FFAs for Acceleration in a Muon Collider

J. Scott Berg, Brookhaven National Laboratory
Colliders of Tomorrow, Fermilab

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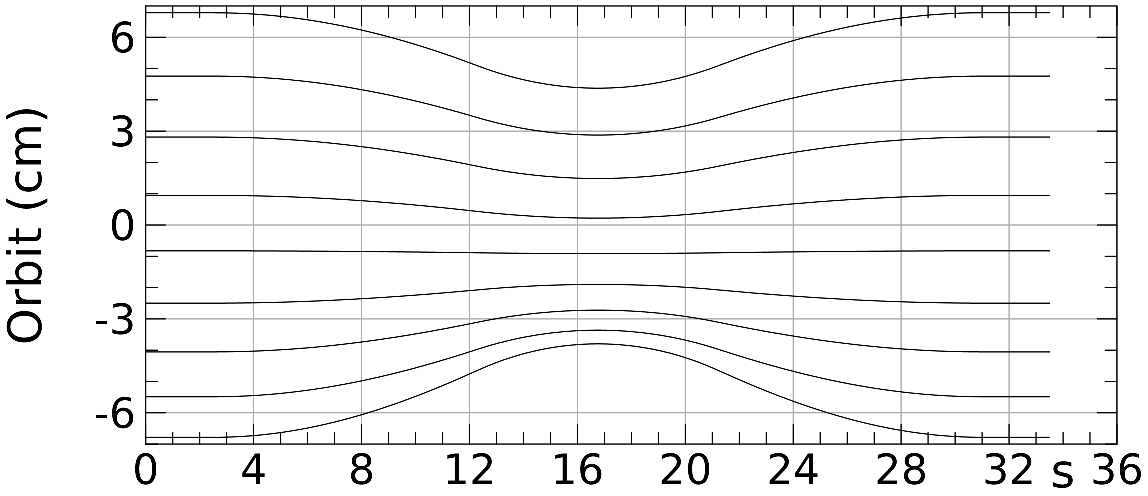
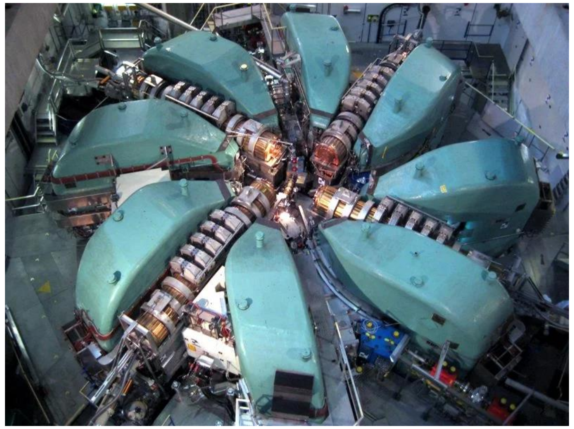
What is an FFA?

- **F**ixed **F**ield **A**lternating gradient accelerator
- Large energy range (e.g., factor of 2) in a single beamline
- Magnet fields do not vary with time
- Each energy follows a different orbit
- Alternating gradient focusing in compact cells for small orbit excursion



FFAs Compared to Other Accelerators



- Cyclotrons
  - Magnet fields don’t change while accelerating
  - Rely on intrinsic focusing from dipoles and their edges, potentially with a small gradient (weak focusing)
  - Require large magnets
- Synchrotrons
  - Magnet fields change proportional to beam momentum
  - Alternating gradient focusing, small beams
  - Compact magnets
  - Small energy acceptance

