Chiral magnetohydrodynamics in the early Universe cosmology

Related works of mine:

KK, PRD97 (2018) 103506 [arXiv:1802.03055 (hep-ph)];

V. Domcke (CERN), KK, K. Mukaida (KEK), K. Schmitz (Münster), M. Yamada (Tohoku), PRL130 (2023) 261803 [arXiv: 2208.03237 (hep-ph)];

F. Uchida (Tokyo), M. Fujiwara (TUM), KK, J. Yokoyama (Tokyo), PLB843 (2023) 138002 [arXiv: 2212.14355 (astro-ph.CO)] arXiv: 2405.06194 (astro-ph.CO);

A. Brandenburg (Nordita), KK, J. Schober (EPFL), PRR 5 (2023) 2, L022028 [arXiv: 2302.00512 (physics.plasma-ph)];

A. Brandenburg, KK, K. Mukaida, K. Schmitz, J. Schober, PRD108 (2023) 063529 [arXiv: 2304.06612 (hep-ph)].

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(Hangzhou Institute for Advanced Study, UCAS)

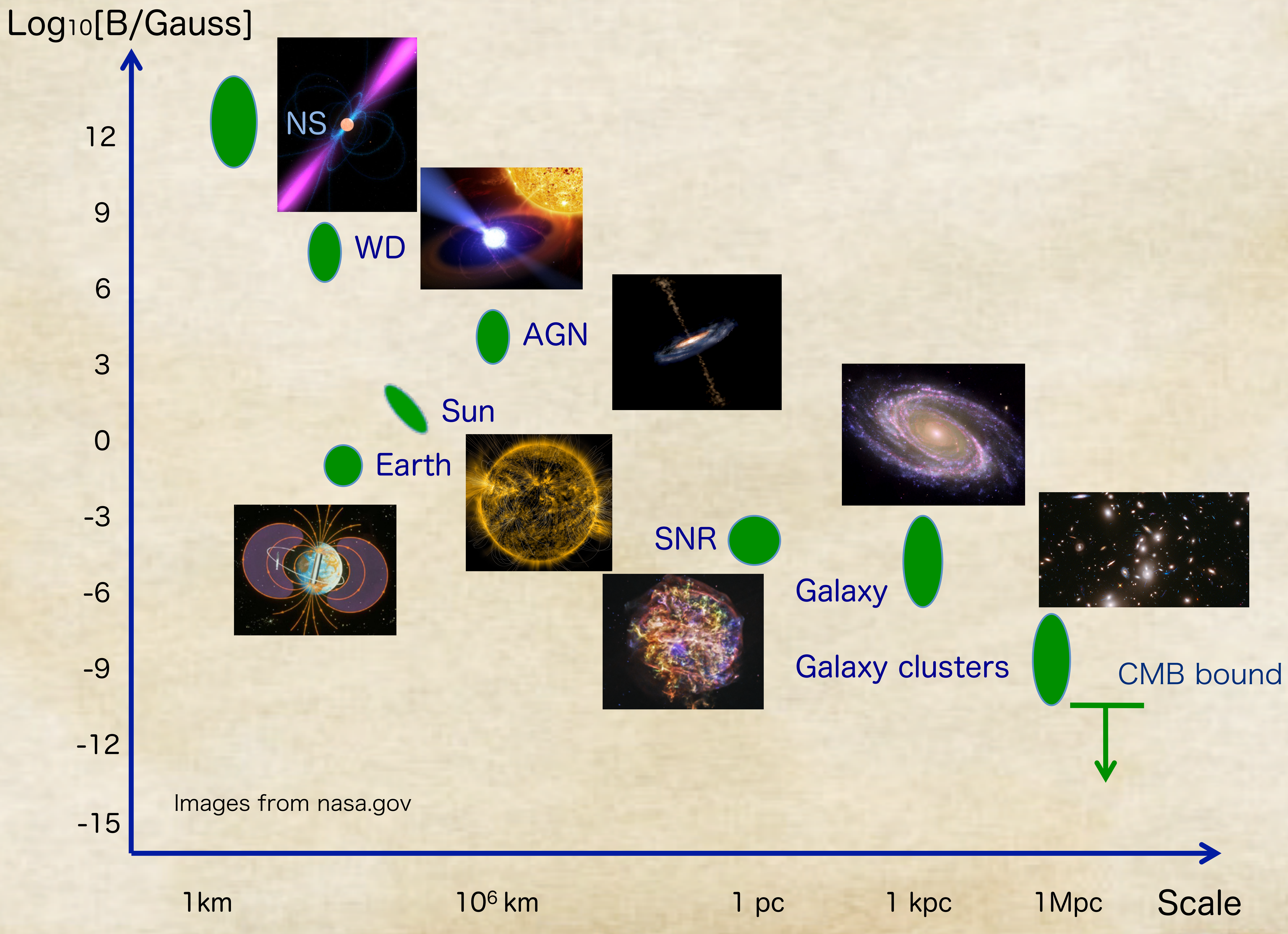
West lake workshop on nuclear physics 2024

Zhejiang University, 10/19/2024

1. Introduction — Why primordial magnetic fields? —
2. Magntohydrodynamics (MHD) and chiral magnetic effect
3. Application of chiral MHD in the early Universe
   i) Chiral plasma instability in the early Universe
   ii) Chiral MHD with zero total chirality
4. Summary

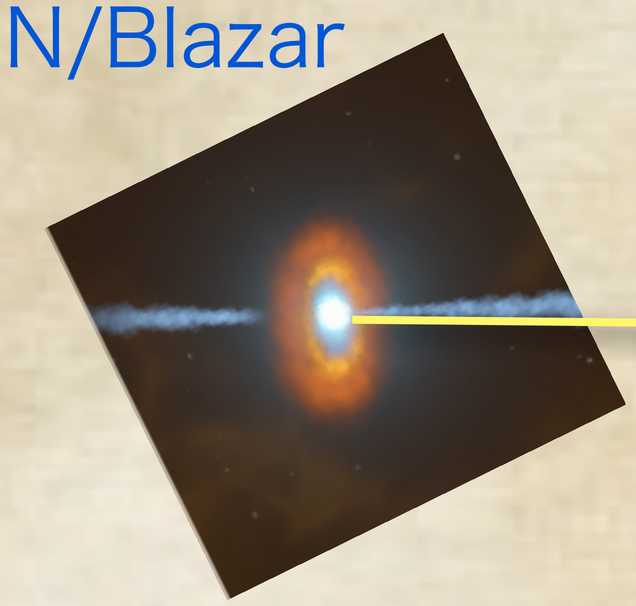
Introduction — Why primordial magnetic fields? —

Magnetic fields (MFs)are ubiquitous in the Universe.



Observations of the intergalactic magnetic fields

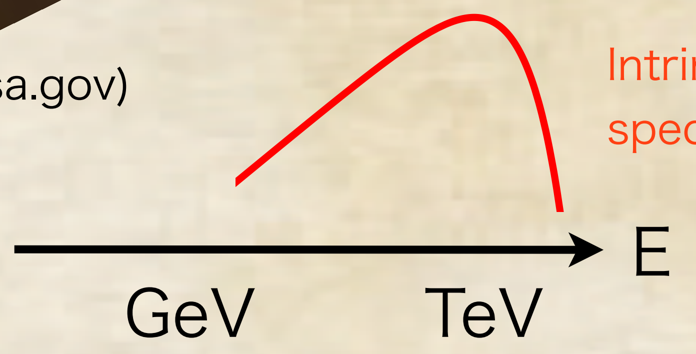
AGN/Blazar

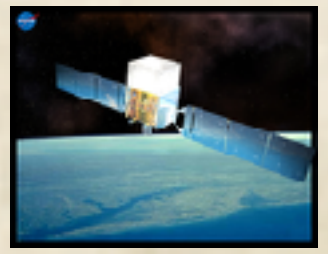


(from nasa.gov)

~TeV γ

~ 100 Mpc





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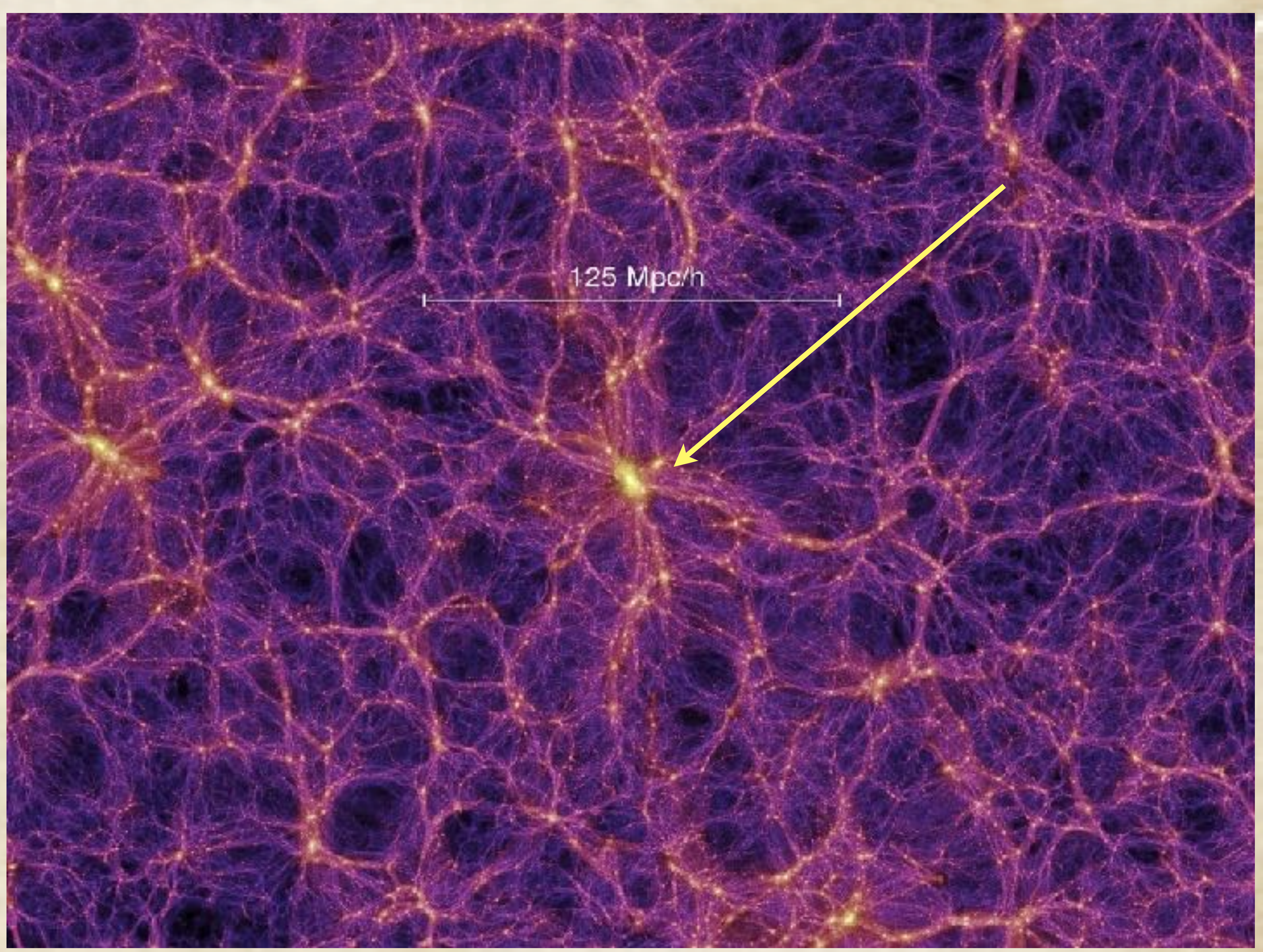
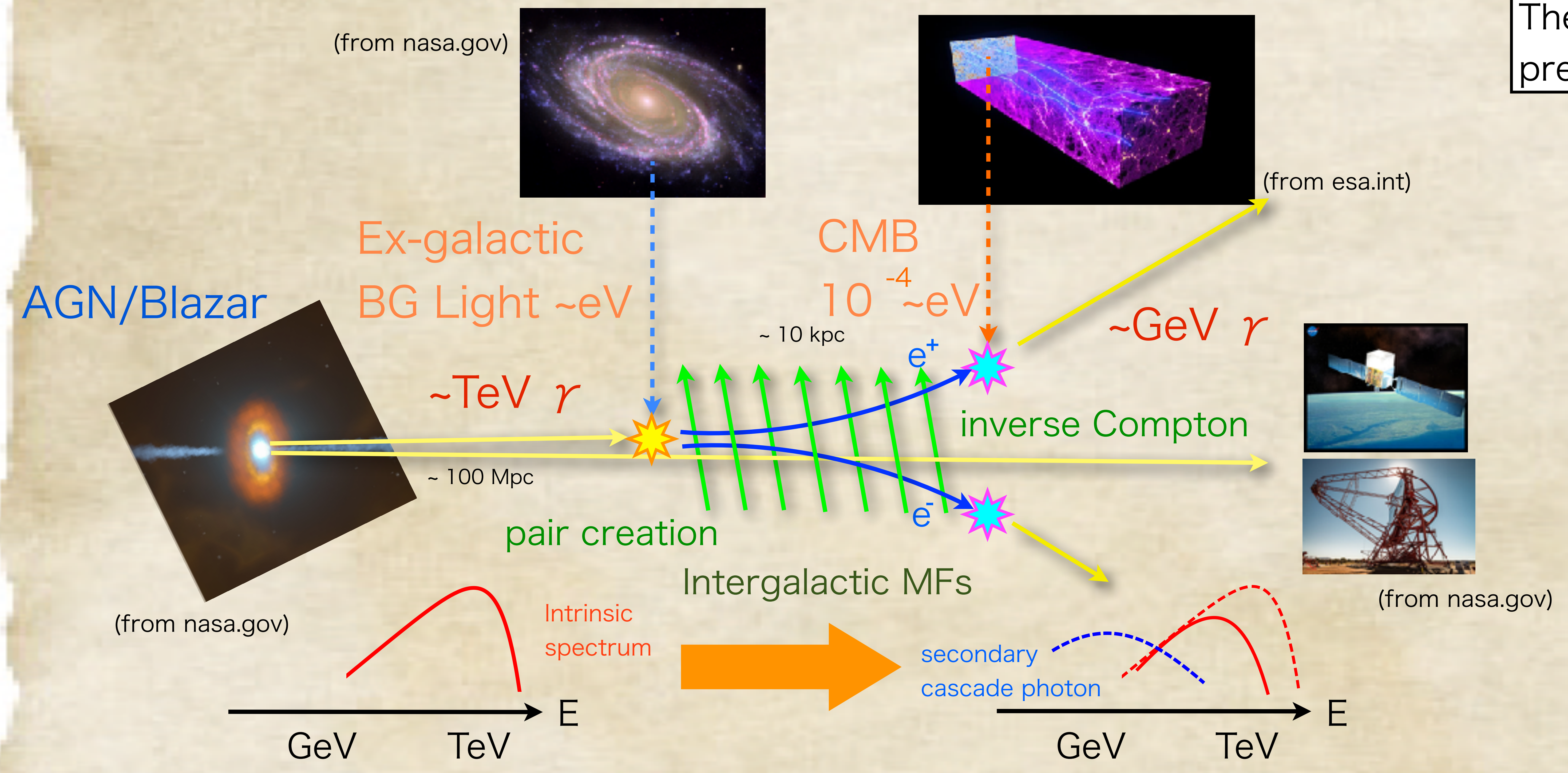
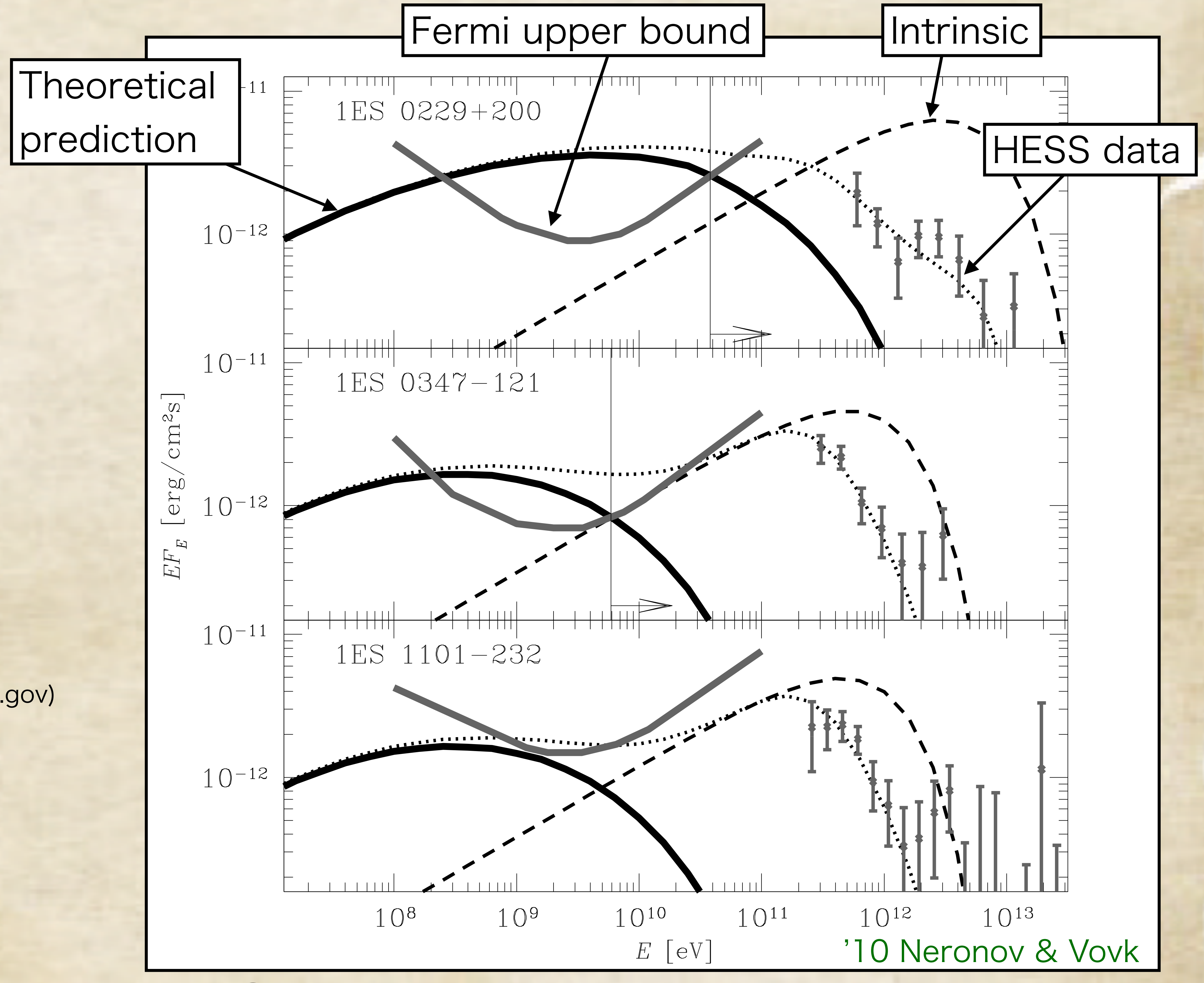


Simulation by Volker Springel, Virgo Consotium

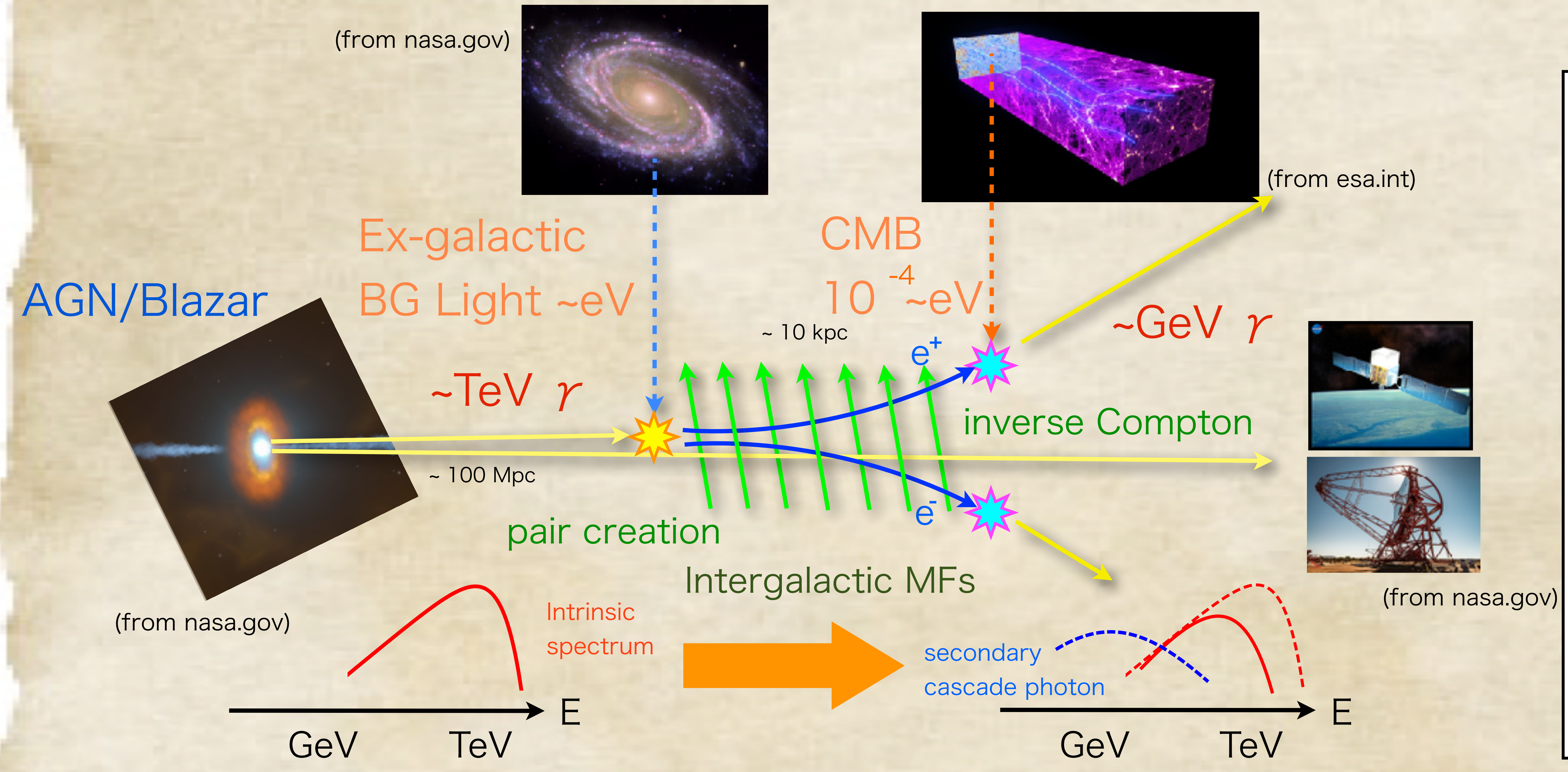
Observations of the intergalactic magnetic fields

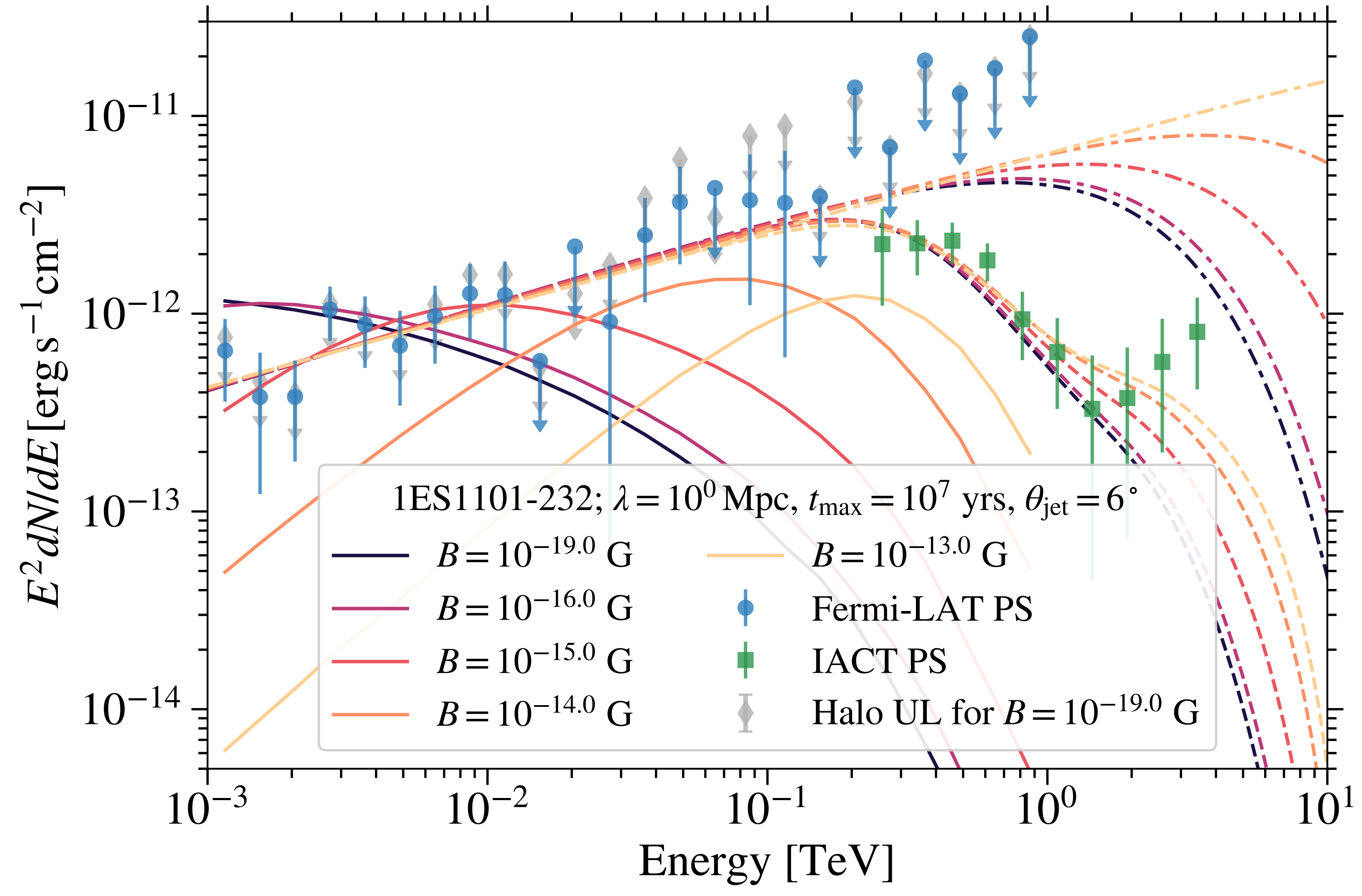




Non-observation of the secondary cascade GeV photon can give the lower bound of the intergalactic magnetic fields (indirect implication)

Observations of the intergalactic magnetic fields

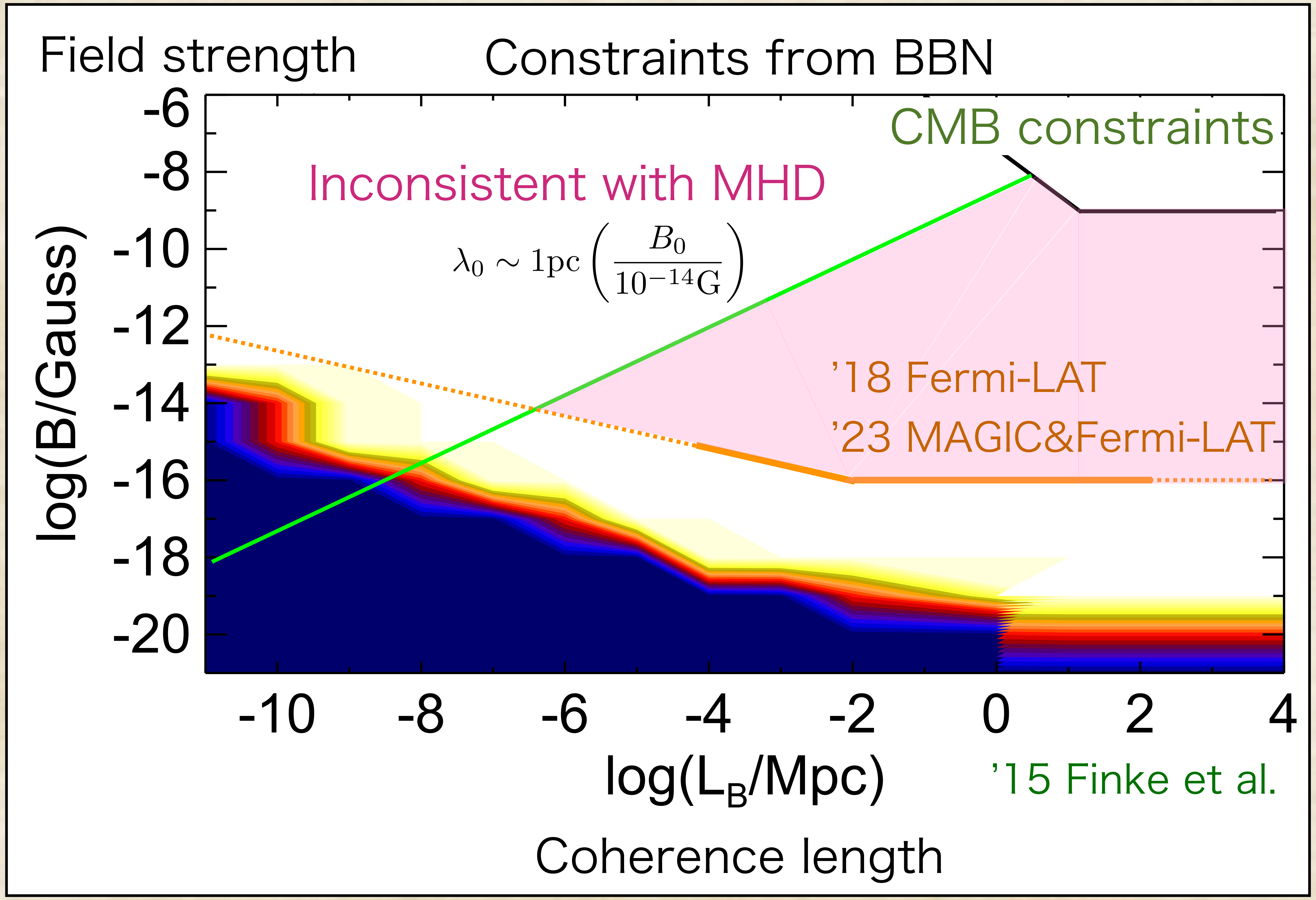




1804.08035 (Fermi-LAT collaboration)

Non-observation of the secondary cascade GeV photon can give the lower bound of the intergalactic magnetic fields (indirect implication)

