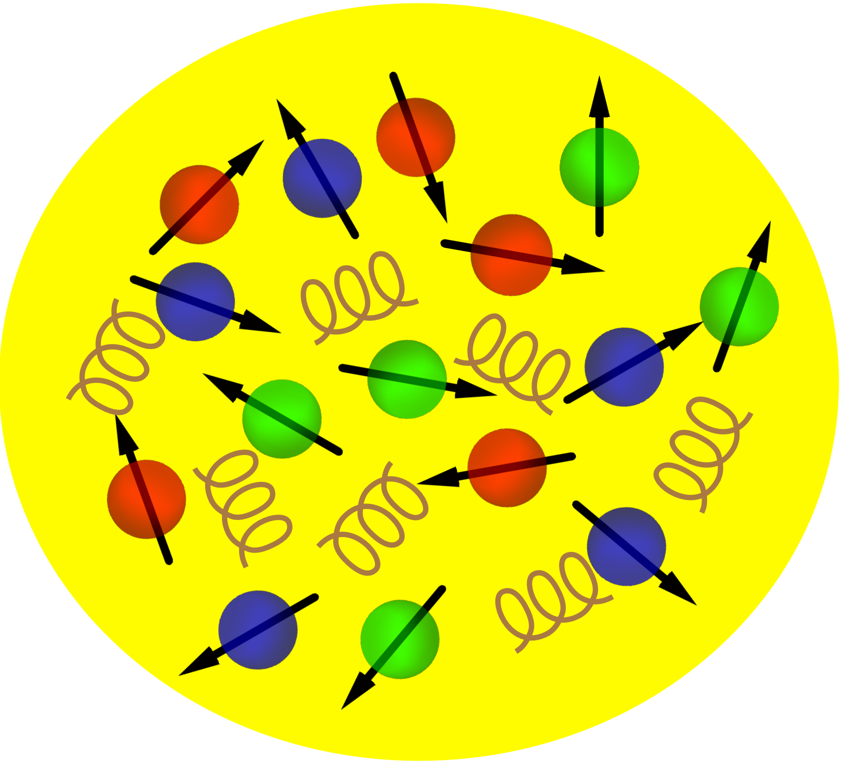
Spin relaxation dynamics in hot QCD matter



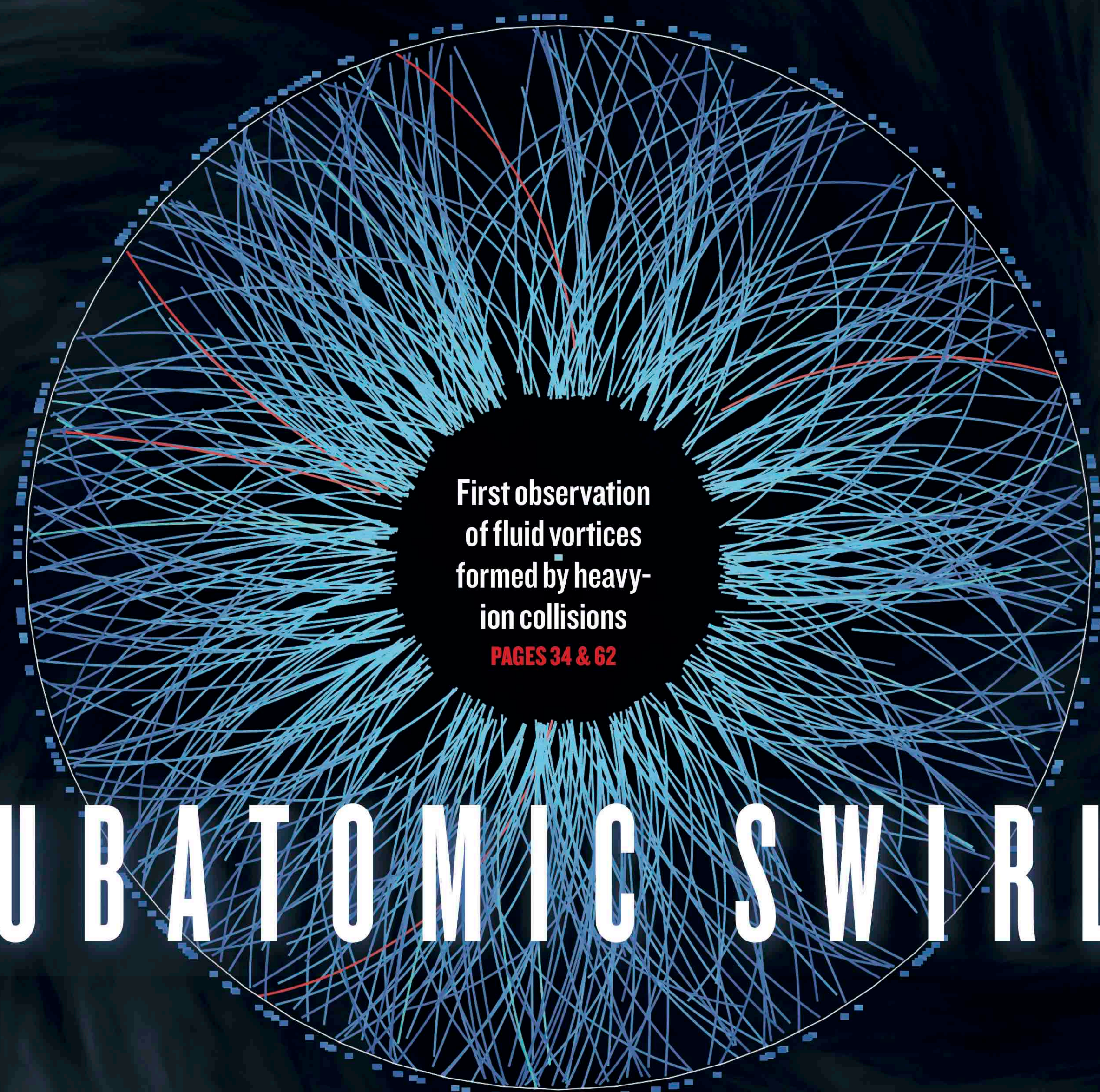
Masaru Hongo (Niigata University/RIKEN iTHEMS)

2024/10/18, West Lake Workshop on Nuclear Physics 2024

Hattori-MH-Huang-Matsuo-Taya PLB (2019), MH-Huang-Kaminski-Stephanov-Yee JHEP (2021)
MH-Huang-Kaminski-Stephanov-Yee JHEP (2021), JHEP (2022), Hidaka-MH- Stephanov-Yee PRC (2024)

nature

THE INTERNATIONAL WEEKLY JOURNAL OF SCIENCE



First observation of fluid vortices formed by heavy-ion collisions

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CLIMATE CHANGE

PARIS AGREEMENT

Time for nations to match words with deeds

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SUMMER SELECTION

Recommended reading for the holiday season

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STEM CELLS

YOUTHFUL SECRETS

How the hypothalamus helps to control the ageing process

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3 August 2017

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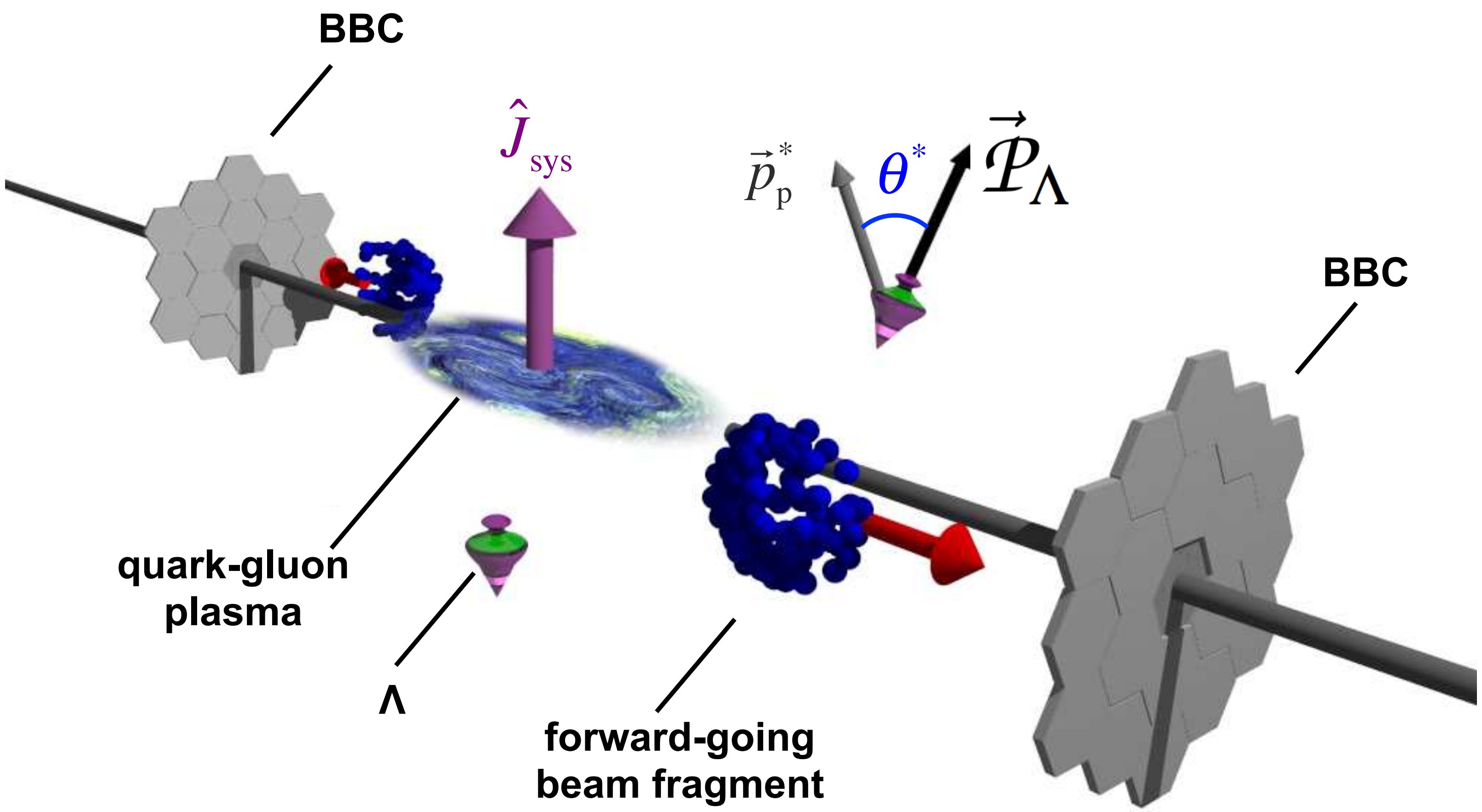
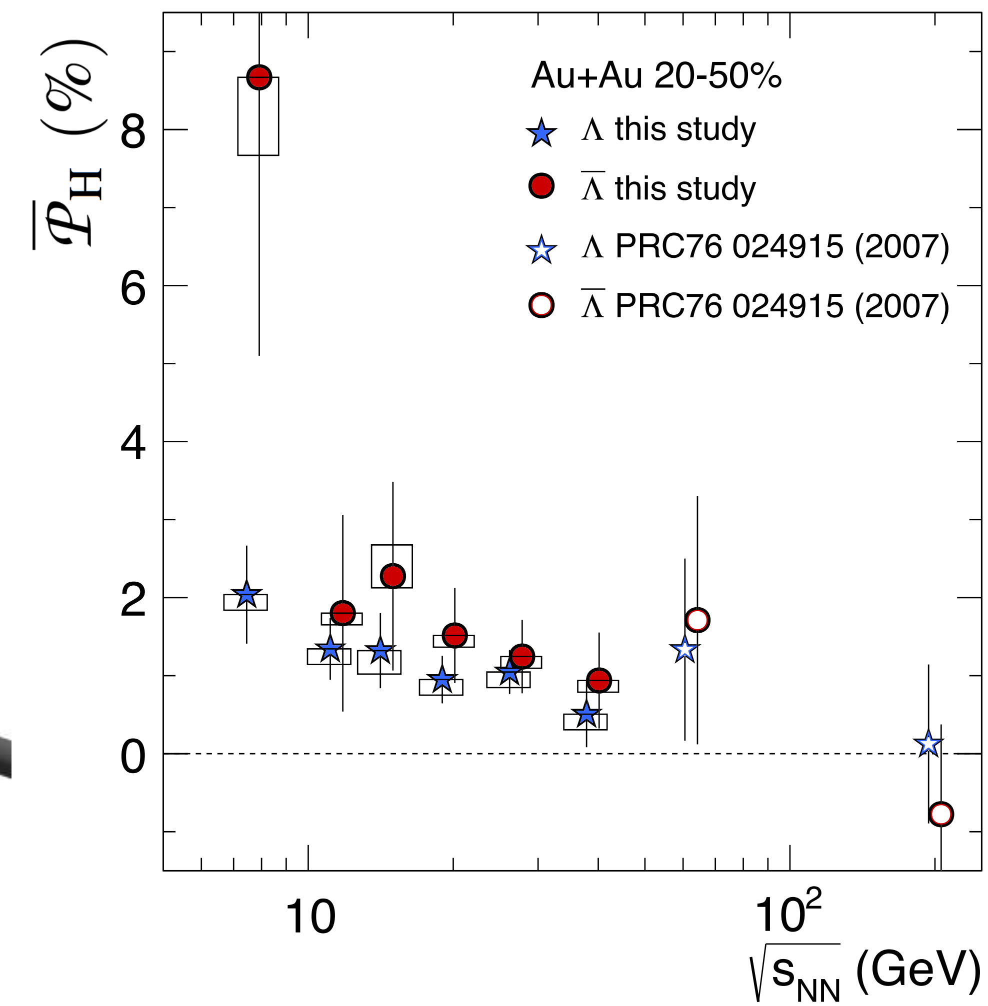
Spin in heavy-ion collisions

