The road less travelled: NNLL for event shapes with PanScales

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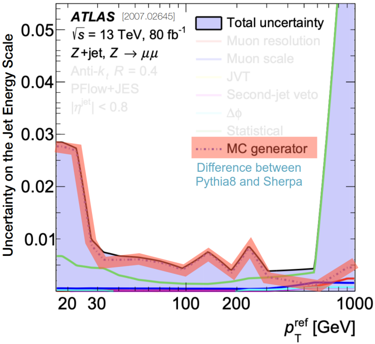
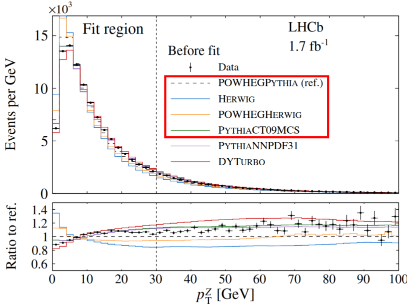
Shower uncertainties

- Differences between parton showers / MC event generators
  ⇝ large systematic uncertainties in e.g. jet calibration
- These ultimately propagate to most analyses

[ATLAS, 2007.02645]

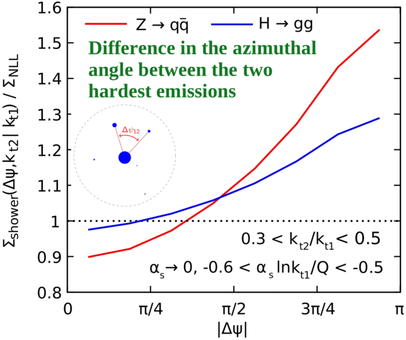


[LHCb, 2109.01113]

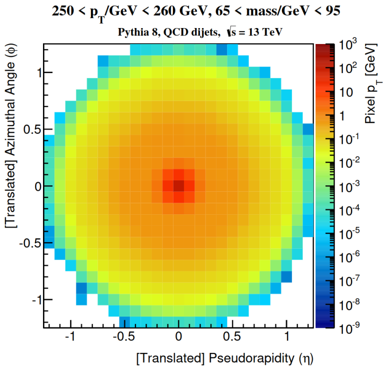


Shower uncertainties

- Theory prediction (= shower) used as input for a large set of machine-learning tools
- ML will learn unphysical features



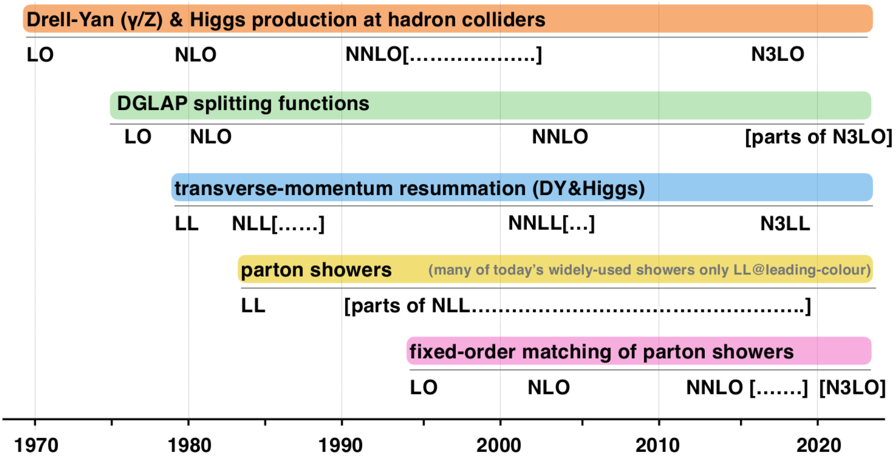
[courtesy F. Dreyer]



[Nachman et al. 1511.05190]

slide from G. Salam

selected collider-QCD accuracy milestones



slide from G. Salam

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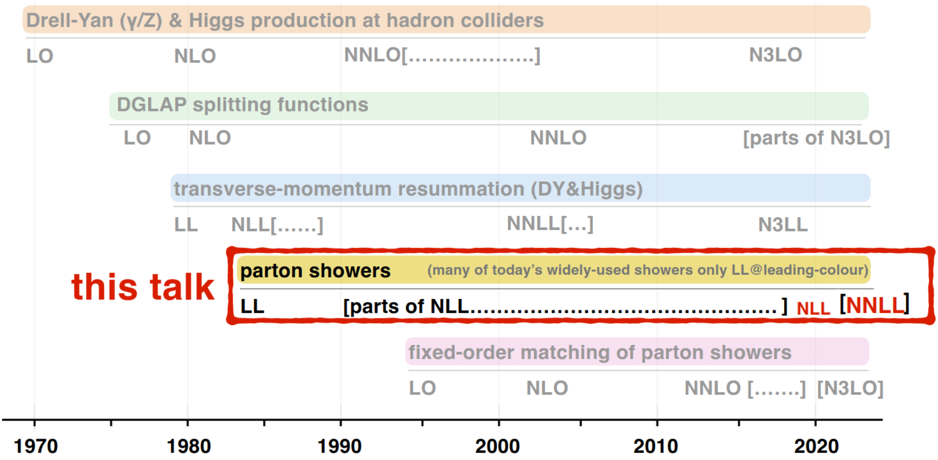
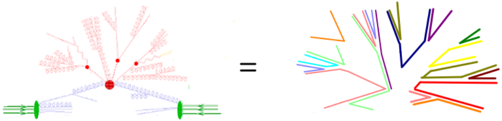
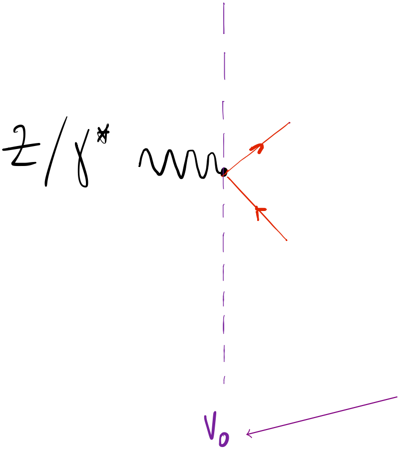


