

# Discussion: "Connections to Fundamental Theory"

# Gary Shiu

# Cosmic Expansion and Fundamental Theory

In realizing cosmic expansion, having a fundamental theory is both a blessing and a curse:

• String theory has too many scalars: if not stabilized, they could lead to varying coupling constants, 5-th force, and mess up BBN (unless  $m \gtrsim 30 \text{ TeV}$ ).

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- Cosmic acceleration is often driven by scalars (inflaton, quintessence,…). Fundamental theory such

as string theory has many scalars (and axions, hence axiverse).

• These scalars can also alter cosmic expansion histories (moduli dominated non-thermal history, EMD, stasis, …). Cosmology before BBN is the Wild West!

• Why most scalars are stabilized while one (or a few) is dynamical? Who order the mass hierarchy? Naturalness? Does string theory suggest a departure from the simple single field scenarios?

• String theory provides a UV complete framework to address various naturalness problems: Snowmass white papers: [2203.07629 \[hep-th\]](https://inspirehep.net/literature/2052446), [2204.01742 \[hep-th\]](https://inspirehep.net/literature/2063384), Review: [2401.01939 \[hep-th\]](https://inspirehep.net/literature/2743274).

• **Scalars in string theory have a second role: Dine-Seiberg problem.**

![](_page_1_Picture_2.jpeg)

![](_page_1_Picture_4.jpeg)

![](_page_1_Picture_6.jpeg)

![](_page_1_Picture_8.jpeg)

![](_page_1_Picture_10.jpeg)

![](_page_1_Picture_12.jpeg)

# Dine-Seiberg Problem

• There are no free parameters in string theory: coupling constants are vevs of scalar fields.

• A vacuum exists only if terms of different order compete. A de Sitter vacuum requires at least 3

If different order terms compete to give a minimum, why aren't higher order terms important?

- 
- competing terms.
- 
- 

• The vevs of scalar fields are the perturbative expansion parameter, e.g.,  $g \sim \langle \exp(-\phi) \rangle$ .

• The Dine-Seiberg problem: difficulty in finding parametrically controlled vacua. (LVS? KKLT? DGKT?).

![](_page_2_Picture_12.jpeg)

![](_page_2_Figure_2.jpeg)

#### Cosmic Acceleration inflaton field (*t*) government the energy density of the energy den

- of inflationary expansion still to occur. By the uncertainty principle, arbitrarily principle, arbitrarily precise timing is  $\mathcal{L}$ notus on **cosmic acceleration**, icaving alconative cosmic cxp
- variance, so the inflation will have speaked inflation (only).
	-
- **Inflation:** Assuming that other than the inflaton, all moduli are stabilized, are we done?

![](_page_3_Figure_5.jpeg)

• I'll focus on **cosmic acceleration**, leaving alternative cosmic expansion histories to Jim. • Observations suggest two accelerating phases: **inflation** (early), & **dark energy** (now). • Hurdles to embed these two accelerating phases into string theory are somewhat different. local di↵erences in the time when inflation ends, *t*(x), so that di↵erent regions of space inflate

**Slow roll conditions:**

$$
\epsilon_V = \frac{M_P^2}{2} \left(\frac{V'}{V}\right)^2 \ll 1, \quad |\eta_V| = |M_P^2 \frac{V''}{V}| \ll 1
$$

**Dimension 6 operators**

$$
\mathcal{O}_6 = c_1 \frac{V(\phi)}{\Lambda_{UV}^2} \quad \rightarrow \quad \Delta \eta \sim \mathcal{O}\left(\frac{M_P}{\Lambda_{UV}}\right)^2,
$$

 $\Lambda_{UV}$  = KK scale, string scale, Planck scale,...

# Primordial Gravitational Waves

• Models of inflation that generate detectable gravitational waves require  $V(\phi)$  to be nearly flat over a super-Planckian field range:

 $\Delta \phi \gtrsim 1$ 

• Near future experiments e.g. CMB-S4, Simons Observatory, LiteBIRD are reaching the  $10^{-3}$ level.

$$
\mathcal{L}_{\text{eff}}[\phi] = \frac{1}{2}(\partial \phi)^2 - \frac{1}{2}m^2 \phi^2 \left(1 + \sum_{i=1}^{\infty} c_i \frac{\phi^{2i}}{\Lambda^{2i}} + \cdots \right)
$$

$$
\left(\frac{r}{0.01}\right)^{1/2} M_{\text{Pl}} \qquad \qquad \text{[Lyth '96]}
$$

But Riess suspects that the mystery can't be solved by observations alone. "We won't really resolve it until some brilliant person, the next Einstein-like person, is able to get the idea of what's going on," he said.