Global Λ polarization [STAR Collaboration, Nature (2017)]





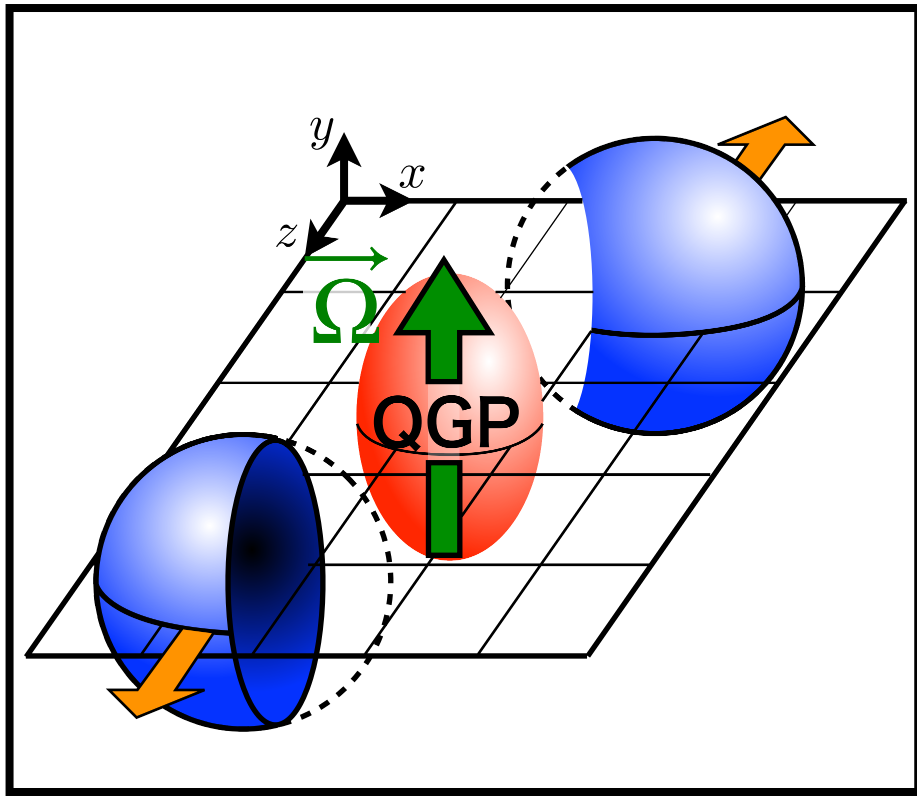
"Non-central collisions have angular momentum on the order of 1000$\hbar$, and the resulting fluid may have a strong vortical structure…"

The extreme condition with a huge angular momentum!

[In terms of vorticity, it reaches $\omega \sim 9 \times 10^{21}\,\text{s}^{-1} \simeq 10\,\text{MeV}$]

Spin polarization from rotation

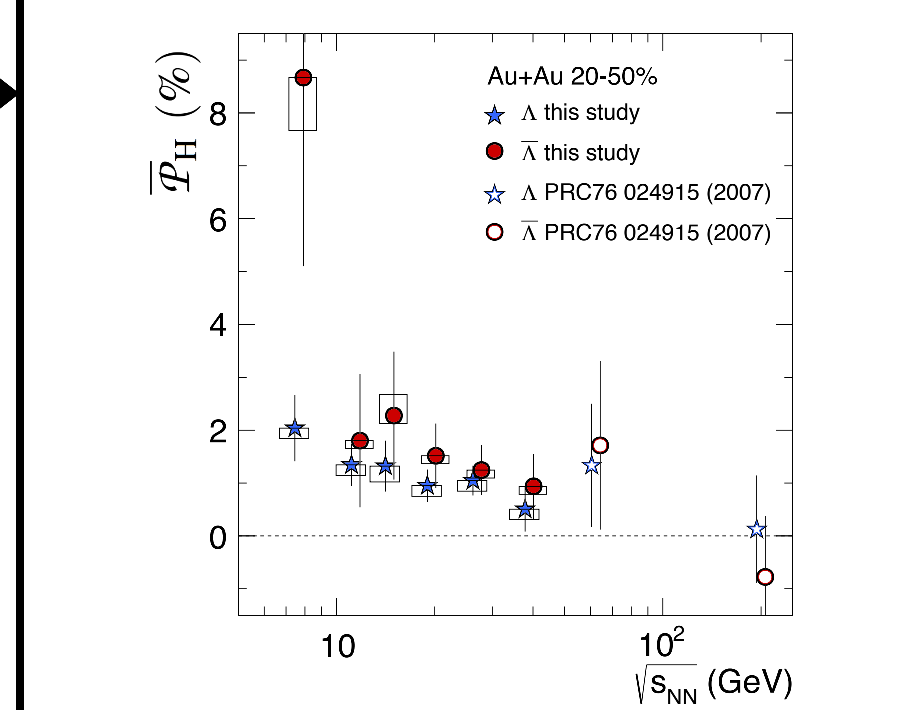
Assume created QGP has angular momentum at initial stages!



Theoretical model

1. Hydrodynamic time-evolution
2. Translate hydro into particle distributions
3. Hadronic evolution

[STAR, Nature (2017)]



Rotating velocity profiles after (usual) hydrodynamic evolutions is expected to induce observed spin polarization!

[Liang-Wang, Becattini, ..., see recent reviews, e.g., Becattini arXiv:2204.01144]

Spin polarization from rotation

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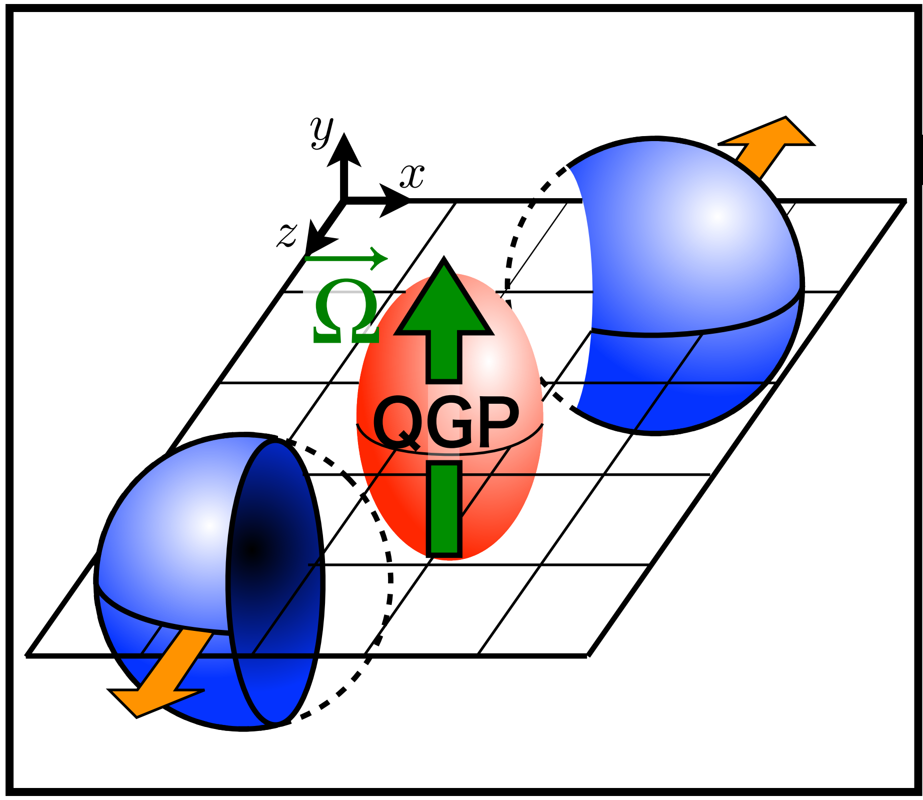

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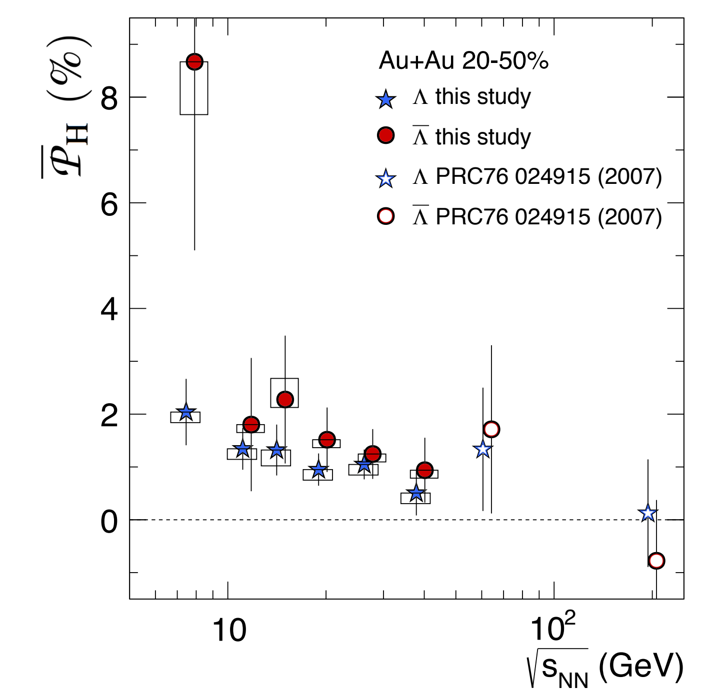

Rotating velocity profiles after (usual) hydrodynamic evolutions is expected to induce observed spin polarization!

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◆ Questions

- What is intrinsic spin dynamics at a hydrodynamics stage?
- What is intrinsic spin dynamics at a hadronic stage?

Outline

◆ Spin relaxation at a hydrodynamics stage

[MH-Huang-Kaminski-Stephanov-Yee, JHEP (2021), JHEP (2022)]

◆ Spin dynamics at a hadronic stage

[Hidaka-MH- Stephanov-Yee, PRC (2024)]

Outline

◆ Spin relaxation at a hydrodynamics stage

[MH-Huang-Kaminski-Stephanov-Yee, JHEP (2021), JHEP (2022)]

◆ Spin dynamics at a hadronic stage

[Hidaka-MH- Stephanov-Yee, PRC (2024)]

<u>Warm up</u>

Derivation of diffusion eq.

