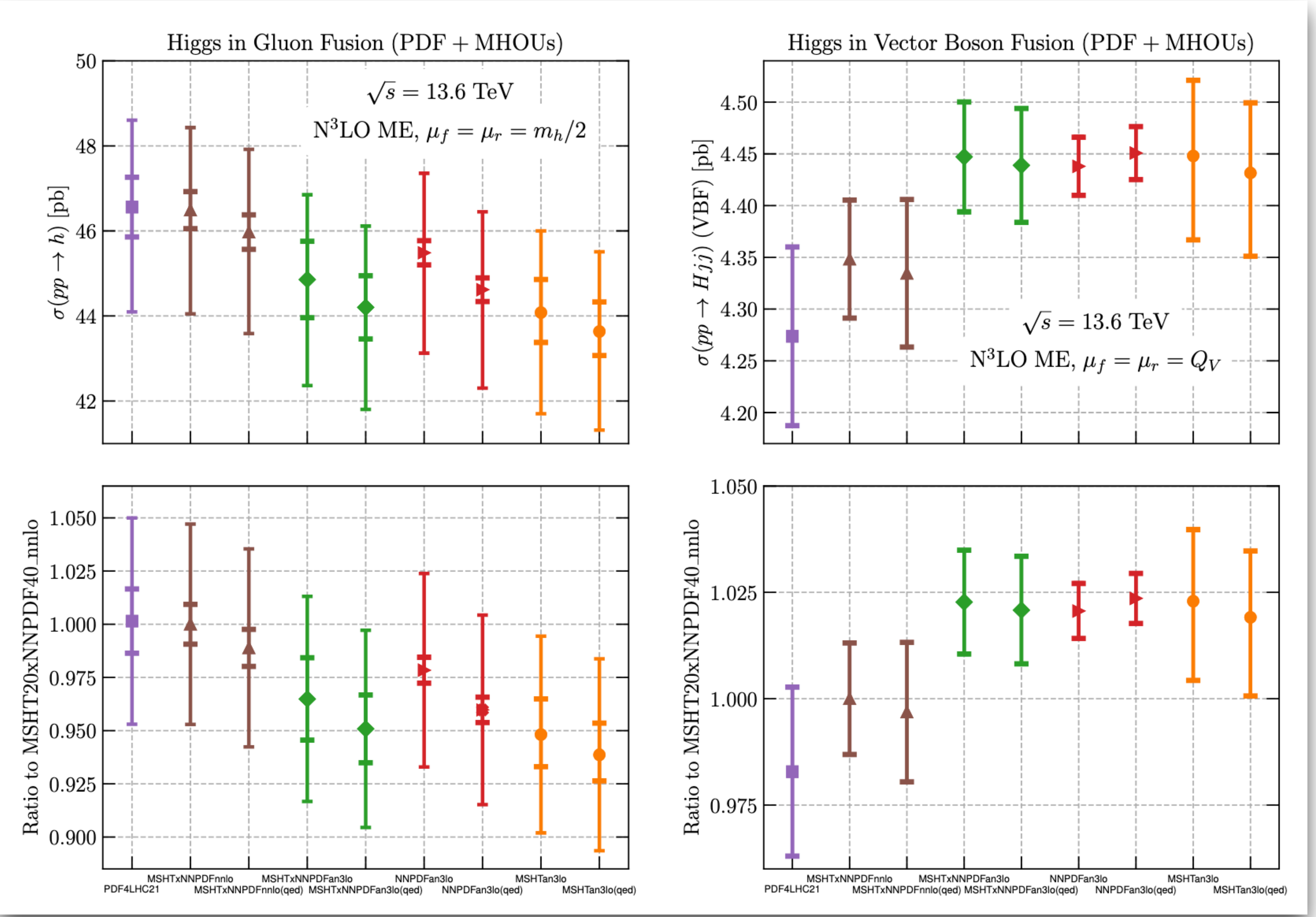
aN^{3}LO PDFs and Higgs cross-sections

In **Higgs gluon fusion**:

- At NNLO: PDF4LHC21 very close to MSHTxNNPDF
- aN^{3}LO (+QED) PDF corrections: **-3.5% (-5%)**
- MSHT20 aN^{3}LO and NNPDF40 aN^{3}LO are within 2σ (PDFu).

In **Higgs VBF**:

- At NNLO: PDF4LHC21 is $-1.8\,\%$ lower than MSHTxNNPDF
- aN^{3}LO (+QED) PDF corrections: **+2.5% (+2.5%)**
- MSHT20 aN^{3}LO and NNPDF40 aN^{3}LO are within 1σ (PDFu).



Predicting higher order effects

aN^{3}LO PDFs can be used:

- To **compute N^{3}LO cross sections** more precisely.
- To **evaluate evaluate the effect of using NNLO PDFs in a N^{3}LO calculation**. For example, if we consider:

$$\Delta^{\text{exact}}_{\text{NNLO}} \equiv \left| \frac{\sigma^{\text{N}^3\text{LO}}_{\text{N}^3\text{LO}-\text{PDF}} - \sigma^{\text{N}^3\text{LO}}_{\text{NNLO}-\text{PDF}}}{\sigma^{\text{N}^3\text{LO}}_{\text{N}^3\text{LO}-\text{PDF}}} \right|$$

previously extimated with:

$$\Delta^{\text{app}}_{\text{NNLO}} \equiv \frac{1}{2} \left| \frac{\sigma^{\text{NNLO}}_{\text{NNLO}-\text{PDF}} - \sigma^{\text{NNLO}}_{\text{NLO}-\text{PDF}}}{\sigma^{\text{NNLO}}_{\text{NNLO}-\text{PDF}}} \right|$$

In **Higgs production**, both gluon fusion and VBF:

- the approximate estimate $\Delta^{\text{app}}_{\text{NNLO}}$ is very unreliable, and specifically it underestimates $\Delta^{\text{exact}}_{\text{NNLO}}$.
- difference between weighted and unweighted combination aN^{3}LO PDF combination $\mathcal{O}(1\%)$ is smaller than the shift from NNLO to aN^{3}LO PDFs $\mathcal{O}(3\%)$.

PDF set	$\sigma(gg \to h)$	$\sigma(h \text{ VBF})$
$\Delta^{\text{exact}}_{\text{NNLO}}$ (NNPDF4.0)	2.2%	1.3%
$\Delta^{\text{exact}}_{\text{NNLO}}$ (MSHT20)	5.3%	2.3%
$\Delta^{\text{exact}}_{\text{NNLO}}$ (combination)	3.3%	2.3%
$\Delta^{\text{app}}_{\text{NNLO}}$ (NNPDF4.0)	0.2%	0.2%
$\Delta^{\text{app}}_{\text{NNLO}}$ (MSHT20)	1.4%	1.3%
$\Delta^{\text{app}}_{\text{NNLO}}$ (combination)	1.6%	0.5%

Including newer N^{3}LO terms

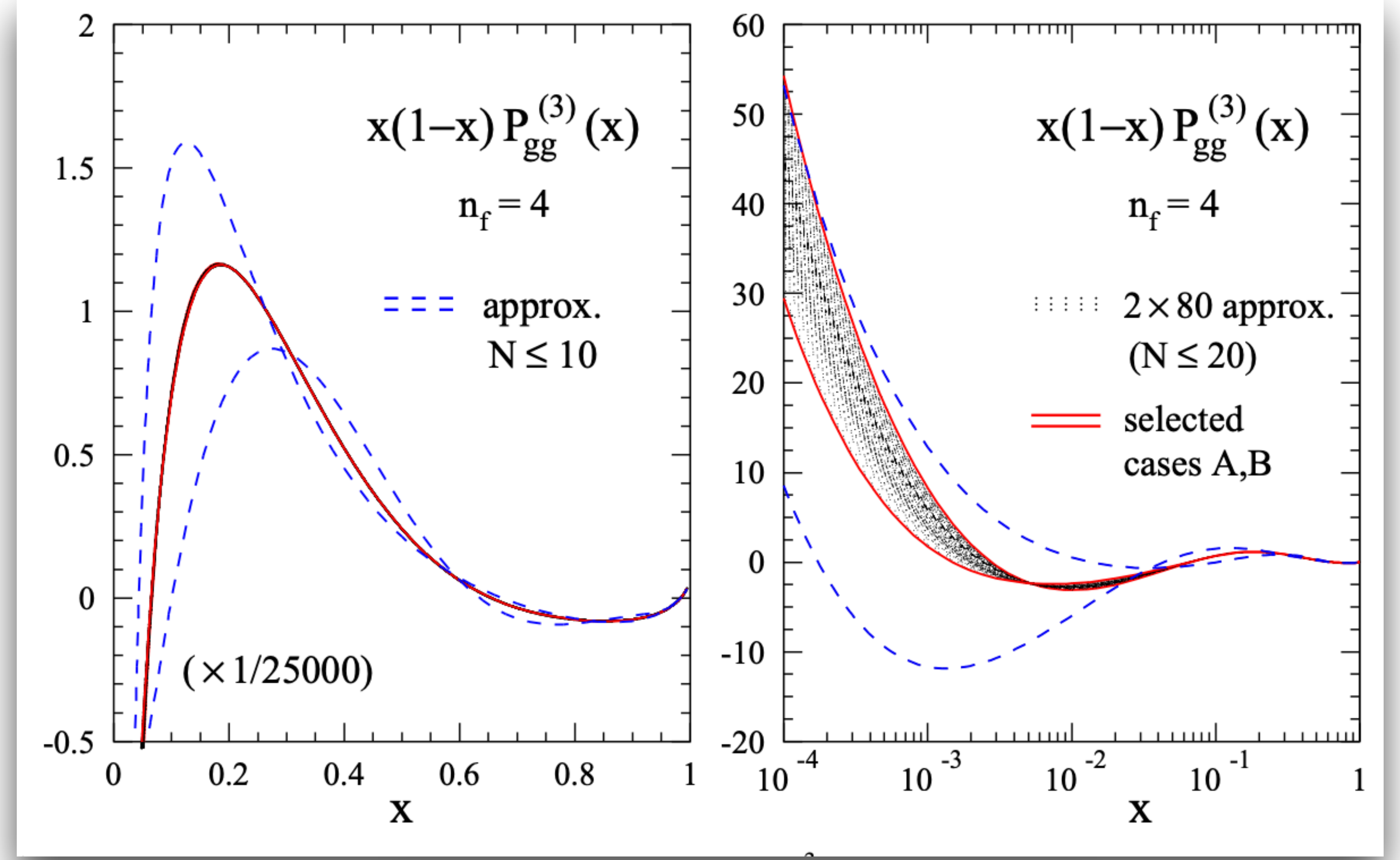
See J. Blümlein talk

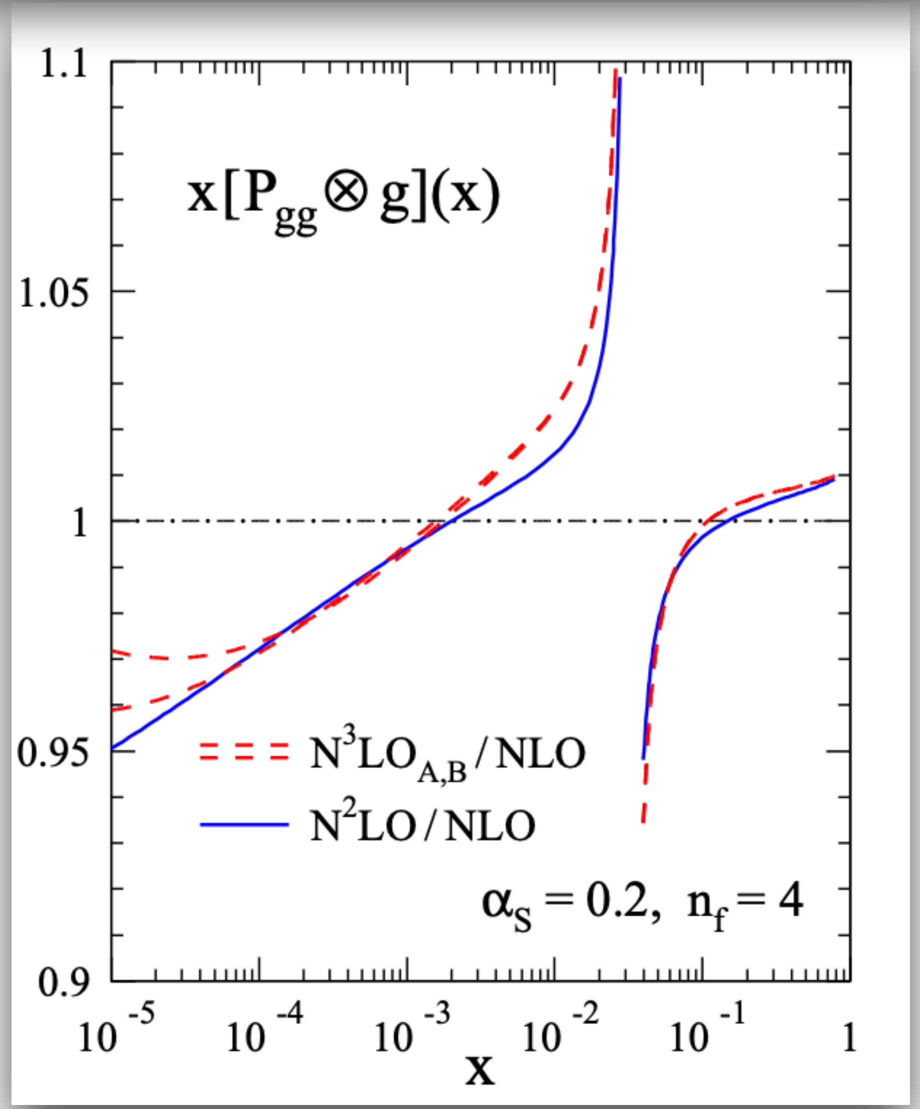
In the last year, huge progress on the N^{3}LO splitting functions and and matching. For eg. NNPDF40 aN^{3}LO does not include:

