Accuracies of the Free Precession Phase Measurements

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Outline

Introduction to Free Precession Method

Basics of a Longitudinal Polarimeter

Monte Carlo simulation code for sawtooth and synchrotron motion

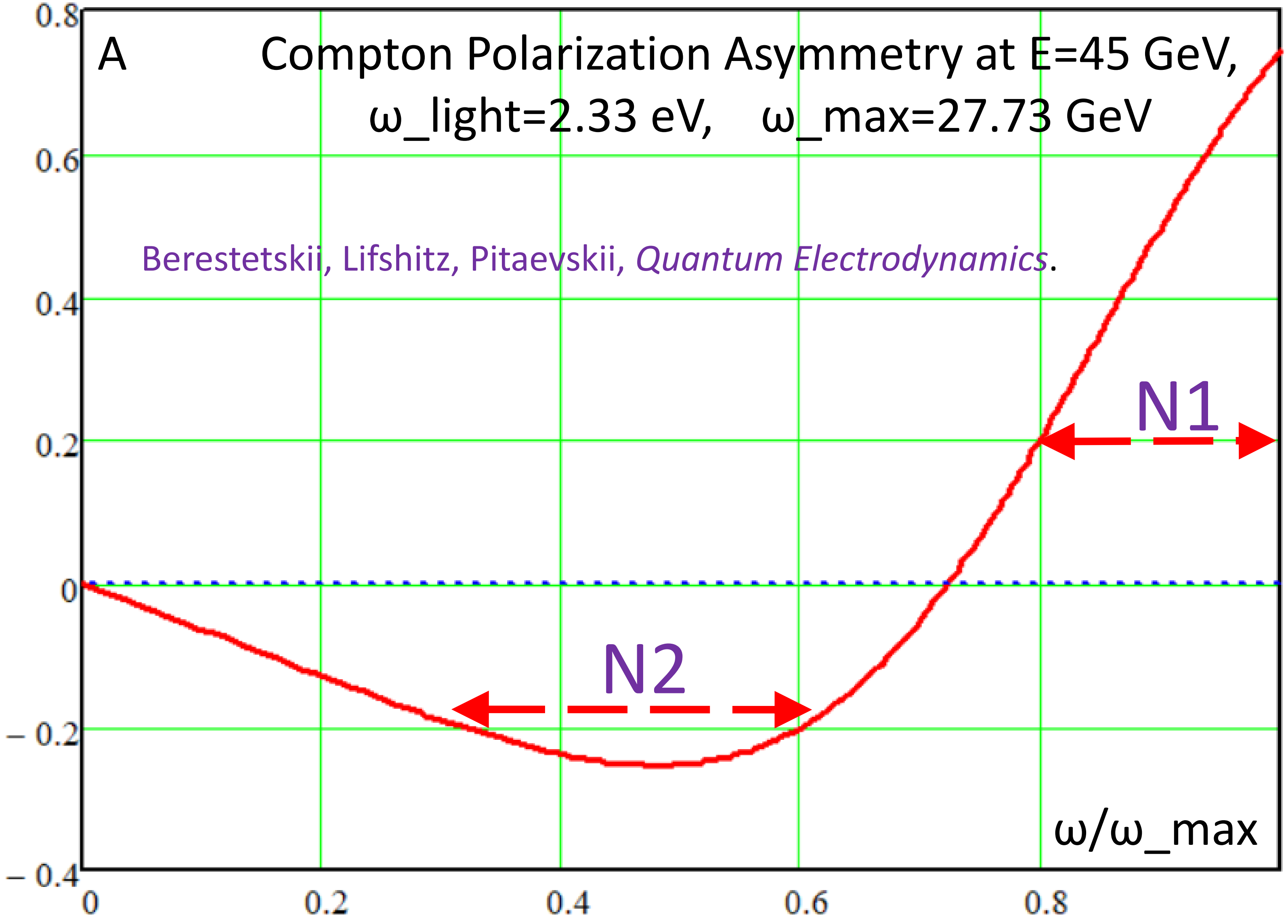
Statistical analysis of spin rotation tracking

Local orbit bump for a Flipper

Flipper harmonic strength evaluations at Z beam energy

Discussion of results

Compton polarimeter asymmetry to longitudinal polarization at Z



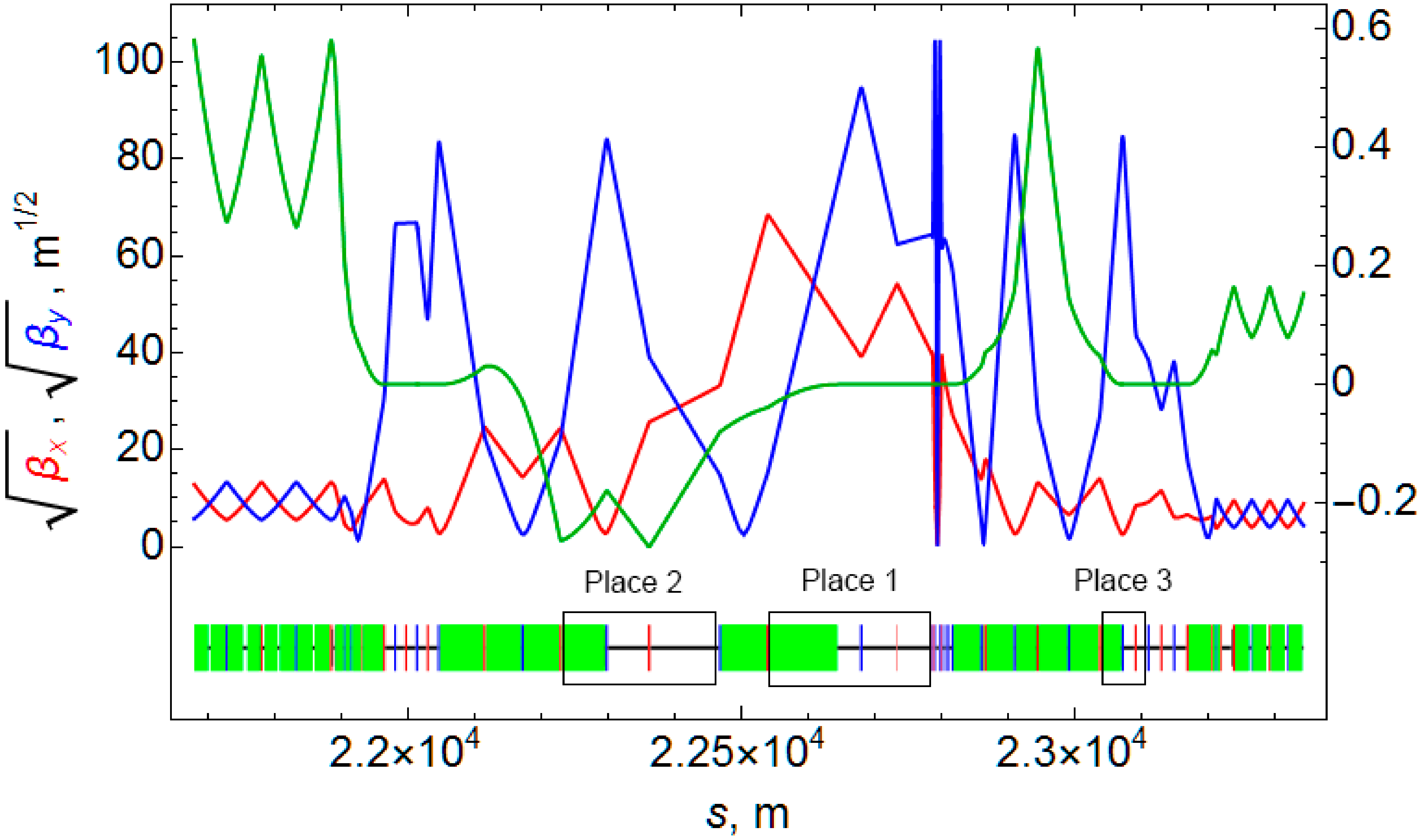
In case of coherent spin precession we can explore large asymmetry *A* to the longitudinal spin component of the ICS cross-section, selecting events from two regions: $\omega/\omega\_max > 0.8$ (N1) and $0.3 < \omega/\omega\_max < 0.6$ (N2). Then do FFT analysis of a signal: (N1-N2)/(N1+N2), modulated by spin precession.

