STUDY OF 3+1D SPACETIME EVOLUTION OF GLASMA IN RELATIVISTIC HEAVY-ION COLLISION

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Space-Time Evolution of HIC at RHIC and LHC





Phenomenological Analysis of the Evolution





Phenomenological Analysis of the Evolution





Initial State Model

- *Non-dynamical model*: providing hydrodynamic initial conditions directly
- Monte Carlo Glauber model [M. L. Miller, et al. (2007)]
- Reduced Thickness Event-by-event Nuclear Topology (TRENTo) [J. S. Moreland, et al. (2015), W. Ke, et al. (2017)]

Dynamical Model: simulating pre-hydrodynamic stage + switching it into hydrodynamics fluid

- IP-glasma model [B. Schenke, et al. (2012)]
 - * Degrees of freedom focused: the dense "soft" gluon matter (glasma)
 - * Description of glasma:

classical Yang-Mills (CYM) equation of motion



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IP-glasma model ①

Theoretical Background:

- ✓ Nucleus at high energy
 - * Gluon saturation: A number of gluons with small momentum fraction is very large and is characterized by Q_{S}
 - * HIC liberates a large amount of soft gluons (glasma)
- Color Glass Condensate (CGC)
 - * a theoretical state that captures features of a relativistic heavy-ion
 - * internal soft gluons = classical Yang-Mills (CYM) field (A, E)
 - * internal hard colored particles
 - = classical color charge density (ρ) generated as event-by-event random number according to weight function ($W_{Y}[\rho]$)

→ momentum rapidity for separation between "hard" and "soft"

* Glasma is modeled using CYM field as well, which is generated by collision of two CGCs

$$\begin{bmatrix} D_{\mu}, F^{\mu\nu} \end{bmatrix} = \delta^{\nu+}\rho_{A} + \delta^{\nu-}\rho_{B}, \qquad \begin{bmatrix} D_{\mu}, \delta^{\pm}\rho_{A/B} \end{bmatrix} = 0$$

*light cone coordinates, $x^{\mp} = (x \mp z)/\sqrt{2}$



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Impact-Parameter-Saturation (IP-sat) model: ulletReferring to phenomenological value of saturation scale $Q_s(x_{\text{Bioken}}, \vec{x}_{\perp})$ to construct initial condition of CGC

Boost invariant approximation: ullet

1. $W_V[\rho]$, given as Gaussian function (McLerran-Venugopalan (MV) model)

2. Shockwave approximation: (Infinitely thin) $\rho^A \propto \delta(x^-)$ and $\rho^B \propto \delta(x^+)$

 $\partial_+ \rho^A = 0$ and $\partial_- \rho^B = 0$ (Frozen)





Study of Rapidity Dependence from Experiment and Theory

Experiment: rapidity dependent observables, and their decorrelation in rapidity direction



Theory:

development of phenomenological models where rapidity dependent effect is taken into acount

* AMPT(a multiphase transport model)





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1. W_Y[ρ] for wide values of Y obtained by solving JIMWLK equation
* B. Schenke, et al. (2016, 2022), S. McDonald, et al. (2019, 2023)

2. Relax shockwave approximation

- Finite thin ρ
 - T. Altinoluk, *et al.* (2014,2016, 2021,2022), G.A. Chirilli (2019,2021), P. Agostini, *et al.* (2019,2021,2022,2023), C. S. Lam, *et al.* (2000), S. Özönder, *et al.* (2014)
- Both finite thin and dynamical ρ
 - D. Gelfand, *et al*. (2016), A. Ipp, *et al*. (2017,2021,2024), S. Schlichting, *et al*. (2021), H. M. and X.-G. Huang (2023,2024)

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Analytic calculation: limited up to leading order

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Computational calculation: require much numerical resource

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 - D. Gelfand, et al. (2016), A. Ipp, et al. (2017,2021,2024), S. Schlichting, et al. (2021), H. M. and X.-G. Huang (2023,2024) **3D glasma simulation beyond shockwave approximation have**

not yet reached stage of phenomenological applications!!