NNLL = next-to-next-to-leading logarithms

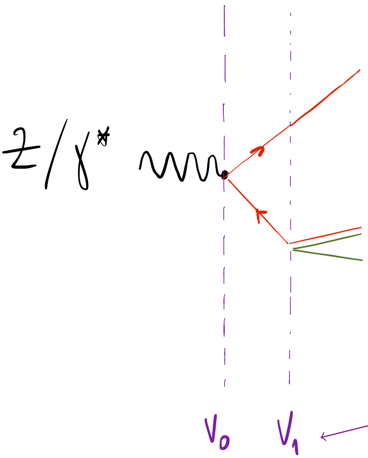
Final-state dipole shower





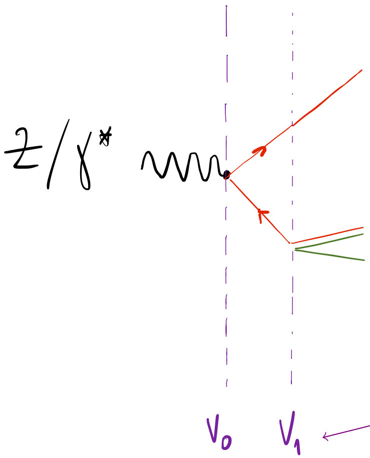
Start with a $q\bar{q}$ final state, which spans a colour dipole

Shower starting scale



$$\frac{\mathrm{d}\mathcal{P}_2(v)}{\mathrm{d}v} = -f_{2\to 3}^{q\bar{q}}(v)\mathcal{P}_2(v)$$

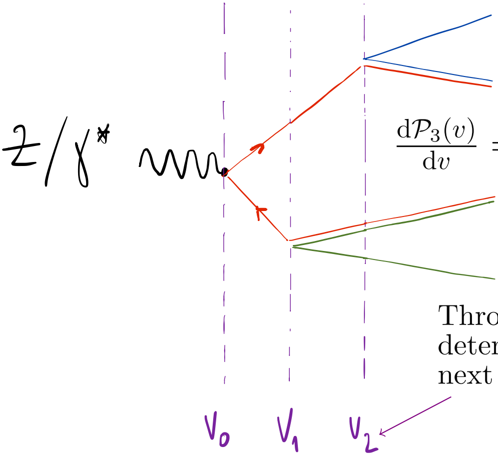
Throw a random number to determine the scale of the next emission, v_1



$$\frac{\mathrm{d}\mathcal{P}_2(v)}{\mathrm{d}v} = -f^{q\bar{q}}_{2\to3}(v)\mathcal{P}_2(v)$$

⟵ *Emission of g, part of two dipoles (qg, $g\bar{q}$)*

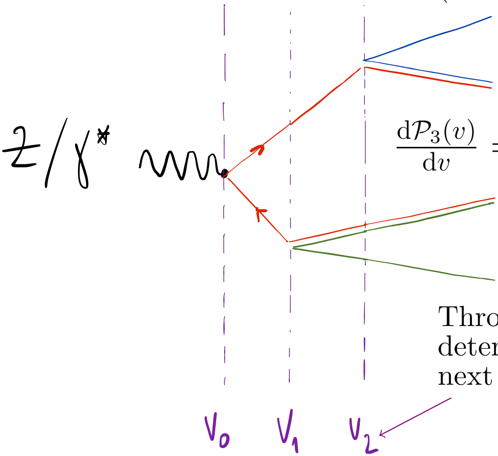
Throw a random number to determine the scale of the next emission, v_1



$$\frac{\mathrm{d}\mathcal{P}_3(v)}{\mathrm{d}v} = -[f^{qg}{}_{2\to3}(v) + f^{g\bar{q}}{}_{2\to3}(v)]\mathcal{P}_3(v)$$

Throw a random number to determine the scale of the next emission, v_2

Dipoles emit independently (incoherently) $\sim$ large-N_C limit



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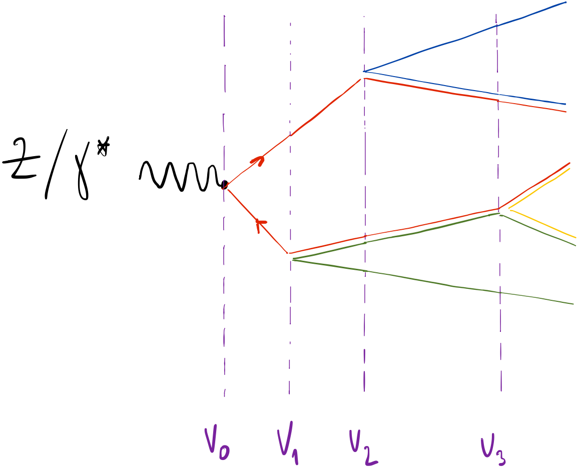


→ down to some cutoff

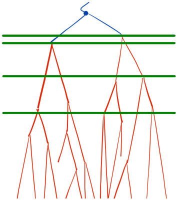
$v_{\text{cut}} \sim 1\,\text{GeV}$

at which point the shower stops

Dipole/antenna showers construction

Evolution variable:

- Transverse momentum k_t
- Opening angle θ
- Virtuality m^2_{ik}, ...