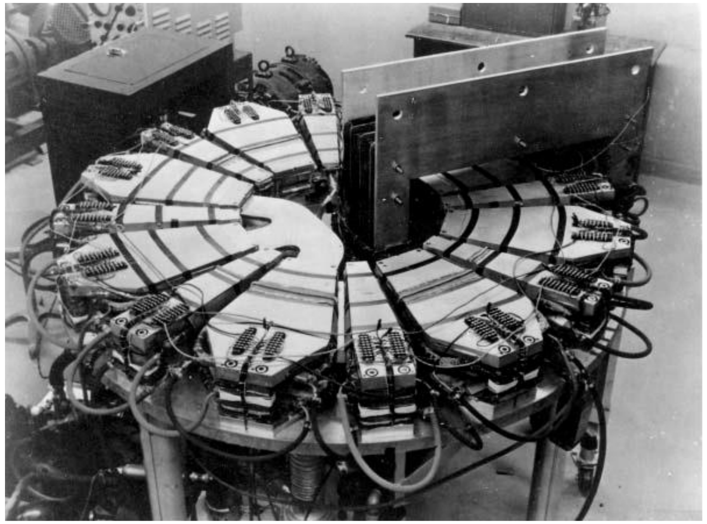
FFAs Compared to Other Accelerators

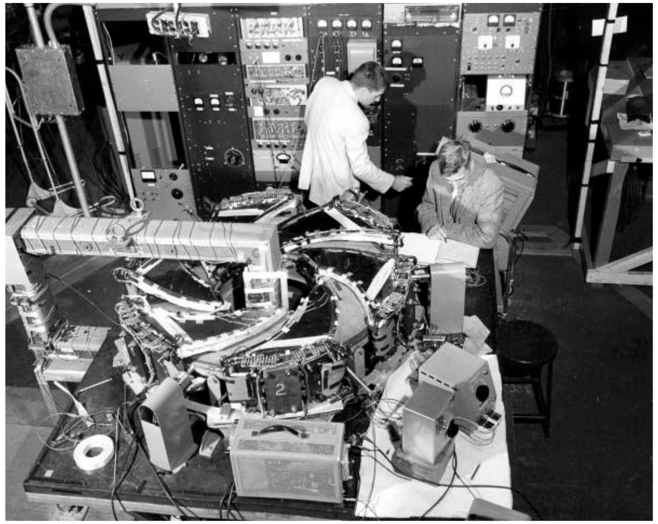
- FFAs are like cyclotrons:
  - Fields in FFA magnets do not vary during acceleration
  - The beamline accepts a wide range of energies
- FFAs are like synchrotrons:
  - Alternating gradient focusing to keep orbit excursion and beam size small
- FFAs are between the two:
  - Magnets are more compact than for cyclotrons, but larger than for synchrotrons

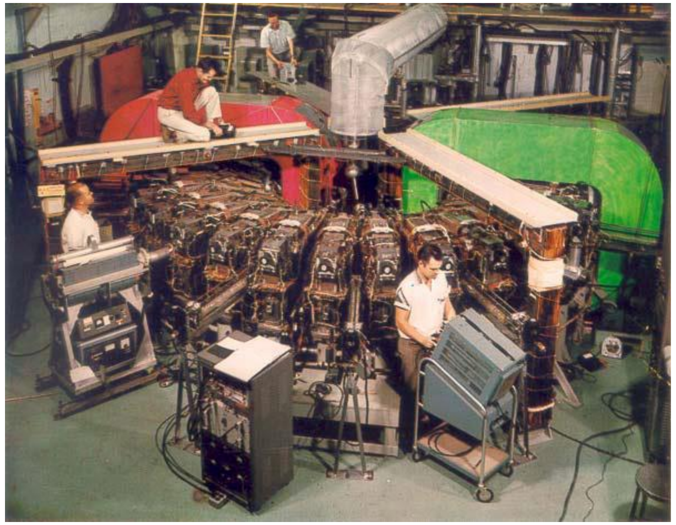
A Brief, Incomplete History of FFAs

The First FFAs: MURA

- FFAs (nee FFAGs) first described in 1954 at MURA in Wisconsin, with contemporaneous development in Japan and Russia
- Three FFAs were operated at MURA in 1956, 1957, and 1959–1961, accelerating electrons







Proton FFAs in Japan

- In 2000, a group led by Mori began building proton FFAs, first at KEK, later at KURRI
- In 1999, they began a series of FFA workshops that continues to this day
- A strong interest in FFAs in Japan continues to this day, including accelerators built at universities

Brookhaven National Laboratory

A Detour: “Scaling” FFAs

- In every FFA mentioned thus far, the design field in the magnets follows a power law

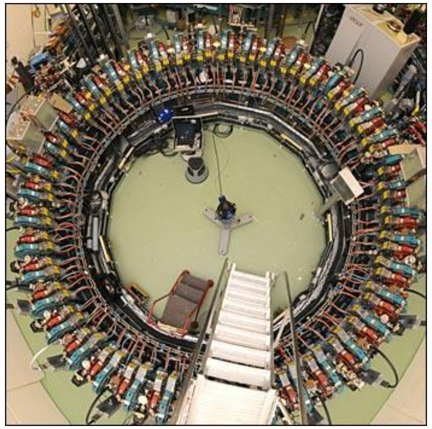
$$B_y(r,0,\theta) = B_{y0}\left(\theta - \int_{r_0}^{r} \frac{\tan\zeta(\bar{r})}{\bar{r}}\,d\bar{r}\right)\left(\frac{r}{r_0}\right)^k$$

- $B_{y0}(\theta)$ has alternating sign
  - Alternating gradient focusing
  - Reverse bending (increased radius for a given field)
- Tunes are independent of energy
  - Avoids resonance crossing during acceleration, eliminating an important loss mechanism
- Increasing k:
  - Smaller orbit excursion and magnets, but
  - More nonlinearity, smaller dynamic aperture, higher fields

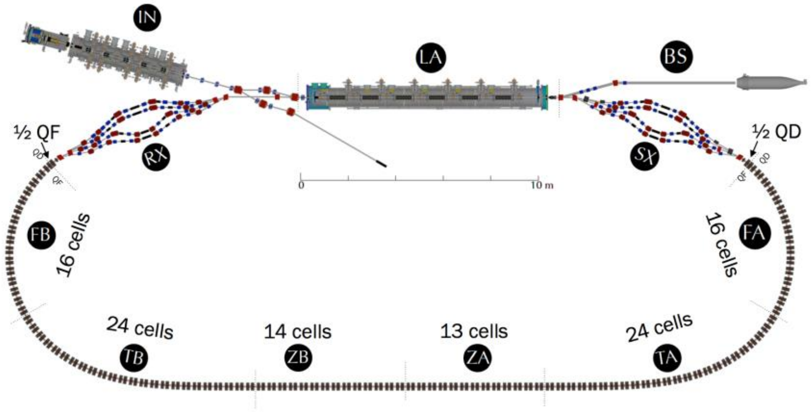
Non-Scaling FFAs

- In 1999, Johnstone and Mills propose non-scaling FFAs, using linear magnets for the purpose of accelerating muons
  - Linear magents to give large dynamic aperture
  - Less reverse bending
  - Orbits and magnets more compact
- But: tunes vary with energy
  - Linear magnets avoid nonlinear resonances
  - Need high periodicity to avoid linear resonances
- 2001: Berg demonstrates you can accelerate in a non-scaling FFA without varying the RF frequency: serpentine acceleration

