”Progress in $b \to s\gamma$”

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“Christophest: Precision Predictions for FCNC Processes”,
Institute for Theoretical Physics, University of Bern, Bern, Switzerland, October 25th, 2024

1. **Introduction & Motivation**

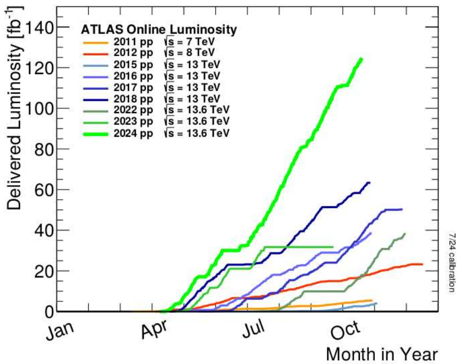
2. **Perturbative QCD effects:**
   Travelling from LO to NNLO with Christoph Greub

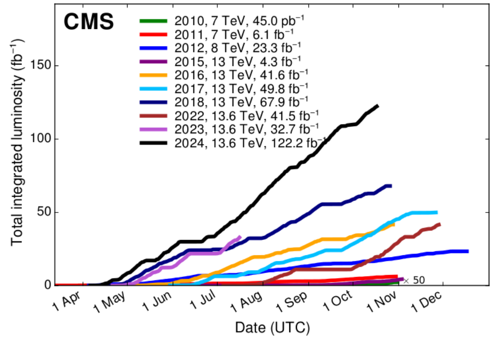
3. **Non-perturbative QCD effects [$\to$ talk of Tobias Hurth today]**

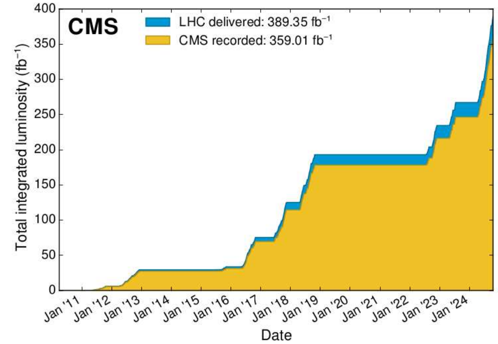
4. ~~**Electroweak and CKM-suppressed corrections.**~~

5. **Summary and outlook**

The current (October 2024) ATLAS and CMS luminosity plots:



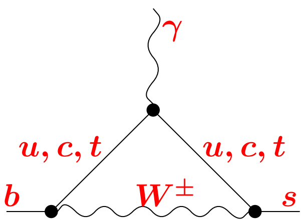


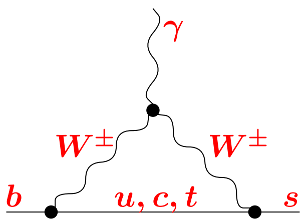


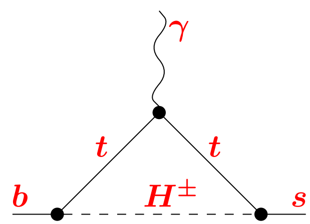
Direct searches give strong bounds on strongly interacting particles (e.g., $m_{\text{gluino}} \gtrsim 2\,\text{TeV}$), but rather weak bounds on only weakly interacting ones.

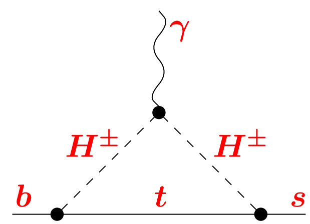
For instance, in the 2HDM-II, the direct-search limit on $M_{H^\pm}$ is still below m_t.

On the other hand, the current (preliminary) bound on $M_{H^\pm}$ from $b \to s\gamma$ is around 650 GeV.



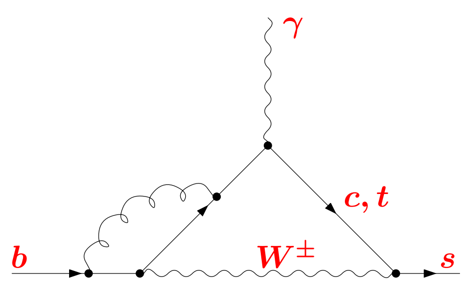






This bound would not be so stringent if Christoph and c.o. have not taken care of continuous accuracy improvement in the SM contribution.

Importance of perturbative QCD effects in $b \to s\gamma$:



S. Bertolini, F. Borzumati and A. Masiero, PRL 59 (1987) 180,
N. G. Deshpande, P. Lo, J. Trampetic, G. Eilam and P. Singer, PRL 59 (1987) 183.

Determination of $\mathcal{B}(\bar{B} \to X_s\gamma)$ in the SM:

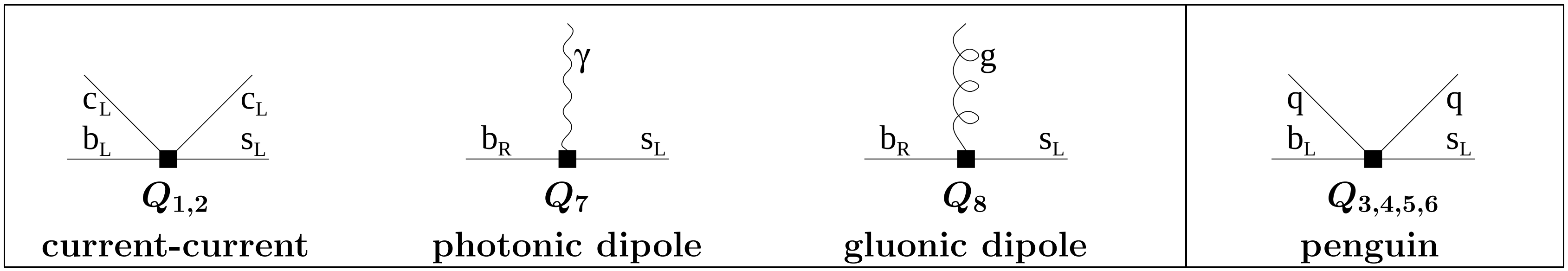
$$\mathcal{B}(\bar{B} \to X_s\gamma)_{E_\gamma > E_0} = \mathcal{B}(\bar{B} \to X_c e\bar{\nu})_{\rm exp} \left|\frac{V_{ts}^* V_{tb}}{V_{cb}}\right|^2 \frac{6\alpha_{\rm em}}{\pi\, C} \left[P(E_0) + N(E_0)\right]$$