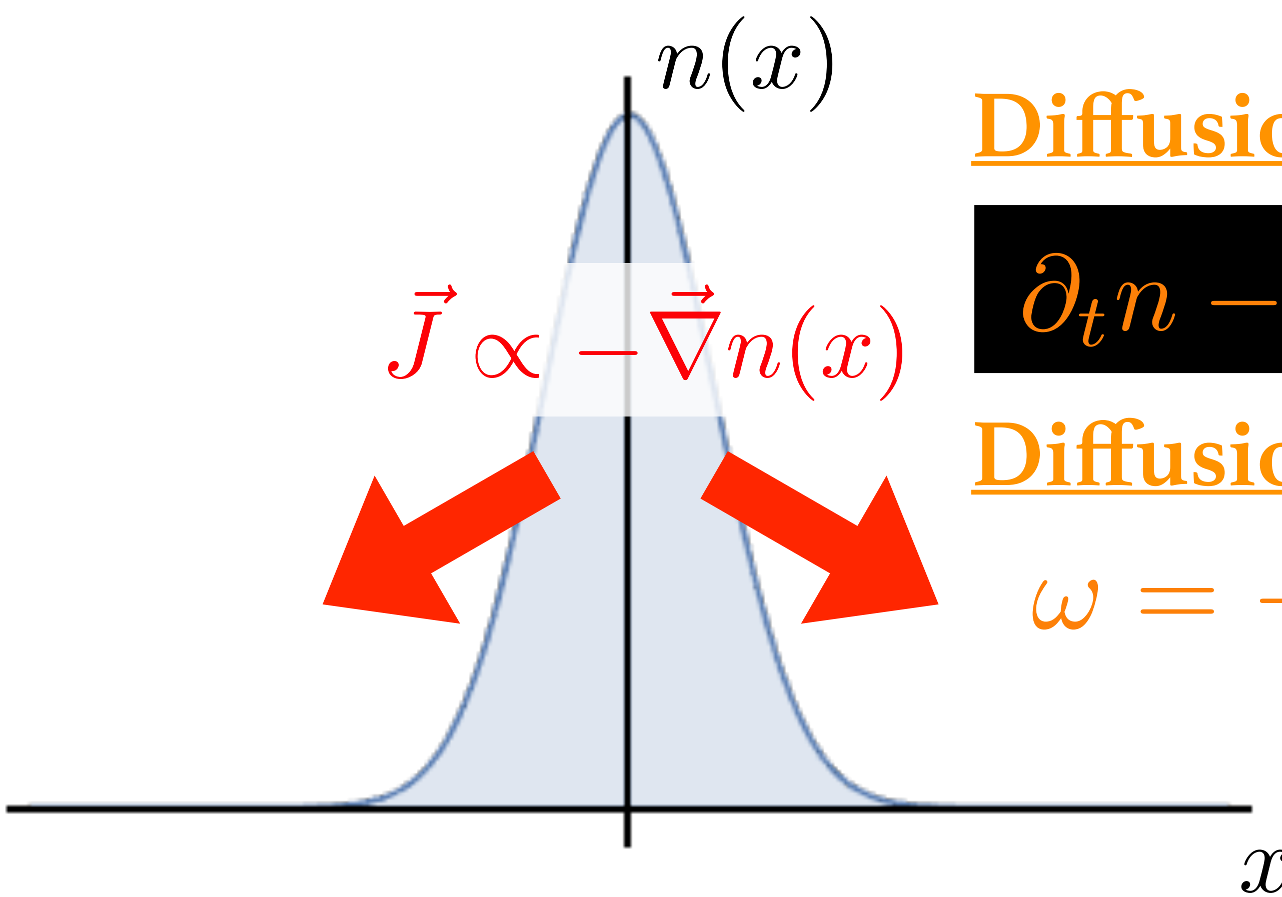
Prototype: Charge diffusion

◆ **Bulding blocks of hydrodynamic equation**

(1) **Conservation law:** $\partial_t n + \vec{\nabla} \cdot \vec{J} = 0$

(2) **Constitutive relation:** $\vec{J} = -T\kappa_n \vec{\nabla}(\beta\mu) \simeq -D\vec{\nabla} n$

(3) **Physical properties:** Values of $\kappa_n, \ \chi_n \quad (D = \kappa_n/\chi_n)$

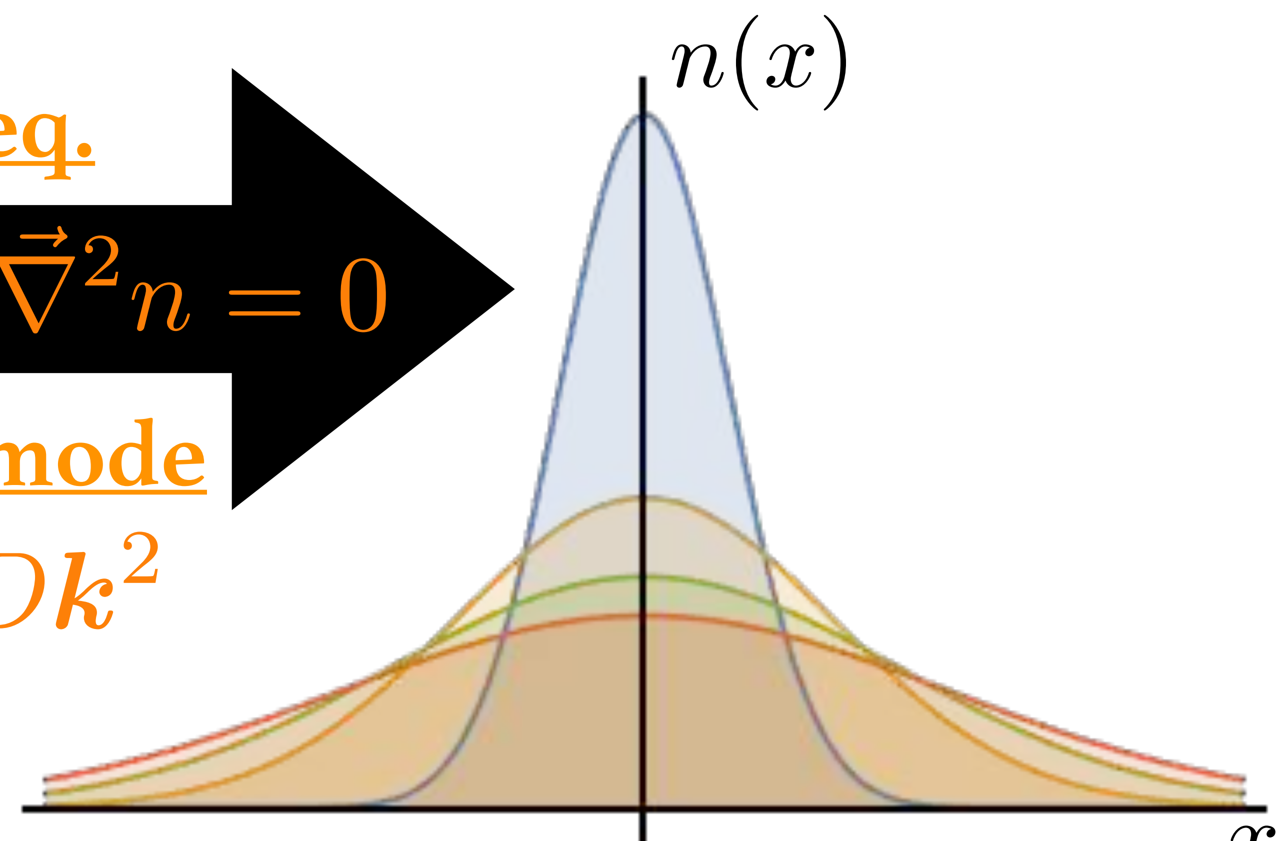


Diffusion eq.

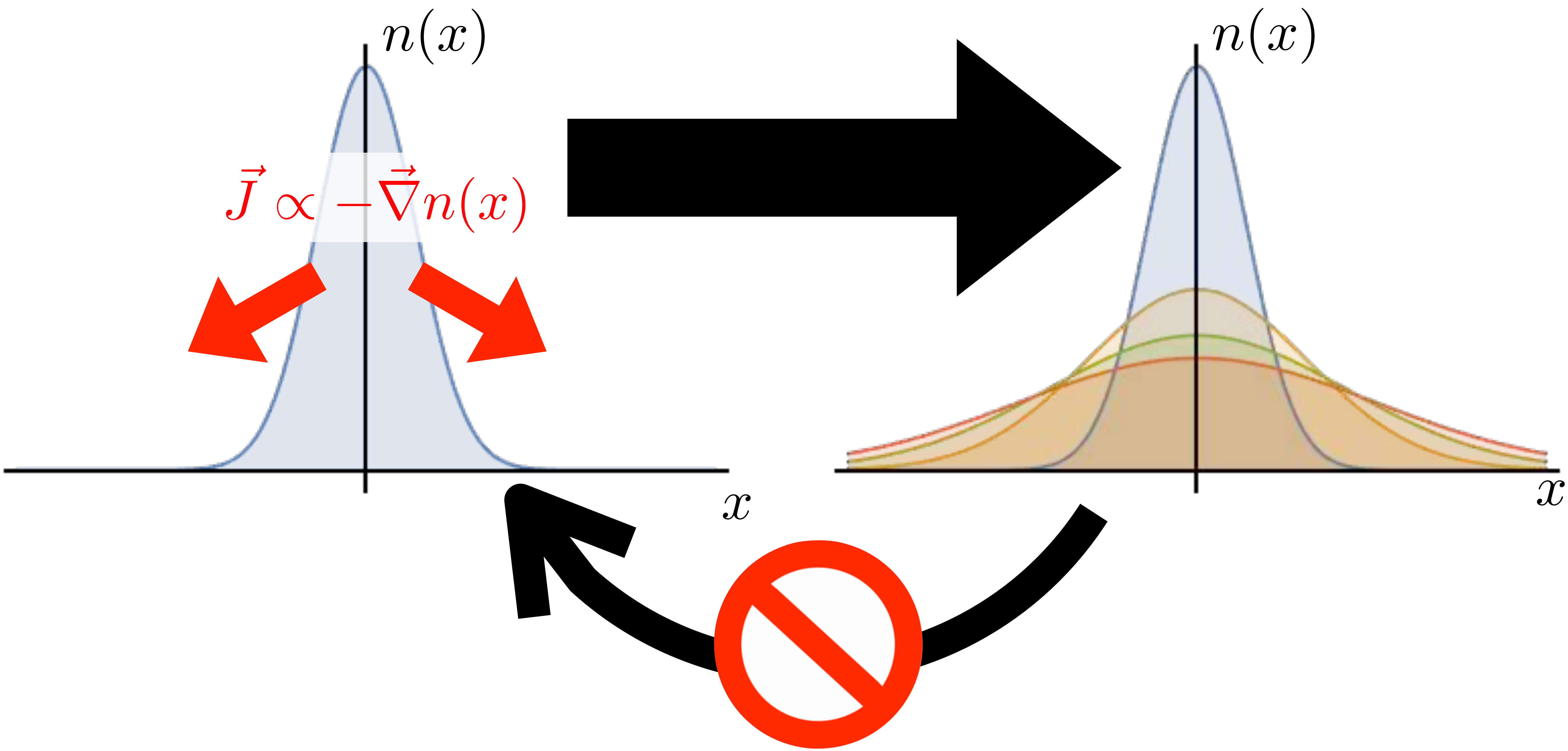
$$\partial_t n - D\vec{\nabla}^2 n = 0$$

Diffusion mode

$$\omega = -\mathrm{i}D\boldsymbol{k}^2$$



Irreversiblity of diffusion



No-go for time-reversal process!

Thermodynamic concepts, especially, The 2nd law, should be there!