Latest constraints from Fermi



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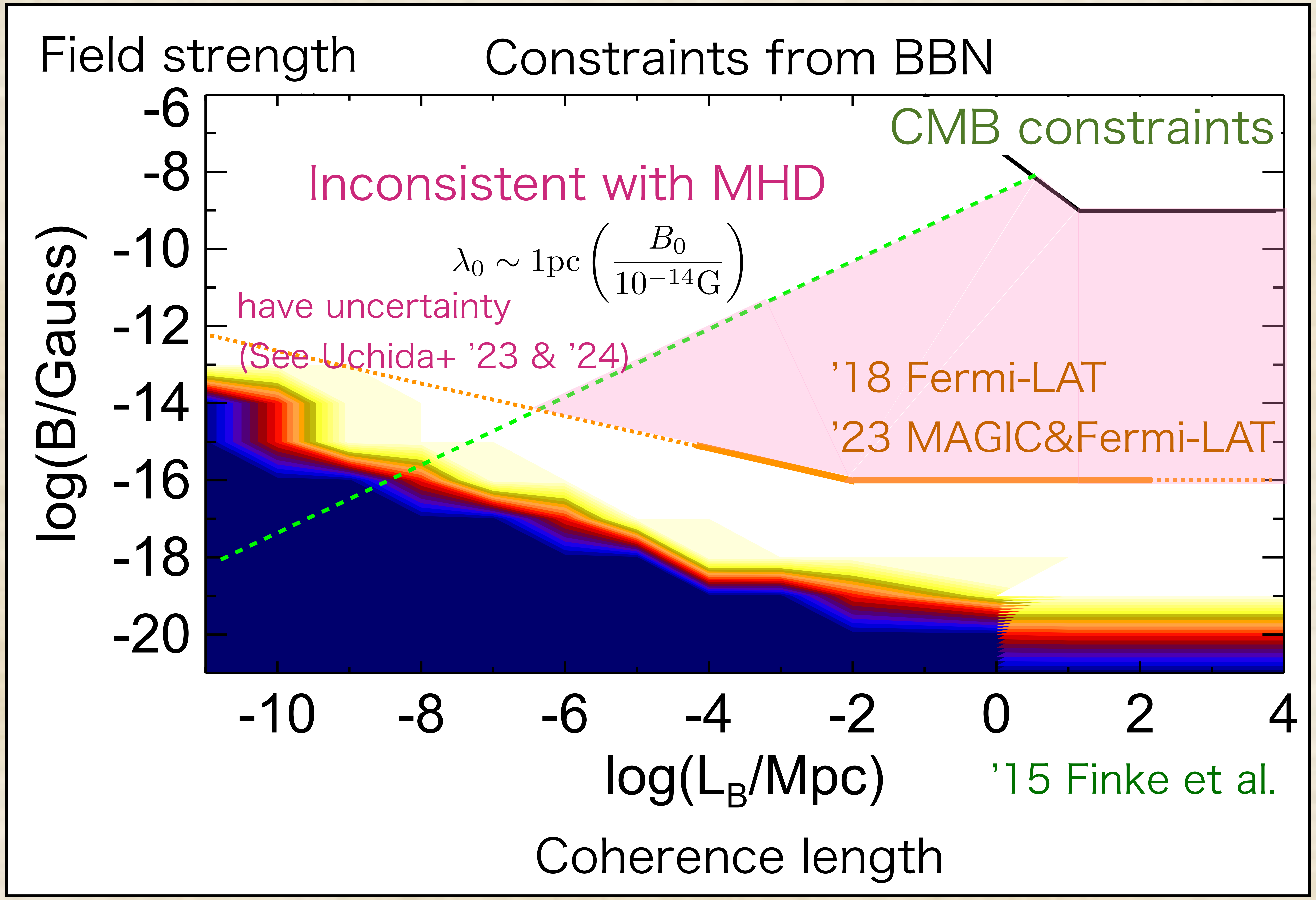
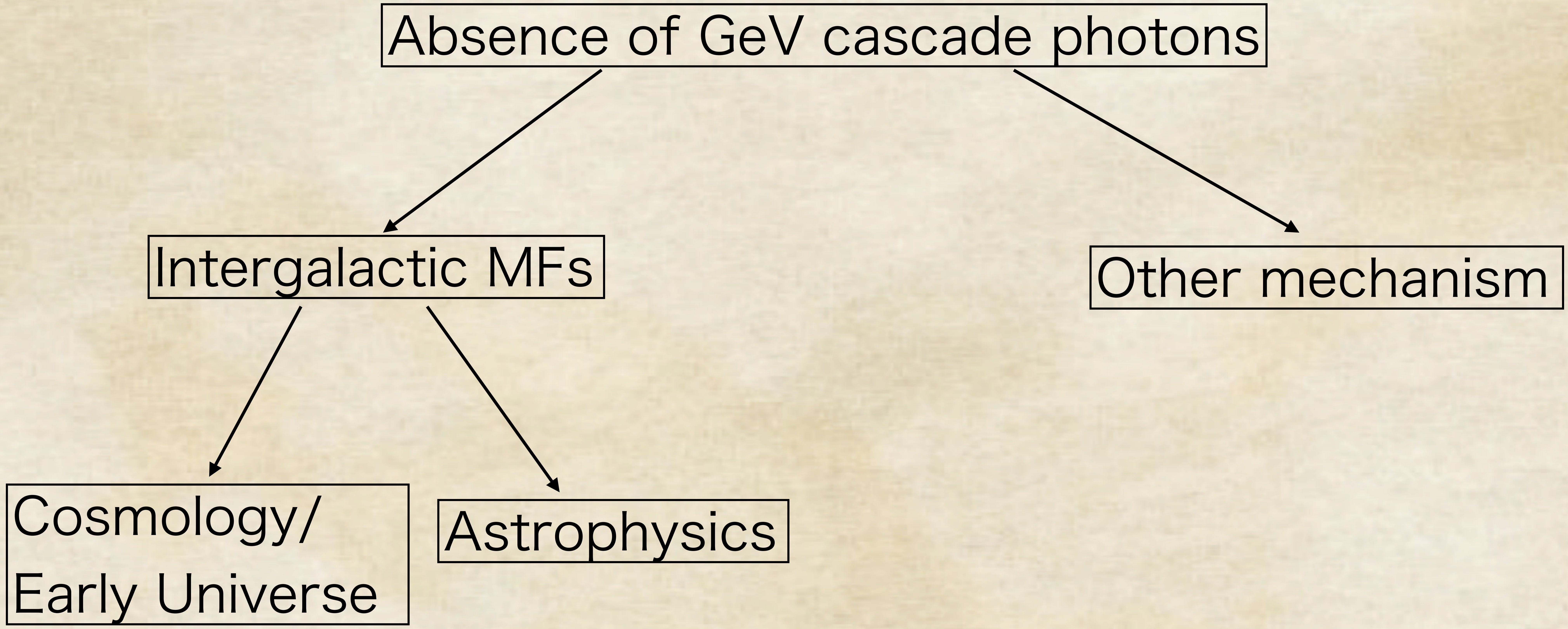
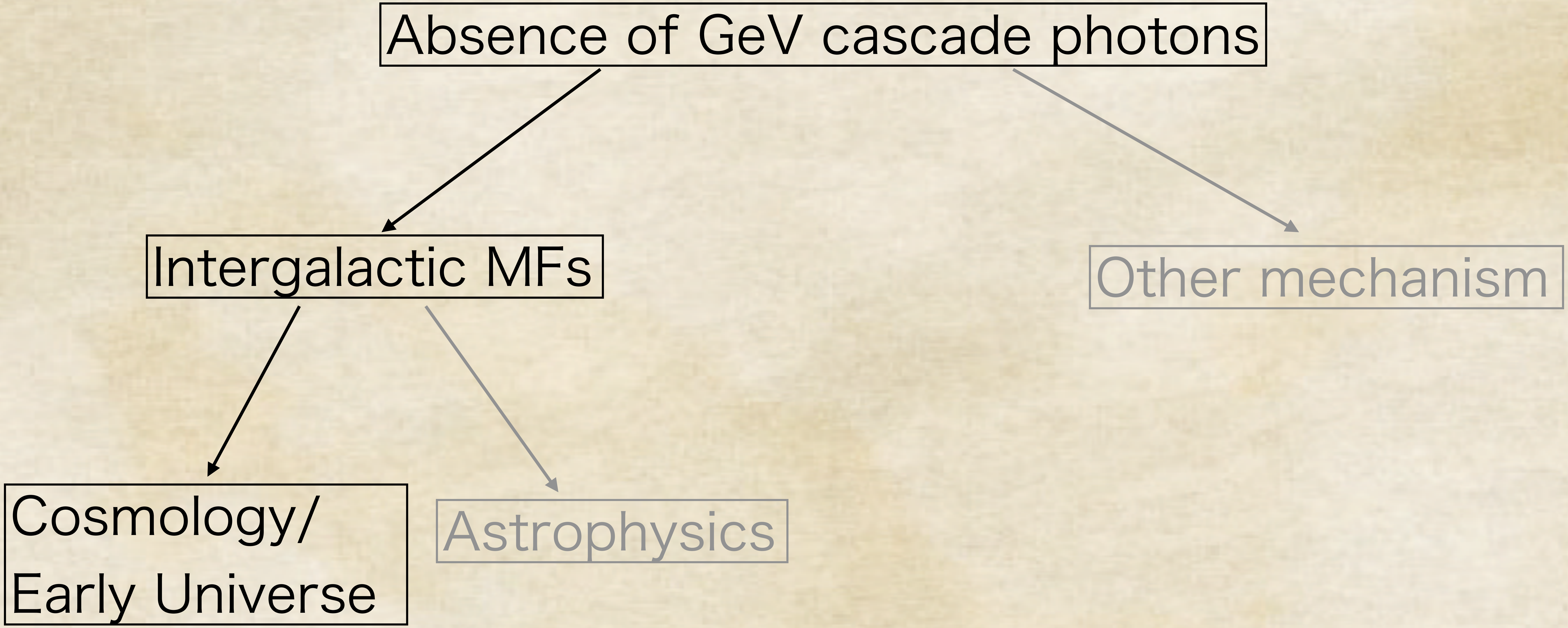



Absence of GeV cascade photons
Intergalactic MFs
Other mechanism
Cosmology/
Early Universe
Astrophysics


Absence of GeV cascade photons
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We might expect that they are relics from the early Universe.

1. Long range MFs are not in thermal equilibrium but keep their long-range spectrum (no “thermal” mass for the MFs). => Carry the information before the recombination?

2. Generation mechanism (magnetogenesis) may need new physics beyond the SM. => Target for the phenomenological model builders, such as axion inflation or phase transition.

3. Chiral effects may play an important role of their generation and/or evolution. => Interest for field theorists.

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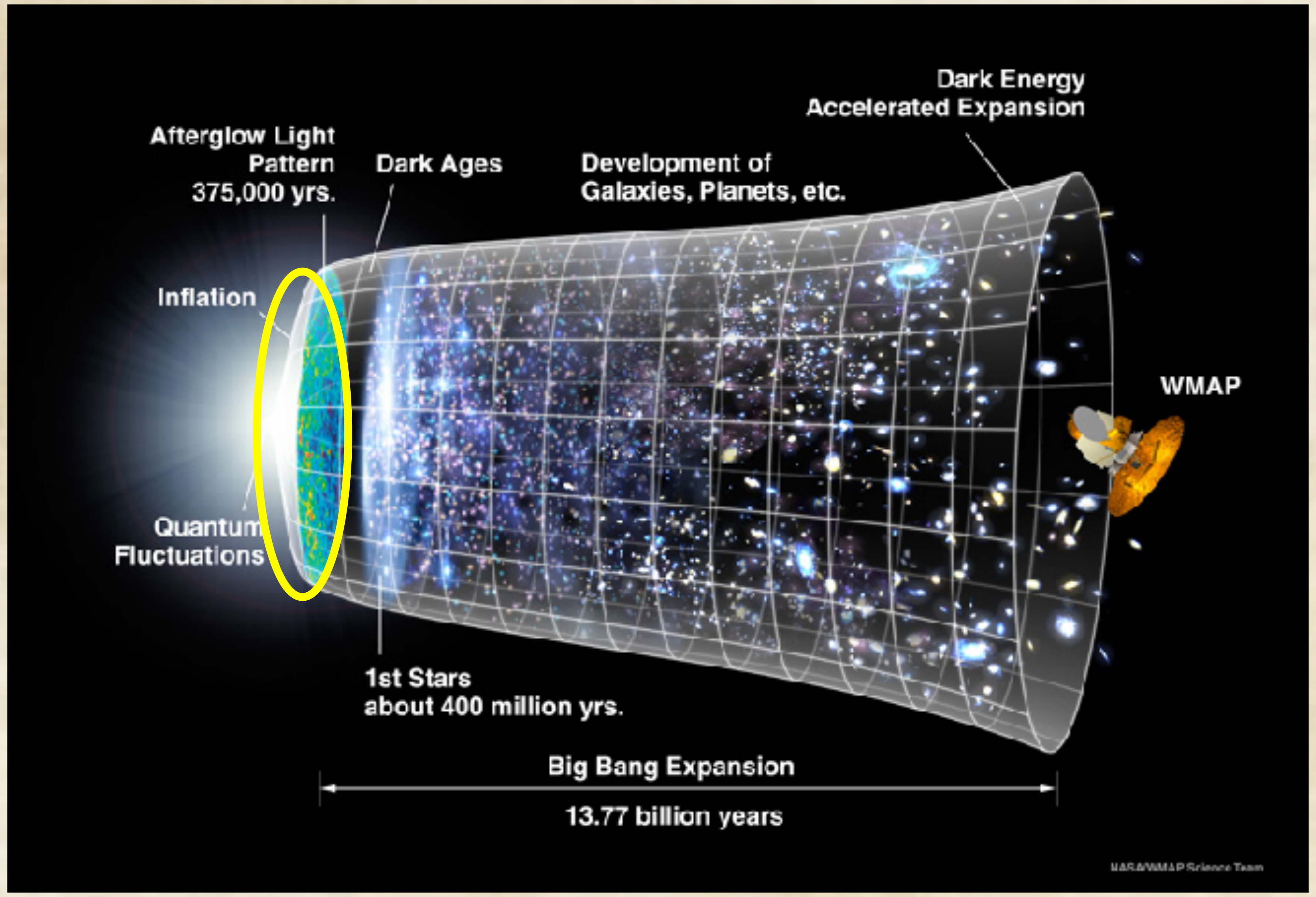
Baryon asymmetry of the Universe can be also explained!

('98 Giovannini & Shaposhnikov,'16 Fujita & KK, KK & Long)

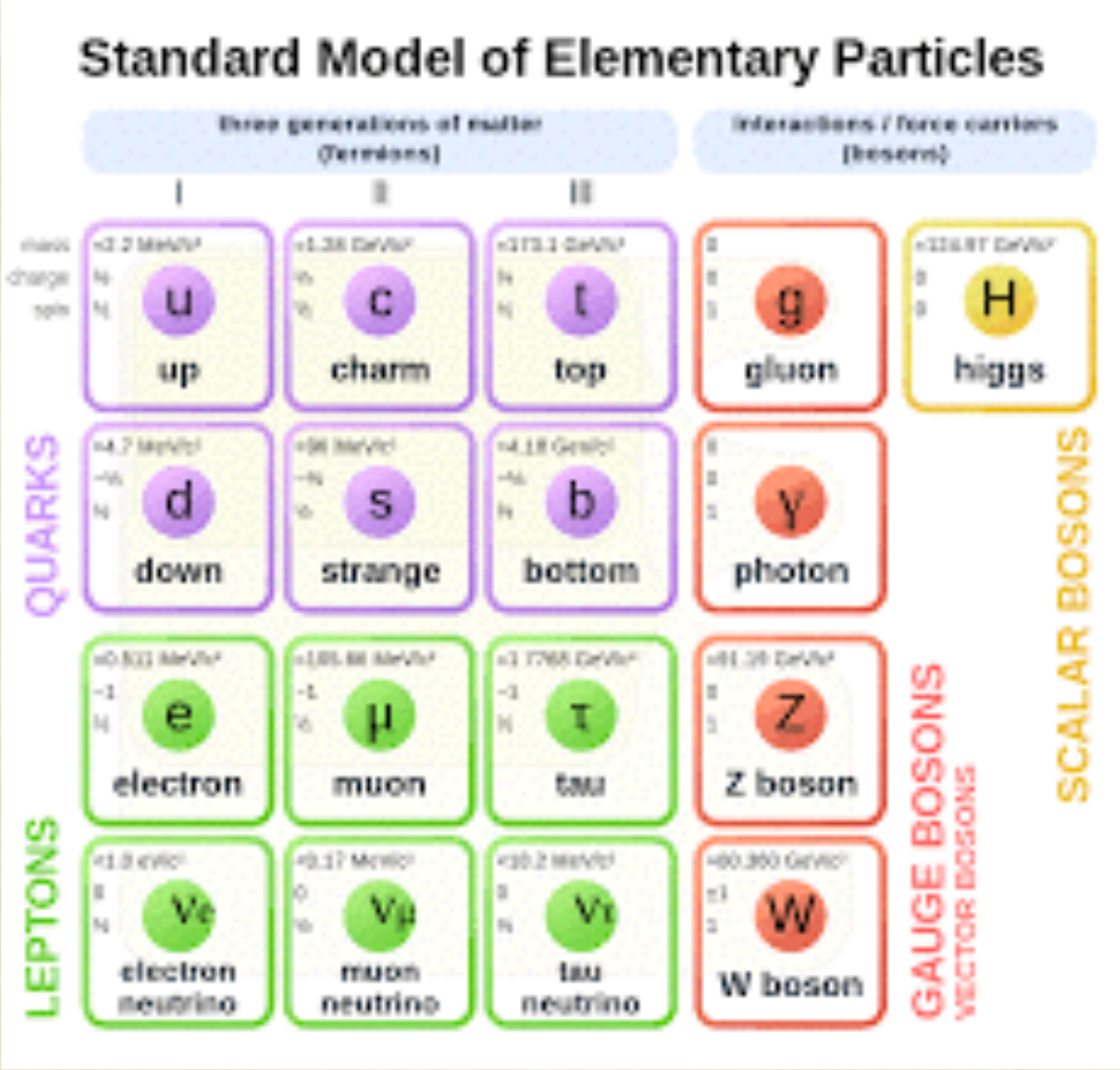
But I will not explain that much in detail in this talk….

Magnetohydrodynamics (MHD) and chiral magnetic effect

Now I have in mind the evolution of magnetic fields in the radiation dominated, very early Universe



Universe filled with thermal plasma of the relativistic particles of the Standard Model of Particle Physics



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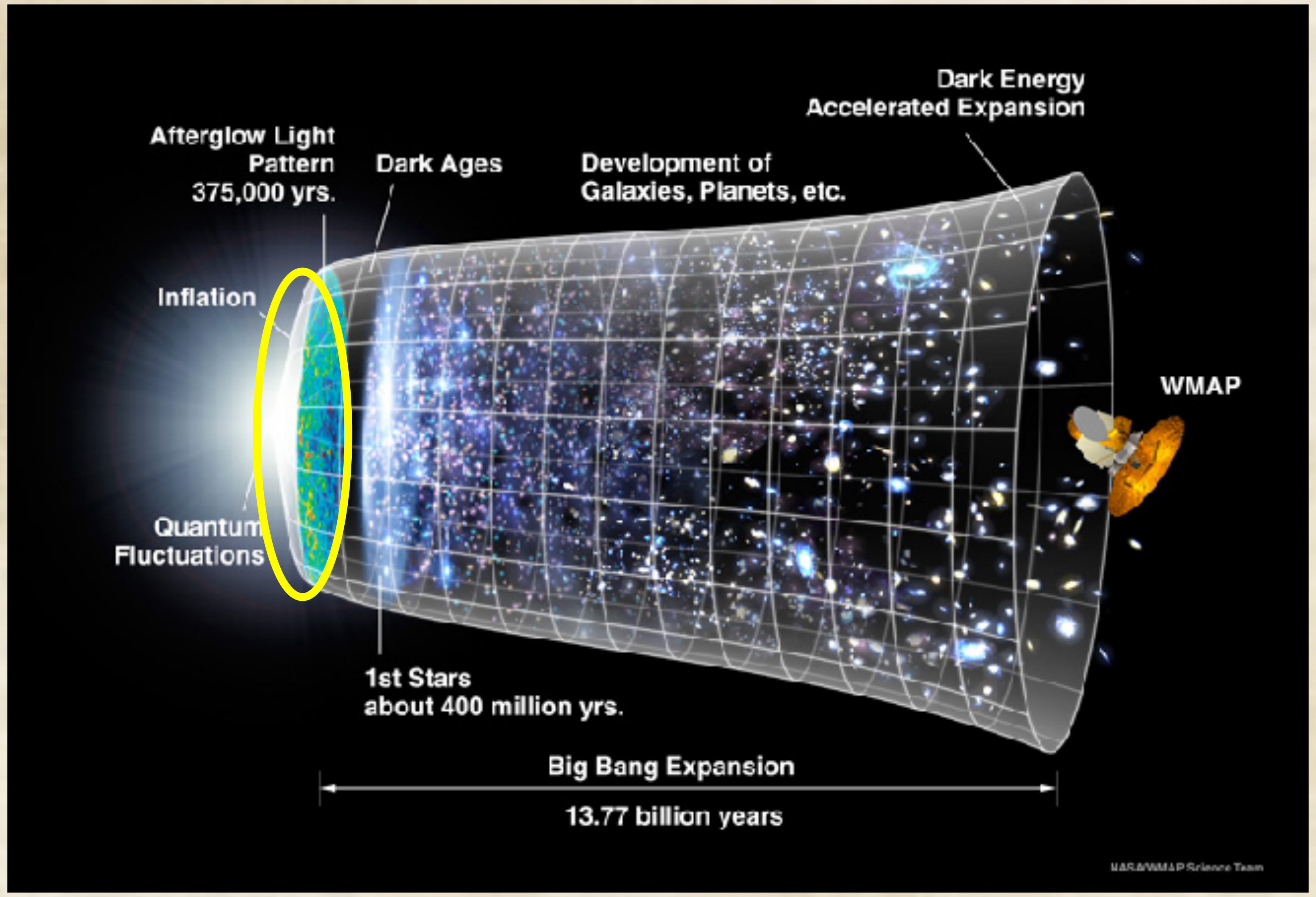

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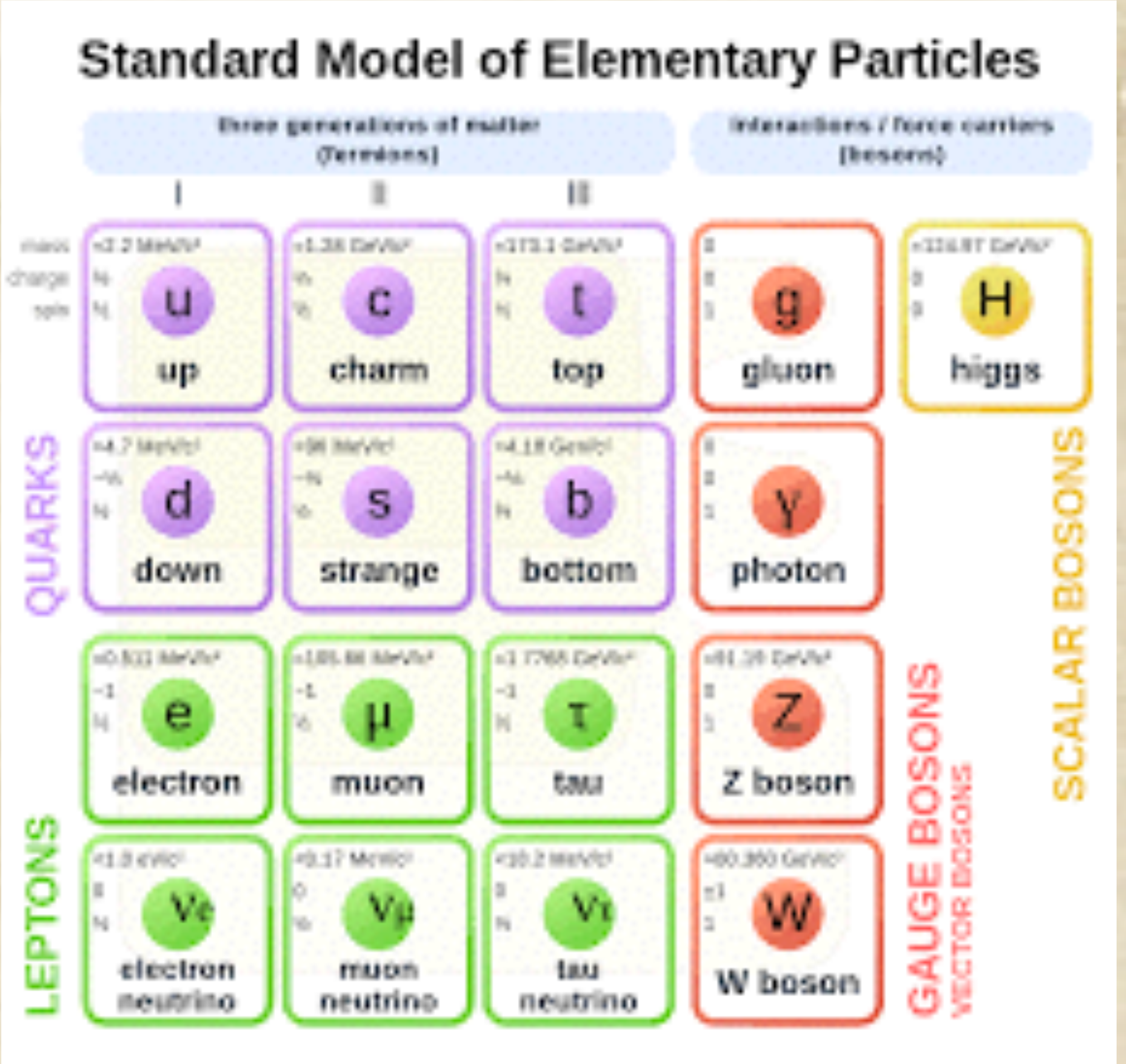

Electric fields are screened while long-wave magnetic fields exist with a coherence length longer than the Debye screening scale $\sim (gT)^{-1}$ $\quad (n \sim T^3)$

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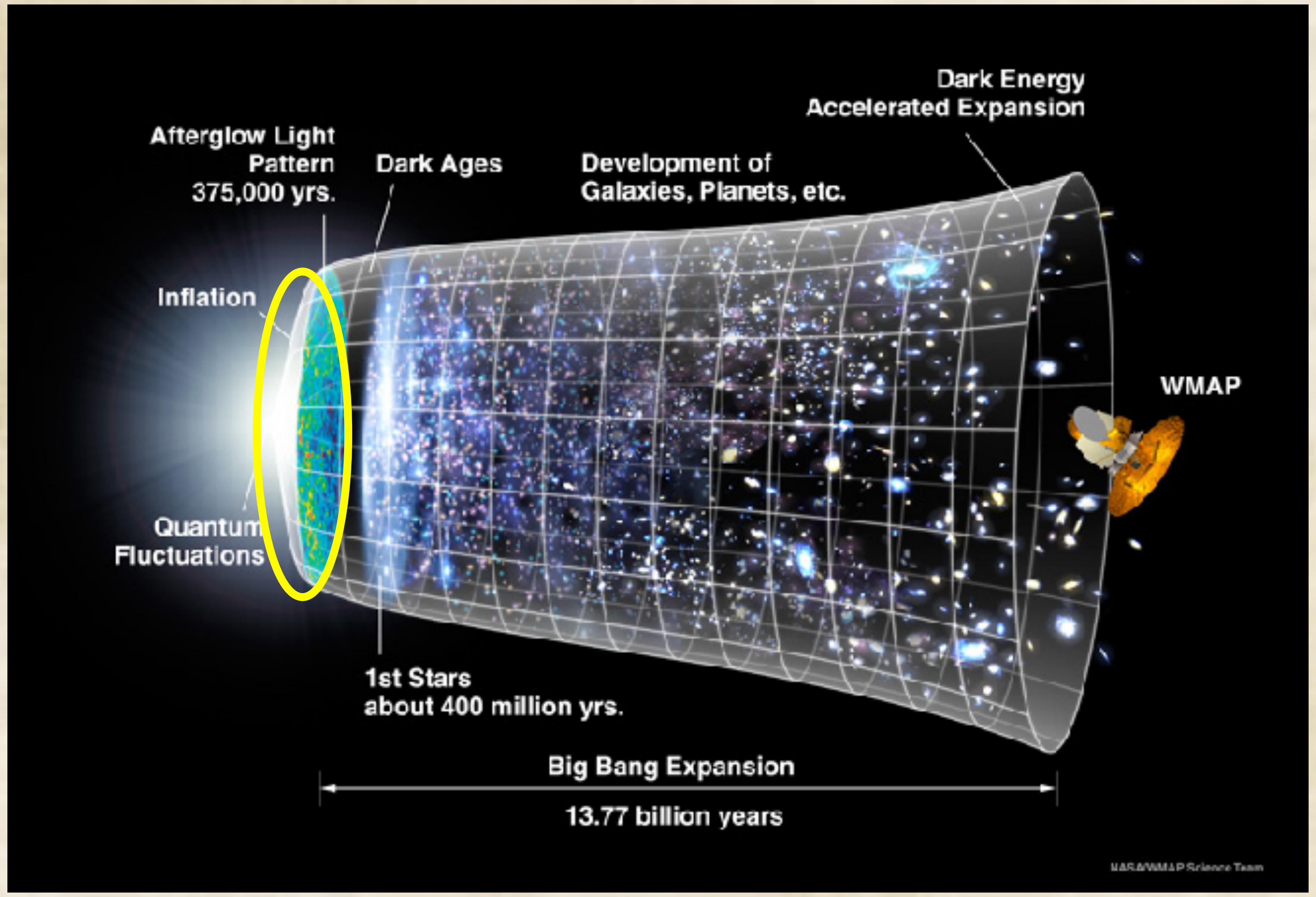

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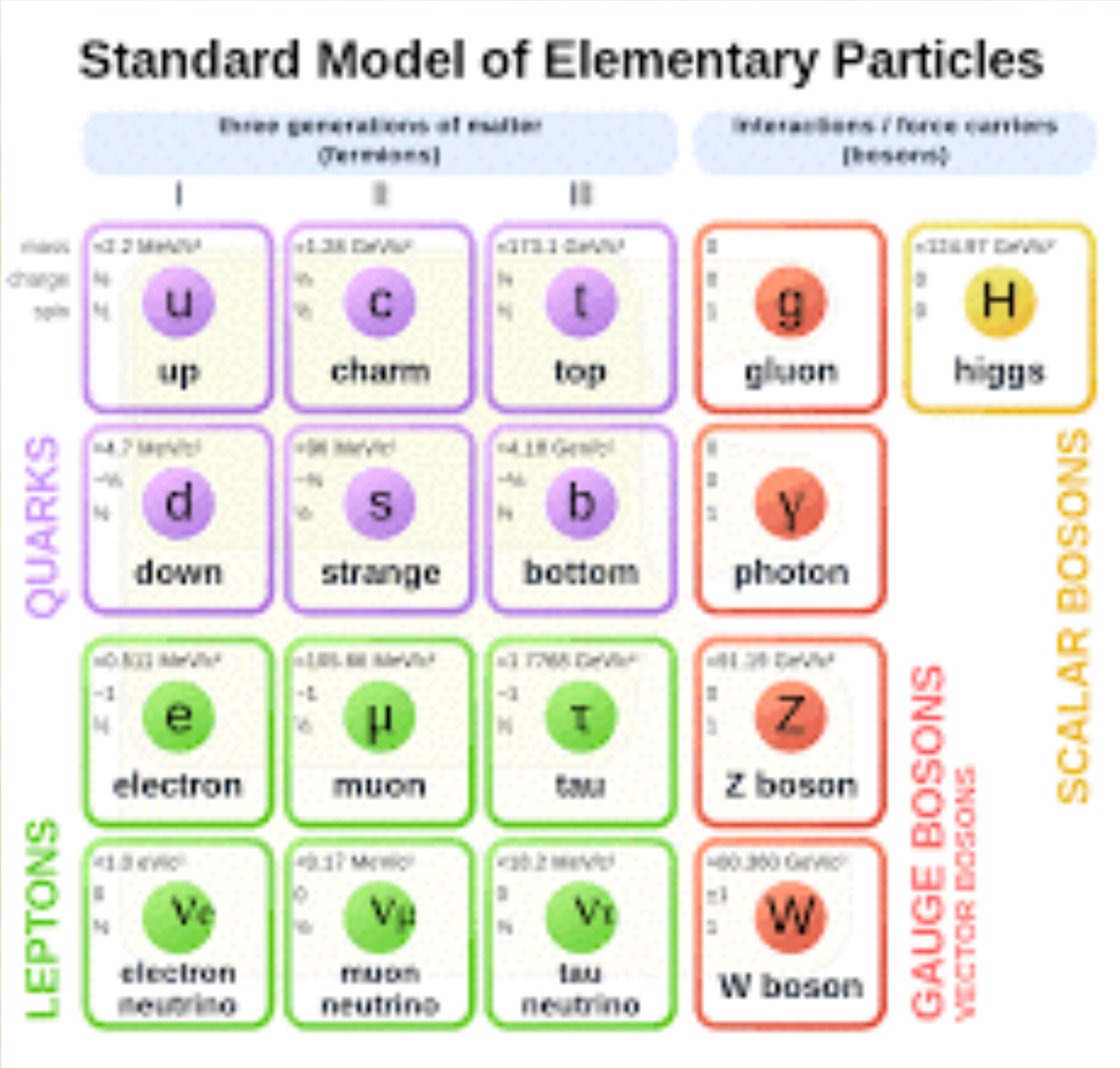

Electric fields are screened while long-wave magnetic fields exist with a coherence length longer than the Debye screening scale $\sim (gT)^{-1}$ $\quad (n \sim T^3)$

=> It is appropriate to describe it with magnetohydrodynamics (MHD).

MHD equations

The dynamical degrees of freedom:

Magnetic field: $\boldsymbol{B} = \boldsymbol{\nabla} \times \boldsymbol{A}$, Plasma velocity: $\boldsymbol{u}$, Energy density: ρ

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η, ν : resistivity/viscosity

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η, ν : resistivity/viscosity

Hard to solve analytically -> Solve numerically and find the physics.