pert. $\sim 96\%$ — non-pert. $\sim 4\%$

Conventionally $E_0 = 1.6\,\mathrm{GeV}$.

$$\frac{\Gamma[b \to X_s^p \gamma]_{E_\gamma > E_0}}{|V_{cb}/V_{ub}|^2\, \Gamma[b \to X_u^p e\bar{\nu}]} = \left|\frac{V_{ts}^* V_{tb}}{V_{cb}}\right|^2 \frac{6\alpha_{\rm em}}{\pi} P(E_0) \qquad C = \left|\frac{V_{ub}}{V_{cb}}\right|^2 \frac{\Gamma[\bar{B} \to X_c e\bar{\nu}]}{\Gamma[\bar{B} \to X_u e\bar{\nu}]}$$

Eight WET operators Q_i matter for $\mathcal{B}_{s\gamma}^{\rm SM}$ when the NLO EW & CKM-suppressed effects are neglected:



$$\Gamma(b \to X_s^p \gamma) = \frac{G_F^2\, m_{b,\,\rm pole}^5\, \alpha_{\rm em}}{32\pi^4} \left|V_{ts}^* V_{tb}\right|^2 \sum_{i,j=1}^{8} C_i(\mu_b) C_j(\mu_b) G_{ij}, \qquad (G_{ij} = G_{ji}, \quad \mu_b \sim m_b)$$

RGEs for the WCs : $\mu \frac{d}{d\mu} \vec{C}(\mu) = \hat{\gamma}^T \vec{C}(\mu)$, ADM: $\hat{\gamma} = \frac{\alpha_s}{4\pi}\hat{\gamma}^{(0)} + \left(\frac{\alpha_s}{4\pi}\right)^2 \hat{\gamma}^{(1)} + \left(\frac{\alpha_s}{4\pi}\right)^3 \hat{\gamma}^{(2)} + \ldots$,

Initial (matching) conditions: $\vec{C}(\mu_0) = \vec{C}^{(0)}(\mu_0) + \frac{\alpha_s}{4\pi}\vec{C}^{(1)}(\mu_0) + \left(\frac{\alpha_s}{4\pi}\right)^2 \vec{C}^{(2)}(\mu_0) + \ldots,$ $(\mu_0 \sim m_t, M_W)$

Perturbative expansion of $\hat{G}$: $\hat{G} = \hat{G}^{(0)} + \frac{\alpha_s}{4\pi}\hat{G}^{(1)} + \left(\frac{\alpha_s}{4\pi}\right)^2 \hat{G}^{(2)} + \ldots,$

In the following, focus on G_{77}, G_{78} and G_{27} (the latter stands for "G_{17} and G_{27}").

However, remember the calculation of $C_7^{(1)}(\mu_0)$ by Christoph and Tobias in hep-ph/9703349.

Calculations of G_{77}, G_{78} and G_{27} up to the NNLO in QCD.

$$G_{77} = 1 + \frac{\alpha_s}{4\pi} G^{(1)}_{77} + \left(\frac{\alpha_s}{4\pi}\right)^2 G^{(2)}_{77} + \ldots,$$

$$G_{78} = \frac{\alpha_s}{4\pi} G^{(1)}_{78} + \left(\frac{\alpha_s}{4\pi}\right)^2 G^{(2)}_{78} + \ldots,$$

$$G_{27} = \frac{\alpha_s}{4\pi} G^{(1)}_{27} + \left(\frac{\alpha_s}{4\pi}\right)^2 G^{(2)}_{27} + \ldots,$$

NLO

A. Ali and C. Greub, ZPC 49 (1991) 431, PLB 259 (1991) 182, PLB 361 (1995) 146 [hep-ph/9506374],
C. Greub, T. Hurth and D. Wyler, hep-ph/9602281, hep-ph/9603404.

