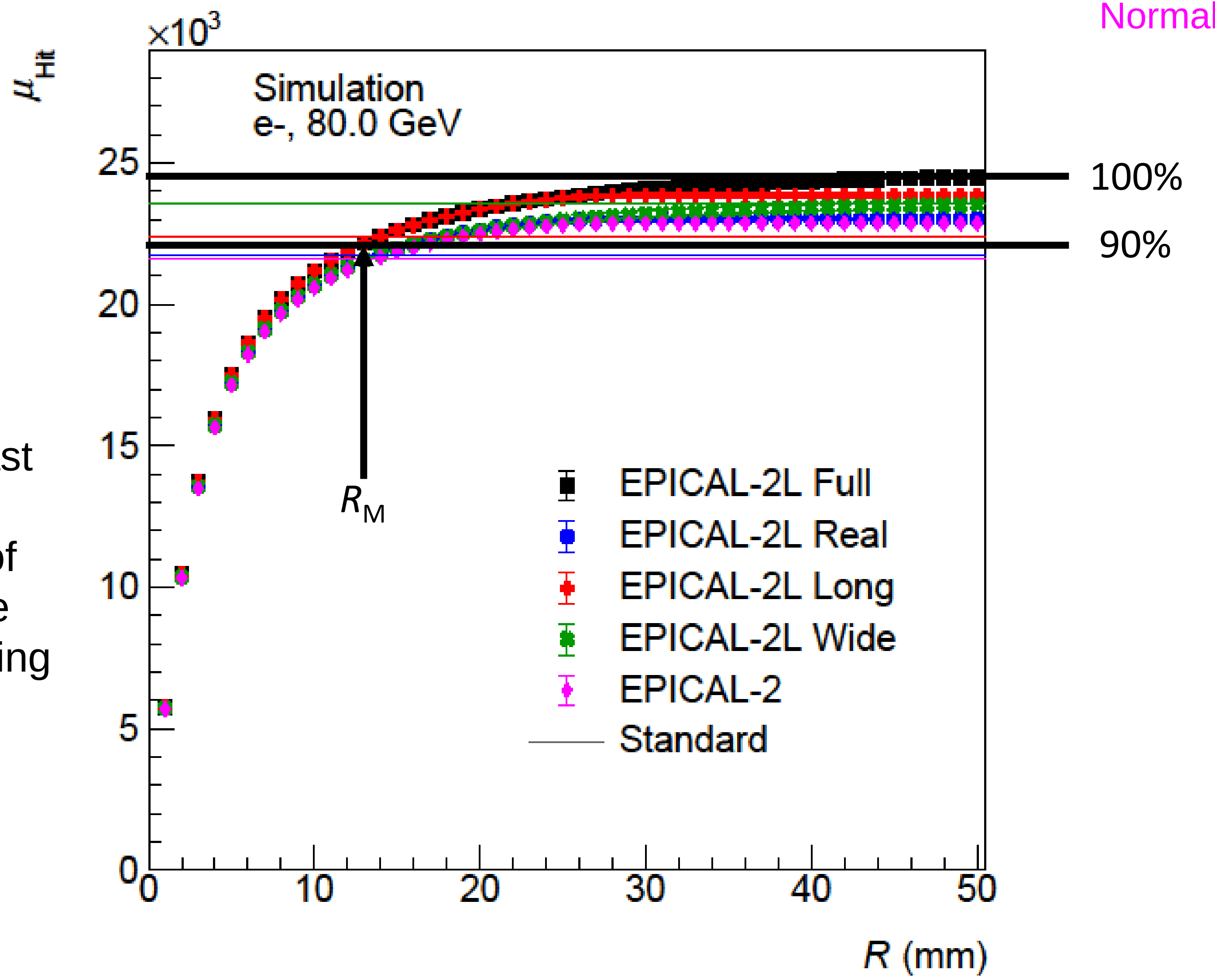
Molière Radius

Johannes Keul

Reminder: Molière radius method 1

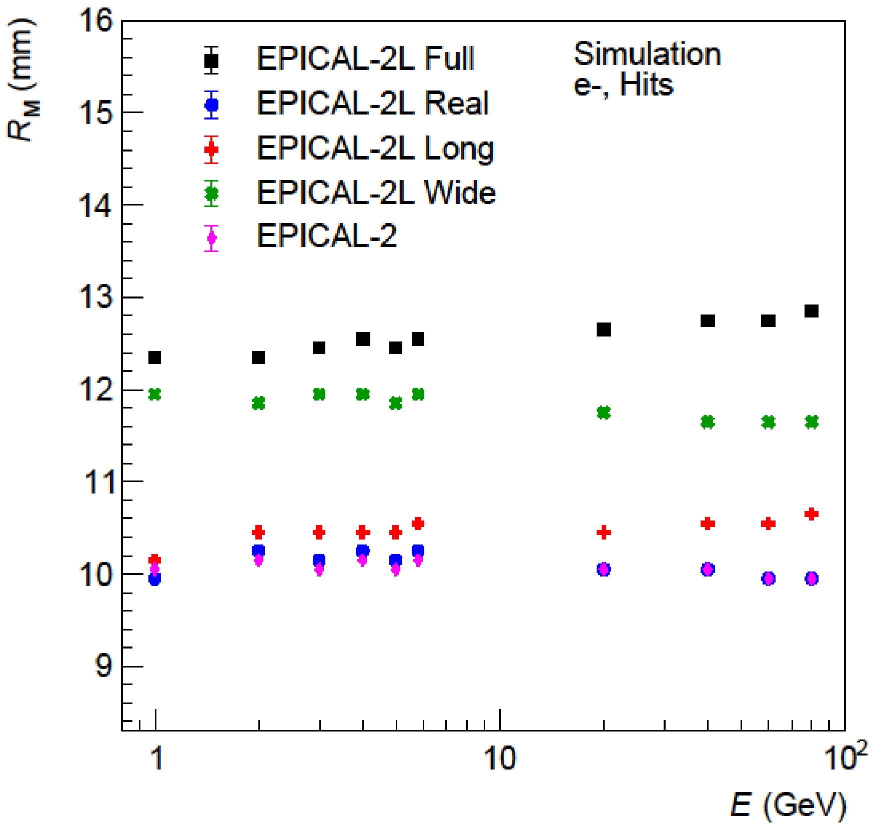
Full: 4096 x 4096 pixel, 96 layers
Real: 1024 x 1024 pixel, 24 layers
Long: 1024 x 1024 pixel, 96 layers
