

Impact of beam-beam effects and linear coupling resonance on the absolute luminosity calibration at the CERN Large Hadron Collider

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Outline

- Introduction and Background
 - LHC and transverse dynamics
 - Luminosity and van der Meer scans
 - Beam-beam effects and tune footprints
 - Beam-beam effects on luminosity
- Studies
 - Aims of the project
 - Benchmarking of Xsuite luminosity numerical integrator to COMBI results
 - Linear coupling
 - Linear coupling and beam-beam effects on luminosity
 - Linear coupling and beam-beam effects on sigma visible
- Conclusions

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Large Hadron Collider (LHC)

- Synchrotron with two separate beam lines
- Dipoles bend the beams and quadrupoles focus and defocus the beams (FODO cells)
- Two 6.8 TeV counter-rotating proton beams
- Four experiments at interaction points (IPs) at four different insertion regions
- ATLAS and CMS are high-luminosity general purpose experiments
- ALICE and LHCb are more focused, lower luminosity experiments
- Most notable discovery is the confirmation of the Higgs Boson



Transverse dynamics: phase space

• Hill's equation, where K(s) is a measure of focusing strength:

$$x'' + K(s)x = 0.$$

• The transverse position and momenta of the particles follow:

$$x(s) = \sqrt{2\beta_x(s)J_x}\cos(\varphi_x(s) - \varphi_0),$$

$$p_x(s) = -\sqrt{\frac{2J_x}{\beta_x(s)}}(\sin(\varphi_x(s)) + \alpha_x \cos(\varphi_x(s))).$$

• Beam size:

$$ext{beam r.m.s.}_{x,y} = \sigma_{x,y}(s) = \sqrt{\epsilon_{x,y} \cdot \beta_{x,y}(s)} \qquad arepsilon_{x,y} = < J_{x,y} > 0$$



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• Beam size:

b

$$ext{eam r.m.s.}_{x,y} = \sigma_{x,y}(s) = \sqrt{\epsilon_{x,y} \cdot eta_{x,y}(s)} \qquad arepsilon_{x,y} = < J_{x,y} >$$



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Transverse dynamics: tune and resonance

• Q_x and Q_y are the tunes of the accelerator: number of oscillations per turn that the particles perform under the influence of the focusing system:

$$Q_{x,y} = \frac{1}{2\pi} \oint \frac{ds}{\beta_{x,y}(s)}.$$

• The resonance condition must be avoided:

$$n \cdot Q_x + m \cdot Q_y = l,$$

where n, m and l are integers.

• LHC tunes are (62.31, 60.32)



Transverse dynamics: tune and resonance

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Luminosity

• Luminosity (cm⁻²s⁻¹) is the proportionality factor between the rate of events detected and production cross section for a given event. It is a parameter of the machine:

$$R = L_{inst} \cdot \sigma_{ev}.$$



• The integral equation for luminosity:

$$L = 2N_1 N_2 f_{rev} N_b \int \int \int \int_{-\infty}^{\infty} \rho_{1x}(x) \rho_{1y}(y) \rho_{1s}(s-s_0) \rho_{2x}(x) \rho_{2y}(y) \rho_{2s}(s+s_0) dx dy dx ds_0.$$

• Under the Gaussian approximation there is an analytical solution:

$$L_{inst} = \frac{N_1 N_2 f_{rev}}{4\pi \sigma_x \sigma_y}$$

Van der Meer scans for absolute luminosity calibration

- Beams are scanned transversely across each other
- Reduction factor W is introduced from separation:

$$L = L_{ho} \cdot W, \qquad W = e^{-\frac{\Delta_i^2}{4\sigma_i^2}}.$$

• Interaction rates are fitted as a function of separation for an event with a known cross section:

$$\Sigma_x = \frac{1}{\sqrt{2\pi}} \frac{\int R(\Delta_x, 0) d\Delta_x}{R(0, 0)}, \quad \Sigma_y = \frac{1}{\sqrt{2\pi}} \frac{\int R(0, \Delta_y) d\Delta_y}{R(0, 0)}.$$



• Sigma visible is the luminometer-dependent constant used to calibrate the luminosity:

$$\sigma_{vis} = \frac{2\pi \Sigma_x \Sigma_y R^{vis}(0,0)}{N_1 N_2} \qquad L_{inst} = \frac{R}{\sigma_{vis}}$$