Non-Scaling FFAs



- From 2007–2011 the EMMA ring at Daresbury
  - Linear non-scaling FFA, similar a low-energy muon accelerator
  - Confirmed expected behavior, serpentine acceleration
- CBETA at Cornell University
  - ERL with a single FFA return arc covering 4 energies, 42–150 MeV
  - Successfully operated 4-pass ERL



Vertical FFAs

- For previous examples, closed orbits for each energy are in a common plane
- In 1955, FFAs with closed orbits vertically displaced for different energies were first discussed (Ohkawa), with more detailed work by other authors in from 1959–1962.
- Brooks rediscovered the idea in 2009, and there has been interest in applying it in the UK and Japan
- Because orbits are displaced vertically, they can have identical lengths at all energies, avoiding RF synchronization problems at high energy

When to use FFAs

Comparing to Synchrotrons

- Synchrotrons won over FFAs
- Synchrotrons have no reverse bending, FFAs usually require it: machine is shorter for a given maximum field
- Synchrotron magnets only need to be large enough to pass a single beam; FFA magnets need aperture for both the beam size and the orbit variation with energy
- At low energy, both machines need to vary RF frequency as the beam accelerates; at high energy, that variation is small in a synchrotron, larger in the FFA
- In a synchrotron, magnet fields must be varied during acceleration

When to use FFAs

- You primarily use FFAs when you are in a big hurry to accelerate
  - Very high repetition rates
  - Unstable particles!
- Linacs accelerate quickly, but are very expensive. So an FFA can save you money
  - Multpass RF must give sufficient savings over more expensive magnets
  - And FFAs generally require lower frequency RF than the corresponding linac would have
- You can manage any required RF frequency variation
  - Possibly with phase space tricks: e.g., serpentine acceleration

Muon Acceleration

General Principles for Muon Acceleration

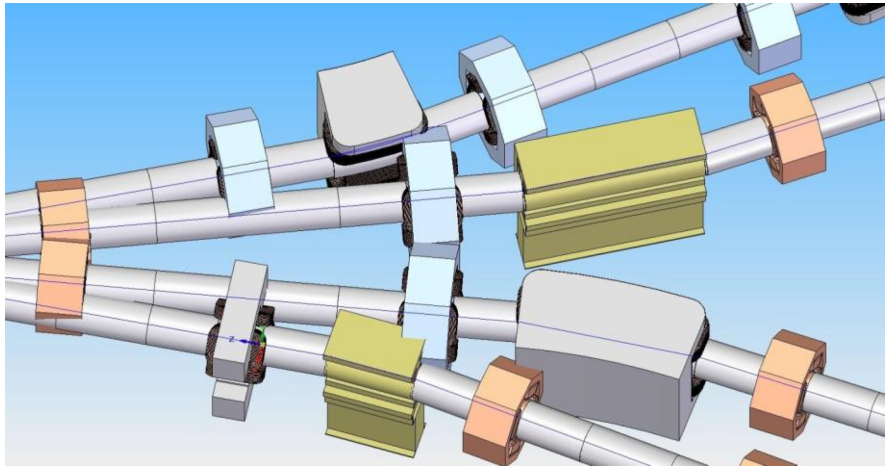
- Acceleration must be fast to avoid muon decays
- RF drives costs and power consumption
    - Make multiple passes through RF, as many as possible
        - Use a ring or RLA where possible
        - More time for magnet and RF manipulations at higher energies
    - Use high RF frequencies
- Keep decays reasonably low
    - High average accelerating gradient
    - High average bend field in recirculating machines, to get more passes
- Avoid emittance growth and losses

