



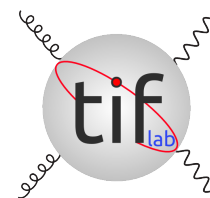
# CLOSURE TESTING

## THE IMPACT OF DATA INCONSISTENCY

STEFANO FORTE  
UNIVERSITÀ DI MILANO & INFN



UNIVERSITÀ DEGLI STUDI DI MILANO  
DIPARTIMENTO DI FISICA



PDF4LHC MEETING

CERN, DECEMBER 3, 2024

## CLOSURE TESTS

- ASSUME TRUTH UNDERLYING PDF  $\Rightarrow$  E.G. SOME RANDOM PDF REPLICA
- GENERATE DATA DISTRIBUTED ACCORDING TO EXPERIMENTAL COVARIANCE MATRIX
- RUN WHOLE UNMODIFIED METHODOLOGY ON THESE DATA
- DO STATISTICS ON “RUNS OF THE UNIVERSE”

# STATISTICAL INDICATORS

## PDFS AND THEIR UNCERTAINTY (REMINDER)

$i$ -TH DATA PREDICTION (REPLICA AVERAGE):  $f_i^{\text{pred}} = \frac{1}{N_{\text{rep}}} \sum_n f_i^{(n)}$

PDF UNCERTAINTY (REPLICA COVARIANCE):  $C_{ij}^{\text{PDF}} = \frac{1}{N_{\text{rep}}-1} \sum_n \left( f_i^{(n)} - f_i^{\text{pred}} \right) \left( f_j^{(n)} - f_j^{\text{pred}} \right)$

## BIAS-VARIANCE RATIO

**BIAS:**  $b^2 = \frac{1}{N_{\text{dat}}} \sum_{ij} \left( f_i^{\text{pred}} - f_i^{\text{true}} \right) C_{ij}^{-1} \left( f_j^{\text{pred}} - f_j^{\text{true}} \right)$ ;  $C_{ij}$  exp. covmat

**VARIANCE:**  $\sigma^2 = \frac{1}{N_{\text{dat}}} \frac{1}{N_{\text{rep}}-1} \sum_{ij} \left( f_i^{(n)} - f_i^{\text{pred}} \right) C_{ij}^{-1} \left( f_j^{(n)} - f_j^{\text{pred}} \right)$

$R_{bv} = b/v$  DEVIATION FROM TRUTHS IN UNITS OF UNCERTAINTY

IN DATA EIGENVECTOR BASIS

## N- $\sigma$ QUANTILE

$\xi_n$ : FRACTION OF BIAS DISTN. WITHIN  $n\sigma$  OF TRUTH

# NNPDF4.0 CLOSURE TEST RESULTS: NUMBERS

## BIAS/VARIANCE RATIO AND ONE- $\sigma$ QUANTILE

DATA-SPACE, DATA COVARIANCE MATRIX, OUT-OF-SAMPLE

PDF-SPACE & COV MATRIX

Dataset	$\sqrt{b/v}$	$\xi_{1\sigma}^{(\text{data})}$	$\text{erf}(R_{bv}/\sqrt{2})$	flavour	$\xi_{1\sigma}^{(\text{pdf})}$
DY	$0.99 \pm 0.08$	$0.69 \pm 0.02$	$0.69 \pm 0.04$	$\Sigma$	$0.82 \pm 0.04$
Top-pair	$0.75 \pm 0.06$	$0.75 \pm 0.03$	$0.82 \pm 0.03$	$g$	$0.70 \pm 0.05$
Jets	$1.14 \pm 0.05$	$0.63 \pm 0.03$	$0.62 \pm 0.02$	$V$	$0.65 \pm 0.05$
Dijets	$0.99 \pm 0.07$	$0.70 \pm 0.03$	$0.69 \pm 0.04$	$V_3$	$0.63 \pm 0.05$
Direct photon	$0.71 \pm 0.06$	$0.81 \pm 0.03$	$0.84 \pm 0.03$	$V_8$	$0.72 \pm 0.04$
Single top	$0.87 \pm 0.07$	$0.69 \pm 0.04$	$0.75 \pm 0.04$	$T_3$	$0.71 \pm 0.05$
Total	$1.03 \pm 0.05$	$0.68 \pm 0.02$	$0.67 \pm 0.03$	$T_8$	$0.71 \pm 0.05$
				Total	$0.71 \pm 0.02$