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Beam-beam interactions



• Beam-beam force:

$$F_r(r) = -\frac{ne^2(1+\beta^2)}{2\pi\epsilon_0} \cdot \frac{1}{r} \left[1 - \exp\left(-\frac{r^2}{2\sigma^2}\right) \right]$$

• Amplitude dependent detuning:

$$\Delta Q(r) = \frac{Nr_0\beta}{4\pi\gamma\sigma^2} \cdot \frac{1}{(\frac{r}{2})^2} \cdot (\exp(-(\frac{r}{2})^2 I_0(\frac{r}{2})^2 - 1)$$

• Beam-beam parameter:

$$\Delta Q_{r\to 0} \approx \xi = \frac{N r_0 \beta^*}{4\pi \gamma \sigma^2}$$

Tune Footprints

• Beam dynamics can be investigated using tune footprints, where the amplitude dependent detuning can be visualised



1IP - Head-on collision footprint

Tune Footprints

• In vdM scans tune footprints are changed and distorted with distortions increasing with separation



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Tune Footprints

• In vdM scans tune footprints move towards and away from different resonance lines



- Three main changes due to beam-beam effects resulting in two effects:
 - 1) Orbit deflection (orbit shift)
 - 2) Transverse beam sizes (optical distortion)
 - 3) Non-Gaussian beams (optical distortion)
- The full effect can only be captured with a numerical integrator

• The luminosity bias from beam-beam effects is pictured showing different contributions from two different effects



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 - 3) Non-Gaussian beams (optical distortion)
- The full effect can only be captured with a numerical integrator

- An additional orbit deflection is seen in the direction of scanning
- Orbit deflection matches analytical expression



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• The green line shows the orbit deflection leading to a bias from orbit shift



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 - 1) Orbit deflection (orbit shift)
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- The full effect can only be captured with a numerical integrator
- Beta-beating is seen in the plot, where beta-star is reduced from the beam-beam force for LHC
- This translates to the transverse beam sizes being focused by the beam-beam force
- As a result, luminosity is enhanced (LHC case)



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- The full effect can only be captured with a numerical integrator

• The red line shows the change in transverse beam size and non-gaussian beam distributions leading to a bias from optical distortion



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 - 1) Orbit deflection (orbit shift)
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 - 3) Non-Gaussian beams (optical distortion)
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- If luminosity bias is calculated with the assumption of Gaussian beam distributions the purple line would be obtained
- In reality beams are distorted by having different actions and are no longer Gaussian



- Three main changes due to beam-beam effects resulting in two effects:
 - 1) Orbit deflection (orbit shift)
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• These two biases will add up to make the full bias in black from beam-beam effects



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Aims of the project

- 1.Develop a numerical luminosity integrator in the new beam simulation framework Xsuite and reproduce results from previous studies [6].
- 2.Use the numerical integrator to study the impacts of beam-beam effects and linear coupling resonance on luminosity
- 3.Quantify the impact of beam-beam effects and linear coupling resonance on the sigma visible

Luminosity Bias: 1 IP

- A numerical integrator was developed in Xsuite based on previous studies [6]
- There is good agreement between the two results, and clear differences with the Gaussian calcualtion



Luminosity Bias: 2 IPs

- Results for 2 IPs are similar to those for 1 IP
- Slight deviations at the tails for 2 interaction points.



Sigma visible comparisons

- Sigma visible is comparable despite deviations in the tails for 2IPs
- Differences are below the significance level of 0.1%



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Linear coupling

- Coupling between the horizontal and vertical transverse directions
- Comes from errors and tilted installations of quadrupole magnets in the lattice
- Controlled with skew quadrupoles
- Can be quantified in terms of the minimum tune approach C⁻:

$$C^{-} = \frac{1}{2\pi R} \int_0^{2\pi} \sqrt{\beta_x \beta_y} K e^{i[(\phi_x - \phi_y) - (Q_x - Q_y - p)\theta]} d\theta$$

Coupling resonances are known to have an impact on tune spreads



[6]

Closest tune approach

• Numerical minimization to find *C*⁻ which aligns with skewness K:



Closest tune approach

• Numerical minimization to find *C*⁻ which aligns with skewness K:



Closest tune approach

• Numerical minimization to find *C*⁻ which aligns with skewness K:



Phase space

- Normalized phase space with and without beam-beam and linear coupling resonance effects
- The effects combine to produce a distorted phase space
- Simulated using the vdM beam-beam parameter and coupling parameter $C^- = 8 \times 10^{-3}$