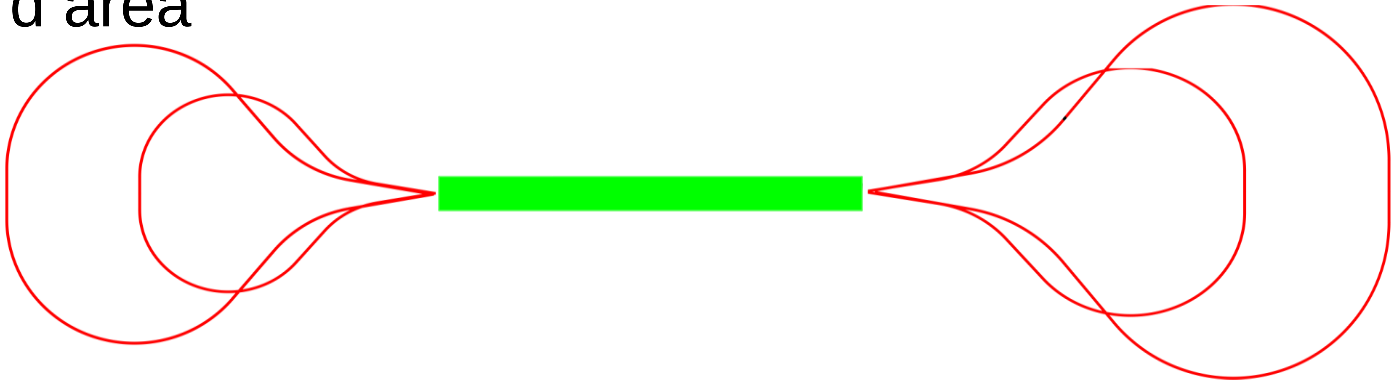
Stages of Muon Acceleration

- Low energy just after cooling
  - The cooling channel, somewhat reversed, without absorbers
- Initial superconducting linac to a couple GeV
  - Once beam is small enough
- Low energy recirculating acceleration
  - No time for changing magnet fields, adding power to RF to replace stored energy, shift phase
- High energy acceleration

Low Energy Conventional Acceleration

- Recirculating linear accelerator (RLA)
- “Dogbone” configuration for efficiency
- Baseline: acceleration to around 63 GeV, in multiple stages
- Challenge: limited number of passes
  - Complexity in the switchyard area
  - Large beam size, particularly driven by longitudinal



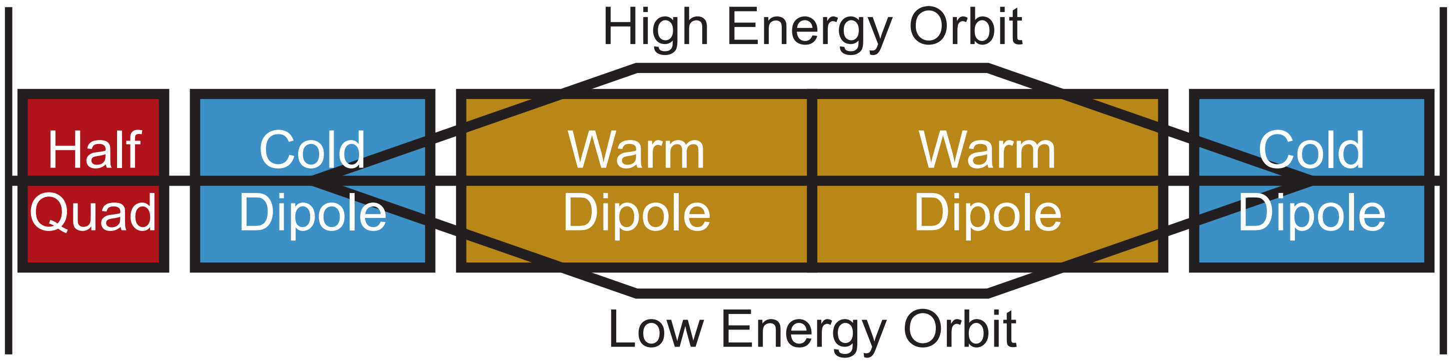


High Energy “Conventional” Acceleration: Pulsed Synchrotrons

- Once we reach high energy, we have time to change magnet fields
  - Use a synchrotron: magnet fields increase with beam energy
  - Repetition rate is low (5–10 Hz)
  - Acceleration times are short: < 1 ms at lower energies, a few ms at higher energies
- Only iron-dominated magnets can ramp on these time scales
  - Iron field limited to about 1.8 T
  - Such a low average bend field means either a low average accelerating gradient and thus lots of decay, or a small number of turns and thus a high RF voltage

The Hybrid Synchrotron

- Increase the average bend field while using pulsed iron magnets by interleaving fixed field superconducting dipoles, and *bipolar* pulsed iron magnets
- The catch: even for infinitely high superconducting magnet fields, average bend field is bounded by a number depending only on the energy gain factor and the iron magnet field!
  - E.g., factor of 2 energy gain, 1.75 T iron magnets, average bend is 5.1 T



Powering Pulsed Magnets

- Pulsed magnet power supplies are a significant cost
  - Delivering GW of power to magnets
  - Energy is recovered and stored in capacitors, but need to watch efficiency
- There are unavoidable losses in iron, plus conductors
  - Can be kept down to a few MW
  - Water cooling of magnets is required

Accelerating to High Energy on the Fermilab Site

- Goal is to accelerate to 5 TeV per beam in a 16.5 km tunnel
- Current designs envision 2 (or even 3) acceleration stages in that tunnel
  - Last stage may only accelerate from 4.2 to 5 TeV
  - Stage before that unlikely to even cover a factor of 2
  - This will only get worse as we make the designs more realistic
- We are unlikely to reach 5 TeV with pulsed synchrotrons on the Fermilab site, but we will get close
- Could FFAs let us reach higher energies?

Replacing Conventional Muon Acceleration with FFAs

Replacing Acceleration with FFAs

- The aforementioned “conventional” acceleration techniques should work for low and high energy acceleration
- But I’ve pointed out issues that
  - Make them costly
  - Limit their possible efficiency
  - Limit their capabilities in some circumstances
- Could replacing these accelerators with FFAs address some of these issues and ultimately be a better choice?