Phenomenological derivation

Step 1. Determine dynamical d.o.m (& its equation of motion)

Charge density: $n(x)$ EoM: $\partial_t n + \vec{\nabla}\cdot\vec{J} = 0$

Step 2. Introduce entropy & conjugate variable

Entropy density: $s(n)$ $\xrightarrow{T\mathrm{d}s = -\mu \mathrm{d}n}$ Chemical pot.: $\beta\mu \equiv -\dfrac{\partial s}{\partial n}$

Step 3. Write down all possible terms with finite derivatives

Current: $\vec{J} = 0 - T\kappa_n \vec{\nabla}(\beta\mu) + O(\vec{\nabla}^2) = -T\kappa_n \vec{\nabla}\dfrac{\partial s}{\partial n} + O(\vec{\nabla}^2)$

Step 4. Restrict terms to be compatible with local 2nd law

$\exists\, s^\mu$ such that $\partial_t s + \vec{\nabla}\cdot\vec{s} \geq 0$ $\Rightarrow$ $\kappa_n \geq 0$ with $\vec{s} = \beta\mu\vec{J}$

One way to determine κ_n

◆ **Linearized constitutive relation**

$$\vec{J} = -T\kappa_n \vec{\nabla}(\beta\mu) \simeq -D\vec{\nabla}n \quad \text{with} \quad D \equiv \frac{\kappa_n}{\chi_n}$$

➡ **Diffusion equation:** $\partial_t n - D\vec{\nabla}^2 n = 0$

➡ **Dispersion relation:** $\omega(\boldsymbol{k}) = -\mathrm{i}D\boldsymbol{k}^2$

◆ **Green's function interpretation of the result**

$$\widetilde{G}_R^{nn}(\omega, \boldsymbol{k}) = \frac{\mathrm{i}\chi_n D\boldsymbol{k}^2}{\omega + \mathrm{i}D\boldsymbol{k}^2} \qquad \left(\chi_n = \lim_{\boldsymbol{k}\to 0} \widetilde{G}_R^{nn}(\omega = 0, \boldsymbol{k})\right)$$

➡ **Charge density correlator enables us to obtain *D***

Semi-phenomenology

◆ **Bulding blocks of hydrodynamic equation**

(1) Conservation law: $\partial_t n + \vec{\nabla} \cdot \vec{J} = 0$

(2) Constitutive relation: $\vec{J} = -T\kappa_n \vec{\nabla}(\beta\mu) \simeq -D\vec{\nabla}n$

(3) Physical properties: Values of $\kappa_n, \ \chi_n \ \ (D = \kappa_n/\chi_n)$

(1) Conservation law — Ward-Takahashi identity resulting from symmetry of systems

(2) Constitutive relation — Phenomenological analysis based on local thermodynamics laws

(3) Physical properties — Matching the hydrodynamic result with the field-theoretical correlator

Application to spin hydrodynamics

Semi-phenomenology

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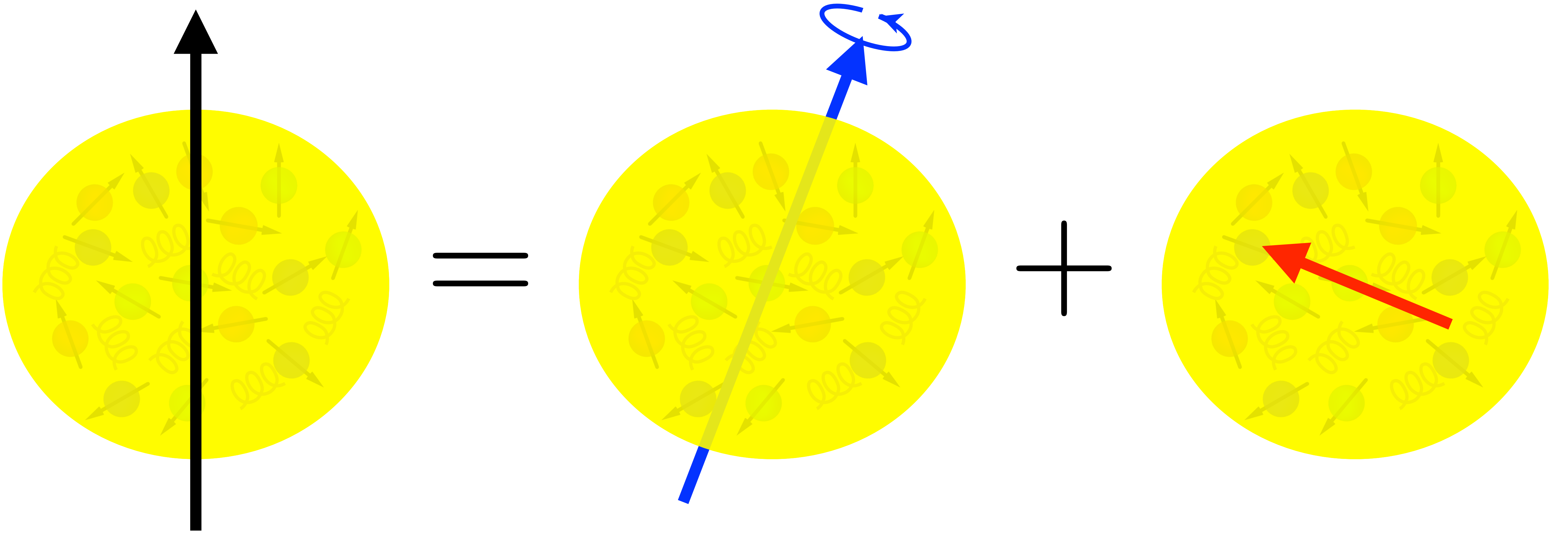
Angular momentum conservation

◆ **Ward-Takahashi identities**

Conservation law: $\partial_\mu \Theta^{\mu\nu} = 0, \;\; \partial_\mu J^{\mu\nu\rho} = 0$

Decomposition: $J^{\mu\nu\rho} = x^\nu \Theta^{\mu\rho} - x^\rho \Theta^{\mu\nu} + \Sigma^{\mu\nu\rho}$

Total AM **Orbital AM** **Spin AM**



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$\exists\, s^\mu$ such that $\partial_t s + \vec{\nabla} \cdot \vec{s} \geq 0$ $\Rightarrow$ $\kappa_n \geq 0$ with $\vec{s} = \beta\mu\vec{J}$

Result in flat spacetime

◆ **Equation of motion**

$$\partial_\mu \Theta^\mu{}_\nu = 0, \qquad \partial_\mu \Sigma^\mu{}_{\nu\rho} = -(\Theta_{\nu\rho} - \Theta_{\rho\nu})$$

◆ **Constitutive relation**

$$\Theta^\mu_\nu = \epsilon u^\mu u_\nu + p\Delta\mu_\nu - \eta^\mu{}_\nu{}^\rho{}_\sigma \partial_\rho u^\sigma - (\eta_s)^\mu{}_\nu{}^\rho{}_\sigma (\partial_\rho u^\sigma - \mu_\rho{}^\sigma)$$

$$\Sigma^\mu{}_{\nu\rho} = \varepsilon^\mu{}_{\nu\rho\sigma}\sigma^\sigma$$

◆ **Transport coefficient: η, ζ, ηs**

$$\eta^\mu{}_\nu{}^\rho{}_\sigma = 2\eta\left(\frac{1}{2}(\Delta^{\mu\rho}\Delta_{\nu\sigma} + \Delta^\mu_\sigma\Delta^\rho_\nu) - \frac{1}{3}\Delta^\mu_\nu\Delta^\rho_\sigma\right) + \zeta\Delta^\mu_\nu\Delta^\rho_\sigma$$

$$(\eta_s)^\mu{}_\nu{}^\rho{}_\sigma = \frac{1}{2}(\Delta^{\mu\rho}\Delta_{\nu\sigma} - \Delta^\mu_\sigma\Delta^\rho_\nu)$$

Semi-phenomenology

◆ Bulding blocks of hydrodynamic equation

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Linear-mode analysis on spin-hydro to compute η_s

Linearized spin-hydro

Perturbation on the top of **global static** **thermal equilibrium:**

$$\begin{cases} \epsilon(x) = \epsilon_0 + \delta\epsilon(x) \\ v^i(x) = 0 + \delta v^i(x) \\ \sigma^{\hat{a}}(x) = 0 + \delta\sigma^{\hat{a}}(x) \end{cases}$$

with the flat background

Pickup $O(\delta)$-terms only

◆ **Linearized spin-hydrodynamic equations:**

$$0 = \partial_0 \delta\epsilon + \partial_i \delta\pi^i,$$

$$0 = \partial_0 \delta\pi_i + c_s^2 \partial_i \delta\epsilon - \gamma_{\parallel} \partial_i \partial^j \delta\pi_j - (\gamma_\perp + \gamma_s)(\delta_i^j \boldsymbol{\nabla}^2 - \partial_i \partial^j)\delta\pi_j + \frac{1}{2}\Gamma_s \varepsilon_{0ijk} \partial^j \delta\sigma^k,$$

$$0 = \partial_0 \delta\sigma_i + \Gamma_s \delta\sigma_i + 2\gamma_s \varepsilon_{0ijk} \partial^j \delta\pi^k,$$

with a set of parameters:

$$\begin{cases} c_s^2 \equiv \dfrac{\partial p}{\partial \epsilon}, \quad \gamma_\parallel \equiv \dfrac{1}{\epsilon_0 + p_0}\left(\zeta + \dfrac{4}{3}\eta\right), \quad \gamma_\perp \equiv \dfrac{\eta}{\epsilon_0 + p_0}, \\ \chi_s \delta_{ij} \equiv \dfrac{\partial \sigma_i}{\partial \mu^j}, \quad \gamma_s \equiv \dfrac{\eta_s}{2(\epsilon_0 + p_0)}, \quad \Gamma_s \equiv \dfrac{2\eta_s}{\chi_s}. \end{cases}$$

Linear-mode analysis

Linearized eom can be solved by the use of Fourier tr.!

$$\delta\mathcal{O}(x) = e^{-\mathrm{i}(\omega t - \boldsymbol{k}\cdot\boldsymbol{x})}\delta\tilde{\mathcal{O}}(k) \quad \Rightarrow \quad \textbf{EoM:}\ A(\omega,\boldsymbol{k})\delta\tilde{\mathcal{O}}(k) = 0$$

$$(A(\omega,\boldsymbol{k}) : 7\times 7\mathrm{matrix})$$

Characteristic equation: $\det A(\omega,\boldsymbol{k}) = 0$

◆ **Dispersion relation**

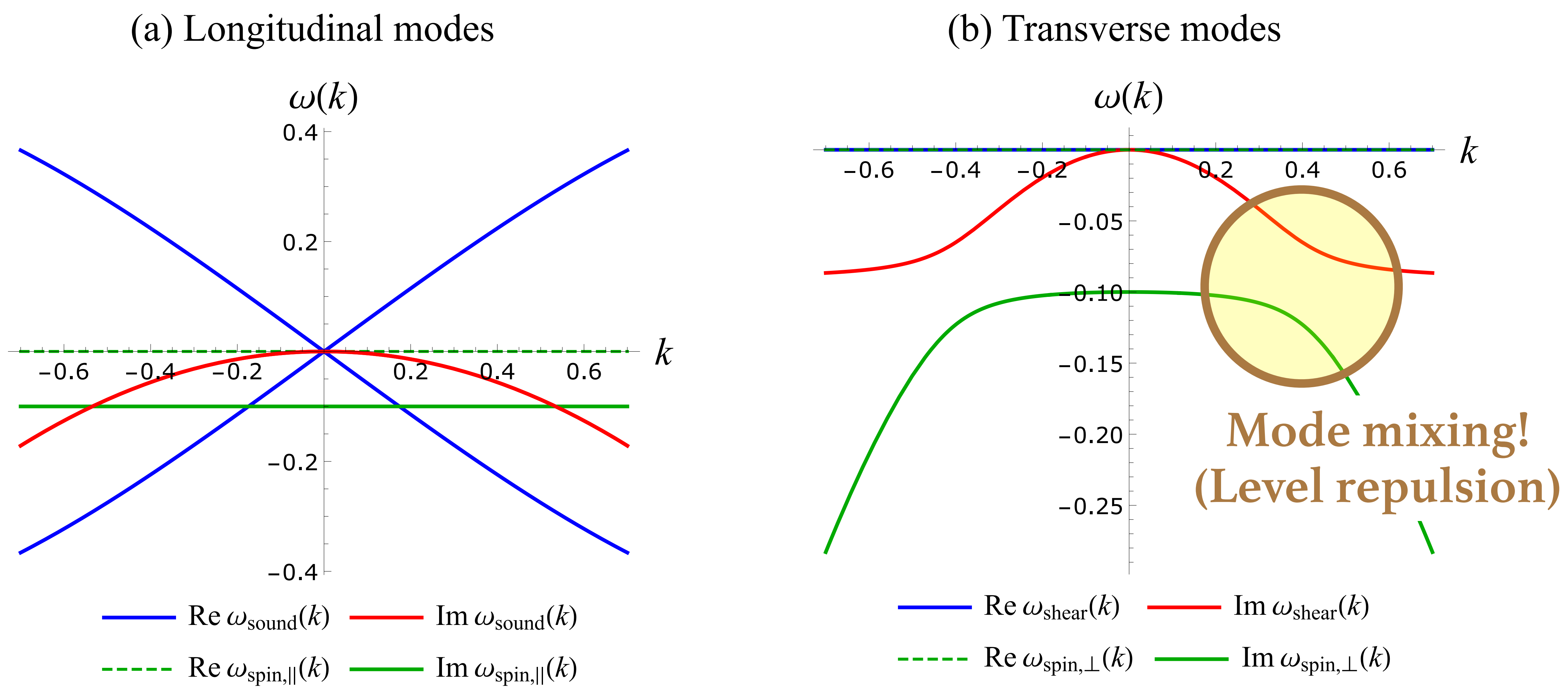
$$\omega_{\mathrm{sound}}(\boldsymbol{k}) = \pm c_s|\boldsymbol{k}| - \frac{\mathrm{i}}{2}\gamma_{\parallel}\boldsymbol{k}^2 + O(\boldsymbol{k}^3),$$

