# Nuclear wave functions for HIC: *ab initio* PGCM

Benjamin Bally

CERN - 13/11/2024

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- Nuclear deformation impacts initial conditions and final state observables
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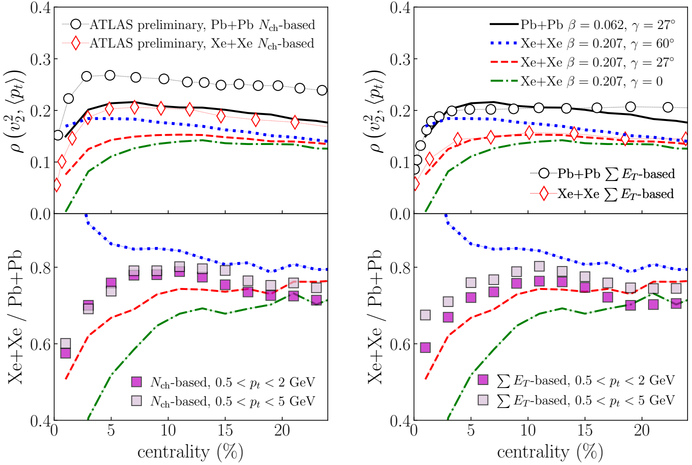
- Use high-energy experiments to gain knowledge about atomic nuclei properties

- Projected Generator Coordinate Method (PGCM) in this context

# Previous works on heavy-ion collisions

- PGCM calculations with a phenomenological interaction: $^{129}$Xe, $^{197}$Au, $^{208}$Pb




Bally, PRL 128, 082301 (2022)

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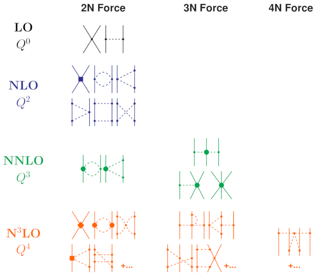
- Construct nuclear Hamiltonian through Effective Field Theory (EFT)
  Hammer, RMP 92, 025004 (2020)
  - Consistent with symmetries of QCD
  - Power counting $\left(\frac{Q}{\Lambda}\right)^n$
  - Different versions: chiral, pionless, deltafull



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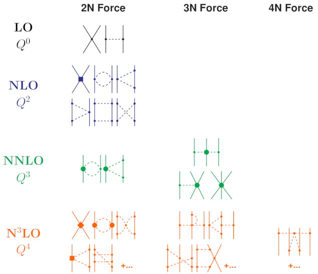
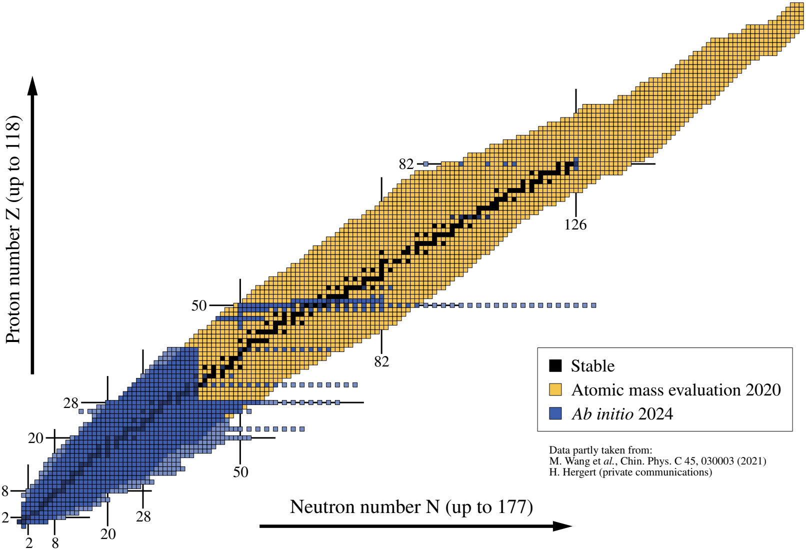



- Solve the many-body Schrödinger equation in a controlled manner to a target accuracy

$$H|\Psi\rangle = E|\Psi\rangle$$

# Reach of *ab initio* methods




Data partly taken from:
M. Wang et *al.*, Chin. Phys. C 45, 030003 (2021)
H. Hergert (private communications)

# Projected Generator Coordinate Method (PGCM)

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  - Low-energy spectroscopy (excitation energies, electromagnetic transitions)
  - Nuclear matrix elements for the neutrinoless double-beta decay