Basic Structure of a Muon FFA

- (Linear) non-scaling FFA
- Simple cells with a “long” drift for an RF cavity
    - Most cells have an RF cavity: adiabatic orbit shifts
    - Some drifts will replace that with injection/extraction hardware
- Have muon beams circulating in both directions
    - Use a reflection symmetric cell structure: triplets
- Generally small factors (2–3) in energy gain
    - Magnet apertures grow quickly for large factors

FFA Design Principles

- Compact cells to reduce orbit excursions and apertures
- Use combined function magnets: both bending and focusing
- Strong horizontal focusing most important, since orbit excursion is horizontal
  - Generally keep vertical focusing weak to make best use of magnet focusing

Why a non-Scaling FFA Rather than Scaling

- Less reverse bending, so better average bend fields
- Smaller apertures
- Nonlinear fields in scaling FFAs grow much more quickly at larger apertures, leading to higher required magnet fields
- Scaling FFAs require lower frequency RF
  - Large beam excursion
  - Large time of flight range
- NufactJ studies (2001) of scaling FFA neutrino factory to 20 GeV
  - A rough cost comparison indicated the scaling FFA solution was significantly more expensive than a non-scaling solution

Why a non-Scaling Rather Than a Vertical FFA?

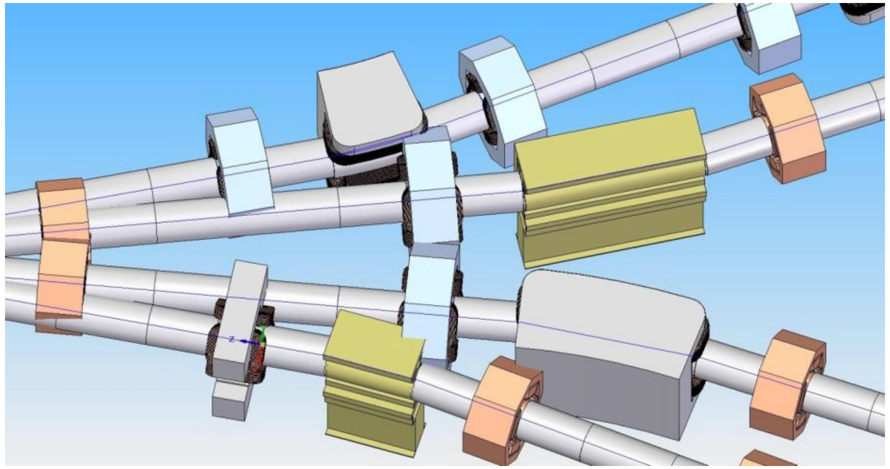
- Orbit lengths independent of energy make vertical FFAs an attractive solution
    - Other solutions must adjust RF frequency make design compromises to enable needed longitudinal phase space dynamics for acceleration (e.g., serpentine acceleration)
- Vertical FFA magnets appear to be very complex to make
- Vertical orbit excursions are larger than horizontal excursions in non-scaling FFAs. Require large RF apertures, lower frequencies.
- Ongoing work in UK and Japan trying to address these issues

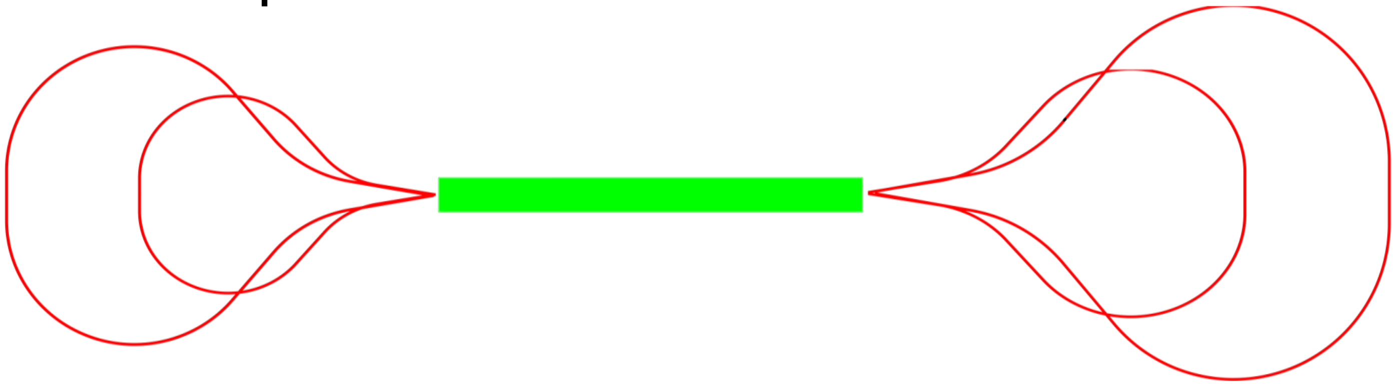
Low Energy FFAs

- Characteristic defining “low” vs. “high” is that at low energies, there is insufficient time to replace stored energy or shift phase in RF cavities
  - Primary limitation is input power coupler
  - Discussed mechanical and other methods to rapidly change RF frequency, nothing appeared viable
- Stored energy in cavities must be sufficient to accelerate for all turns
  - Requires low frequencies (e.g., 325 MHz)
- Serpentine acceleration to accelerate with varying orbit period

Why FFAs for Low Energy Acceleration?