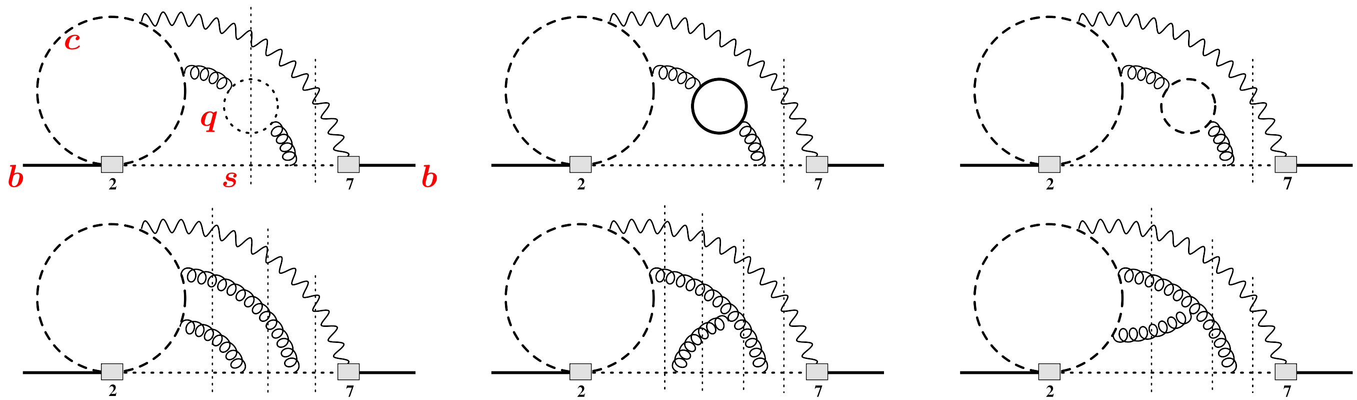
NNLO

$G^{(2)}_{77}$: K. Melnikov and A. Mitov, hep-ph/0505097,
I. Blokland, A. Czarnecki, MM, M. Ślusarczyk and F. Tkachov, hep-ph/0506055,
H. M. Asatrian, A. Hovhannisyan, V. Poghosyan, T. Ewerth, C. Greub and T. Hurth, hep-ph/0605009,
H. M. Asatrian, T. Ewerth, A. Ferroglia, P. Gambino and C. Greub, hep-ph/0607316,
H. M. Asatrian, T. Ewerth, H. Gabrielyan and C. Greub, hep-ph/0611123.

$G^{(2)}_{78}$: H. M. Asatrian, T. Ewerth, A. Ferroglia, C. Greub and G. Ossola, arXiv:1005.5587.

$G^{(2)}_{27}$: Z. Ligeti, M.E. Luke, A.V. Manohar and M.B. Wise, hep-ph/9903305, large-β_0, 4-body,
K. Bieri, C. Greub and M. Steinhauser, hep-ph/0302051, large-β_0, 2-body,
MM and M. Steinhauser, hep-ph/0609241, arXiv:1005.1173, $m_c \gg m_b$,
R. Boughezal, M. Czakon and T. Schutzmeier, arXiv:0707.3090, large-β_0 and massive loops, 2-body,
M. Czakon, P. Fiedler, T. Huber, MM, T. Schutzmeier and M. Steinhauser, arXiv:1503.01791, $m_c = 0$,
MM, A. Rehman and M. Steinhauser, arXiv:1702.07674, arXiv:2002.01548, counterterms, large-β_0 and massive loops,
C. Greub, H. M. Asatrian, F. Saturnino and C. Wiegand, arXiv:2303.01714, physical m_c, partial 2-body,
M. Fael, F. Lange, K. Schönwald and M. Steinhauser, arXiv:2309.14706, physical m_c, full 2-body,
M. Czaja, M. Czakon, T. Huber, MM, M. Niggetiedt, A. Rehman,
K. Schönwald and M. Steinhauser, arXiv:2309.14707, physical m_c, full 2-body,
C. Greub, H. M. Asatrian, H. H. Asatryan, L. Born and J. Eicher, arXiv:2407.17270, physical m_c, full 2-body,
M. Czaja, M. Czakon, T. Huber, MM, M. Niggetiedt, A. Rehman,
K. Schönwald and M. Steinhauser, arXiv:2411.nnnnn, physical m_c, fully inclusive, renormalized result.

Sample propagator diagrams with cuts contributing to G_{27} @ NNLO:



1. Generation of diagrams and performing the Dirac algebra to express everything in terms of (a few) $\times 10^5$ four-loop two-scale scalar integrals with unitarity cuts ($\mathcal{O}(500)$ families).

2. Reduction to master integrals (MIs) with the help of Integration By Parts (IBP) [`KIRA`]. $\mathcal{O}(1\,\text{TB})$ RAM and weeks of CPU needed for the most complicated families.

3. Extending the set of master integrals M_k so that it closes under differentiation with respect to $z = m_c^2/m_b^2$. This way one obtains a system of differential equations

$$\frac{d}{dz} M_k(z,\epsilon) = \sum_l R_{kl}(z,\epsilon)\, M_l(z,\epsilon), \qquad (*)$$

   where R_{nk} are rational functions of their arguments.

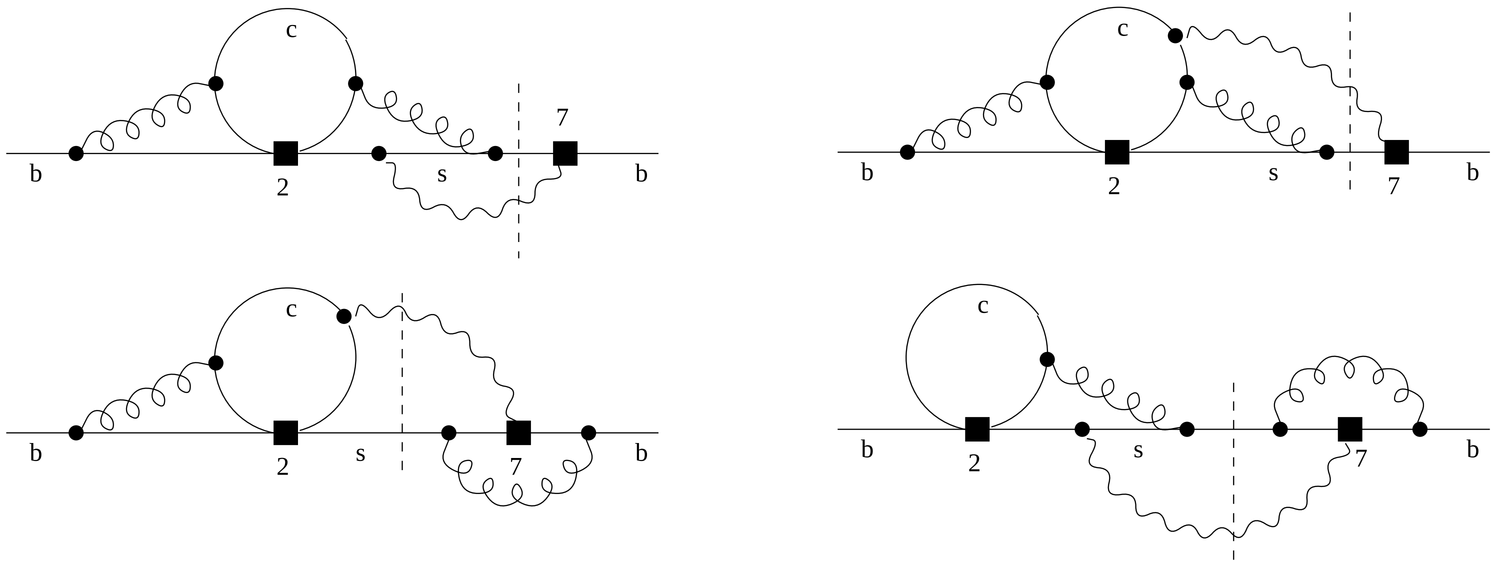
4. Calculating boundary conditions for $(*)$ using automatized asymptotic expansions at $m_c \gg m_b$.