Kinematic map ($n \to n+1$)

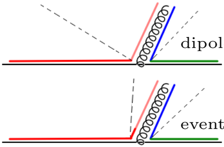
- Global recoil
- Local recoil

$$p_k = a_k \tilde{p}_i + b_k \tilde{p}_j + k_\perp \,,$$
$$p_i = a_i \tilde{p}_i + b_i \tilde{p}_j - f k_\perp \,,$$
$$p_j = a_j \tilde{p}_i + b_j \tilde{p}_j - (1-f) k_\perp$$

$$\mathbb{B}^\mu_{\ \nu}\,, \quad Q' = r\,\bar{Q}$$

Recoil attribution

- Emitter ↔ spectator partitioning
- Event ↔ dipole COM frame



Logarithmic accuracy in the PanScales showers

Oxford

Gavin Salam

Jack Helliwell

Silvia Zanoli

NIKHEF

Melissa van Beekveld

Manchester

Mrinal Dasgupta

UCL

Keith Hamilton

CERN

Alexander Karlberg

Silvia Ferrario

Pier Monni

Monash

Ludo Scyboz

Basem El-Menoufi

IPhT

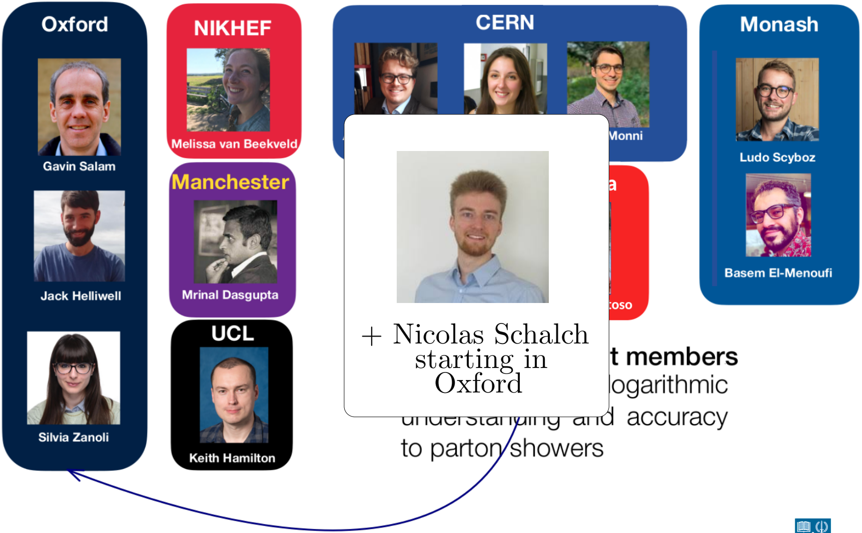
Gregory Soyez

Granada

Alba Soto Ontoso

PanScales current members

A project to bring logarithmic understanding and accuracy to parton showers



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Monni
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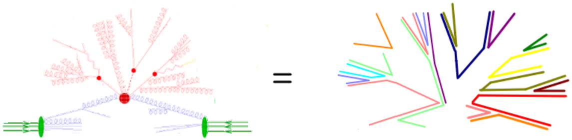
Basem El-Menoufi
+ Nicolas Schalch starting in Oxford
t members
logarithmic
accuracy
to parton showers

- Showers **resum** large logarithms, L, of ratios of scales (e.g. $L = \ln \frac{\mathcal{O}}{Q}$)

  ... by iterating soft (eikonal) and collinear kernels over an ensemble of dipoles



Logarithmic accuracy

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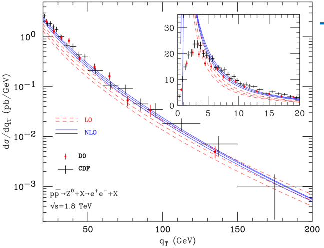
- Dominant terms are called *leading* logarithms (LL), subdominant *next-to-leading* (NLL), ...

Logarithmic accuracy

[Bozzi, Catani et al.]



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- Dominant terms are called *leading* logarithms (LL), subdominant *next-to-leading* (NLL), ...

$$\Sigma(\mathcal{O}) = \mathcal{P}\left(\mathcal{O} < e^L\right) \simeq \exp\left[\underbrace{Lg_1(\alpha_s L)}_{\text{LL}} + \underbrace{g_2(\alpha_s L)}_{\text{NLL}} + \underbrace{\alpha_s g_3(\alpha_s L)}_{\text{NNLL}} + \dots\right]$$

$$\mathcal{O}(\alpha_s^n L^{n+1}) \qquad \mathcal{O}(\alpha_s^n L^n) \qquad \mathcal{O}(\alpha_s^n L^{n-1})$$

(large logarithm $L \simeq \ln \mathcal{O}/Q$ of the value of the observable over some hard scale Q)

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[Bozzi, Catani et al.]

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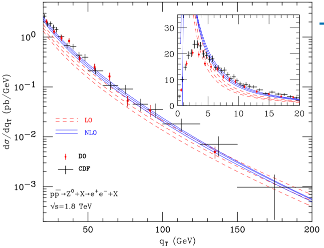

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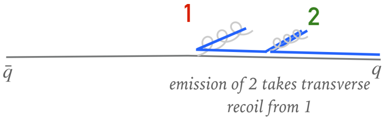
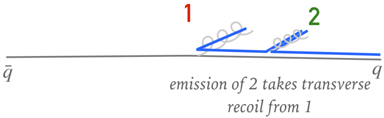
Standard partons showers reproduce LL terms, but ...

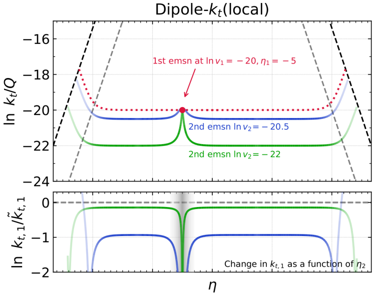
... they differ from NLL on

- An emission that is “far away” from another should not significantly modify the latter’s kinematics



- An emission that is "far away" from another should not significantly modify the latter's kinematics
- Here, "far" means in either $\ln k_t$,, or $\eta \simeq \ln \frac{1}{\theta}$





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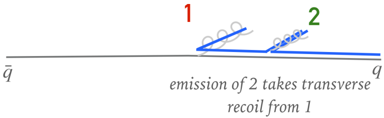
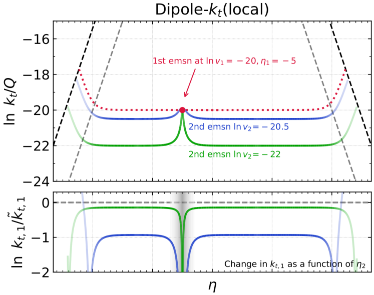
This ensures that the QCD matrix element is reproduced in all singular limits that contribute at NLL