Computational calculation: require much numerical resource

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We have recently developed efficient 3D glasma simulation method
 Today, we apply our method to early stage of Au-Au collisions at RHIC

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"Usual" and "Modified" Milne Coordinates







1. Usual Milne coordinates for observation:

$$\tau = \sqrt{2(x^{-} - x_{c} - \frac{R}{\sqrt{2}\gamma})(x^{+} - x_{c} - \frac{R}{\sqrt{2}\gamma})}$$
$$\eta = \frac{1}{2} \ln \frac{x^{+} - x_{c} - R/[\sqrt{2}\gamma]}{x^{-} - x_{c} - R/[\sqrt{2}\gamma]}$$

2. Modified Milne coordinates for simulation: $\tilde{\tau} = \sqrt{2x^- x^+}, \quad \tilde{\eta} = \frac{1}{2} \ln \frac{x^+}{x^-}$

Strategy



4 steps in numerical calculation

1. Put initial condition of two incoming nuclei on lattice before the collision [†]

 $(A, E, \rho^i)\Big|_{\tilde{\tau}=\tilde{\tau}_{\mathrm{in}}}$

2. Evolve the CYM field and classical color charges numerically

$$\left[D_{\mu}, F^{\mu \ i=1,2,\widetilde{\eta}}\right] = J^{i=1,2,\widetilde{\eta}}, \left[D_{\mu}, J^{\mu}\right] = 0$$

3. Get observables for Usual Milne coordinates

 $T^{\widetilde{\mu}\widetilde{\nu}}(\widetilde{\tau},\widetilde{\eta}) \rightarrow T^{\mu\nu}(\tau,\eta)$

4. Repeat $1 \sim 3$ and take event-average

 $\langle T^{\mu\nu} \rangle_{\text{eve}}$ (~ensemble average over W_Y)





• Incoherent sum of solutions for each single nuclei + small modification for $E^{\widetilde{\eta}}$

 $\begin{array}{l} A_{i} = A_{i}^{A} + A_{i}^{B}, \quad E^{i} = E^{Ai} + E^{Bi} \\ A_{\widetilde{\eta}} = 0, \quad E^{\widetilde{\eta}} = i[A_{i}^{A}, A_{i}^{B}] \\ & ^{* \text{Fock-Schwinger gauge: } A^{\widetilde{\tau}} = 0 \end{array} \end{array} \begin{array}{l} A_{i}^{A/B}, E^{A/B\,i}: \text{ Solution of E.O.M. for single nucleus} \\ E^{\widetilde{\eta}} = i[A_{i}^{A}, A_{i}^{B}]: \\ & \text{made to satisfy Gauss law}\left(\left[D_{\mu}, F^{\mu\widetilde{\tau}}\right] = x^{-}\rho^{A} + x^{+}\rho^{B}\right) \end{array}$

3D Nucleus Model: Formulation

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2D IP-glasma initial condition

• MV model:
$$< \rho_i^{A/B,a}(\overline{x_{\perp}}) \rho_j^{A/B,b}(\overline{x_{\perp}'}) > = \delta^{ij} \delta^{ab} \left(g^2 \mu \left(\frac{\overline{x_{\perp}} + \overline{x'_{\perp}}}{2} \right) \right)^2 \delta^2(\overline{x_{\perp}} - \overline{x'_{\perp}})$$

• Nucleus saturation scale based on IP-sat model: $(g^2 \mu(\overrightarrow{x_\perp}))^2 \propto (Q_{s,A}(\overrightarrow{x_\perp}))^2$

3D IP-glasma initial condition

- Nucleus color charge density = incoherent sum of nucleon's one: $\rho_{tot}^{A/B} = \sum_{i=1}^{N_A=197} \rho_i^{A/B}$
- MV model: $< \rho_i^{A/B,a}(x^{\mp}, \overline{x_{\perp}}) \rho_j^{A/B,b}(x'^{\mp}, \overline{x_{\perp}}') >$

$$= \delta^{ij} \delta^{ab} \left(g^2 \mu_{3\mathrm{D}} \left(\frac{x^{\mp} + x'^{\mp}}{2} - b_i^{\mp}, \frac{\overline{x_{\perp}} + \overline{x'_{\perp}}}{2} - \overline{b_{\perp,i}} \right) \right)^2 \underbrace{N_{1\mathrm{D}}(x^- - x'^-; l_{\mathrm{L}})}_{\text{Longitudinal correlation length}} \delta^2 (\overline{x_{\perp}} - \overline{x'_{\perp}})$$