5. Calculating three-loop single-scale master integrals for the boundary conditions.

6. Solving the system $(*)$ numerically [e.g., A.C. Hindmarsch, `http://www.netlib.org/odepack`] along an ellipse in the complex z plane.

Another approach to bare 2-body contributions in arXiv:2309.14707

[M. Czaja, M. Czakon, T. Huber, MM, M. Niggetiedt, A. Rehman, K. Schönwald, M. Steinhauser]



1. The MIs are numerically calculated at the physical value of m_c using `AMFlow` [arXiv:2201.11669].
2. Thus, no expansions in the limit $m_c \gg m_b$ need to be determined. We have tested them though.
3. UV and IR divergences are dimensionally regulated. The 2-body contributions alone are not IR safe.
4. Sample result: $\Delta_{21}\hat{G}_{27}^{(2)2P}(z) = \frac{368}{243\epsilon^3} + \frac{736-324f_0(z)}{243\epsilon^2} + \frac{1}{\epsilon}\left(\frac{1472}{243} + \frac{92}{729}\pi^2 - \frac{8f_0(z)+4f_1(z)}{3}\right) + p(z),$

   where $p(z=0.04) \simeq 144.959811.$

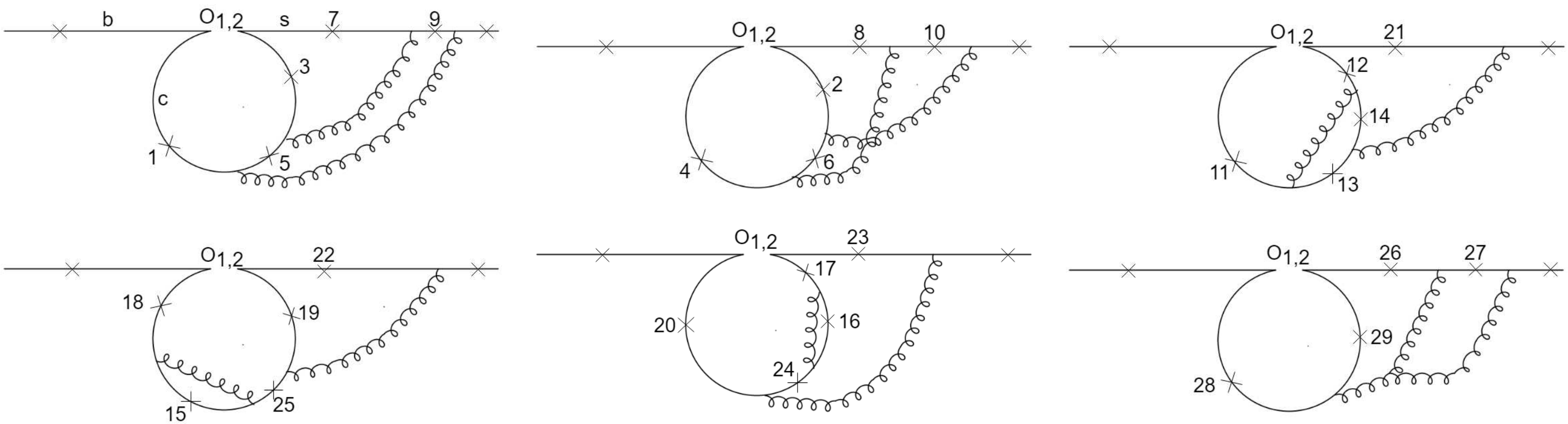
The large-z expansion of $p(z)$ reads:

$$p(z) = \frac{138530}{6561} - \frac{3680}{729}\zeta(3) - \frac{6136}{243}L + \frac{5744}{729}L^2 - \frac{1808}{729}L^3 + \frac{1}{z}\left(-\frac{4222952}{1366875} - \frac{602852}{273375}L + \frac{34568}{18225}L^2 - \frac{532}{1215}L^3\right)$$
$$+ \frac{1}{z^2}\left(-\frac{33395725469}{26254935000} - \frac{111861263}{93767625}L + \frac{156358}{178605}L^2 - \frac{172}{1215}L^3\right) + \mathcal{O}\left(\frac{1}{z^3}\right), \quad \text{with } L = \log z.$$

2-body contributions from vertex diagrams

in arXiv:2303.01714 [C. Greub, H.M. Asatrian, F. Saturnino, C. Wiegand], arXiv:2407.17270 [C. Greub, H. M. Asatrian, H. H. Asatryan, L. Born and J. Eicher],

and arXiv:2309.14706 [M. Fael, F. Lange, K. Schönwald, M. Steinhauser]



1. Amplitudes rather than interference terms.
2. In arXiv:2303.01714: only diagrams with no gluon-(b-quark) couplings.
3. IBP as usual. Then either `AMFlow` or differential equations starting from $m_c \gg m_b$.
4. Simplifying the differential equations and solving them analytically in many cases.
5. Fully analytical solutions at the two-loop level in arXiv:2309.14706.