Compton Polarimeter: Rates

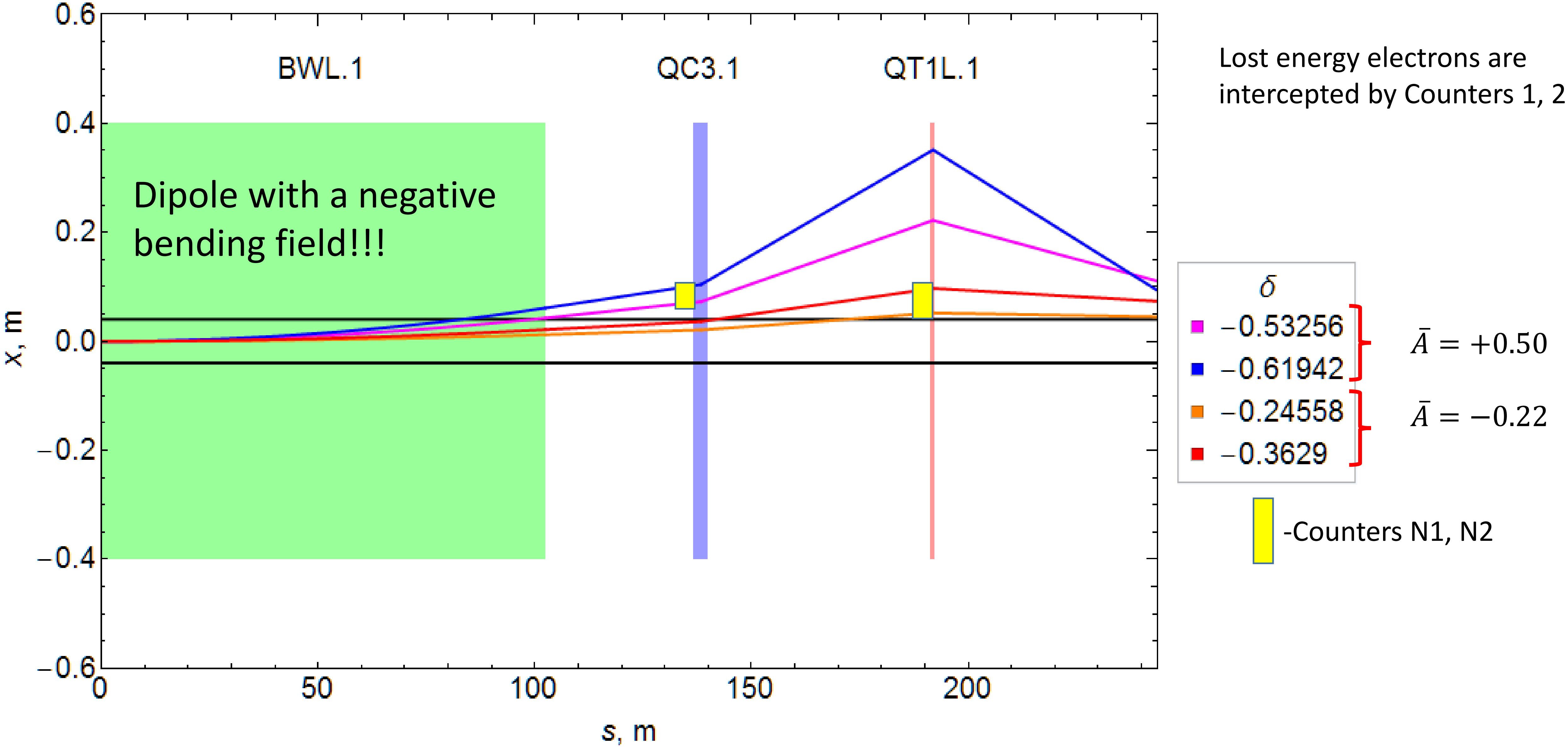
- Laser wavelength $\lambda = 532$ nm.
- Waist size $\sigma_0 = 0.250$ mm. Rayleigh length $z_R = 148$ cm.
- Far field divergence $\theta = 0.169$ mrad
- Interaction angle $\alpha = 1.000$ mrad
- Compton cross section correction 0.5
- Pulse energy: $E_L = 1$ [mJ]; τ_L=5 [ns] (sigma)
- Pulse power: $P_L = 80$ [kW]
- Ratio of angles $R_a = 5.905249$
- Ratio of lengths $R_l = 0.984208$
- $P_L/P_c = 1.1 \cdot 10^{-6}$
- "efficiency" $= 0.13$
- Scattering probability $W \simeq 7 \cdot 10^{-8}$

- With 10^{10} electrons and 3 kHz rep. rate: $\dot{N}_\gamma \simeq 2 \cdot 10^6$

