$B - \overline{B}$ mixing

Christophest: Precision Predictions for FCNC Processes — Bern, October 25, 2024

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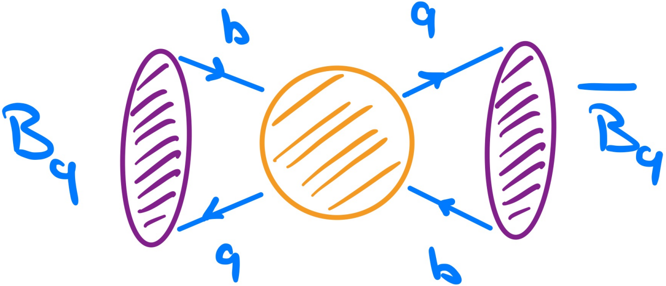

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Next-to-leading order QCD corrections to the lifetime difference of B_s mesons

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Abstract

We compute the QCD corrections to the decay rate difference in the B_s–$\bar{B}_s$ system, $\Delta\Gamma_{B_s}$, in the next-to-leading logarithmic approximation using the heavy quark expansion approach. Going beyond leading order in QCD is essential to obtain a proper matching of the Wilson coefficients to the matrix elements of local operators from lattice gauge theory. The lifetime difference is reduced considerably at next-to-leading order. We find $(\Delta\Gamma/\Gamma)_{B_s} = (f_{B_s}/210\,\mathrm{MeV})^2[0.006\,B(m_b) + 0.150\,B_S(m_b) - 0.063]$ in terms of the bag parameters B, B_S in the NDR scheme. As a further application of our analysis we also derive the next-to-leading order result for the mixing-induced CP asymmetry in inclusive $b \to u\bar{u}d$ decays, which measures $\sin 2\alpha$.

PACS: 12.38.Bx; 13.25.Hw; 14.40.Nd

1. Introduction

The width difference $(\Delta\Gamma/\Gamma)_{B_s}$ of the B_s meson CP eigenstates [1] is expected to be about 10–20%, among the largest rate differences in the b-hadron sector [2], and might be measured in the near future. A measurement of a sizeable $(\Delta\Gamma/\Gamma)_{B_s}$ would open up the possibility of novel CP violation studies with B_s mesons [3,4]. In principle, a measured value for $\Delta\Gamma_{B_s}$ could also give some information on the mass difference ΔM_s [5], if the theoretical prediction for the ratio $(\Delta\Gamma/\Delta M)_s$ can be sufficiently well con-

larger) than expected in the standard model. For this reason a lower bound on the standard model prediction is of special interest.

The calculation of inclusive non-leptonic b-hadron decay observables, such as $\Delta\Gamma_{B_s}$, uses the heavy quark expansion (HQE). The decay width difference is expanded in powers of Λ_{QCD}/m_b, each term being multiplied by a series of radiative corrections in $\alpha_s(m_b)$. In the case of $(\Delta\Gamma/\Gamma)_{B_s}$, the leading contribution is parametrically of order $16\pi^2(\Lambda_{QCD}/m_b)^3$. In the framework of the HQE the main ingredients for a reliable prediction (less than 10% uncer-

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The $B^+ - B_d^0$ lifetime difference beyond leading logarithms

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Abstract

We compute perturbative QCD corrections to the lifetime splitting between the charged and neutral B meson in the framework of the heavy quark expansion. These next-to-leading logarithmic corrections are necessary for a meaningful use of hadronic matrix elements of local operators from lattice gauge theory. We find the uncertainties associated with the choices of renormalization scale and scheme significantly reduced compared to the leading-order result. We include the full dependence on the charm-quark mass m_c without any approximations. Using hadronic matrix elements estimated in the literature with lattice QCD we obtain $\tau(B^+)/\tau(B_d^0) = 1.053 \pm 0.016 \pm 0.017$, where the effects of unquenching and $1/m_b$ corrections are not yet included. The lifetime difference of heavy baryons Ξ_b^0 and Ξ_b^- is also briefly discussed.