- $P_{gq}^{(3)}, P_{gg}^{(3)}$ higher moments: [arxiv:2404.09701] [arxiv:2410.08089].
- Improved parametrization of $a_{H,g}^{(3)}$ [arxiv:2403.00513]

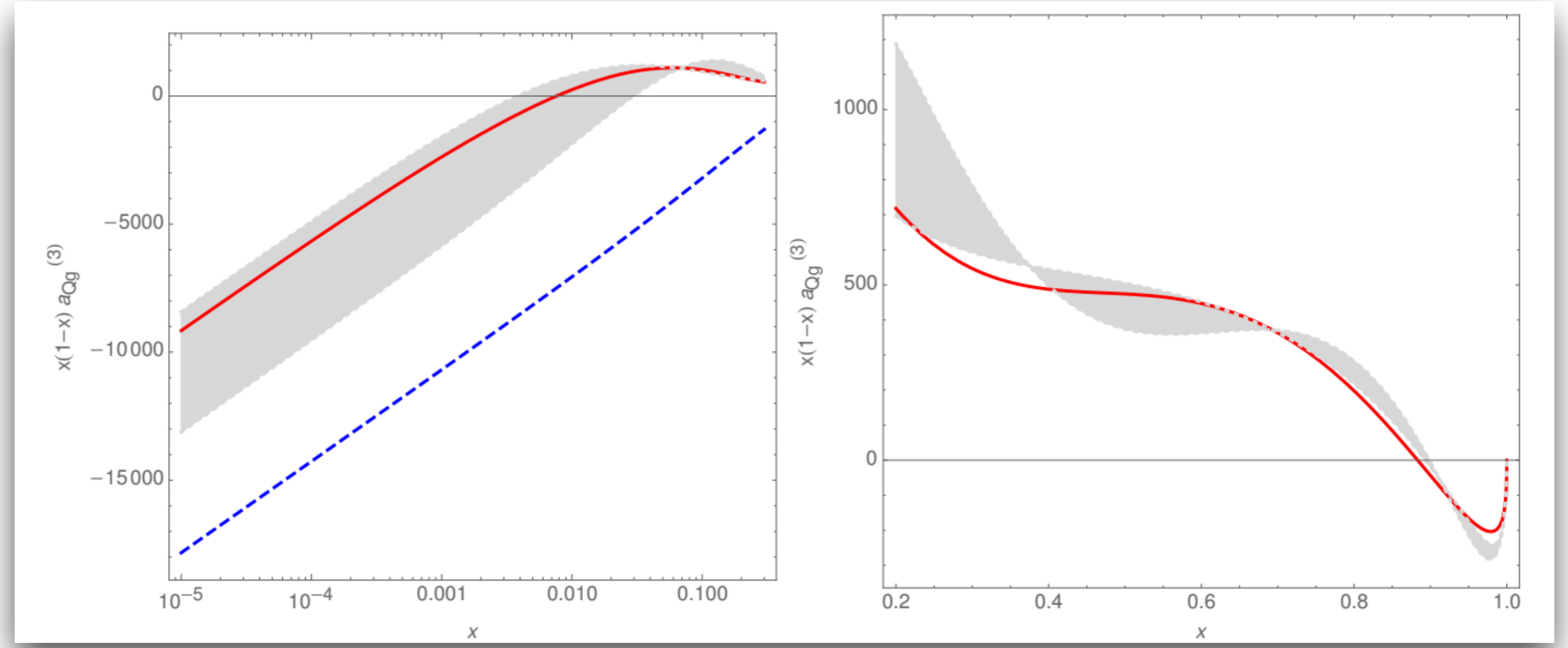
These results make the N^{3}LO approximation error irrelevant down to $x \approx 10^{-4}$

From [arxiv:2410.08089]:
red new parametrization, blue older approximation [arxiv:2310.05744]





From [arxiv:2403.00513]: red new parametrization, grey older approximation [arxiv:1205.5727]



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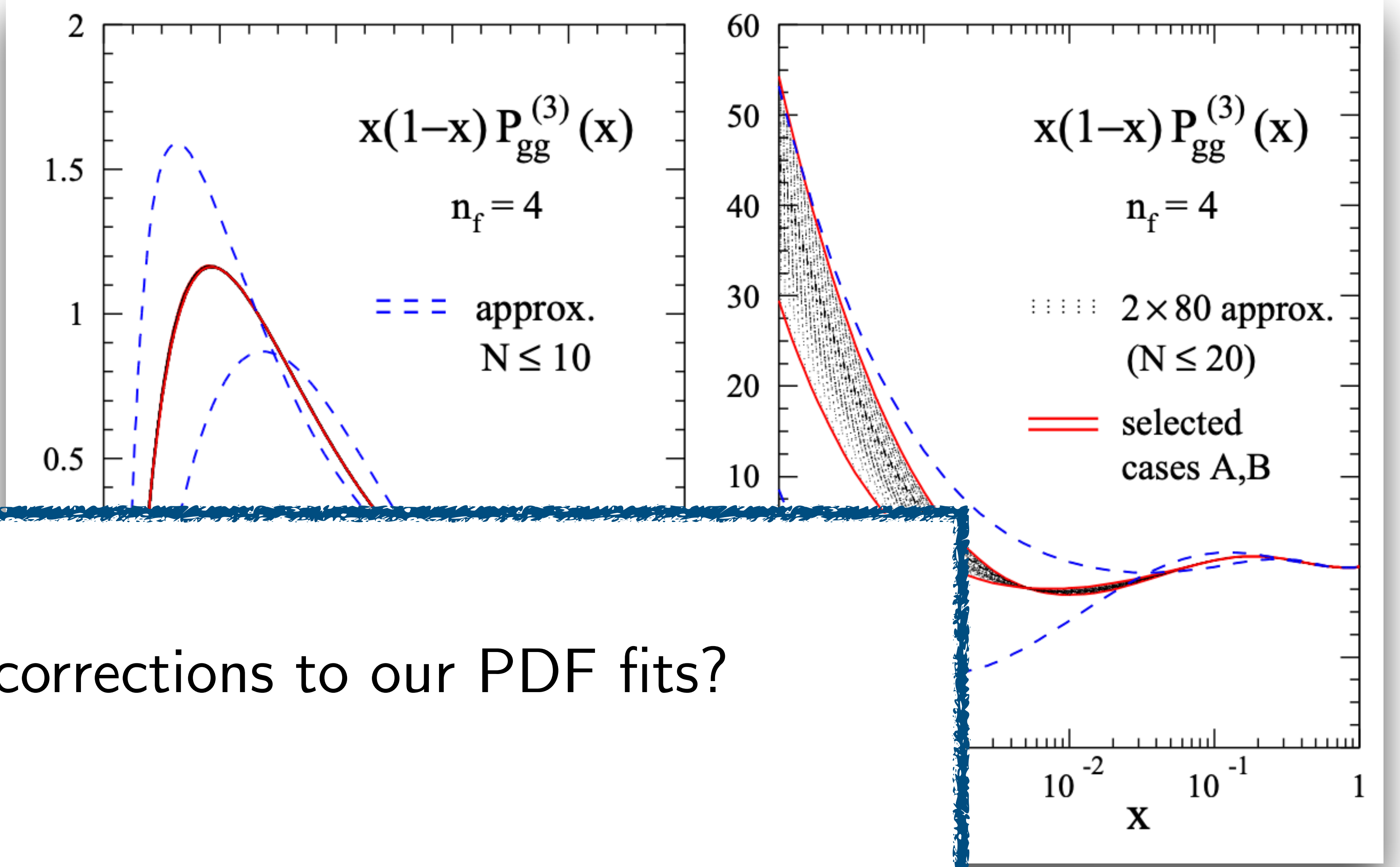
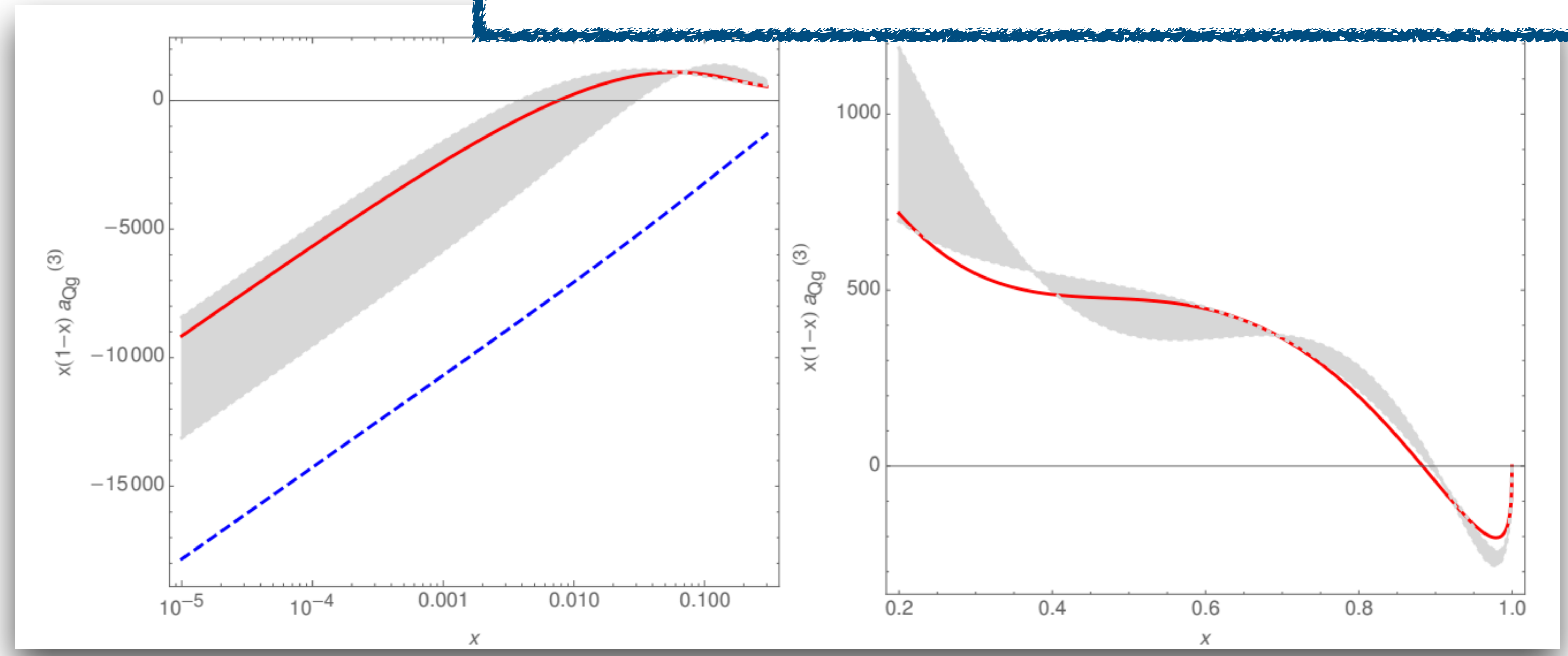
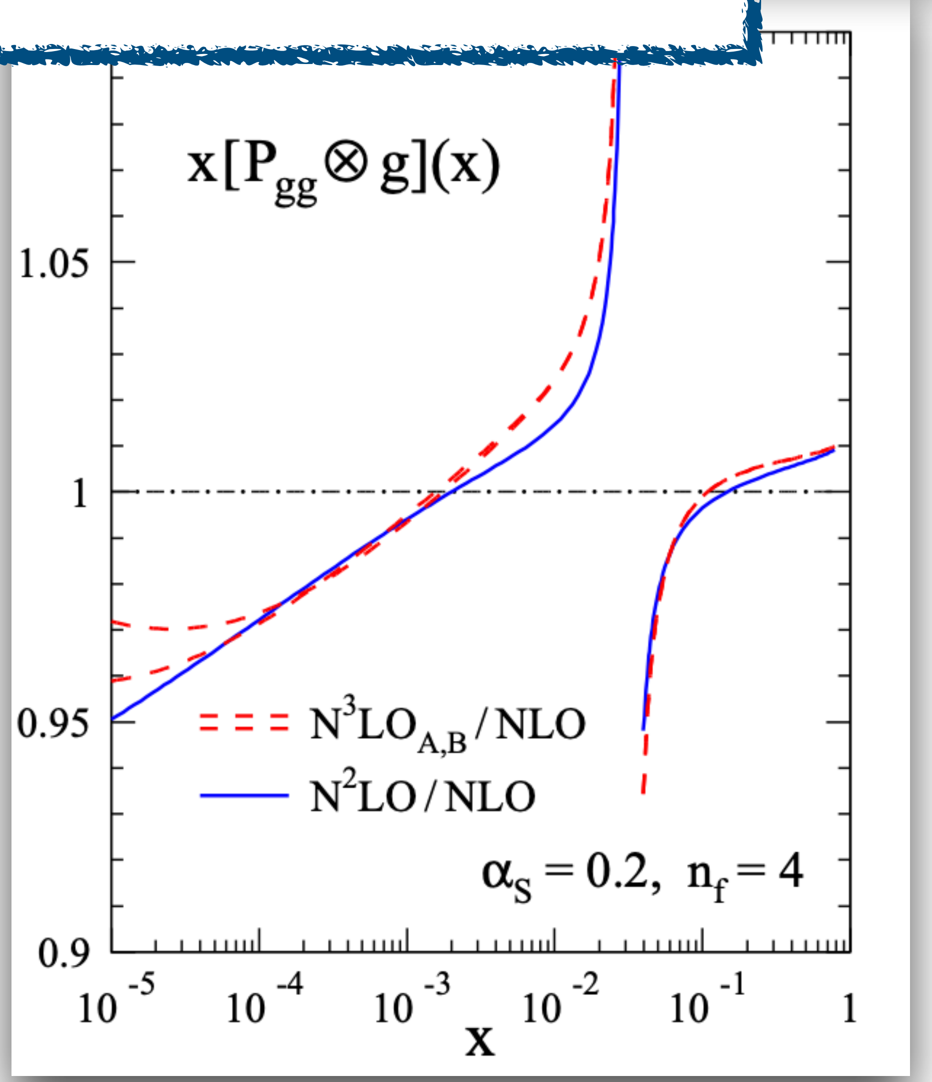


What is the **effect of more recents DGLAP N^{3}LO** corrections to our PDF fits?