(cosmic expansion is hidden in the “comoving” frame, $\boldsymbol{B}_{\mathrm{p}} = a^{-2} \boldsymbol{B}_{\mathrm{c}}$)

Cosmological MHD (supposing a generation mechanism)

=> homogeneous and isotropic magnetic (and velocity) fields

Set the configuration such that the spectrum satisfies

$$\langle B_i(\boldsymbol{k})\rangle = 0 \qquad \langle B_i(\boldsymbol{k})B_j(\boldsymbol{k}')\rangle = (2\pi)^3\left((\delta_{ij}-\hat{k}_i\hat{k}_j)\underline{S(k)} + i\epsilon_{ijk}\hat{k}_k\underline{A(k)}\right)\delta(\boldsymbol{k}-\boldsymbol{k}')$$

$$(S(k)\geq A(k))$$

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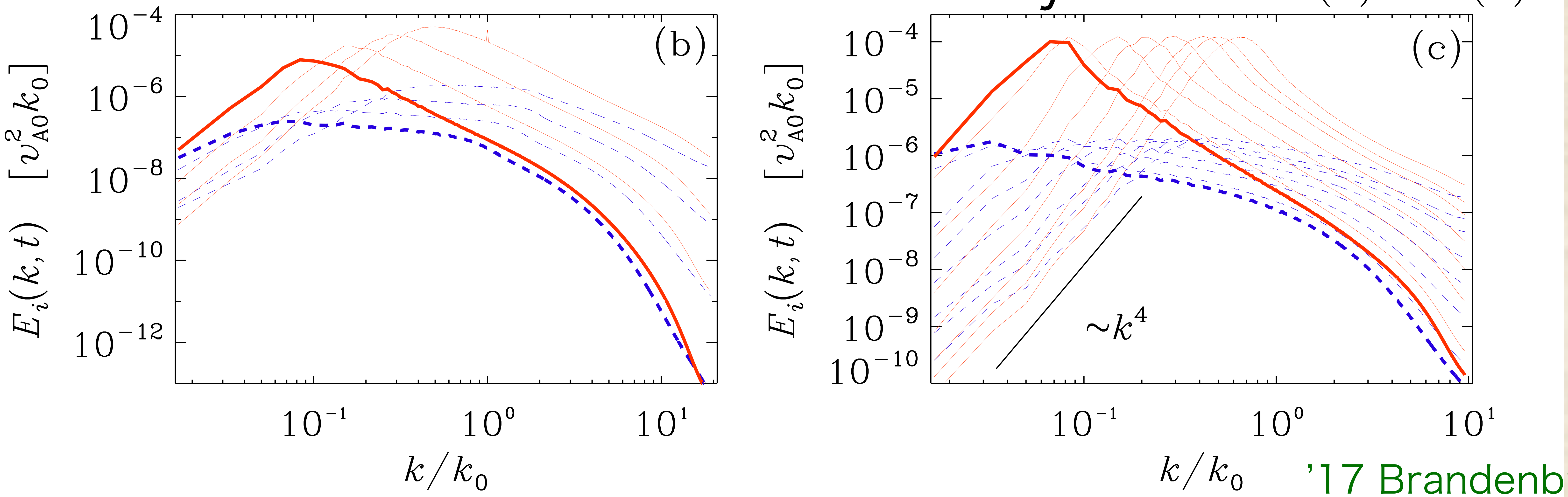
non-helical: $A(k) = 0$ maximally-helical: $A(k) = S(k)$

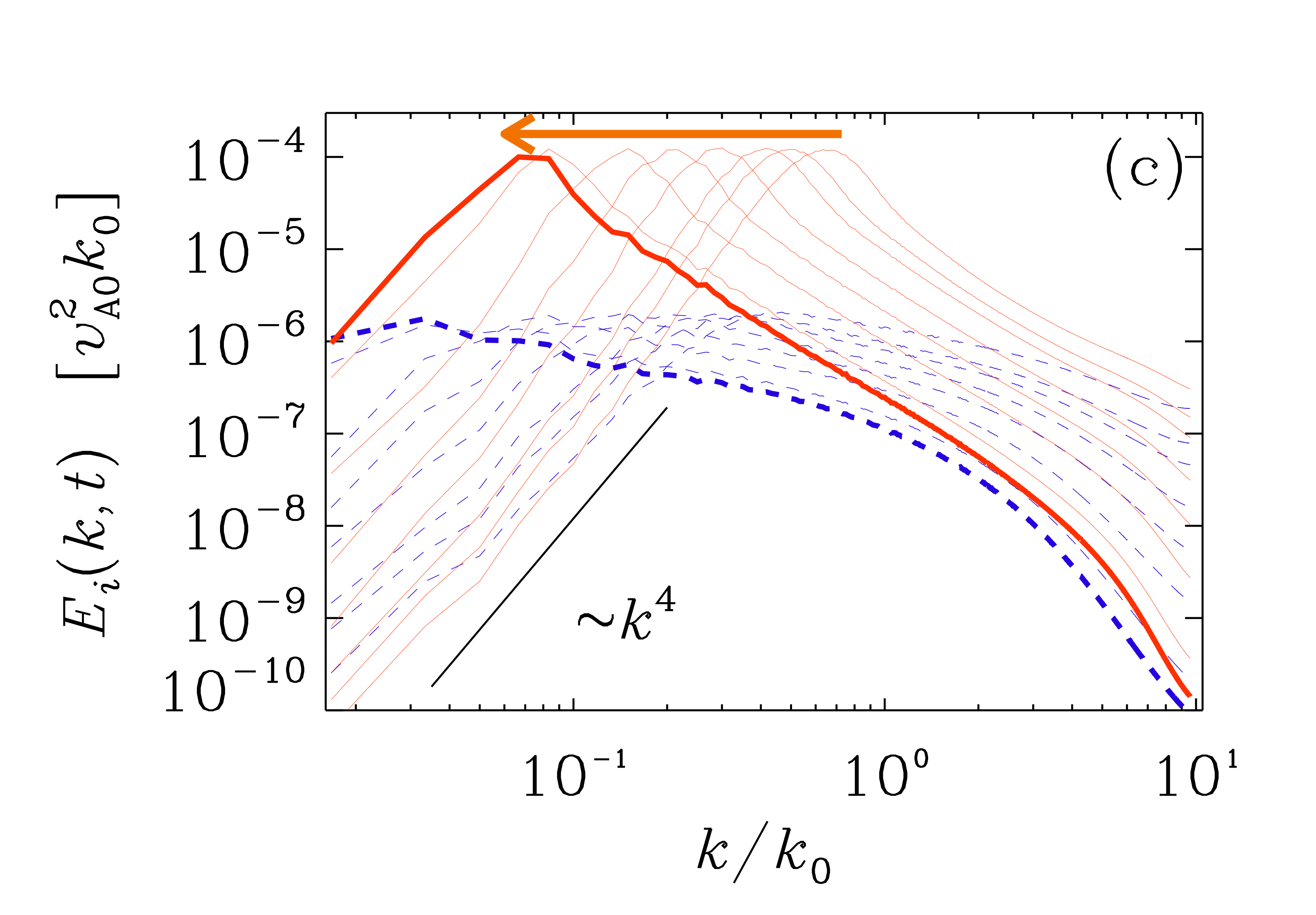


$$E(k) = kS(k)$$

'17 Brandenburg & Kahniashvili

Numerical simulation finds self-similar evolution of magnetic and velocity fields.

Maximally-helical magnetic fields

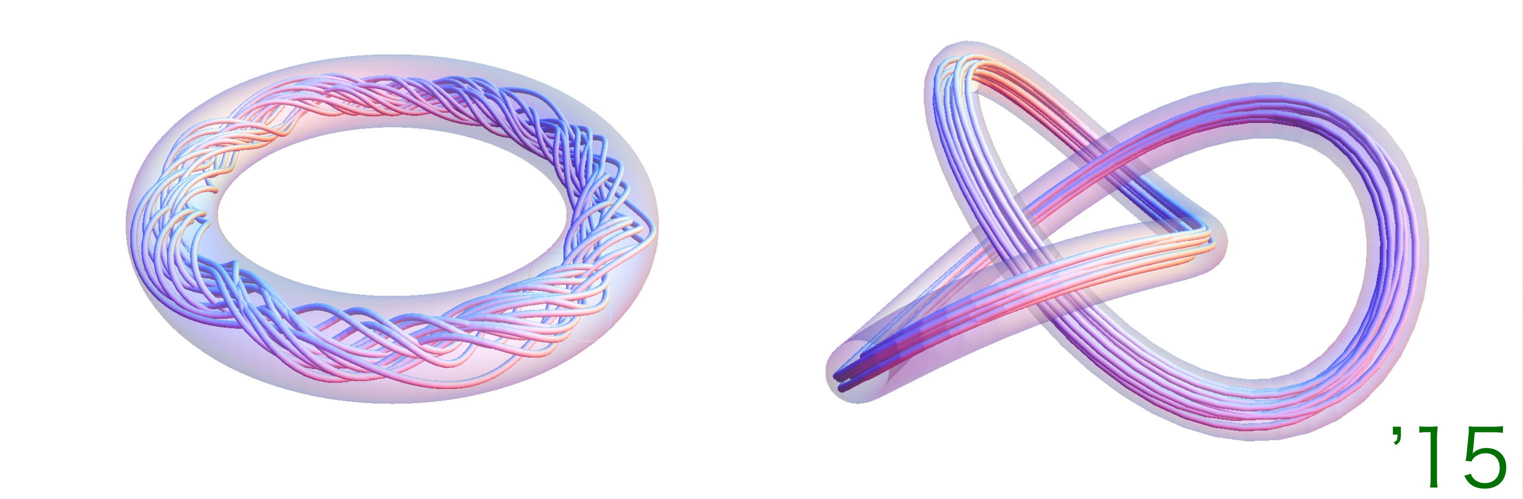


'17 Brandenburg & Kahniashvili

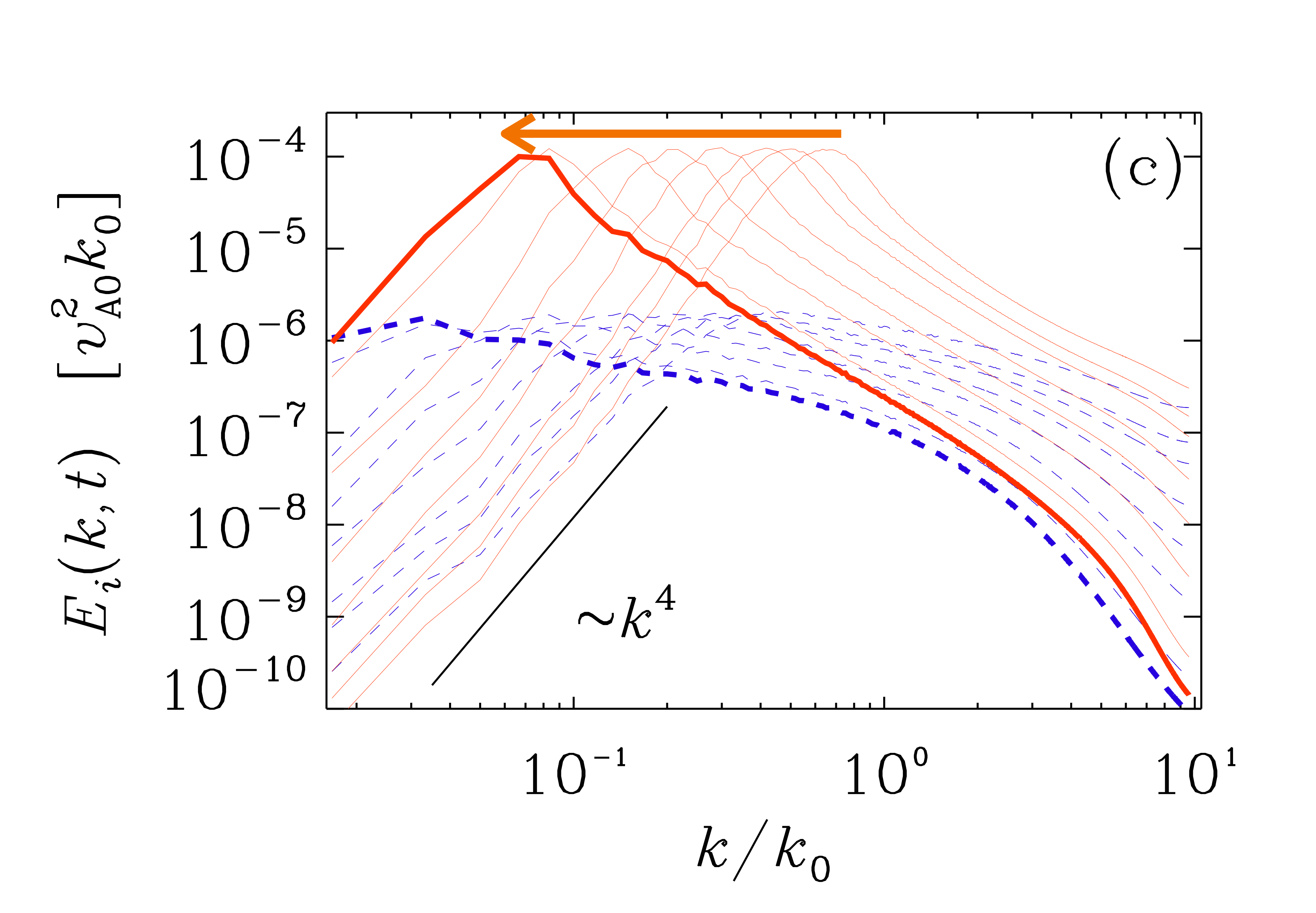
Maximally-helical magnetic fields

Evolution can be understood by the conservation of magnetic helicity

Magnetic helicity
⋯ difference between right- and left-circular polarization modes; describes twist and linkage of magnetic field lines



'15 Hirono+



'17 Brandenburg & Kahniashvili

$$H_Y \simeq k_{\rm peak} A(k_{\rm peak}) \sim k_{\rm peak} S(k_{\rm peak}) \sim E(k_{\rm peak}) = {\rm const.}$$

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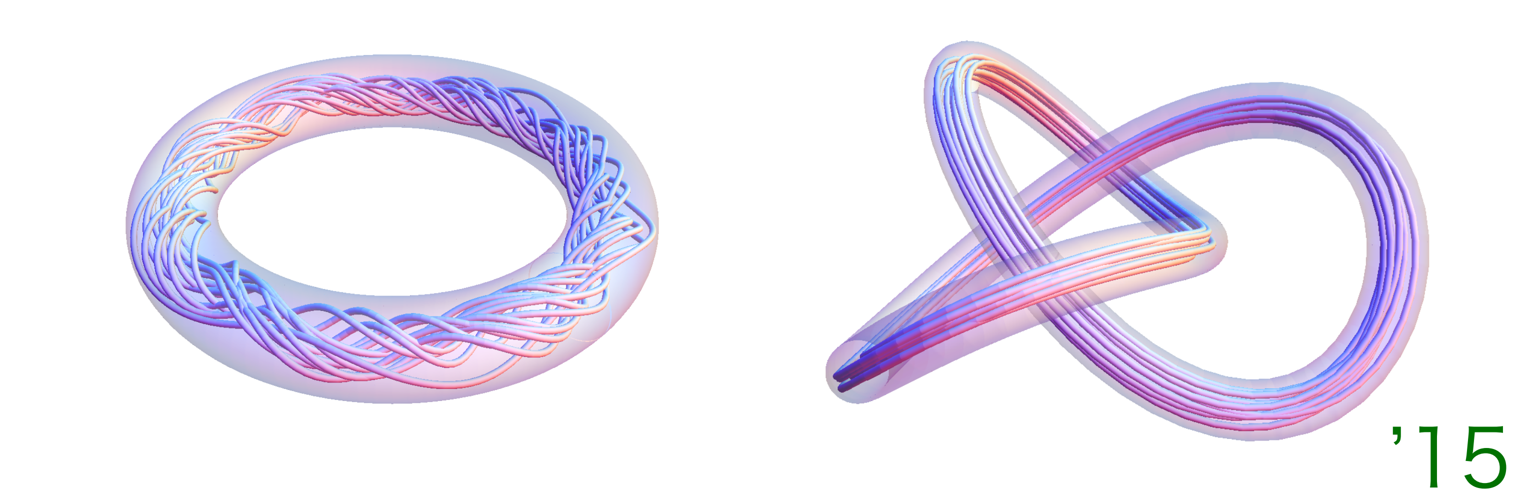

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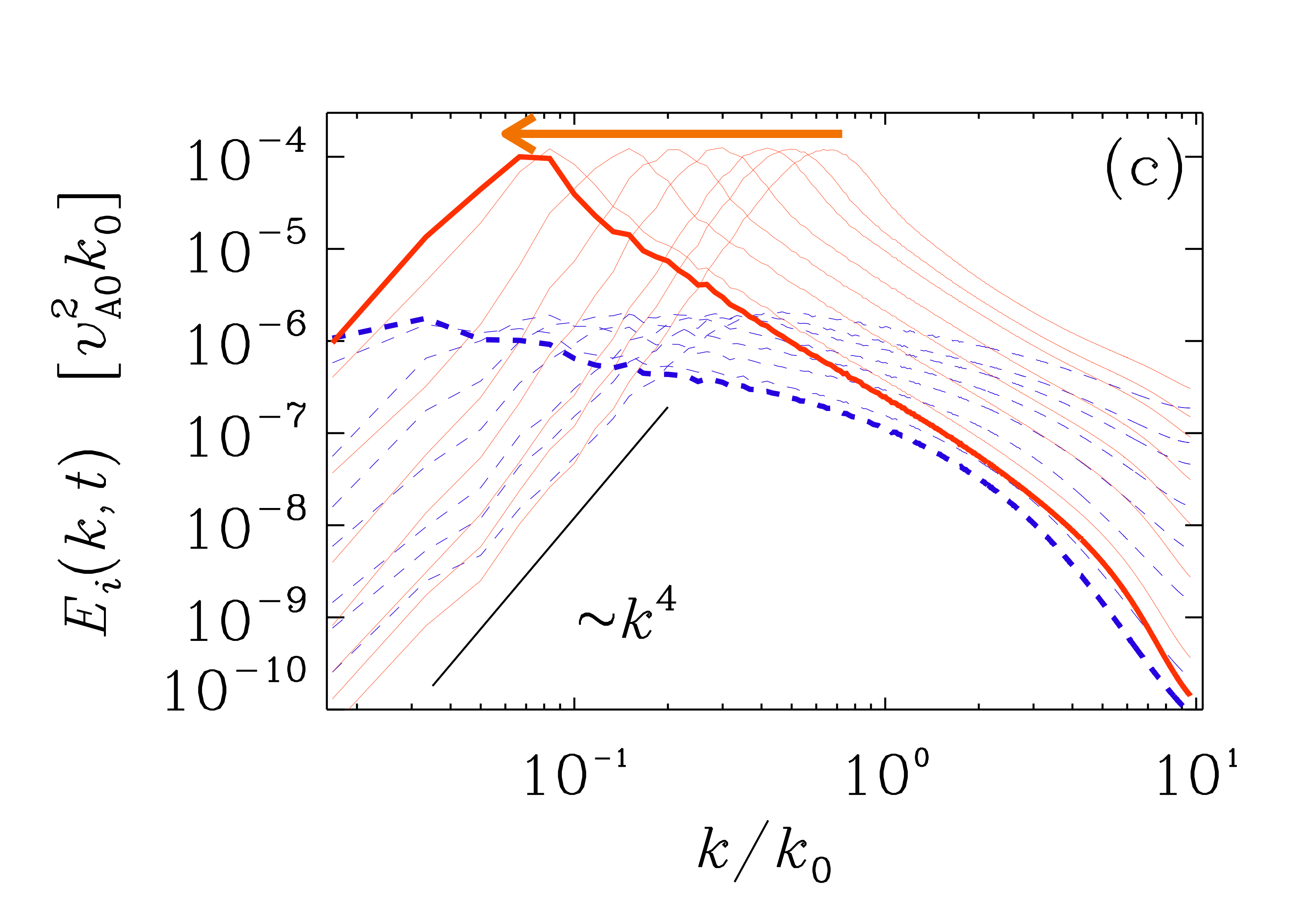


’17 Brandenburg & Kahniashvili

$$H_Y \simeq k_{\rm peak} A(k_{\rm peak}) \sim k_{\rm peak} S(k_{\rm peak}) \sim E(k_{\rm peak}) = {\rm const.}$$

Together with the time scale of the evolution,

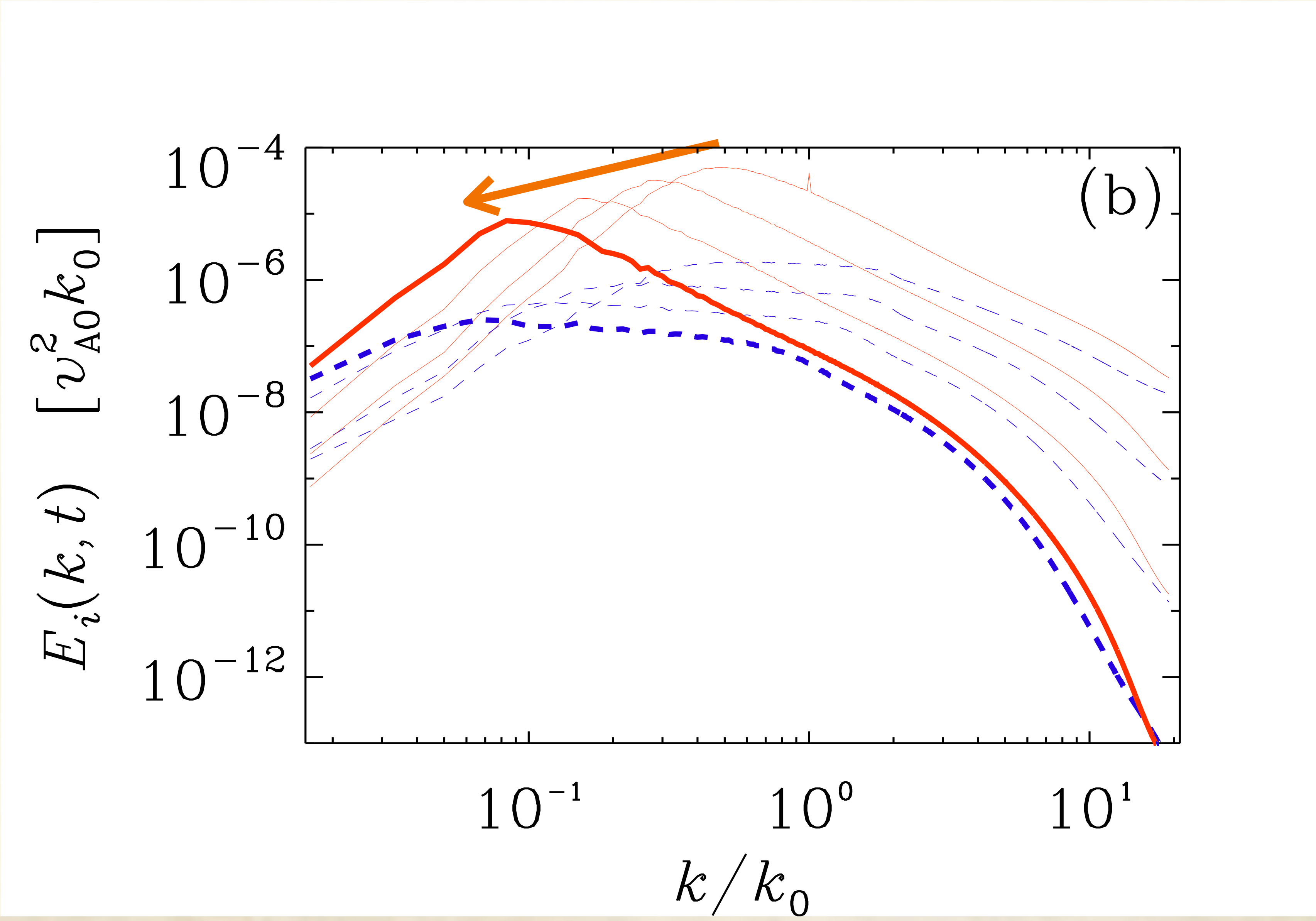
$$t \sim 1/(k_{\rm peak} v) \propto 1/(k_{\rm peak} B) \sim 1/(k_{\rm peak}\sqrt{k_{\rm peak} E(k_{\rm peak})})$$

we obtain the scaling solution,

$$k_{\rm peak} \propto t^{-2/3}$$

’04 Banerjee & Jedamzik, ’24 Uchida, KK+

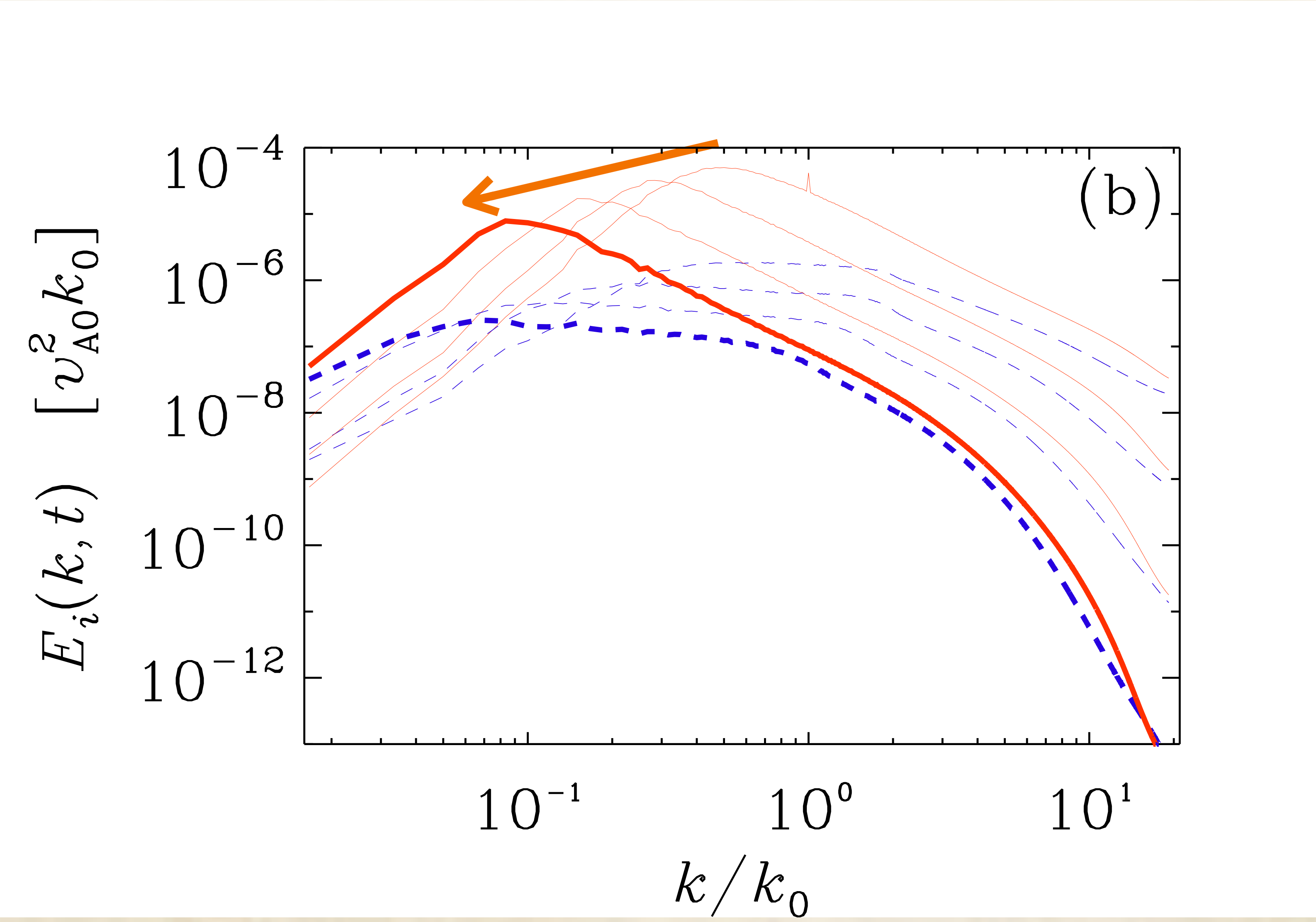
Non-helical magnetic fields



'17 Brandenburg & Kahniashvili

Non-helical magnetic fields

No conserved quantity? How to understand???



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Non-helical magnetic fields

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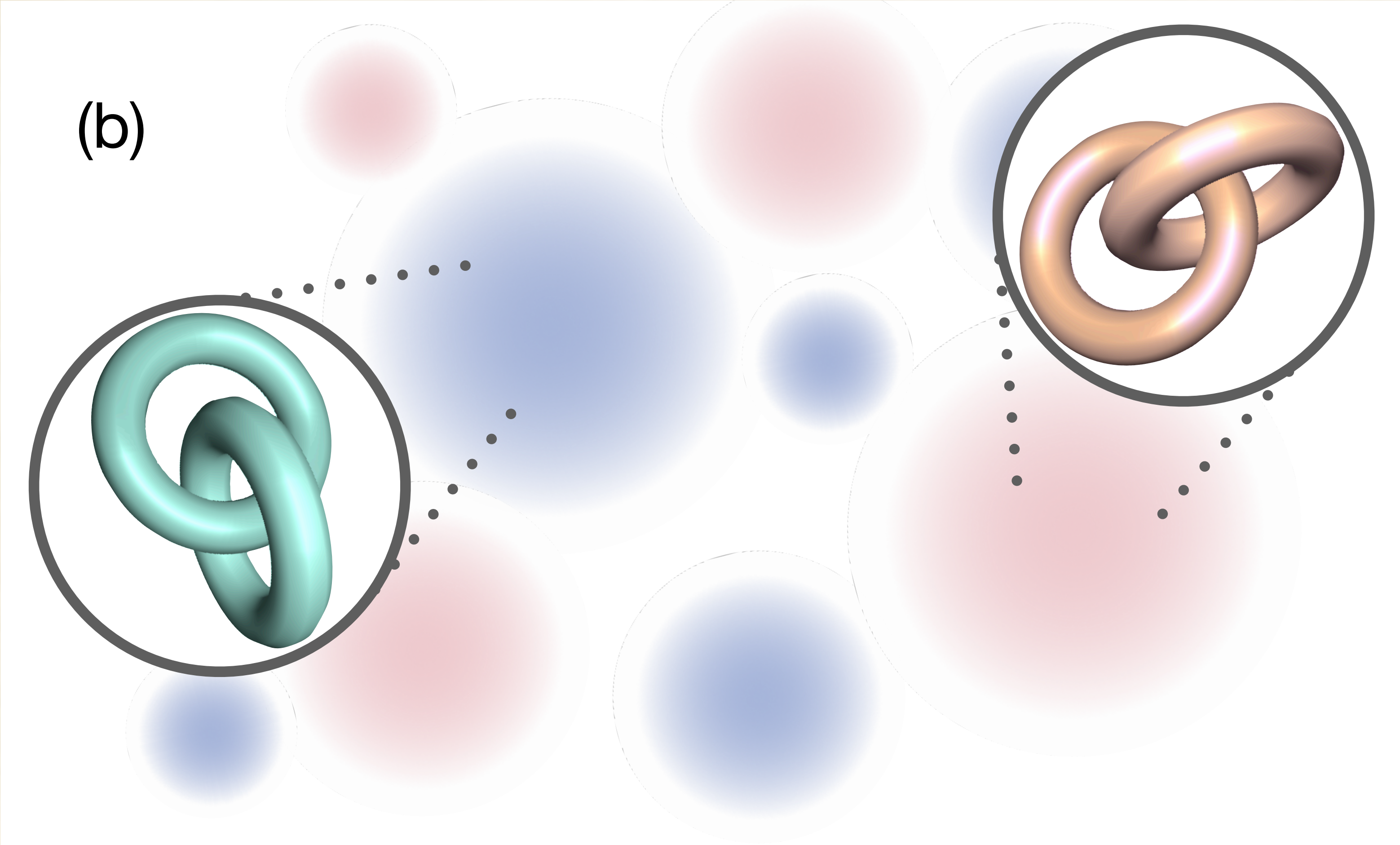
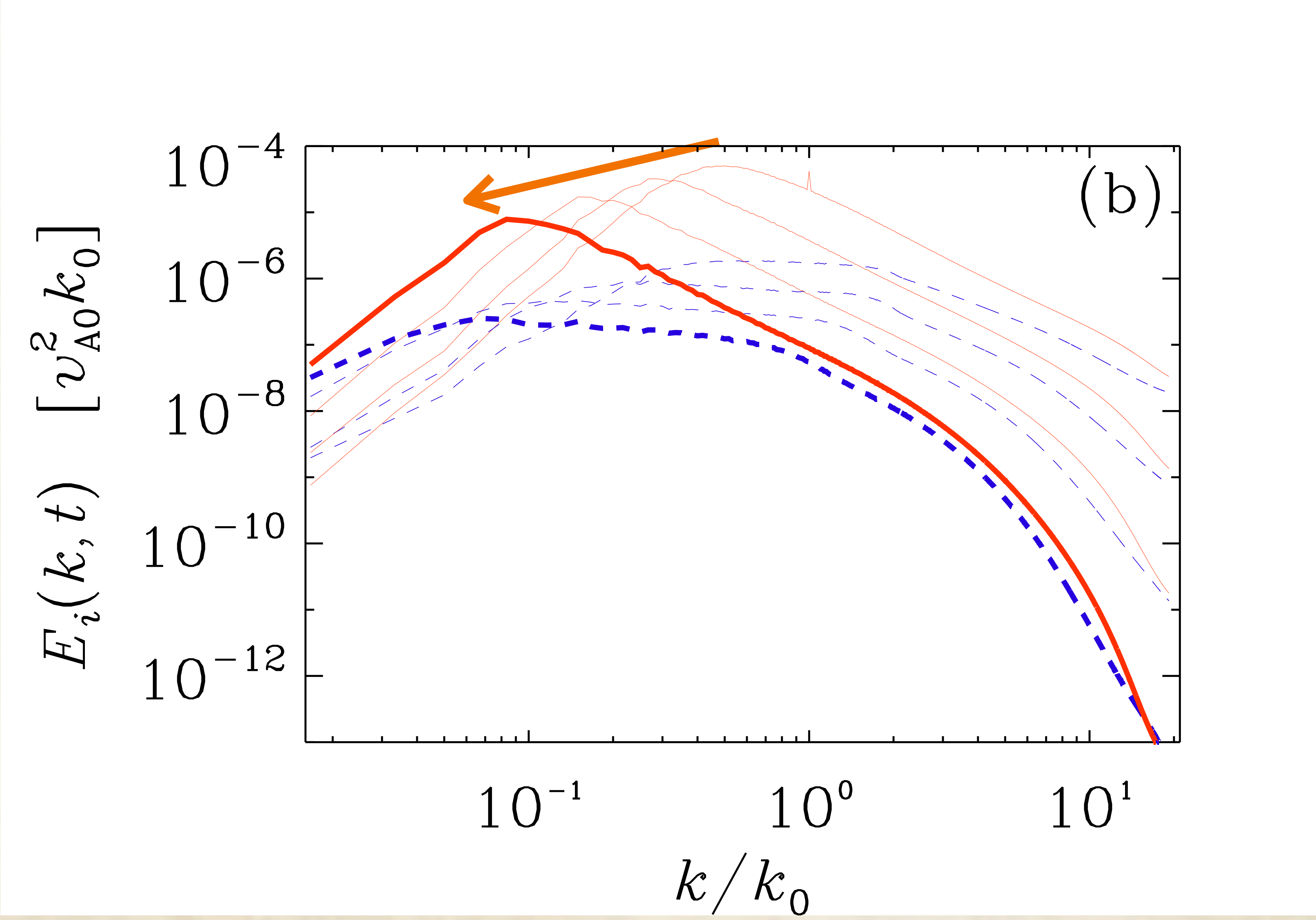
Recently, new conserved quantity is found