PACS: 12.38.Bx; 13.25.Hw; 14.40.Nd

1. Preliminaries

The *Heavy Quark Expansion* (HQE) technique provides a well-defined QCD-based

The $B^+ - B_d^0$ Lifetime Difference Beyond Leading Logarithms #1

Martin Beneke (Aachen, Tech. Hochsch.), Gerhard Buchalla (CERN), Christoph Greub (Bern U.), Alexander Lenz (Regensburg U.), Ulrich Nierste (CERN and Fermilab) (Feb, 2002)

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Next-to-leading order QCD corrections to the lifetime difference of B(s) mesons #2

M. Beneke (CERN), G. Buchalla (CERN), C. Greub (Bern U.), A. Lenz (Munich, Max Planck Inst.), U. Nierste (DESY) (Aug, 1998)

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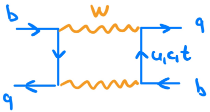
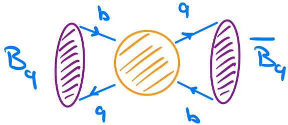
Lifetime differences

- neutral B mesons: B_d, B_s
- weak interaction
- $\Delta B = 2$:
  $B_q \sim (\bar{b}, q) \leftrightarrow (b, \bar{q}) \sim \overline{B}_q$, $q = d, s$

  mass matrix: M^q decay matrix: Γ^q



- M^q_{12}: dominated by top quarks
  Γ^q_{12}: internal u, c quarks



$$\frac{\Delta\Gamma_q}{\Delta M_q} = -\mathrm{Re}\frac{\Gamma^q_{12}}{M^q_{12}}$$

$$\Delta M_q = M^q_H - M^q_L \qquad \Delta\Gamma_q = \Gamma^q_L - \Gamma^q_H$$

$$|B_{q,L}\rangle = p|B_q\rangle + q|\bar{B}_q\rangle \qquad |B_{q,H}\rangle = p|B_q\rangle - q|\bar{B}_q\rangle$$

Experiment

[. . . ,CLEO,BABAR,Belle,CDF,D0,ATLAS,CMS,LHCb] [HFLAV'22]

$$\Delta M_s^{\mathrm{exp}} = (17.765 \pm 0.006)\ \mathrm{ps}^{-1}$$
$$\Delta M_d^{\mathrm{exp}} = (0.5065 \pm 0.0019)\ \mathrm{ps}^{-1}$$

$$\Delta \Gamma_s^{\mathrm{exp}} = (0.083 \pm 0.005)\ \mathrm{ps}^{-1}$$
$$\Delta \Gamma_d^{\mathrm{exp}} = (0.001 \pm 0.010)\ \mathrm{ps}^{-1}$$

Experiment

Karlsruhe Institute of Technology

[. . . ,CLEO,BABAR,Belle,CDF,D0,ATLAS,CMS,LHCb] [HFLAV'22]

$$\Delta M_s^{\mathrm{exp}} = (17.765 \pm 0.006)\ \mathrm{ps}^{-1}$$
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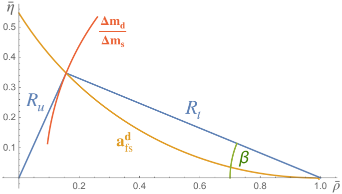
$$\Delta\Gamma_s^{\mathrm{exp}} = (0.083 \pm 0.005)\ \mathrm{ps}^{-1}$$
$$\Delta\Gamma_d^{\mathrm{exp}} = (0.001 \pm 0.010)\ \mathrm{ps}^{-1}$$

- ΔM_q sensitive to NP with masses $> \mathcal{O}(100)$ TeV
- $\Delta\Gamma_q$ probes light new particles; "small" ➪ sensitive to NP
- $\Delta\Gamma_s/\Delta M_s$: robust; compare theory and experiment
- needed: small perturbative and non-perturbative uncertainties
- ΔM_q [Buras,Jamin,Weisz'90] here: $\Delta\Gamma_q$

$B-\overline{B}$ mixing and the Unitarity Triangle

- Information only from mixing parameters
- 2 measurements ➪ fix apex
  3$^{\rm rd}$ measurement ➪ check
- $\Delta M_d/\Delta M_s$ ➪ R_t
- $a_{CP}(B_d(t)\to J/\psi K_S)$ ➪ β
- Independent 3$^{\rm rd}$ observable:
  - $R_u \sim |V_{ub}/V_{cb}| \leftrightarrow$ "exclusive vs. inclusive"
  - But: $a_{\rm fs}^d \propto \frac{\sin\beta}{R_t} = \frac{\bar\eta}{\sqrt{(1-\bar\rho)^2+\bar\eta^2}}$