* Gaussian shape in longitudinal direction: $g^2 \mu(x^{\mp}, \overline{x_{\perp}}) \propto N_{1D}(x^{\mp^2}, r_L)$ * $(g^2 \mu(\overline{x}))^2 = \int dx^{\mp} (g^2 \mu_{3D}(\overline{x}))^2 \propto (Q_{s,n}(\overline{x_{\perp}}))^2$

3D Nucleus Model: Model Parameters

Determination of four parameters for mimicking Au-Au collision at RHIC

1. Bjorken x for determination of $Q_{s,n}$

$$x_{\text{Bjorken}} \sim \frac{p_{\perp}}{\sqrt{s_{NN}}}$$
 [relation for Y ~ 0]
< $p_{\perp} > \sim 1 \text{ GeV}$
 $\sqrt{s_{NN}} = 200 \text{ (GeV)}$

2. Longitudinal extent of ρ , r_L

 ρ consists of hard partons with larger x_{Bjorken}

$$r_L \sim \left[2x_{\rm Bjorken}\sqrt{s_{NN}}\right]^{-1}$$

3. Ratio, $\lambda = g^2 \mu / Q_{s,n}$ 4. Longitudinal correlation length, l_L



tuned by referring to observables



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3-1. Results: Initial Condition of Single Nucleus (13-14P)

3-2. Results: Central Collisions (15-21P) \leftarrow \mathcal{E} , N_{CS} , n_{S}

3-3. Results: Non-Central Collisions (22-30P)

4. Summary and Outlook (31P)



$Q_{\rm s}$ of Single Nucleus

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Saturation scale of nucleus $Q_{s,A}$

• Wilson line correlator for transverse translational invariant glasma [T. Lappi (2008)]

$$C(k_t) = \int d^2 x_t \langle \left| \operatorname{Tr} \left[U^{\dagger}(\boldsymbol{x}_t + \boldsymbol{y}_t) U(\boldsymbol{y}_t) \right] \right|^2 - 1 \rangle e^{i \boldsymbol{k}_t \cdot \boldsymbol{x}_t} \quad \left(U(\mathbf{x}_T) = P \mathrm{e}^{\mathrm{i} \int \mathrm{d} \boldsymbol{x}^- A^+} \right)$$

• Q_s can be defined as value of transverse momentum where peak structure appears



$Q_{\rm s}$ of Single Nucleus







$$Q_{s,A}^{\text{est.}} = 0.65 \ g^2 \mu_{2D} = 0.346 \lambda \text{ GeV}$$

for $l_{\text{L}}/2r_{\text{L}}$ =0.4-1.0 and $\lambda = 2 - 4$

✓ $\lambda = 3.18$ reproduces phenomenological value of $Q_{s,A} = 1.1$ GeV [N. Armesto (2002)]



Rapidity Profile of ε_{LRF} at Central Collisions



Energy density in local rest frame: $T^{\mu}_{\nu}u^{\nu} = \varepsilon_{\rm LRF}u^{\mu}$

- We assume $u_{\perp} = 0$ and extract ε_{LRF} from $\varepsilon_{\text{LRF}} = \frac{1}{2} \left([T^{\tau\tau} T^{\eta\eta}] + \sqrt{[T^{\tau\tau} + T^{\eta\eta}]^2 4[T^{\tau\eta}]^2} \right)$
- Note: CYM field of single nucleus (Weizsäcker-Williams field) strictly yields zero $\varepsilon_{\rm LRF}$

✓ As $l_{\rm L}$ decreases, shape of $\varepsilon_{\rm LRF}$ changes from a rounded one to dip structure

 \checkmark Later, we use $l_{\rm L}=0.6$ and 1.0



Transverse Profile of ε_{LRF} at Central Collisions







- * ε_{LRF} in the center of figures: $\varepsilon_{\text{LRF}} \sim 6$ GeV/ fm³
 - * Boost-invariant hydrodynamic simulations [U. W. Heinz *et al.* (2002)]: $\varepsilon_{\rm LRF} \sim 11$ GeV/ fm³
 - * Boost-invariant glasma simulation with MV model [A. Krasnitz *et al.* (2003)]: 7.1 GeV/ fm³ $\leq \varepsilon_{LRF} \leq 40$ GeV/ fm³
- ✓ This discrepancy should be improved when $N_c = 2 \rightarrow 3$.