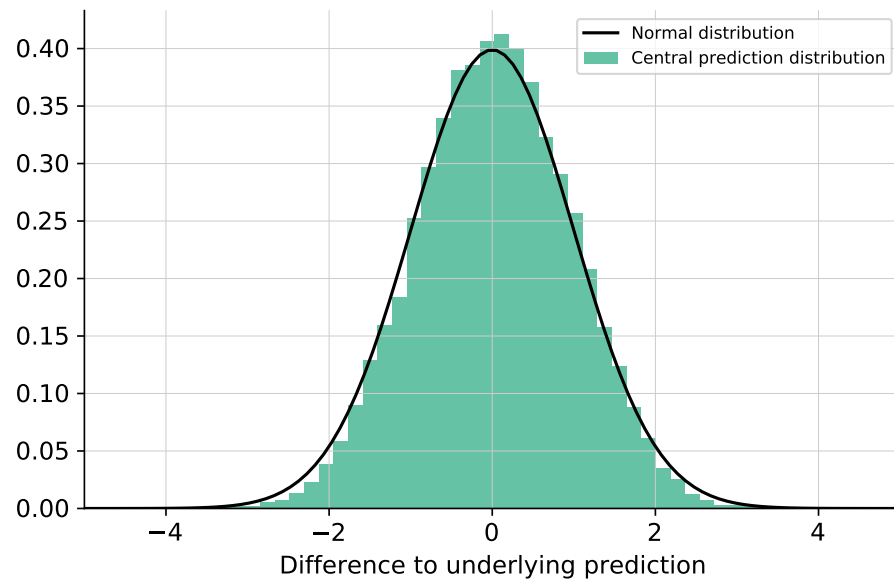
- 25 “UNIVERSE RUNS”, 45 REPLICAS EACH
- IN-SAMPLE DATA: PRE 2015
- OUT OF SAMPLE DATA: 2015-2020, MOSTLY LHC
- PDFs HIGHLY CORRELATED  $\Rightarrow$  SAMPLED AT 4 POINTS EACH

# NNPDF4.0 CLOSURE TEST RESULTS:

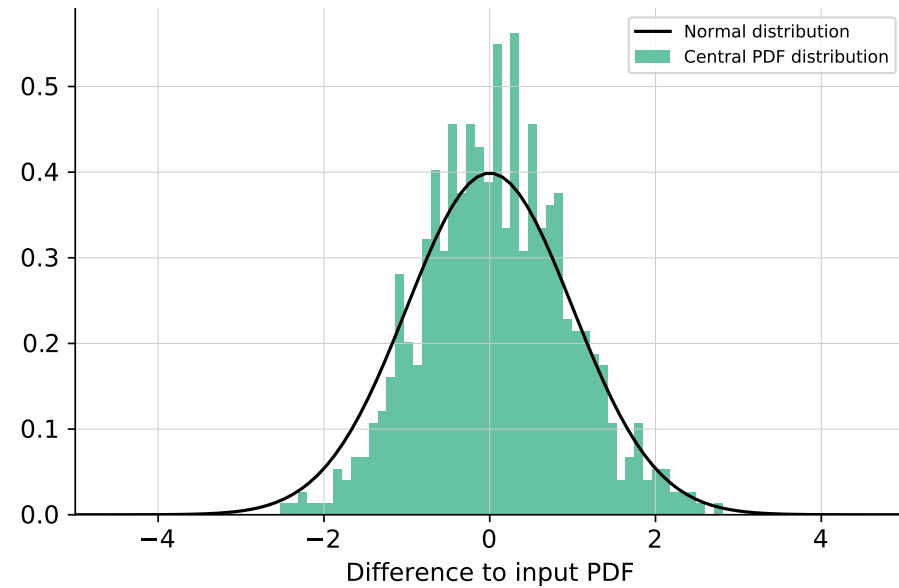
## PICTURES

### DISTRIBUTION OF DEVIATIONS FROM TRUTH

DATA SPACE (OUT OF SAMPLE)



PDF SPACE



- PDF-SPACE MORE NOISY THAN DATA SPACE

# NEW STATISTICAL INDICATORS

A. Barontini, M. Costantini, G. De Crescenzo, S.F., M. Ubiali, in preparation

## PDFS AND THEIR UNCERTAINTY (REMINDER)

$i$ -TH DATA PREDICTION (REPLICA AVERAGE):  $f_i^{\text{pred}} = \frac{1}{N_{\text{rep}}} \sum_n f_i^{(n)}$

PDF UNCERTAINTY (REPLICA COVARIANCE):  $C_{ij}^{\text{PDF}} = \frac{1}{N_{\text{rep}} - 1} \sum_n \left( f_i^{(n)} - f_i^{\text{pred}} \right) \left( f_j^{(n)} - f_j^{\text{pred}} \right)$

## NORMALIZED BIAS

**BIAS:**  $b^2 = \frac{1}{N_{\text{dat}}} \sum_{ij} \left( f_i^{\text{pred}} - f_i^{\text{true}} \right) C_{ij}^{-1} \left( f_j^{\text{pred}} - f_j^{\text{true}} \right)$ ;  $C_{ij}$  exp. covmat

**NORMALIZED BIAS:**  $R_b^2 = \frac{1}{N_{\text{dat}}} \sum_{ij} \left( f_i^{\text{pred}} - f_i^{\text{true}} \right) C_{ij}^{\text{PDF}^{-1}} \left( f_j^{\text{pred}} - f_j^{\text{true}} \right)$ ;

DEVIATION FROM TRUTHS IN UNITS OF UNCERTAINTY IN **PDF EIGENVECTOR BASIS**

- **CORRELATES** EXPERIMENTALLY UNCORRELATED **DATA SENSITIVE TO SAME PDF**
- **WEIGHS** EXPERIMENTS BASED ON **NUMBER OF DATA POINT**