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Tune shifts introduced by coupling

- Coupling pushes the tunes away from the resonance line, although this will often be corrected for in operation up to a certain level
- Coupling distorts the tune spread, a narrowing effect in this head-on collision case



Tune Footprints – 1IP with coupling

- Different configurations of scans were observed
- Sometimes tune footprints are narrowed, sometimes widened
- Pushed away from the linear coupling resonance line



Luminosity bias with coupling: 1 IP Horizontal

- Luminosity biases for different effects and with different normalizations are shown for 1 IP
- There is a difference in luminosity bias by introducing linear coupling resonance



Luminosity bias with coupling: 1 IP Vertical

- Luminosity biases for different effects and with different normalizations are shown for 1 IP
- There is a difference in luminosity bias by introducing linear coupling resonance



Luminosity bias with coupling: 2 IPs

- Luminosity biases for different effects and with different normalizations are shown for 2 IP
- The results are very similar to the 1 IP case with small changes in the shape



Luminosity bias with coupling: diagonal scans

- Luminosity biases for different effects with different normalizations are shown for diagonal scans
- A combination of horizontal and vertical results are seen for diagonal scans



Sigma visible bias – 1 IP

- Sigma visible is calculated for one and two interaction points
- The sigma visible bias is calculated under different conditions and normalisations
- Four cases of coupling are tested: no coupling, $C^- = 5 \times 10^{-3}$, $C^- = 8 \times 10^{-3}$, and $C^- = 16 \times 10^{-3}$



Sigma visible bias - 2 IPs

- Sigma visible is calculated for one and two interaction points
- The sigma visible bias is calculated under different conditions and normalisations

• Four cases of coupling are tested: no coupling, $C^- = 5 \times 10^{-3}$, $C^- = 8 \times 10^{-3}$, and $C^- = 16 \times 10^{-3}$



Sigma visible bias: changed working point

- Sigma visible bias is calculated where the working point is shifted from (62.31, 60.32) to (62.312, 60.316), closer to the linear resonance line
- There is a stronger effect on sigma visible bias when the working point is not corrected



Sigma visible bias: uncorrected tune shifts

- An uncorrected working point is used to investigate the different components of linear coupling resonance effects
- A greater change in sigma visible is seen when tune shifts are not corrected



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Conclusions

- The numerical integrator in 4D was developed in Xsuite and successfully benchmarked against state-of-the-art results from COMBI
- A simplified model of the interplay between beam-beam and linear coupling resonance (using a single skew quadrupole) has been developed in Xsuite
- An extensive simulation campaign of impacts on phase space, footprints and luminosity has been carried out
- Linear coupling resonance is shown to modify beam-beam footprints in terms of tune shifts and tune spreads
- Linear coupling has been proven to modify the luminosity bias, and consequently the sigma visible during van der Meer scans for the first time
 - 1. This study proves the dependence of this effect on coupling strength C^-
 - 2. Shown that tunes when moved closer to the diagonal $Q_x = Q_y$ there is a stronger effect
 - 3. If the linear coupling tune shift is uncorrected the effect is even stronger

References

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[4] ATLAS Collaboration. ATLAS event at 900GeV. CERN. 2015

[5] Wanćzyk J. Precision Luminosity Measurement at Hadron Colliders. EPFL; 2024.

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Questions

Additional slides

Orbit deflection

- An additional orbit deflection is seen in the direction of scanning
- Orbit deflection matched analytical expectations:
- Slightly different scales for x and y from different tunes Q_x and Q_y



 $\Delta_x^{BB} = \frac{\theta_x \beta_x^*}{2 \tan(\pi Q_x)}.$

Frequency spectrum

- Fourier transforms of the beam centroids are plotted
- There are two modes of oscillations: 0 and π -mode
- For van der Meer scans the distance between the modes will change following the tune shifts for different separations





Frequency spectrum analysis

- Plotting the distance between the 0 and π –mode for different directions of vdM scans
- Slight deviations in x and y directions from the different tunes Q_x and Q_y
- Shape is dictated by the beam-beam force



Beam-beam models

Weak-strong model:

• Calculates the kicks on the particles in one bunch from a constant opposing bunch, with a Gaussian transverse distribution

Strong-strong model:

• Calculates the kicks on the particles in one bunch from an evolving opposing bunch



FFT error

• With increased coupling more particles are erroneously mapped onto the resonance line, causing a loss of shape in the tune spread viewed