Topological charge density

$$n_{\rm T} \equiv \frac{1}{8\pi^2} {
m Tr} \boldsymbol{E} \cdot \boldsymbol{B}$$

What happens for 3D glasma?

 2D glasma initial condition shows strong parallel longitudinal colorelectric and colormagnetic fields emerge between WW fields just after collisions



Generation of $\langle n_{\rm T}^2 \rangle$!!

 $(\tau = 0 +)$



Rapidity Profile of Squared N_{cs} at Central Collisions





Topological charge fluctuations are generated around mid-rapidity by the collision, with a shape similar to the LRF energy density

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Axial charge density

 $n_5 \equiv \tau j_5^{\tau}$

• Adler-Bell-Jackiw anomaly equation tells us that the topological charge density is related to the divergence of the axial current

$$\partial_{\tau}(\tau j_5^{\tau}) - \tau \sum_{i=1,2,\eta} \partial_i j_5^i = \tau n_{\mathrm{T}}$$

• To estimate n_5 , we focus on the central area the transverse plane and at mid-rapidity ($\eta = 0$), and assume $\langle j_5^{i=1,2,\eta} \rangle = 0$

$$n_5 = \int^{\tau} d\tau \ \tau n_{\rm T}$$



Transverse Correlation of n_5 at Central Collisions







✓ D_⊥ decreases in Q_sr_⊥ < 3, and is consistent with zero within the error in Q_sr_⊥ < 3
 ✓ This behavior of D_⊥ does not change over proper time evolution
 ✓ This behavior and even magnitude of D_⊥ are in qualitative agreement with previous studies on the boost-invariant glasma [M. R. Jia *et al.* (2021)]

Transverse Profile of ε_{LRF} at Non-Central Collisions



✓ ε_{LRF} at central collision ($b_{imp}/R = 0$) are isotropic in transverse plane

✓ ε_{LRF} at non-central collision with $b_{imp}/R = 1$ have an anisotropic shape that looks elliptical

✓ The ε_{LRF} deformation is expected to reflect shape of overlap region of colliding nuclei

Mid-rapidity!!



Eccentricity ε_n at Non-Central Collisions



Eccentricity

$$\varepsilon_n = \frac{\int d^2 \boldsymbol{x}_{\perp} \varepsilon_{\text{LRF}} r_{\perp}^n e^{in\phi}}{\int d^2 \boldsymbol{x}_{\perp} \varepsilon_{\text{LRF}} r_{\perp}^n} \qquad \qquad \phi \equiv \arctan\left(x^2/x^1\right)$$
$$r_{\perp} \equiv \sqrt{(x^1)^2 + (x^2)^2}$$

- ϵ_n characterizes spatial anisotropy of a produced matter in the transverse plane perpendicular to the collision axis
- ε_2 characterizes elliptical deformation
- ε_n is expected to be converted into anisotropic flow (such as the elliptic flow which is a response to ε_2 during the system's collective evolution in the hydrodynamic stage

Rapidity Profile of ε_n at Non-Central Collisions



Eccentricity

- ✓ In central collisions, $\operatorname{Re}[\varepsilon_2]$ and $\operatorname{Re}[\varepsilon_4]$ are consistent with zero within the error
- As b_{imp} increases Re[ε₂] and Re[ε₄] become non-zero, indicating the formation of an anisotropic shape of the glasma



 $[\]checkmark$ consistent with the shape of $\varepsilon_{\rm LRF}$

Rapidity Correlation of ε_n at Non-Central Collisions



- $\langle \operatorname{Re}\varepsilon_n(\eta)\operatorname{Re}\varepsilon_n(4)\rangle$
- ✓ For $b_{imp} = 0$, the rapidity correlation is found to be a monotonic increasing function of η , which is expected to reflect the decorrelation effect that increases with the distance between two observed rapidities
- As b_{imp} increases, an enhancement around $\eta = 0$ gradually emerges, reflecting the increase in the magnitude of the eccentricity



Reaction Plane Profile of ε_{LRF} at Non-Central Collisions



- Glasma generated in non-central collisions seems to expand diagonally
 - → Pointing vector along collision axis create angular moment perpendicular to reaction plane?