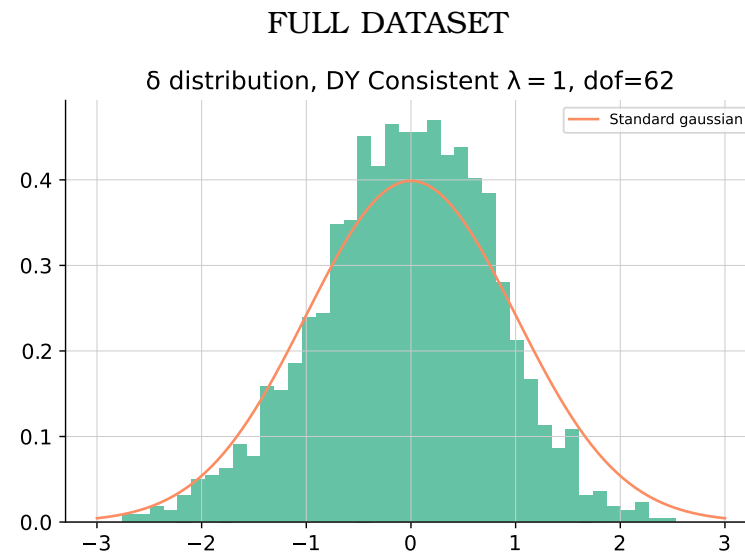
## N- $\sigma$ QUANTILE

$\xi_n$ : FRACTION OF **NORMALIZED BIAS** DISTN WITHIN  $n\sigma$  OF TRUTH

# NEW CONSISTENT CLOSURE TEST

- 25 “UNIVERSE RUNS”, 100 REPLICAS EACH
- FULL NNPDF4.0 DATASET, VARIOUS IN-SAMPLE OUT OF SAMPLE PARTITIONS BASED ON REPRESENTATIVE FOLDS
- PDF COVARIANCE MATRIX HIGHLY CORRELATED  $\Rightarrow$  PCA REDUCTION

## NORMALIZED BIAS DISTRIBUTION



$$R_b = 0.88 \pm 0.02; \quad \xi_{1\sigma} = 0.75 \pm 0.02$$

NNPDF4.0 UNCERTAINTIES SLIGHTLY OVERESTIMATED

# THE INCONSISTENCY

## THE MODEL

- GENERATE DATA WITH STAT+SYST UNCERTAINTIES  $C = C^{\text{stat}} + C^{\text{systr}}$ ;

$$C_{ij}^{\text{systr}} = \sum_k \Delta_i^k \Delta_j^k; \Delta_i^k \Rightarrow k\text{-TH SYSTEMATICS ON } i\text{-TH DATAPOINT}$$

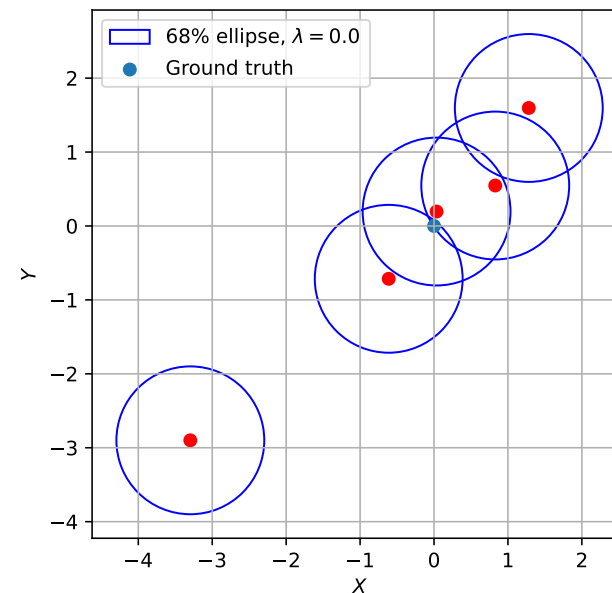
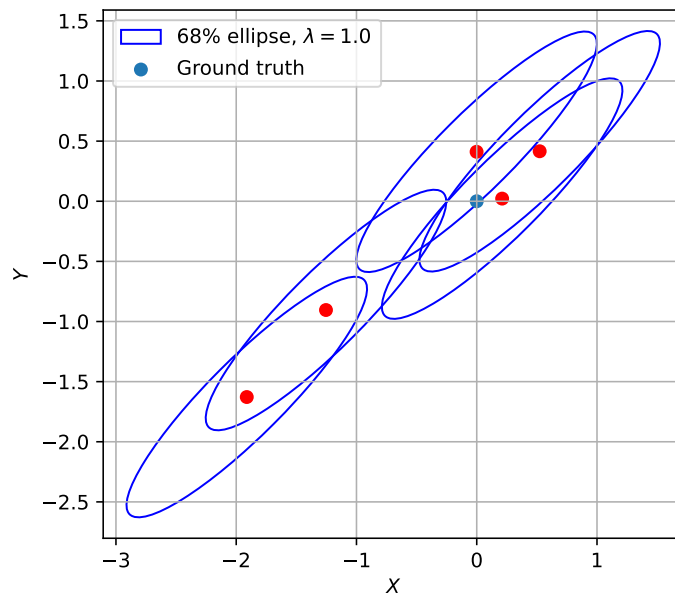
- ASSUME SYSTEMATICS UNDERESTIMATED  $\Rightarrow$  PERFORM FIT WITH  $\Delta_i^k \rightarrow \lambda \Delta_i^k$