Fully inclusive (2-, 3- and 4-body), renormalized results for $G_{17}^{(2)}$ and $G_{27}^{(2)}$.

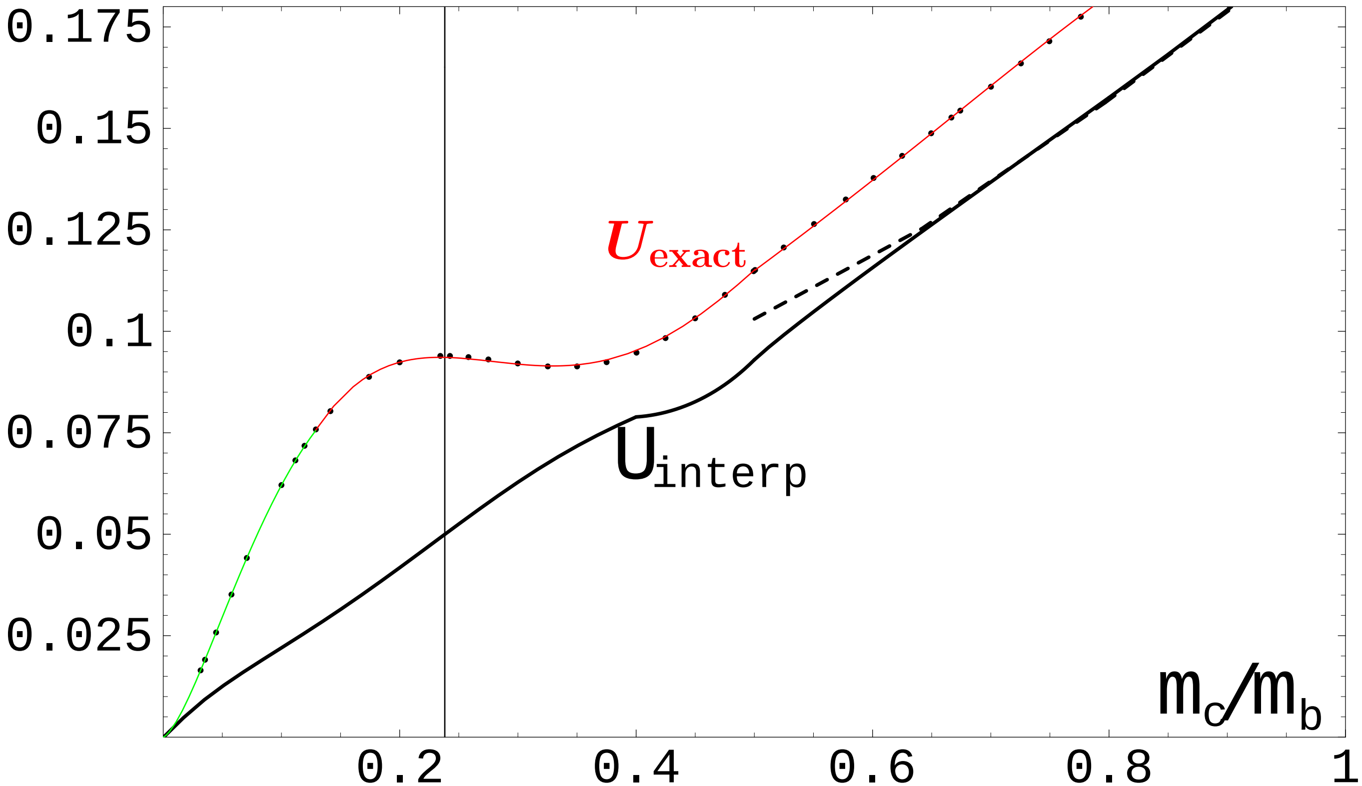
M. Czaja, M. Czakon, T. Huber, MM, M. Niggetiedt, A. Rehman, K. Schönwald and M. Steinhauser, arXiv:2411.nnnnn.

All the $\frac{1}{\epsilon^n}$ poles have cancelled with better than 10^{-55} accuracy.

Comparison to the interpolated NNLO correction in arXiv:1503.01791.