- Evolution variable:

$$v \simeq k_t \theta^{\beta_{\rm PS}}\,, \qquad 0 \le \beta_{\rm PS} < 1$$

- Recoil scheme:
  - Local [as in standard showers]
  - Global [recoil shared across the whole event through (rescaling and) boost]

$$\lim_{k_\perp \to 0} \mathbb{B}^\mu_\nu\, p^\nu \sim p^\mu - \frac{Q \cdot p}{Q^2} k_\perp^\mu$$

similarly, Alaric [Herren et al. '22,'24],
Manchester-Vienna [Forshaw et al. '20]
Apollo [Preuss '24]

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- Recoil scheme:
  - Local [as in standard showers]
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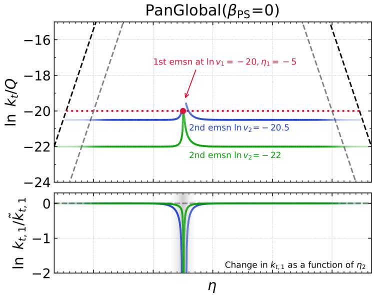
PanLocal

$k_t\sqrt{\theta}$ ordered

Dipole/antenna

PanGlobal

k_t or $k_t\sqrt{\theta}$ ordered

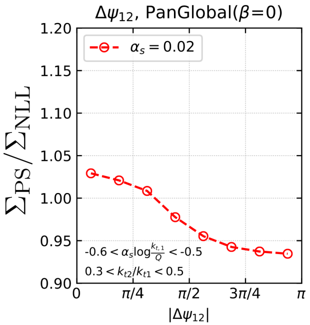


Recipe for all-order logarithmic tests

1. Run full shower with specific $\alpha_s = \alpha_s(Q)$

   Should $\Sigma_{\mathrm{PS}}/\Sigma_{\mathrm{NLL}} = 1$?

   No! There are shower-generated NNLL terms still...



$$\Sigma_{\mathrm{PS}} = \exp\left[L g_1(\lambda) + g_2(\lambda) + \alpha_s g_3(\lambda) + \ldots\right]$$

$$\Sigma_{\mathrm{NLL}} = \exp\left[L g_1(\lambda) + g_2(\lambda)\right]$$

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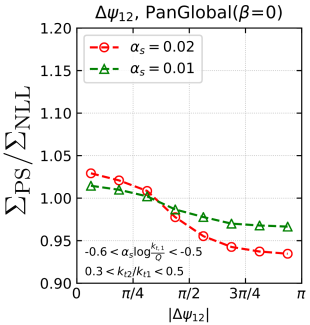
   - (NLL effects $\sim \alpha_s^n L^n$, but NNLL $\sim \alpha_s^{n+1} L^n$)



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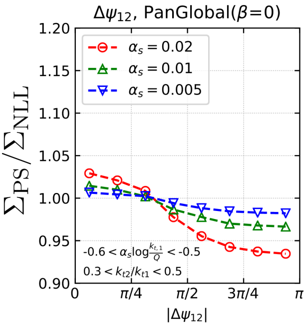

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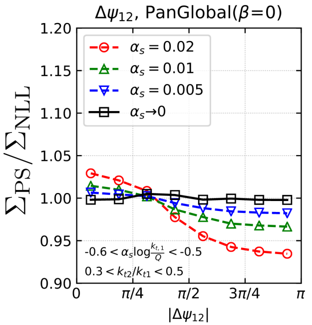
   - (NLL effects $\sim \alpha_s^n L^n$, but NNLL $\sim \alpha_s^{n+1} L^n$)
3. Extrapolate $\alpha_s \to 0$



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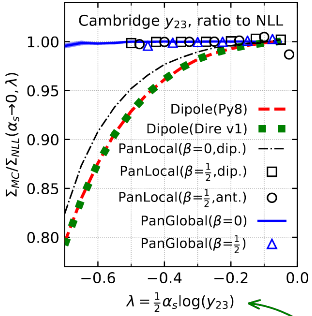
Numerical NLL accuracy tests



$\alpha_s L \equiv$ argument of $g_1(\lambda)$, $g_2(\lambda)$, ...

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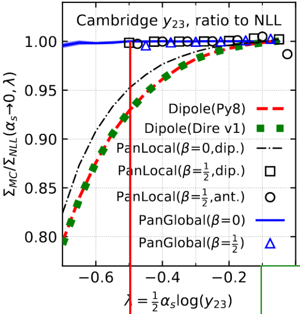
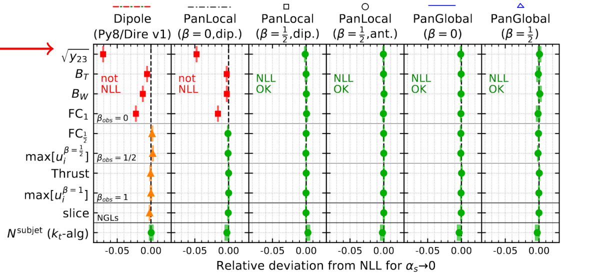


$\lambda = \alpha_s L = -0.5$



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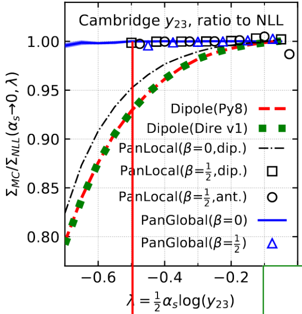
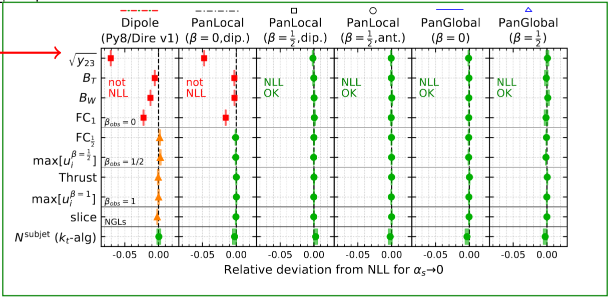


NLL accurate PanScales showers for many classes of observables!