Are aN^{3}LO PDFs stable?

From [arxiv:2403.00513]: re

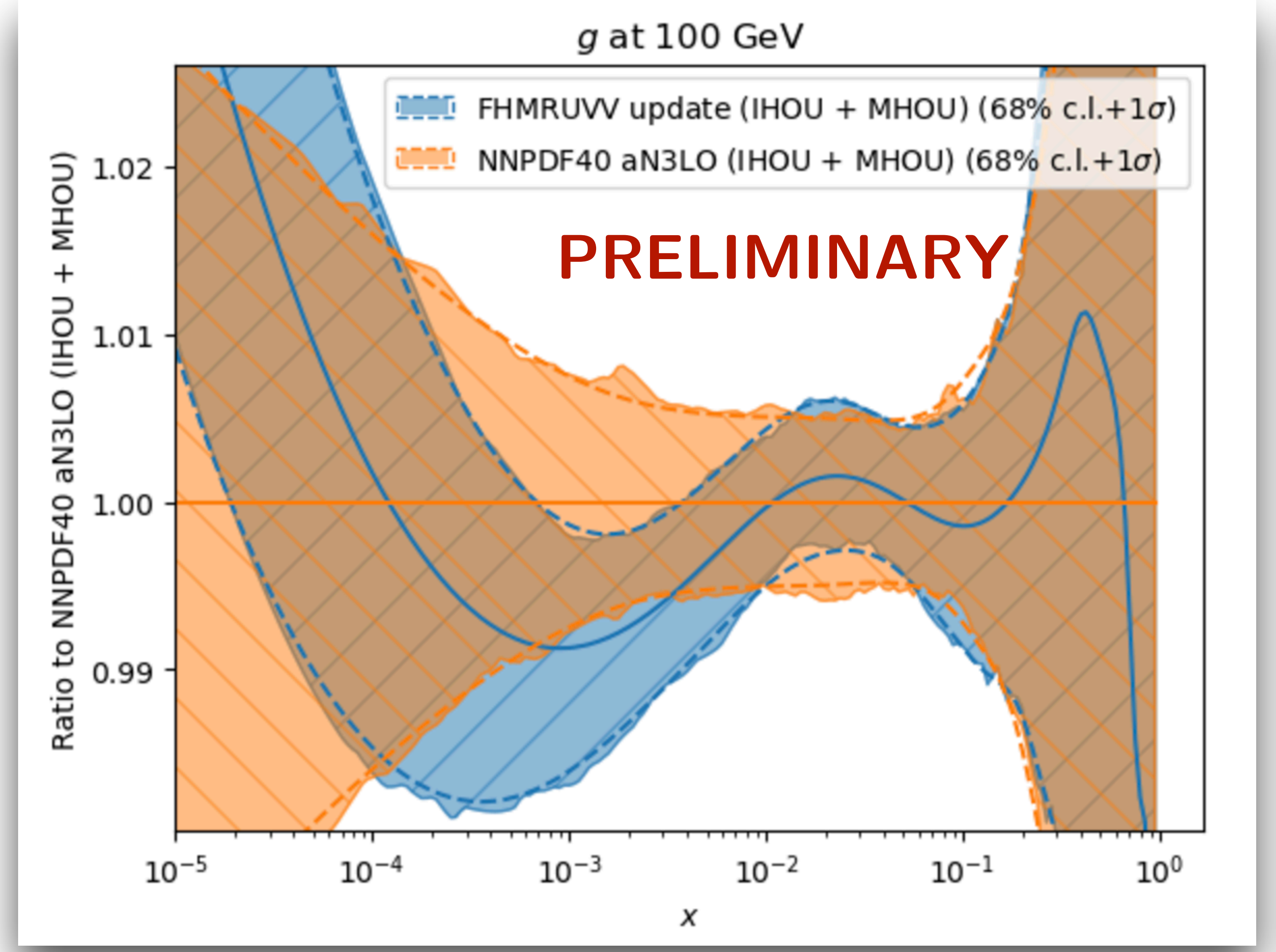


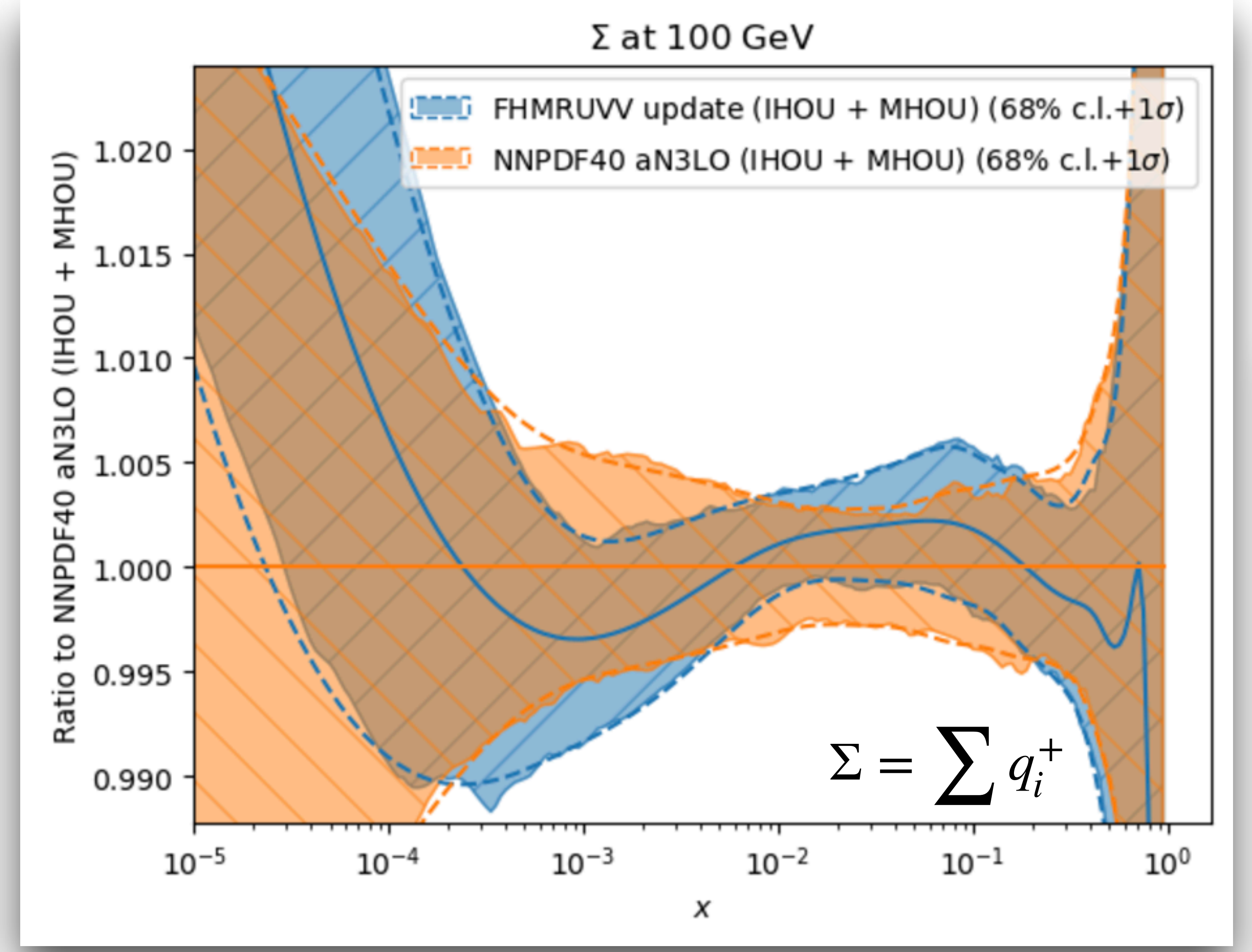


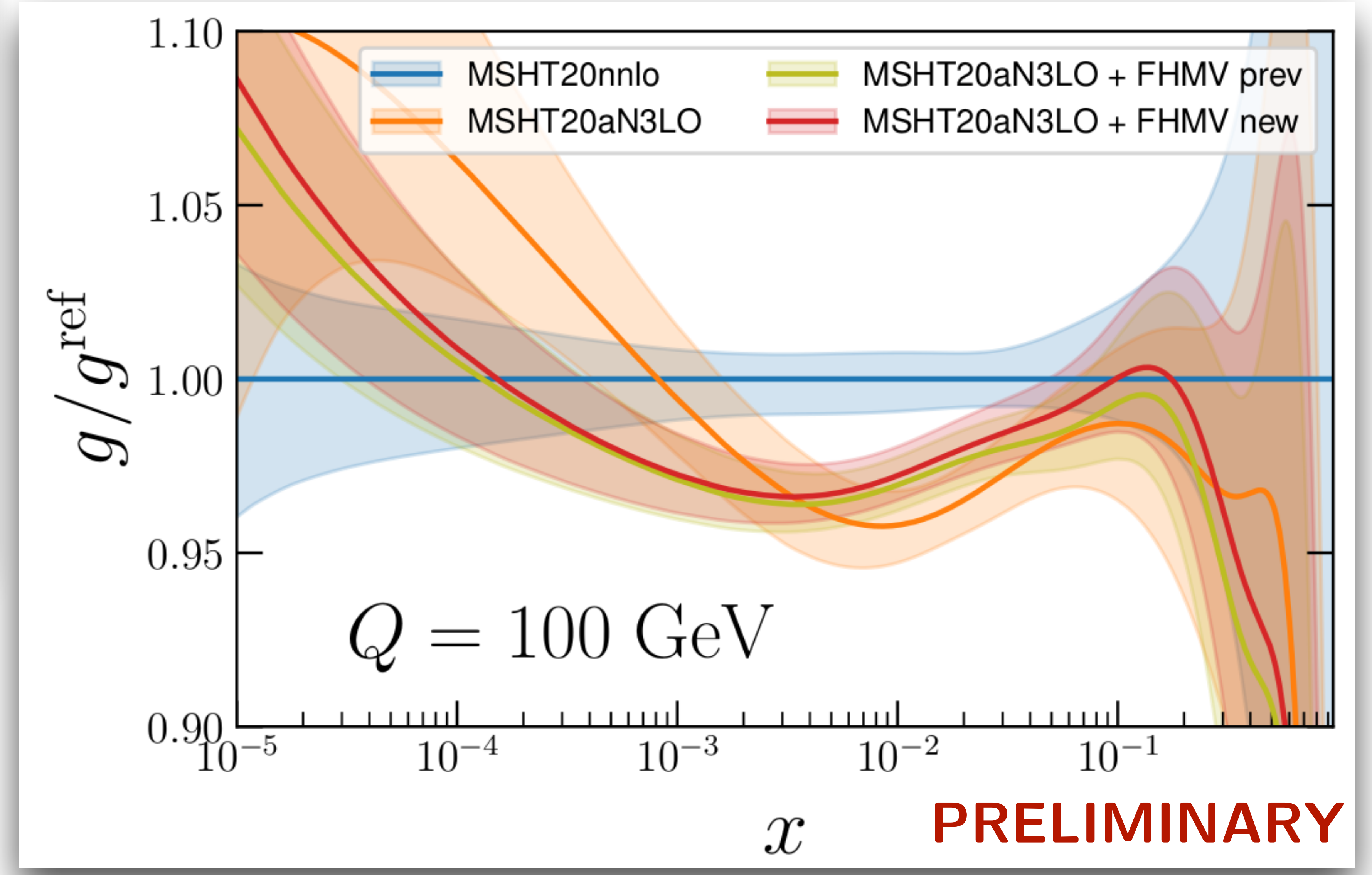
Including newer N^3LO terms

What is the **effect of more recents DGLAP N^3LO** corrections to our PDF fits?