So he issued **a plea to the theorists**: "Keep working," he said. "We need your help. ... It's a very juicy problem, it's hard, and **you'll win a Nobel Prize if you figure it out. In fact, I'll give you mine.**"

![](_page_5_Picture_9.jpeg)

Photo: Belinda Pratten, Australian National University

**Brian P. Schmidt** 

![](_page_5_Picture_12.jpeg)

Photo: Homewood Photograph

Adam G. Riess

Nobel Prize 2011

**Saul Perlmutter** 

# Dark Energy

![](_page_5_Picture_1.jpeg)

![](_page_5_Picture_2.jpeg)

Photo: Roy Kaltschmidt. Courtesy: Lawrence Berkeley National Laboratory

- a de Sitter minimum,
- a de Sitter maximum, or
- a runaway potential with  $\epsilon \equiv -$ .<br>-<br>-*H*  $\frac{1}{H^2}$  < 1

**Current** cosmic acceleration can be realized by:

![](_page_6_Figure_9.jpeg)

If the universe underwent a rolling phase before, why not again? (main hurdle: 5-th force constraint)

![](_page_6_Picture_10.jpeg)

![](_page_6_Picture_11.jpeg)

Unlike inflation which needs to last 60 e-folds to solve the flatness & horizon problems, the current acceleration may last only an e-fold or less.

Recent DESI results gave a tantalizing hint of varying dark energy, though it is too early to tell.

Generally  $\epsilon \neq \epsilon_V$  due to non-negligible kinetic energy. How do we bound  $\epsilon$  w/o knowing on-shell solutions?

## Bounds on late-time acceleration and cosmological attractors

![](_page_7_Picture_1.jpeg)

## Flavio Tonioni  $UW$ -Madison Physics  $\rightarrow$  KU Leuven

Hung V. Tran UW-Madison Math

![](_page_7_Figure_6.jpeg)

[STT1]: ``Accelerating universe at the end of time,'' PRD **108**, no.6, 063527 (2023) [\[2303.03418\]](https://inspirehep.net/literature/2639032). [STT2]: ``Late-time attractors and cosmic acceleration," PRD **108**, no.6, 063528 (2023) [\[2306.07327\]](https://inspirehep.net/literature/2668778). [STT3]: ``Collapsing universe before time," JCAP **05**, 124 (2024) [\[2312.06772\].](https://inspirehep.net/literature/2735879) [STT4]: ``Analytic bounds on late-time axion-scalar cosmologies," [\[2406.17030\]](https://inspirehep.net/literature/2802035).

![](_page_7_Picture_4.jpeg)

### Asymptotic Dark Energy Field-space boundaries

- Could the current acceleration be realized by rolling towards the asymptotic regions of the landscape?
- Does not require terms of different order to compete, in contrast to the Dine-Seiberg problem for vacua.
- A tower of states becomes light as we approach the asymptotic. Entropy bound suggest that the potential has an exponential falloff [Ooguri, Palti, GS, Vafa].
- But solving multi-field dynamics is much more difficult than taking derivatives of potential!
- As in many dynamical systems, the late-time regime exhibits some universal behaviors. This allows us to prove bounds on acceleration [GS, Tonioni, Tran].

![](_page_8_Figure_6.jpeg)

![](_page_8_Figure_7.jpeg)

explain small numbers in Nature?

• Given a multi field quintessence model, how do we diagnose if it can support acceleration

- $\Lambda$   $\alpha$  donou  $\frac{1}{l}$ ,  $\frac{1}{l}$ ,  $\frac{1}{l}$   $\frac{1}{l}$   $\frac{1}{l}$ sible critical points of the dynamical system of inter
	- without solving for the time-dependent solutions? ([STT1, STT2].
	-
	- models, the bounds we derived continue to apply [STT4].