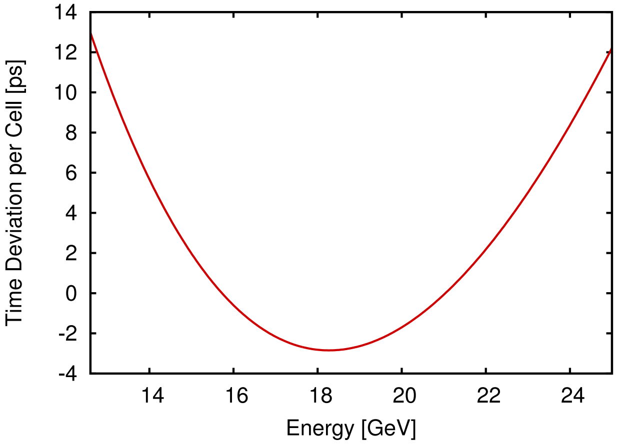
- Switchyard limits number of passes
  - FFA has no real limit on number of passes
  - But the FFA will have a cost optimal number of passes
- Have a single arc rather than several
  - But magnets in FFA arc are more expensive

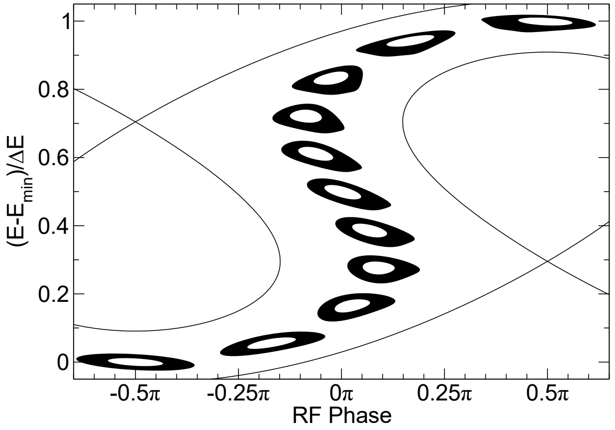




Serpentine Acceleration

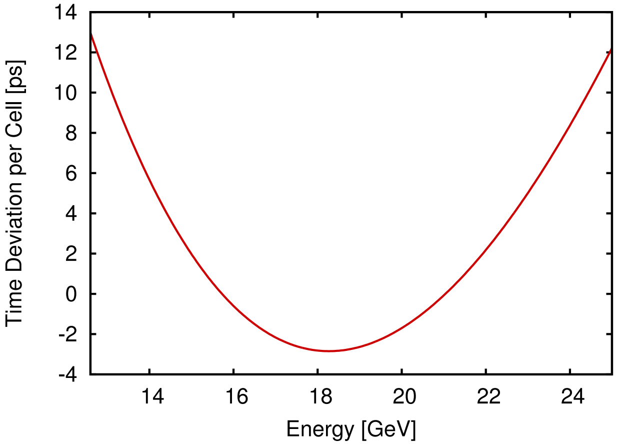
- Time of flight vs. energy can be adjusted to be parabolic-like
    - Results in higher peak magnetic fields
- Acceleration in serpentine channel
    - RF frequency doesn’t change
    - Time of flight synchronized to RF for two energies
    - RF crest is crossed 3 times
    - Bunch passes through channel between two separatrices
- EMMA experiment used serpentine acceleration

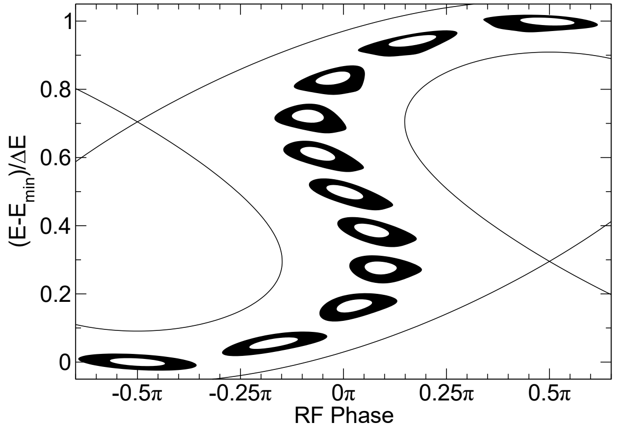




Serpentine Acceleration

- Channel width related to RF voltage divided by the time of flight range: $a = \frac{V}{\omega \Delta T \Delta E}$
- a sufficiently small, channel collapses
- Smaller a, greater longitudinal emittance growth
- Larger a, fewer RF passes, more cost
- Lower RF frequency, lower a but higher cost
- Reduce ΔT by
    - Shorter cells, but requires higher magnet strength
    - Less bend per cell, leading to longer ring
        - More decays for given voltage, or more voltage & fewer turns