Possible longitudinal polarimeter locations in FCC-ee

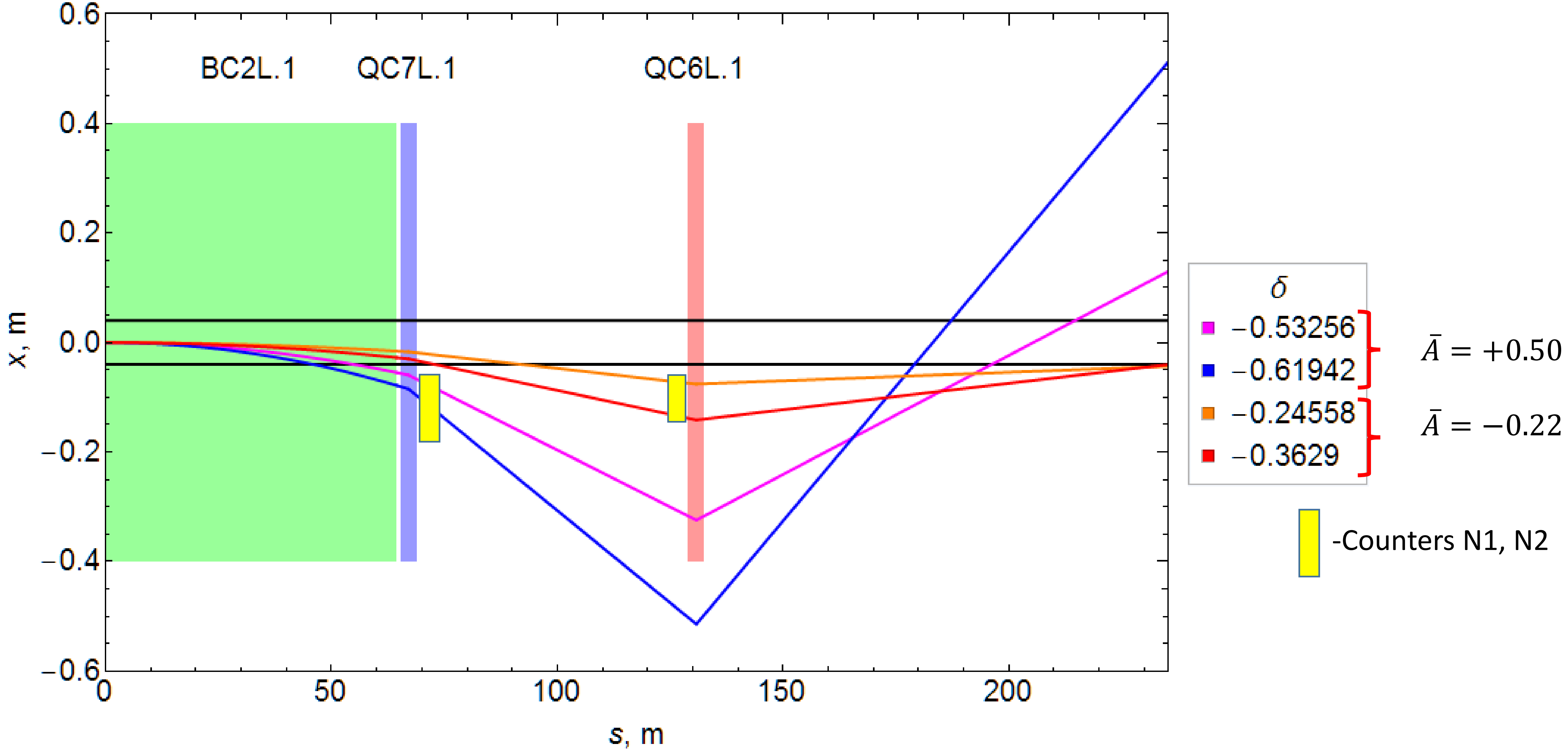


Trajectories with different energy losses at E=45.6 GeV, place1:

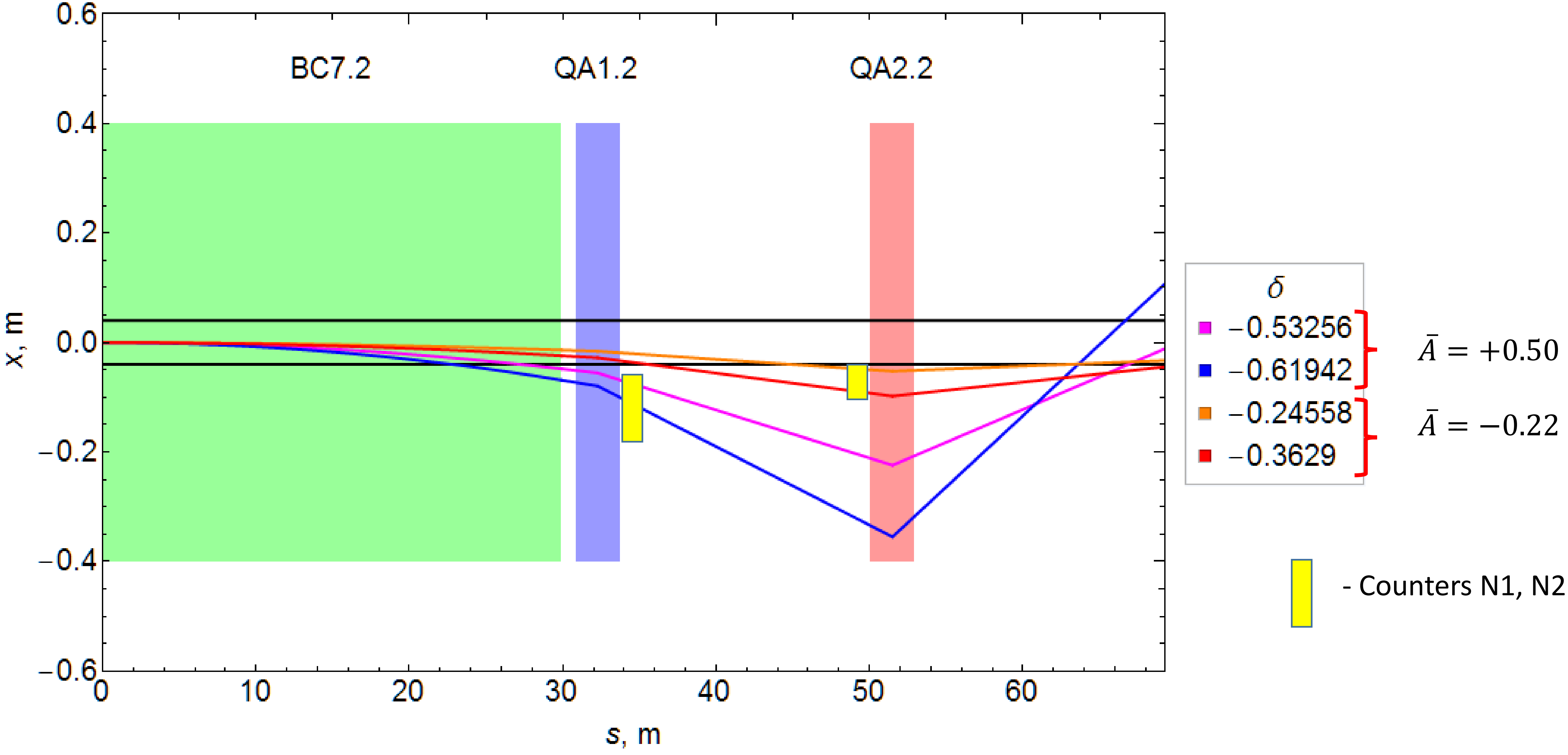


Lost energy electrons are intercepted by Counters 1, 2

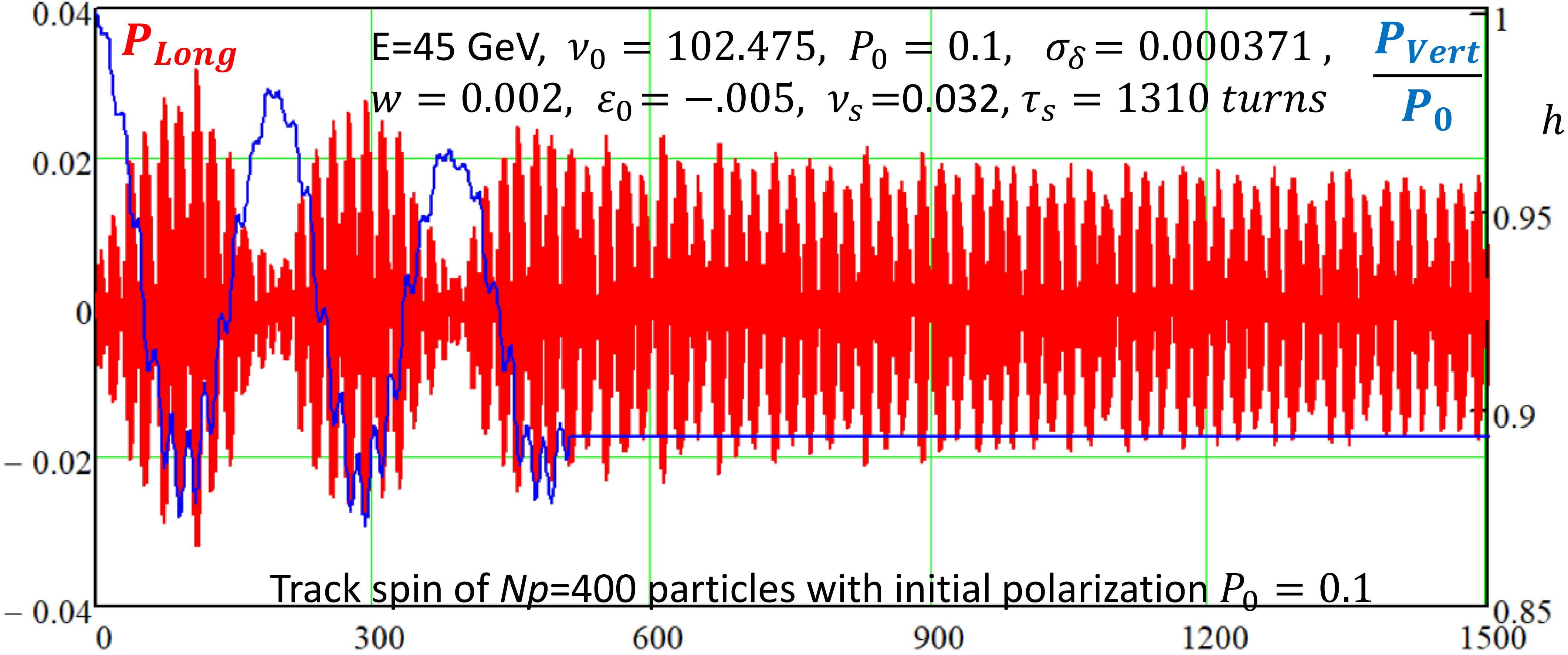
Trajectories with different energy losses at E=45.6 GeV, place 2:



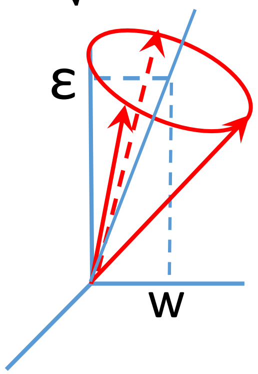
Trajectories with different energy losses at E=45.6 GeV, place 3:



Organizing of the coherent spin precession by Flipper (at Z)



$h = \sqrt{\varepsilon^2 + w^2}$



Spin precession in resonance frame

Coherent rotation of the total spin ensemble is done by powerfull Flipper device: w=0.002. Its frequency is shifted from the resonance by small detuning factor: $\varepsilon_0 = -.005$. Flipper is on 512 turns. After that we observe free spin precession during 2048 turns. Polarization loss is only 10%. In principle, Flipper kicks effectively spin only first 100 turns, or so!

Saw tooth model of a ring (quite simplistic)

A ring with 4 IP, spaced by 90^0.
All bending magnets have homogeneous field and parallel edges.
Synchrotron radiation power: $P_{SR} \sim H^2 E^2$.
Let's assume that the damping parameter $\lambda = const$:

$$\frac{dE}{dx} = -\lambda E^2 \quad \rightarrow \quad E(x) = \frac{E(0)}{1+\lambda E(0)x}$$

Here x – is a fraction of a turn, thus $\lambda = \frac{1}{E_{min}} - \frac{1}{E_{max}}$, $E_{max} - E_{min} = (\Delta E)_{turn}$.

Average energy of a particle: $\langle E \rangle = \frac{1}{x}\int_0^x \frac{E(0)dx}{1+\lambda E(0)x} = \frac{1}{\lambda x}\ln(1 + \lambda E(0)x) \approx E(0)\left[1 - \frac{\lambda E(0)x}{2} + \frac{(\lambda E(0)x)^2}{3} - \cdots\right]$

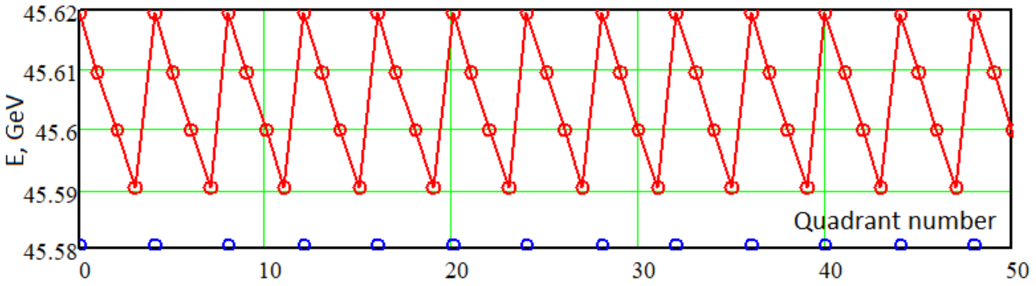
I took E_{max} =45.62 GeV, $E_{min} = 45.581$ GeV, $(E_{max}+E_{min})/2 = 45.6$ GeV and $E_{max} - E_{min} = 0.039$ GeV.

I placed an RF cavity at the beginning of the ring, with $V_{RF} = 0.079$ GV and harmonic number $h = 121200$.
Momentum compaction was defined as $\alpha_C = 29.2 \cdot 10^{-6}$.
The RF phase shift accumulates according to the energy difference with the reference particle. For simplicity, the particle energy decays smoothly along the quadrant and makes quantum jumps only at their ends.

In my tracking code I got the true value of the synchrotron tune $\nu_s = 0.029$, see next slide.

The synchrotron motion

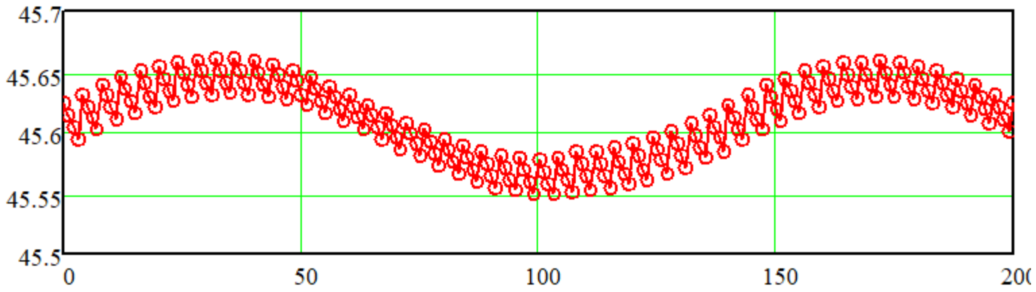
Saw tooth of a reference particle



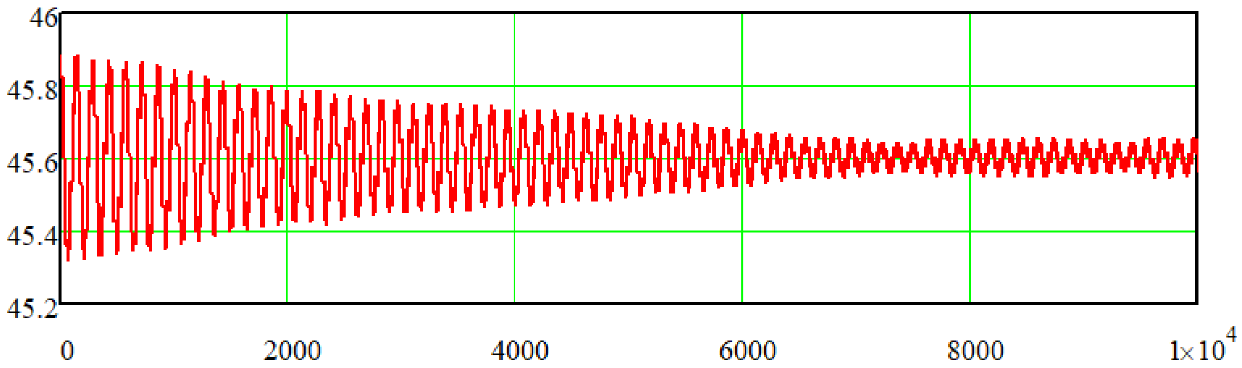
Quantum fluctuations occur at the beginning of each quadrant. Their relative sigma value is:

$\Delta = \sigma_\delta / \sqrt{\tau_s}$ with $\sigma_\delta = 3.9 \cdot 10^{-4}$ and $\tau_s = 1169$ turns.