  [Beneke,Buchalla,Lenz,Nierste'02]
- Note: $a_{fs}^d = \mathcal{O}(m_c^2/m_b^2)$
- $a_{\rm fs}^q = \frac{\Gamma(\overline{B}_q(t)\to f)-\Gamma(B_q(t)\to f)}{\Gamma(\overline{B}_q(t)\to f)+\Gamma(B_q(t)\to f)} = {\rm Im}\frac{\Gamma_{12}^q}{M_{12}^q}$
  CP asymmetry in flavour-specific $B_q \to f$ decays
  (i.e. $\overline{B}_q \to f$ and $B_q \to \bar f$ are forbidden)

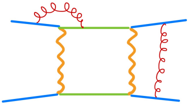


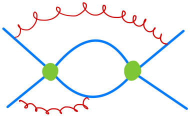
$a_{\rm fs}^{d,\rm exp} = -0.0021 \pm 0.0017$
$a_{\rm fs}^{s,\rm exp} = -0.0006 \pm 0.0028$

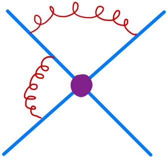
[HFLAV'22]

Effective theories

$$\text{SM} \quad \rightarrow \quad \mathcal{H}_{\text{eff}}^{|\Delta B|=1} \quad \rightarrow \quad \mathcal{H}_{\text{eff}}^{|\Delta B|=2}$$



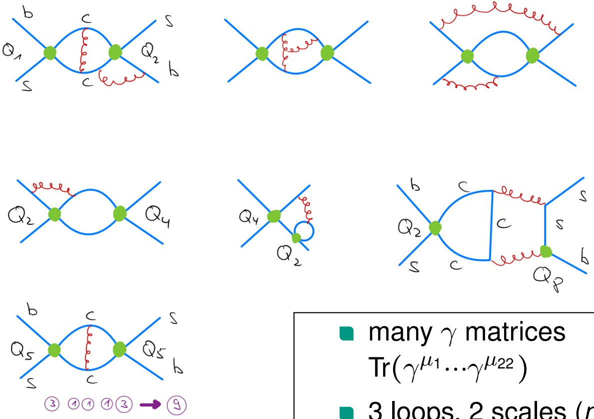




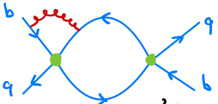
$$\Delta\Gamma \quad \sim \quad H^{ab}(m_c/m_b)\langle B_s|Q|\bar{B}_s\rangle + \widetilde{H}_S^{ab}(m_c/m_b)\langle B_s|\widetilde{Q}_S|\bar{B}_s\rangle + \ldots$$

$\Delta B = 1$

$$\mathcal{H}_{\text{eff}}^{|\Delta B|=1} = \frac{4G_F}{\sqrt{2}}\left[-\lambda_t^s\left(\sum_{i=1}^{6} C_i Q_i + C_8 Q_8\right) - \lambda_u^s \sum_{i=1}^{2} C_i (Q_i - Q_i^u) + \ldots\right.$$



- many γ matrices
  $\text{Tr}(\gamma^{\mu_1} \ldots \gamma^{\mu_{22}})$
- 3 loops, 2 scales (m_c, m_b)
  ➪ expansions



$Q_1, Q_2 \qquad z = m_c^2/m_b^2$

$$\sigma = \frac{1-\sqrt{1-4z}}{1+\sqrt{1-4z}}$$

and similarly for $F_S(z)$. The leading order functions $F_{ij}^{(0)}$, $F_{S,ij}^{(0)}$ read explicitly

$$F_{11}^{(0)}(z) = 3\sqrt{1-4z}\,(1-z)\,,$$
$$F_{S,11}^{(0)}(z) = 3\sqrt{1-4z}\,(1+2z)\,, \qquad (18)$$
$$F_{12}^{(0)}(z) = 2\sqrt{1-4z}\,(1-z)\,,$$
$$F_{S,12}^{(0)}(z) = 2\sqrt{1-4z}\,(1+2z)\,, \qquad (19)$$
$$F_{22}^{(0)}(z) = \tfrac{1}{2}(1-4z)^{3/2}\,,$$
$$F_{S,22}^{(0)}(z) = -\sqrt{1-4z}\,(1+2z)\,. \qquad (20)$$