• We consider scalars rolling towards the field space boundary: axions with a compact field space are assumed to be stabilized above. The saxions can then be canonically normalized.

• In the presence of dynamical axions, the field space metric is curved but in certain classes of

![](_page_9_Picture_16.jpeg)

## Multi-field Quintessence canonically-normalized scalar fields *<sup>a</sup>*, for *a* = 1*,...,n*,

• String theoretical potentials generically take the form (also argument by *[Ooguri, Palti, GS, Vafa]*):

- $\sum_{i=1}^{n}$ rameter with only knowledge of the dimension of spaceare subject to a scalar potential of the form of the
	- $V = \sum$ *i*=1

after canonically normalizing the scalar fields to  $\phi^a$ ,  $a = 1,...,n$ .  $T_{\rm eff}$  main results of our paper are the following. (i)  $\epsilon$ and canonically normalizing the scalar helds to  $\psi$ ,  $u = 1,...,n$ .

$$
\sum_{i=1}^{m} \Lambda_i e^{-\kappa_d \gamma_{ia} \phi^a}.
$$

•  $\Lambda_i$ ,  $\gamma_{ia}$  depend on the microscopic origin of  $V_i$ ,  $\kappa_d = d$ -dim. gravitational coupling. Potentials from e.g. internal curvature, fluxes, branes/O-planes, Casimir-energy, etc take this form.  $\mathbf{r}_i$  the microscopic origin of  $V$   $\kappa_i-d$ -dim aravitational  $\epsilon$  $\frac{d}{d}$  and morooopic dright of  $\frac{1}{d}$ ,  $\frac{d}{d}$   $-\frac{d}{d}$  and gravitational coupling. curvature, includes, branes/O-pianes, Gasimin-energy, etc tan

# Summary of Results

• Treating the universe as a dynamical system, we bound the rate of time variation of the Hubble parameter at late time [STT1]. The bound provides a useful diagnostic for dark energy models.

• Our bound when applied to string theoretic constructions identifies a generic obstacle to acceleration if the  $d$ -dim. dilation is one of the rolling fields. We also suggest several ways out.

• We prove conditions under which scaling solutions are **late-time attractors**. Moreover, we

- 
- 
- prove that scaling solutions **saturate** our bound on  $\epsilon$  [STT2].
- 
- 
- As a spinoff, we derived analogous bounds on ekpyrosis [STT3].

• Our results apply irrespective of whether the potential is generated classically or quantum mechanically, whether the kinetic term is negligible, & whether some potential term dominates.

• This program can be extended to quintessence models with dynamical axions as well [STT4].

![](_page_10_Figure_13.jpeg)

![](_page_10_Figure_14.jpeg)

![](_page_10_Figure_15.jpeg)

![](_page_10_Figure_16.jpeg)

## **Cosmological Equations** general class of potentials subsumes e.g. generalized as-

 $\cdot$  Non-compact  $d$ -dim. spacetime is characterized by the FLRW metric: sisted inflation [5, 6]. Let the non-compact *d*-dimensional lct *d*-dim. spacetime is characterized by the FLRW metric:

• Scalar field equations and Friedmann equations: atione and Eriodmann caugtione: quachono and if no annanni o quachono. Friedmann equations: The scalar-field and Friedmann equations re $\dot H$  $-\frac{1}{D} < 1$ Casimir-energy terms. In fact, here the *d*-dimensional

<u>.</u><br>-

$$
d \widetilde{s}_d^2 = - \mathrm{d} t^2 + a^2(t) \, \mathrm{d} l_{\mathbb{R}^{d-1}}^2,
$$

**• Hubble parameter:**  $H \equiv -$ . The proper diagnostic for cosmic **acceleration** is  $\boldsymbol{\dot{\chi}}$ *a a* with the  $\dot{a}$  =  $\dot{a}$  =  $\dot{a}$ . reformulation and proper diagnosity for cosmic acceleration is *Department of Mathematics, University of Wisconsin-Madison, 480 Lincoln Drive, Madison, WI 53706, USA*  $\boldsymbol{a}$ 

to be distinguished from the slow-roll parameter 
$$
\epsilon_V = \frac{d-2}{4} \kappa_d^2 \left(\frac{\nabla V}{V}\right)^2
$$
.