$\lambda = 1 \Rightarrow$  CONSISTENT;  $\lambda = 0 \Rightarrow$  EXTREME INCONSISTENCY

## PREDICTED UNCERTAINTY ON GENERATED DATA

CONSISTENT

EXTREME INCONSISTENCY





# THE INCONSISTENCY

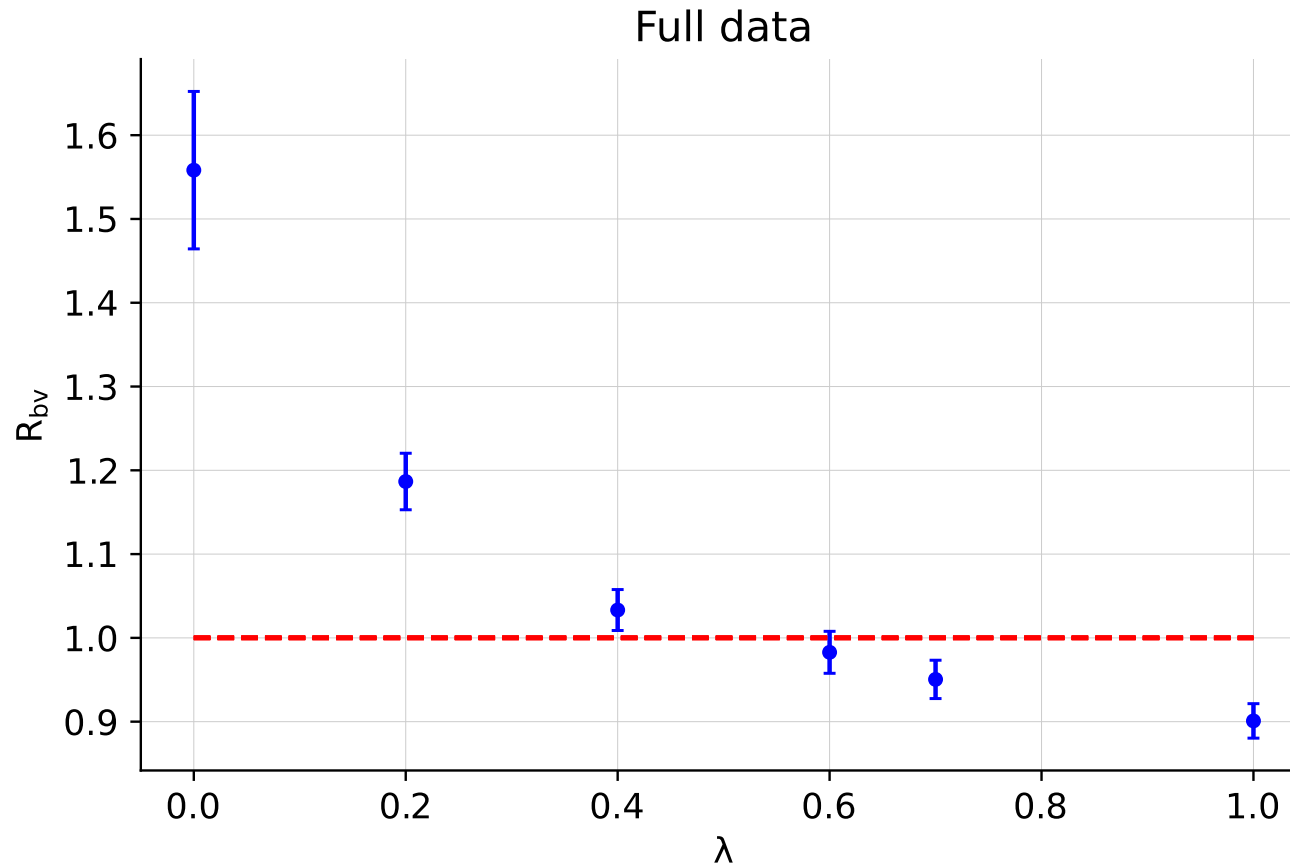
## SCENARIOS

- “DIS”: DIS ONLY FIT, IN-SAMPLE HERA DATA INCONSISTENT  
BULK INCONSISTENCY
- “DY”: GLOBAL FIT, HIGH-MASS ATLAS 2D DY INCONSISTENT  
SINGLE DATASET INCONSISTENCY
- “JETS”: GLOBAL FIT, ALL IN-SAMPLE SINGLE-INCLUSIVE JETS INCONSISTENT  
INCONSISTENCY OF DATA WITH HIGH-IMPACT ON A PDF

# THE NORMALIZED BIAS

DIS: **BULK INCONSISTENCY**

BIAS VS INCONSISTENCY (FULL DATASET)

