



Chiral transport phenomena in core-collapse supernovae

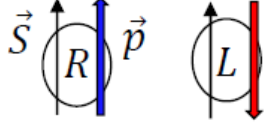


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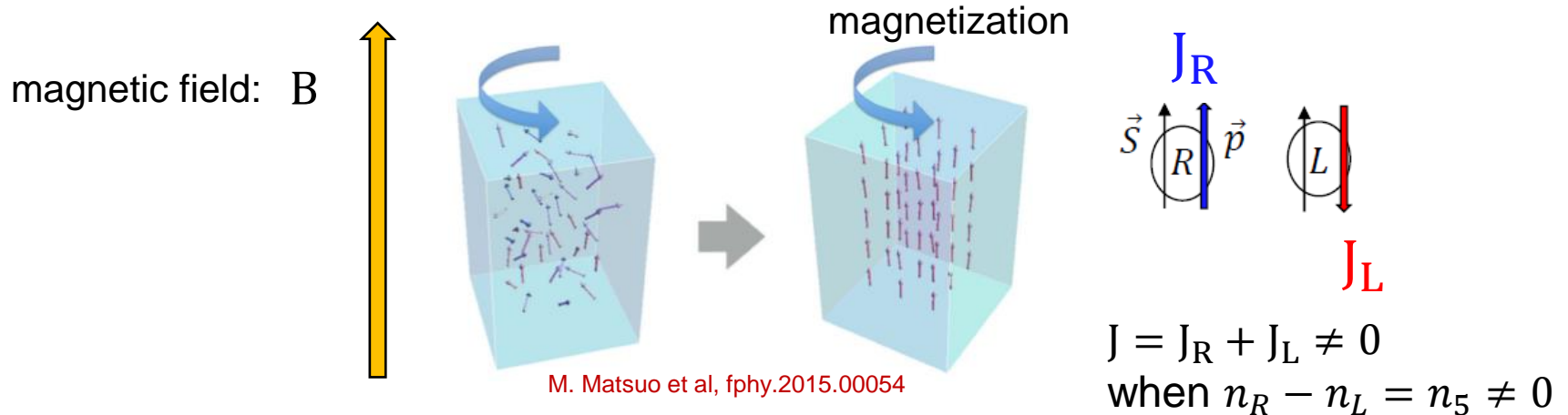
(West lake workshop on nuclear physics 2024, Oct. 19, 2024)

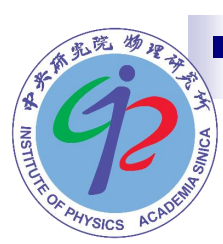
Exotic transport in chiral matter

- Chiral fermions (massless fermions) : $J^\mu = J_R^\mu + J_L^\mu$, $J_5^\mu = J_R^\mu - J_L^\mu$.

chirality=helicity ($\vec{S} \cdot \hat{p}$)  classically : $\partial_\mu J_{R/L}^\mu = 0$
- Chiral anomaly : $\partial_\mu J_{R/L}^\mu = \pm \frac{\mathbf{E} \cdot \mathbf{B}}{4\pi^2} \Rightarrow \partial_\mu J_5^\mu = \frac{\mathbf{E} \cdot \mathbf{B}}{2\pi^2}$ S. Adler, J. Bell, R. Jackiw, 69
- Chiral magnetic effect (CME) : $J^\mu = \xi_B B^\mu$, $\xi_B = \frac{\mu_5}{2\pi^2}$. axial chemical potential
parity odd ($\mu_5 = (\mu_R - \mu_L)/2$)
parity violation
(in a quasi-equilibrium cond.)

A. Vilenkin, PRD 22, 3080 (1980)
 K. Fukushima, D. Kharzeev, H. Warringa, PRD78, 074033 (2008)

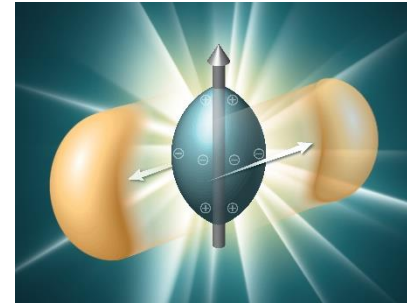




Where to find chiral transport in the real world?