Are aN^3LO PDFs stable ?





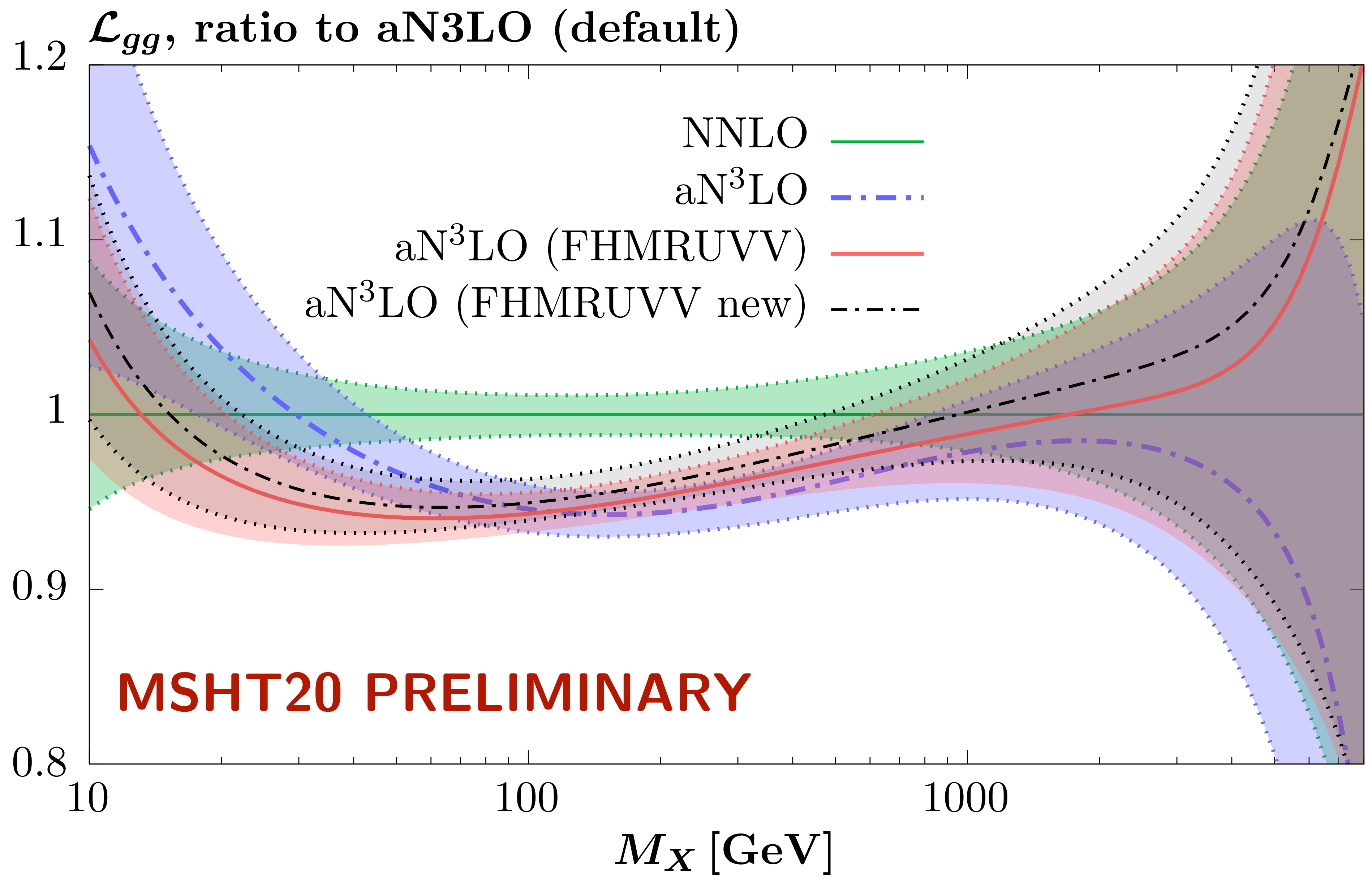


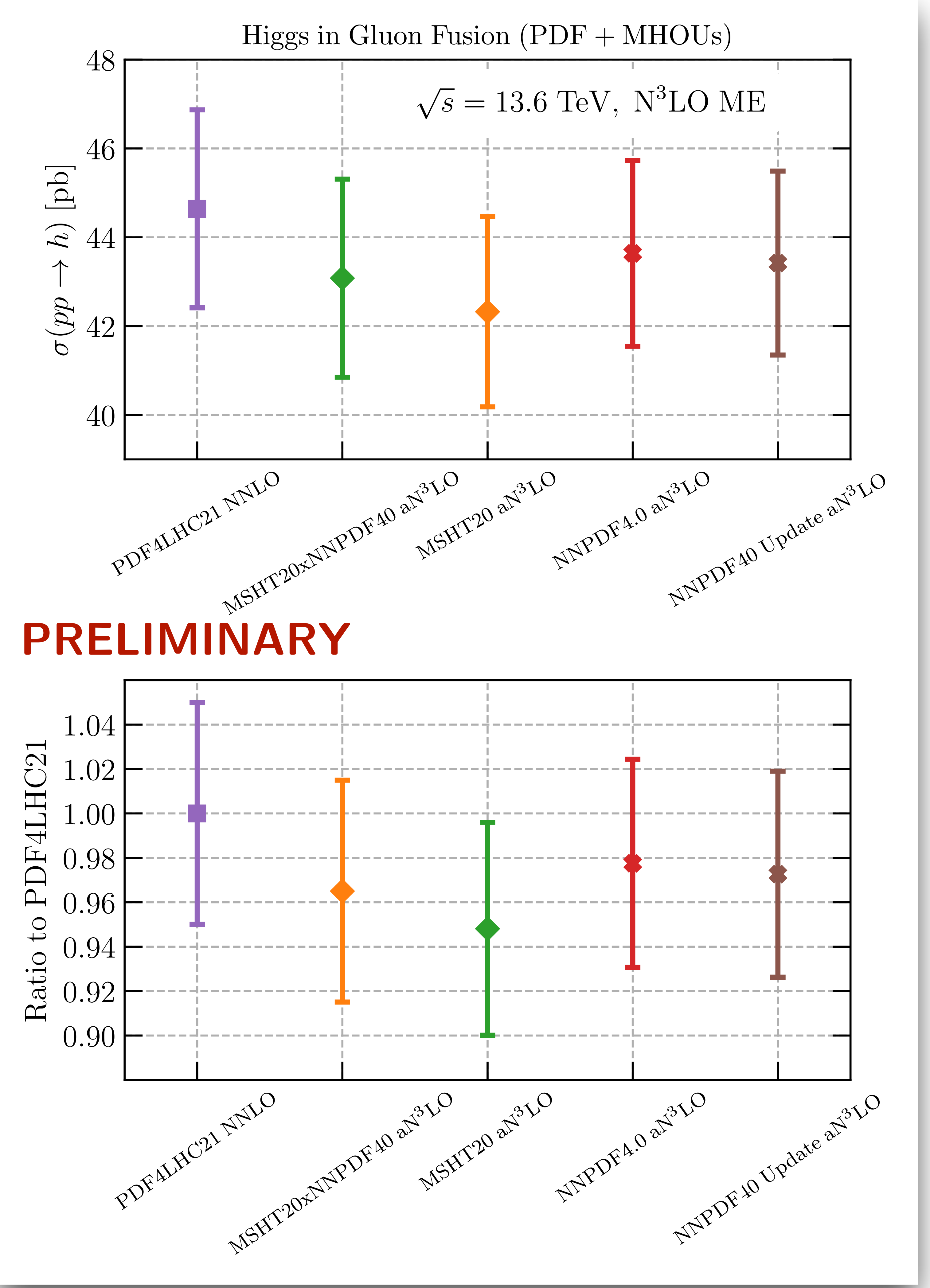
- N^3LO approximation error on DGLAP evolution is now negligible.
- Newer elements only have a significant impact at small-x.
- Shifts are within within 1σ (2σ) of the published NNPDF40 aN^3LO (MSHT20 aN^3LO).
- Gluon aN^3LO / NNLO ratios moves closer in the 2 fits.

Including newer N^{3}LO terms

What is the **effect of more recents DGLAP N^{3}LO** corrections to our PDF fits?

Are aN^{3}LO PDFs stable ?





- N^{3}LO approximation error on DGLAP evolution is now negligible.
- For MSHT20: impact on Higgs gluon fusion cross-sections $\approx 1.0\ \%$.
- For NNPDF40: impact on Higgs gluon fusion cross-sections $\approx 0.5\ \%$. Below $0.1\ \%$ for VBF.

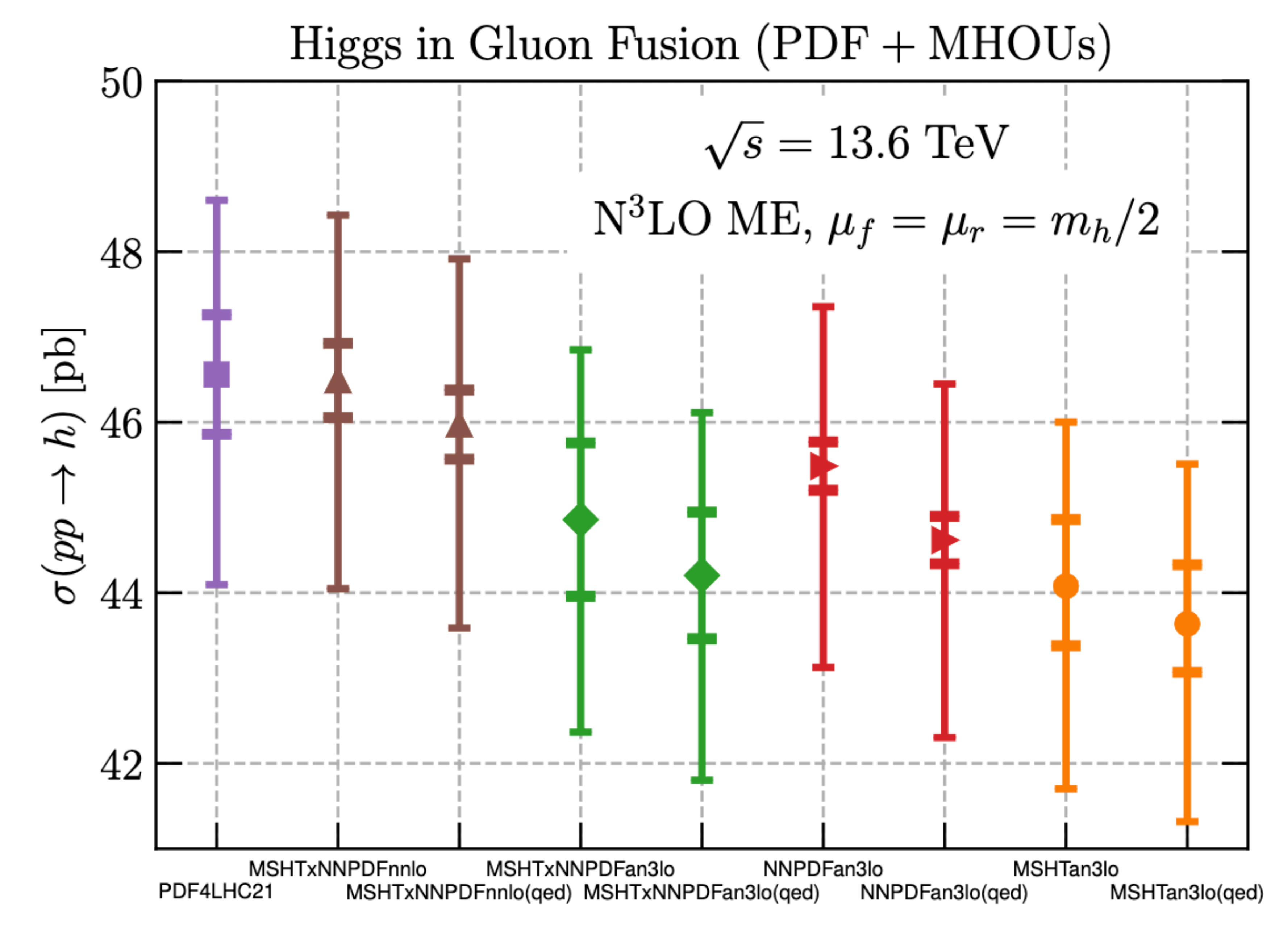
Results with MSHT20 and NNPDF40 aN^{3}LO move closer.

Summary

- Inclusion of corrections beyond NNLO accuracy in PDF fits is now possible. **N^{3}LO QCD** corrections to DGLAP are now accurately known (up to $x \approx 10^{-4}$).
- These corrections are mandatory for **precise LHC phenomenology** (in particular Higgs boson production).
- We have proposed a first **combination set** based on 2 independent studies:

  `MSHT20xNNPDF40_an3lo, MSHT20xNNPDF40_an3lo_qed`

- Working on inclusion of recent developments on N^{3}LO splitting functions and matching conditions. Preliminary results **suggest stability of our aN^{3}LO fits**.