The next-to-leading order expressions $F_{ij}^{(1)}$, $F_{S,ij}^{(1)}$ are

$$\begin{aligned} F_{11}^{(1)}(z) &= 32(1-z)(1-2z)\big(\mathrm{Li}_2(\sigma^2) \\ &\quad + \ln^2\sigma + \tfrac{1}{2}\ln\sigma\ln(1-4z) - \ln\sigma\ln z\big) \\ &\quad + 64(1-z)(1-2z) \\ &\quad \times\big(\mathrm{Li}_2(\sigma) + \tfrac{1}{2}\ln(1-\sigma)\ln\sigma\big) \\ &\quad - 4(13-26z-4z^2+14z^3)\ln\sigma \\ &\quad + \sqrt{1-4z}\,\big[4(13-10z)\ln z \\ &\quad - 12(3-2z)\ln(1-4z) \\ &\quad + \tfrac{1}{6}(109-226z+168z^2)\big] \\ &\quad + 2\sqrt{1-4z}\,(5-8z)\ln\frac{\mu_2}{m_b}\,, \end{aligned} \qquad (21)$$

$$\begin{aligned} F_{S,11}^{(1)}(z) &= 32(1-4z^2)\big(\mathrm{Li}_2(\sigma^2) \\ &\quad + \ln^2\sigma + \tfrac{1}{2}\ln\sigma\ln(1-4z) - \ln\sigma\ln z\big) \\ &\quad + 64(1-4z^2) \\ &\quad \times\big(\mathrm{Li}_2(\sigma) + \tfrac{1}{2}\ln(1-\sigma)\ln\sigma\big) \\ &\quad - 16(4-2z-7z^2+14z^3)\ln\sigma \\ &\quad + \sqrt{1-4z}\,\big[64(1+2z)\ln z \\ &\quad - 48(1+2z)\ln(1-4z) \\ &\quad - \tfrac{8}{3}(1-6z)(5+7z)\big] \\ &\quad - 32\sqrt{1-4z}\,(1+2z)\ln\frac{\mu_2}{m_b}\,, \end{aligned} \qquad (22)$$

$$\begin{aligned} F_{12}^{(1)}(z) &= \tfrac{64}{3}(1-z)(1-2z)\big(\mathrm{Li}_2(\sigma^2) \\ &\quad + \ln^2\sigma + \tfrac{1}{2}\ln\sigma\ln(1-4z) - \ln\sigma\ln z\big) \\ &\quad + \tfrac{128}{3}(1-z)(1-2z) \\ &\quad \times\big(\mathrm{Li}_2(\sigma) + \tfrac{1}{2}\ln(1-\sigma)\ln\sigma\big) \end{aligned}$$

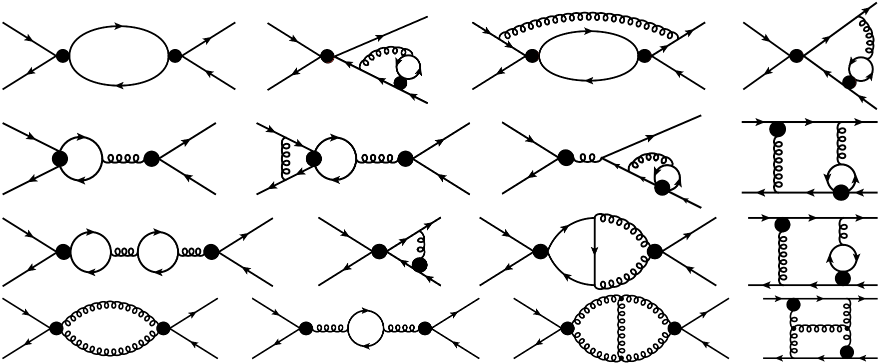
$$\begin{aligned} &\quad + (2-259z+662z^2 \\ &\quad - 76z^3 - 200z^4)\frac{\ln\sigma}{6z} \\ &\quad - \sqrt{1-4z}\,\Big[(2-255z+316z^2)\frac{\ln z}{6z} \\ &\quad + 8(3-2z)\ln(1-4z) \\ &\quad + \tfrac{2}{9}(127-199z-75z^2)\Big] \\ &\quad - 2\sqrt{1-4z}\,(17-26z)\ln\frac{\mu_1}{m_b} \\ &\quad + \tfrac{4}{3}\sqrt{1-4z}\,(5-8z)\ln\frac{\mu_2}{m_b}\,, \end{aligned} \qquad (23)$$