'17 Brandenburg & Kahniashvili

Hosking integral: ~ Two-point function of helicity

'21, '22 Hosking & Schekochihin

$$\int d^3r \langle h(\boldsymbol{x}) h(\boldsymbol{x}+\boldsymbol{r}) \rangle \sim (E(k_{\rm peak}))^2 k_{\rm peak}^{-3} = {\rm const.}$$

Non-helical magnetic fields

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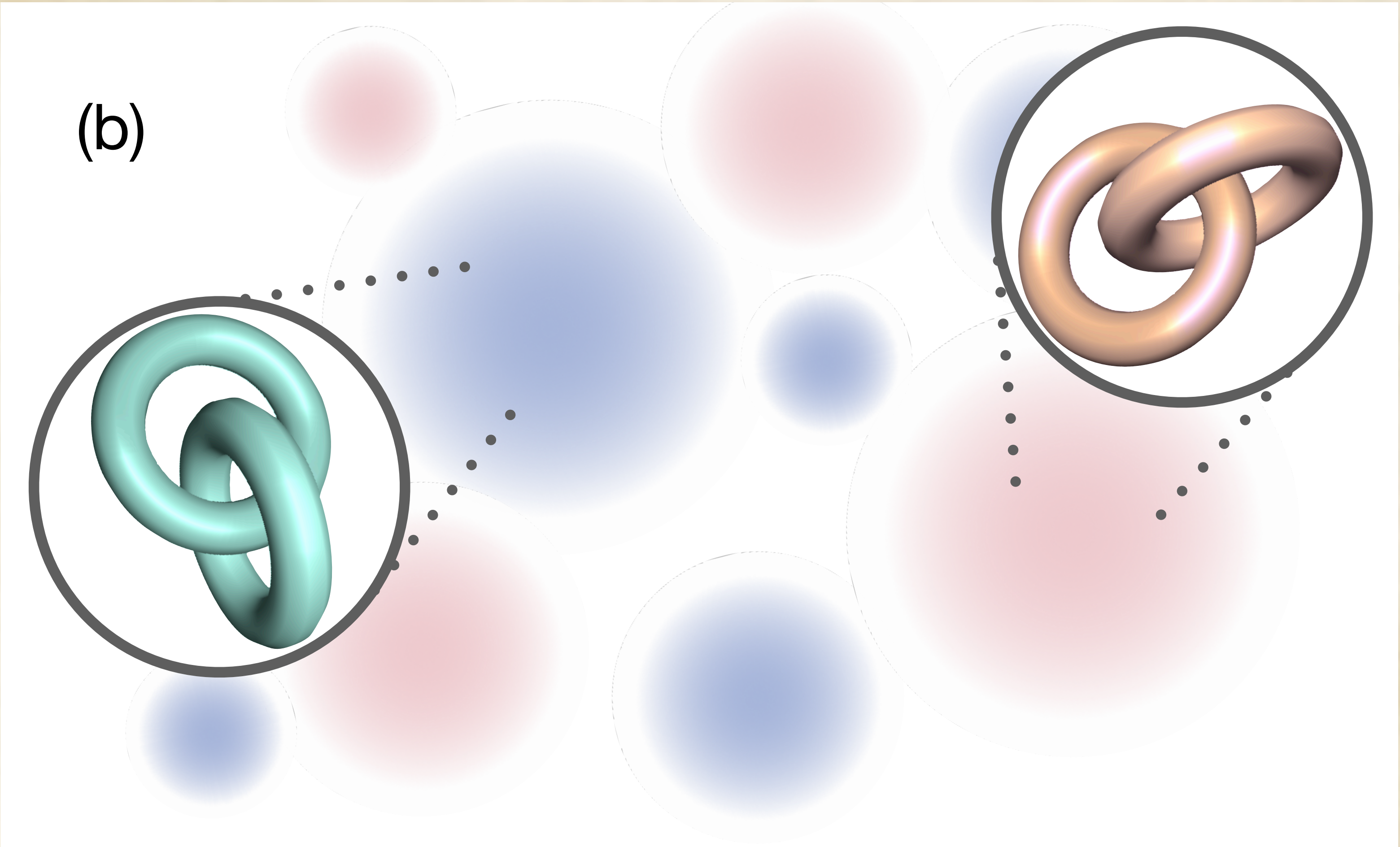

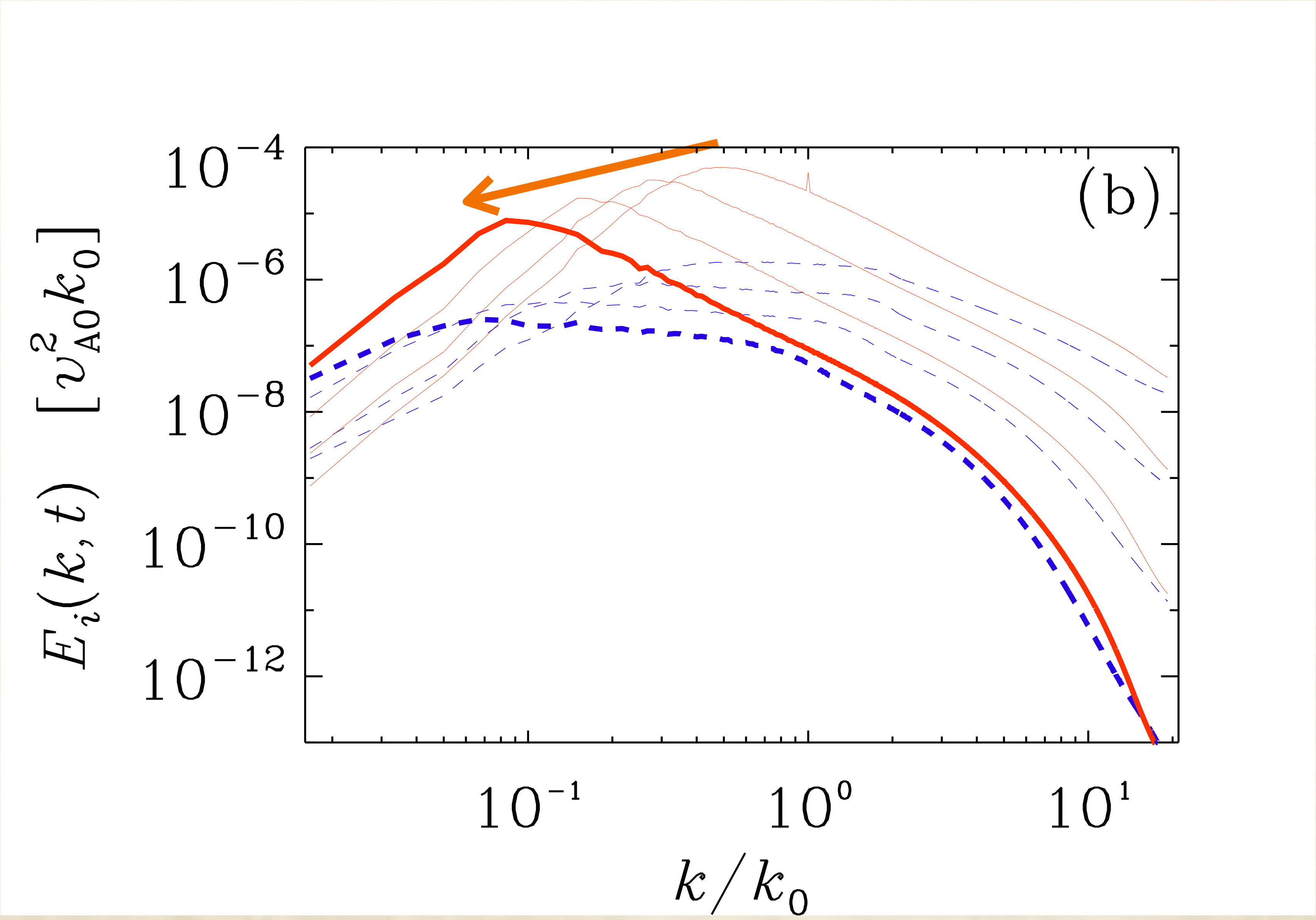

'17 Brandenburg & Kahniashvili

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$$\int d^3r \langle h(\boldsymbol{x})h(\boldsymbol{x}+\boldsymbol{r})\rangle \sim (E(k_{\rm peak}))^2 k_{\rm peak}^{-3} = {\rm const.}$$

Time scale argument e.g. reconnection

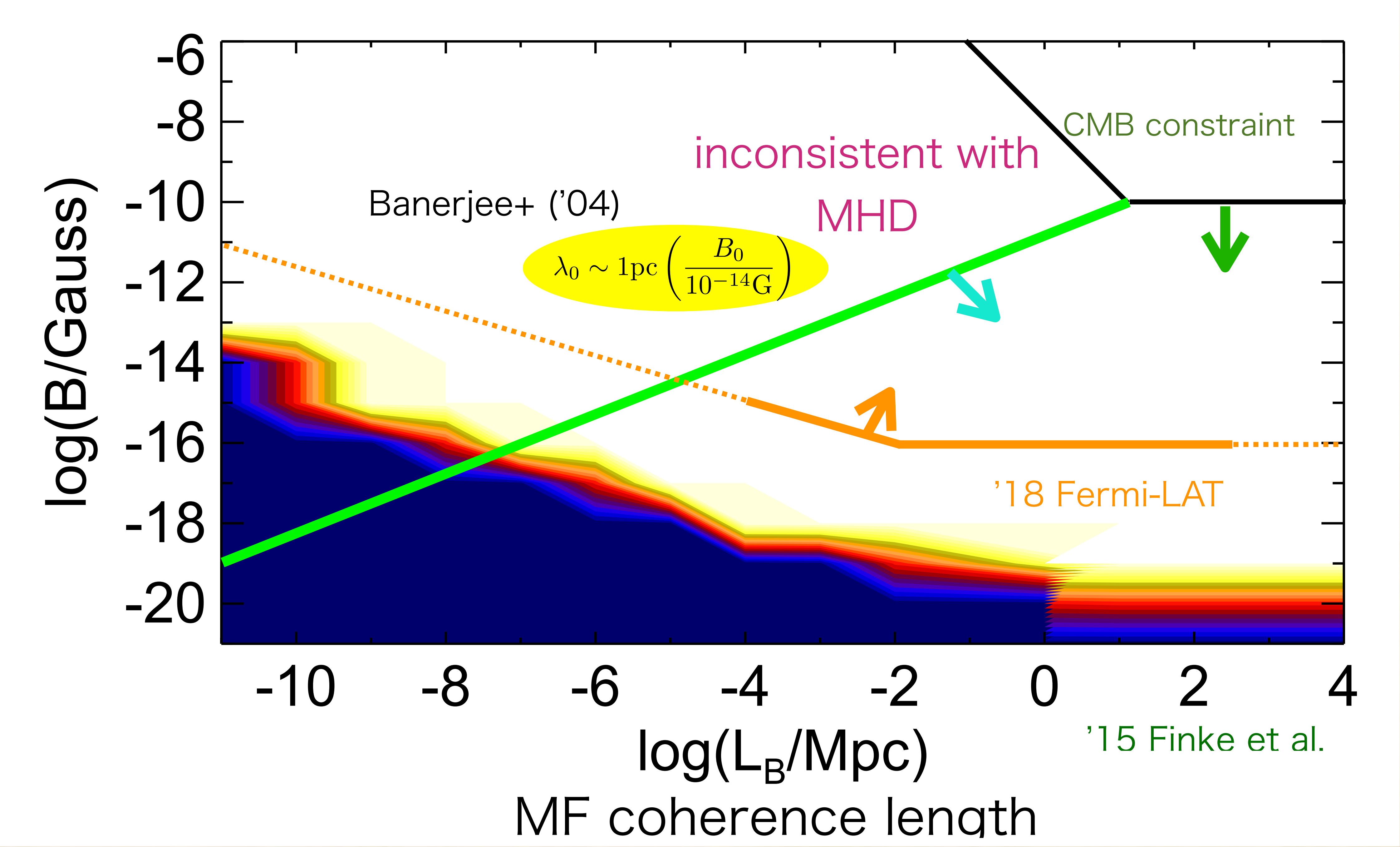
=> e.g., $E(k_{\rm peak}) \propto t^{-12/17}, \quad k_{\rm peak} \propto t^{-8/17}$

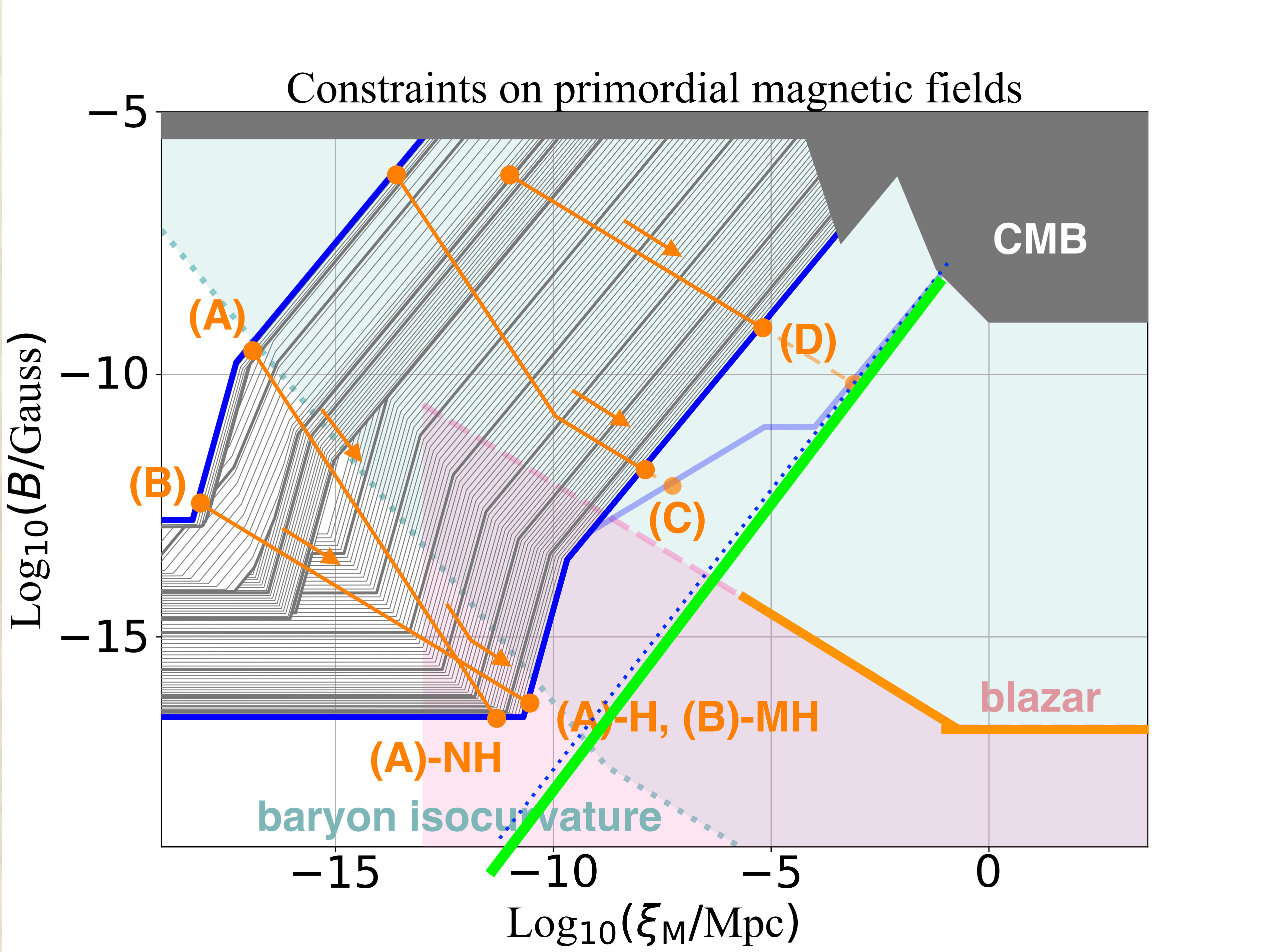
but depends the parameters.

'23, '24 Uchida, KK+

Regime dependent analysis…

'24 Uchida, KK+





'24 Uchida, KK+

Is it complete to describe the magnetic field evolution in the early Universe?

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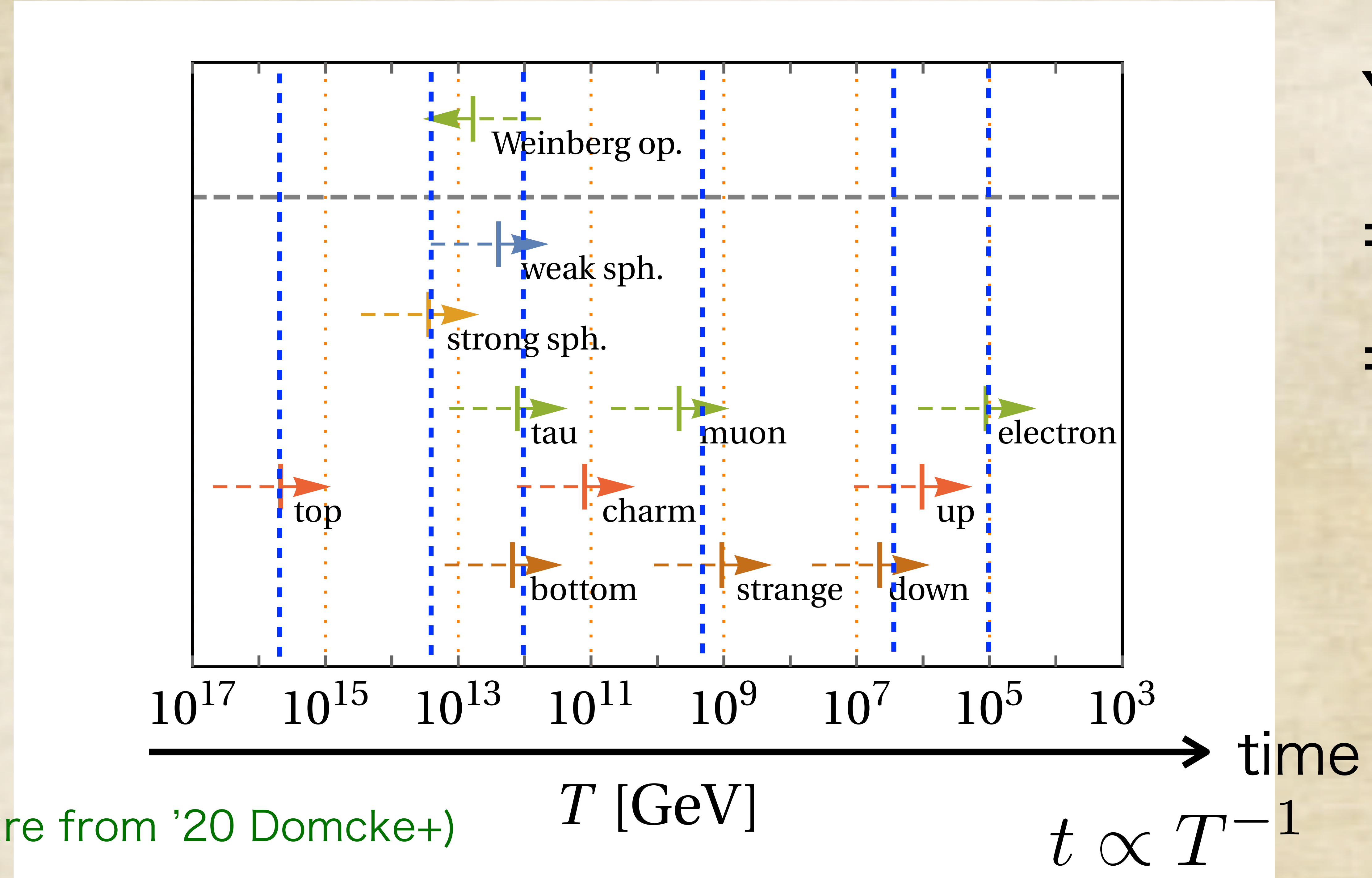
No, in the hot early Universe, we need to take into account the chiral asymmetry.



(Figure from '20 Domcke+)

Equilibrium temperature of Yukawa/sphalerons

Yukawa interaction is ineffective

= approximate conserved quantity

=> Chirality !

$$\mu_5^Y = \sum_i \epsilon_i c_i y_i^2 \mu_i$$

Another dynamical DOF for MHD.

In the presence of chirality,
we are interested in the chiral magnetic effect.

$$\boldsymbol{j} = \frac{2\alpha}{\pi}\mu_5 \boldsymbol{B}$$

MHD equations

The dynamical degrees of freedom:

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η, ν : resistivity/viscosity

MHD equations are extended to chiral MHD

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Anomaly eq.: $$\frac{D\mu_5}{Dt} = D_5 \boldsymbol{\nabla}^2 \mu_5 + \lambda\eta[\boldsymbol{B} \cdot (\boldsymbol{\nabla} \times \boldsymbol{B}) - C\mu_5 \boldsymbol{B}^2]$$

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$$C \sim \frac{g^2}{2\pi}, \quad \lambda \sim \frac{6C}{T^2}, \quad \left(n_5 \simeq \frac{\mu_5 T^2}{3}\right)$$

η, ν : resistivity/viscosity

Slide Background Courtesy: H. Oide

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$$C \sim \frac{g^2}{2\pi}, \quad \lambda \sim \frac{6C}{T^2}, \quad \left(n_5 \simeq \frac{\mu_5 T^2}{3}\right)$$

More non-trivial evolution is expected.

η, ν : resistivity/viscosity

Application of chiral MHD in the early Universe

Chiral plasma instability in the early Universe

Chiral plasma instability

Maxwell's equation in the momentum space:

$$\frac{d\boldsymbol{B}_k^{\pm}}{dt} = \eta\left(-k^2\boldsymbol{B}_k^{\pm} \quad \pm C\mu_5 k\boldsymbol{B}_k^{\pm} \quad \right) + \left(\boldsymbol{\nabla}\times(\boldsymbol{v}\times\boldsymbol{B}^{\pm})\right)_k$$

Chiral plasma instability

Maxwell's equation in the momentum space:

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=> one helicity mode feels instability

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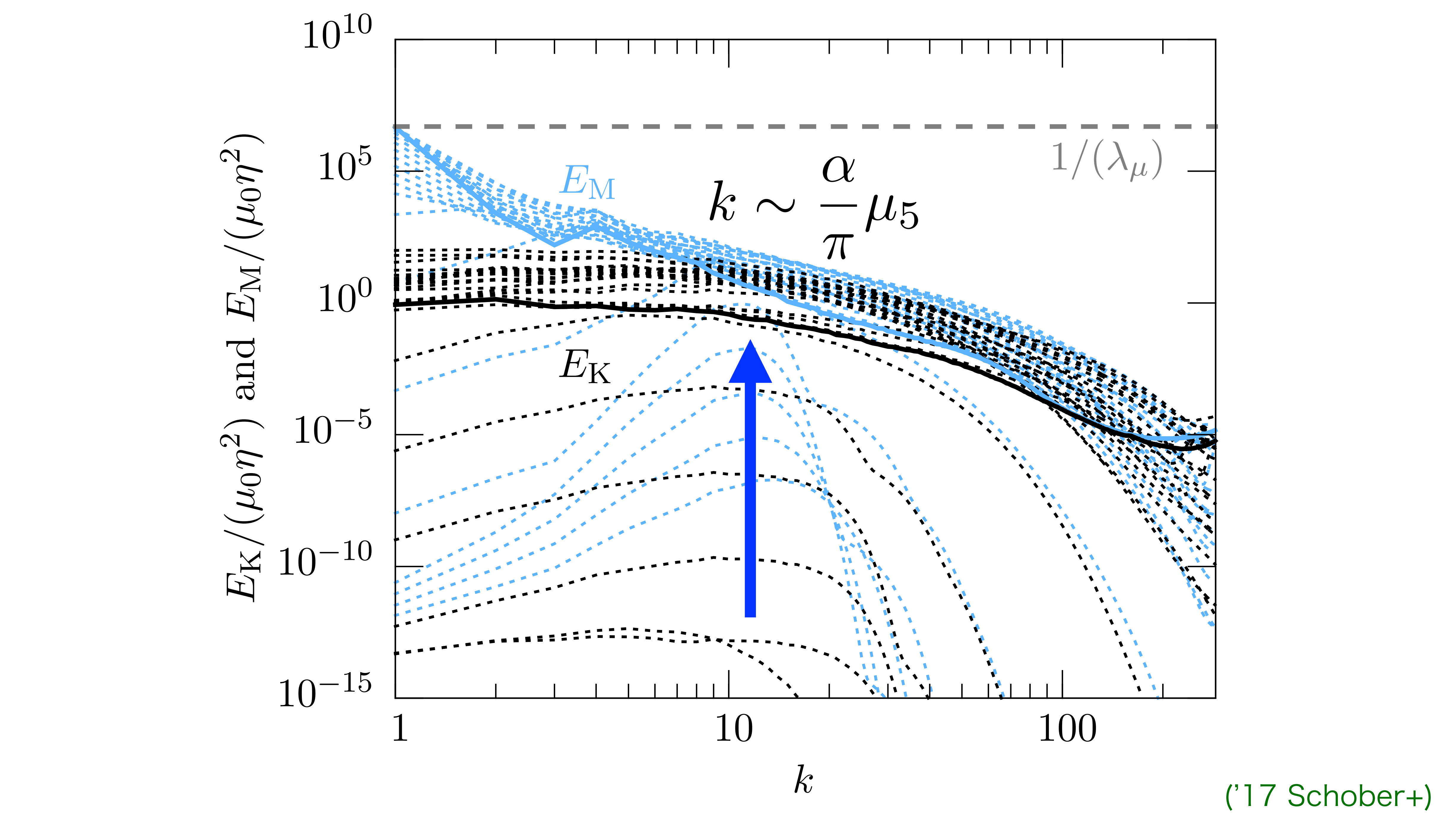
Note: total helicity is conserved $\partial_\mu j_5^\mu = -\frac{q^2 g'^2}{32\pi^2} Y_{\mu\nu}\tilde{Y}^{\mu\nu} \Rightarrow \partial_t\left(Q_5 + \frac{q^2}{16\pi^2}\mathcal{H}\right) = 0$

Numerical MHD results



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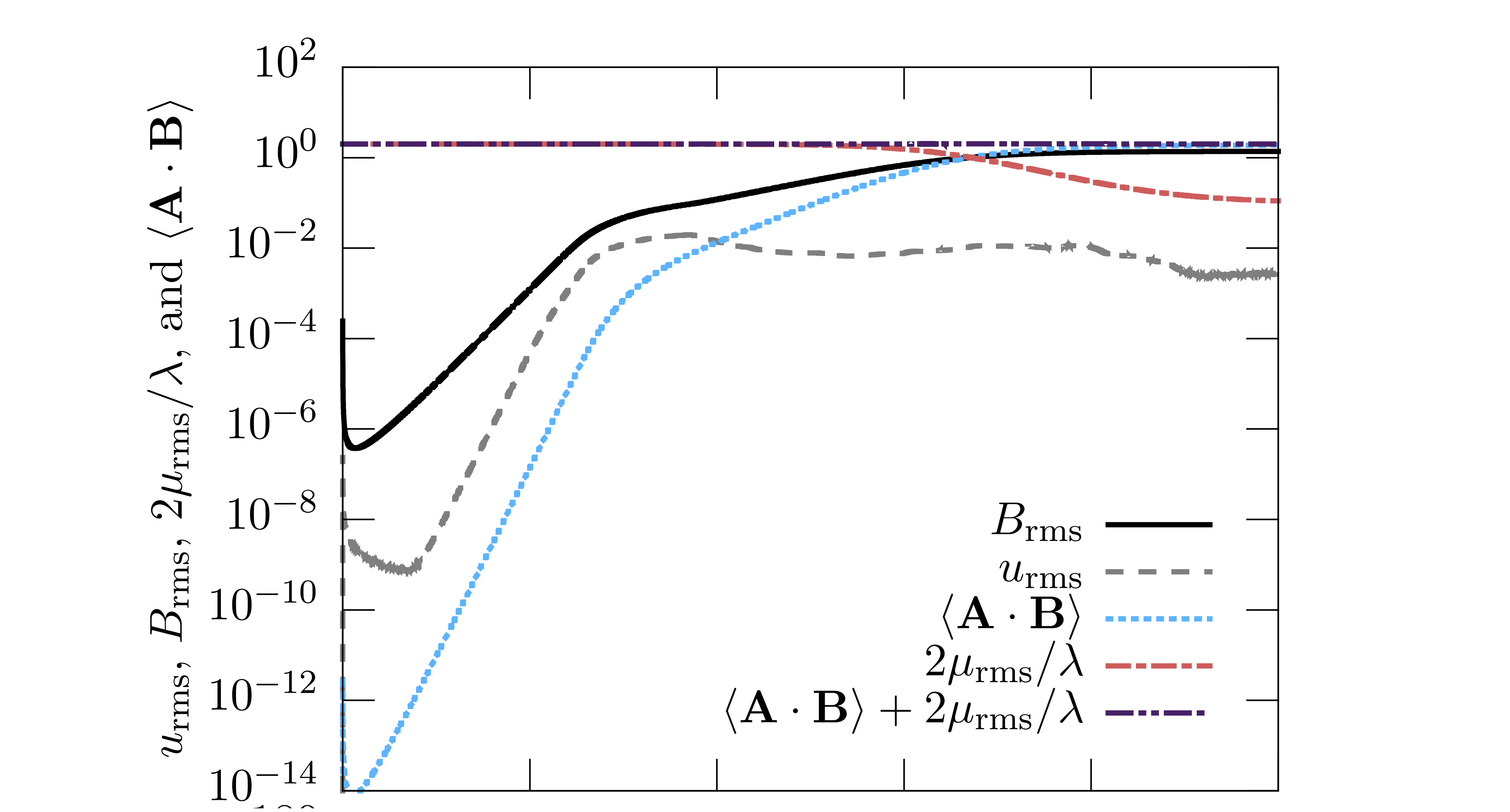