OUTLOOK

- **Newer contributions from N^{3}LO QCD** corrections can be included once available.
- **Inclusion of theory error** (both from incomplete terms and higher orders) is mandatory.
- **QED corrections should also be taken into account**.
- Working on the removal of NNLO k-factors and usage of full NNLO grids for hadronic processes.

Thank you for the attention!

N^3LO corrections and PDF fits

DGLAP Splitting functions:

$$P_{ij} = \alpha_s P_{ij}^{(0)} + \alpha_s^2 P_{ij}^{(1)} + \alpha_s^3 P_{ij}^{(2)} + \alpha_s^4 P_{ij}^{(3)}, \quad i,j = q,g$$

- **Large-n_f**: $\mathcal{O}(n_f^3)$, $P_{NS}^{(n_f^2)}$ [arxiv:1610.07477]; $P_{qq,PS}^{(n_f^2)}$[arxiv:2308.07958]; $P_{gq}^{(n_f^2)}$ [arxiv:2310.01245]
- **NS small-x** [arxiv:2202.10362]: $P_{NS}^{(3)} \supset \sum_{k=0}^{6} \ln^k(x)$
- **Singlet small-x** [arxiv:1805.06460]: $P_{ij}^{(3)} \supset \sum_{k=0}^{3} \frac{\ln^k(x)}{x}$
- **Large-x** [arxiv:2205.04493], [arxiv:1911.10174], [arxiv:0912.0369]:

$$P_{ii}^{(3)} \approx A_{4,i} \frac{1}{(1-x)_+} + B_{4,i}\delta(1-x) + C_{4,i}\ln(1-x) + D_{4,i}$$

$$P_{ij}^{(3)} \approx \sum_{k}^{6} \ln^k(1-x)$$

- 10 lowest **Mellin Moments** [arxiv:1707.08315] [arxiv:2111.15561], [arxiv:2302.07593],[arxiv:2307.04158],[arxiv:2310.05744]. More recent studies: [arxiv:2404.09701] [arxiv:2410.08089].

PDFs matching conditions:

$$\begin{pmatrix} g \\ \Sigma \\ h^+ \end{pmatrix}^{n_f+1} (\mu_h^2) = \mathbf{A}_{S,h^+}^{(n_f)}(\mu_h^2) \cdot \begin{pmatrix} g \\ \Sigma \\ h^+ \end{pmatrix}^{n_f} (\mu_h^2)$$

analytic results for all the terms are available: [arxiv:0904.3563], [arxiv:1008.3347],[arxiv:1402.0359], [arxiv:1409.1135],[arxiv:1406.4654], [arxiv:2211.0546],[arxiv:2311.00644],
approximated results for $a_{H,g}^{(3)}$ [arxiv:2403.00513]

DIS coefficient functions:

$$C_{i,j} = \alpha_s^0 C_{i,j}^{(0)} + \alpha_s^1 C_{i,j}^{(1)} + \alpha_s^2 C_{i,j}^{(2)} + \alpha_s^3 C_{i,j}^{(3)}, \quad j = q,g$$

are known at N^3LO in the **massless limit**. γ/Z: [arxiv:9605317] [arxiv:0411112] [arxiv:2208.14325], $W^\pm$: [arxiv:1606.08907].

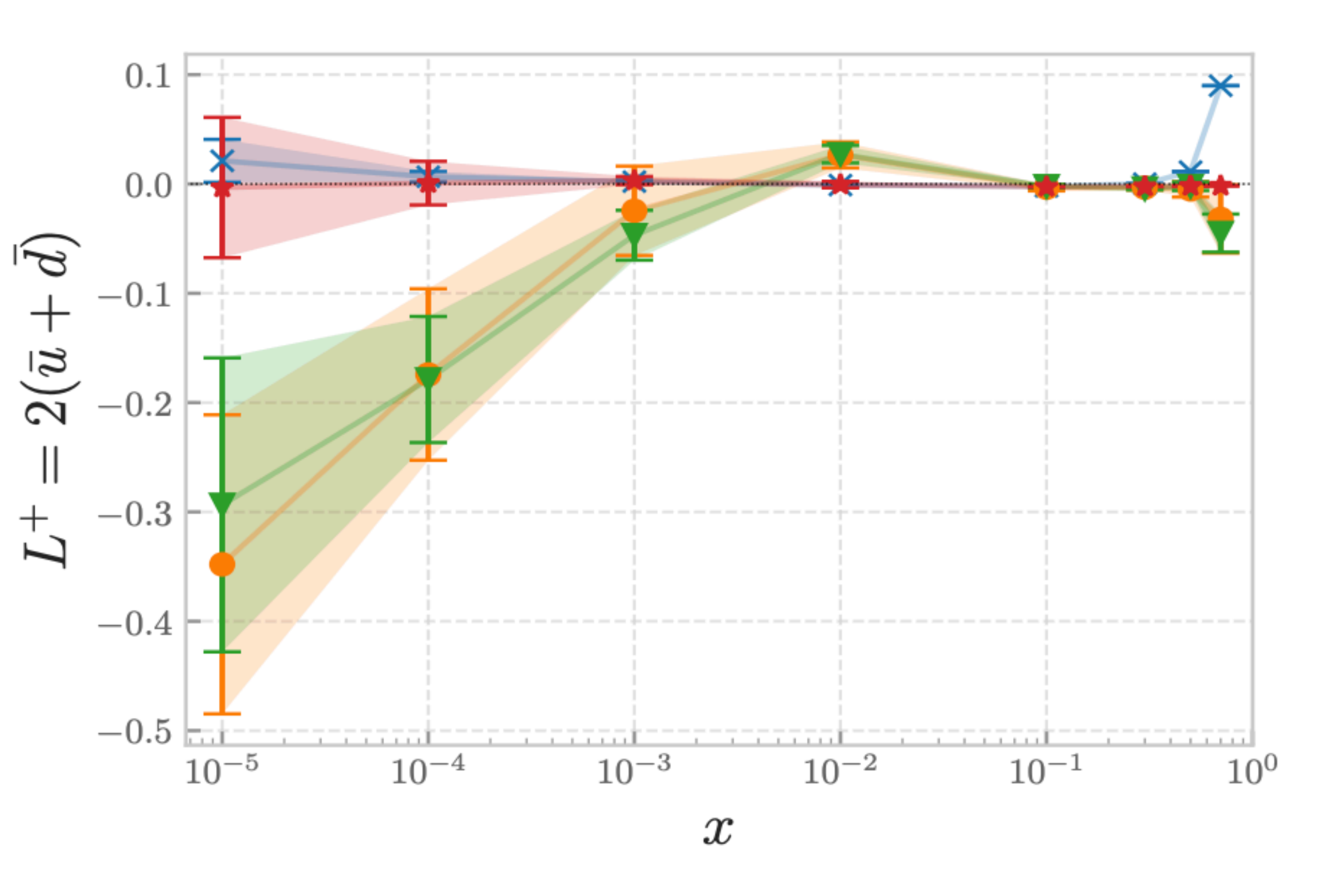
Massive contributions can be approximated from different known kinematic limit.

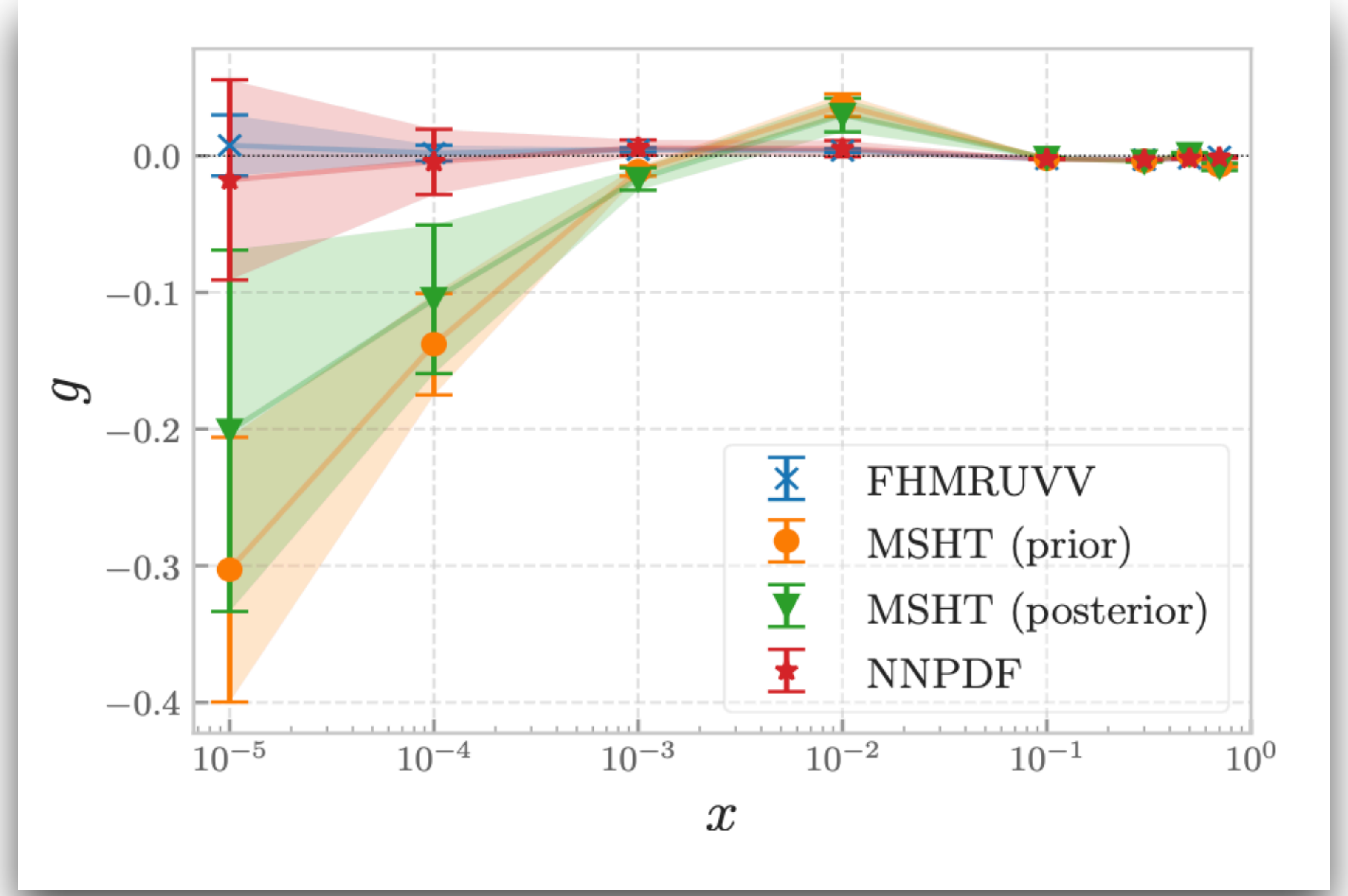
aN^{3}LO DGLAP evolution benchmark

Cooper-Sarkar, Cridge, Giuli, Harland-Lang, Hekhorn, Huston, GM, Moch, Thorne
[arxiv:2406.16188]