- Nuclear physics : relativistic heavy ion collisions

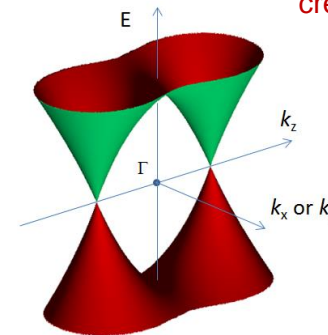
Review : D. Kharzeev, et.al, Prog. Part. Nucl. Phys. 88, 1 (2016)



credit : BNL

- Condensed matter : Weyl semimetals

Textbook : E. V. Gorbar, et.al,
“Electric Properties of Dirac and Weyl Semimetals”
(World Scientific, 2021)



Borisenko et al., Phys. Rev. Lett. 113, 027603 (2014)

- Astrophysics : core-collapse supernovae

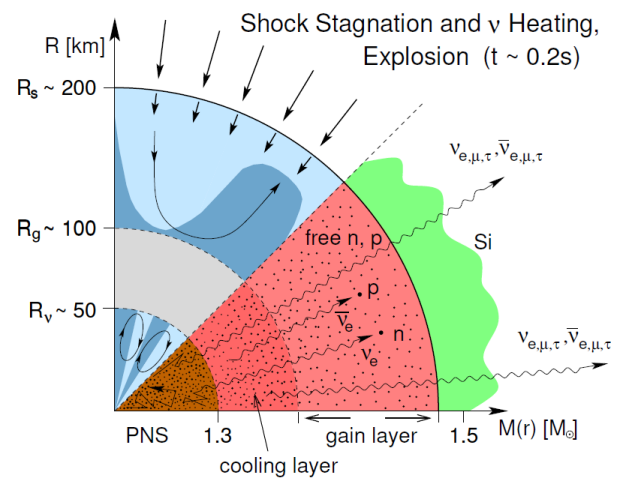
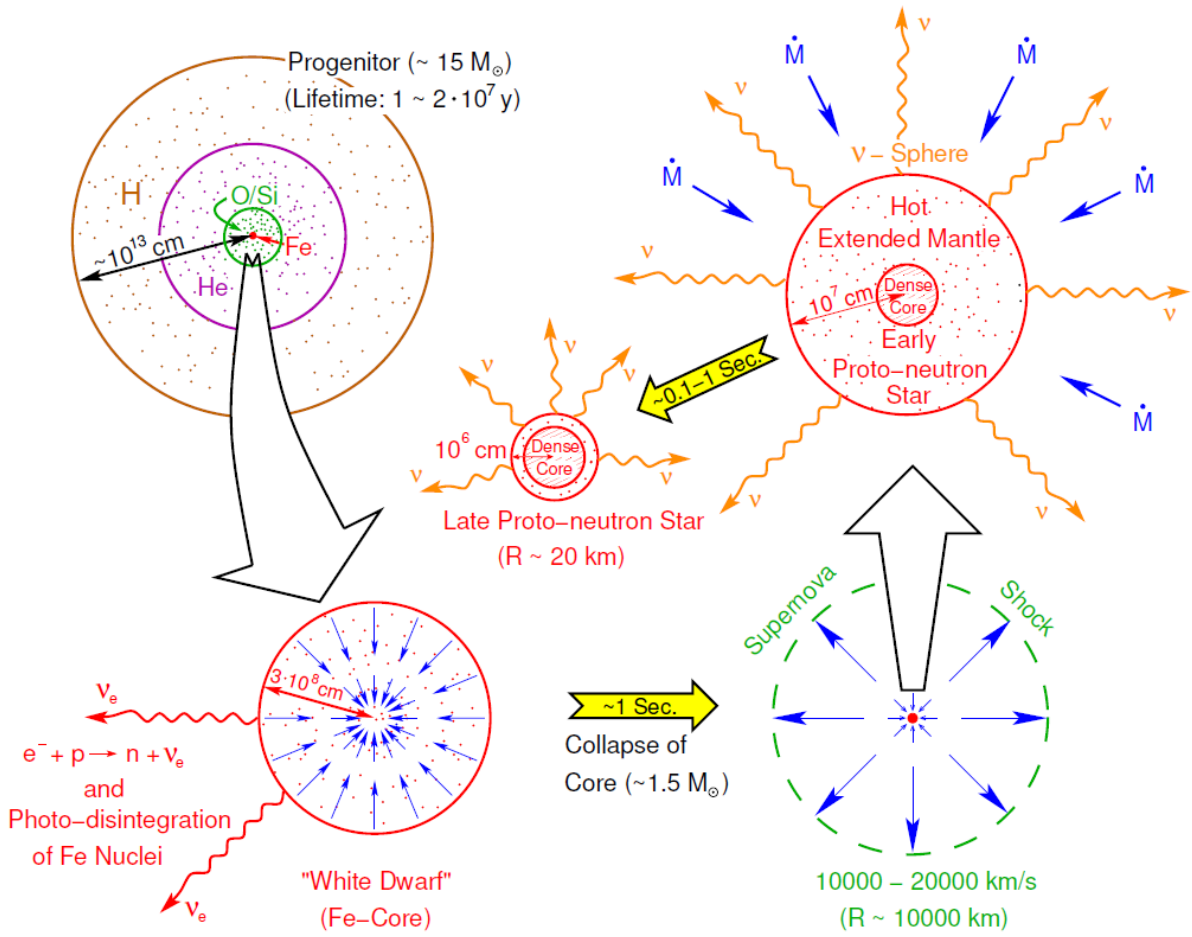
Review : K. Kamada, N. Yamamoto, DY,
Prog. Part. Nucl. Phys. 129 (2023) 104016