Wide: 4096 x 4096 pixel, 24 layers
Normal: 1024 x 1024 pixel, 24 layers

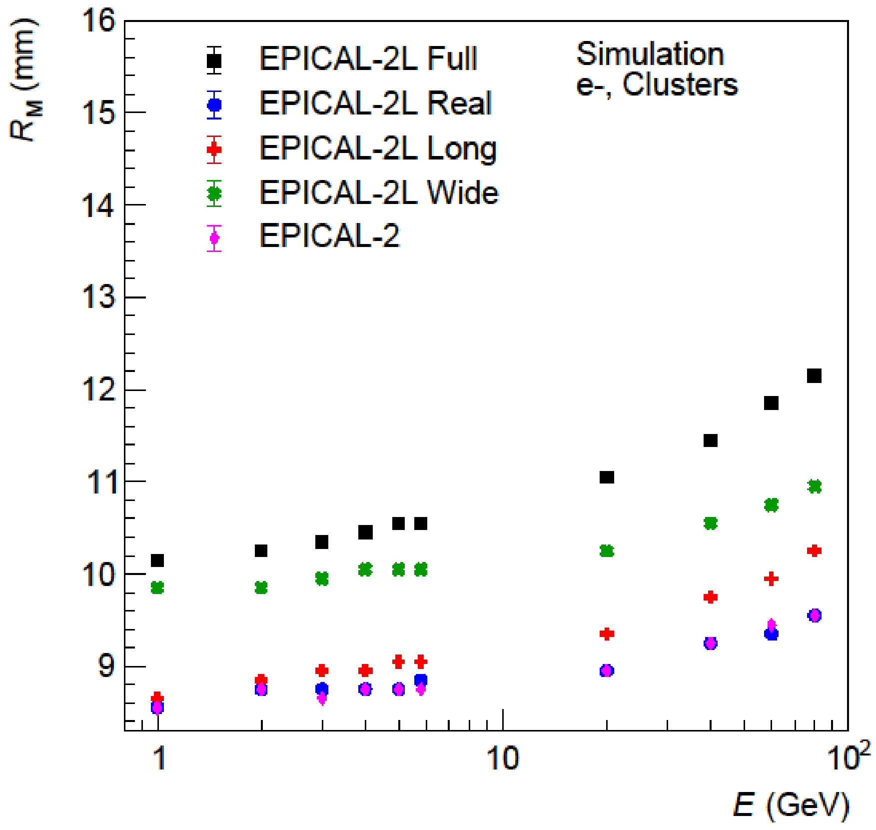
- Use maximum as 100% value
- R_M: center of the last bin where μ_{Hit} is smaller than 90% of the maximum value
- Using 0.1 mm binning for calculation