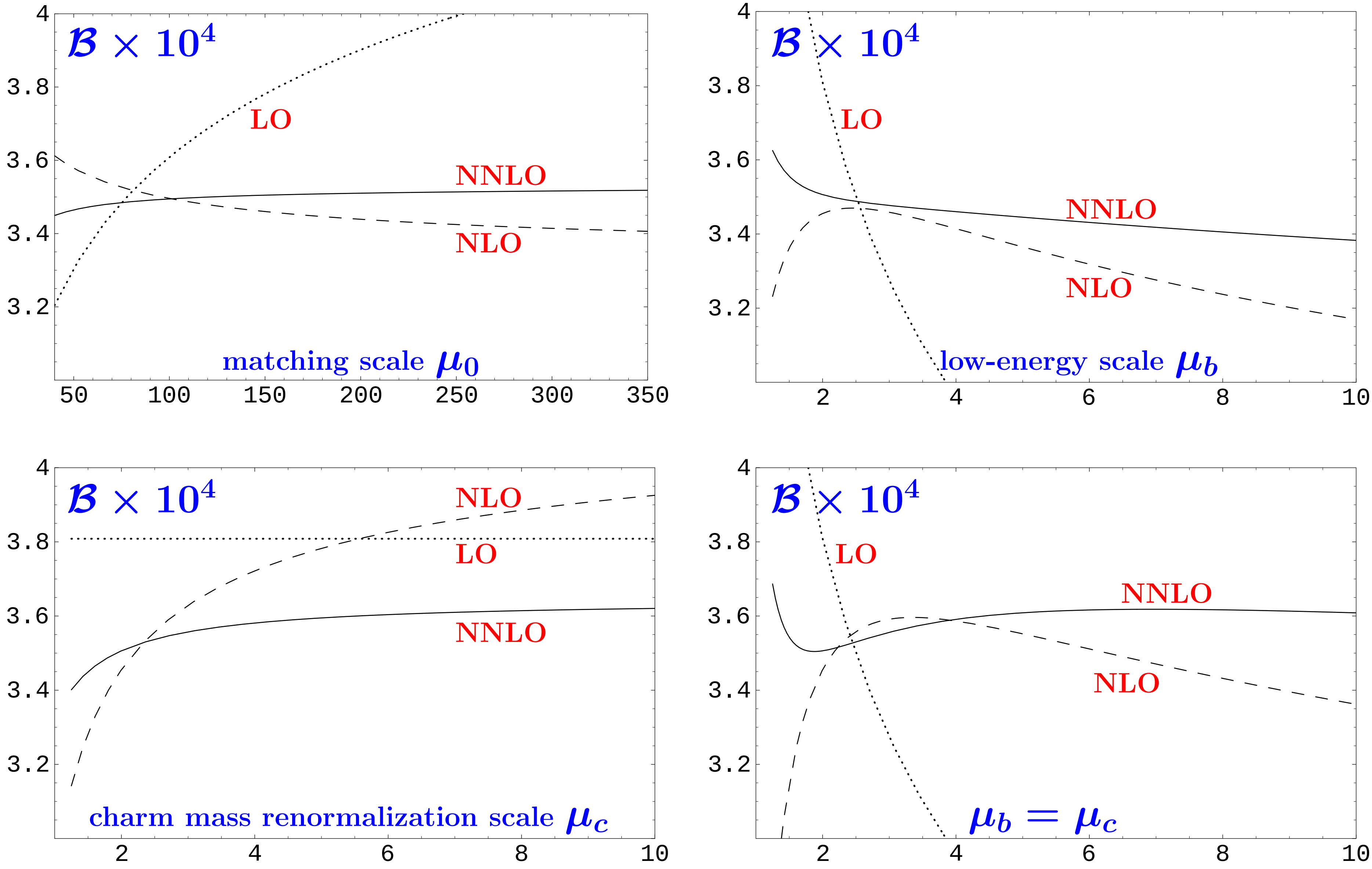
$$\frac{\Delta\mathcal{B}_{s\gamma}}{\mathcal{B}_{s\gamma}} \simeq U(z,\delta) \equiv \frac{\alpha_s^2(\mu_b)}{8\pi^2} \; \frac{C_1^{(0)}(\mu_b)F_1(z,\delta)+\left(C_2^{(0)}(\mu_b)-\frac{1}{6}C_1^{(0)}(\mu_b)\right)F_2(z,\delta)}{C_7^{(0)\text{eff}}(\mu_b)} \qquad \left(z=\frac{m_c^2}{m_b^2}\right)$$

Interpolated and exact results for $\delta = 1$ (no cut on E_γ):



Renormalization scale dependence of $\mathcal{B}(\bar{B} \to X_s\gamma)_{E_\gamma > 1.6\ \mathrm{GeV}}$

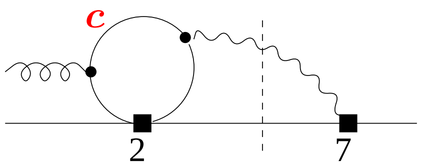
(The global normalization is "preliminary" due to an outdated fit formula for the semileptonic ratio C.)



"Central" values: $\mu_0 = 160$ GeV, $\mu_b = 2$ GeV, $\mu_c = 2$ GeV.

Resolved photon contribution to the Q_7-$Q_{1,2}$ interference.

M.B. Voloshin, hep-ph/9612483; A. Khodjamirian, R. Rückl, G. Stoll and D. Wyler, hep-ph/9702318;
Z. Ligeti, L. Randall and M.B. Wise, hep-ph/9702322; G. Buchalla, G. Isidori, G. Rey, hep-ph/9705253;
M. Benzke, S.J. Lee, M. Neubert, G. Paz, arXiv:1003.5012; A. Gunawardana, G. Paz, arXiv:1908.02812;
M. Benzke, T. Hurth, arXiv:2006.00624.

$\langle \bar{B}|$  $|\bar{B}\rangle$

$$\delta \mathrm{N}(\mathrm{E}_0) = (C_2 - \tfrac{1}{6}C_1)C_7 \underbrace{\left[-\frac{\mu_G^2}{27 m_c^2} + \frac{\Lambda_{17}}{m_b}\right]}_{-\frac{\kappa_V \mu_G^2}{27 m_c^2}}$$

$$\Lambda_{17} = \tfrac{2}{3}\mathrm{Re}\int_{-\infty}^{\infty} \frac{d\omega_1}{\omega_1}\left[1 - F\left(\frac{m_c^2 - i\varepsilon}{m_b\omega_1}\right) + \frac{m_b\omega_1}{12 m_c^2}\right] h_{17}(\omega_1,\mu)$$

$\omega_1 \leftrightarrow$ gluon momentum, $\quad F(x) = 4x \arctan^2\left(1/\sqrt{4x-1}\right)$

The soft function h_{17}:

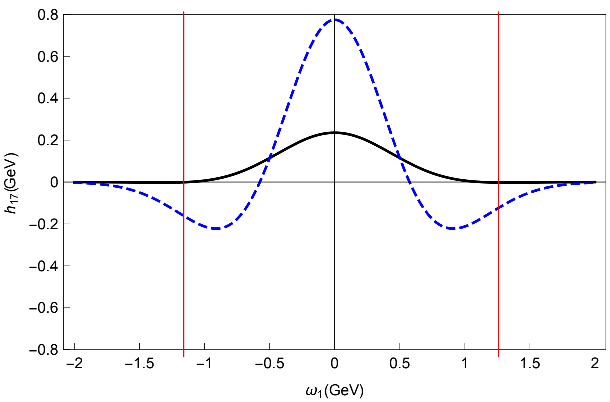
$$h_{17}(\omega_1,\mu) = \int \frac{dr}{4\pi M_B} e^{-i\omega_1 r} \langle \bar{B}|(\bar{h}S_{\bar{n}})(0)\not{\bar{n}}\, i\gamma_\alpha^\perp \bar{n}_\beta (S_{\bar{n}}^\dagger g G_s^{\alpha\beta} S_{\bar{n}})(r\bar{n})(S_{\bar{n}}^\dagger h)(0)|\bar{B}\rangle \qquad (m_b - 2E_0 \gg \Lambda_{\rm QCD})$$

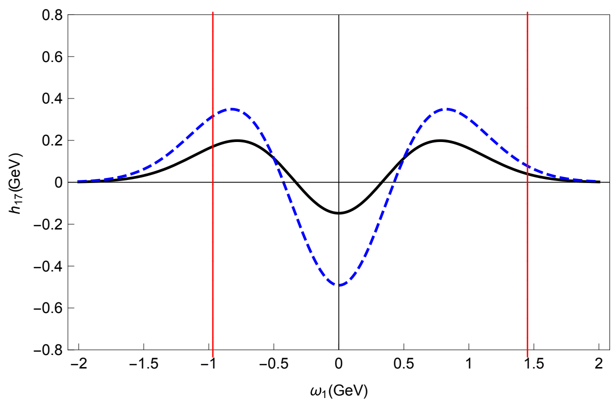
A class of models for h_{17}:

$$h_{17}(\omega_1,\mu) = e^{-\frac{\omega_1^2}{2\sigma^2}} \sum_n a_{2n} H_{2n}\left(\frac{\omega_1}{\sigma\sqrt{2}}\right), \qquad \sigma < 1\ \mathrm{GeV}$$

Hermite polynomials

Constraints on moments (e.g.): $\quad \int d\omega_1 h_{17} = \frac{2}{3}\mu_G^2, \qquad \int d\omega_1 \omega_1^2 h_{17} = \frac{2}{15}(5m_5 + 3m_6 - 2m_9).$





G+P numerically:
$\Lambda_{17} \in [-24, 5]$ MeV for $m_c = 1.17$ GeV.

In our code: $\kappa_V = 1.2 \pm 0.3$.
Warning: scheme for m_c!

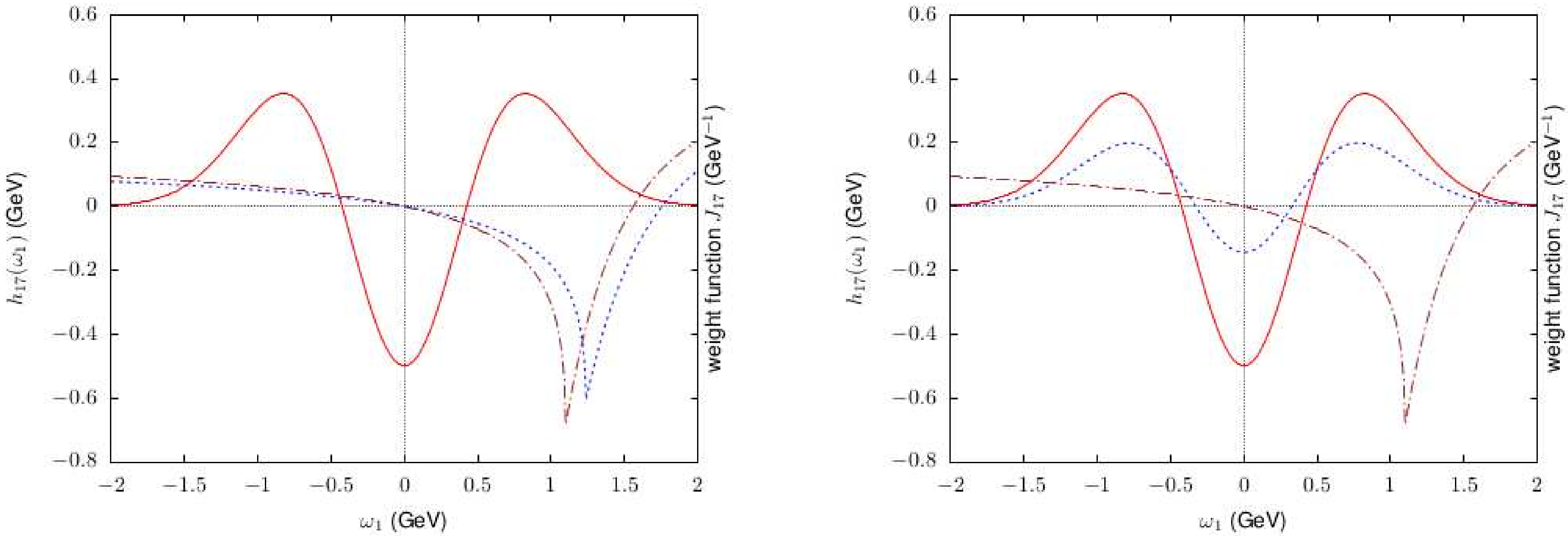
Moment constraints vs. models of h_{17}

M. Benzke, S.J. Lee, M. Neubert, G. Paz, arXiv:1003.5012 – only the leading moment included.

A. Gunawardana, G. Paz, arXiv:1908.02812 – estimates of the subleading moments from LLSA included.

M. Benzke, T. Hurth, arXiv:2006.00624 – as above but with more generous modeling and partial $1/m_b^2$ corrections.

Plots from the latter article:



Another recent contribution: clarifying the SCET treatment of resolved photons in the Q_8-Q_8 interference; T. Hurth and R. Szafron, arXiv:2301.01739.