$$\omega_{\mathrm{spin},\parallel}(\boldsymbol{k}) = -\mathrm{i}\Gamma_s$$

$$\omega_{\mathrm{shear}}(\boldsymbol{k}) = -\frac{\mathrm{i}\Gamma_s + \mathrm{i}(\gamma_\perp+\gamma_s)\boldsymbol{k}^2 - \mathrm{i}\sqrt{\Gamma_s^2 - 2\Gamma_s(\gamma_\perp-\gamma_s)\boldsymbol{k}^2 + (\gamma_\perp+\gamma_s)^2\boldsymbol{k}^4}}{2}$$

$$\omega_{\mathrm{spin},\perp}(\boldsymbol{k}) = -\frac{\mathrm{i}\Gamma_s + \mathrm{i}(\gamma_\perp+\gamma_s)\boldsymbol{k}^2 + \mathrm{i}\sqrt{\Gamma_s^2 - 2\Gamma_s(\gamma_\perp-\gamma_s)\boldsymbol{k}^2 + (\gamma_\perp+\gamma_s)^2\boldsymbol{k}^4}}{2}$$

Dispersion relation



$$\omega_{\text{shear}}(\boldsymbol{k}) = -\frac{\mathrm{i}\Gamma_s + \mathrm{i}(\gamma_\perp + \gamma_s)\boldsymbol{k}^2 - \mathrm{i}\sqrt{\Gamma_s^2 - 2\Gamma_s(\gamma_\perp - \gamma_s)\boldsymbol{k}^2 + (\gamma_\perp + \gamma_s)^2\boldsymbol{k}^4}}{2}$$

$$\omega_{\text{spin},\perp}(\boldsymbol{k}) = -\frac{\mathrm{i}\Gamma_s + \mathrm{i}(\gamma_\perp + \gamma_s)\boldsymbol{k}^2 + \mathrm{i}\sqrt{\Gamma_s^2 - 2\Gamma_s(\gamma_\perp - \gamma_s)\boldsymbol{k}^2 + (\gamma_\perp + \gamma_s)^2\boldsymbol{k}^4}}{2}$$

Spin-spin correlator

Taking $k=0$, all spin modes has $\omega_{\rm spin}(k=0) = -\mathrm{i}\Gamma_s$

Spin densities shows a gapped relaxation dynamics with a characteristic time scale $\tau_s = \Gamma_s^{-1}$

◆ **Green's function interpretation of the result**

Spin-spin correlator: $$\widetilde{G}_{\rm R}^{\sigma^i \sigma^j}(\omega, \boldsymbol{k}) = \frac{\mathrm{i}\chi_s \Gamma_s + \cdots}{\omega + \mathrm{i}\Gamma_s + O(\boldsymbol{k}^2)} \delta^{ij}$$

(Definition of spin susceptibility: $\lim_{\boldsymbol{k}\to 0} \widetilde{G}_{\rm R}^{\sigma_i \sigma_j}(\omega = 0, \boldsymbol{k}) = \chi_s \delta^{ij}$)

Spin-spin correlator enables us to obtain $\Gamma_s \equiv \dfrac{2\eta_s}{\chi_s}$

Semi-phenomenology

◆ Bulding blocks of hydrodynamic equation

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√(2) Constitutive relation — Phenomenological analysis based on local thermodynamics laws

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√(3) Physical properties — Matching the hydrodynamic result with the field-theoretical correlator

Summary of our result

◆ Equation of motion

$$\partial_\mu \Theta^\mu{}_\nu = 0, \qquad \partial_\mu \Sigma^\mu{}_{\nu\rho} = -(\Theta_{\nu\rho} - \Theta_{\rho\nu})$$

◆ Constitutive relation

$$\Theta^\mu_\nu = \epsilon u^\mu u_\nu + p\Delta\mu_\nu - \eta^\mu{}_\nu{}^\rho{}_\sigma \partial_\rho u^\sigma - (\eta_s)^\mu{}_\nu{}^\rho{}_\sigma (\partial_\rho u^\sigma - \mu_\rho{}^\sigma)$$

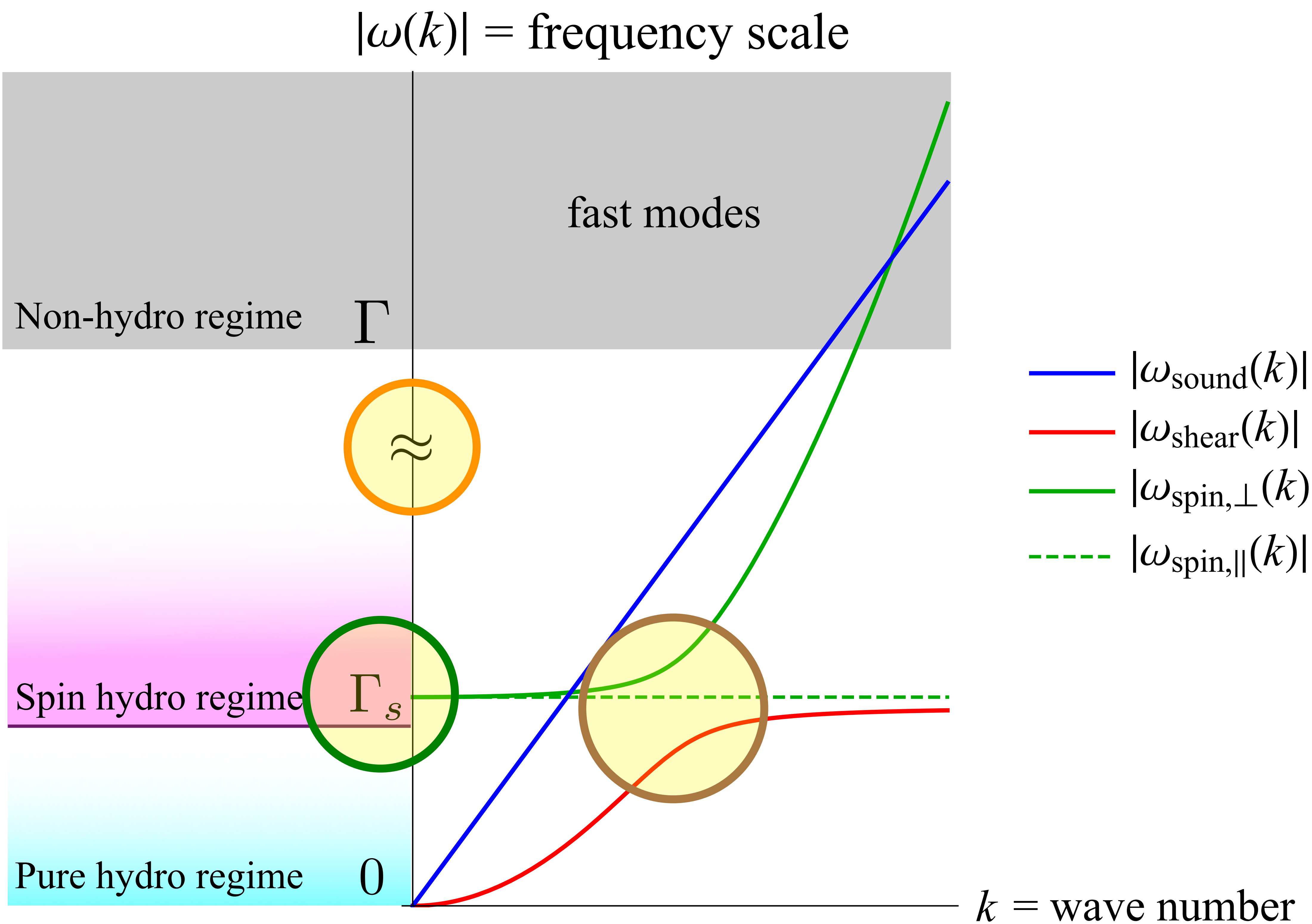
$$\Sigma^\mu{}_{\nu\rho} = \varepsilon^\mu{}_{\nu\rho\sigma}\sigma^\sigma$$

◆ Green-Kubo-formula for rotational viscosity η_s

$$(\eta_s)^\mu{}_{\hat{a}}{}^\nu{}_{\hat{b}} = \frac{1}{2}\eta_s(\Delta^{\mu\nu}\Delta_{\hat{a}\hat{b}} - \Delta^\mu_{\hat{b}}\Delta^\nu_{\hat{a}})$$

$$\eta_s = 2 \lim_{\Gamma_s \ll \omega \ll \Gamma} \lim_{\boldsymbol{k}\to 0} \frac{1}{\omega} \mathrm{Im}\, \widetilde{G}_{\mathrm{R}}^{\Theta^x{}_{\hat{y}}|_{(a)}, \Theta^x{}_{\hat{y}}|_{(a)}}(\omega, \boldsymbol{k})$$

Sketch of our result

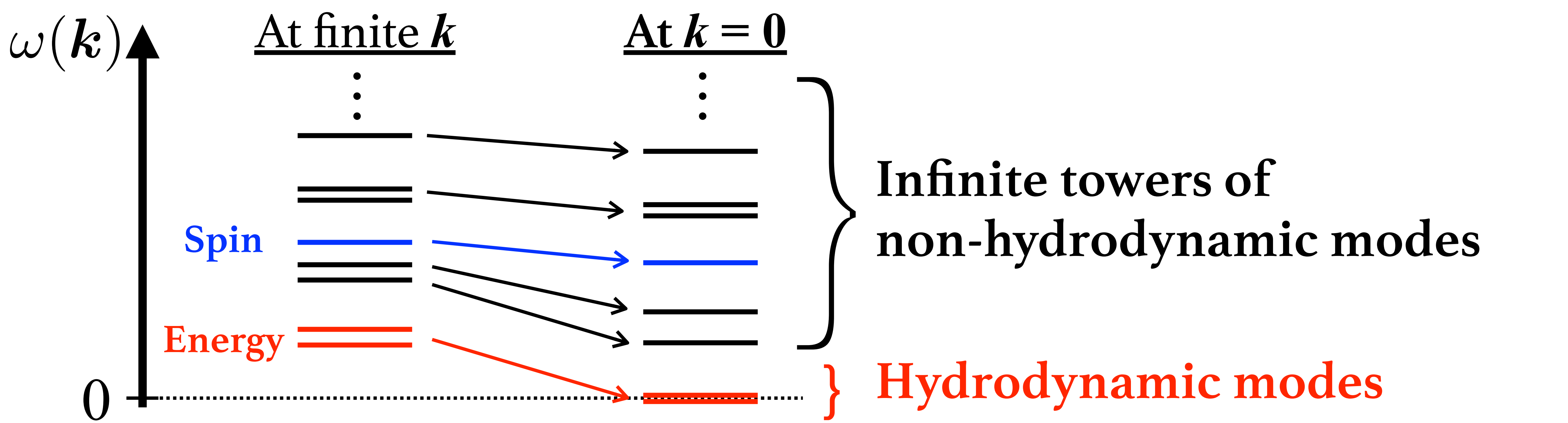


When **scale separation** occurs?