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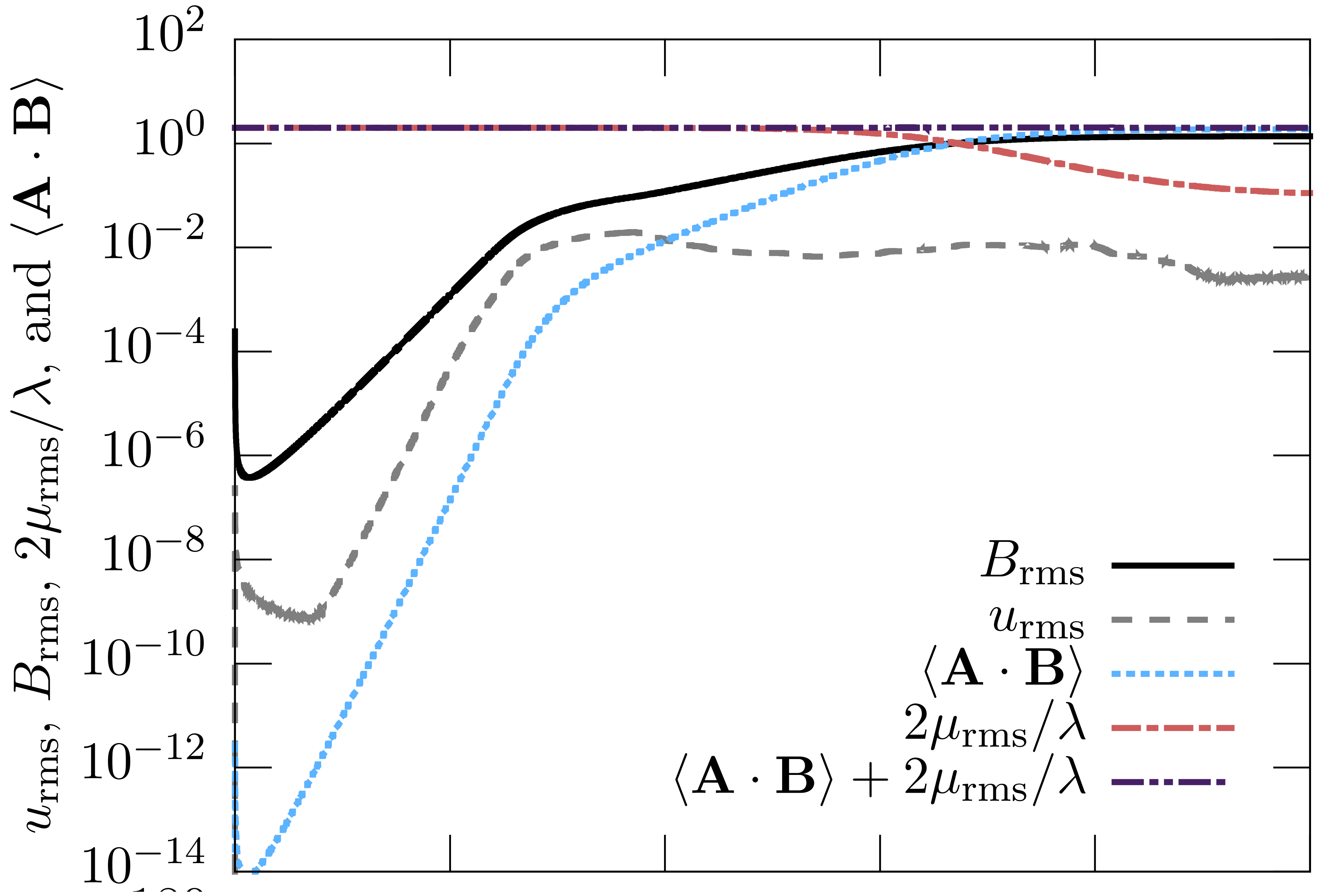

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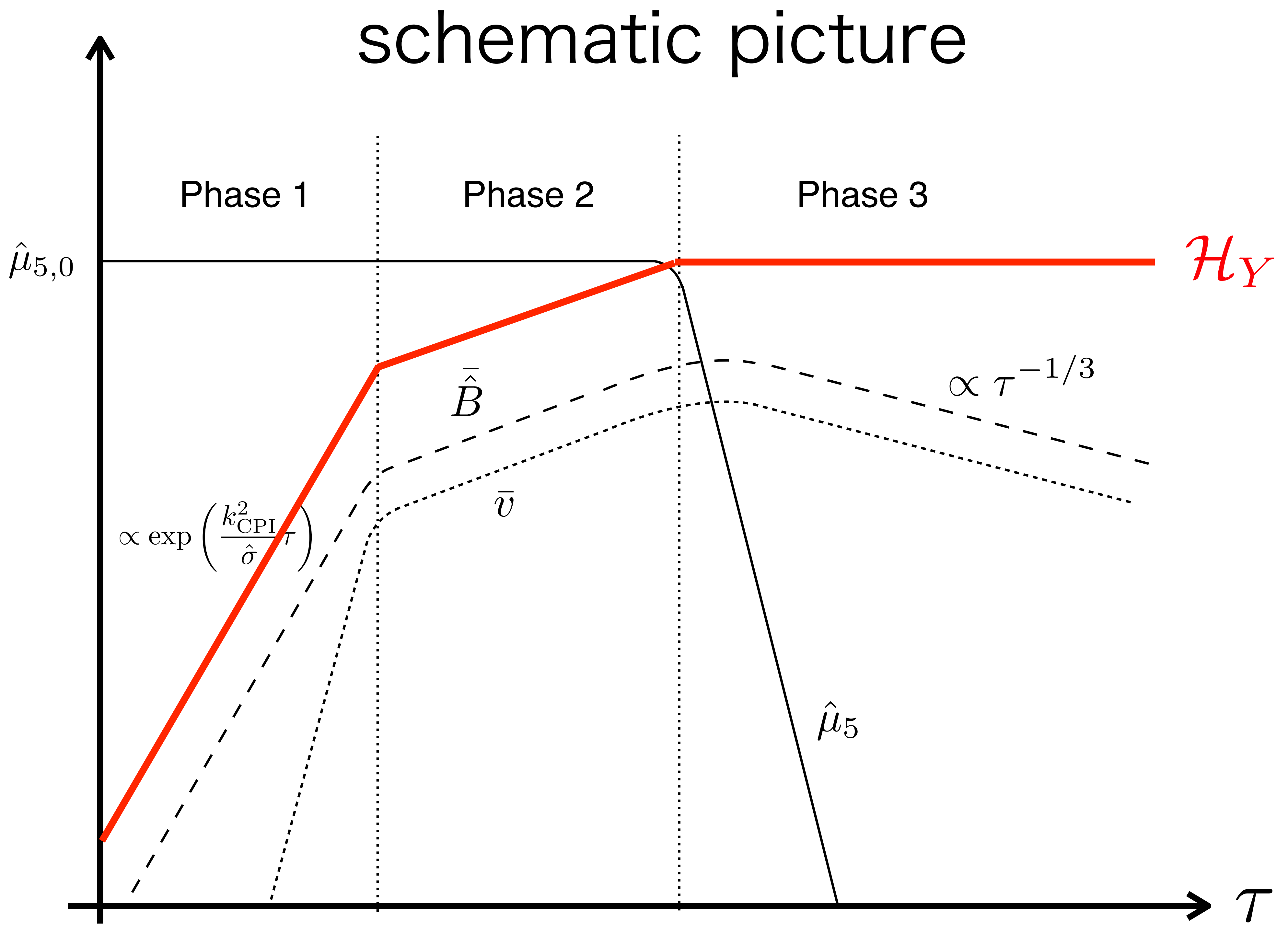

('23 Kamada, Yamamoto, Yang)

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- An asymmetry (does not have to baryon) generation mechanism leads to CPI

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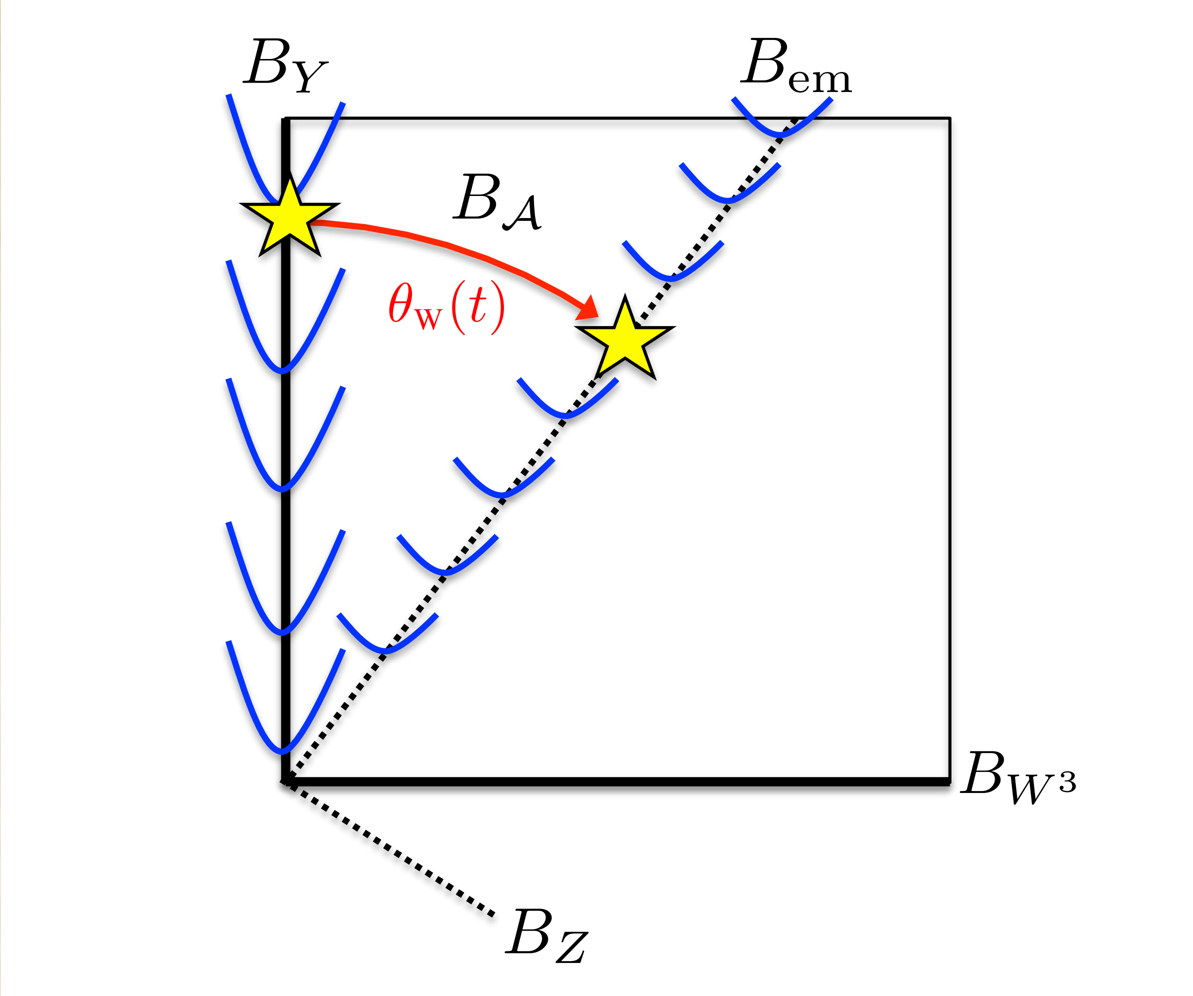

Gauge group

$$SU(2)_W \times U(1)_Y \to U(1)_{\rm em}$$

Large-scale (massless) MFs

$$B_Y \to B_{\rm em} = \cos\theta_W B_Y + \sin\theta_w B_{W^3}$$

Magnetic helicity

$$H_Y^{\rm before} \to H_{\rm em}^{\rm after} = H_Y^{\rm before}$$

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BAU: $\Delta H_Y = -\sin^2\theta_W H_Y^{\rm before}$, $\Delta N_{\rm CS} \sim \sin^2\theta_W H_Y^{\rm before}$ ➡ $\Delta Q_B = \#\Delta N_{\rm CS} - \#\Delta H_Y \sim \sin^2\theta_W H_Y^{\rm before}$

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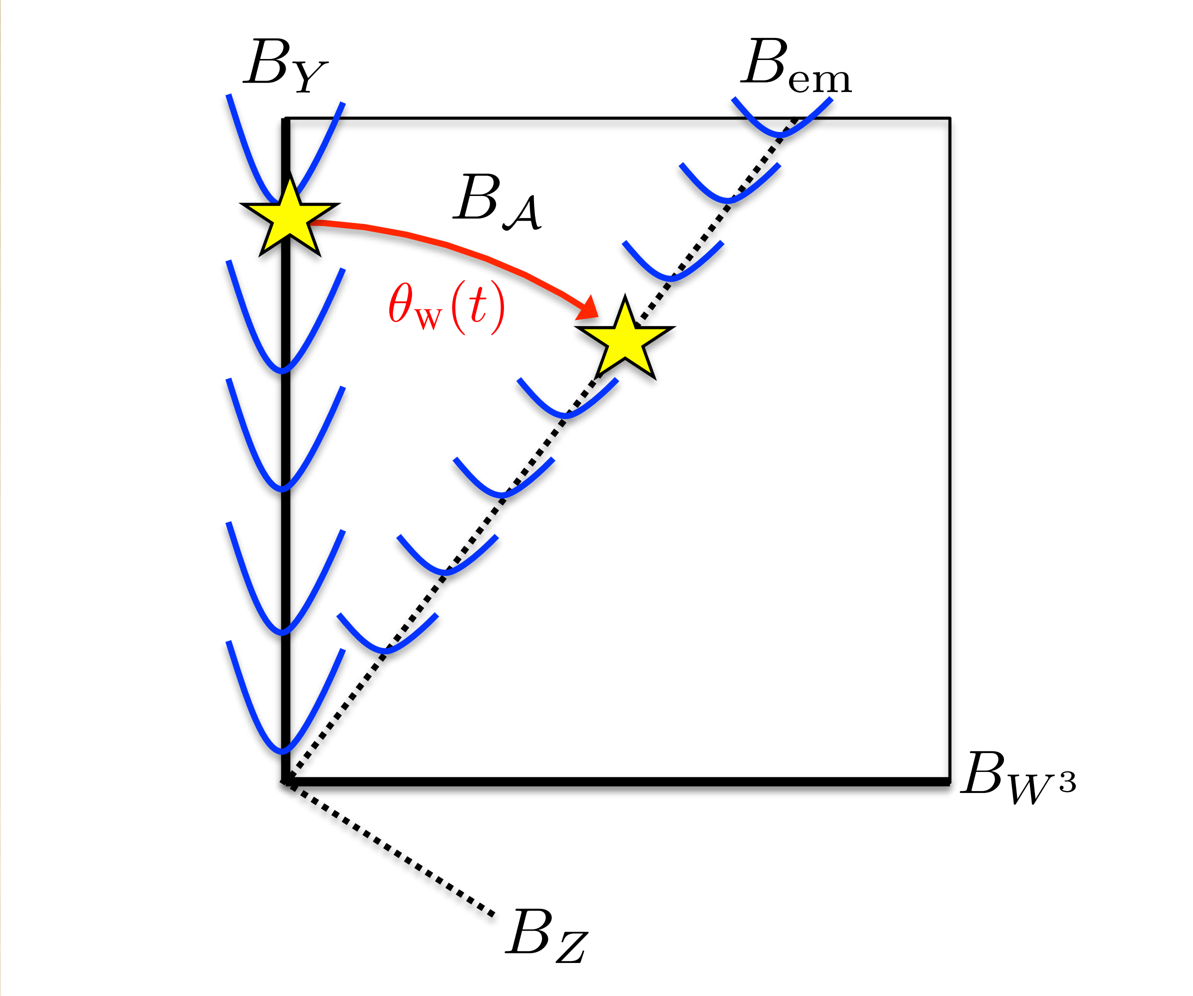

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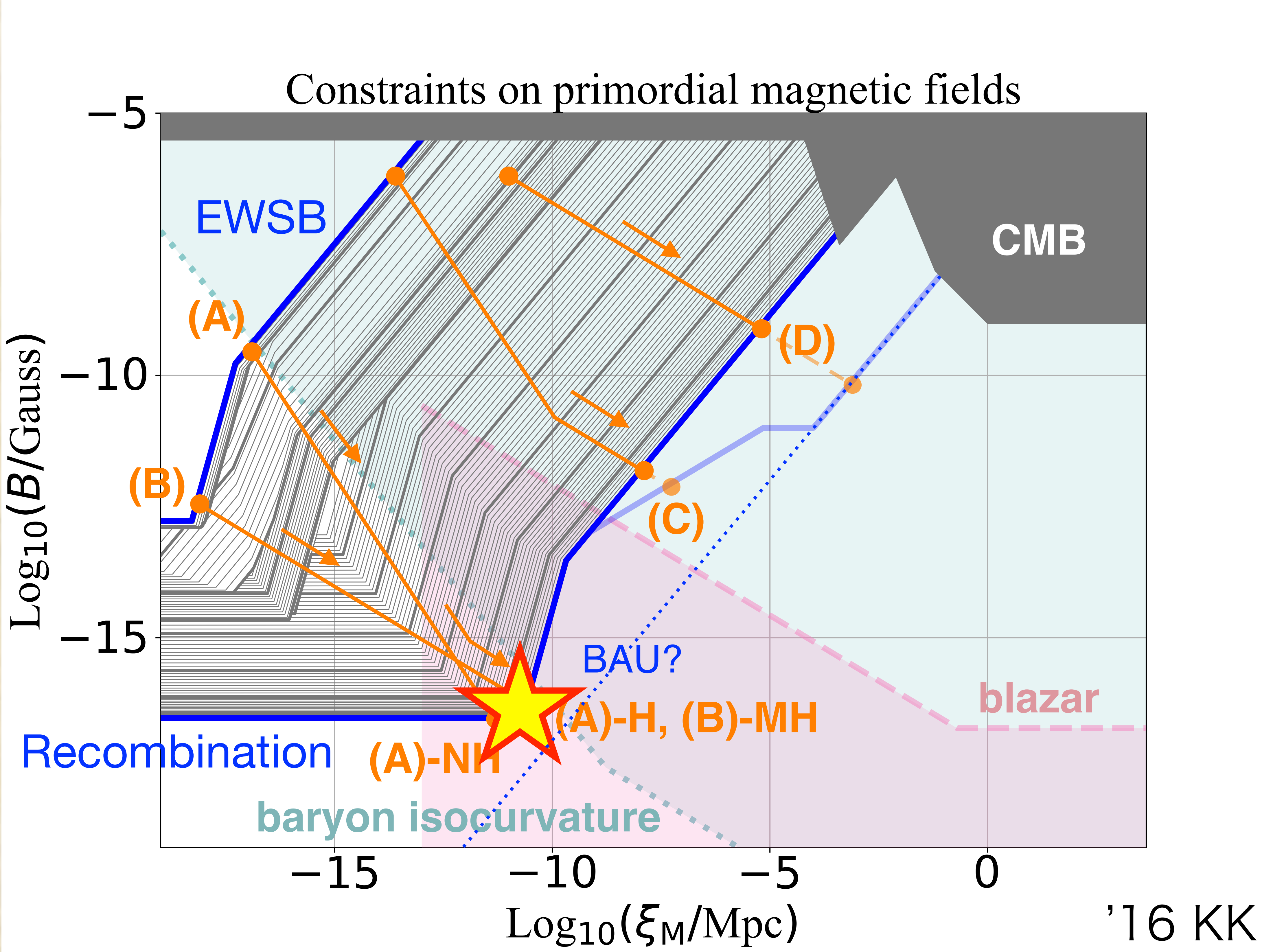
- A large lepton flavor asymmetry, $\frac{\mu_{\Delta_{\rm f}}}{T} \gtrsim 4\times 10^{-3}$ (thought to be harmless),
  is ruled out otherwise we suffer from baryon overproduction. ('23 Domcke, KK+)

Slide Background Courtesy: H. Oide

Intergalactic MFs cannot be explained by primordial MFs before EWSB.



’16 KK & Long, ’24 Uchida, KK+

Chiral MHD with zero total chirality

What happens if we start from a balanced initial condition?

$$Q_5 + \frac{\alpha}{4\pi}\mathcal{H} = 0$$

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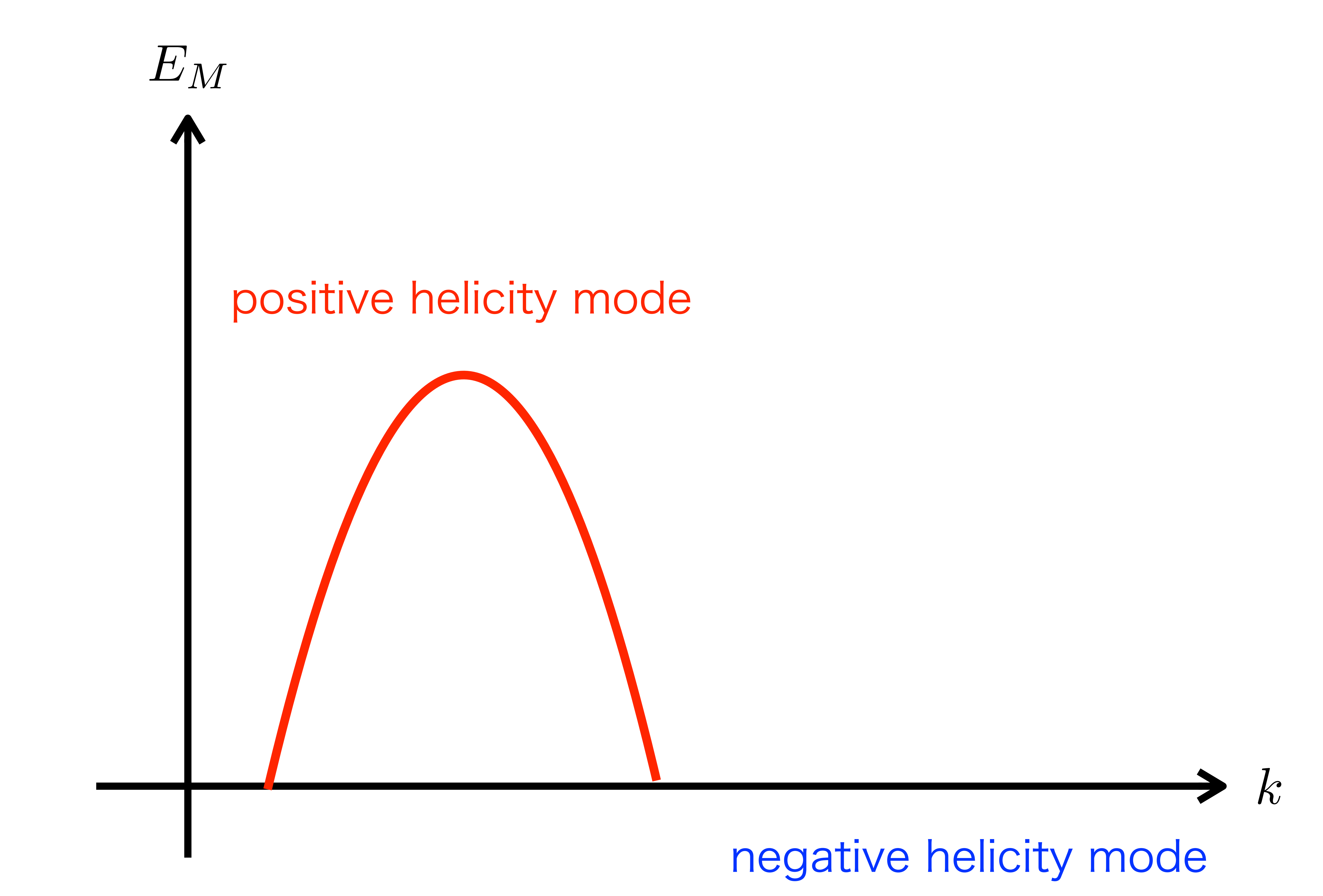

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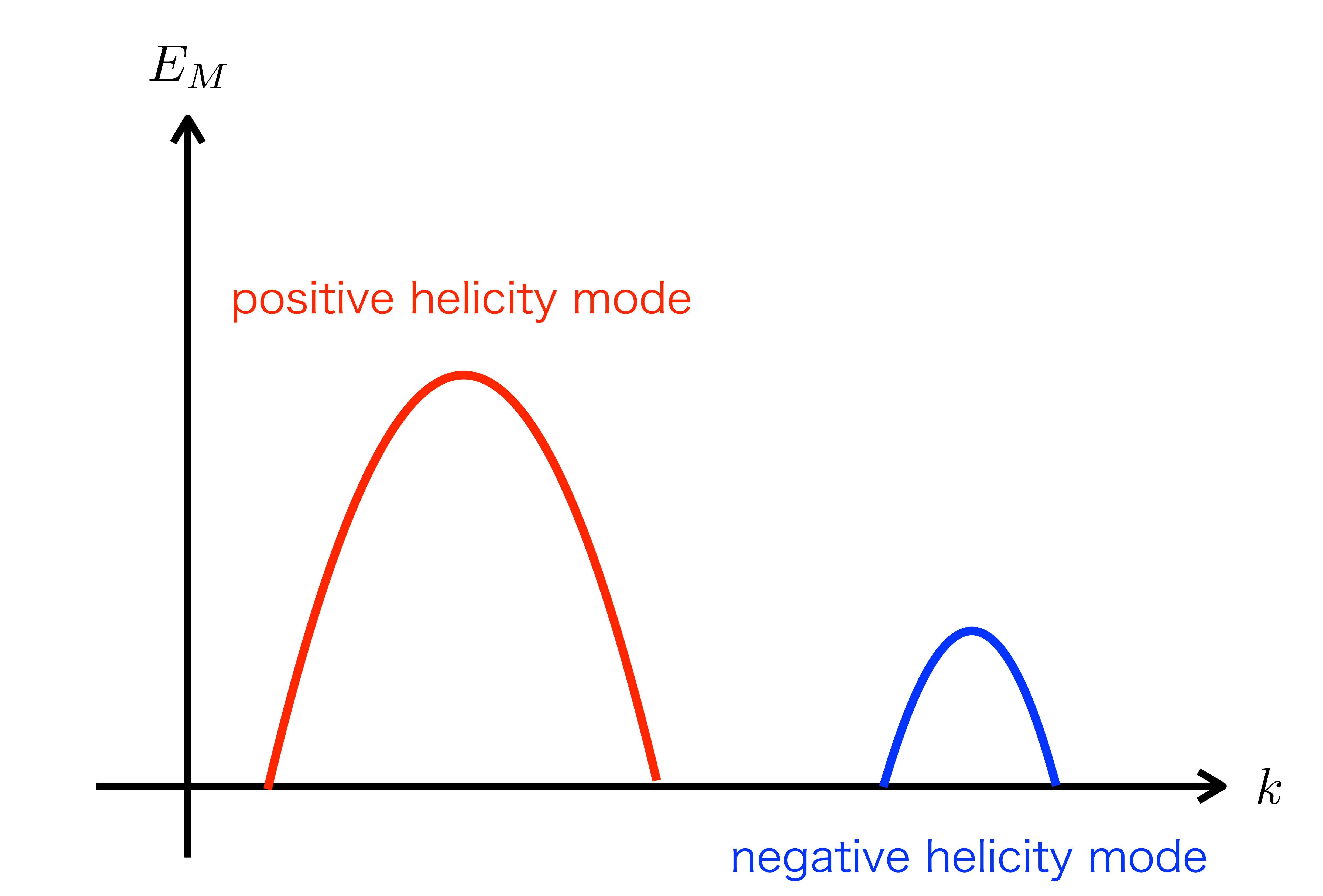

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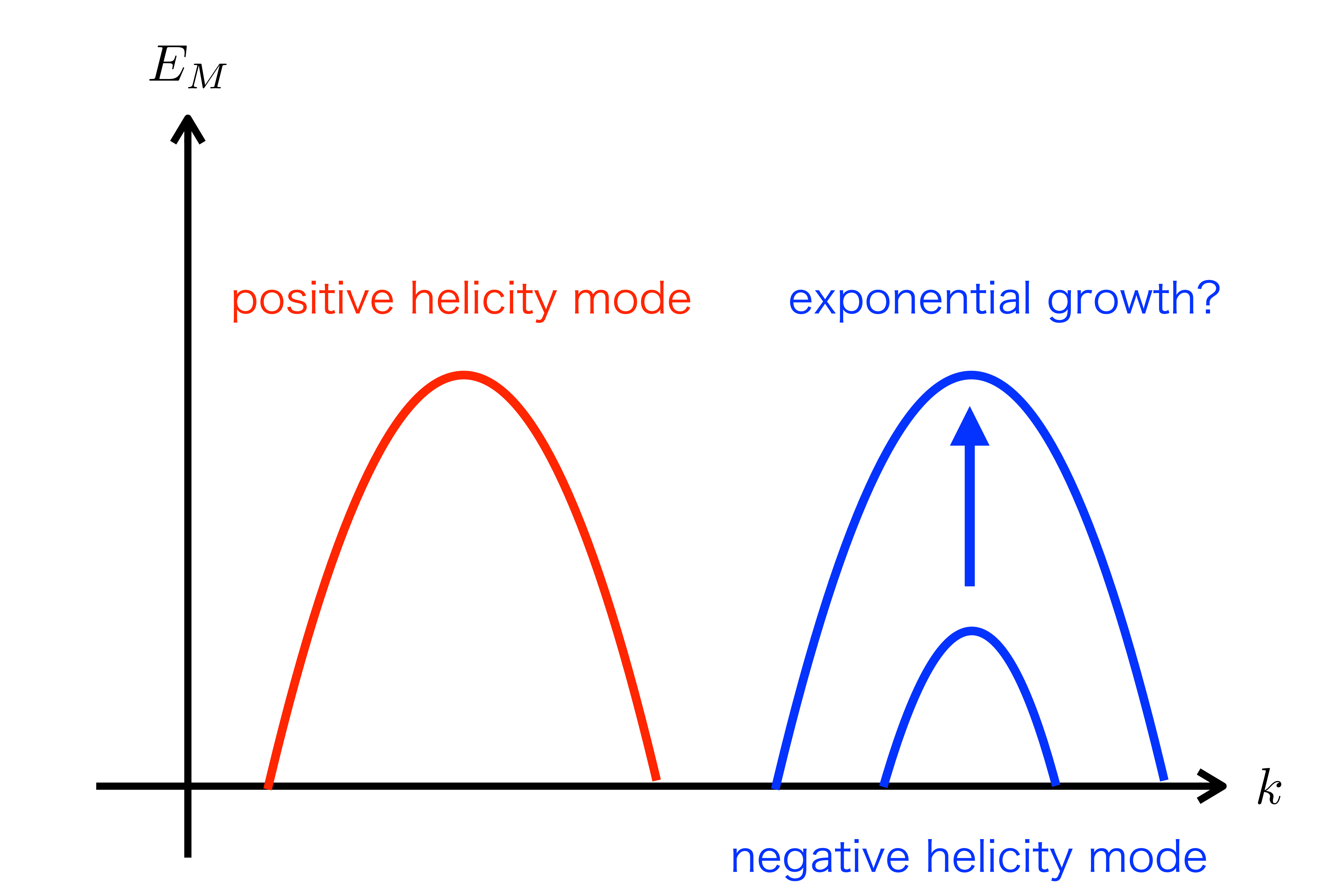

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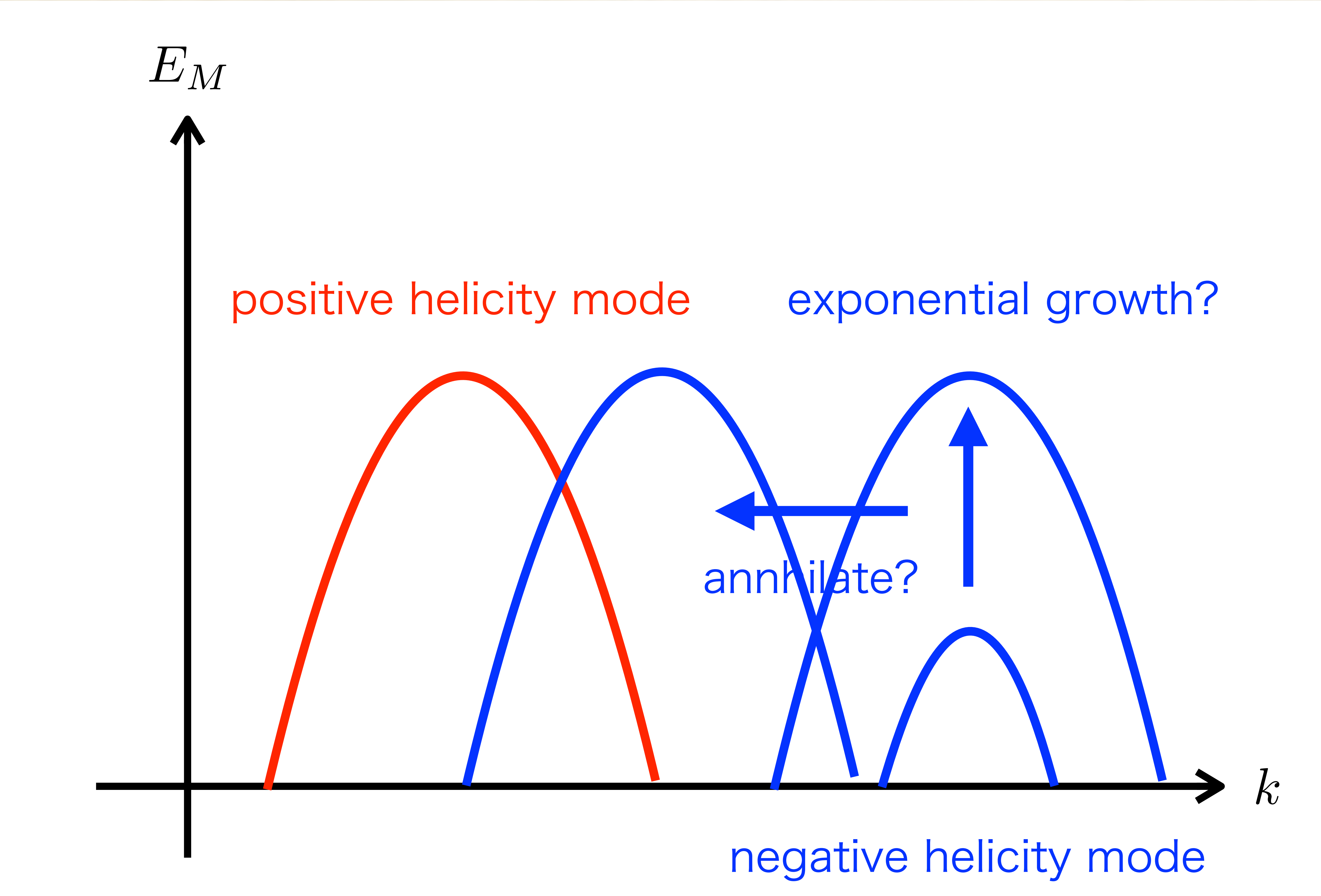