Reminder: Molière radius method 1

Full: 4096 x 4096 pixel, 96 layers
Real: 1024 x 1024 pixel, 24 layers
Long: 1024 x 1024 pixel, 96 layers
Wide: 4096 x 4096 pixel, 24 layers
Normal: 1024 x 1024 pixel, 24 layers





Molière radius method 2

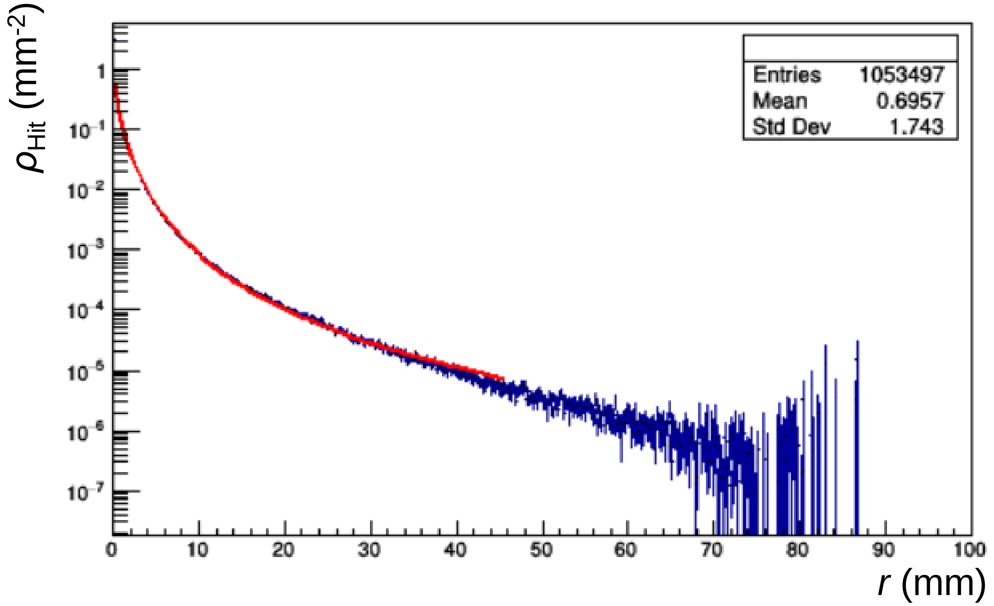
1. Fit the lateral hit (cluster) density profile
2. Integrate over the Fit function
3. Determine the point where the integral reaches 90% of the full integral:

$$\int_0^{R_M}\int_0^{2\pi}\rho(r)r\,d\varphi dr = 0.9\int_0^{\infty}\int_0^{2\pi}\rho(r)r\,d\varphi dr$$