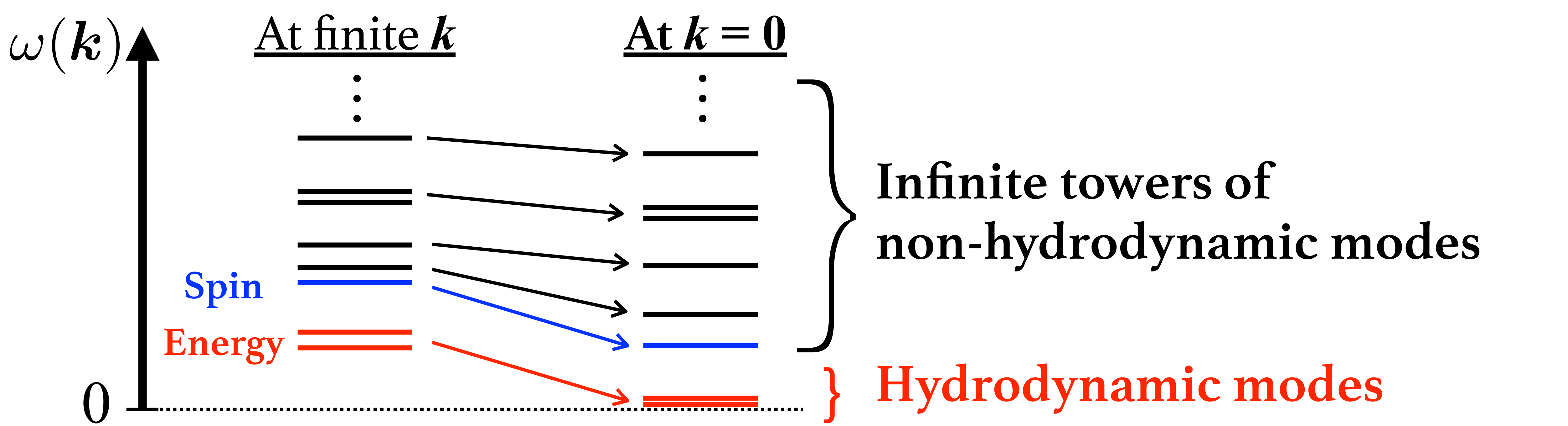
$$(\Gamma_s \ll \Gamma)$$

Spin hydro is ill-defined

◆ Scenario 1 (Bad: Spin hydro = Hydro++++?)

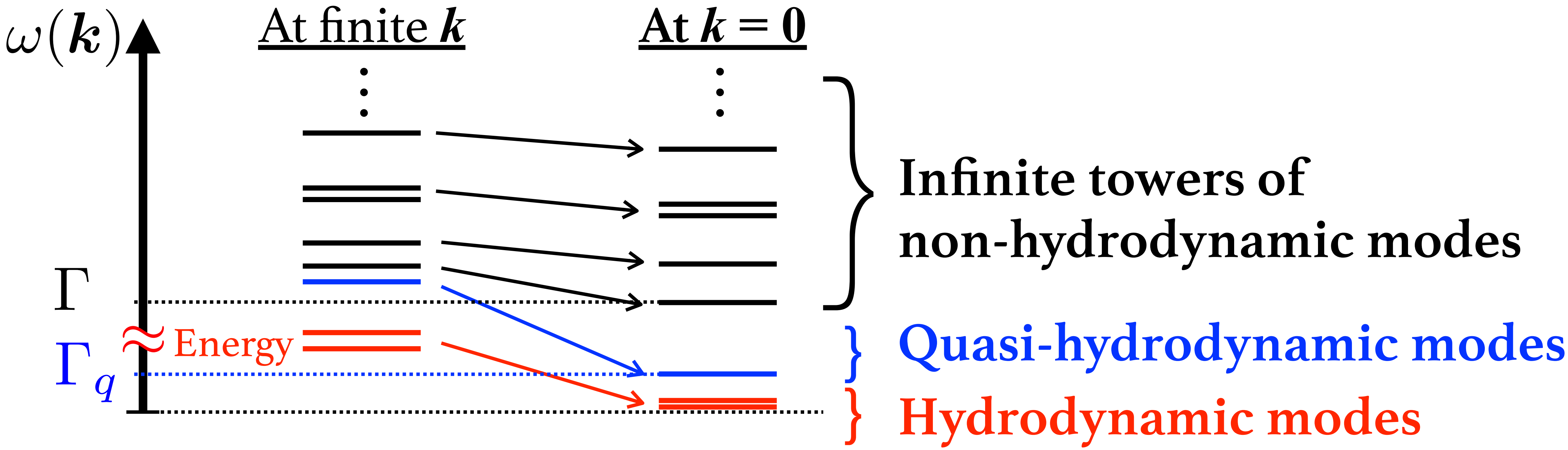


◆ Scenario 2 (Better but still not good: Spin hydro = Hydro+?)



Well-defined Hydro+

◆ When Hydro+ is well-defined



If $\Gamma_q \ll \Gamma$ is satisfied, Hydro+ becomes well-defined!!

This generally happens when

emergent symmetry appears by tuning parameters (T, m, …)!

- Critical fluid: Scale symmetry emerges at $T = T_c$
- SU(2)$_A$ chiral fluid: SU(2)$_A$ symmetry emerges at $m_q = 0$

HQ-spin hydro is well-defined

When we consider **heavy quark limit: $M \to \infty$,**
emergent heavy quark symmetry appears!

◆ Heavy quark spin dynamics

Heavy quark spin relaxation rate is suppressed by M^{-1},
so that HQ-spin hydro is well-defined Hydro+!

◆ Heavy quark spin hydro & its relaxation rate

(1) Hydrodynamic equation for HQ-spin density

(2) Evaluation of the HQ-spin relaxation rate at high-T limit

(i) Kubo formula (ii) HQ-spin correlator (iii) Spin kinetic theory

$$\Gamma_s = C_2(R) \frac{g^2 m_D^2 T}{6\pi M^2} \log(1/g)$$

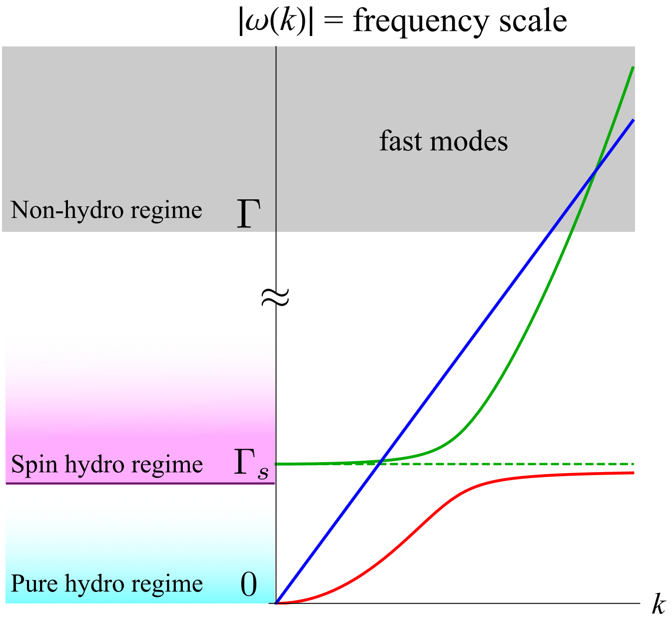
[MH-Huang-Kaminski-Stephanov-Yee, JHEP (2022)]

Outline

◆ Spin relaxation at a hydrodynamics stage

- Semi-phenomenological approach
- Heavy quark spin relaxation at $T \gg \Lambda_{QCD}$

$$\Gamma_s = C_2(R)\frac{g^2 m_D^2 T}{6\pi M^2}\log(1/g)$$



[MH-Huang-Kaminski-Stephanov-Yee, JHEP (2021), JHEP (2022)]

◆ Spin dynamics at a hadronic stage

[Hidaka-MH- Stephanov-Yee, PRC (2024)]

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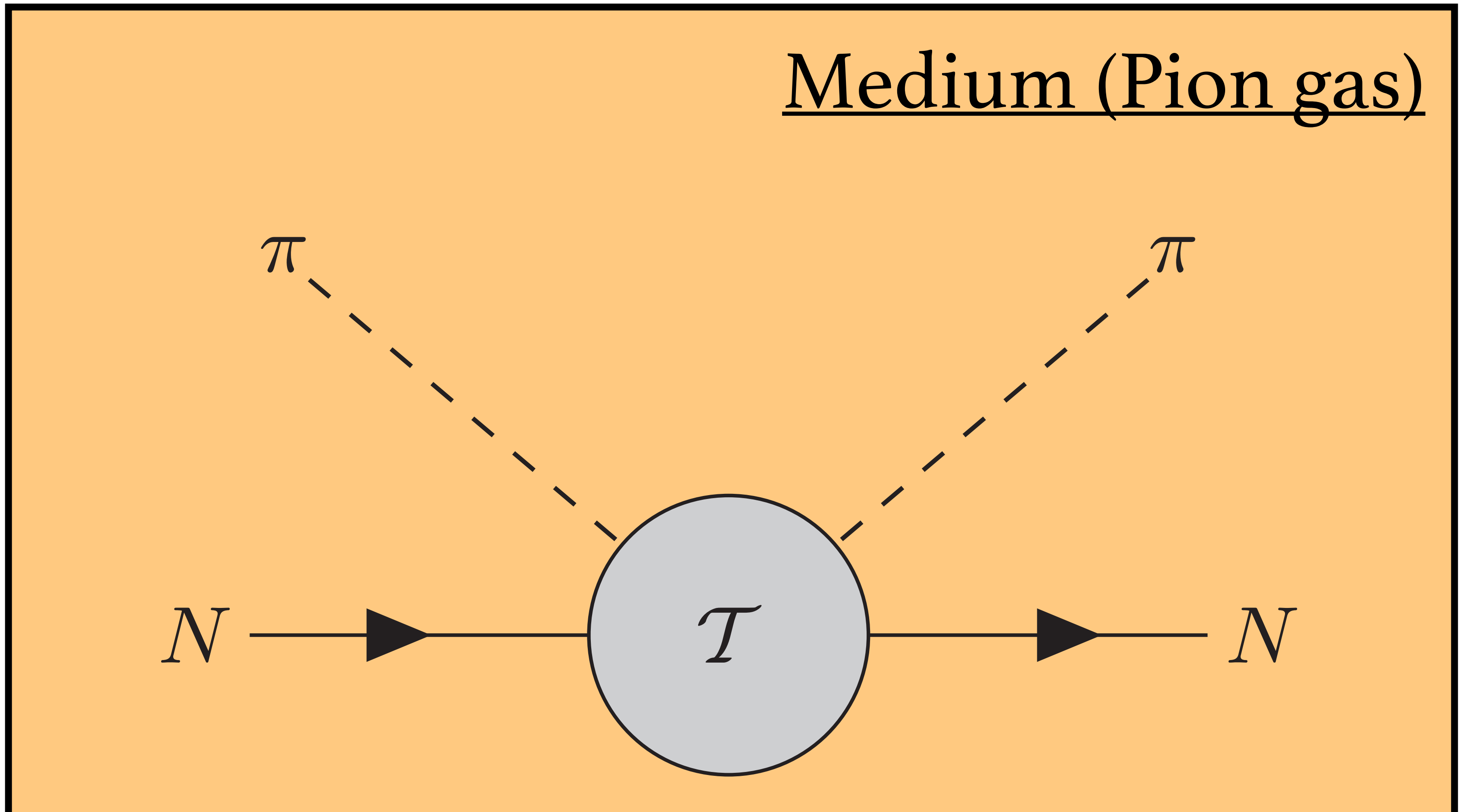
Baryon spin relaxation



$$N^s_\alpha(\boldsymbol{p}) + \pi^a(\boldsymbol{k}) \to N^{s'}_\beta(\boldsymbol{p}') + \pi^b(\boldsymbol{k}')$$

Dynamics of baryons (p, n, Λ, ...) at low tempearture $T < f_\pi$ is dominated by scattering with pions in a medium!

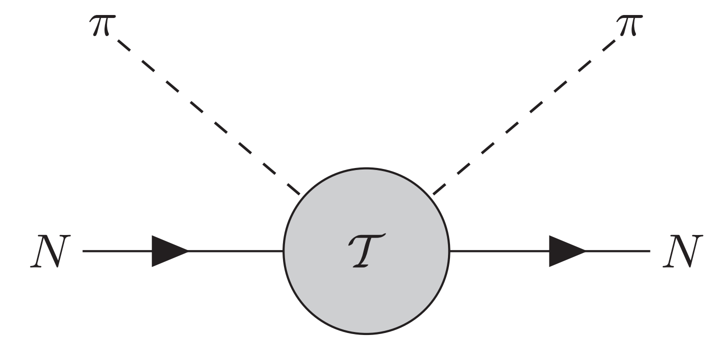
Spin flipping ampritude induces spin relaxation!