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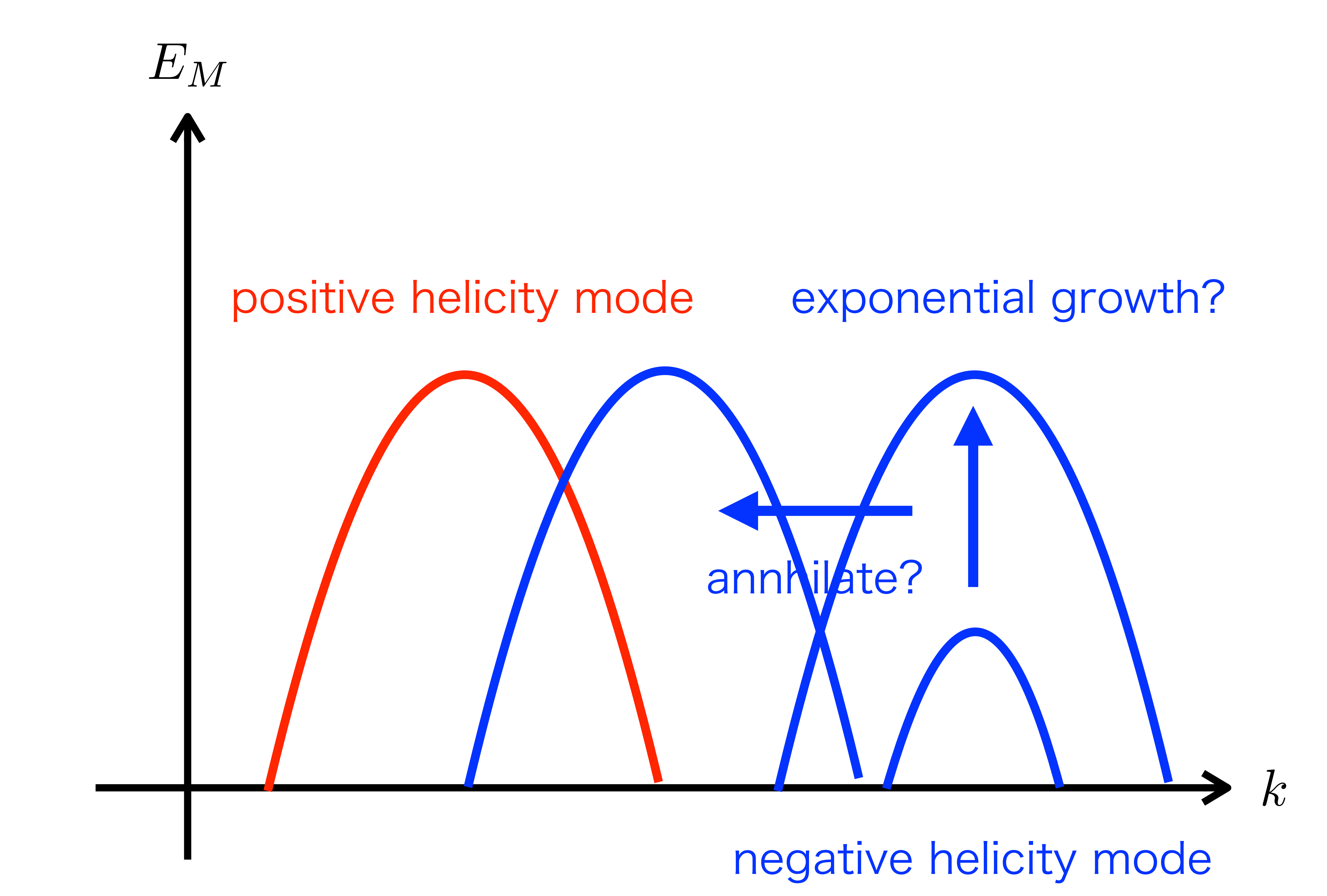

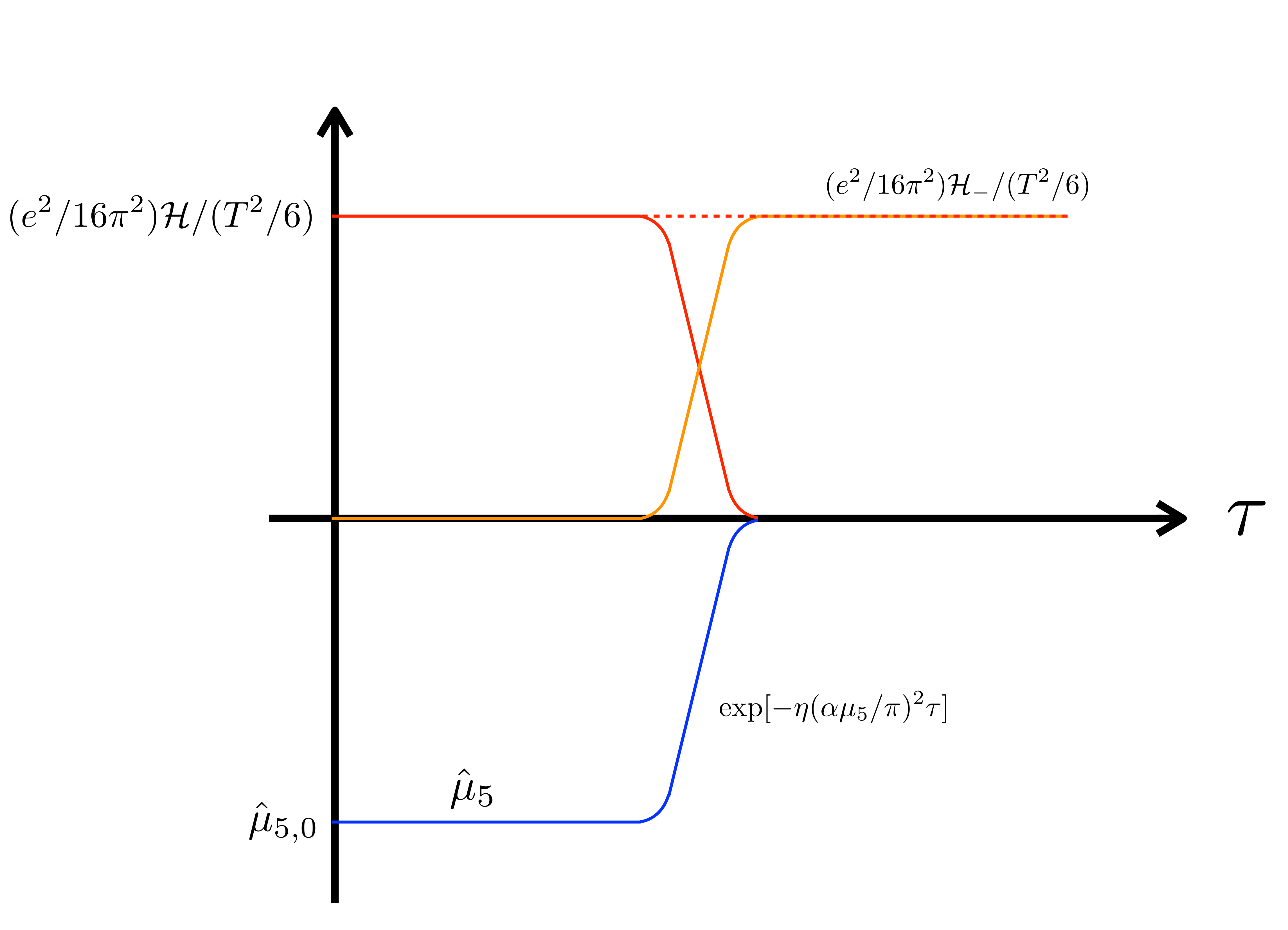

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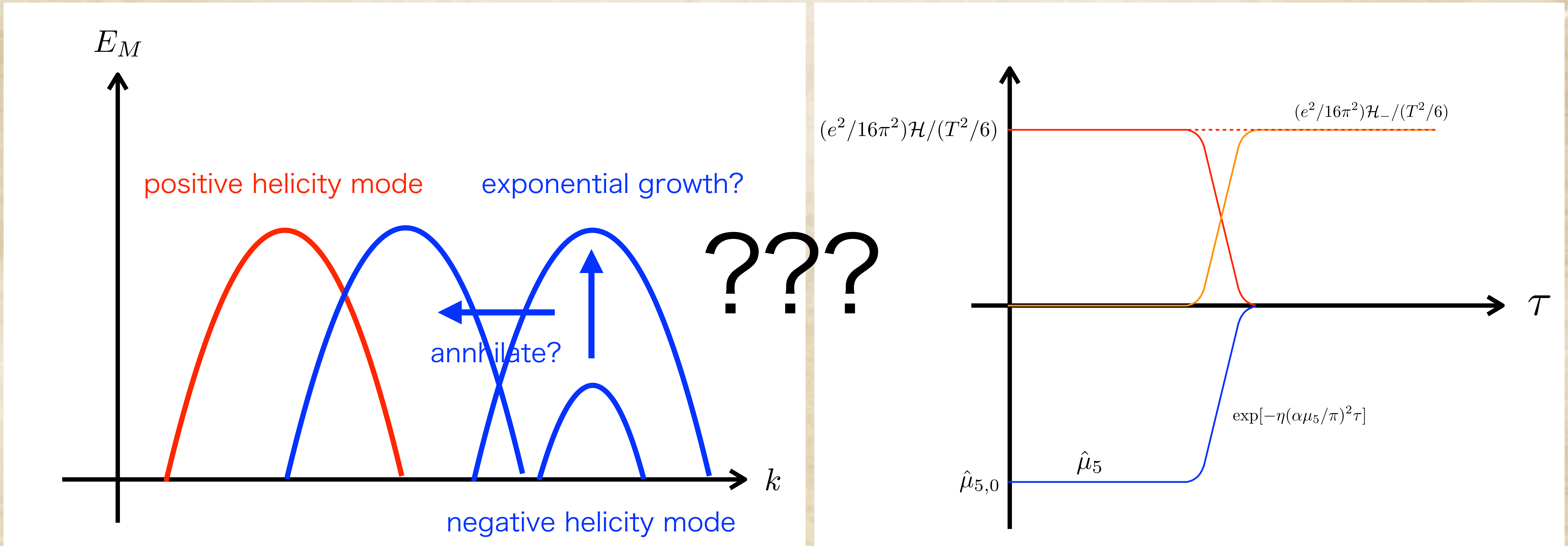
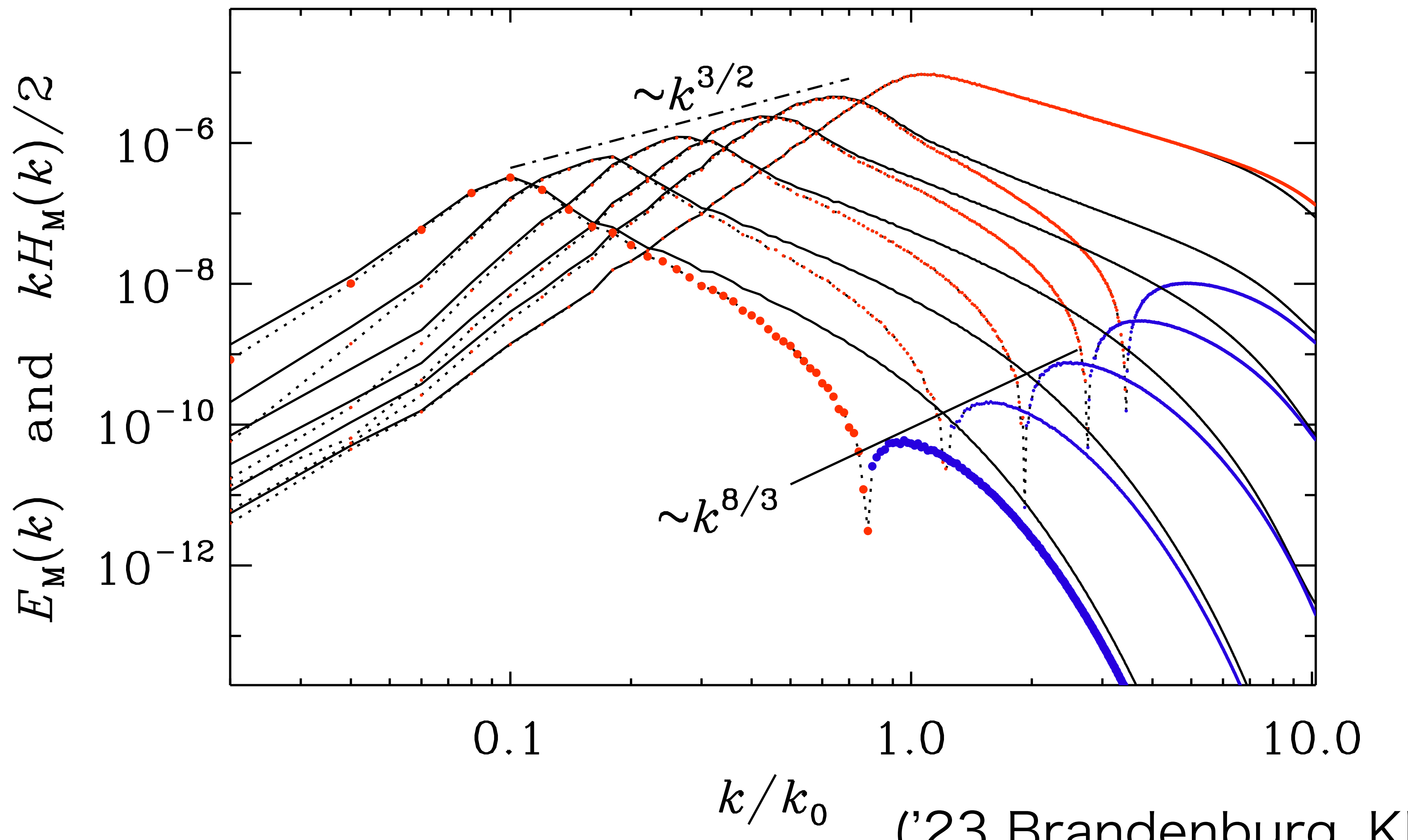
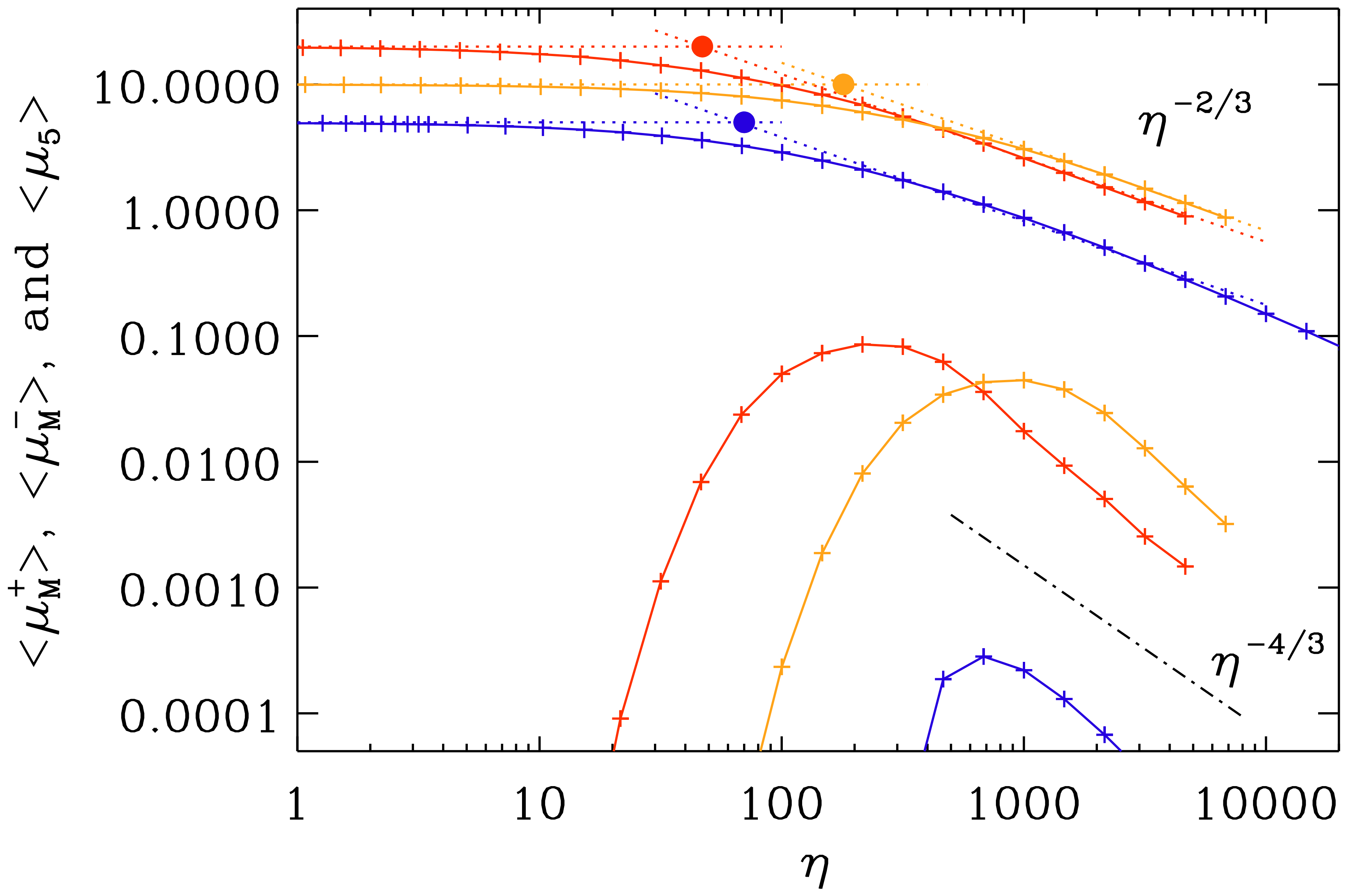


The result turned out to be…



('23 Brandenburg, KK+)



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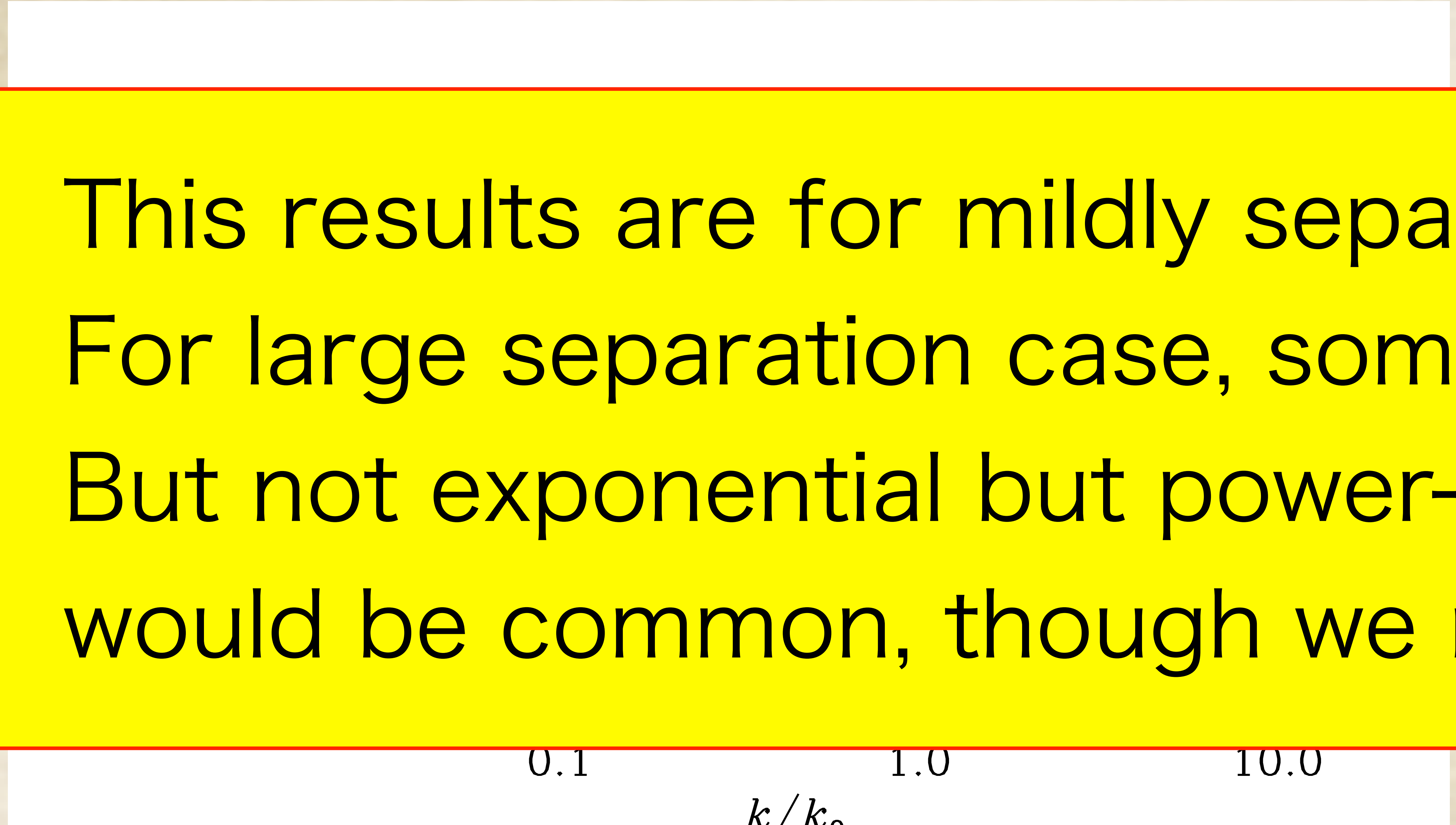

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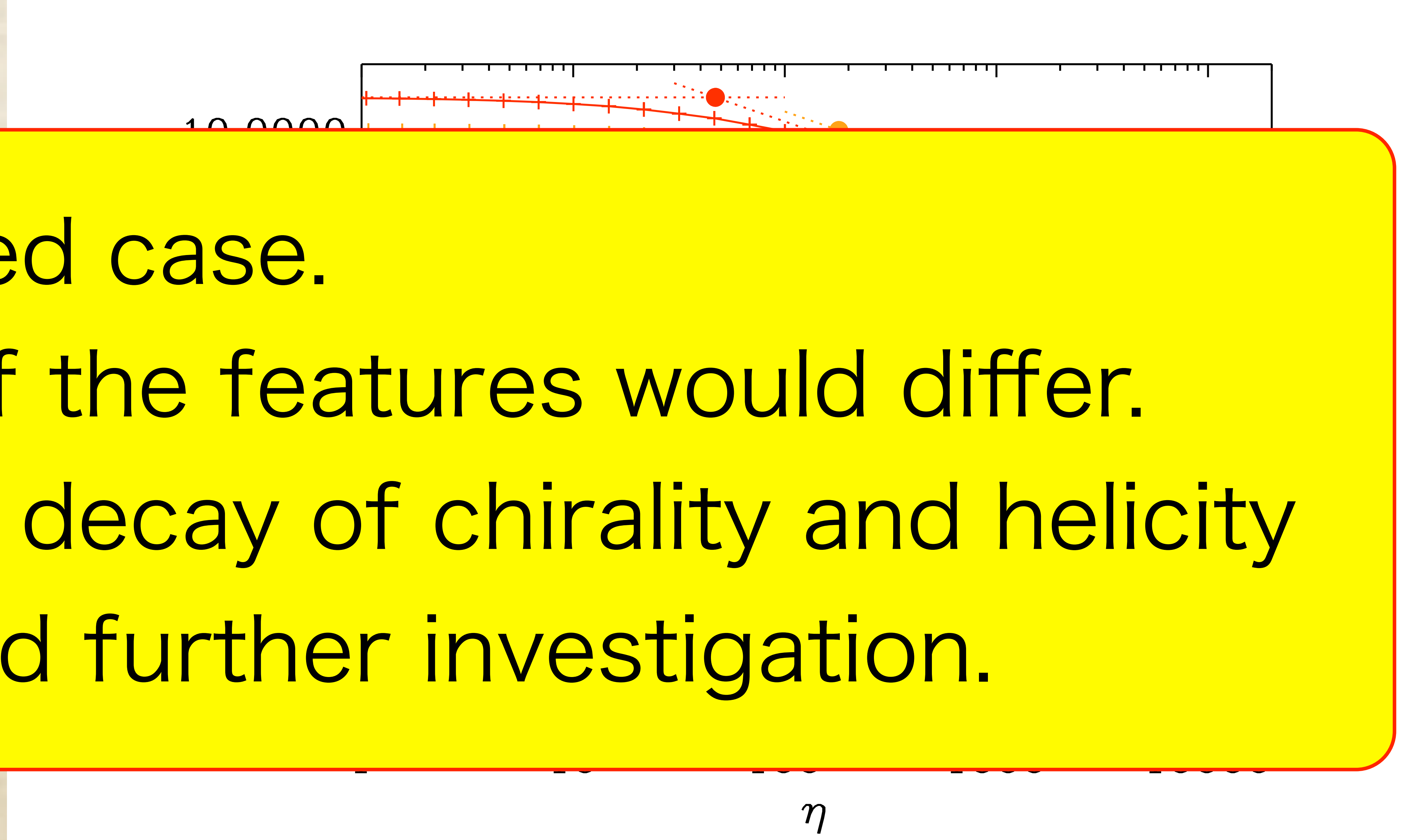

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This results are for mildly separated case.
For large separation case, some of the features would differ.
But not exponential but power-law decay of chirality and helicity would be common, though we need further investigation.

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Cosmologically interesting consequences?

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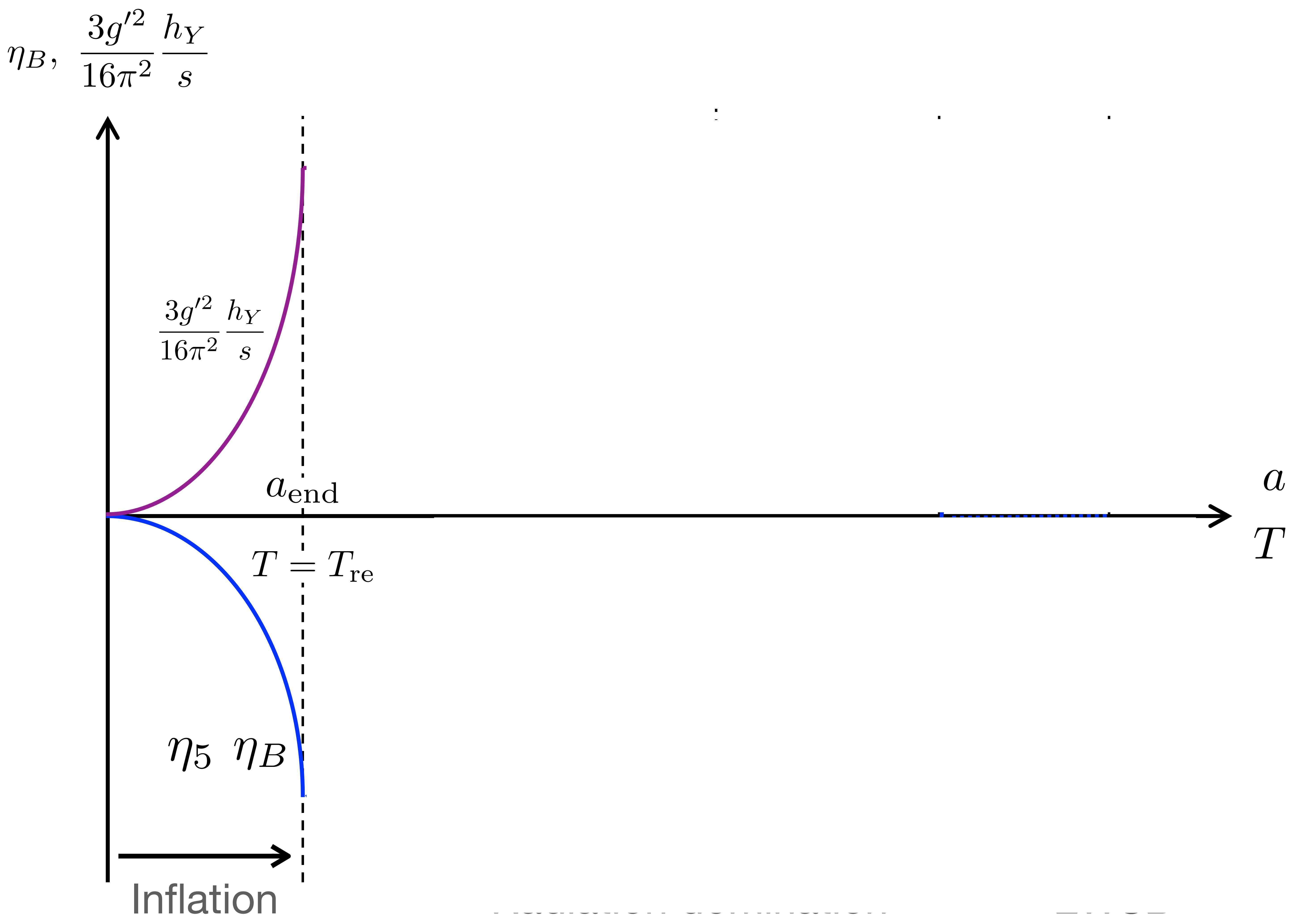
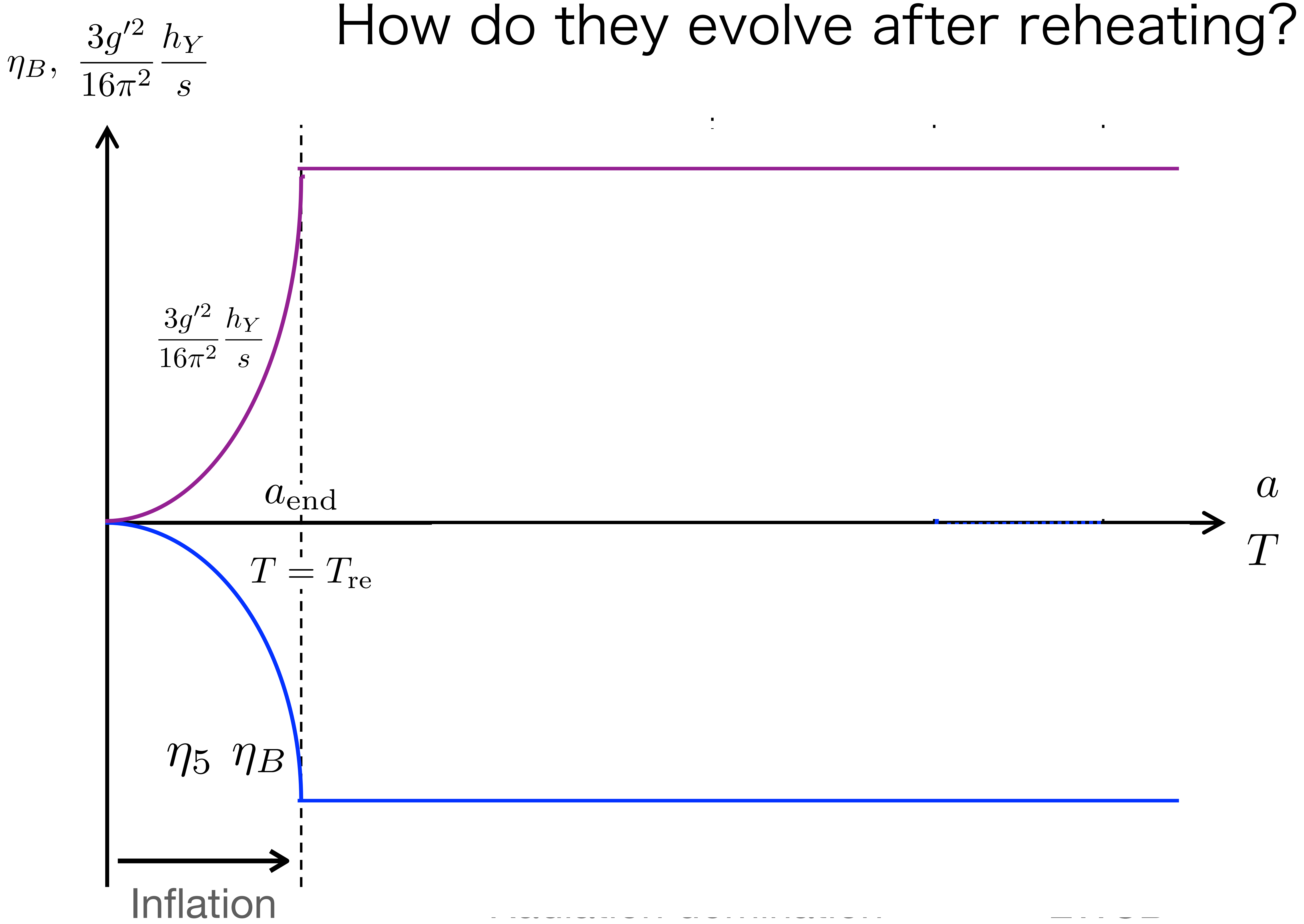
Dynamics after axion inflation.