Fitting the layer integrated lateral profile

Trying Jan's function:

$$\rho(r) = p_0\left(\frac{p_1{}^2 - 3p_1 + 2}{2\pi p_1{}^2 p_2{}^2}\right)\left(1 + \frac{r}{p_1 p_2}\right)^{-p_1}$$



For all Fits: X^2/NDF > 30

- Fits don't work well
- ➢ Function is intended to fit one layer, not the integrated profile

- Trying the same function multiple times:

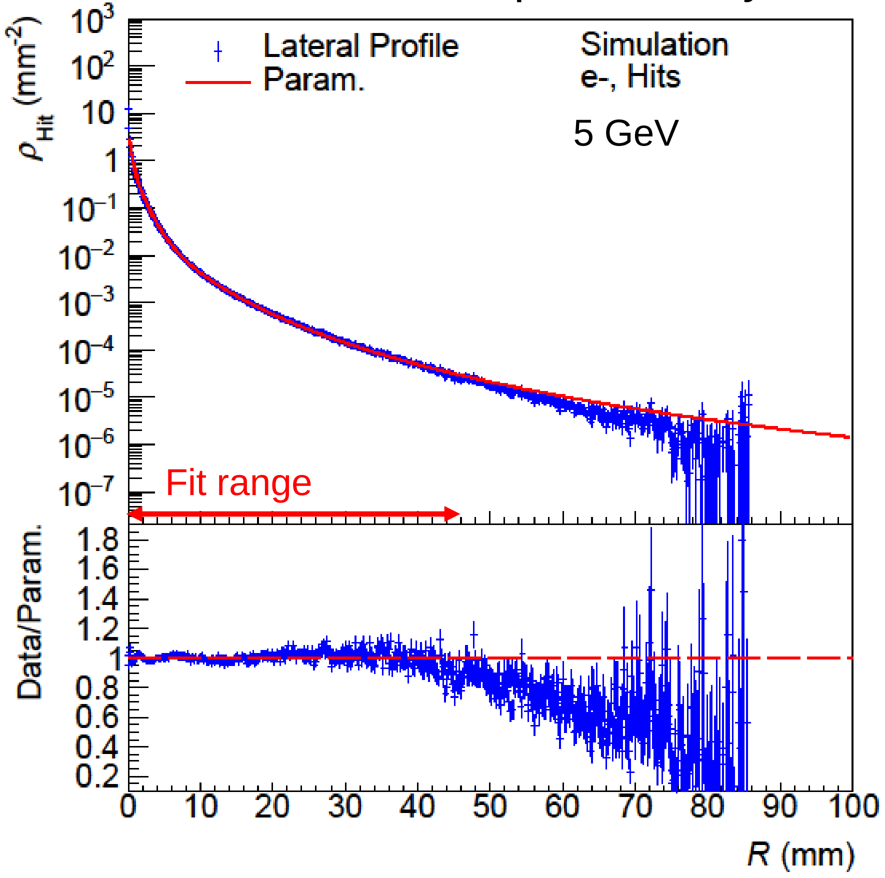
$$\rho(r) = \sum_{i=1}^{n} \rho_i(r)$$

- Idea: each function in the sum can describe a part of the shower, ideally $n = N_{\mathrm{layer}}$
- Even with n = 2 the X^2/NDF gets better, but the function diverges for small and large r

Fitting the layer integrated lateral profile

- Trying a simpler function inspired from DOI 10.1088/1674-1137/32/3/006:
$$\rho(r) = a(r^2 + b)^{-2}$$
- Easy to calculate integral:
$$\int_0^{\infty}\int_0^{2\pi}\rho(r)r\, d\varphi dr = \frac{\pi a}{b}$$
- This function is also intended to fit a single layer
- Trying the same function multiple times:
$$\rho(r) = \sum_{i=1}^{n}\rho_i(r)$$
- Good fit results for $n = 4$
- $X^2/NDF \approx 2$ (in fit range)