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- Traditionally, employed with phenomenological nuclear interactions

- Recently, use of chiral interaction obtained from EFT

# Symmetry-breaking mean-field reference states

- Minimization of the energy

$$\delta\left(\langle\Phi|H|\Phi\rangle\right) = 0$$

- $|\Phi\rangle \equiv$ product state (Slater determinant or Bogoliubov quasi-particle state)

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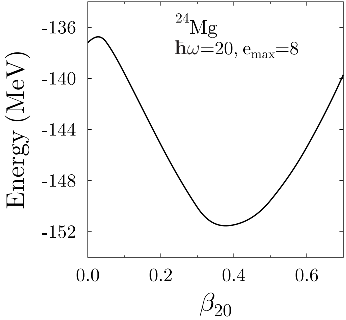

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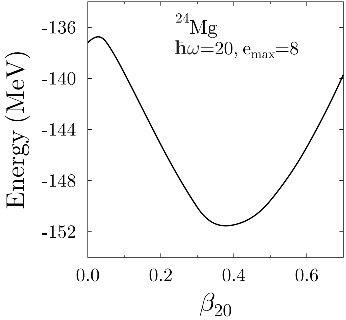


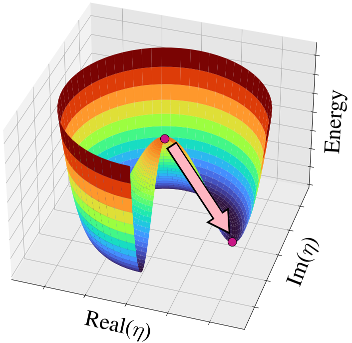

# Symmetry-breaking mean-field reference states

- Explore the energy surface performing constrained minimization

$$\delta\left(\langle\Phi(q)|H - \lambda Q|\Phi(q)\rangle\right) = 0 \quad \text{with} \quad \langle\Phi(q)|Q|\Phi(q)\rangle = q$$

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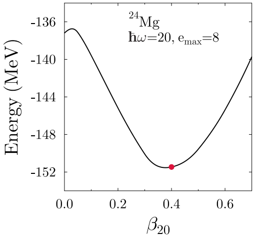

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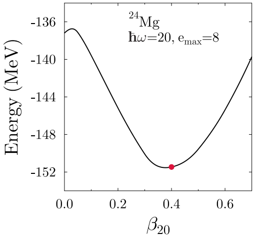


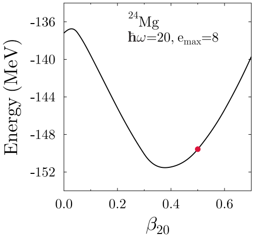

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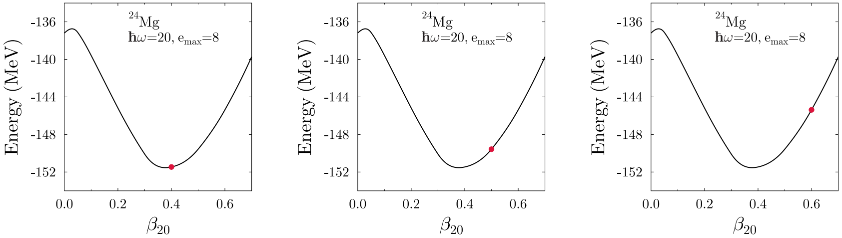
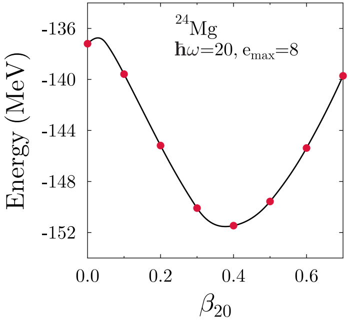
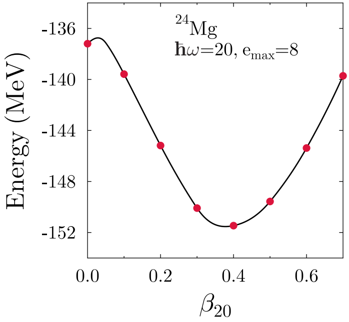


# Linear superposition of symmetry-projected states



- We consider the more general wave functions

$$|\Psi^{\sigma M}_{\epsilon}\rangle = \sum_{qK} f^{\sigma M}_{\epsilon,qK} P^{\sigma}_{MK} |\Phi(q)\rangle \quad \text{where} \quad \sigma \equiv Z, N, J, \pi$$