At reheating, electric fields are screened while magnetic fields remain, keeping the total (hyper)magnetic helicity.

How do they evolve after reheating?



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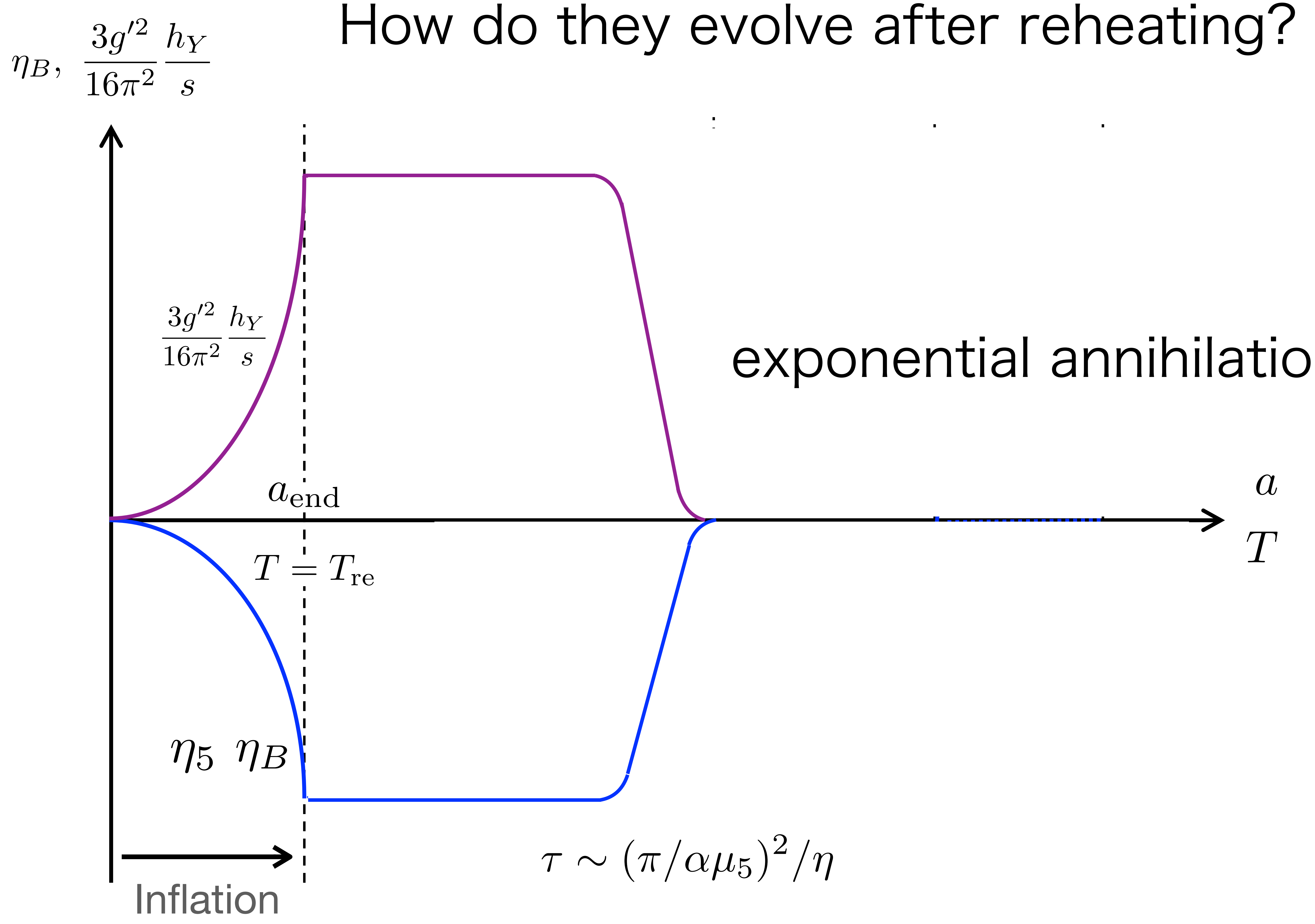

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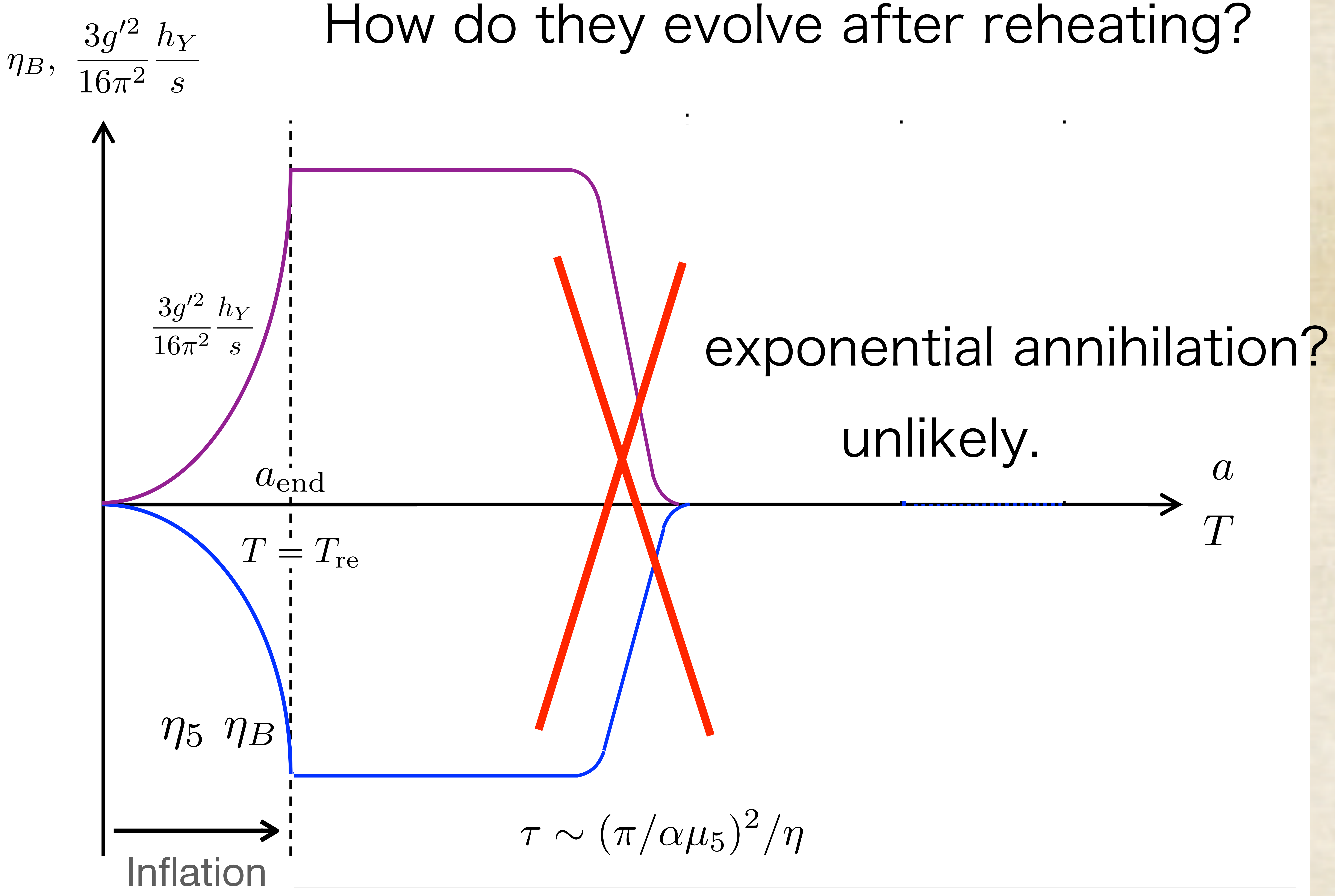

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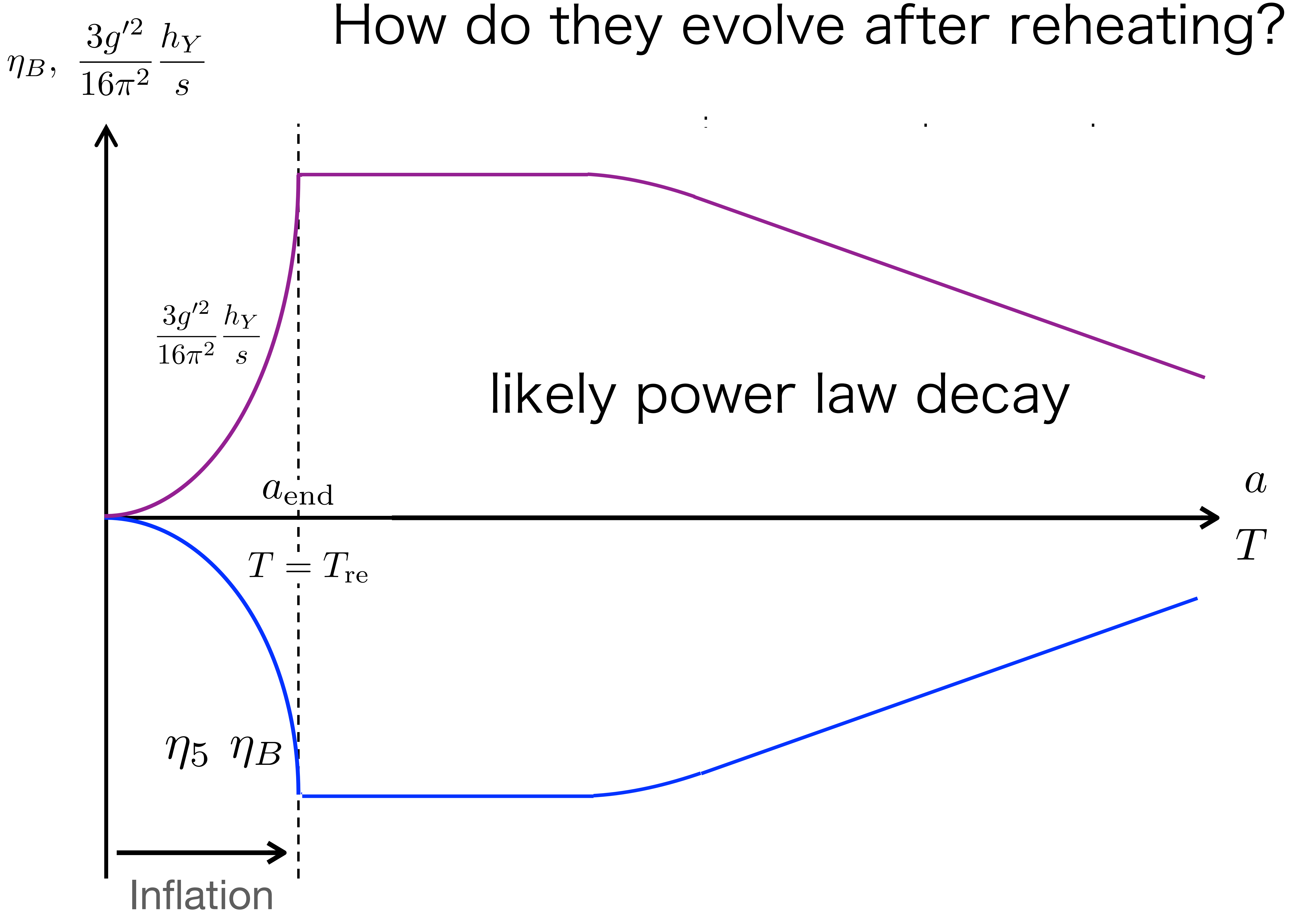
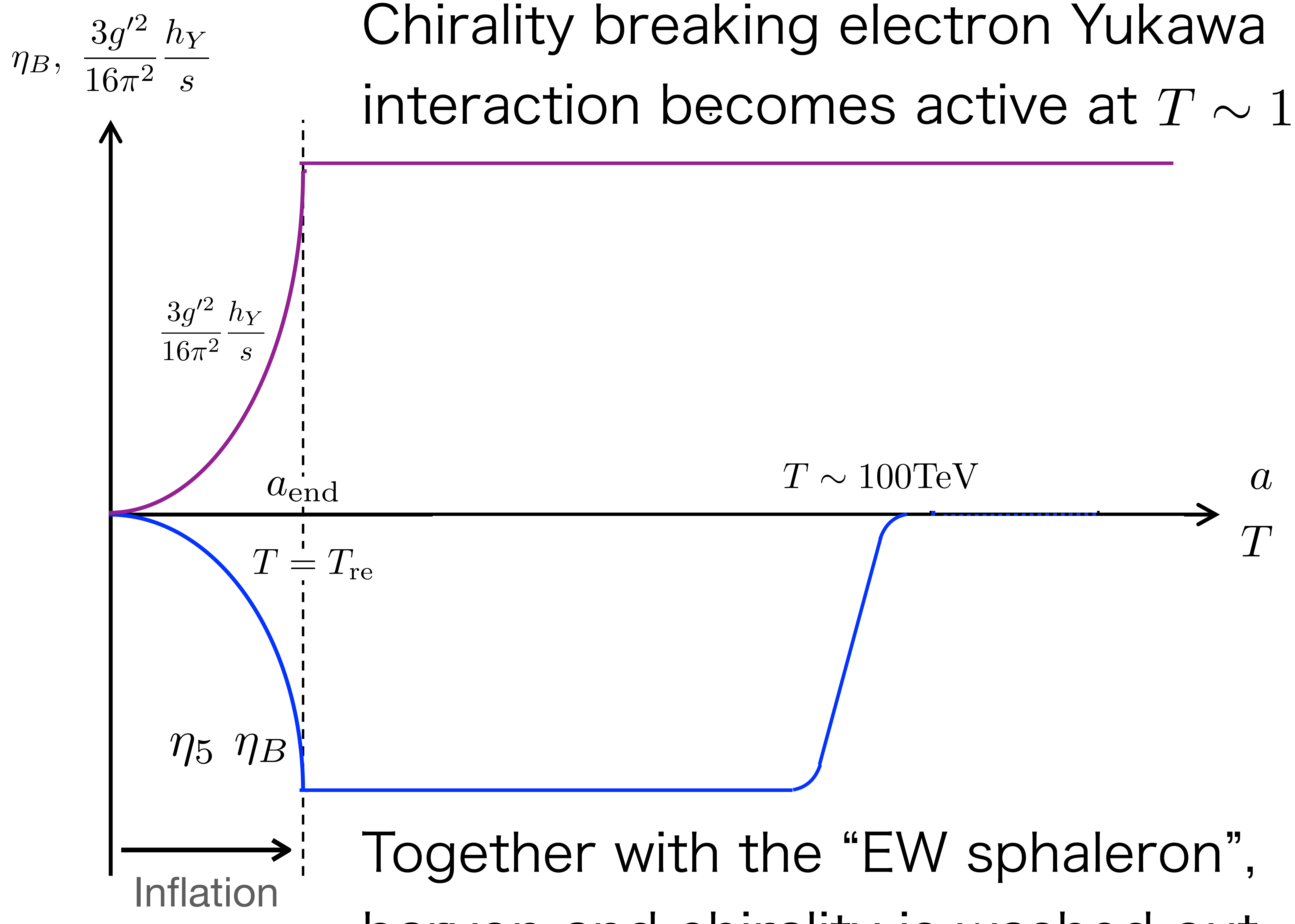


Chirality breaking electron Yukawa interaction becomes active at $T \sim 100\mathrm{TeV}$ ('92 Campbell+)



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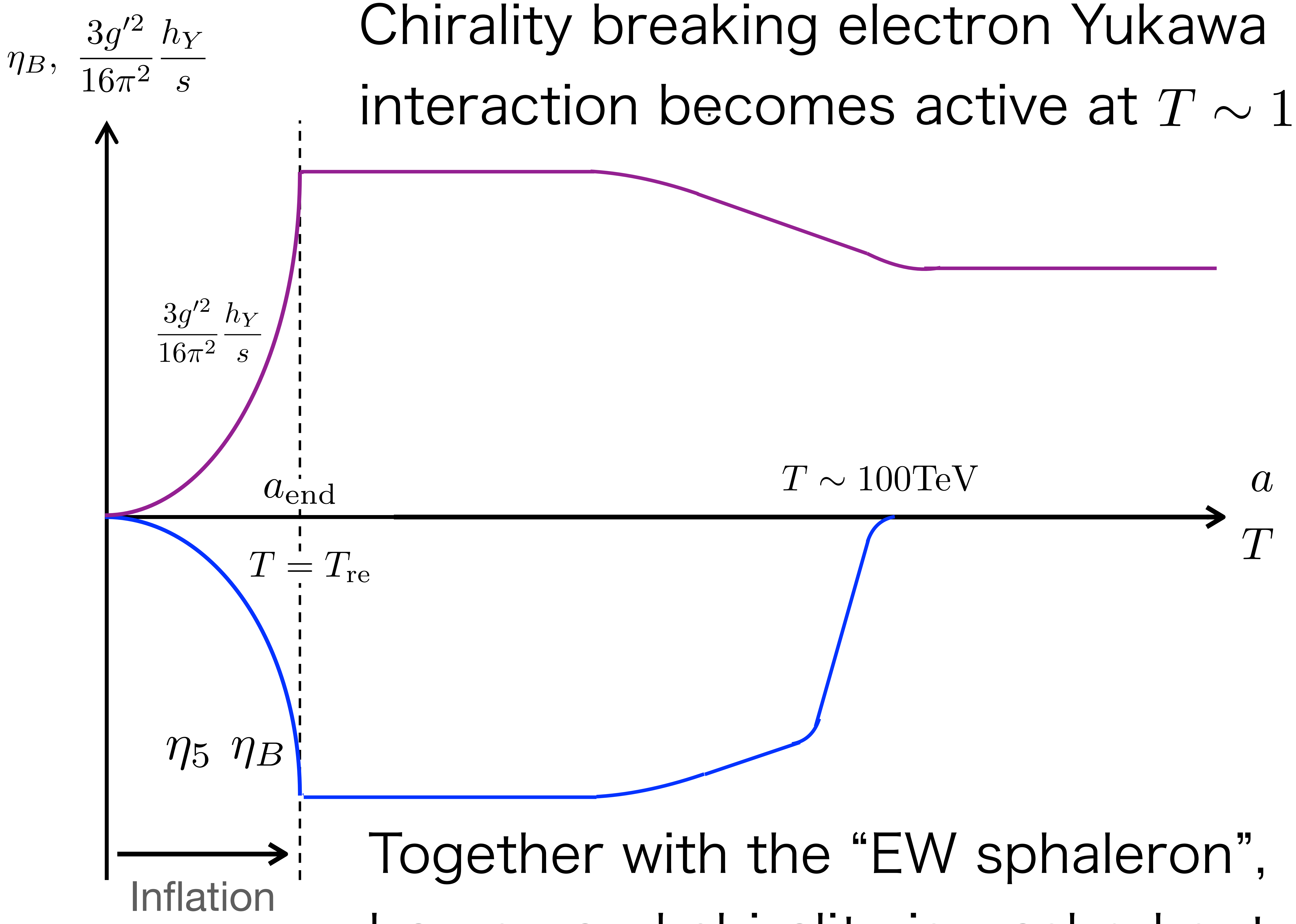

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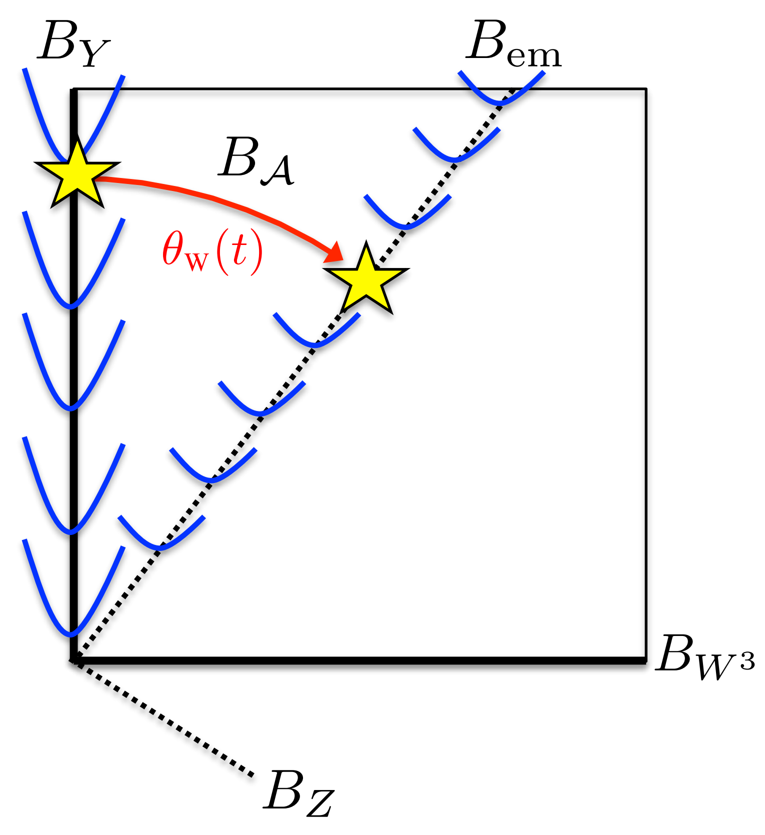
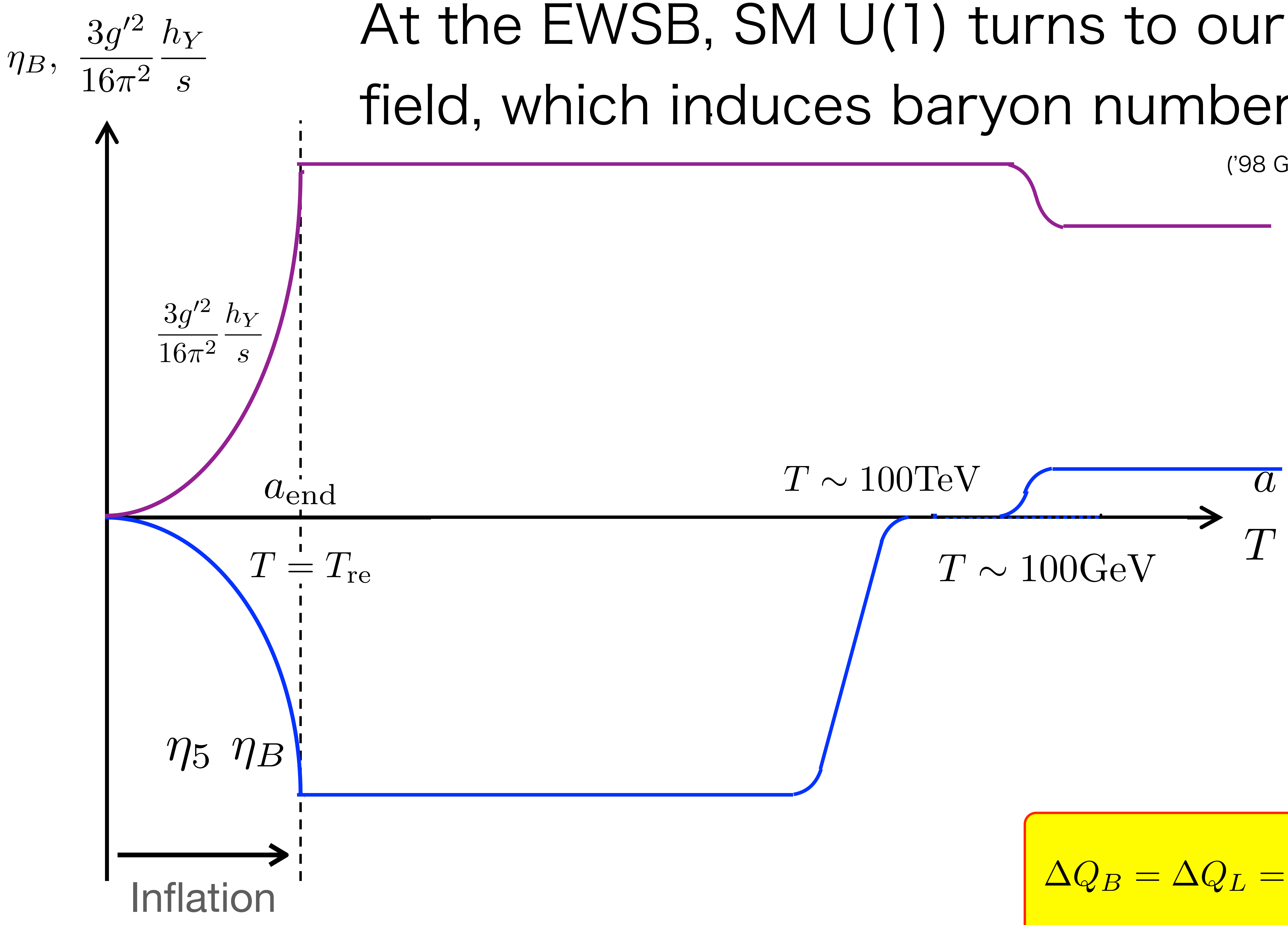
At the EWSB, SM U(1) turns to our electromagnetic field, which induces baryon number.

('98 Giovannini&Shaposhnikov, '16 KK & Long)





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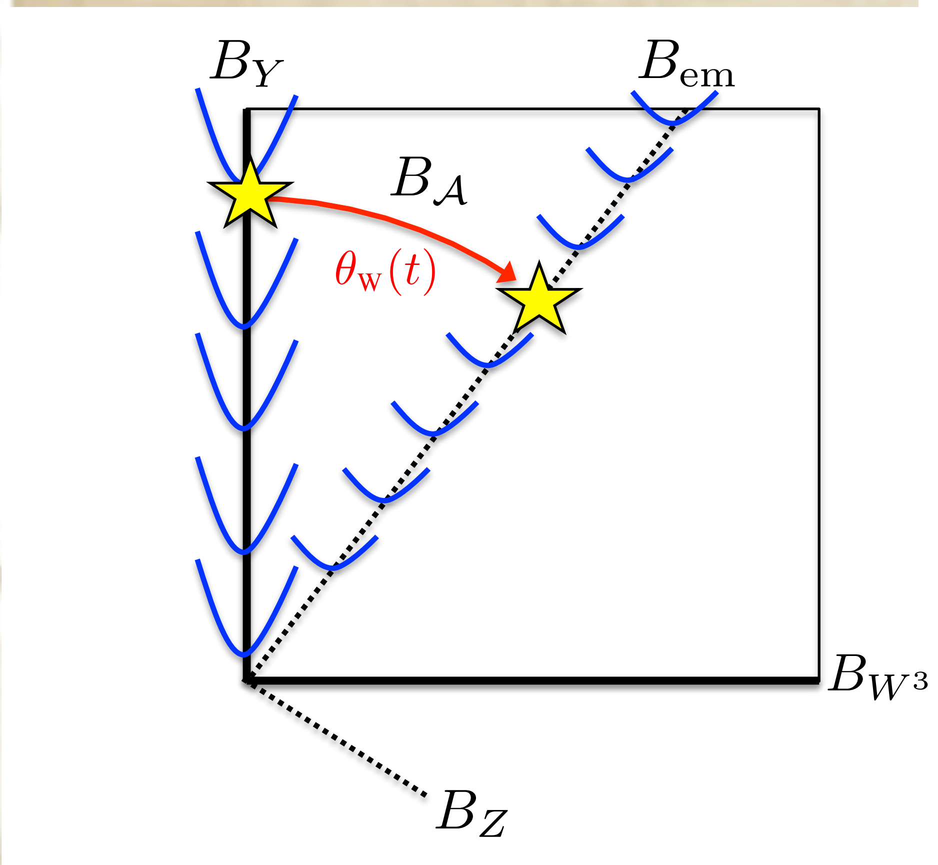

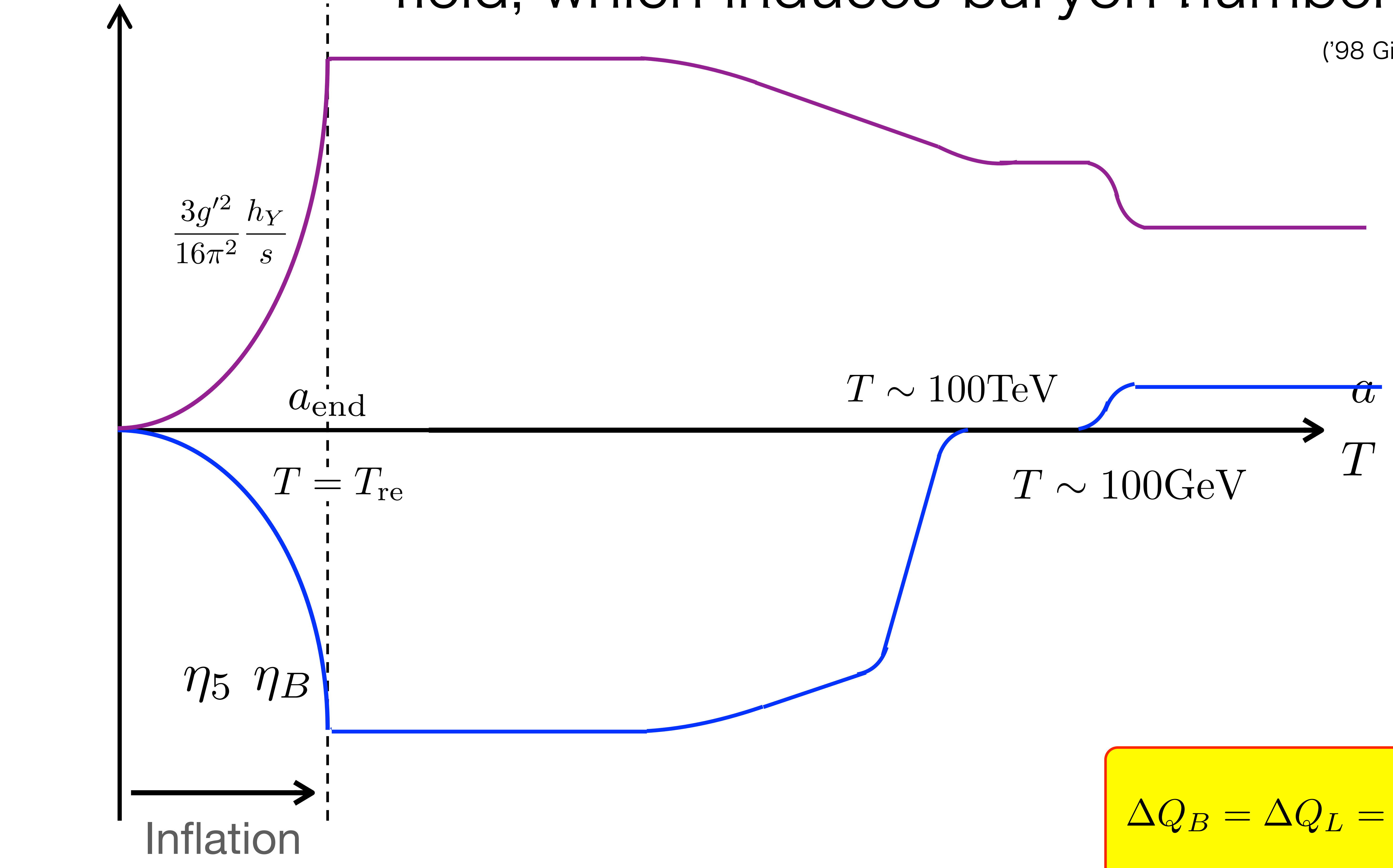

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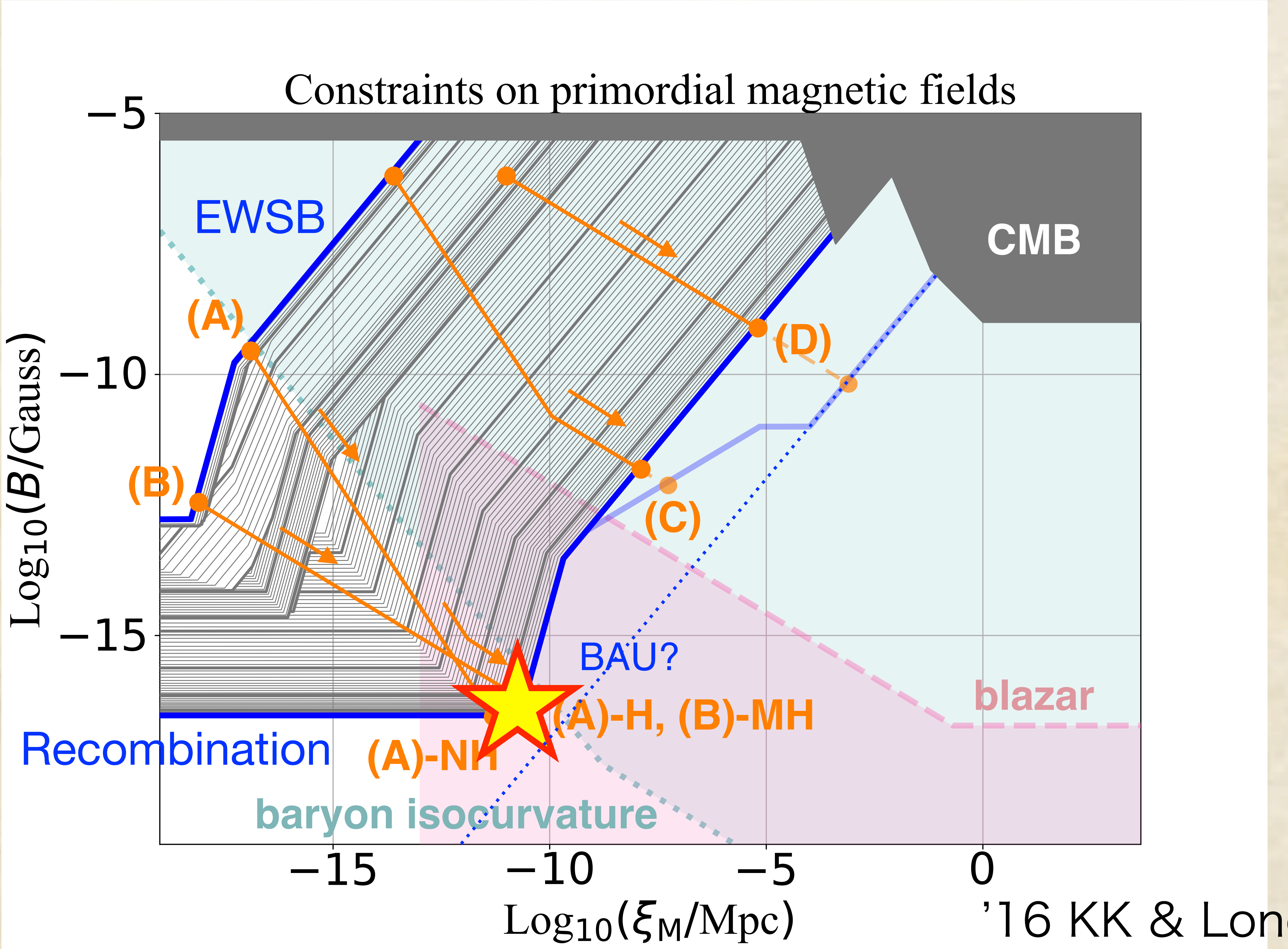
Still difficult to reconcile the BAU and intergalactic MFs…



'16 KK & Long, '24 Uchida, KK+ to appear

But axion inflation can generate helical primordial MFs as the origin of BAU.