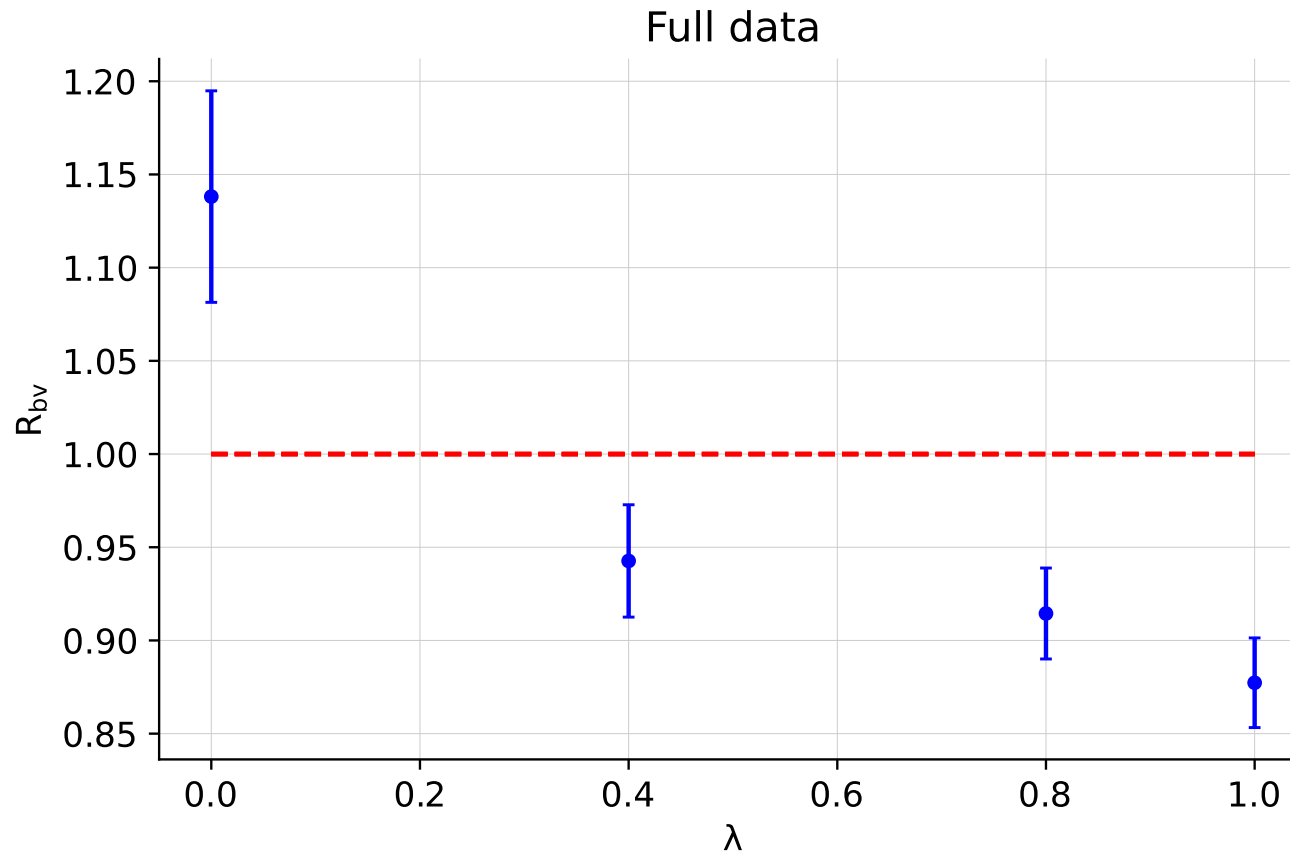


- $\lambda \gtrsim 0.3 \Rightarrow$  **MODEL PARTLY CORRECTS** INCONSISTENCY
- $\lambda = 0$  NONLINEAR GROWTH: **MODEL POOR** (UNCERTAINTIES UNDERESTIMATED)

# THE NORMALIZED BIAS

DY: SINGLE DATASET INCONSISTENCY

BIAS VS INCONSISTENCY (FULL DATASET)