Baryon spin density matrix $\rho^{2\times2}(p)$

[Hidaka-MH- Stephanov-Yee, PRC (2024)]

◆ Kinetic equation for Baryon spin density matrix

$$\rho^{2\times2}(\boldsymbol{p}) \;\Rightarrow\; \begin{cases} n(\boldsymbol{p}) = \sum_s \rho(\boldsymbol{p}; s, s) = \mathrm{Tr}[\rho^{2\times2}(\boldsymbol{p})] \\ \boldsymbol{S}(\boldsymbol{p}) = (\hbar/2)\mathrm{Tr}[\boldsymbol{\sigma}\rho^{2\times2}(\boldsymbol{p})] \end{cases}$$



$N^s_\alpha(\boldsymbol{p}) + \pi^a(\boldsymbol{k}) \to N^{s'}_\beta(\boldsymbol{p}') + \pi^b(\boldsymbol{k}')$

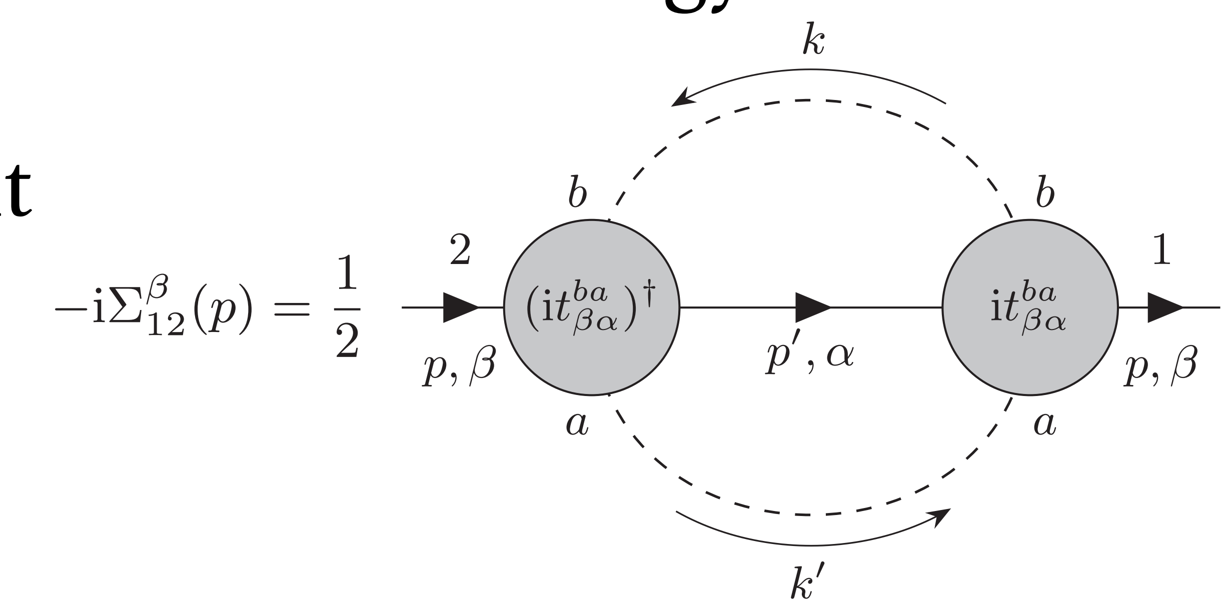
$$\frac{\partial \rho^{2\times2}_p(\boldsymbol{p})}{\partial t} = \int_{\boldsymbol{p}',\boldsymbol{k},\boldsymbol{k}'} \sum_{a,b,\beta} \mathcal{T}^{ba}_{p\beta}(\boldsymbol{p},\boldsymbol{k};\boldsymbol{p}',\boldsymbol{k}')\rho^{2\times2}_\beta(\boldsymbol{p}')\left[\mathcal{T}^{ba}_{p\beta}(\boldsymbol{p},\boldsymbol{k};\boldsymbol{p}',\boldsymbol{k}')\right]^\dagger$$
$$\times\, n_B(\epsilon_{\boldsymbol{k}'})[1+n_B(\epsilon_{\boldsymbol{k}})](2\pi)^4\delta^{(3)}(\boldsymbol{p}+\boldsymbol{k}-\boldsymbol{p}'-\boldsymbol{k}')\delta(E_{\boldsymbol{p}}+\epsilon_{\boldsymbol{k}}-E_{\boldsymbol{p}'}-\epsilon_{\boldsymbol{k}'}) - \widetilde{\gamma}_N(\boldsymbol{p})\rho^{2\times2}_p(\boldsymbol{p}),$$

Derivation [See, e.g., Appendix of Hidaka-MH- Stephanov-Yee, PRC (2024)]

1. Write down Kadanoff-Baym eq. with the self energy shown below

2. Consider the nonrelativistic limit

3. Take the dilute baryon limit by neglecting the $O(\rho^2)$-terms



Baryon spin density matrix $\rho^{2\times2}(p)$

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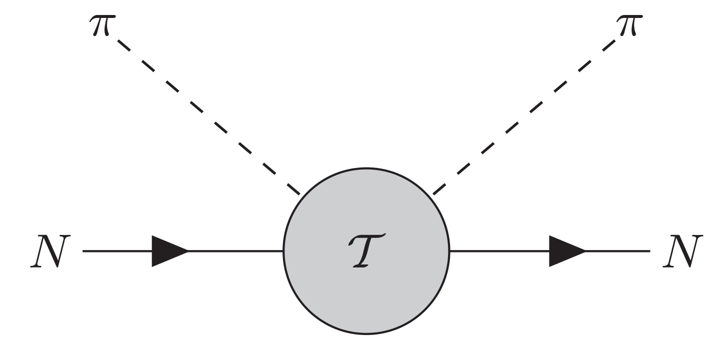


$N^s_\alpha(\boldsymbol{p}) + \pi^a(\boldsymbol{k}) \to N^{s'}_\beta(\boldsymbol{p}') + \pi^b(\boldsymbol{k}')$

$$\frac{\partial \rho_p^{2\times2}(\boldsymbol{p})}{\partial t} = \int_{\boldsymbol{p}',\boldsymbol{k},\boldsymbol{k}'} \sum_{a,b,\beta} \mathcal{T}^{ba}_{p\beta}(\boldsymbol{p},\boldsymbol{k};\boldsymbol{p}',\boldsymbol{k}')\rho_\beta^{2\times2}(\boldsymbol{p}')\left[\mathcal{T}^{ba}_{p\beta}(\boldsymbol{p},\boldsymbol{k};\boldsymbol{p}',\boldsymbol{k}')\right]^\dagger$$

$$\times\, n_B(\epsilon_{\boldsymbol{k}'})[1+n_B(\epsilon_{\boldsymbol{k}})](2\pi)^4\delta^{(3)}(\boldsymbol{p}+\boldsymbol{k}-\boldsymbol{p}'-\boldsymbol{k}')\delta(E_{\boldsymbol{p}}+\epsilon_{\boldsymbol{k}}-E_{\boldsymbol{p}'}-\epsilon_{\boldsymbol{k}'}) - \widetilde{\gamma}_N(\boldsymbol{p})\rho_p^{2\times2}(\boldsymbol{p}),$$

We can evaluate the baryon spin relaxation rate once we know the scattering amplitude $[\mathcal{T}^{ba}_{\beta\alpha}(\boldsymbol{p}',\boldsymbol{k}';\boldsymbol{p},\boldsymbol{k})]_{s',s}$

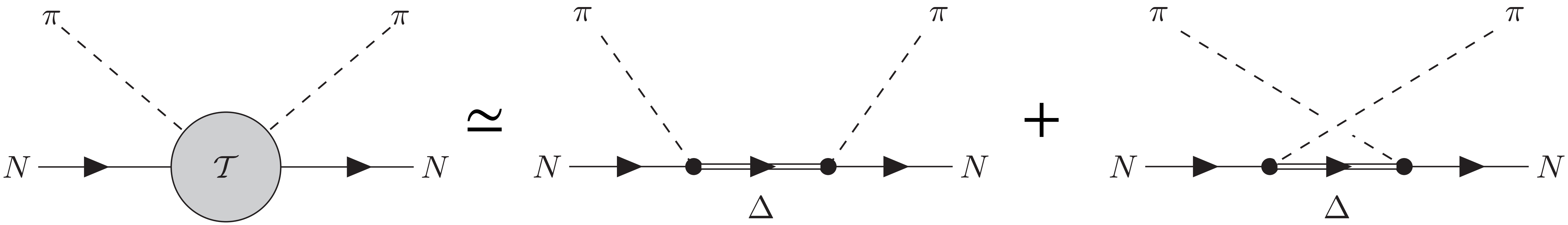
- $N\pi$-scattering: Enough experimental data (phase shifts)
- $\Lambda\pi$-scattering: Experimental input/SU(3)-Chiral perturbation

⇒ Evaluate spin relaxation rates of nucleon & Λ-baryon!

Nucleon spin relaxation rate γ_{N_s}

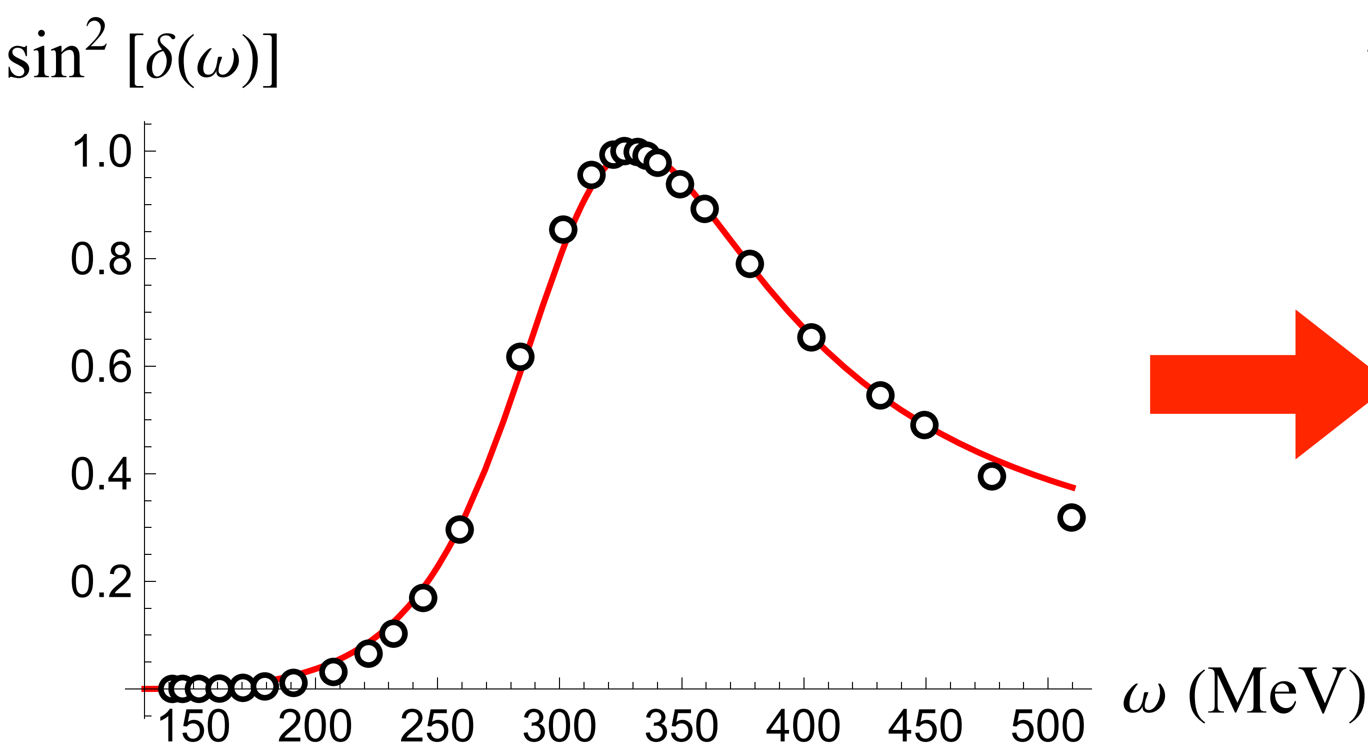
[Hidaka-MH- Stephanov-Yee, PRC (2024)]