$$\begin{aligned} F_{S,12}^{(1)}(z) &= \tfrac{64}{3}(1-4z^2)\big(\mathrm{Li}_2(\sigma^2) \\ &\quad + \ln^2\sigma + \tfrac{1}{2}\ln\sigma\ln(1-4z) - \ln\sigma\ln z\big) \\ &\quad + \tfrac{128}{3}(1-4z^2) \\ &\quad \times\big(\mathrm{Li}_2(\sigma) + \tfrac{1}{2}\ln(1-\sigma)\ln\sigma\big) \\ &\quad + (1-35z+4z^2 \\ &\quad + 76z^3 - 100z^4)\frac{4\ln\sigma}{3z} \\ &\quad - \sqrt{1-4z} \\ &\quad \times\Big[(1-33z-76z^2)\frac{4\ln z}{3z} \\ &\quad + 32(1+2z)\ln(1-4z)\,. \\ &\quad + \tfrac{4}{9}(68+49z-150z^2)\Big] \\ &\quad - 16\sqrt{1-4z}\,(1+2z)\ln\frac{\mu_1}{m_b} \\ &\quad - \tfrac{64}{3}\sqrt{1-4z}\,(1+2z)\ln\frac{\mu_2}{m_b}\,, \end{aligned} \qquad (24)$$

$$\begin{aligned} F_{22}^{(1)}(z) &= \tfrac{4}{3}(4-21z+2z^2)\big(\mathrm{Li}_2(\sigma^2) \\ &\quad + \ln^2\sigma + \tfrac{1}{2}\ln\sigma\ln(1-4z) - \ln\sigma\ln z\big) \\ &\quad + \tfrac{4}{3}(1-2z)(5-2z) \\ &\quad \times\big(\mathrm{Li}_2(\sigma) + \tfrac{1}{2}\ln(1-\sigma)\ln\sigma\big) \\ &\quad - (7+13z-194z^2 \\ &\quad + 304z^3 - 64z^4)\frac{\ln\sigma}{6z} \end{aligned}$$

2-loop $\Delta B = 1$ Feynman diagrams

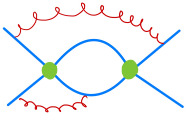
$+Q_3, Q_4, Q_5, Q_6, Q_8$



$\Delta B = 2$

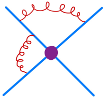
- Heavy Quark Expansion [Khoze,Shifman'83; ...; Manohar,Wise'94]

$$\Gamma_{12}^{s} = \frac{1}{2M_{B_s}} \mathrm{Abs}\langle B_s | i \int \mathrm{d}^4x \; T \, \mathcal{H}_{\mathrm{eff}}^{\Delta B=1}(x) \mathcal{H}_{\mathrm{eff}}^{\Delta B=1}(0) | \bar{B}_s \rangle$$



- $\Delta\Gamma_s$ in terms of $|\Delta B| = 2$ operators [Beneke,Buchalla,Greub,Lenz,Nierste'99; ...]

$$\Gamma_{12}^{s} = -(\lambda_c^s)^2 \Gamma_{12}^{cc} - 2\lambda_c^s \lambda_u^s \Gamma_{12}^{uc} - (\lambda_u^s)^2 \Gamma_{12}^{uu}$$

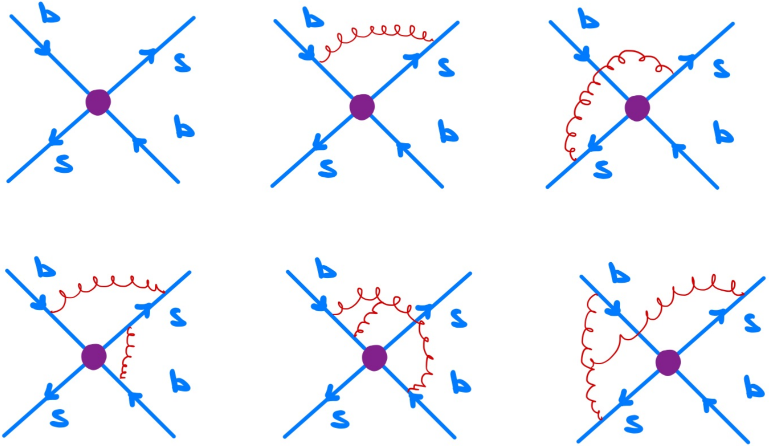


$$\Gamma_{12}^{ab} = \frac{G_F^2 m_b^2}{24\pi M_{B_s}} \left[H^{ab}(z) \langle B_s | Q | \bar{B}_s \rangle + \widetilde{H}_S^{ab}(z) \langle B_s | \widetilde{Q}_S | \bar{B}_s \rangle \right] + \mathcal{O}(\Lambda_{\mathrm{QCD}}/m_b)$$

$$Q = \bar{s}_i \gamma^\mu (1-\gamma^5) b_i \, \bar{s}_j \gamma_\mu (1-\gamma^5) b_j \qquad \widetilde{Q}_S = \bar{s}_i (1+\gamma^5) b_j \, \bar{s}_j (1+\gamma^5) b_i$$