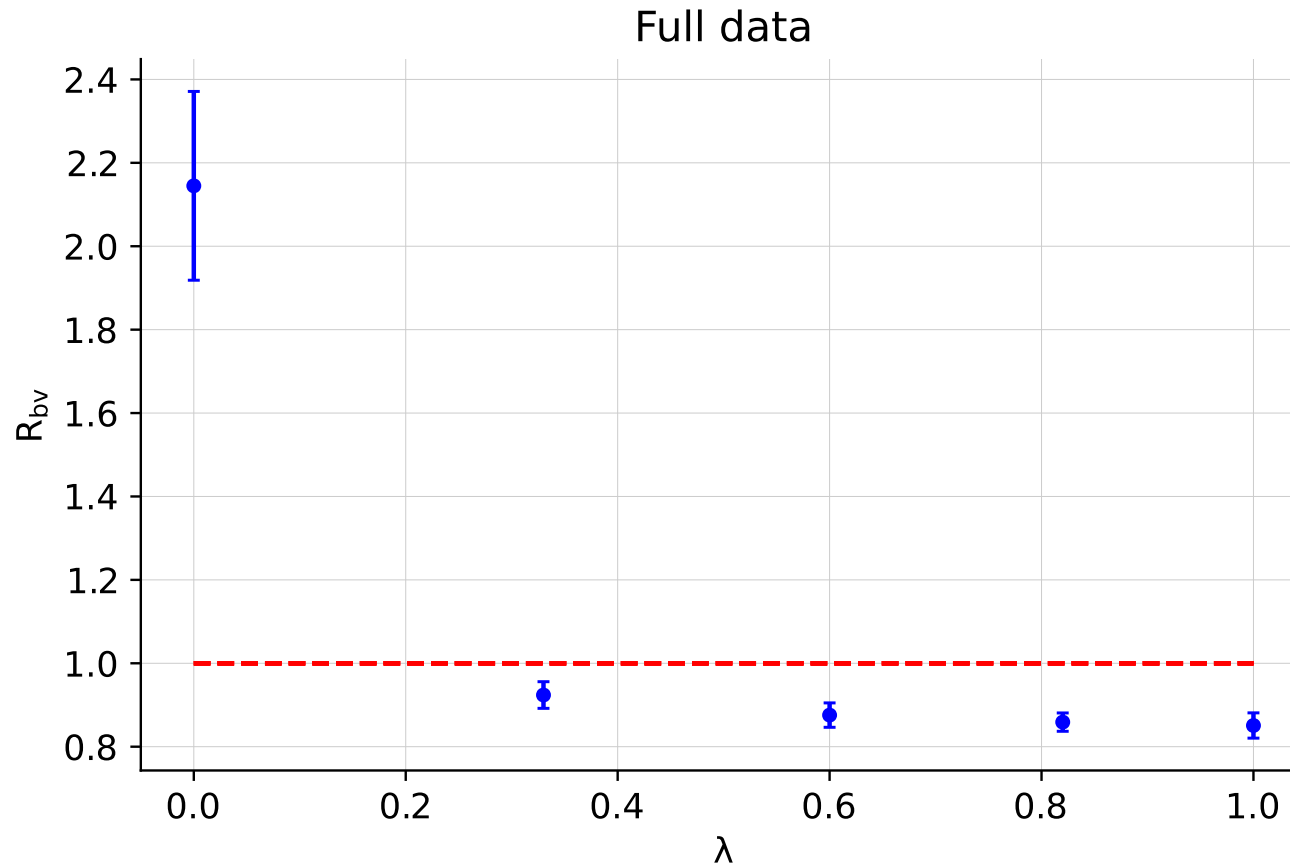


- $\lambda \gtrsim 0.2 \Rightarrow$  MODEL CORRECTS INCONSISTENCY
- $\lambda = 0$  LEAP: MODEL INACCURATE (UNCERTAINTIES SOMEWHAT UNDERESTIMATED)

# THE NORMALIZED BIAS

JETS: HIGH-IMPACT INCONSISTENCY

BIAS VS INCONSISTENCY (FULL DATASET)



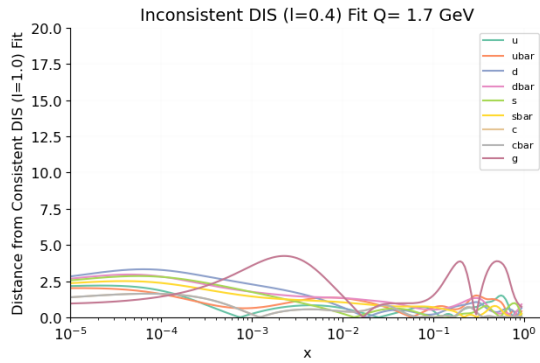
- $\lambda \gtrsim 0.3 \Rightarrow$  MODEL FULLY CORRECTS INCONSISTENCY
- $\lambda = 0$  PHASE TRANSITION: MODEL FAILS (UNCERTAINTIES COMPLETELY WRONG)

# THE PDFs

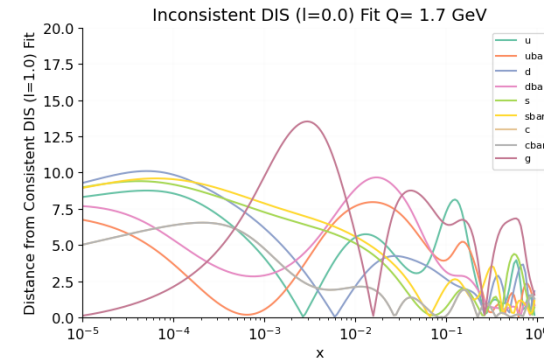
DISTANCES:  $\frac{f^{\text{consistent}} - f^{\text{inconsistent}}}{\sigma^{\text{PDF}} / \sqrt{N_{\text{rep}}}}$ ;  $d \sim 1 \Rightarrow$  STATISTICAL EQUIVALENCE