$$
\ddot{\phi}^a + (d-1)H\dot{\phi}^a + \frac{\partial V}{\partial \phi_a} = 0,
$$
  

$$
\frac{(d-1)(d-2)}{2}H^2 - \kappa_d^2 \left[\frac{1}{2}\dot{\phi}_a\dot{\phi}^a + V\right] = 0,
$$
  

$$
\dot{H} = -\frac{\kappa_d^2}{d-2} \left[\frac{1}{2}\dot{\phi}_a\dot{\phi}^a - V\right] - \frac{d-1}{2}H^2,
$$

$$
\ddot{\phi}^a + (d-1)H\dot{\phi}^a + \frac{\partial V}{\partial \phi_a} = 0,
$$
  

$$
\frac{(d-1)(d-2)}{2}H^2 - \kappa_d^2 \left[\frac{1}{2}\dot{\phi}_a\dot{\phi}^a + V\right] = 0,
$$
  

$$
\dot{H} = -\frac{\kappa_d^2}{d-2} \left[\frac{1}{2}\dot{\phi}_a\dot{\phi}^a - V\right] - \frac{d-1}{2}H^2,
$$

 $\epsilon \equiv -$ *H*  $\frac{1}{H^2}$  < 1

Hung V. Tran*‡*

#### Cosmology as a Dynamical System Cosmology as  $\frac{1}{x}$ lynamica u ∪yo 1 <u>ici il provincia di controlle di</u><br>Electronic

 $\sim$  1000110093 do d Dyns<br>• It is convenient to work with the rescaled variables:

given schematically as follows:

- 
- Friedmann equation also takes a simple form:

$$
\frac{d\vec{z}}{dt} = g(\vec{z}) , \qquad \text{where } \vec{z} \equiv (x^1, ..., x^n, y^1, ..., y^m, H)
$$

$$
x^{a} = \frac{\kappa_{d}}{\sqrt{d-1}\sqrt{d-2}} \frac{\dot{\phi}^{a}}{H}, y_{i} = \frac{\kappa_{d}\sqrt{2}}{\sqrt{d-1}\sqrt{d-2}} \frac{\sqrt{V_{i}}}{H}
$$

$$
\left( x\right) ^{2}
$$

 $= 20$ 

• The cosmological equations can be formulated in terms of an autonomous system of ODEs  $\overline{1}$  $\overline{2}$ −<br>−2 ÷ −2 → ted in tern

• Among the above ODEs is  $\epsilon = -\dot{H}/H^2 = (d-1)x^2$ ; strategy is to bound the kinetic energy. gy is:

$$
2 + \left(y\right)^2 = 1
$$

# Geometric Bound on Cosmic Acceleration

• Define  $m$  vectors  $\mu_i$ , one for each potential term with components  $\left(\mu_i\right)_a = \gamma_{ia}$ 

![](_page_13_Figure_2.jpeg)

# **Obstruction by the Dilaton**

• String-theoretical potentials take the form: **String-theoretical potentials take the form** 

- least three terms, not all of the same sign (e.g., from loop corrections).  $\widetilde{\widehat{\mathcal{S}}}$ *δ*  $\widetilde{\widehat{\mathcal{S}}}$ Ways out: 1)  $\tilde{\delta}$  is stabilized; 2)  $\tilde{\delta}$  is rolling but not in the asymptotic ways out:  $\bullet$  $\frac{1}{2}$ −2 Stac • lower bound on : ≥ t three terms, not all of the same sign (e.g., from loop corrections). • Ways out: 1)  $\tilde{\delta}$  is stal −<br>−2  $(\text{red}; 2) \tilde{\delta}$  is n<br>ef the earne −<br>artisti
	- <sup>4</sup> (∞)<sup>2</sup> <sup>≥</sup> <sup>4</sup> <sup>2</sup> • Non-universal couplings for other moduli: can  $N \sim 10^{10}$ theory coupling coupling coupling coupling