Full: 4096 x 4096 pixel, 96 layers



Fitting the layer integrated lateral profile

- Trying a simpler function inspired from DOI 10.1088/1674-1137/32/3/006:

$$\rho(r) = a(r^2 + b)^{-2}$$

- Easy to calculate integral:

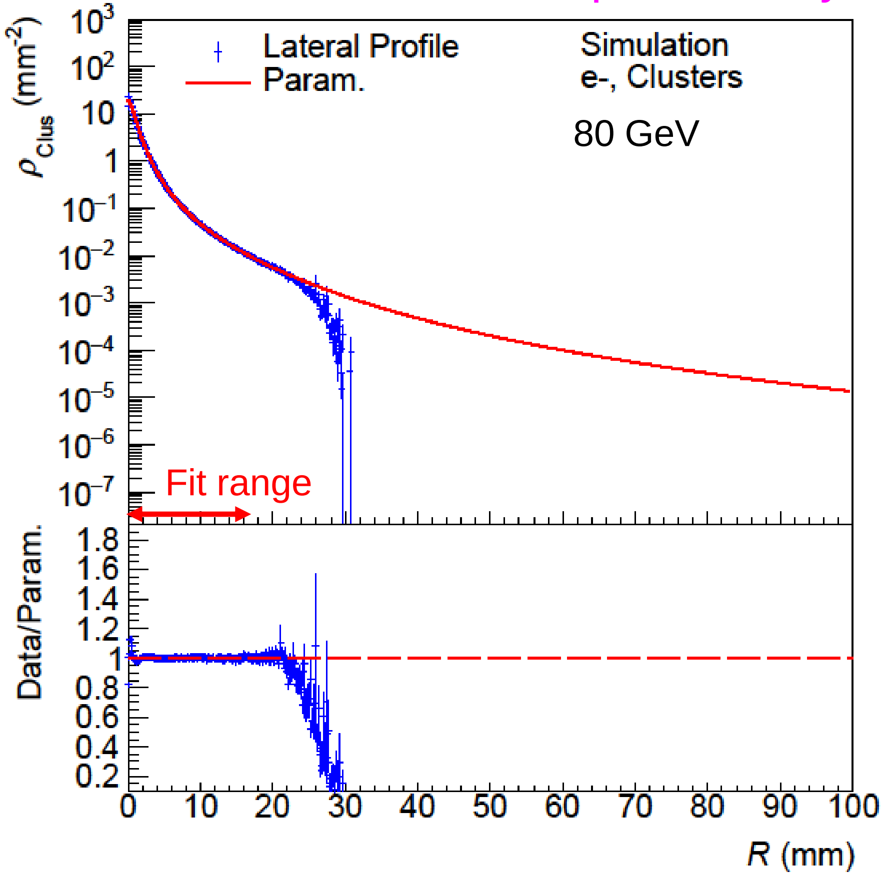
$$\int_0^{\infty}\int_0^{2\pi} \rho(r) r \, d\varphi dr = \frac{\pi a}{b}$$

- This function is also intended to fit a single layer
- Trying the same function multiple times:

$$\rho(r) = \sum_{i=1}^{n} \rho_i(r)$$

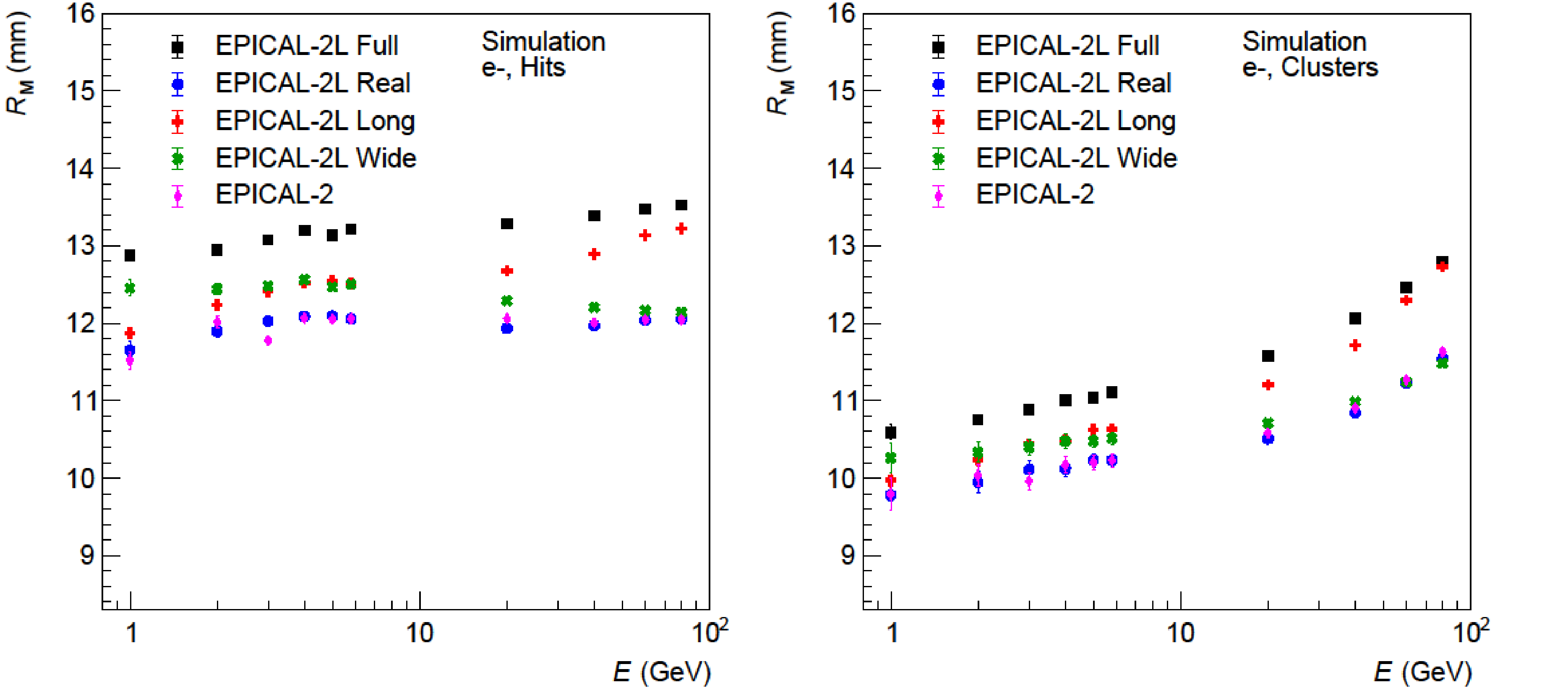
- Good fit results for n = 4
- $X^2/NDF \approx 2$ (in fit range)

Normal: 1024 x 1024 pixel, 24 layers



Molière radius method 2

Full: 4096 x 4096 pixel, 96 layers
Real: 1024 x 1024 pixel, 24 layers
Long: 1024 x 1024 pixel, 96 layers
Wide: 4096 x 4096 pixel, 24 layers
Normal: 1024 x 1024 pixel, 24 layers