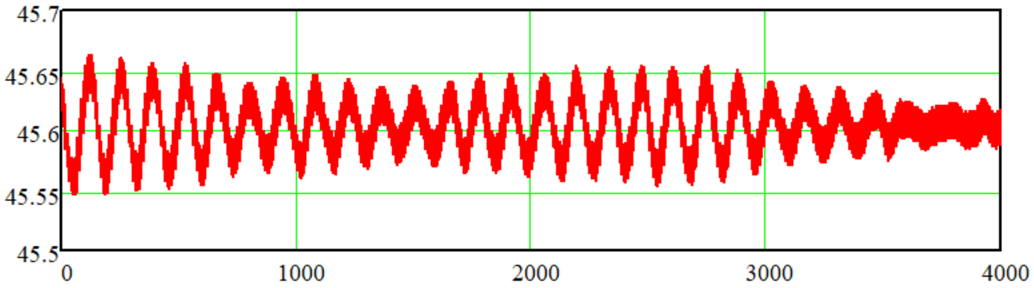
Saw tooth of nonequilibrium particle



Damping of a large fluctuation



Saw tooth with quantum fluctuations



Large oscillations do not decay exponentially, but make continuous small oscillations, finally approaching equilibrium.

Spin tracking approach

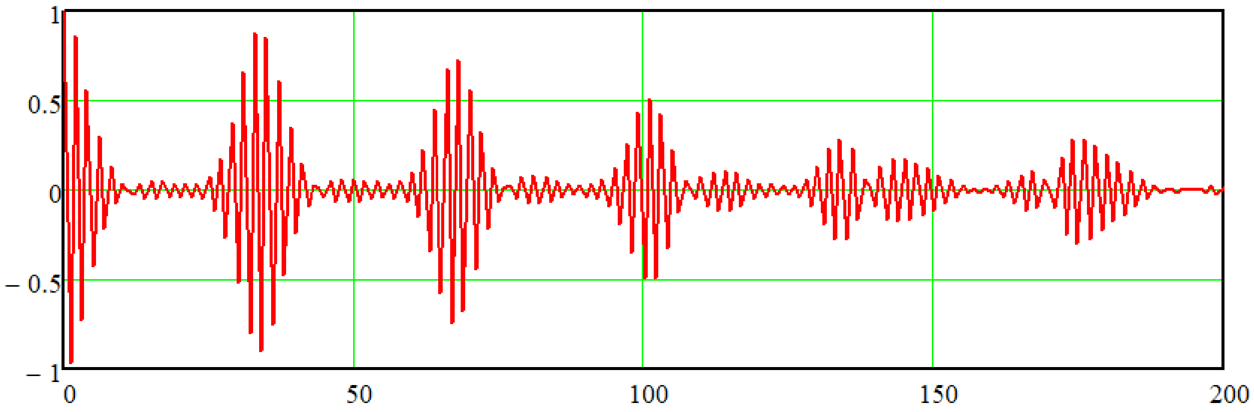
I tracked the synchrotron motion of an ensemble of 1000 particles subjected to quantum fluctuations and damping.
I also tracked their spins under the assumption that all they initially are oriented along the longitudinal axis.

The spin rotates around the vertical axis in accordance with the energy integral for each quadrant. This integral is determined by the energy of the particle at the entrance to the quadrant (which corresponds to a quarter turn):

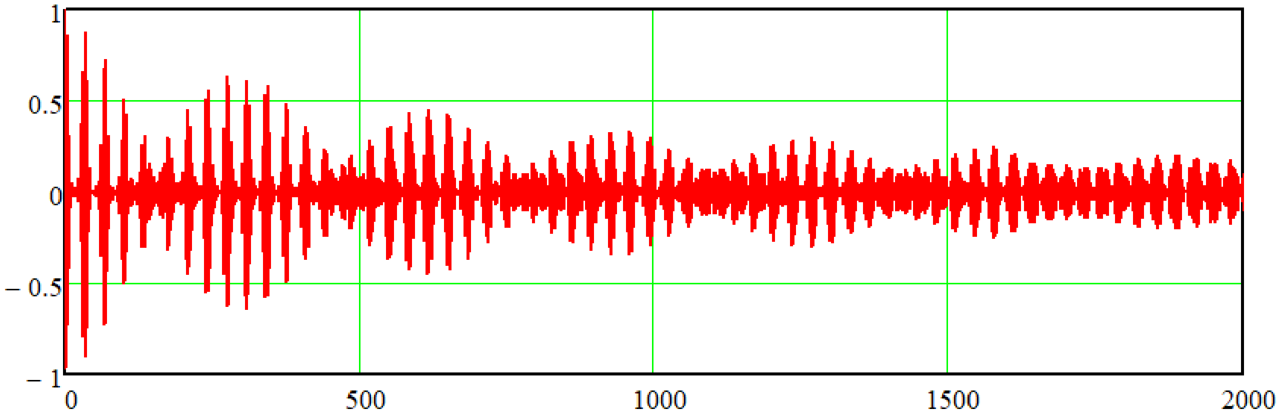
$$\langle E \rangle = \frac{1}{0.25}\int_0^{0.25} \frac{E(0)dx}{1+\lambda E(0)x} = \frac{1}{\lambda \cdot 0.25}\ln(1+\lambda E(0)\cdot 0.25)$$

As a result of quantum fluctuations of SR, the spin precession phases are blurred relative to each other in a certain interval, which grows with time as the square root of the number of turns. So, decoherence of the spin ensemble occurs. But for the first few hundred turns we observe strong beating of the longitudinal spin component which occurs with the synchrotron tune (the synchrotron period is 34.5 turns).

Spin precession of the spin ensemble during first 200 turns



Spin precession of the spin ensemble during 2000 turns



Longitudinal Compton polarimeter data

A longitudinal Compton polarimeter counts lost electrons scattered due to interaction with the circularly polarized laser light. The effective cross-section asymmetry to the difference in lost particle's momentum is relatively large and at Z is equal to approximately $A = 0.5$.

By performing a Monte Carlo simulation, I generate a Poisson distribution, assuming that the number of events is :

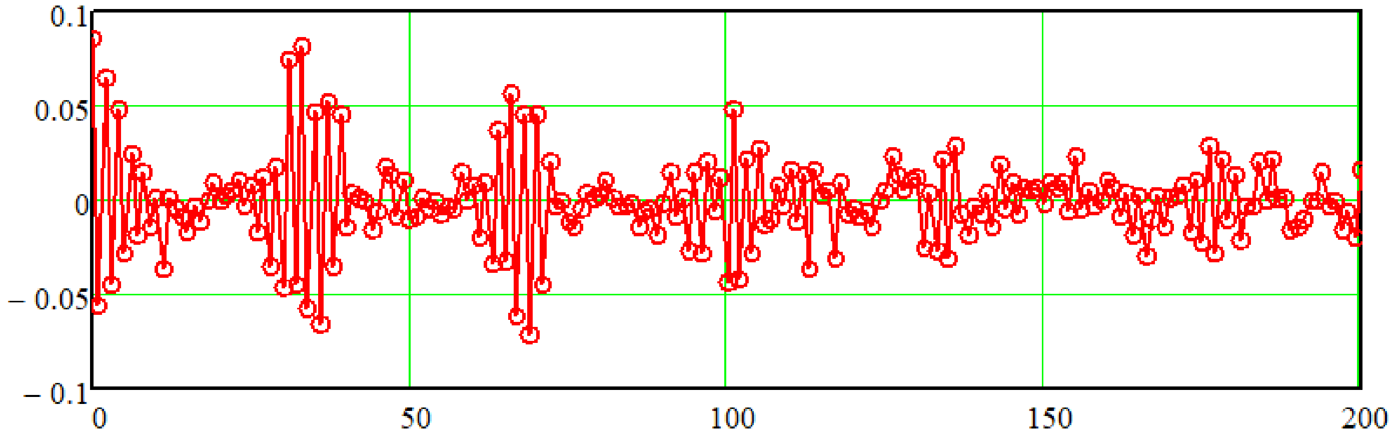
$$N = \langle N \rangle \left(1 + A \cdot P \cdot S_y\right)$$

where $\langle N \rangle$ is the average number of the scattered electrons from a single bunch per turn (expected 2000 to 20000), P is the initial polarization degree (expected 0.05 to 0.15) and S_y is the longitudinal spin component of a bunch, assuming fully polarized beam in the beginning of free spin precession.