 $\mathrm{X}_{1, d-1}$  $\delta k \sigma-\chi_{\rm E} \Phi \ =\ -\ \int_{{\bf x}_{\rm F}}\ \widetilde{*}_{1,d-1} \Lambda\, {\rm e}^{\kappa_d[\gamma_{\widetilde{\delta}}(\chi_{\rm E}) \delta-\gamma_{\widetilde{\sigma}}(\chi_{\rm E},r,k) \sigma]}\,,$ ̃ ̃  $\rho$  $\delta-\gamma_{\tilde{\sigma}}($  $J_{{\rm X}_{1,d-1}}$ 

• The  $d$ -dim. dilaton  $\delta$  is a linear combination of the 10d dilaton  $\Phi$  and Einstein frame volume. Φ  $\ddot{\cdot}$ √ Einstein f ran ⊧ volu

 $\cdot$  While the field basis choice is not unique, d-dimensional dilaton  $\delta$  has **universal properties**:  $\widetilde{\widehat{\mathcal{S}}}$  $\overline{r}$ nsional dila<br> aton  $\widetilde{\delta}$  has  $d-c$ √<br>∽∕ unique, d-dimensional dilaton  $\delta\,$  has  ${\bf u}$  $\overline{\mathbf{r}}$ **1**∣

• Ways out: 1)  $\delta$  is stabilized; 2)  $\delta$  is rolling but not in the asymptotic regions; 3) V contains at *V* ̃  $\frac{1}{\sqrt{1-\frac{1$ ng but<br>n (e a **T** olling but not

• Non-universal couplings for other moduli: can use our bound to **constrain compactifications**. l: can use our pound to constrain compact

![](_page_14_Picture_14.jpeg)

![](_page_14_Picture_15.jpeg)

$$
S=-\int_{\mathcal{X}_{1,9}} [A_r\wedge\star_{1,9}A_r]\,\Lambda_{10,r}\,\mathrm{e}^{-k\sigma-\chi_{\rm E}\Phi}=-\int_{\mathcal{X}}
$$

RR fields are not weighed by  $e^{-\chi_E \Psi}$  (effectively set  $\chi_E=0$ ) but would not affect our argument.  $\alpha$ re not weighed by  $e^{-\chi_E\Phi}$  (effectively set  $\chi_E^{}=0$ ) but would not  $\alpha$ RR fields are not weighed by  $e^{-\lambda E^\varphi}$  (effectively set  $\chi_E=0$  $\tau^{\chi_E\Phi}$  (effectively set  $\chi_F=0$ ) but would not affect our argument.  $\ddot{\phantom{a}}$ 

- $\widetilde{\widehat{\mathcal{S}}}$ + The d-dim. dilaton  $\tilde{\delta}$  is a linear combination of the 10d dilaton  $\Phi$  and Einstein **T**
- field basis choice is not unique, d-din  $\overline{a}$  $\sim$  string-frame: , string-frame: , string-frame: ,  $\sim$  ln The Theorem is chosen is not unit Ñ,  $\cdot$  While the field basis choice is  $\mathbf{r}$ √ ot unique,

$$
\gamma_{\widetilde{\delta}} = \frac{d}{\sqrt{d-2}} - \frac{1}{2}\chi_{\mathrm{E}}\sqrt{d-2} \quad \geq \frac{2}{\sqrt{d-2}} \quad \implies \qquad \epsilon \geq \frac{d-2}{4} \, (\gamma_{\infty})^2 \geq \frac{d-2}{4} \, \gamma_{\widetilde{\delta}}^2 \geq 1
$$