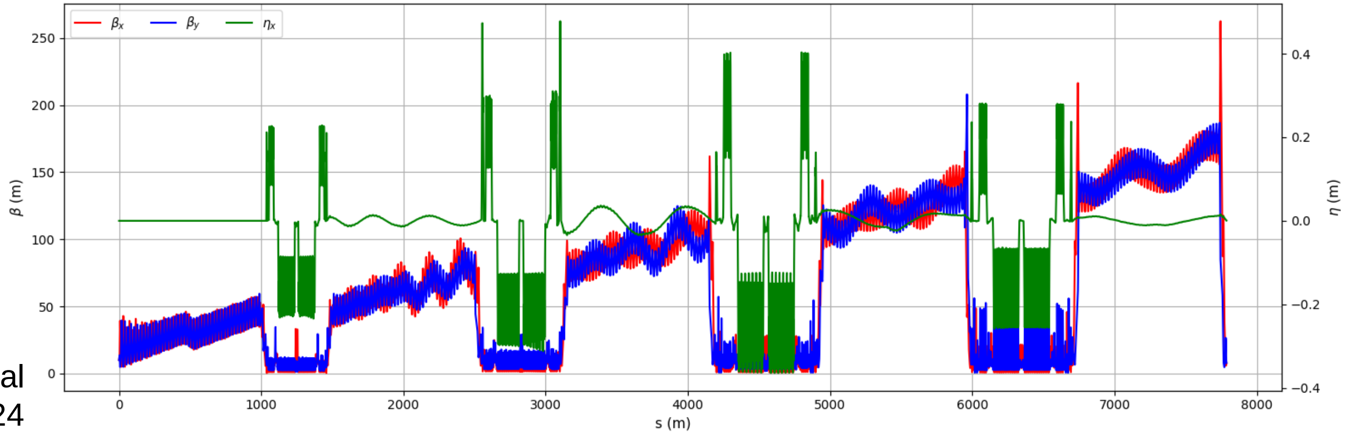


Conclusions from Earlier Studies

- FFAs studied for neutrino factory muon acceleration, to 5 or 10 GeV
  - Larger transverse emittance but smaller longitudinal emittance than for a muon collider
  - A neutrino factory was considered a first stage of a muon collider facility; unclear if there is currently physics interest for this
- Did not appear to have a cost advantage
  - Longitudinal emittance preservation forced FFAs to relatively few turns
  - Transverse beam size gave magnets large aperture
  - More FFA rings than RLA stages required

Should FFAs for Low Energy Acceleration be Revisited?

- After MAP, studies by Berg and IMCC designs indicated that RLA arcs become long and complex for emittance preservation
- With smaller transverse and larger longitudinal emittance for collider beams, the design tradeoffs are different
  - In particular, problems related to large transverse emittance and chromaticity would be reduced significantly



A. Aksoy, IMCC/MuCol Annual Meeting, 2024

Should FFAs for Low Energy Acceleration be Revisited?

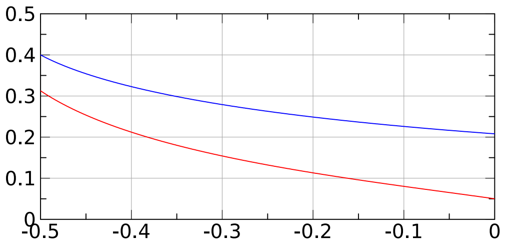
- Could nonlinear fields help?
  - Nonlinear fields may increase energy range or improve serpentine acceleration channel width
  - Even small nonlinearities gave dynamic aperture problems for neutrino factory beams, but probably OK for muon collider emittances
- Nonlinear fields were simply added to non-scaling FFAs in earlier studies, no attempt to consider any cost impacts from higher fields

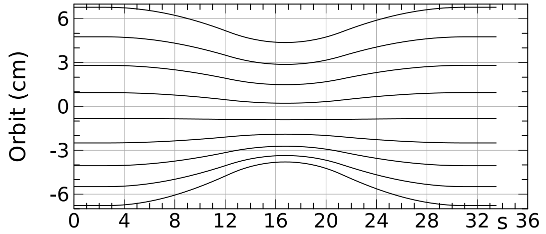
FFAs for High Energy Muon Acceleration

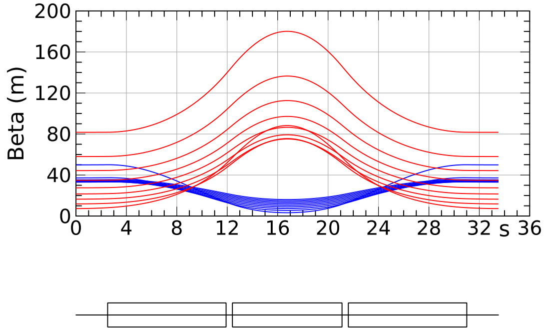
- Acceleration times are long enough that RF manipulations could be possible
  - Less power required into input coupler to shift phase and replace stored energy in cavities
  - Higher frequency RF may be possible
- May not be constrained to use serpentine acceleration
  - Can optimize design for lower magnet fields, important for reaching high energies
- Energy reach of a given footprint proportional to achievable superconducting magnetic field, unlike hybrid synchrotrons

First Study of High Energy FFA Acceleration

- Ring that just fits on Fermilab site
- 3.5 MV/m average accelerating gradient
- Minimize maximum field at magnet coil
  - Trying to get maximum energy reach
  - Defined so that 4.5σ beam is at 2/3 coil radius
- Optimizing for maximum field produces designs that differ from what has been found for past studies (neutrino factory, EMMA, CBETA, etc.)