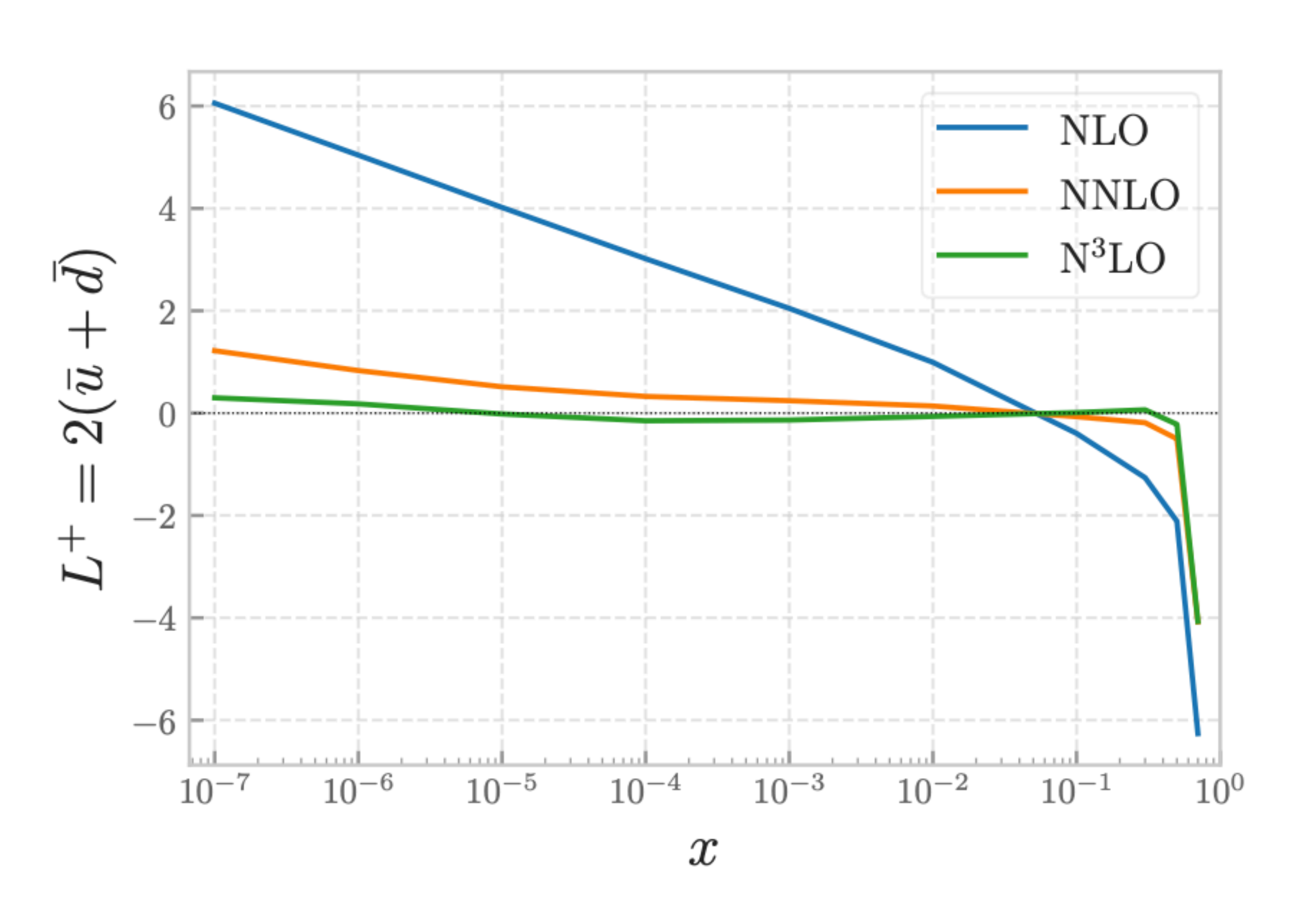
- **Benchmark**: comparison of aN^{3}LO DGLAP evolution **using different splitting function** approximation, and different evolution codes:
  MSHT [arxiv:2207.04739], FHMRUVV [arxiv:1707.08315] [arxiv:2111.15561], [arxiv:2302.07593],[arxiv:2307.04158], [arxiv:2310.05744] ([arxiv:2404.09701])
- Effect of aN^{3}LO is within the 2%, except for small and large-x regions.
- Good agreement of our in-house approximations and FHMRUVV.
- Stability of different DGLAP solution methods has also been checked.

Relative difference w.r.t NNLO evolution, VFNS $Q = 2 \to 100$ GeV



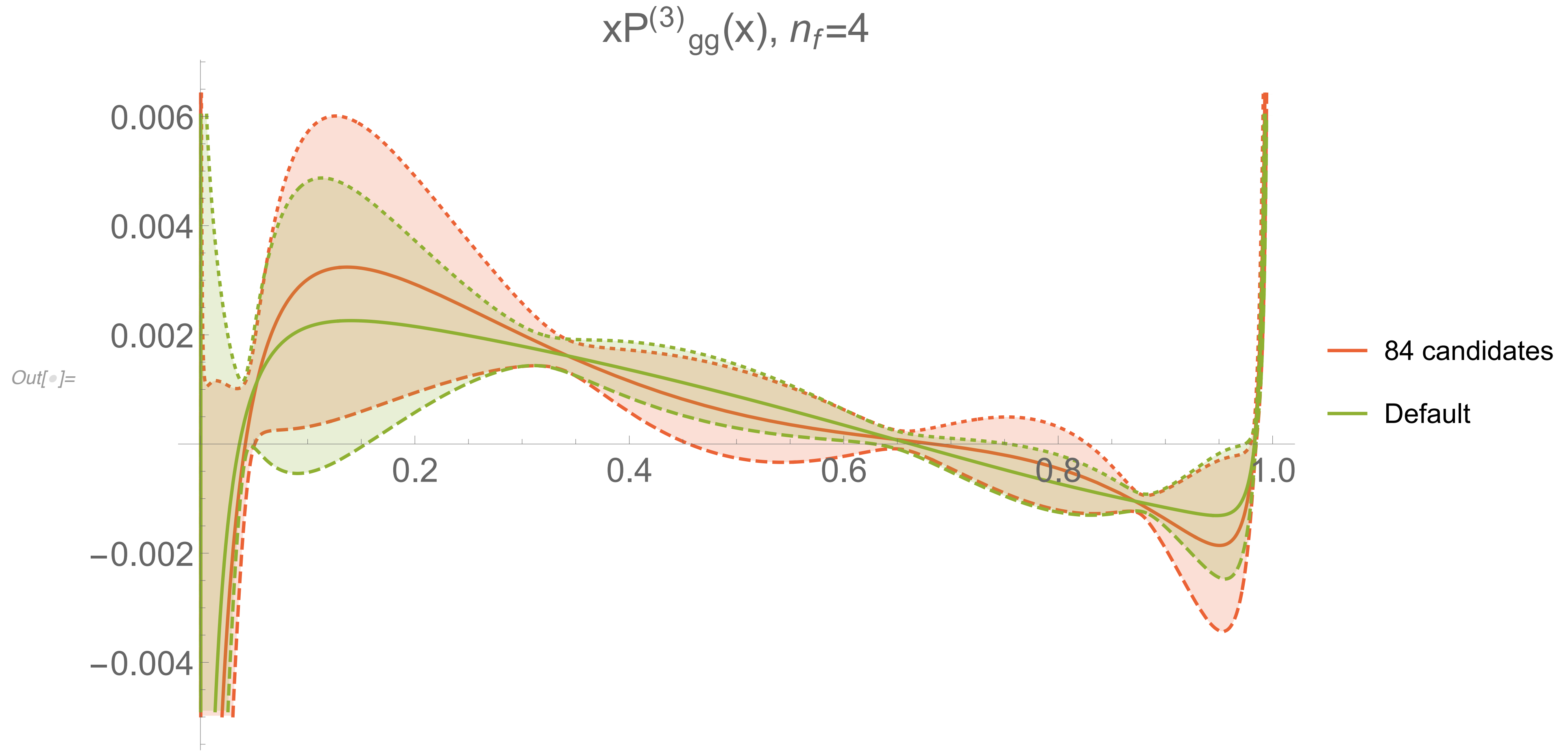


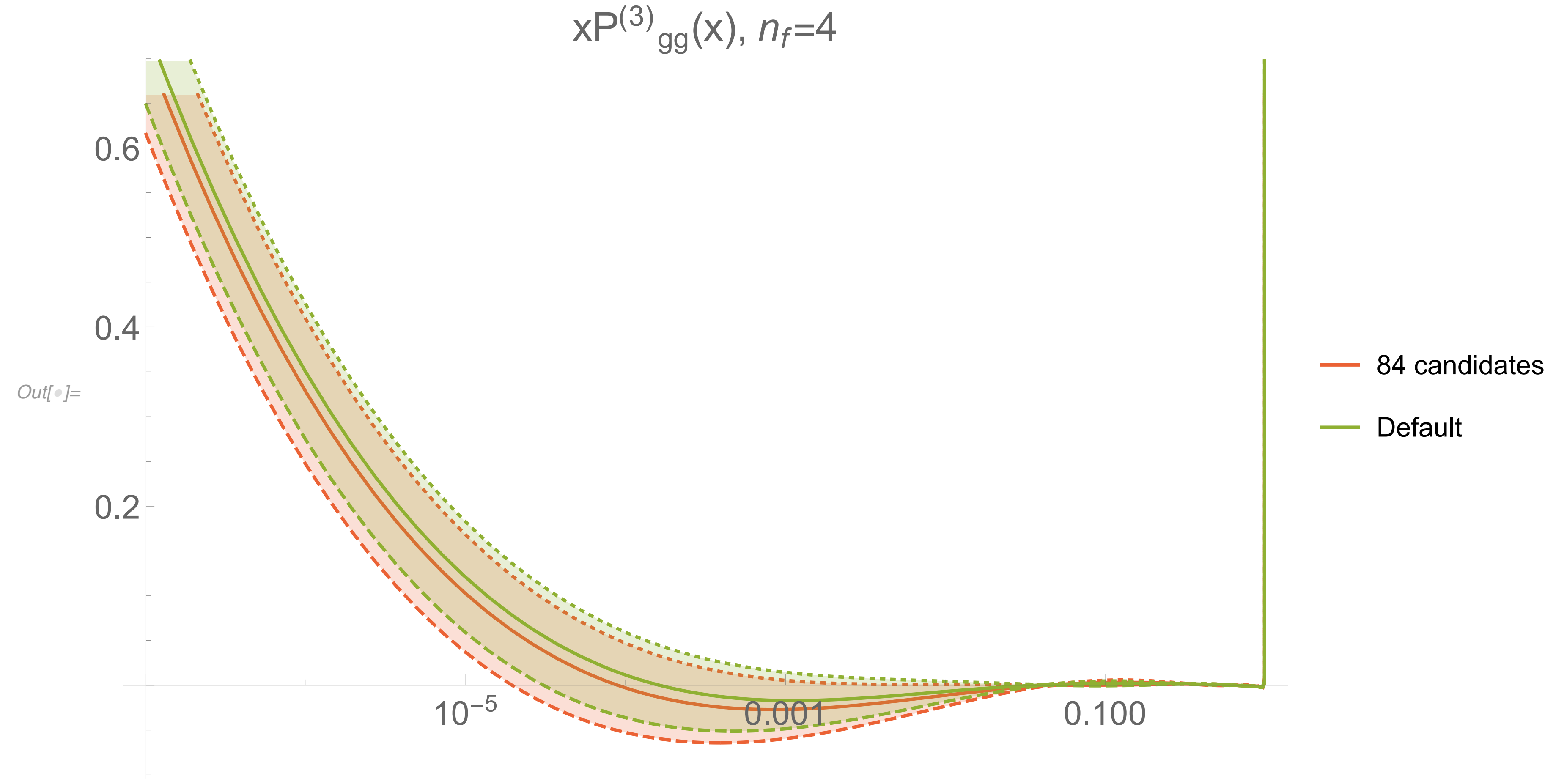
Relative difference Truncated vs Exact DGLAP evolution