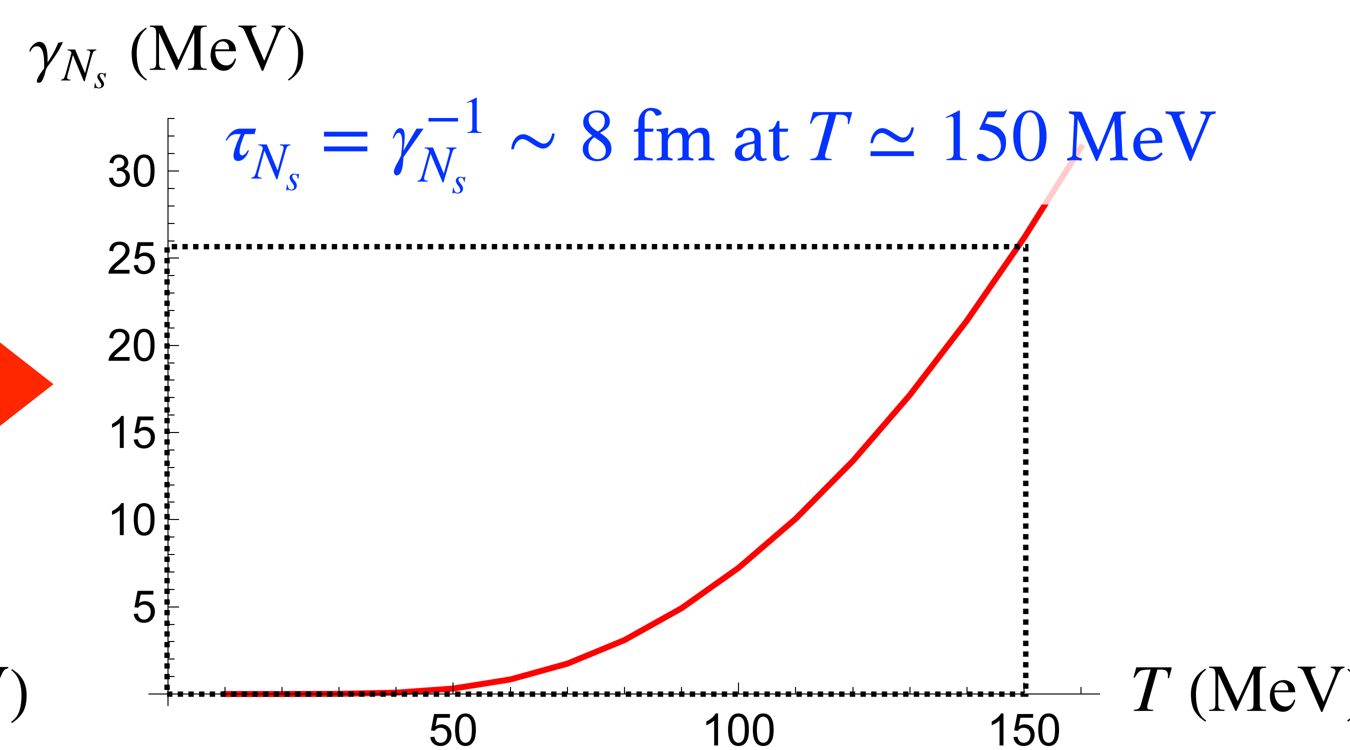
◆ Nucleon spin relaxation in Δ-resonance approximation



- Fit the experimental phase shift $\delta(\omega)$ for $I = J = 3/2$ channel

- Use that to evaluate $\gamma_{N_s} = \dfrac{8}{9\pi}\displaystyle\int_{m_\pi}^{\infty} d\omega |e^{2i\delta(\omega)} - 1|^2 n_B(\omega)[1 + n_B(\omega)]$



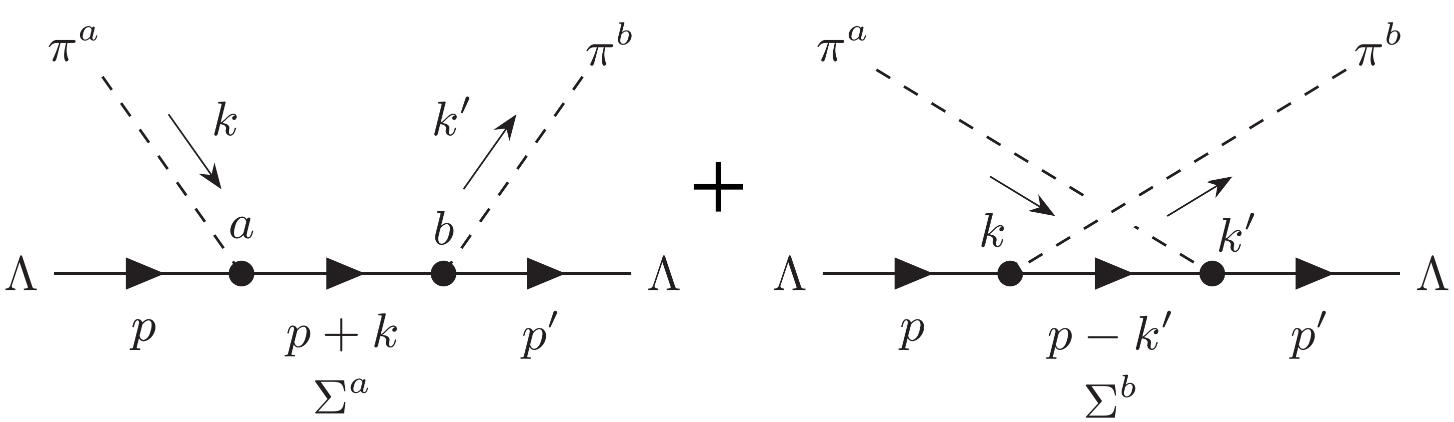


Λ-baryon spin relaxation rate γ_{Λ_s}

[Hidaka-MH- Stephanov-Yee, PRC (2024)]

◆ Two ways to obtain the scattering amplitude

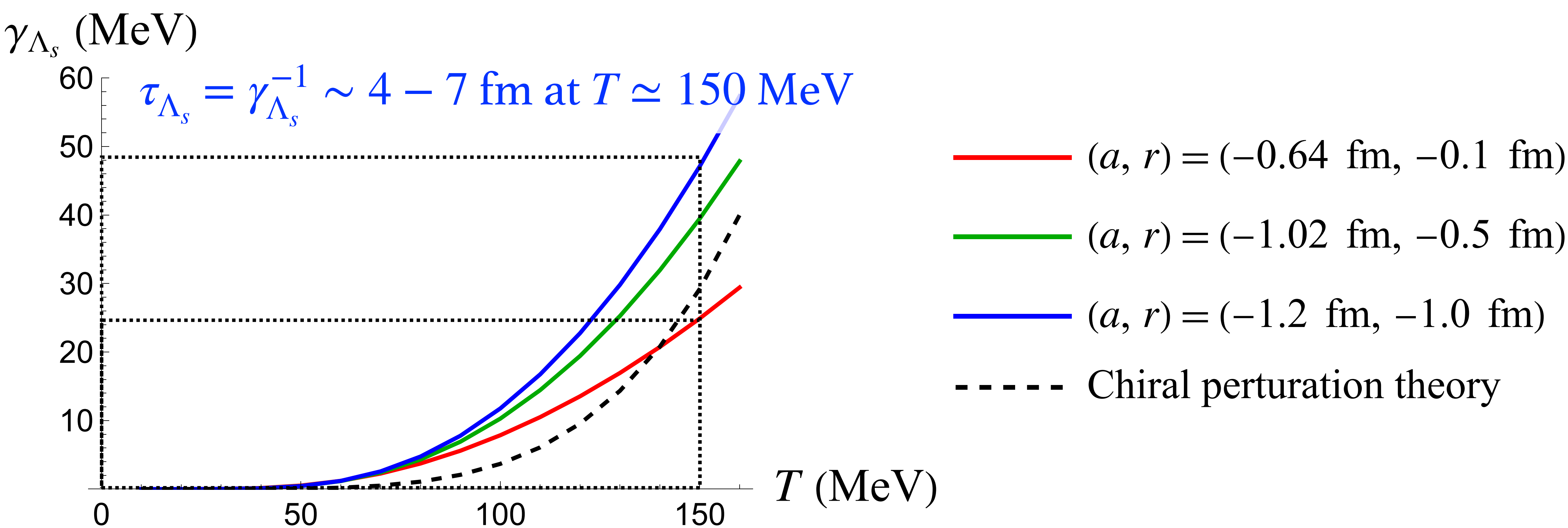
1. SU(3) chiral perturbation



2. Low-energy scattering

Reasonable parametrization reproducing spectrum of $\Lambda, \Sigma, \Sigma^*$

$$e^{2i\delta_{1/2}(k)} \simeq \frac{ik - \frac{1}{a^3k^2} - \frac{1}{r}}{-ik - \frac{1}{a^3k^2} - \frac{1}{r}}$$

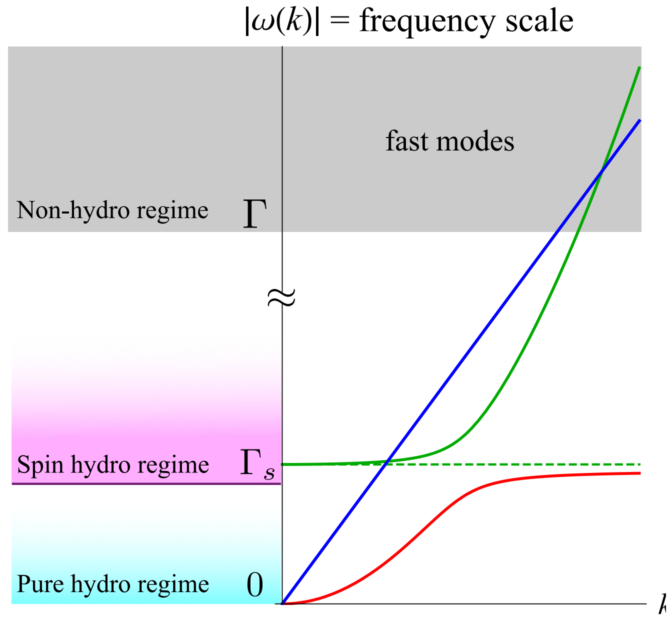


Summary

◆ Spin relaxation at a hydrodynamics stage

- Semi-phenomenological approach
- Heavy quark spin relaxation at $T \gg \Lambda_{QCD}$

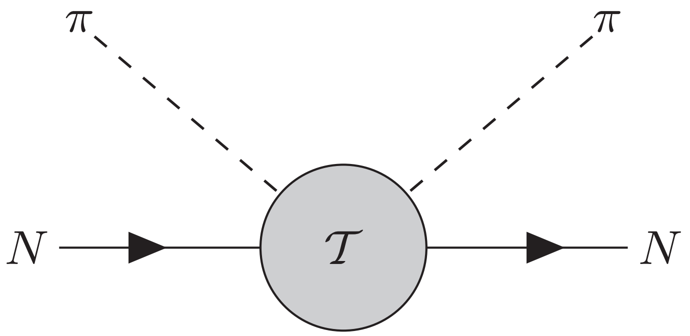
$$\Gamma_s = C_2(R)\frac{g^2 m_D^2 T}{6\pi M^2}\log(1/g)$$



[MH-Huang-Kaminski-Stephanov-Yee, JHEP (2021), JHEP (2022)]

◆ Spin dynamics at a hadronic stage

- Kinetic eq. for Baryon spin density matrix
- Experimental input/chiral perturbation



$$\tau_{N_s} = \gamma_{N_s}^{-1} \sim 8 \text{ fm at } T \simeq 150 \text{ MeV}, \quad \tau_{\Lambda_s} = \gamma_{\Lambda_s}^{-1} \sim 4-7 \text{ fm at } T \simeq 150 \text{ MeV}$$

[Hidaka-MH- Stephanov-Yee, PRC (2024)]