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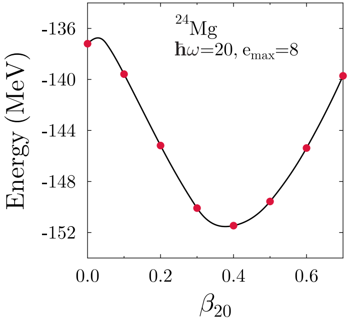

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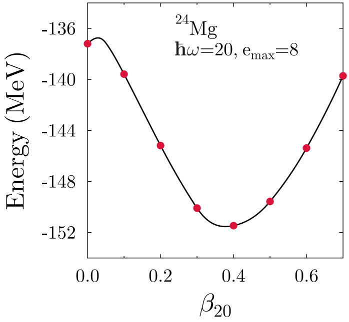


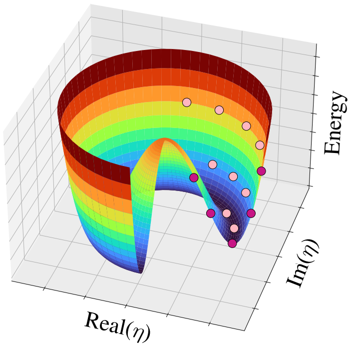

# Recent developments

- The method is exact for the deuteron

  Bally, arXiv.2410.03356 (2024)

- Relative error of 1-2% for $^{3}$H and $^{3,4}$He (preliminary)

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- New methods:
  - PGCM + Perturbation Theory (PT)
    Frosini, EPJA 58, 62-63-64 (2022)
  - PGCM + In-Medium Similarity Renormalization Group (IMSRG)
    Zhou, arXiv:2410.23113 (2024)

# Sampling of nucleons

- **Here:** deformed one-body density at the average deformation $\bar{q}$ of the PGCM ground state

$$\rho^{(1)}_{\bar{q}}(r_1) = \frac{\langle \Phi(\bar{q}) | a^{+}_{r_1} a_{r_1} P^Z P^N | \Phi(\bar{q}) \rangle}{\langle \Phi(\bar{q}) | P^Z P^N | \Phi(\bar{q}) \rangle}$$

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- **Ultimate goal:** $A$-body correlated density (as in NLEFT)

$$\rho^{(A)}(r_1, \dots, r_A) = \frac{\langle \Psi^{\sigma M}_{\epsilon} | a^+_{r_1} \dots a^+_{r_A} a_{r_A} \dots a_{r_1} | \Psi^{\sigma M}_{\epsilon} \rangle}{\langle \Psi^{\sigma M}_{\epsilon} | \Psi^{\sigma M}_{\epsilon} \rangle}$$

# Preprints: arXiv:2402.05995 and arXiv:2405.20210

CERN-TH-2024-021

**The unexpected uses of a bowling pin: exploiting $^{20}$Ne isotopes for precision characterizations of collectivity in small systems**

Giuliano Giacalone,[1,*] Benjamin Bally,[2] Govert Nijs,[3] Shihang Shen,[4] Thomas Duguet,[5,6] Jean-Paul Ebran,[7,8] Serdar Elhatisari,[9,10] Mikael Frosini,[11] Timo A. Lähde,[12,13] Dean Lee,[14] Bing-Nan Lu,[15] Yuan-Zhuo Ma,[14] Ulf-G. Meißner,[10,16,17] Jacquelyn Noronha-Hostler,[18] Christopher Plumberg,[19] Tomás R. Rodríguez,[20] Robert Roth,[21,22] Wilke van der Schee,[3,23,24] and Vittorio Somà[5]

CERN-TH-2024-074

**Anisotropic flow in fixed-target $^{208}$Pb+$^{20}$Ne collisions as a probe of quark-gluon plasma**

Giuliano Giacalone,[1,*] Wenbin Zhao,[2,3,†] Benjamin Bally,[4] Shihang Shen,[5] Thomas Duguet,[6,7] Jean-Paul Ebran,[8,9] Serdar Elhatisari,[10] Mikael Frosini,[11] Timo A. Lähde,[12,13] Dean Lee,[14] Bing-Nan Lu,[15] Yuan-Zhuo Ma,[14] Ulf-G. Meißner,[16,17,5] Govert Nijs,[18] Jacquelyn Noronha-Hostler,[19] Christopher Plumberg,[20] Tomás R. Rodríguez,[21] Robert Roth,[22,23] Wilke van der Schee,[18,24,25] Björn Schenke,[26,‡] Chun Shen,[27,28,§] and Vittorio Somà[6]