Summary

- Blazar observation motivates us to study cosmological MHD.
- New conserved quantity (Hosking integral) improved our understanding.
- Chiral magnetic effect is an interesting effect for many fields of physics.
- Magnetohydrodynamics is modified to Chiral Magnetohydrodynamics (CMHD) taking into account it.
- Chiral plasma instability can be used to explain the BAU as well as constrain the phenomena in the early Universe.
- Interesting behavior of CMHD is found with the balanced initial condition of the chirality and helicity.
- Axion inflation realizes such an initial condition, in which late time CMHD evolution of the system is important for the baryon asymmetry of the Universe.
- It is difficult to reconcile the blazar observation and BAU, but primordial MFs are interesting as the origin of the BAU.

Appendix

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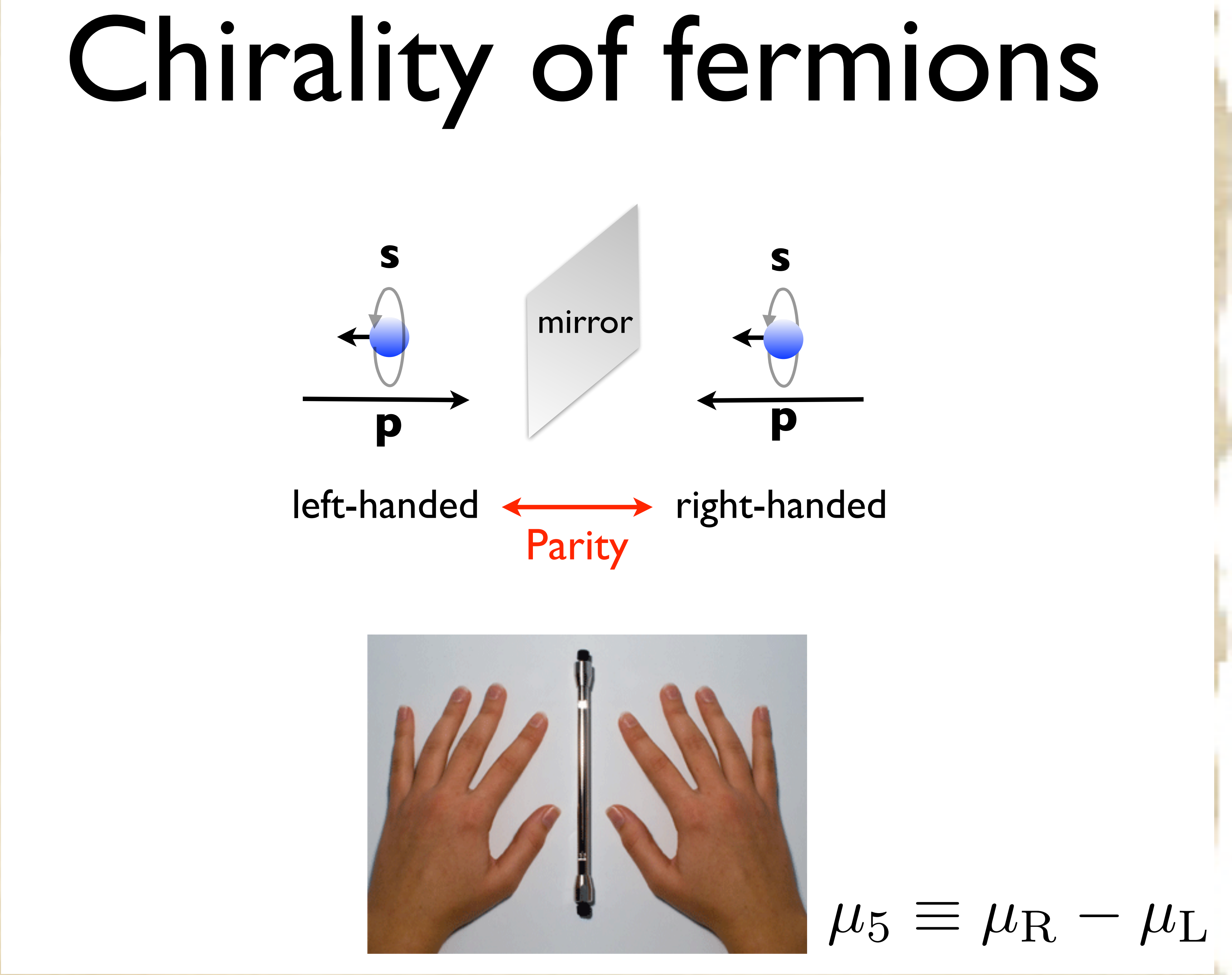

from the slide of N. Yamamoto

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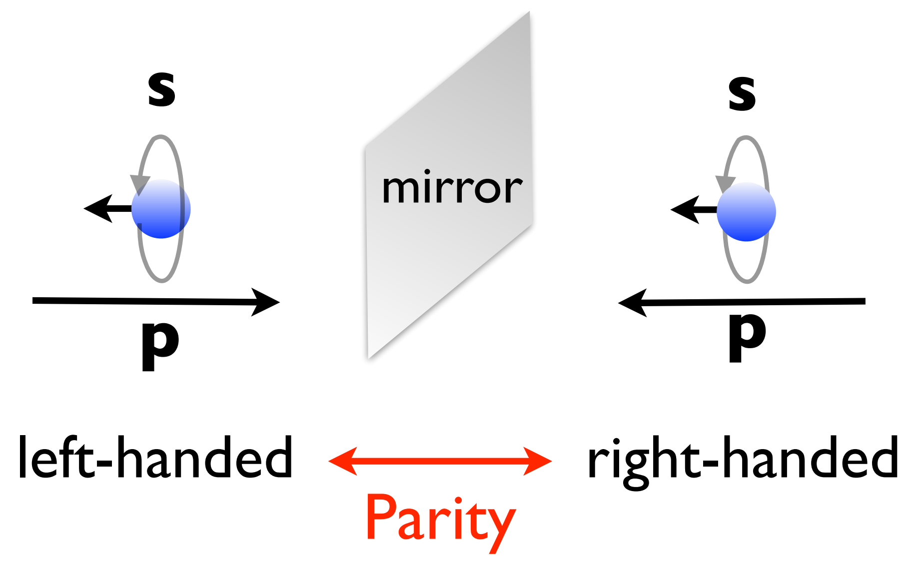
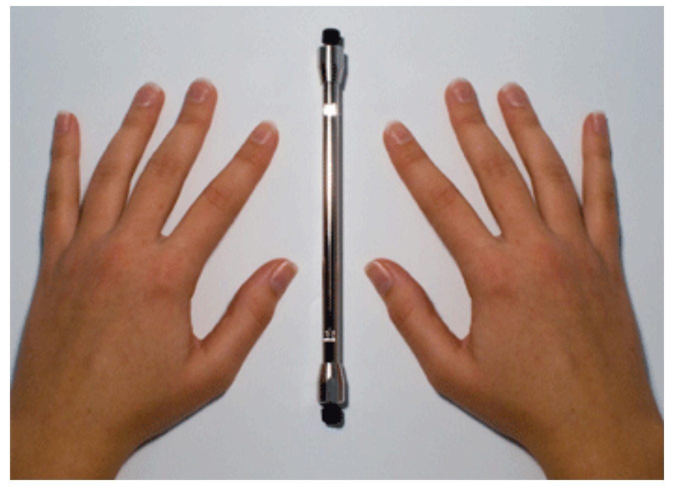
Chiral magnetic effect: $\boldsymbol{j} = \dfrac{2\alpha}{\pi}\mu_5 \boldsymbol{B}$

Chirality of fermions



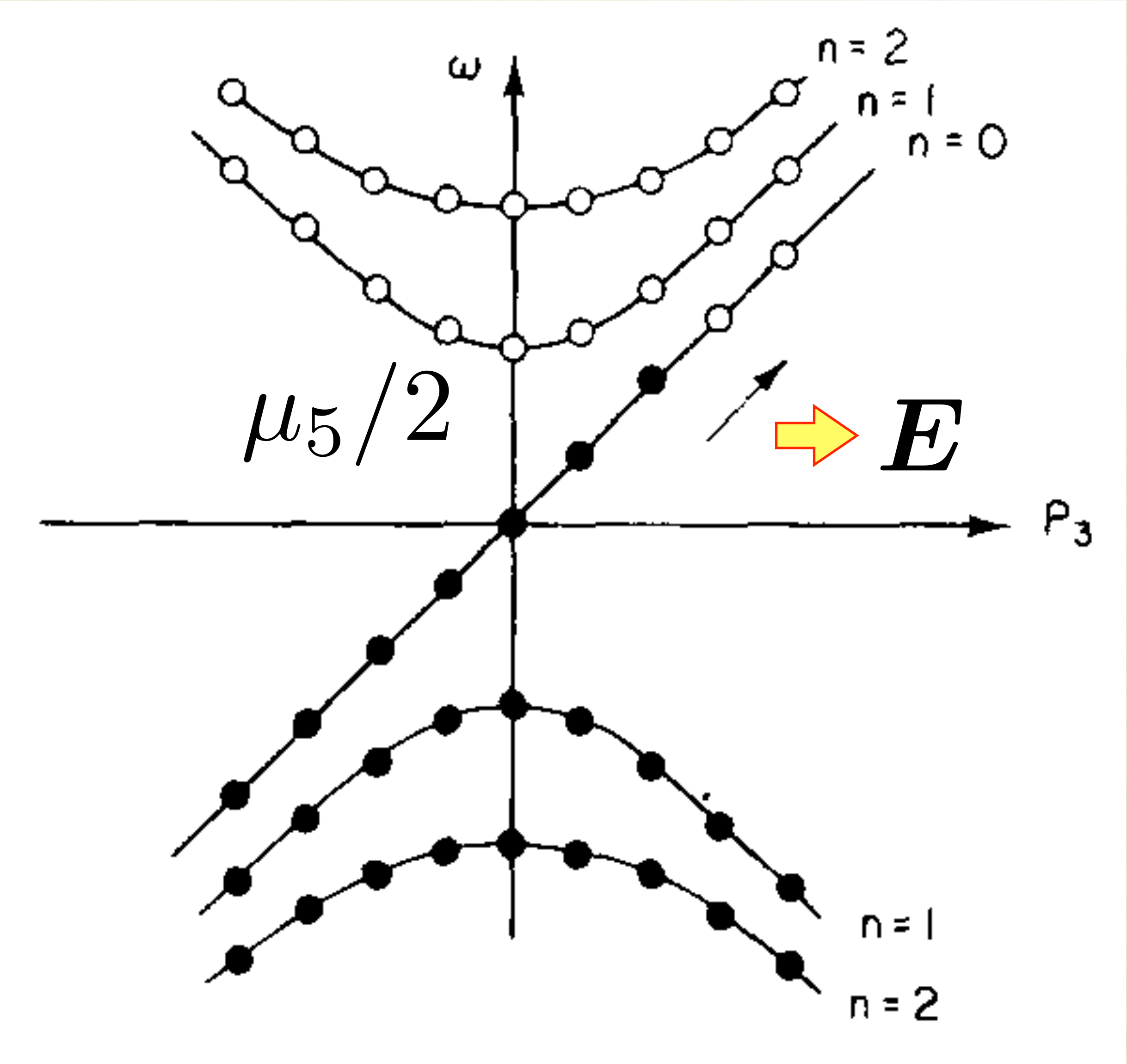


$\mu_5 \equiv \mu_{\rm R} - \mu_{\rm L}$

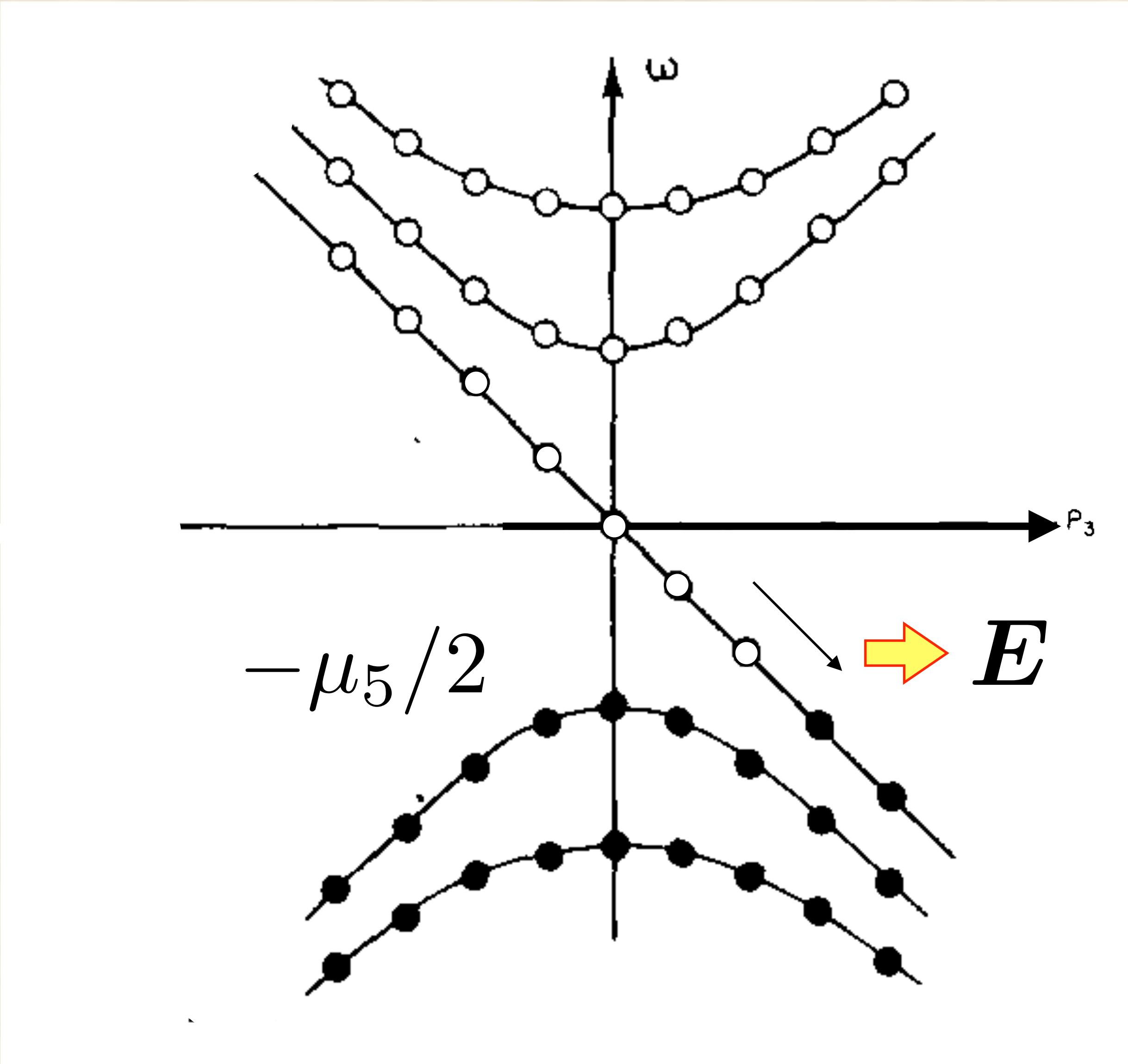
from the slide of N. Yamamoto

The relevance of the CME and chiral anomaly

can be seen by looking at the Landau level



Right-handed fermion



Left-handed fermion

('83 Nielsen&Ninomiya)

The number of states with $p_z > 0$ is large for right-handed fermions with charge +e and vice versa

➡ positive current in z-direction

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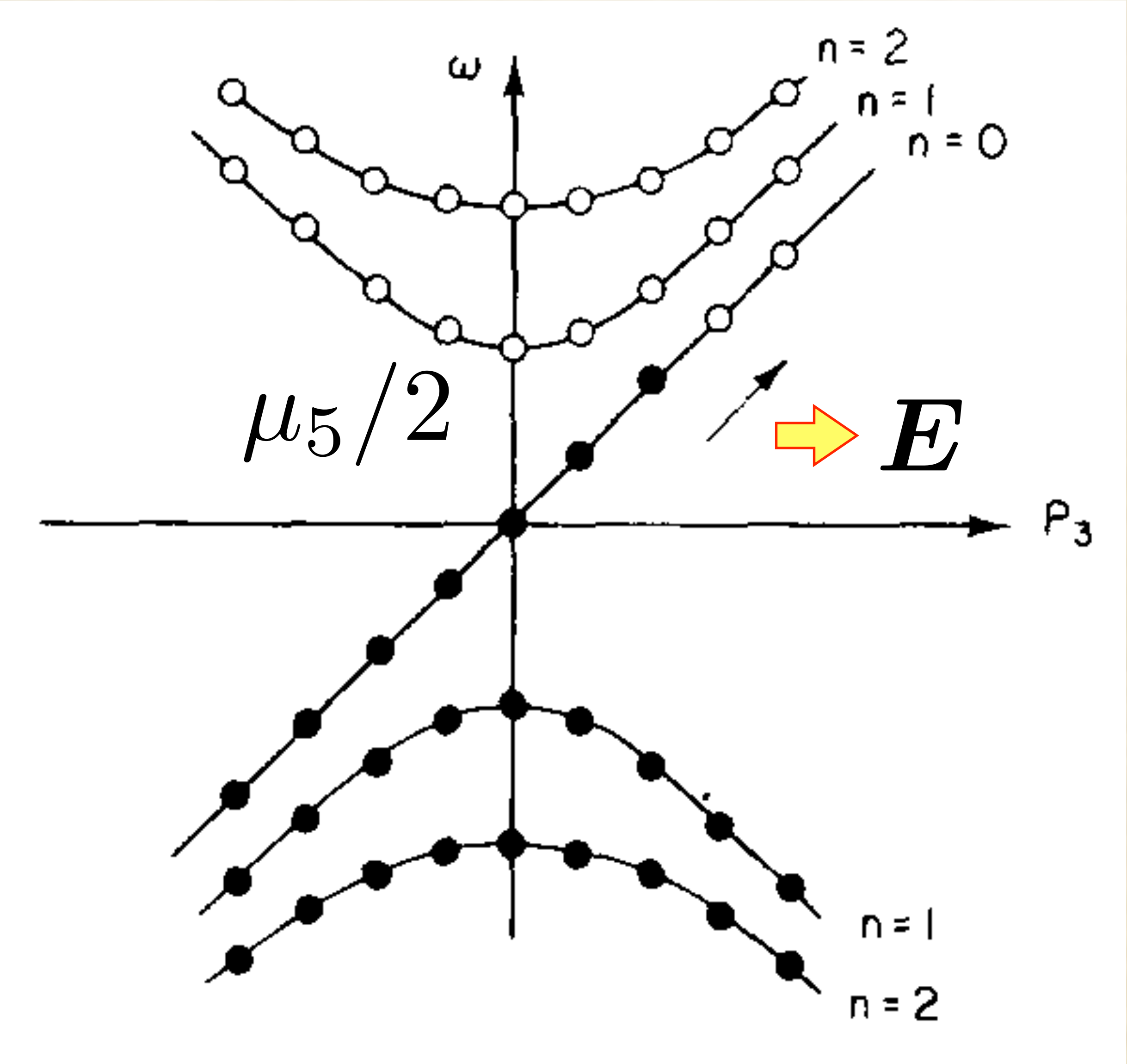

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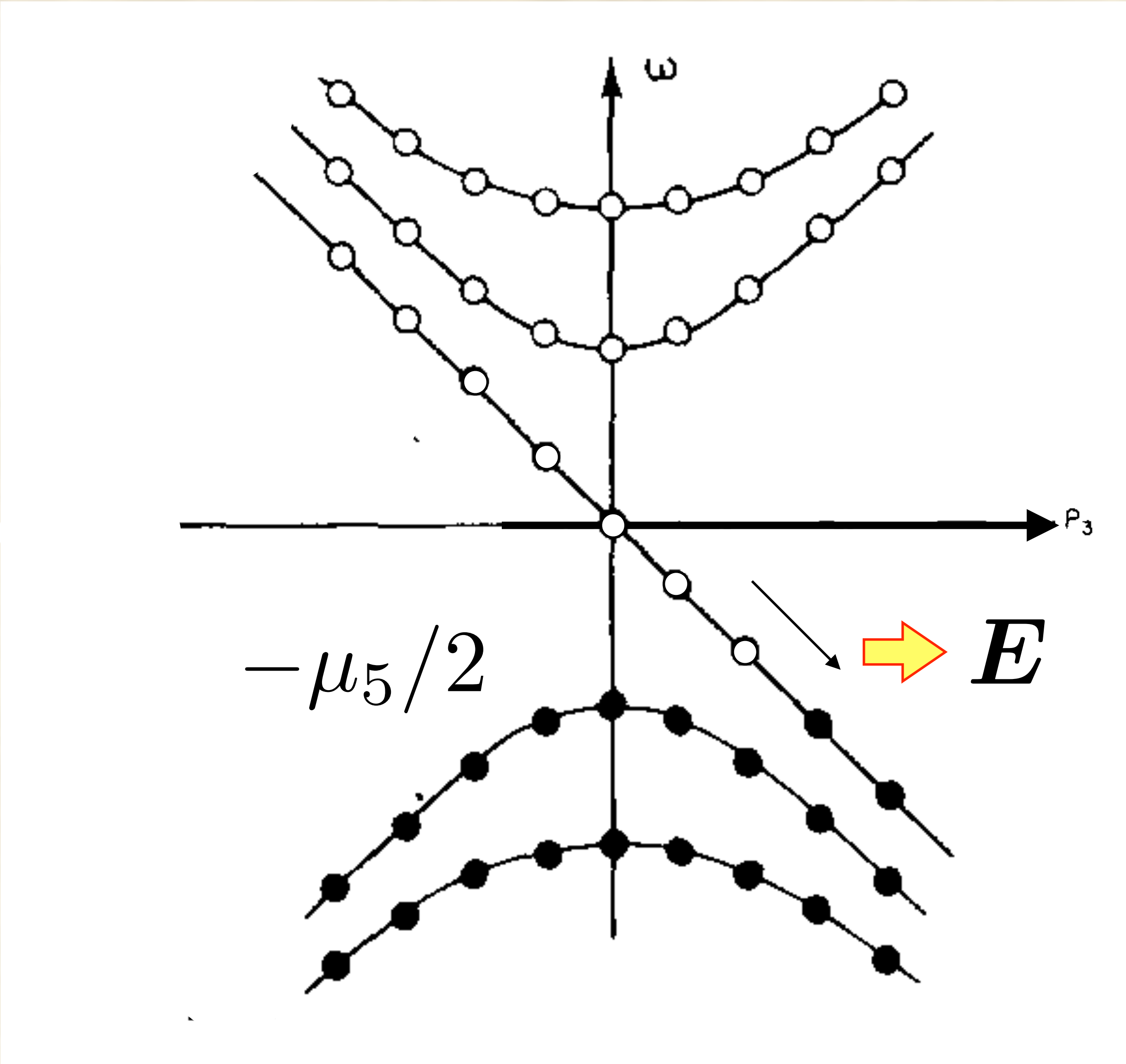

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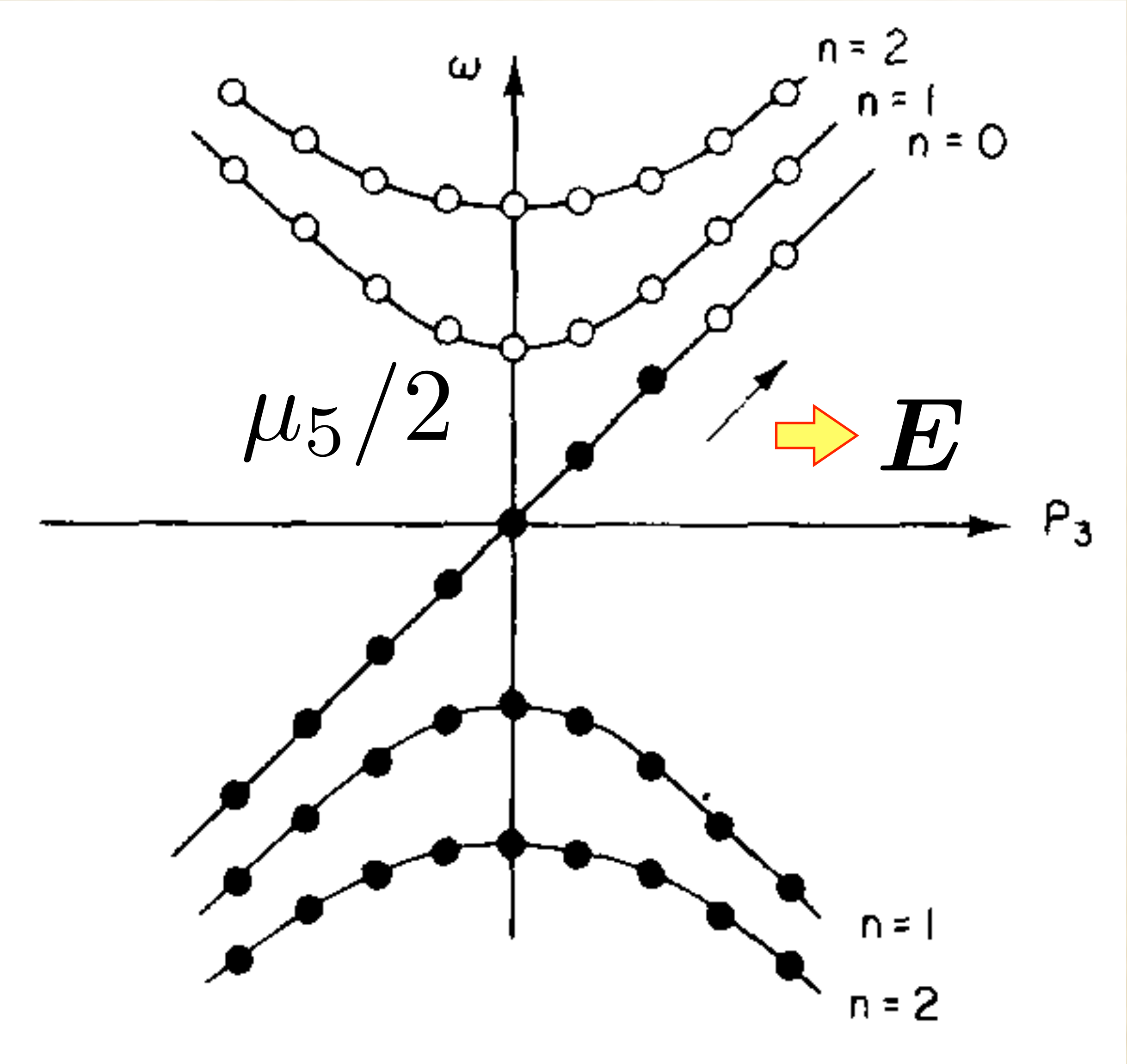

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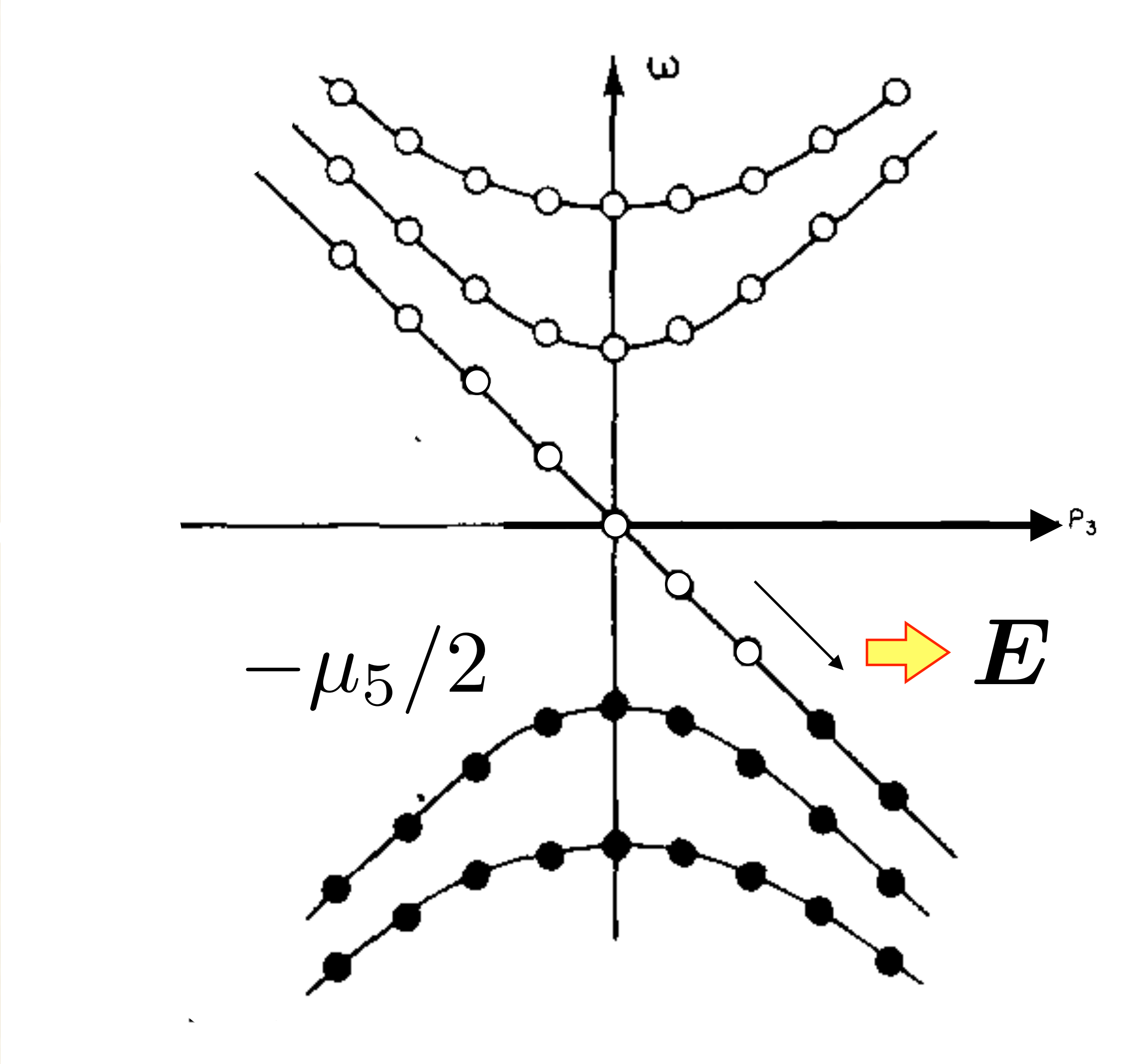

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