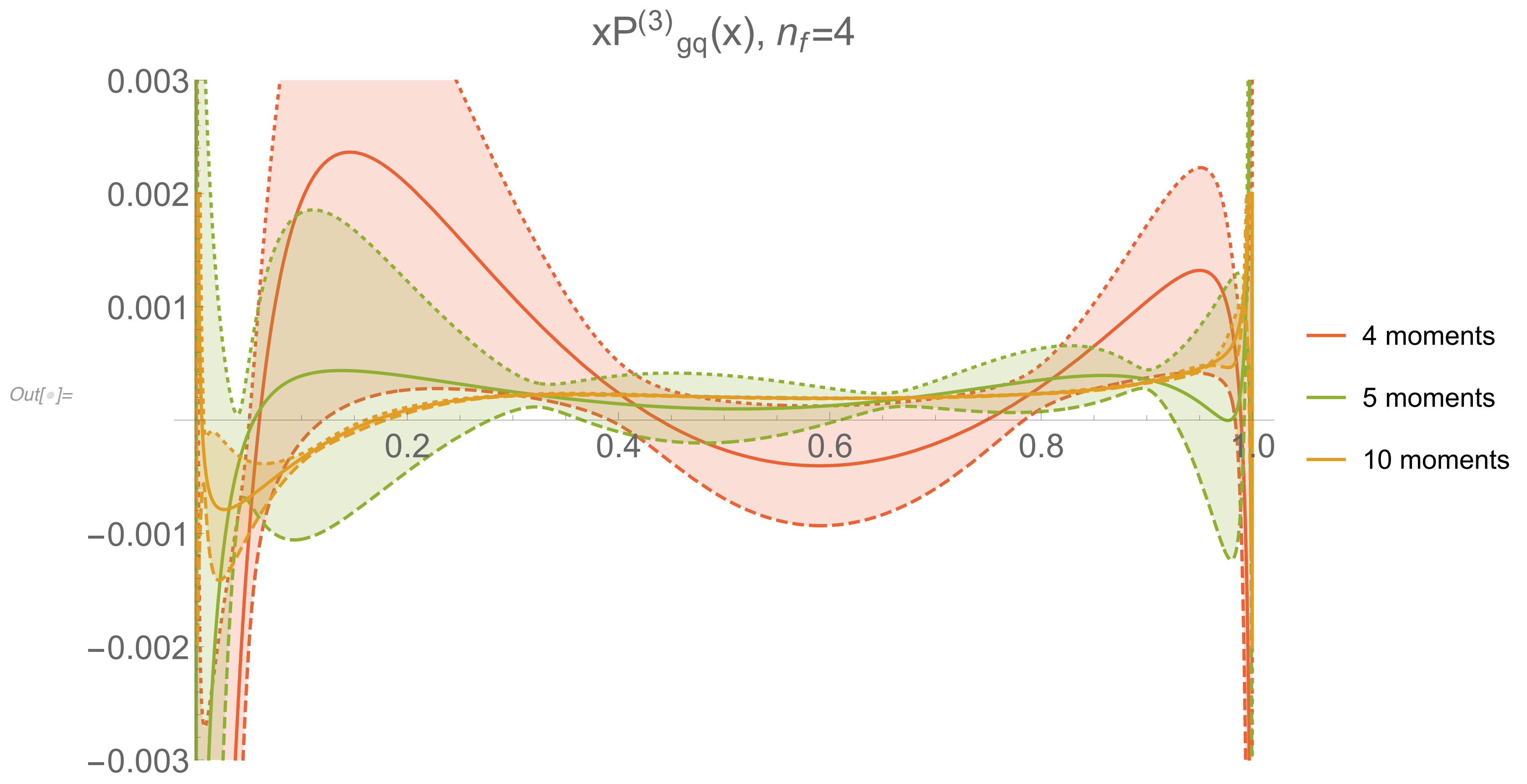
Including newer N^{3}LO terms

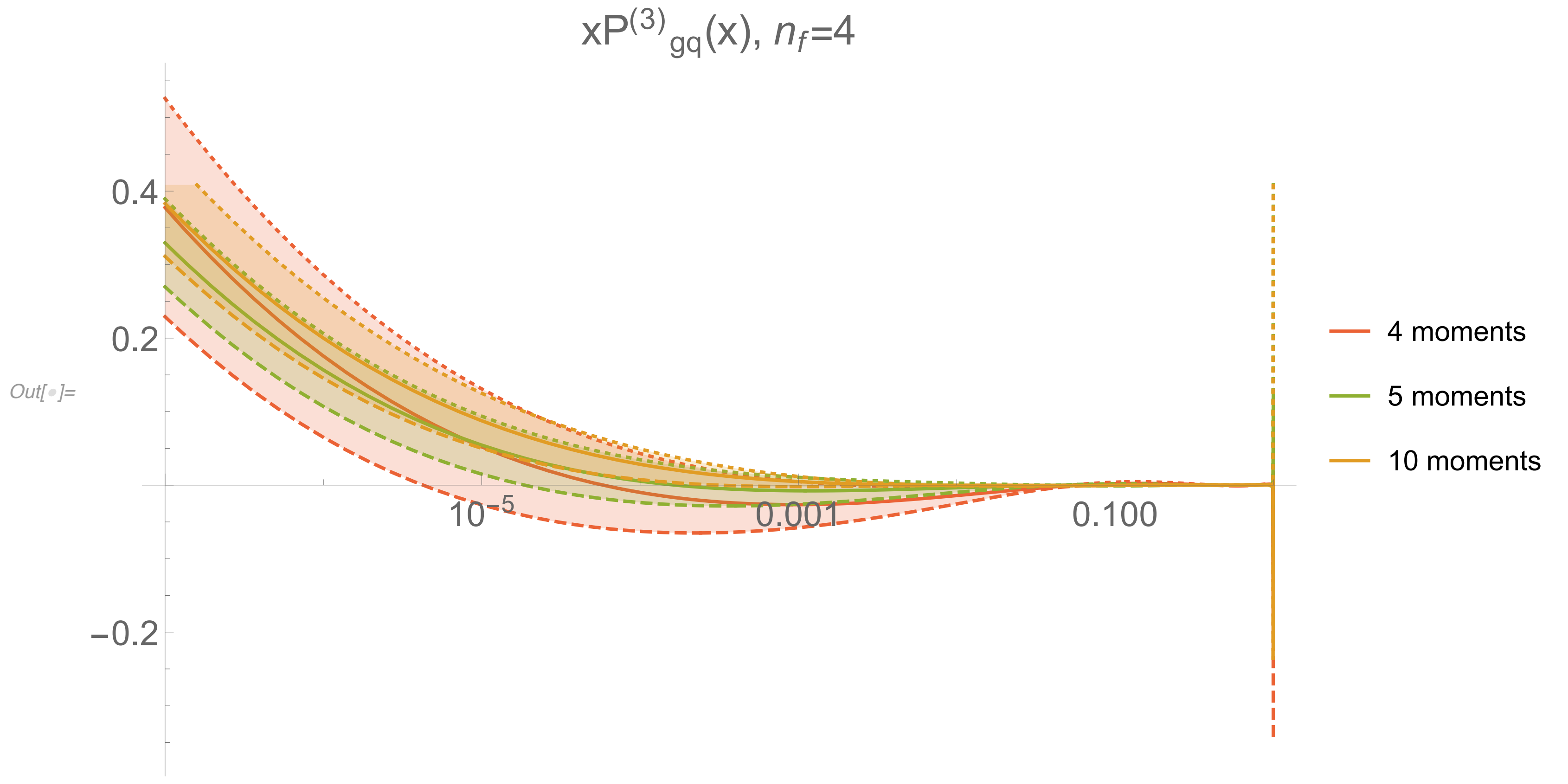
How does the approximation change if we add more test functions?





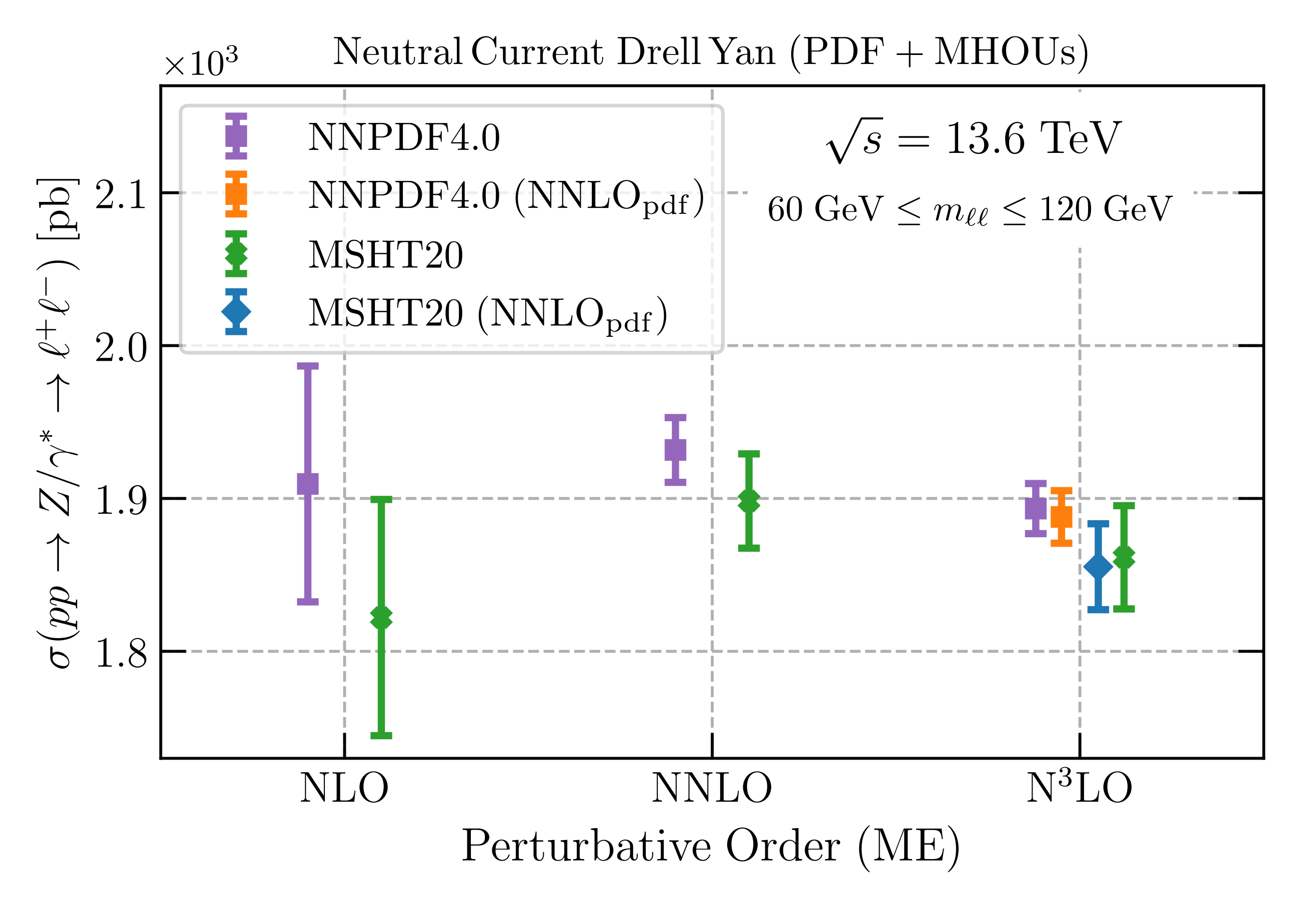
How does the approximation change if we add more Mellin moments?