- Collaboration between low- and high-energy nuclear physics communities
  - Heavy-ion collisions
  - Nuclear structure (PGCM)
  - Nuclear structure (NLEFT)

# Computational aspects

- Chiral Hamiltonian: Hüther N3LO

  Hüther *et al.*, PLB 808, 135651 (2019)

- Collective coordinates *q*: $\beta_{20}, \beta_{22}, \beta_{30}, \beta_{32}$

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- Numerical suite TAURUS
  Bally, EPJA 57, 69 (2021); Bally, EPJA 60, 62 (2024)
  Repository: `https://github.com/project-taurus`

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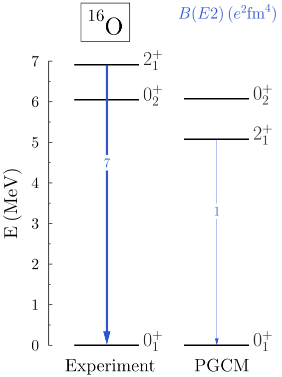
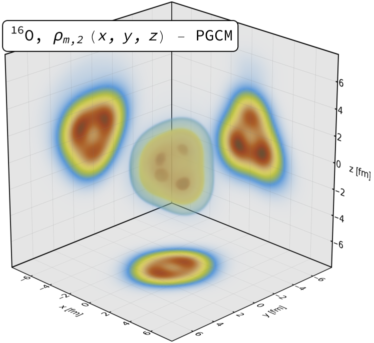
- Topaze supercomputer (CEA/CCRT)

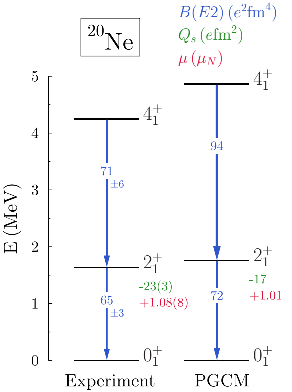
# $^{16}$O: spectroscopy and deformed one-body density

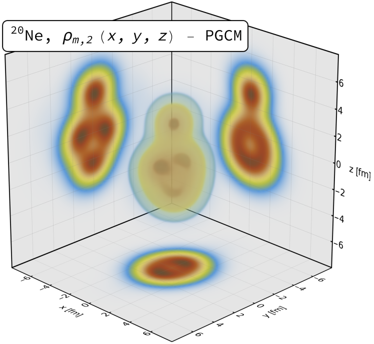



- Relative agreement with experimental data
- Density ~ tetrahedron of four $\alpha$-like clusters

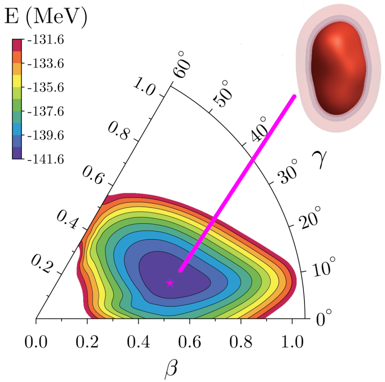
# $^{20}$Ne: spectroscopy and deformed one-body density

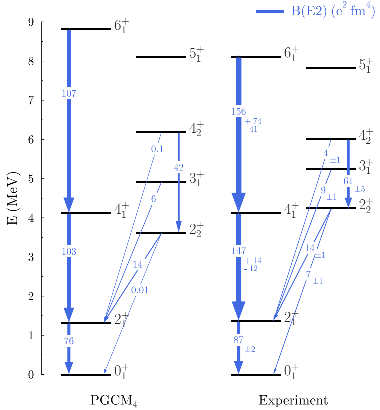



- Good agreement with experimental data
- Spectroscopic moment $Q_s = \langle er^2 Y_{20} \rangle$
- Density $\sim {}^{16}\mathrm{O} + \alpha$

# Other example: $^{24}$Mg




Bally, EPJA 60, 62 (2024)

- Ground state exhibits large intrinsic triaxial deformation
- Excellent description using $\chi$EFT Hamiltonian

# Summary and outlook

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- Can be used to better model the initial conditions of high-energy experiments
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- **Predictions for possible $^{20}$Ne + $^{20}$Ne and $^{20}$Ne + $^{208}$Pb runs at LHC**
- In the future: sampling based on the correlated densities
  - one-body + two-body densities
  - $A$-body density