see Kohei's talk for applications in cosmology



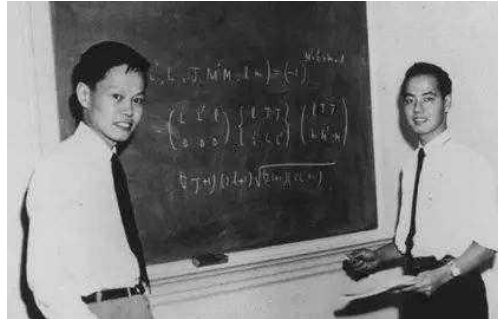
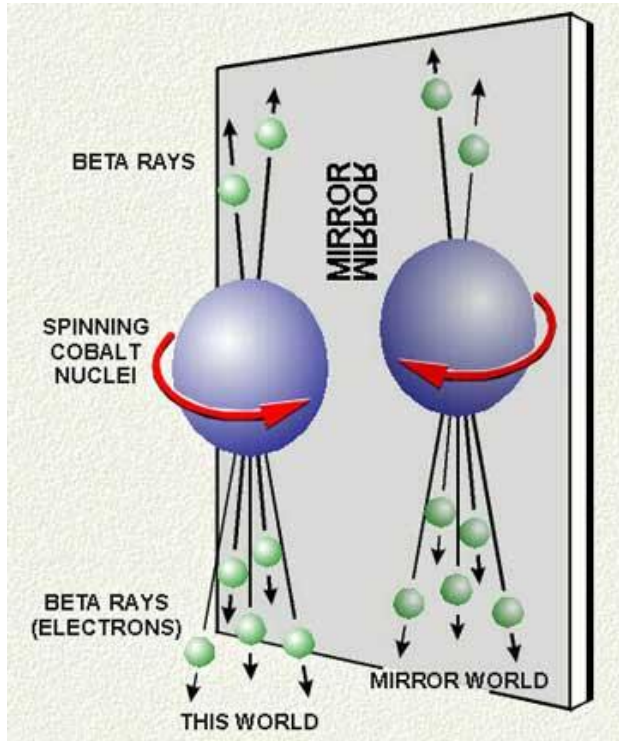
credit : RIKEN 3

Evolution of core-collapse supernovae (CCSN)



H.-Th. Janka et al., astro-ph/0612072

Parity violation & weak interaction



Lee & Yang (th) 1956



Wu et al., (exp) 1957

- Neutrinos play an important role for CCSN and interact with matter via the weak int.
- Intrinsic **parity violation** for weak interaction : essential for chiral effects
- What will be the transport properties for “chiral” leptons under parity (chirality) violation?