also subleading-colour at NLL, spin, ISR, ...
[PanScales '20,'21,'22,'23,'24...]

$$\lambda = \alpha_s L = -0.5$$

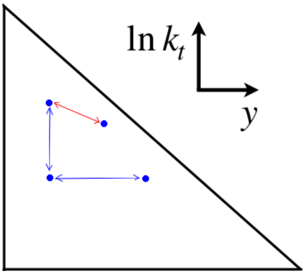


NLL: summary

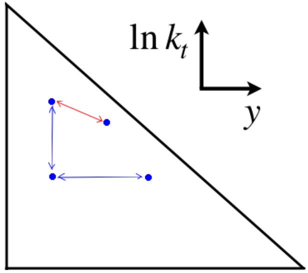
- We "only" need two ingredients:
  - The correct inclusive rate of emission up to $\mathcal{O}(\alpha_s^2)$ (CMW α_s scheme) [Catani et al '91]

$$\alpha_s^{\text{eff}} = \alpha_s \left[1 + \frac{\alpha_s}{2\pi} K_1\right]$$

  - The condition that any emission does **not** affect other emissions "far" in the Lund plane



- We can relate shower ingredients to analytic resummations
  e.g. [Banfi et al, 1807.11487] (ARES)
- We need:
  - an NLL shower to start with
    [PanScales]



NLL → NNLL

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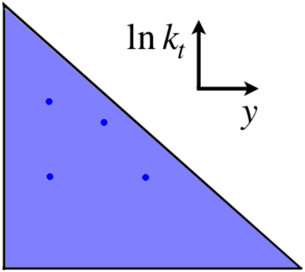
    [Banfi et al '18], [Catani et al '19]

$$\alpha_s^{\text{eff}} = \alpha_s \left[1 + \frac{\alpha_s}{2\pi} K_1 + \frac{\alpha_s^2}{4\pi^2} K_2\right]$$



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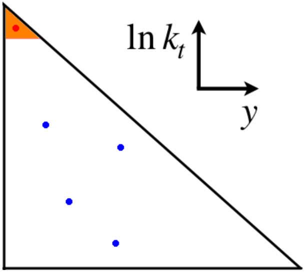
  - inclusive emission rate up to $\mathcal{O}(\alpha_s^3)$
    [Banfi et al '18], [Catani et al '19]
  - 2-jet NLO matching
    [Hamilton et al '22]

$$\Sigma = \sigma_{\text{LO}}(1+\frac{\alpha_s}{\pi}C_1)\exp\left[Lg_1(\alpha_s L) + ...\right]$$

terms $\sim \alpha_s^n L^{n-1}$



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  - radiation pattern for soft partons up to $\mathcal{O}(\alpha_s^2)$ (i.e. double-soft)

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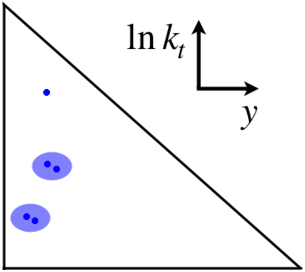

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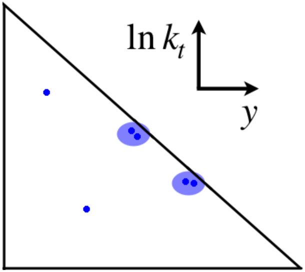
  - radiation pattern for soft partons up to $\mathcal{O}(\alpha_s^2)$ (i.e. double-soft)
  - matrix element for hard radiation up to $\mathcal{O}(\alpha_s^2)$ (i.e. triple-coll. $\equiv B_2(z)$)

terms $\sim \alpha_s^n L^{n-1}$



Double-soft corrections

- First time a shower goes **demonstrably** beyond NLL

Parton showering with higher-logarithmic accuracy for soft emissions

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Gavin P. Salam,[3,4] Ludovic Scyboz,[3] and Gregory Soyez[1,5]

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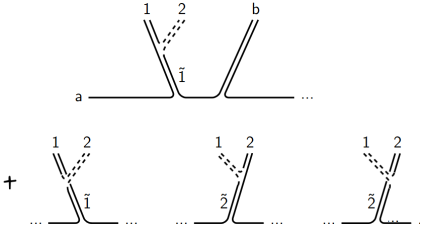
The accuracy of parton-shower simulations is often a limiting factor in the interpretation of data from high-energy colliders. We present the first formulation of parton showers with accuracy one order beyond state-of-the-art next-to-leading logarithms, for classes of observable that are dominantly sensitive to low-energy (soft) emissions, specifically non-global observables and subjet multiplicities. This represents a major step towards general next-to-next-to-leading logarithmic accuracy for parton showers.

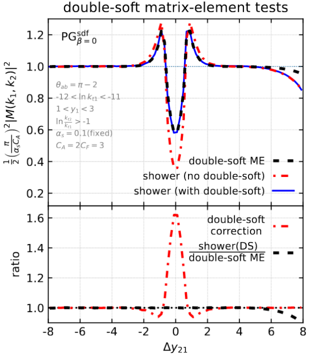
Real matrix element

- One corrects the shower (ps) acceptance probability to recover the exact double-soft (ds) matrix element,

$$P_{\text{accept}} = \frac{|M_{\text{ds}}|^2}{\sum_{h \in \text{hist}} |M^h_{\text{ps}}|^2}$$

- PanGlobal: in general 4 histories





Real ME: colour flow & flavour

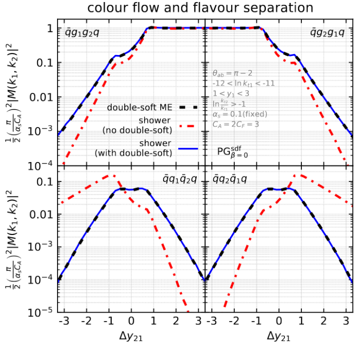
- There are two colour flows: for the emission of a double-soft pair 1, 2 from the dipole (ab),

$$|M|^2 = \underbrace{\sum_{h \in a12b} |M_h^{(12)}|^2}_{F^{(12)}} + \underbrace{\sum_{h \in a21b} |M_h^{(21)}|^2}_{F^{(21)}}$$