- Nonperturbative MEs from lattice or sum rules [...; Kirk,Lenz,Rauh'17; King,Lenz,Rauh'19'21; Bazavov et al.'16; Dowdall,Davies,Horgan,Lepage,Monahan,et al.'19; Di Luzio,Kirk,Lenz,Rauh'19]
- $H^{ab}(z)$, $\widetilde{H}_S^{ab}(z)$: Wilson coefficients from matching

$\Delta B = 2$ **Feynman diagrams**



IR regulator: dimensionally vs. m_{gluon}

R_0

- Subtelty: 3 operators (Q, Q_S, $\widetilde{Q}_S$) but: $R_0 = \frac{1}{2}Q + Q_S + \widetilde{Q}_S$ is suppressed by $1/m_b$ [Beneke,Buchalla,Dunietz'96]
- Guarantee the $1/m_b$-suppression order-by-order in α_s:

$$\langle R_0 \rangle = \frac{1}{2}\alpha_1 \langle Q \rangle + \alpha_2 \langle Q_S \rangle + \langle \widetilde{Q}_S \rangle$$

with

$$\alpha_i = 1 + \frac{\alpha_s}{4\pi}\alpha_i^{(1)} + \left(\frac{\alpha_s}{4\pi}\right)^2 \alpha_i^{(2)} + \ldots$$

- NLO [Beneke,Buchalla,Greub,Lenz,Nierste'99]: $\alpha_1^{(1)} = 1 + \frac{\alpha_s(\mu_2)}{4\pi} C_F \left(6 + 12 \log \frac{\mu_2}{m_b}\right)$
- NNLO [Asatrian,Hovhannisyan,Nierste,Yeghiazaryan'17] [Gerlach,Shtabovenko,Nierste,Steinhauser'22]

Known results

- LO [. . . , Beneke,Buchalla,Greub,Lenz,Nierste'99; Beneke,Buchalla,Dunietz'96]

- NLO [Beneke,Buchalla,Greub,Lenz,Nierste'99;

  Ciuchini,Franco,Lubicz,Mescia,Tarantino'03; Beneke,Buchalla,Lenz,Ulrich'03; Lenz,Nierste'06;

  Asatrian,Asatryan,Hovhannisyan,Nierste,Tumasyan,Yeghiazaryan'20; Gerlach,Nierste,Shtabovenko,Steinhauser'21'22]

- NNLO n_f part: [Asatrian, Hovhannisyan,Nierste,Yeghiazaryan'17]

  full $Q_{1,2} \times Q_{1,2}$: [Gerlach,Nierste,Shtabovenko,Steinhauser'22][Reeck,Shtabovenko,Steinhauser'24]

Renormalization schemes

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- Calculation: $m_b^{\rm pole}$ and $m_c^{\rm pole}$
- $\frac{\Delta\Gamma_q}{\Delta M_q} \sim m_b^2 \times f\left(z = \overline{m}_c^2(\mu_c)/\overline{m}_b^2(\mu_b)\right)$
- Overall prefactor m_b^2:

$\overline{\rm MS}$
pole
PS [Beneke'98]

$$\begin{aligned} m^{\rm PS}(\mu_f) &= m^{\rm OS} - \delta m(\mu_f) \\ \delta m(\mu_f) &= -\frac{1}{2}\int_{|\vec{q}|<\mu_f} \frac{{\rm d}^3 q}{(2\pi)^3} V(\vec{q}\,) \\ &= \mu_f \frac{C_F \alpha_s}{\pi}\left\{1 + \frac{\alpha_s}{4\pi}\left[a_1 + \beta_0\left(2 + \log\frac{\mu^2}{\mu_f^2}\right)\right] + \ldots\right\} \end{aligned}$$