I consider the beat of the counts as a polarimeter signal: $U_n = (N_n - \langle N \rangle)/\langle N \rangle$

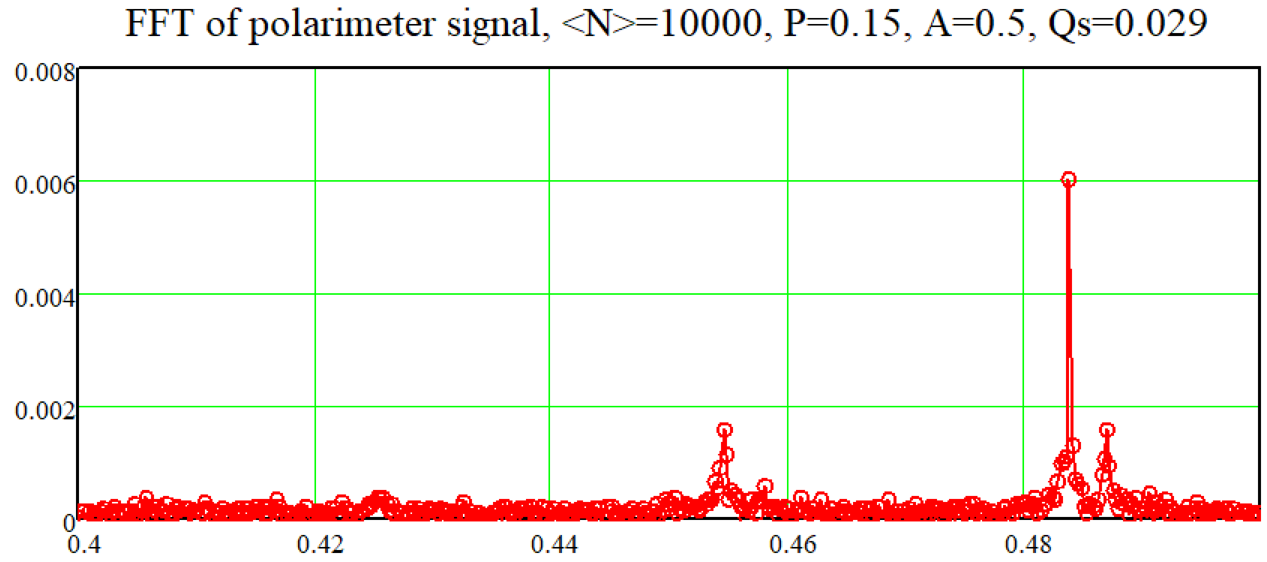
Statistical nature of such a signal lead to a noise in extracting of the longitudinal spin component from the polarimeter data. An example of a single run during 200 turns is presented below:

Compton polarimeter signal for <N>=10000, A=0.5, P=0.15



A more or less clear reproduction of spin precession can be seen during the first 3-5 synchrotron periods, but then the data peaks are blurred due to spin decoherence..

Fourier spectrum of the polarimeter data



The main peak and two side frequencies (one mirror-symmetrical) are clearly visible.

The argument of the largest peak can be thought of as the zeroth approximation of the precessional phase, and its frequency can be thought of as the spin tune.

But you can try to improve the accuracy of phase determination by calculating the correlation of the polarimeter signal with a sinusoid of a given frequency.

$F(\varphi) = \sum_{n=1}^{nmax} U_n \sin(2\pi\nu n + \varphi)$ then solving $F'(\varphi_0)=0 \rightarrow \varphi_0$

If such a correlation analysis of the data from all 4 polarimeters is carried out, assuming a common spin tune, then the errors in determining the phase differences between the 4 quadrants will become minimal.

I performed such an analysis to find the precession phases in all 4 quadrants and determine the statistical precisions and their dependencies on input parameters such as the level of polarization or the number of scattered electrons.

Fitting of phase determination errors

First, I studied the dependence of phase determination accuracy on the length of the data sample. I found that for given beam parameters at Z (synchrotron tune $\mathrm{Qs} = 0.029$, energy spread $\sigma_\delta = 0.00039$, damping time $\tau = 1169$ turns) the optimal sample length is about 4000 turns and this dependence is not sharp. Let's look at some results with 4096 turns.

For a set of input parameters $P = 0.2$, $\langle Nc \rangle = 2000$, $A = 0.5$ I got the following sigmas (with accuracy about 10%):
$\sigma_\varphi = 0.121$ - for phase determination, $\sigma_{\Delta\varphi} = 0.046$ - for phase difference of two quadrants.

We see that the phase difference is determined approximately 3 times better than their individual values!

Now another three sets: $P = 0.0625, \langle Nc \rangle = 2000, A = 0.5 \;\; \rightarrow \;\; \sigma_\varphi = 0.381, \; \sigma_{\Delta\varphi} = 0.163$

$$P = 0.0625, \langle Nc \rangle = 4000, A = 0.5 \;\; \rightarrow \;\; \sigma_\varphi = 0.285, \; \sigma_{\Delta\varphi} = 0.092$$
$$P = 0.0625, \langle Nc \rangle = 10000, A = 0.5 \;\; \rightarrow \;\; \sigma_\varphi = 0.124, \; \sigma_{\Delta\varphi} = 0.069$$

It looks like: $\sigma_{\Delta\varphi} \sim \frac{P}{\sqrt{\langle Nc \rangle}}$ - more or less natural scaling!

General scenario for saw tooth studies

In my opinion, when FCCee launches, we should spend a few weeks or months investigating the sawtooth problem. First, of course, it is necessary to globally tune the machine and maximize the achievable level of polarization.

In special sawtooth runs, we can store up to 11200 non-colliding bunches, polarize them with wigglers for several hours to a high degree (up to 40%), and then rotate spins with a flipper, bunch by bunch, and measure the free spin precession over 4000 revolutions for each bunch. The complete procedure will take several hours, since one measurement takes 1 s.

Let's assume that for single bunch the attainable accuracy of the phase differences determination is about $\sigma_{\Delta\varphi} = 0.06$ radians. Then, after completing the full series, we will receive a 100-fold improvement in the statistical error of phase measurement. Thus, in one beam store the achievable accuracy can reach:

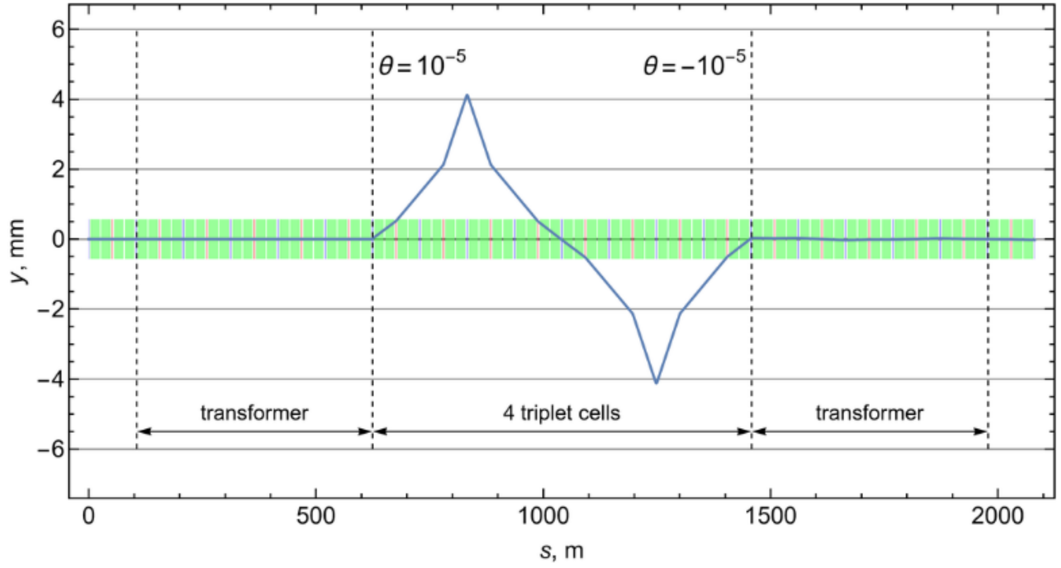
$$\sigma_{\Delta\varphi} = 0.0006 \quad \rightarrow \quad \Delta E = 40 \text{ keV}$$