<http://physics.nist.gov/GenInt/Parity/cover.html>

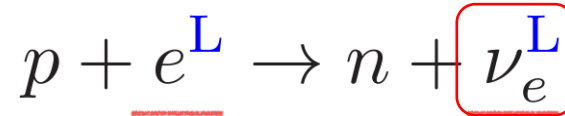
$$n \rightarrow p + e_L + \bar{\nu}_R$$

ultrarelativistic (nearly massless) neutrinos & electrons considered ; no BSM

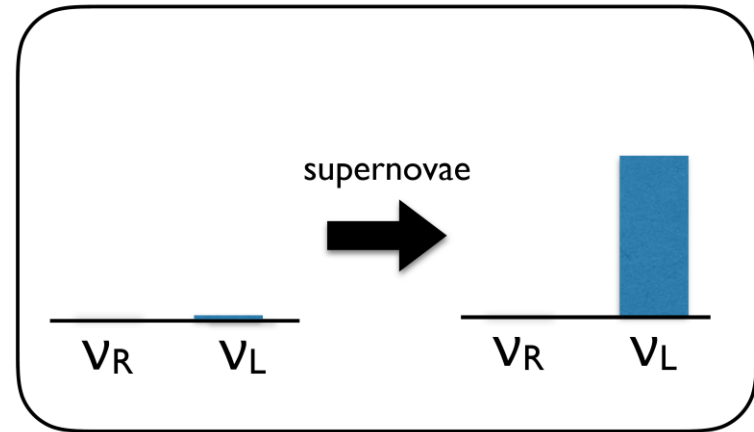
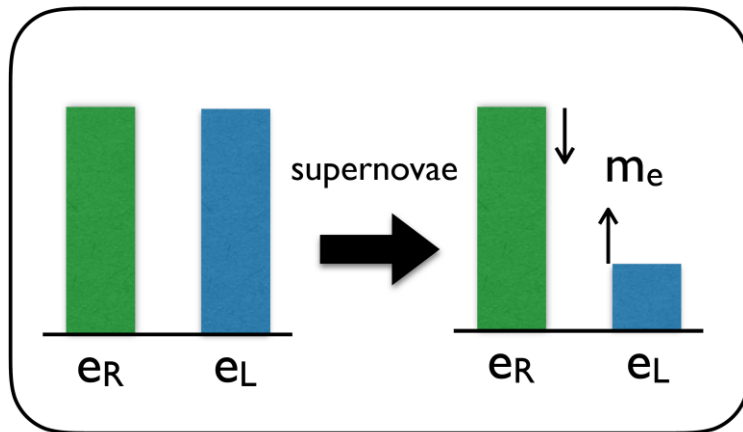
Generation of chirality imbalance

- Electron capture process in supernovae :

A. Ohnishi, N. Yamamoto, 2014, arXiv:1402.4760



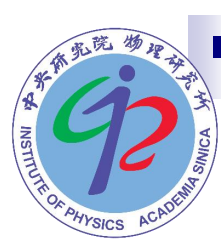
an innate lefthander



D. Grabowska, D.B. Kaplan, S. Reddy, PRD 91 (8) (2015) 085035

- Back-reaction from non-equilibrium neutrinos.

$$\nu_L^e(q) + n(k) \rightleftharpoons e_L(q') + p(k')$$



Chiral effects on dense stars?

- Three long-standing puzzles :
 - Pulsar kicks : the origin of momentum asymmetry for neutron stars? $v \sim 10^2$ km/s
A. Lyne, D. Lorimer, *Nature* 369, 127 (1994).
V. Kaspi, et al., *Nature* 381, 584 (1996).
 - Magnetars : the origin of strong & stable magnetic fields? $B \sim 10^{15}$ G
review: A. K. Harding, D. Lai, *Rept. Prog. Phys.* 69, 2631 (2006)
 - Explosions of CCSN with the observed energy?
- ❖ Chiral effects as a “microscopic” mechanism may provide possible explanations (qualitatively) in a self consistent framework. (disclaimer : not the only solution)
- Radiation hydrodynamics : e.g. S. W. Bruenn, *Astrophys. JI. Suppl.* 58 (1985) 771.
magneto-hydrodynamics (MHD) + relativistic kinetic theory (Boltzmann eq.)
matter (e, n, p) in equilibrium radiation (ν) out of equilibrium
- Constructing chiral radiation hydrodynamics :
chiral magneto-hydrodynamics (ChMHD) + chiral kinetic theory (CKT)
reviews : K. Kamada, N. Yamamoto, *DY, PPNP* 129 (2023) 104016
Y. Hidaka S. Pu, Q, Wang, *DY, PPNP* 127 (2022) 103989

Chiral radiation transport equation

■ CKT for neutrinos :
$$q \cdot D f_q^{(\nu)} = (1 - f_q^{(\nu)}) \Gamma_q^{<} - f_q^{(\nu)} \Gamma_q^{>},$$

Boltzmann eq. in the inertial frame

collision term with quantum corrections

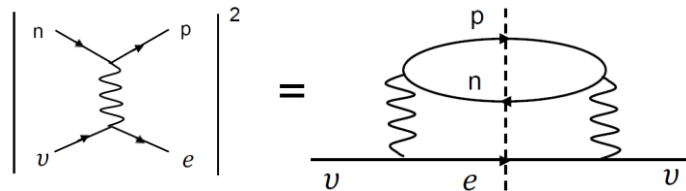
N. Yamamoto & DY, APJ 895 (2020), 1

■ Neutrino absorption :
$$\bar{\Gamma}_q^{\leq} \approx \bar{\Gamma}_q^{(0)\leq} + \bar{\Gamma}_q^{(\omega)\leq}(q \cdot \omega) + \bar{\Gamma}_q^{(B)\leq}(q \cdot B),$$