$V(\vec{q}\,)$: static potential; $\mu_f = 2$ GeV; known to N^{3}LO

Numerical results for $\Delta\Gamma_s/\Delta M_s$

3 loops: expansion up to m_c^2/m_b^2; only Q_1, Q_2 [Gerlach,Nierste,Shtabovenko,Steinhauser'22]

$$\frac{\Delta\Gamma_s}{\Delta M_s} = \left(4.33^{+0.23}_{-0.44}{}_{\mathrm{scale}}{}^{+0.09}_{-0.19}{}_{\mathrm{scale},\,1/m_b} \pm 0.12_{B\tilde{B}_S} \pm 0.78_{1/m_b} \pm 0.05_{\mathrm{input}}\right) \times 10^{-3}\ (\overline{\mathrm{MS}})$$

$$\frac{\Delta\Gamma_s}{\Delta M_s} = \left(4.20^{+0.36}_{-0.39}{}_{\mathrm{scale}}{}^{+0.09}_{-0.19}{}_{\mathrm{scale},\,1/m_b} \pm 0.12_{B\tilde{B}_S} \pm 0.78_{1/m_b} \pm 0.05_{\mathrm{input}}\right) \times 10^{-3}\ (\mathrm{PS})$$

$$\Delta\Gamma_s^{\mathrm{SM}} = (7.6 \pm 1.7) \times 10^{-2}\mathrm{ps}^{-1}$$
$$\Delta\Gamma_s^{\mathrm{exp}} = (8.3 \pm 0.5) \times 10^{-2}\mathrm{ps}^{-1}$$

- $\overline{\mathrm{MS}}$ + PS
- $\mu_1 = \mu_c = \mu_b \in \{2.1, 8.4\}$ GeV
- NLO→ NNLO: scale dependence reduced by factor 2
- uncertainty dominated by $1/m_b$ correction
- pole scheme inadequate
- TODO: NNLO penguin contribution

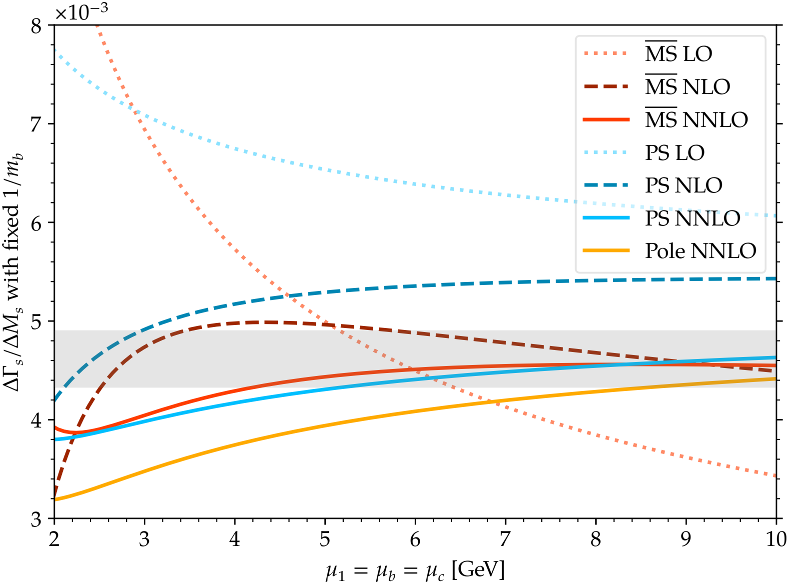
[Albrecht,Bernlochner,Lenz,Rusov'24]:

$$\Delta\Gamma_s^{\mathrm{SM}} = (9.1 \pm 1.5) \times 10^{-2}\mathrm{ps}^{-1}$$

[ratio: $\Delta\Gamma_s^{\mathrm{SM}} = (8.8 \pm 1.4) \times 10^{-2}\mathrm{ps}^{-1}$]