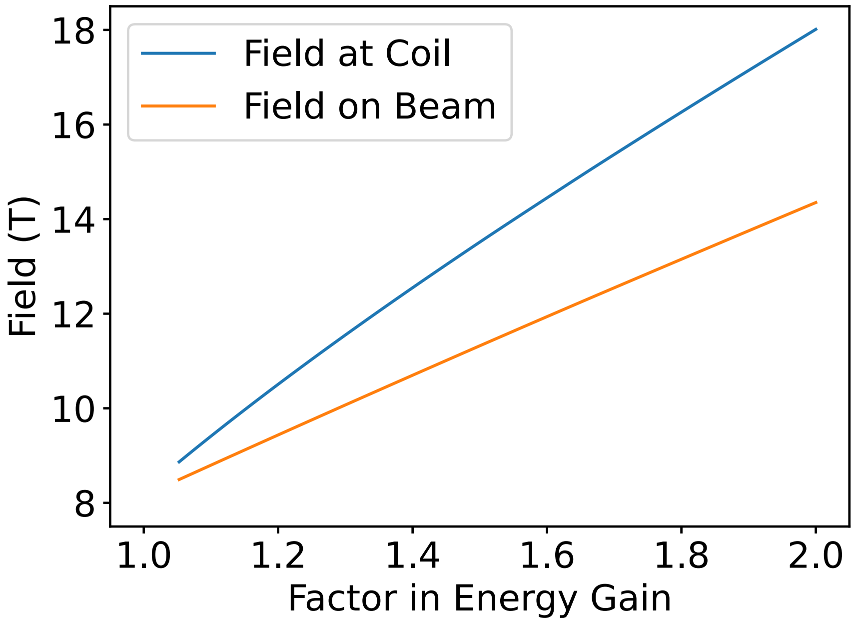






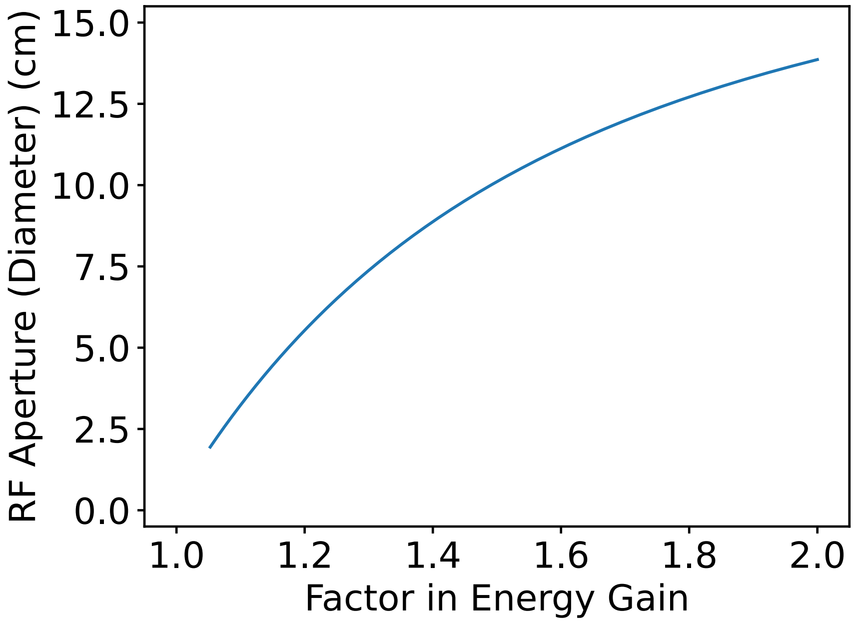
Field and Energy Range

- Assume maximum energy of 5 TeV
- Magnet field depends on minimum energy
- Plot shows field at coil, at 1.5 times beam radius, and field at beam
- Factor of 2 energy gain possible, but high fields
- Limitations similar to pulsed synchrotron
  - Minimum energy 3.1–3.6 GeV for 5 TeV max for 12.5 T max
  - Factor of 2, maximum energy 3.5–4.4 TeV for 12.5 T max
- Similar to (somewhat better than?) pulsed synchrotron numbers



Required Aperture, RF Consequences

- For factor of 2 acceleration, beam too large for Tesla cavity aperture
- 650 MHz probably possible, but gradient may be reduced
- Large apertures for superconducting magnets



Injection/Extraction

- Take advantage of orbit position dependence on energy
- For 0.2 T kickers, about 3 straights for extraction
- Challenge is extraction septum
  - High energies require high fields
  - Straights are short, and have to clear superconducting magnet cryostat
  - Ideas to manage:
    - Generate angle and position at septum
    - Pipe penetrating into aperture
    - Special magnets with larger apertures (higher fields!)
    - Longer straights: maybe taper straight length (oval shaped ring)

Conclusions and Future Directions

Conclusions

- FFAs are an alternative to more conventional acceleration techniques for muon acceleration that can address issues with those conventional solutions
- In particular FFAs may achieve greater energy reach on the Fermilab site as superconducting magnets become capable of reaching higher fields, while pulsed synchrotrons could not take advantage of those advances
- At lower energies, FFAs could provide a less costly alternative for acceleration
- There are still issues to address in FFA designs, in particular injection and extraction at high energies

Future Directions

- Oval shaped FFAs for high energy acceleration
    - Adiabatically transition to longer cells that leave more space for injection/extraction
- Revisit FFAs at lower energies, using muon collider beam emittances, and comparing to latest RLA designs
- Incorporating nonlinearities in non-scaling designs
    - Improve energy range and/or longitudinal machine dynamics at lower energies
    - At high energies, only small nonlinearities expected to help, due to field limitations
- Most studies are very preliminary, more detailed work needed