Summary and outlook

- **Updated SM prediction (preliminary):**

$$\mathcal{B}_{s\gamma}^{\rm SM} = (3.51 \pm 0.14) \times 10^{-4} \quad (\pm 4.0\%).$$

No interpolation in m_c, input parameters of 2024, uncertainty treatment as in arXiv:2002.01548.
Remaining issue: The semileptonic ratio C in the global normalization. See arXiv:2411.nnnnn.

- **Current experimental world average (PDG 2024, HFLAV 2024):**

$$\mathcal{B}_{s\gamma}^{\rm SM} = (3.49 \pm 0.19) \times 10^{-4} \quad (\pm 5.5\%).$$

Belle II prospects: $\pm 2.6\%$, arXiv:1808.10567.

- **Perturbative outlook:**

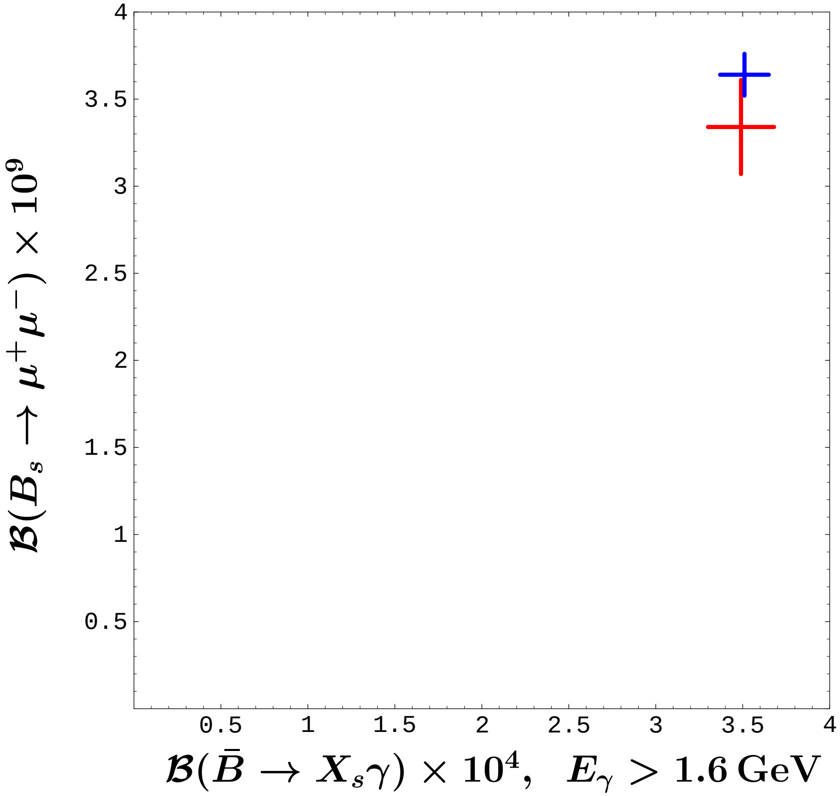
Make the NLO formally complete [being finalized by Tobias Huber & c.o.]
Include $E_\gamma > E_0$ in $G_{17}^{(2)}$ and $G_{27}^{(2)}$.
Calculate other than 2-body contributions in $G_{11}^{(2)}$, $G_{12}^{(2)}$, $G_{22}^{(2)}$, $G_{18}^{(2)}$ and $G_{28}^{(2)}$.
Calculate $G_{ij}^{(2)}$ without neglecting Q_3-Q_6.
N^3LO ?

- **Non-perturbative outlook:**

Complete the $\frac{1}{m_b^2}$ resolved-photon corrections in the $Q_{1,2}$-Q_7 interference.
Use arXiv:2301.01739 to update the Q_8-Q_8 interference.
…?

BACKUP SLIDES

SM predictions vs. measurements for $\mathcal{B}(\bar{B} \to X_s\gamma)$ and $\mathcal{B}(B_s \to \mu^+\mu^-)$



$\mathcal{B}(\bar{B} \to X_s\gamma)^{\mathrm{exp}}_{E_\gamma>1.6} \times 10^4 = 3.49 \pm 0.19 \quad (\pm 5.4\%)$

CLEO, BaBar and Belle measurements combined by PDG [2022,2024] and HFLAV [arXiv:2206.07501].

$\mathcal{B}(\bar{B} \to X_s\gamma)^{\mathrm{SM}}_{E_\gamma>1.6} \times 10^4 = 3.51 \pm 0.14 \quad (\pm 4.0\%)$

Preliminary for 2411.nnnnn.

$\mathcal{B}(B_s \to \mu^+\mu^-)^{\mathrm{exp}} \times 10^9 = 3.34 \pm 0.27 \quad (\pm 8.1\%)$

LHCb, CMS and ATLAS measurements combined by PDG [2024].

$\mathcal{B}(B_s \to \mu^+\mu^-)^{\mathrm{SM}} \times 10^9 = 3.64 \pm 0.12 \quad (\pm 3.3\%)$

arXiv:1311.0903 by C. Bobeth, M. Gorbahn, T. Hermann, MM, E. Stamou, M. Steinhauser with parameter updates and -0.5% QED correction from arXiv:1907.07011 by M. Beneke, C. Bobeth and R. Szafron. See arXiv:2407.03810 by M. Czaja and MM.