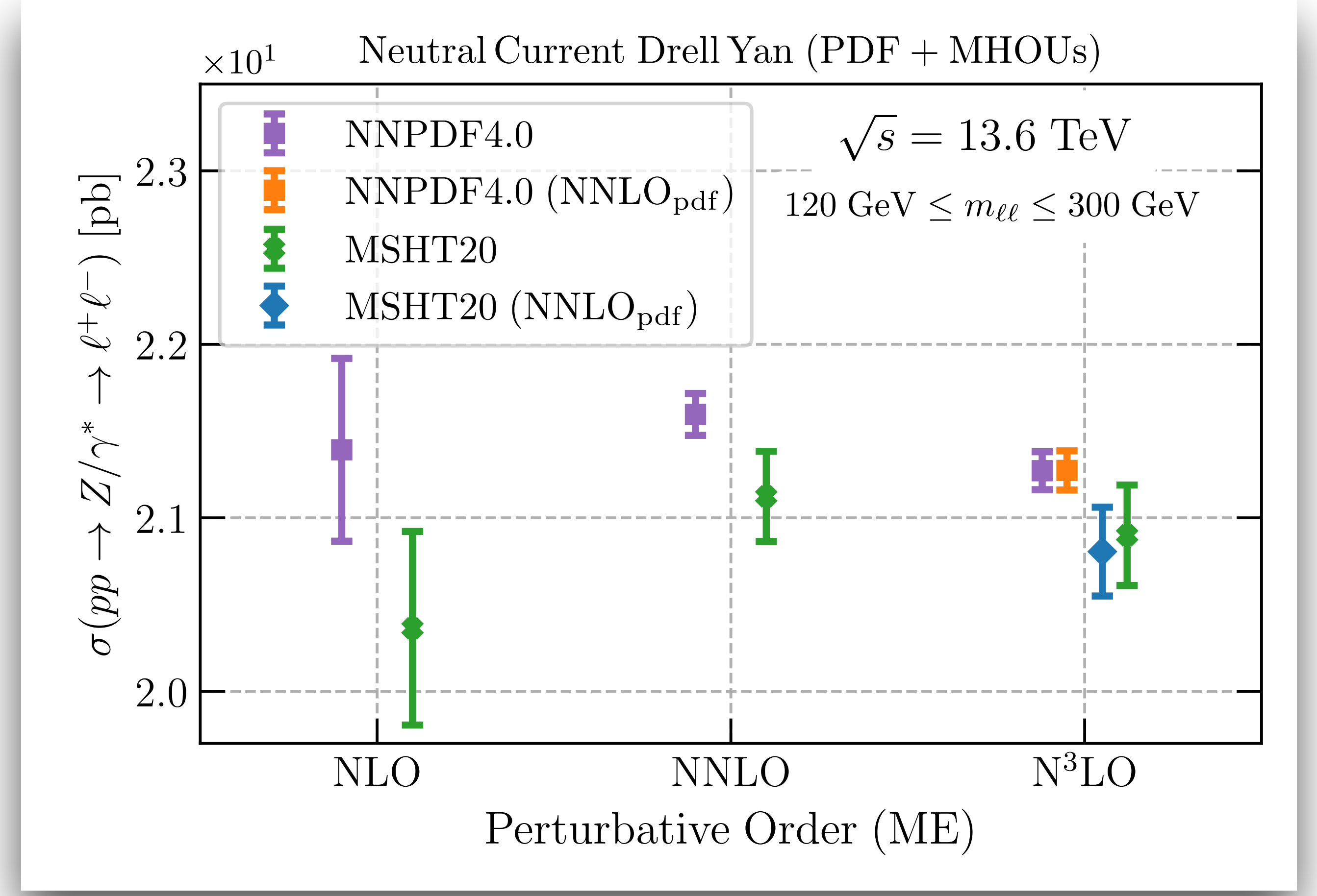




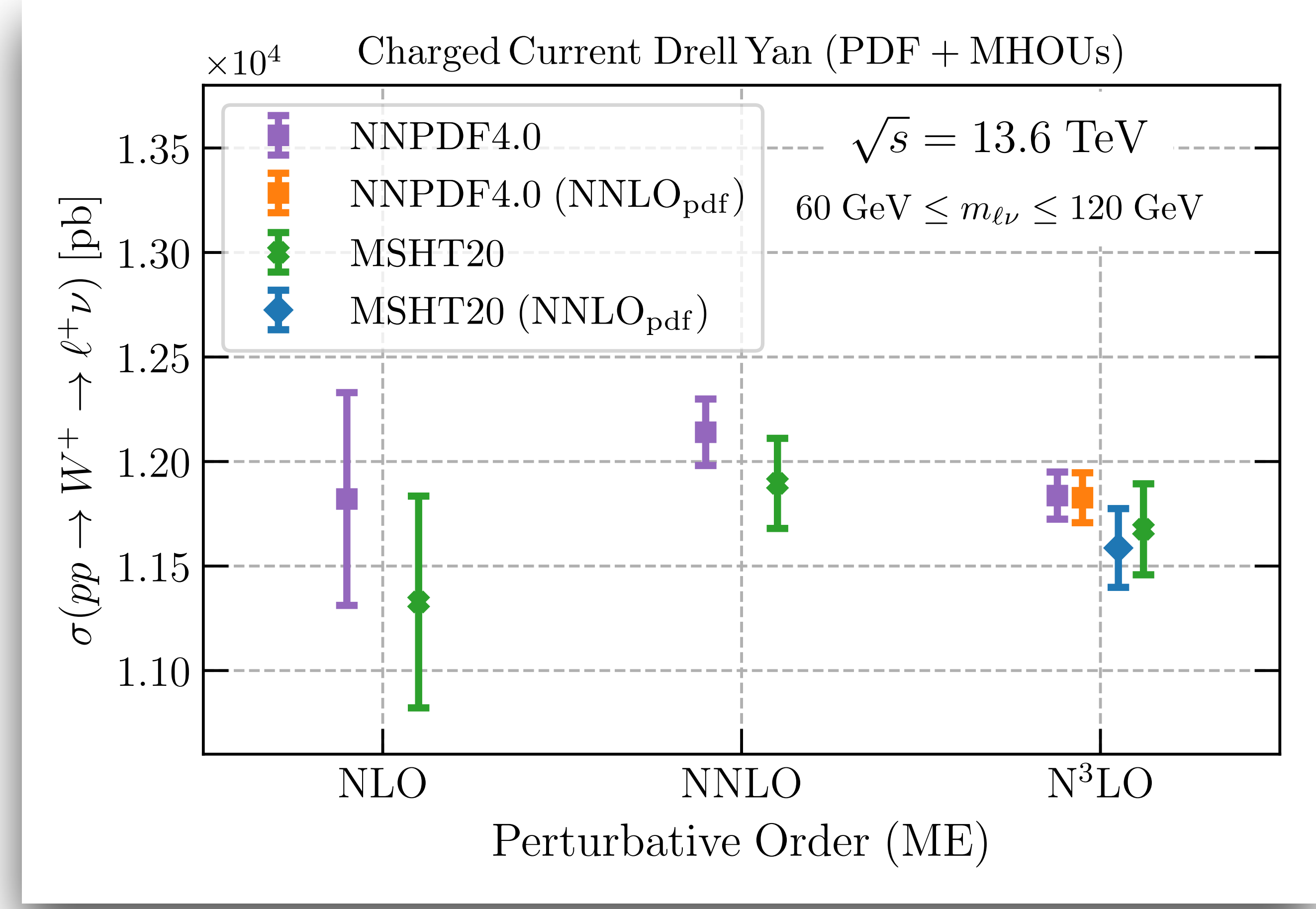
LHC phenomenology: Drell-Yan

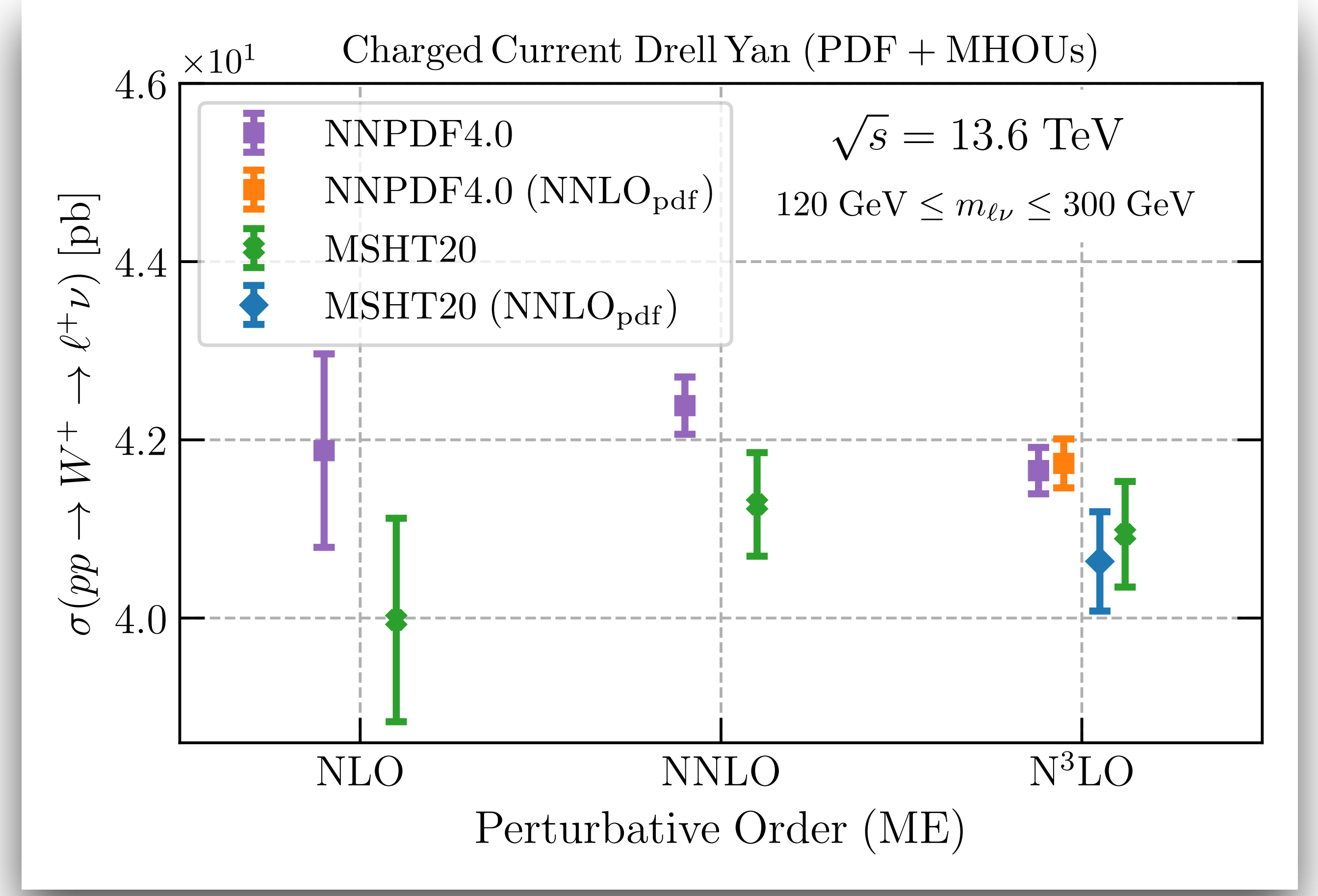
Z production





W^+ production





Process	NNPDF4.0						MSHT20				
	σ (pb)	$\delta_{\rm th}$	$\delta_{\rm PDF}^{\rm noMHOU}$	$\delta_{\rm PDF}^{\rm MHOU}$	$\Delta_{\rm NNLO}^{\rm app}$	$\Delta_{\rm NNLO}^{\rm exact}$	σ (pb)	$\delta_{\rm th}\sigma$	$\delta_{\rm PDF}$	$\Delta_{\rm NNLO}^{\rm app}$	$\Delta_{\rm NNLO}^{\rm exact}$
W^+ (p)	1.2×10^4	1.0	0.5	0.5	1.1	0.1	1.2×10^4	1.9	1.7	2.3	0.8
W^- (p)	8.8×10^3	1.0	0.5	0.5	1.1	0.1	8.7×10^3	1.9	1.6	2.1	0.0
Z (p)	1.9×10^3	0.9	0.4	0.5	1.1	0.3	1.9×10^3	1.8	1.6	2.6	0.3
W^+ (hm)	4.7×10^{-4}	2.8	2.8	3.3	3.2	1.1	4.6×10^{-4}	4.0	3.9	2.0	1.3
W^- (hm)	1.4×10^{-4}	2.9	2.9	3.3	3.3	0.1	1.5×10^{-4}	4.2	4.2	2.0	0.6
Z (hm)	2.1×10^{-4}	2.3	2.3	2.5	3.4	0.3	2.2×10^{-4}	3.6	3.6	2.7	0.2

aN^3LO PDF with QED corrections

`NNPDF40_aN3LO_QED` [arxiv:2406.01779]

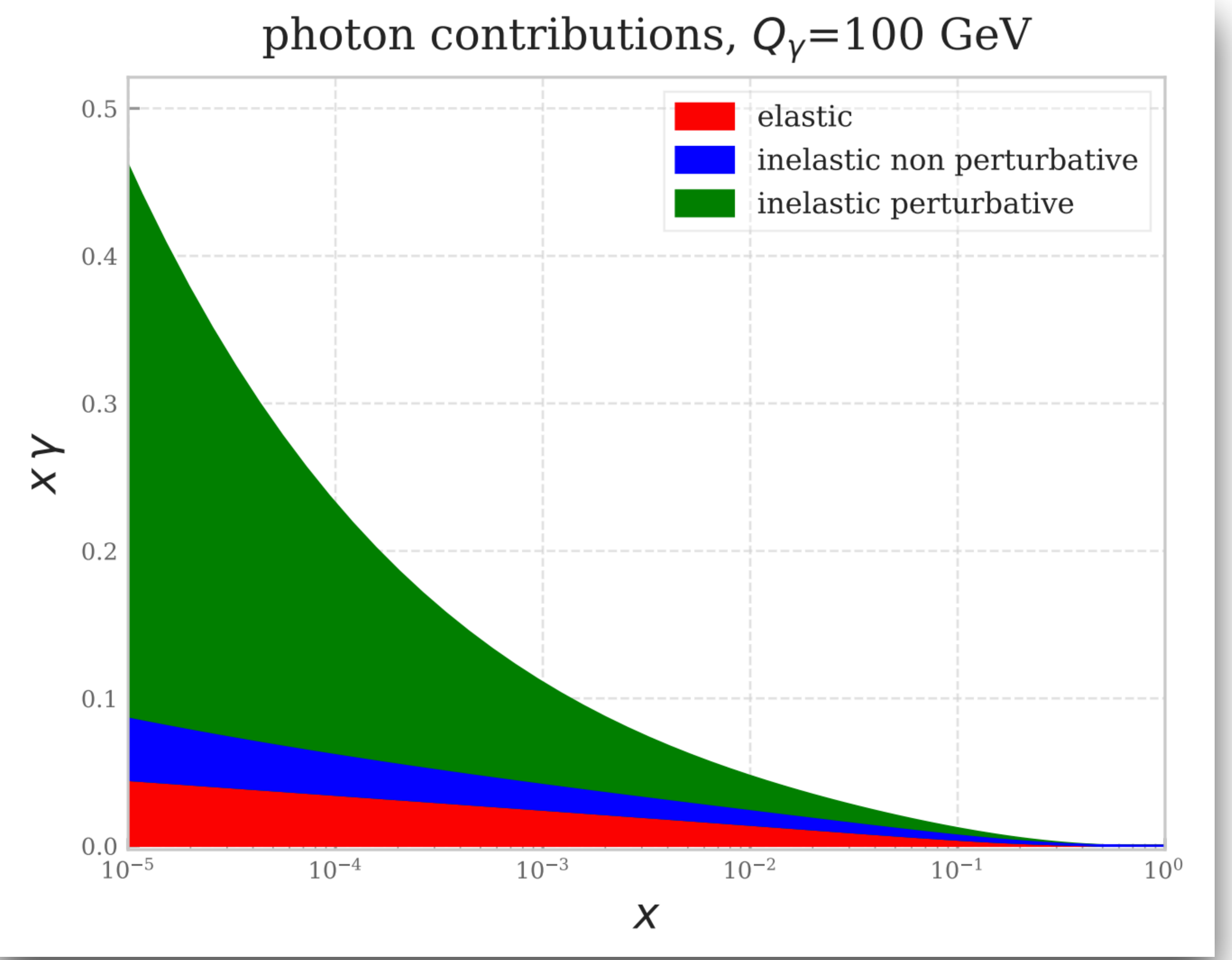
`MSTH20_aN3LO_QED` [arxiv:2312.07665]

The photon **PDF is computed from DIS** structure functions [arxiv:1607.04266] [arxiv:1708.01256]:

$$x\gamma(x,Q^2) = \frac{2}{\alpha_{em}} \int_x^1 \frac{dz}{z} \int_{\frac{M_p x^2}{1-z}}^{\frac{Q^2}{1-z}} \frac{d\mu^2}{\mu^2} \alpha_{em}(\mu^2) \left[(zP_{\gamma q} + \frac{2xM_p}{Q^2})F_2 - z^2 F_L \right] - \alpha_{em}(Q^2) z^2 F_2$$

- Depending on the kinematic region the structure functions F_2, F_L are computed form: **pQCD DIS**, **Inelastic DIS**, **Elastic DIS**.
- DGLAP evolution with mixed $QED \otimes QCD$: $\mathcal{O}(\alpha_s \alpha_{em})$, $\mathcal{O}(\alpha_{em}^2)$ corrections.
- Update the other partons with an iterative procedure from a QCD fit and modifying the momentum sum rule:

$$\int_0^1 xdx \; g(x) + \sum_i q_i^+(x) + \gamma(x) = 1$$



From `NNPDF4.0 NNLO QED` [arxiv:2401.08749]