- If the shower over-populates, $F_{\rm ps}^{(12)} > F_{\rm ds}^{(12)}$, swap colour connection with probability

$$P_{\rm swap} = \frac{F_{\rm ps}^{(12)} - F_{\rm ds}^{(12)}}{F_{\rm ps}^{(12)}}$$

- Same for $g \to gg$ vs. $g \to q\bar{q}$



Virtual corrections

- Part of the virtuals already covered by K_1 $(= K_{\mathrm{CMW}})$, which gets the correct NLO rate in the soft-collinear region

$$\alpha_s \to \alpha_s^{\mathrm{eff}} = \alpha_s \left[1 + \frac{\alpha_s}{2\pi} K_1\right]$$

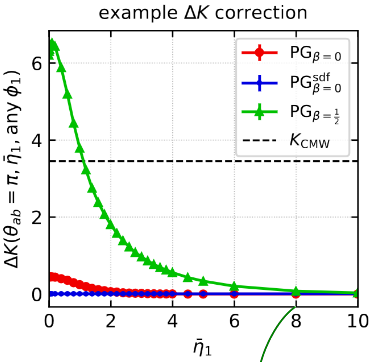
- At large angle, $y \sim 0$, this is not enough. Correct this,

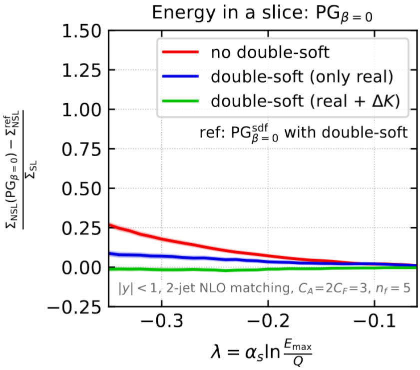
$$\alpha_s \to \alpha_s^{\mathrm{eff}} = \alpha_s \left[1 + \frac{\alpha_s}{2\pi} (K_1 + \Delta K_1(y))\right]$$

- Related to the fact that the shower does not conserve kinematics of the parent, $y_{12} \neq y_{\tilde{1}}$

$$\Delta K_1 = \int d\Phi_{12/\tilde{1}} |M_{12/\tilde{1}}|^2 - \int d\Phi_{12/\tilde{1}_{\mathrm{sc}}} |M_{12/\tilde{1}_{\mathrm{sc}}}|^2$$

- (Shower-dependent) ΔK evaluated and tabulated



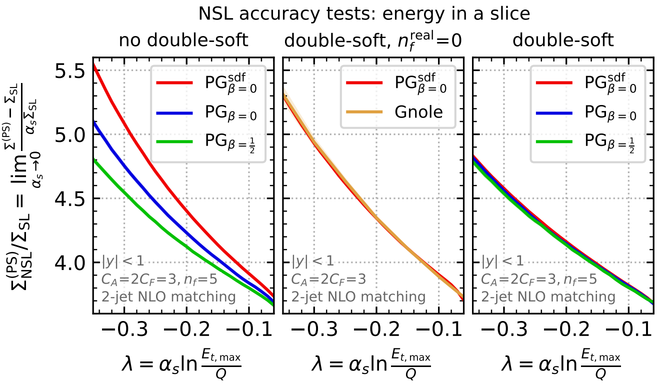


$\Delta K(\bar{\eta}_1) \to 0$ when $\bar{\eta}_1 \to \infty$ (soft-collinear is fine already)

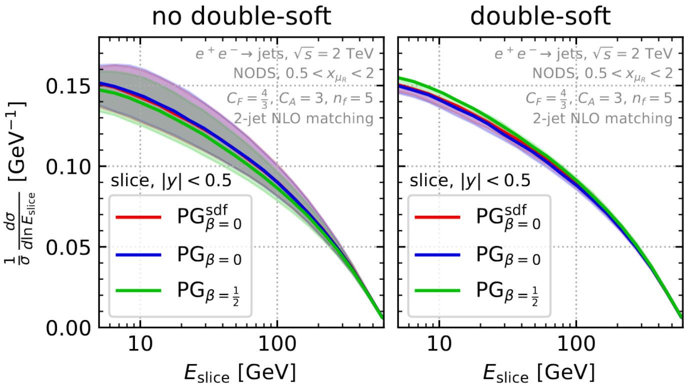
$\Delta K \equiv 0$ for $\mathrm{PG}^{\mathrm{sdf}}_{\beta=0}$

Non-global logarithms @ NSL

- Transverse energy in a slice, $\sum_i E_{T,i}$ with $|y| < 1$
- Comparison with Gnole [Banfi, Dreyer, Monni '21]



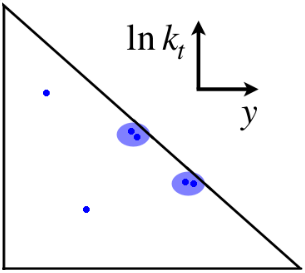
Double-soft: phenomenological impact



- Central value only slightly affected by double-soft corrections
- But scale uncertainties greatly reduced!

NLL → NNLL

- We can relate shower ingredients to analytic resummations
  [Banfi et al, 1807.11487] (ARES)
- We need:
  - an NLL shower to start with ✓
    [PanScales]
  - inclusive emission rate up to $\mathcal{O}(\alpha_s^3)$
    [Catani et al 1904.10365]
  - 2-jet NLO matching ✓
    [Hamilton, Karlberg, Salam, LS, Verheyen]
  - radiation pattern for soft partons up to $\mathcal{O}(\alpha_s^2)$ (i.e. double-soft) ✓
  - matrix element for hard radiation up to $\mathcal{O}(\alpha_s^2)$ (i.e. triple-coll. $\equiv B_2(z)$)

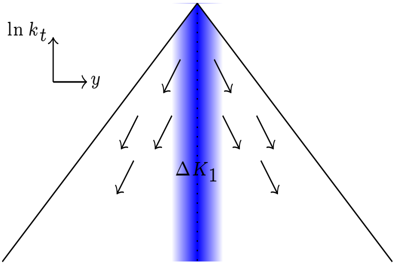


For event shapes @ NNLL, we only need the **integrated** quantities!