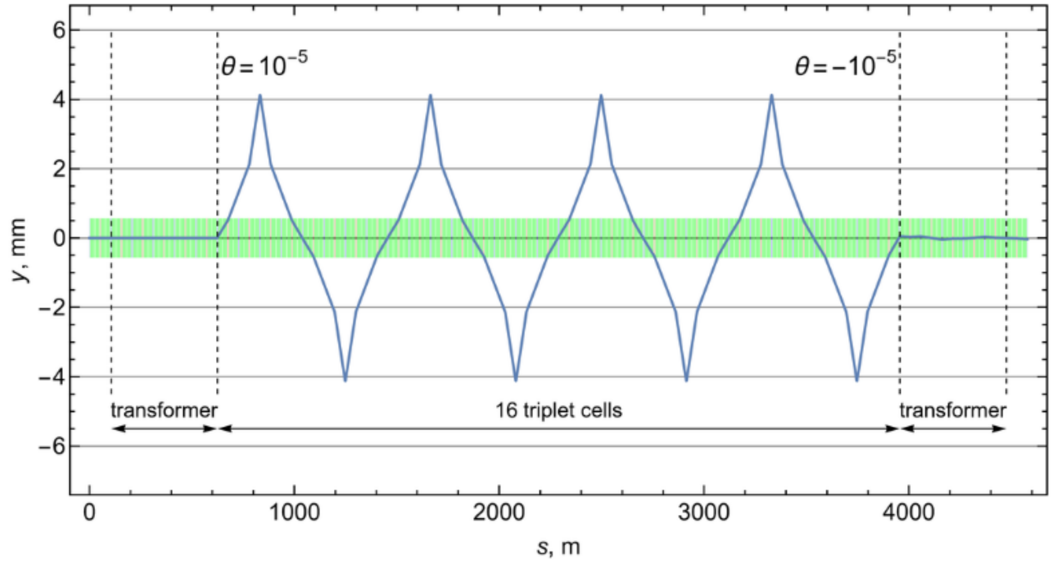
So, about 100 series of such measurements will be enough to limit the accuracy of determining the energy difference between the 4 quadrants to the level of 4 keV. Of course, this is a very optimistic scenario - in reality, many difficulties and surprises await us.

Finally, I would like to remind you that after spin rotation, the bunches are not completely depolarized, they lose only a few percent of the degree of polarization and can continue to be used again after using wigglers for few hours to restore the initial polarization level.

Spin rotation by a flipper

It is generally accepted that the depolarizer or flipper must swing a beam locally consisting of two blocks that interfere in such a way that only a local orbital bump appears. My previous reports on this subject showed that at 45.6 GeV only a half-wave or single wave bump with a regular FODO arc lattice is effective. Flipper blocks separated by longer orbit bumps interfere destructively. To solve this problem, it is necessary to increase the number of regular bending magnets per betatron wave by factor 2. It seems we have found a suitable lattice solution, presented below.

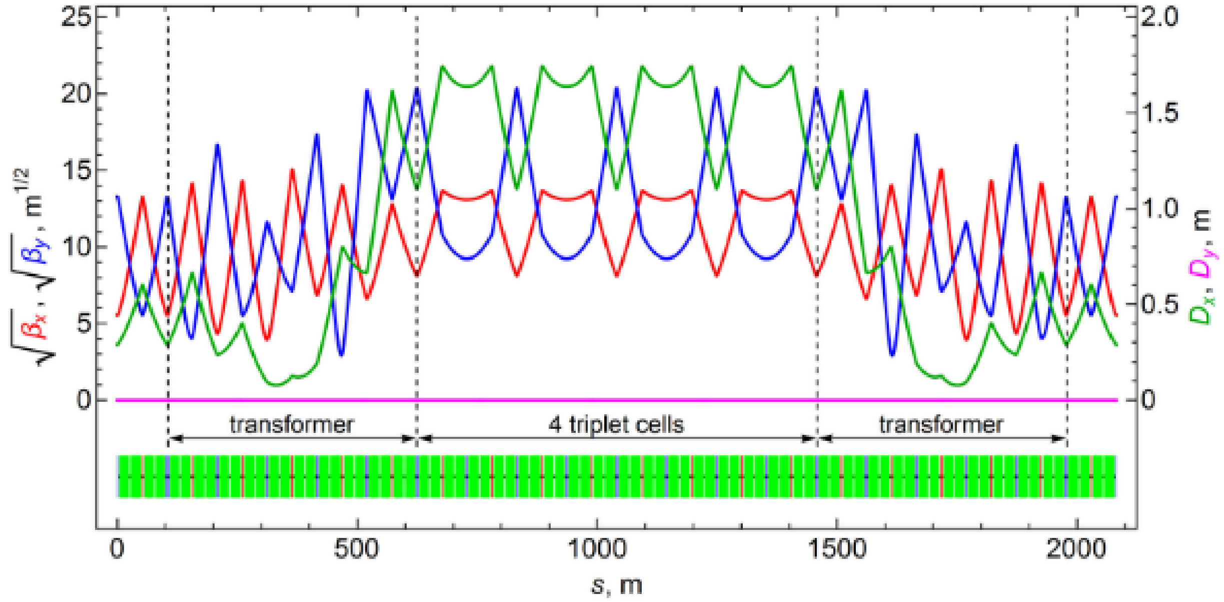


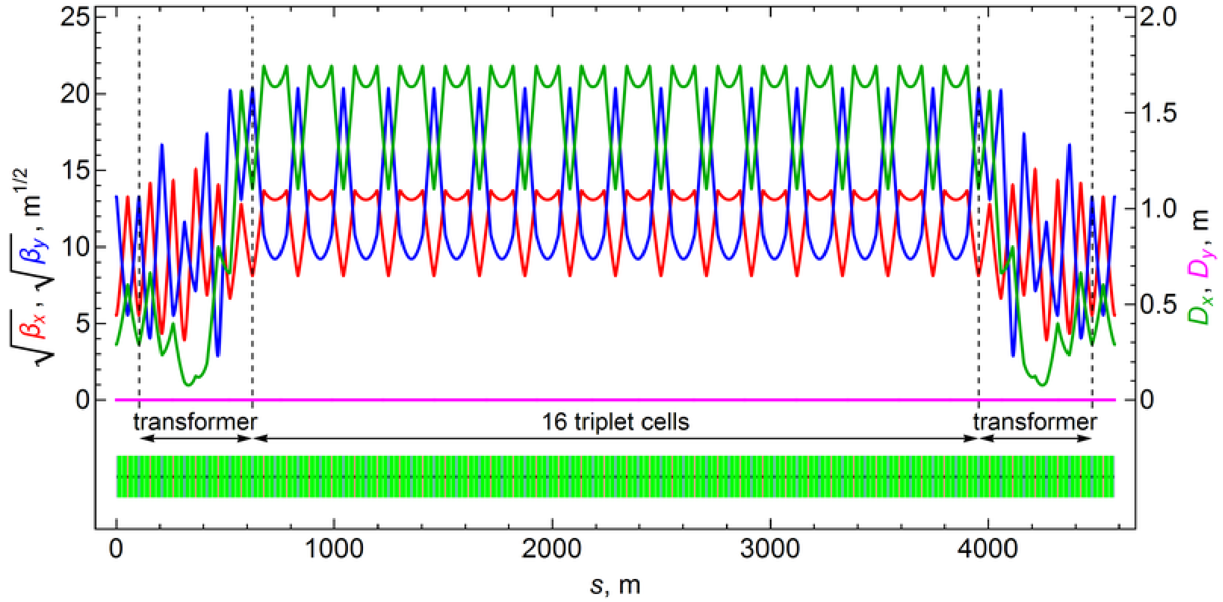


The complete insert consists of 3 sections: a multiple wave local bump and 2 transformers, which match the internal triplet cell structure of the bump to a regular FODO arc lattice. Only a half of regular quads will be powered along a local bump section. This doubles the wavelength. Additionally, instead of FODO, triplets provide better performance of the focusing structure. Details are presented at the next slides.