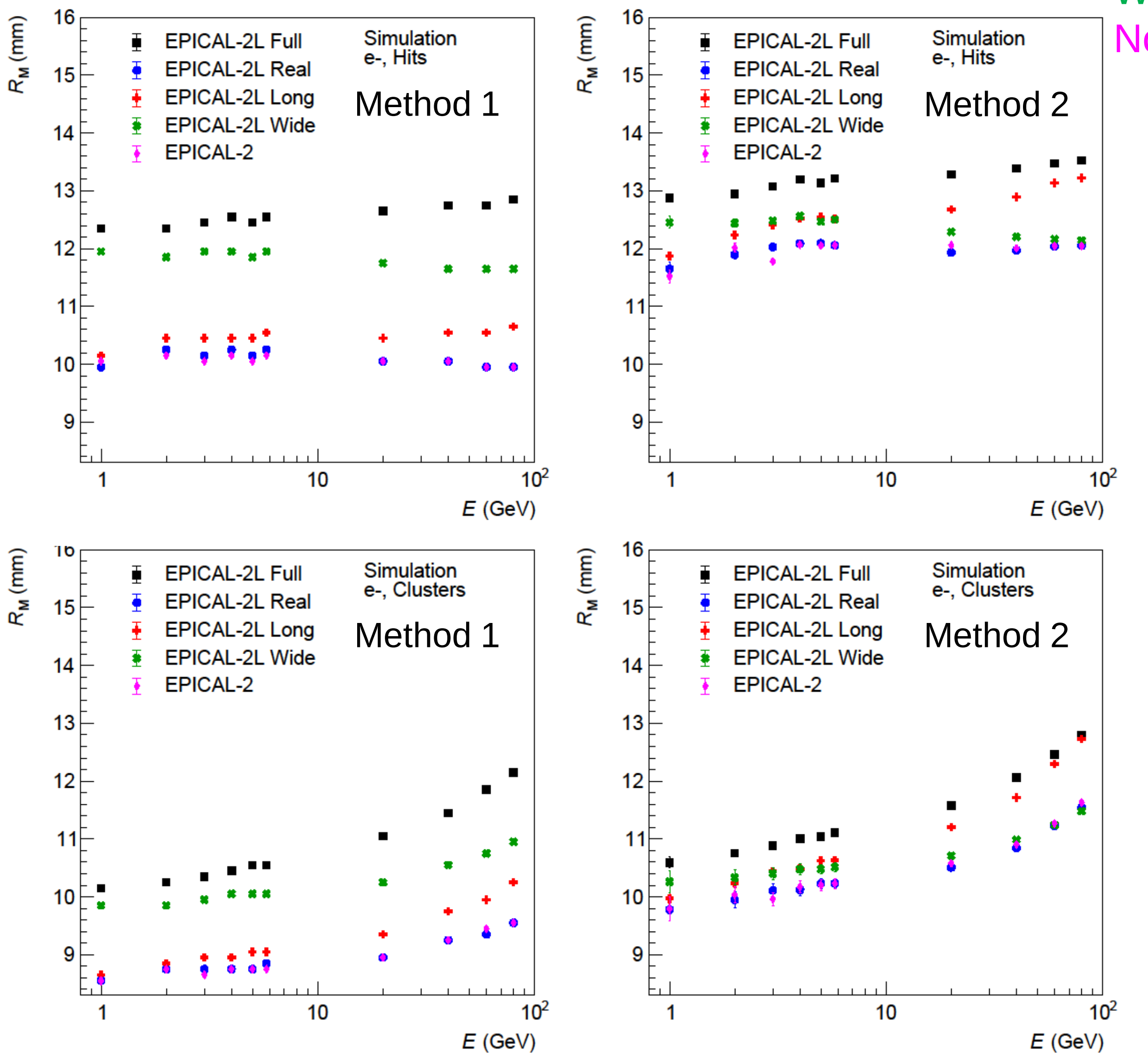


Comparison of the methods

Full: 4096 x 4096 pixel, 96 layers
Real: 1024 x 1024 pixel, 24 layers
Long: 1024 x 1024 pixel, 96 layers
Wide: 4096 x 4096 pixel, 24 layers
Normal: 1024 x 1024 pixel, 24 layers

Method 1: no Fit

Method 2: Fit



- Similar behavior for both methods
- Difference: the non-wide variants are closer to the wide variants for method 2
- ➢ Reason: Fit can model the profile beyond the edges of the detector

- Molière radius analysis is only done in simulation so far
- ➢ Molière radius analysis for data is coming soon (high statistics for DESY data take lot of computing time)

Conclusion

- 2 possible ways to calculate Molière radius: with and without a fit
- Most functions to describe radial profiles are designed to fit a single layer → add the function multiple times
- Simpler function works better
- ➢ Similar behavior for both methods; For the fit method the non-wide variants are closer to the wide variants

- Further studies on the elongated events that we discussed in the last meeting will follow in our next meeting, simulations are still running…