The drift picture

$$\alpha_s^{\text{eff}} = \alpha_s \left[1 + \frac{\alpha_s}{2\pi}(K_1 + \Delta K_1)\right]$$

- The shower does not conserve e.g. the rapidity of the parent gluon, $y_{\tilde{1}} \neq y_{12}$ (assumed to hold in calculating K_1)
  - ⇝ average rapidity drift $\langle \Delta_y \rangle$



- Correct the depleted central region, by *adding* a total $\Delta K_1^{\text{int}} = \int dy \Delta K_1(y)$:

$$\Delta K_1^{\text{int}} = 2\langle \Delta_y \rangle$$

- Note: for event shapes, integrated quantity is enough at NNLL

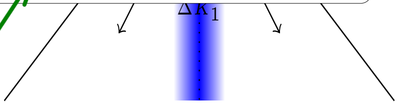
The drift picture

$$\alpha_s^{\text{eff}} = \alpha_s$$

shower	colour	$\frac{1}{4\pi}\Delta K_1^{\text{int,PS}}$	$\frac{1}{2\pi}\langle\Delta_y\rangle$	$\frac{1}{2\pi}\langle\Delta_{\ln k_t}\rangle$
$\text{PG}^{\text{sdf}}_{\beta=0}$	C_F	0	0.000018(39)	-1.953481(1)
	C_A	0	0.000002(2)	1.162602(2)
	$n_f T_R$	0	-0.0000003(3)	-0.1048049(3)
$\text{PG}_{\beta=0}$	C_F	0.04967(3)	0.049576(8)	-1.964624(6)
	C_A	0.0323(5)	0.032107(4)	1.174900(4)
	$n_f T_R$	0.0040(1)	0.003962(1)	-0.104655(1)
$\text{PG}_{\beta=\frac{1}{2}}$	C_F	1.6725(5)	1.672942(9)	-1.749920(5)
	C_A	0.0172(11)	0.015303(5)	1.172042(5)
	$n_f T_R$	0.0535(2)	0.053476(1)	-0.094205(1)

- The shower does not c gluon, $y_{\tilde{1}} \neq y_{12}$ (assum
  - ⇝ average rapidity dri
- Correct the depleted c region, by *adding* a tot $\Delta K_1^{\text{int}} = \int dy \Delta K_1(y)$:

$$\Delta K_1^{\text{int}} = 2\langle\Delta_y\rangle$$



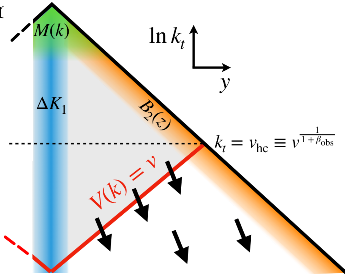
- Note: for event shapes, integrated quantity is enough at NNLL

The drift picture: B_2, K_2 and $\mathcal{F}$

$$\alpha_s^{\text{eff}} = \alpha_s \left[1 + \frac{\alpha_s}{2\pi}(K_1 + \Delta K_1 + B_{2,\text{PS}}^{\text{int}}) + \frac{\alpha_s^2}{4\pi^2} K_{2,\text{PS}}\right]$$

- Similar connection with hard-collinear fron
  [Dasgupta et al. '21, '23, '24]

$$B_{2,\text{PS}}^{\text{int}} = B_{2,\text{NLO}}^{\text{int}} - \langle \Delta_y \rangle - \langle \Delta_{\ln k_t} \rangle + \beta_0 \frac{\pi^2}{12}$$

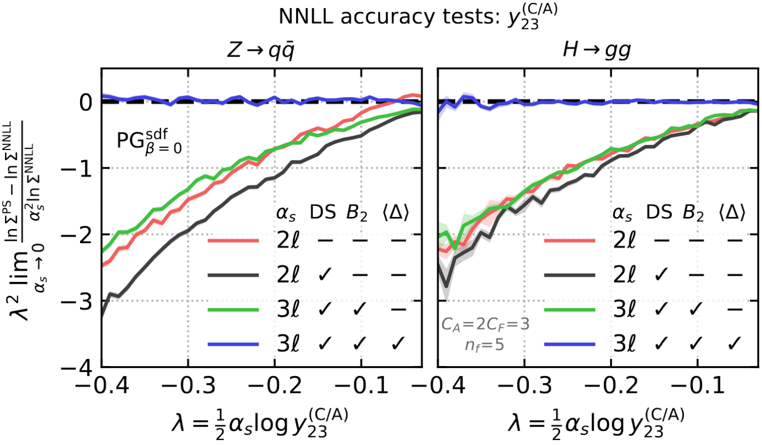


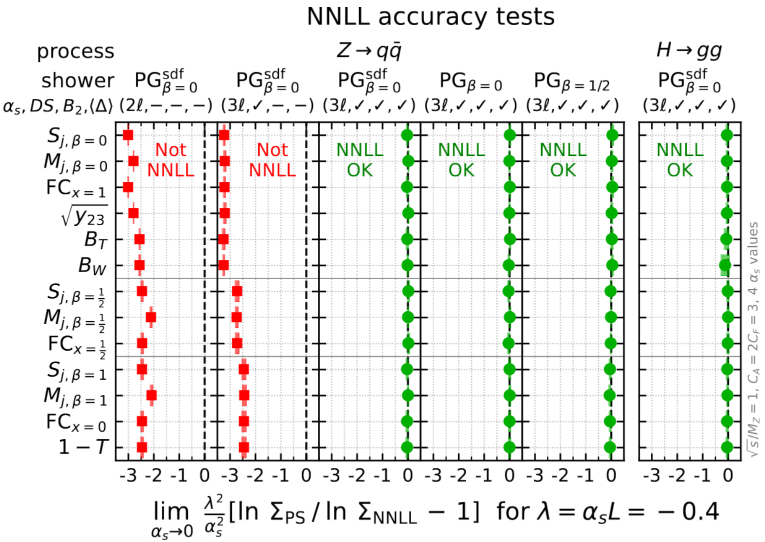
- Inclusive rate correction at $\mathcal{O}(\alpha_s^3)$
  [Banfi et al. '18], [Catani et al. '19]