DIS: BULK INCONSISTENCY

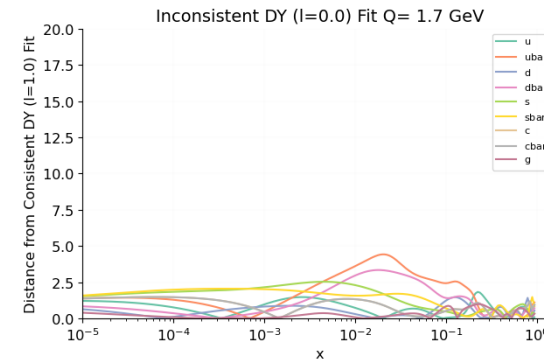
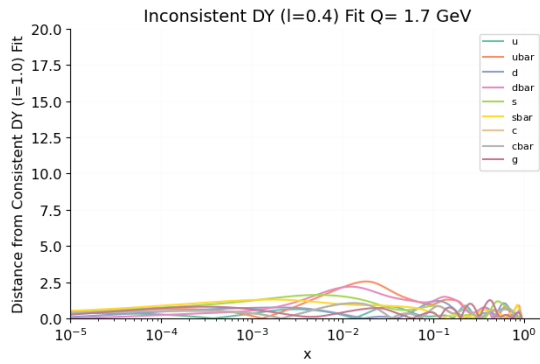
$\lambda = 0.4$ : MODEL CORRECTS



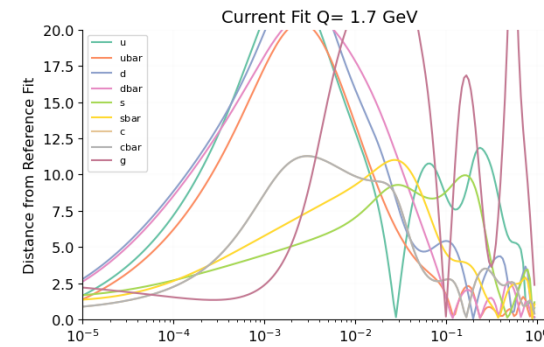
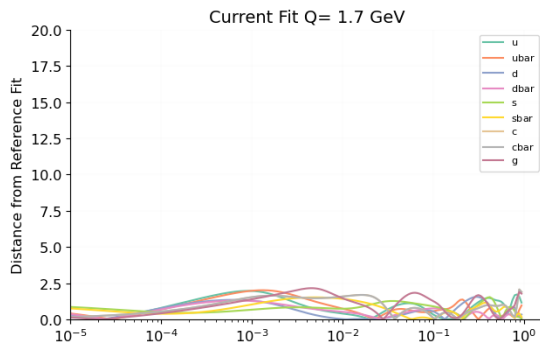
$\lambda = 0$ : MODEL FAILS



DY: SINGLE DATASET INCONSISTENCY



JETS: HIGH-IMPACT INCONSISTENCY

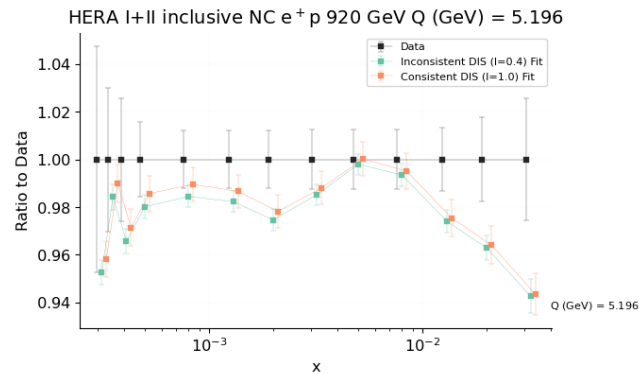


# DATA vs. PREDICTION I

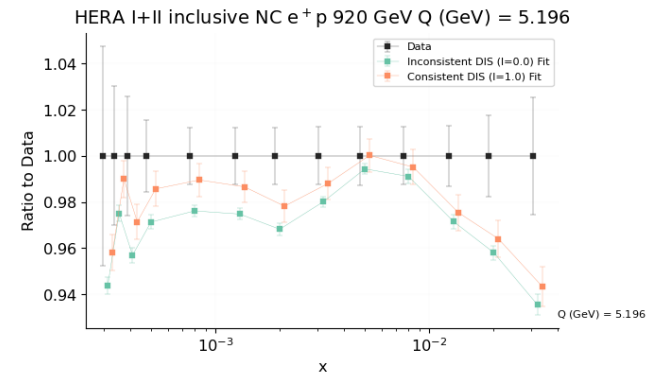
INCONSISTENT VS. CONSISTENT PDF PREDICTIONS FOR INCONSISTENT DATASET

## DIS: BULK INCONSISTENCY

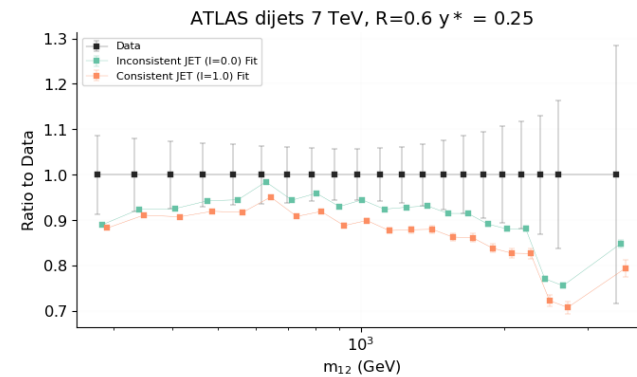
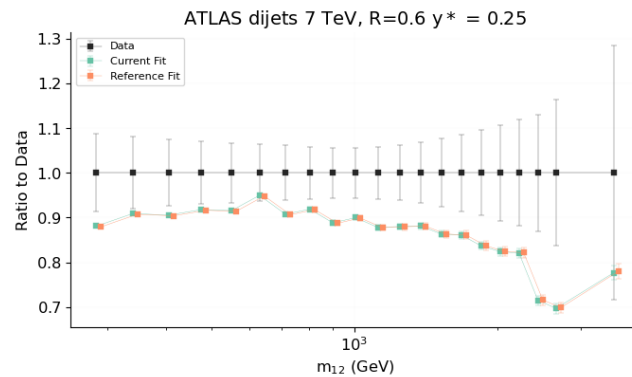
$\lambda = 0.4$ : MODEL CORRECTS