- = 0*.*
	- scale factor takes a power law form: *a*(*t*) ∼ *t p* solutions can be characterized and  $a(t) \sim t^p$ If the rank of the *ia*-matrix matches the number of al<del>o</del> laului tan<del>os</del> d po wer la<mark>y</mark>  $\bm{v}$ r takes a power law form:  $a(t) \thicksim t^{\mu}$  $\overline{\phantom{a}}$
- critical points of the autonomous system:  $\bm{a}$  autonomous system:  $\dot{x}^a = 0$ rolling-scalar solutions are general. Given the matrix @✓ = 0*,*  $\sqrt{2}$ politics ( 10MOU @' = 0*,* the critical is dominated by the potential term, which  $\overline{X}$ *m m*
- **Analytic solution:** for rank  $\gamma_{ia} = m$  $rank v =$  $m<sub>1</sub>$  solutions exist of the solution of the tic solu<sup>.</sup>  $\frac{1}{2}$   $\frac{1}{2}$   $\frac{1}{4}$   $\frac{1}{4}$   $\frac{1}{4}$   $\frac{1}{4}$ = 0*.* ing the '-equation Analytic s **P**: for rank  $\gamma_i$ *d*
- field space trajectory:  $\phi_*^a(t) = \phi_0^a +$ 2 *d*  $\kappa_d$   $\lfloor \sum_{i=1}^{\infty} \frac{1}{j} \rfloor$ the time evolution is consistent with dropping the same  $\cdot$  of time  $\mathsf{H}$ ; trajectory.  $\psi_*(\iota)=\psi_0\mp\frac{\epsilon}{\kappa_d}\left| \right. \angle$
- scale factor:  $\cdot$  scale-factor  $p = \frac{4}{\sqrt{2}} \sum_{l=1}^{n} \sum_{l=1}^{n}$  $\int_a^b d-2 \left(1-\frac{1}{1-\frac{1$  $\mathbf{I}$ such that the axionic term is such that the axionic term is  $\mathcal{S}$  $p =$ 4  $d-2$  $\sqrt{ }$
- The kinetic term & every potential term have the same parametric dependence in time: The kinetic term & every potential term have the same parametric dependence in time: terms at any time, it is consistent to neglect the axions. be kinetic

#### Scaling Solutions all these reasons, although it is hard to prove these reasons, although it is hard to prove that scales in the<br>These reasons, although it is hard to prove that scales in the prove that scales in the proven that scales in ing solutions always capture the inevitable late-time behavior of the complete solutions, they deserve a detailed rank *ia* = *m* and *n>m*, the scalar fields outnumber the scalar-potential terms, but then we can rotate the field-

ottroater behevier o anno amaolor bondviore.<br>**"Oved** in [STT2, STT4],  $T(t) = T(t_0) \begin{pmatrix} 0 \\ 0 \end{pmatrix}$ ,  $V_i(t) = V_i(t_0) \begin{pmatrix} 0 \\ 0 \end{pmatrix}$  proved in [3112, 3114], going beyond earlier analysis<br>of linear stability proved in [STT2, STT4], allidu of linear stability. Late-time attractor behavior

 $\overline{\mathbf{u}}$  under the slow-roll approximation, by which one slow-roll approximation, by which one slow-roll approximation,  $\overline{\mathbf{v}}$ rank *ia* = *m* and *n>m*, the scalar fields outnumber the  $s_{\rm c}$ **a** *i j* . (III.3) . (III.3) . (III.3) . (II.3) .

**No slow-roll:** 
$$
T(t) = T(t_0) \left(\frac{t_0}{t}\right)^2
$$
,  $V_i(t) = V_i(t_0) \left(\frac{t_0}{t}\right)^2$  **Exercise 1.2.12**

\n- \n field space trajectory:\n 
$$
\phi_*^a(t) = \phi_0^a + \frac{2}{\kappa_d} \left[ \sum_{i=1}^m \sum_{j=1}^m \gamma_i^a (M^{-1})^{ij} \right] \ln \frac{t}{t_0}, \qquad M_{ij} = \gamma_{ia} \gamma_j^a
$$
\n
\n- \n scale factor:\n 
$$
p = \frac{4}{d-2} \sum_{i=1}^m \sum_{j=1}^m (M^{-1})^{ij}.
$$
\n
\n- \n [Copeland, Liddle, Wands, '97]\n [Collinucci, Nielsen, Van Riet, '04]\n
\n

ne parametric dependence i m & every potential term have the same parametric dep

**No slow-roll:** 
$$
T(t) = T(t_0) \left(\frac{t_0}{t}\right)^2
$$
,  $V_i(t) = V_i(t_0) \left(\frac{t_0}{t}\right)^2$  **l.e.**

#### • The cosmological autonomous system admits scaling solutions ( $\epsilon =$  constant  $\;>0$ ): and cosmological autonomous system admits scaling solutions ( $\epsilon = \text{constant} > 0$ ). ical autonomous system admits scaling solutions ( $\epsilon=$