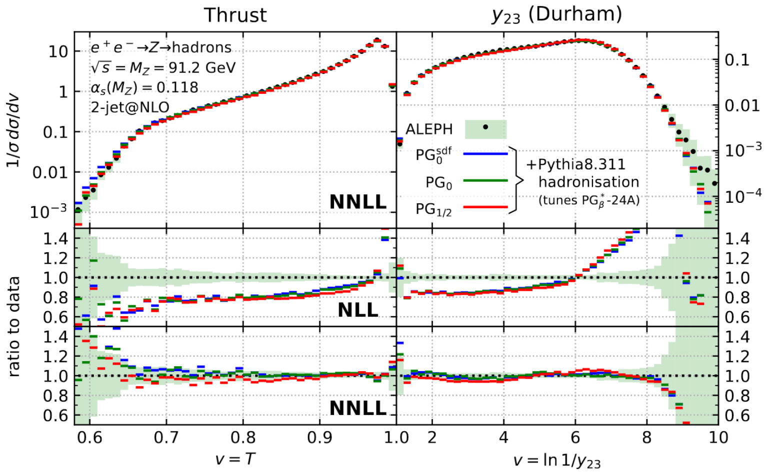
$$K_{2,\text{PS}} = K_2^{\text{analytic}} - 4\beta_0 \langle \Delta_{\ln k_t} \rangle$$

- Similarly for multiple emission constraint $\mathcal{F}$
- Proof of drift equivalence in appendices of
  [2406.02661]

NNLL tests: $\alpha_s \to 0$ limit




NNLL accuracy tests
process
$Z \to q\bar{q}$
$H \to gg$
shower
$\mathrm{PG}^{\mathrm{sdf}}_{\beta=0}$
$\mathrm{PG}^{\mathrm{sdf}}_{\beta=0}$
$\mathrm{PG}^{\mathrm{sdf}}_{\beta=0}$
$\mathrm{PG}_{\beta=0}$
$\mathrm{PG}_{\beta=1/2}$
$\mathrm{PG}^{\mathrm{sdf}}_{\beta=0}$
$\alpha_s, DS, B_2, \langle\Delta\rangle$
$(2\ell,-,-,-)$
$(3\ell,\checkmark,-,-)$
$(3\ell,\checkmark,\checkmark,\checkmark)$
$(3\ell,\checkmark,\checkmark,\checkmark)$
$(3\ell,\checkmark,\checkmark,\checkmark)$
$(3\ell,\checkmark,\checkmark,\checkmark)$
$S_{j,\beta=0}$
$M_{j,\beta=0}$
$\mathrm{FC}_{x=1}$
$\sqrt{y_{23}}$
B_T
B_W
$S_{j,\beta=\frac{1}{2}}$
$M_{j,\beta=\frac{1}{2}}$
$\mathrm{FC}_{x=\frac{1}{2}}$
$S_{j,\beta=1}$
$M_{j,\beta=1}$
$\mathrm{FC}_{x=0}$
$1-T$
Not NNLL
Not NNLL
NNLL OK
NNLL OK
NNLL OK
NNLL OK
$\sqrt{s}/M_Z = 1,\ C_A = 2C_F = 3,\ 4\ \alpha_s$ values
-3 -2 -1 0
$\lim_{\alpha_s \to 0} \frac{\lambda^2}{\alpha_s^2}[\ln \Sigma_{\mathrm{PS}} / \ln \Sigma_{\mathrm{NNLL}} - 1]$ for $\lambda = \alpha_s L = -0.4$

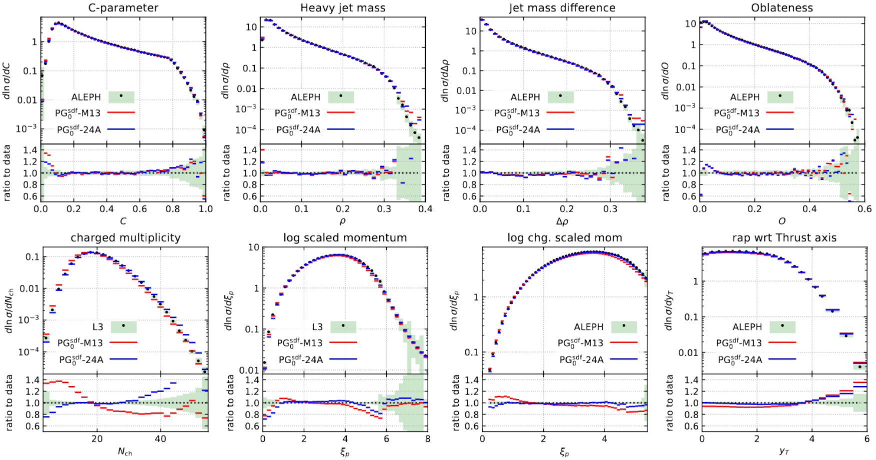


- Some LL/NLL showers require $\alpha_s(M_Z) \sim 0.130$ to agree with data

NNLL: impact of tuning

IRC-safe

IRC-unsafe



Summary & outlook

- Shower accuracy has lagged behind for 40 years, compared to the precision in other contexts
- NLL now **established** [PanScales], [Alaric], [FHP], [Apollo], [Deductor],...
- Major steps towards NNLL accuracy (for now in e^+e^-):
  - Double-soft corrections in PanGlobal
  - Drift picture & NNLL ingredients equivalence "theorems"
    - ⇝ NNLL for event shapes in PanGlobal!

- Work on triple-collinear see e.g. recent [van Beekveld et al, 2409.08316]
- Double-soft & drifts for ISR, as well as for PanLocal $\beta = 1/2$
- NLO matching in pp [24XX.YYYY]
- Quark masses [2XXX.YYYY]
- ...

Backup