Rapidity Profile of $T^{\tau 3}$ at Non-Central Collisions



$T^{\tau 3}$

- $\checkmark T^{\tau 3}$ is found to increase with $|\eta|$ in the large rapidity region
- \checkmark CYM fields exist in the large rapidity region and contribute to $T^{\tau 3}$

Assumption

We assume that within the CYM fields, there is a glasma part, which will eventually become QGP, and a non-glasma part, with the majority of the latter consisting of WW fields

Subtraction Method

Focusing on the fact that ε_{LRF} of the WW fields is zero, while that of the glasma is large, we differentiate between the glasma and non-glasma parts in the CYM fields based on the magnitude of ε_{LRF} , and define the EM tensor of the glasma part as

$$T_{\rm gl}^{\mu\nu} = \theta(\varepsilon_{\rm LRF} - \varepsilon_{\rm c})T^{\mu\nu}$$



Rapidity Profile of $T^{\tau 3}$ at Non-Central Collisions



 $T^{\tau 3}$ with the subtraction method



✓ At $|\eta| < 2$, the newly defined $T_{gl}^{\tau_3}$ agrees well with the original one

 \checkmark At $|\eta| > 2$, in contrast to the original one, $T_{gl}^{\tau 3}$ goes to zero as the rapidity becomes larger

Rapidity Profile of L at Non-Central Collisions



Rapidity distribution of Angular momentum

$$\frac{dL^2}{d\eta} \equiv \int d^2 \boldsymbol{x}_{\perp} \tau \frac{1}{2} \varepsilon^{2ij} M^{\tau}_{ij}$$
$$= \int d^2 \boldsymbol{x}_{\perp} \tau \left(x^3 T^{\tau 1} - x^1 T^{\tau 3} \right)$$

- To focus glasma's contribution, we use $T_{\rm gl}^{\tau 3}$ as $T^{\tau 3}$
- We omit the transverse component T^{τ_1} since it should be tiny compared to the longitudinal component $T_{gl}^{\tau_3}$ in the early stage.

Rapidity Profile of L at Non-Central Collisions



Rapidity distribution of Angular momentum



 \checkmark At $\eta = 0$, the generation of L appears to be zero, regardless of b_{imp}

 \checkmark As η increases, the generation of L is observed, and its magnitude strongly depends on $b_{\rm imp}$

 \checkmark The peak magnitude appears around $\eta = 2 - 3$, which is close to the peak in $T_{\rm gl}^{\tau 3}$

✓ The highest peak appears around $\frac{b_{imp}}{R} = \frac{1}{2} - \frac{3}{4'}$ indicating that *L* depends on b_{imp} non-monotonically



<u>Comment</u>

• Little or no angular L at $\eta = 0$, even for non-zero b_{imp}

• Given that the CGC description is more reliable at higher collision energies, this suggests that little angular momentum would be retained in $\eta = 0$ region in the glasma and subsequently in the QGP at very high energies?

• Effect beyond the high-energy limit's description may be more important for explaining the spin polarization of hadrons observed at mid-rapidity at lower collision energies

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Summary

- We apply the 3D glasma simulation, proposed in our previous work, to the early stage of Au-Au collisions
- We investigate rapidity profiles for a wide range of physical quantities of the glasma in central and non-central collision

Outlook

- The effects not considered in the current calculations, as well as making the model parameters more realistic, should be contemplated to enable more quantitative discussions
 (ex: N_c = 2 → 3)
- Additionally, the next step, which is actually being explored currently, is to use the 3D glasma description as an initial state model to provide the initial conditions for hydrodynamics and make comparisons with experimental data.