Luminosity calculated in the Gaussian approximation

- Luminosity in a horizontal vdM scan is modelled in the Gaussian approximation without beambeam effects
- This is benchmarked against the analytical expression
- Although the Gaussian is not a perfect model it will give good indicators of trends in studies



- Three main changes due to beam-beam effects resulting in two effects:
 - 1) Orbit deflection (orbit shift)
 - 2) Transverse beam sizes (optical distortion)
 - 3) Non-Gaussian beams (optical distortion)
- The full effect can only be captured with the strong-strong model and a numerical integrator
- Beta-beating is seen in the plot, where beta-star is reduced from the beam-beam force for LHC
- This translates to the transverse beam sizes being focused by the beam-beam force
- As a result, luminosity is enhanced (LHC case)

 $\frac{\beta^*}{\beta_0}$ as a function of Tune Q 1.20= 0.003241.15 $\xi = 0.00648$ 1.10 1.05°n € 1.00 0.950.90 0.850.80+0.10.20.30.4 0.0 0.5Tune Q $4\pi\xi \cot(2\pi Q) - 4\pi^2\xi^2$

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Previous results from COMBI

• Previous studies compared luminosity calculated with the numerical integrator (COMBI Integral) with the Gaussian approximated luminosity (COMBI Gaussian), as well as legacy results from MADX and the analytical expression



Luminosity numerical integrator

- Bunches of the beam are divided into macroparticles, each represents a number of particles
- No assumption on beam distributions, 2D histograms are populated with the position of the particles
- Ranges of the histogram encapsulate the overlap region
- In this project a 15 by 15 σ grid is divided into 1200 by 1200 smaller grids

Difference in sigma visible biases

- Difference in sigma visible biases due to different effects are shown in the tables
- For 1 IP there is a consistent reduction in sigma visible bias

$\Delta \sigma_{vis}$ bias	$C^- = 5 \times 10^{-3}$	$C^- = 8 \times 10^{-3}$
$\Delta rac{\sigma_{vis_c}}{\sigma_{vis_0}}$	0.00075%	0.0059%
$\Delta \frac{\sigma_{vis_{bb_c}}}{\sigma_{vis_0}}$	0.00911%	0.0316%
$\Delta rac{\sigma_{vis_{bb_c}}}{\sigma_{vis_c}}$	0.00913%	0.0334%

• For 2 IPs the effect is different, increased in some cases and in the opposite direction

$\Delta \sigma_{vis}$ bias	$C^-=5\times 10^{-3}$	$C^-=8\times 10^{-3}$
$\Delta rac{\sigma_{vis_c}}{\sigma_{vis_0}}$	0.0066%	0.0004%
$\Delta \frac{\sigma_{vis_{bb_c}}}{\sigma_{vis_0}}$	0.0033%	0.0191%
$\Delta \frac{\sigma_{vis_{bb_c}}}{\sigma_{vis_c}}$	-0.0085%	0.0096%

Luminous region beam distribution bias

• The edges are the most impacted by beam-beam effect distortions



Bias due to coupling with C- = 5×10^{-3}

Outlooks

- There is a quantifiable impact of linear coupling and beam-beam effects on the sigma visible
- Subsequent studies should explore a more realistic model of the machine to capture all effects
- Additionally further studies should be made with higher order elements such as sextupoles and octupoles
- Non-localised coupling should be used instead of just a local skew quadrupole
- More interaction points, with the correct phase advances should be used
- Asymmetry can be introduced with coupling only in one beam line

Tune Footprints: offset scans





Tune Footprints: vdM scans with 2 IPs



Luminosity bias with coupling: 1 IP

- Luminosity biases for different effects and with different normalizations are shown for 1 IP
- There is a difference in luminosity bias by introducing linear coupling resonance



Luminosity bias with coupling: 2 IPs

- Luminosity biases for different effects and with different normalizations are shown for 2 IP
- The results are very similar to the 1 IP case with small changes in the shape



Luminosity bias with coupling: diagonal scans

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- A combination of horizontal and vertical curves are seen for diagonal scans



Sigma visible bias

- Sigma visible is calculated for one and two interaction points
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Sigma visible bias: changed working point

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Sigma visible bias: stronger beam-beam parameter

- A stronger beam-beam parameter nearly 3 times the vdM beam-beam parameter is used
- Here the luminosity has been increased to $2.2 \times 10^{34} \text{cm}^{-2} \text{ s}^{-1}$