Local bump optics (MADx files prepared by Alexei Otboev)

The phase shift of each transformer is 0.5. Thus, the total phase advance of the insert is an integer.



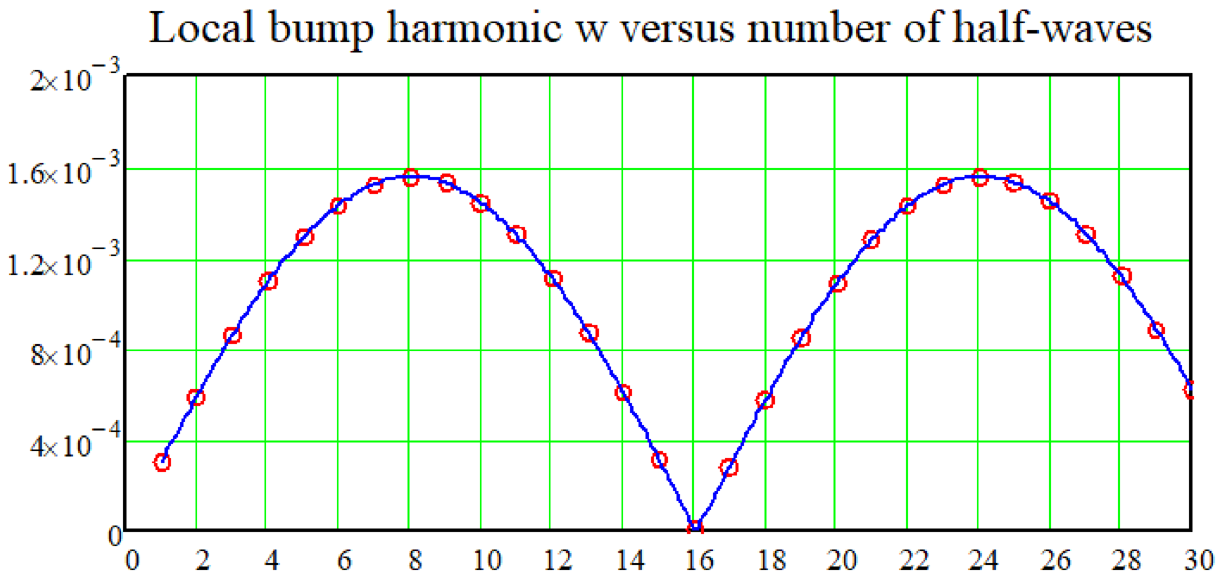


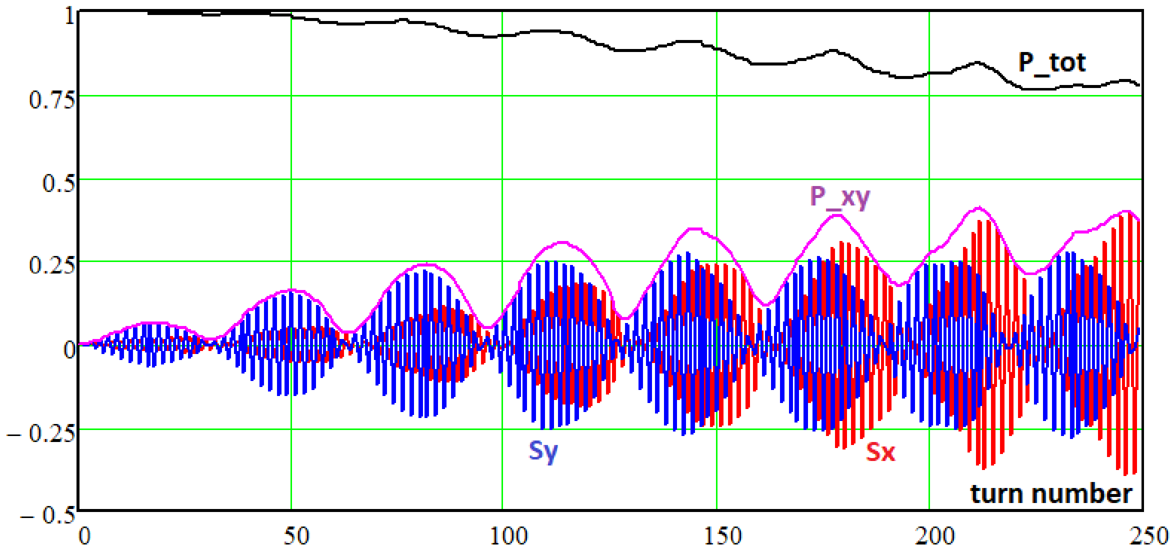
The dispersion increases approximately 2 times. This leads to a slight increase in the momentum compaction ratio of the ring. From my point of view, this is a positive development, since increasing the synchrotron frequency would be very beneficial for polarization in many aspects. As has been repeatedly noted, the synchrotron tune value $Q_s = 0.029$ currently accepted corresponds to an excessively high value $\xi = \nu_0\sigma_\delta/Q_s = 1.4$ of the synchrotron modulation index and any reduction of the momentum compaction is welcome.

In my opinion, chromaticity correction of the insert should not be a problem, since the dispersion function is on average higher than that of a conventional arc and the sextupoles generally becomes weaker. But this requires special study.

Flipper strength evaluation

Each of the two flipper blocks swings the beam vertically with an angular amplitude of $\vartheta = 1 \cdot 10^{-5}$.
The triplet cell structure could be described as: D BB FF BB BB FF BB D, where D is a defocusing half-quad, F is a focusing half-quad, B is a bend in the horizontal plane by 2.134 mr. A half-wave bump consists of two cells, i.e. from 16 regular bends. The interference of two half-wave humps becomes constructive, since the spin phase advance per a half-wave bump is 3.533 radians and the second half of the one wave hump is mirror symmetrical to the first. As a result, the optimal length of the optical structure between two flipper blocks is 8 half-wavelengths.
The effective force of such a local flipper will be $w = 1.6 \cdot 10^{-3}$ and after the particle passes through the entire local bump insert, its spin will rotate through an angle of $\phi = 4\pi w = 20$ mrad. This value of the flipper harmonic is large enough to effectively deflect the entire ensemble of spins from the stable vertical direction, let's say to about $S_{tr} = 0.4$ per 200 turns, see the right plot. The flipper here was turned on exactly at the resonant frequency.





Discussion of results

The free precession method allows one to obtain not only the spin tune value, but also the relative phase differences between the polarimeters installed on the ring.

The accuracy of the phases obtained as a result of measuring one bunch can be on the order of 0.06 radians, or 4 MeV of the difference in the energy integrals of the two quadrants. In a series of experiments specifically devoted to the study of the sawtooth shape, we can compress the uncertainties of the phase differences to the level of 4 keV or even better.

By combining the analysis of data obtained from 4 longitudinal polarimeters and from one special polarimeter-energy spectrometer, it is possible to obtain a real picture of how energy is distributed in 4 IPs.