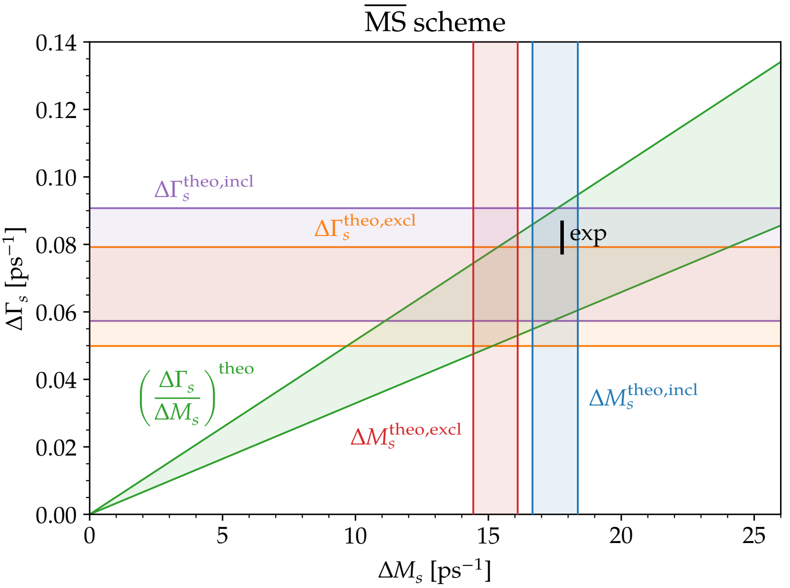
μ dependence

[Gerlach,Nierste,Shtabovenko,Steinhauser'22]



$\Delta\Gamma_s$ vs. ΔM_s

[Gerlach,Nierste,Shtabovenko,Steinhauser'22]



Next steps

- full m_c/m_b dependence (not only $(m_c/m_b)^{0,1,2}$ terms)
  - semi-analytic results for all $\mathcal{O}(500)$ MIs at 3 loops
- penguin operators (Q_3, Q_4, Q_5, Q_6) at NNLO
  - $\mathrm{Tr}(\gamma^{\mu_1} \ldots \gamma^{\mu_{22}})$
  - rank-11 tensor integrals

[Reeck,Shtabovenko,Steinhauser'24]

[Gerlach,Nierste,Reeck,Shtabovenko,Steinhauser in prep.]

Thank you very much for everything!

Thank you very much for everything!

Congratulations

Input parameters

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[PDG; Bazavov et al.'17 (Fermilab-MILC); Dowdall et al.'19 (HPQCD); Chetyrkin et al.'17]

$\alpha_s(M_Z)$	=	0.1179 ± 0.001	$m_c(3\text{ GeV})$	=	0.993 ± 0.008 GeV
m_t^{pole}	=	172.9 ± 0.4 GeV	$m_b(m_b)$	=	4.163 ± 0.016 GeV
M_{B_s}	=	5366.88 MeV	M_{B_d}	=	5279.64 MeV
B_{B_s}	=	0.813 ± 0.034	B_{B_d}	=	0.806 ± 0.041 MeV
$\widetilde{B}'_{S,B_s}$	=	1.31 ± 0.09	$\widetilde{B}'_{S,B_d}$	=	1.20 ± 0.09 MeV
f_{B_s}	=	0.2307 ± 0.0013 GeV	f_{B_d}	=	0.1905 ± 0.0013 MeV

$$\langle B_s|Q(\mu_2)|\overline{B}_s\rangle = \frac{8}{3}M_{B_s}^2 f_{B_s}^2 B_{B_s}(\mu_2)$$
$$\langle B_s|\widetilde{Q}_S(\mu_2)|\overline{B}_s\rangle = \frac{1}{3}M_{B_s}^2 f_{B_s}^2 \widetilde{B}'_{S,B_s}(\mu_2)$$

$$\langle B_s|R_0|\bar{B}_s\rangle = -(0.43 \pm 0.17) f_{B_s}^2 M_{B_s}^2 ,$$

[Davies et al.'19; Dowdall et al.'19]

$$\langle B_s|R_1|\bar{B}_s\rangle = (0.07 \pm 0.00) f_{B_s}^2 M_{B_s}^2 , \qquad \langle B_s|\widetilde{R}_1|\bar{B}_s\rangle = (0.04 \pm 0.00) f_{B_s}^2 M_{B_s}^2 ,$$
$$\langle B_s|R_2|\bar{B}_s\rangle = -(0.18 \pm 0.07) f_{B_s}^2 M_{B_s}^2 , \qquad \langle B_s|\widetilde{R}_2|\bar{B}_s\rangle = (0.18 \pm 0.07) f_{B_s}^2 M_{B_s}^2 ,$$
$$\langle B_s|R_3|\bar{B}_s\rangle = (0.38 \pm 0.13) f_{B_s}^2 M_{B_s}^2 , \qquad \langle B_s|\widetilde{R}_3|\bar{B}_s\rangle = (0.29 \pm 0.10) f_{B_s}^2 M_{B_s}^2$$

[Kirk,Lenz,Rauh'17; King,Lenz,Rauh'19'21; . . .]