$\lambda = 0$ : MODEL FAILS



## JETS: HIGH-IMPACT INCONSISTENCY



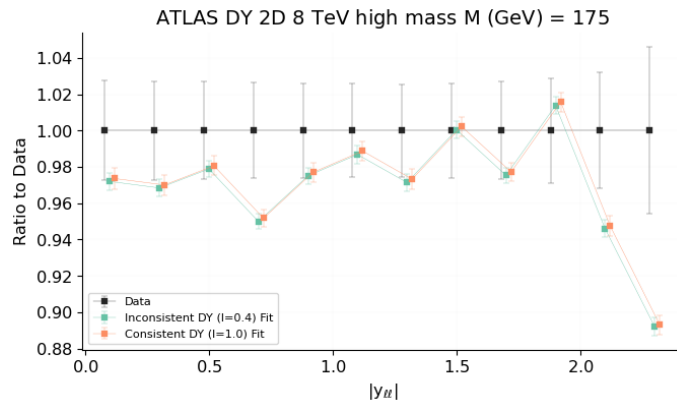
- $\lambda = 0.4$ : MODEL CORRECTS, PREDICTION DOES NOT MOVE
- $\lambda = 0$ : MODEL FAILS, PREDICTION OFF WITH UNCHANGED UNCERTAINTY

# DATA vs. PREDICTION II

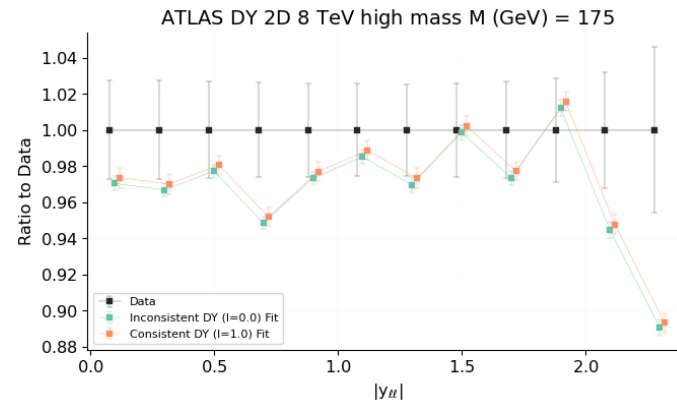
INCONSISTENT vs. CONSISTENT PDF PREDICTIONS FOR INCONSISTENT DATASET

## DY: SINGLE DATASET INCONSISTENCY

$\lambda = 0.4$ : MODEL CORRECTS



$\lambda = 0$ : MODEL FAILS



- PREDICTION DOES NOT MOVE
- $\lambda = 0.4$ : MODEL CORRECTS, UNCERTAINTY UNCHANGED  
DESPITE SMALLER DATA UNCERTAINTY
- $\lambda = 0$ : MODEL FAILS, UNCERTAINTY SHRINKS DRIVEN BY SMALLER DATA UNCERTAINTY

## FLAGGING INCONSISTENCIES

### VALIDATING THE NNP4.0 DATASET SELECTION METHOD

- LARGE  $R_b \Rightarrow$  LARGE  $\chi_p^2$  OF INCONSISTENT DATASET
- $\chi_p^2$  OF CONSISTENT DATASETS SENSITIVE TO SAME PDFs ALSO LARGE

#### EXAMPLE:

INCONSISTENT ATLAS JET DATA WITH IN-SAMPLE CMS JET DATA:

- $n_\sigma = \frac{\chi_p^2 - 1}{\sqrt{N_{\text{dat}}/2}}$ : ATLAS TRUE POSITIVE:  $n_\sigma = 4.8$ ; CMS FALSE POSITIVE:  $n_\sigma = 2.7$

REPEAT FIT WITH LARGE WEIGHT TO PUTATIVE INCONSISTENT DATASETS

- WEIGHTED FIT:
  - ATLAS INCONSISTENT TRUE POSITIVE:  $n_\sigma = 2.8$  REMAINS INCONSISTENT
  - CMS CONSISTENT FALSE POSITIVE:  $n_\sigma = 0.7$  BECOMES CONSISTENT



## CONCLUSIONS

- NNPDF4.0 PDF UNCERTAINTIES **SOMEWHAT OVERESTIMATED** ( $\sim 10\%$ )
- ML MODEL **CORRECTS FOR INCONSISTENCY** EXCEPT IN EXTREME CASES
- **EXTREME INCONSISTENCY** SIGNALLED BY LARGE  $\chi^2$ ,  
**DETECTABLE** BY WEIGHTED FIT