 $\nu_L^e(q) + n(k) \rightleftharpoons e_L(q') + p(k')$

vorticity & magnetic field corrections :

breaking spherical symmetry & axisymmetry



analytic expressions :
$$\bar{\Gamma}_q^{(0)>} \approx \frac{(g_V^2 + 3g_A^2)}{\pi} G_F^2 (q \cdot u)^3 (1 - f_{0,q}^{(e)}) \left(1 - \frac{3q \cdot u}{Mc^2}\right) \frac{n_n - n_p}{1 - e^{\beta(\mu_p - \mu_n)}},$$

$$\bar{\Gamma}_q^{(\omega)>} \approx \frac{(g_V^2 + 3g_A^2)}{2\pi M} G_F^2 (q \cdot u)^2 (1 - f_{0,q}^{(e)}) \left(\frac{2}{E_i} + \beta f_{0,q}^{(e)}\right) \frac{n_n - n_p}{1 - e^{\beta(\mu_p - \mu_n)}},$$

$$\bar{\Gamma}_q^{(B)>} \approx \frac{(g_V^2 + 3g_A^2)}{2\pi M} G_F^2 (q \cdot u) (1 - f_{0,q}^{(e)}) \left(1 - \frac{8q \cdot u}{3Mc^2}\right) \frac{n_n - n_p}{1 - e^{\beta(\mu_p - \mu_n)}}.$$

$\bar{\Gamma}_q^{(0)>}$: S. Reddy, M. Prakash, J. M. Lattimer, PRD58:013009,1998

Neutrino flux driven by magnetic fields

- Energy-momentum tensor & neutrino current :

$$T_{(\nu)}^{\mu\nu} = \int_q 4\pi\delta(q^2) \left(q^\mu q^\nu f_q^{(\nu)} - q^{\{\mu} S_q^{\nu\}\rho} \mathcal{D}_\rho f_q^{(\nu)} \right),$$

$$J_{(\nu)}^\mu = \int_q 4\pi\delta(q^2) \left(q^\mu f_q^{(\nu)} - S_q^{\mu\rho} \mathcal{D}_\rho f_q^{(\nu)} \right), \quad \mathcal{D}_\mu f_q^{(\nu)} \equiv D_\mu f_q^{(\nu)} - \mathcal{C}_\mu[f_q^{(\nu)}]$$

- The momentum kick from neutrinos **near equilibrium** :

N. Yamamoto & DY,
PRD 104, 123019 (2021)

$$\Delta T_\nu^{i0} = -\kappa(\nabla \cdot \mathbf{v}) \mu_\nu B^i, \quad \kappa = \frac{1}{72\pi M G_F^2 (g_V^2 + 3g_A^2)} \frac{e^{2\beta(\mu_n - \mu_p)}}{n_n - n_p}$$

for $\mu_n - \mu_p \gg T = \beta^{-1}$.

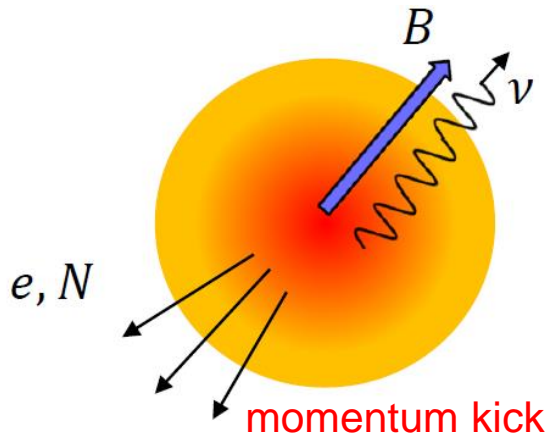
❖ Pulsar kicks : $\Delta T_e^{i0} = -\Delta T_\nu^{i0}$

(momentum con.)

❖ Effective CME (w/o μ_5) : $\Delta T_e^{i0} = \mu_e \Delta J_e^i$

⇒ $\Delta J_e^i = \xi_B B^i$, (effective CME out of equilibrium)

$$\xi_B = -\kappa(\nabla \cdot \mathbf{v}) \frac{\mu_\nu}{\mu_e}. \quad (\text{effective } \mu_5)$$



see also

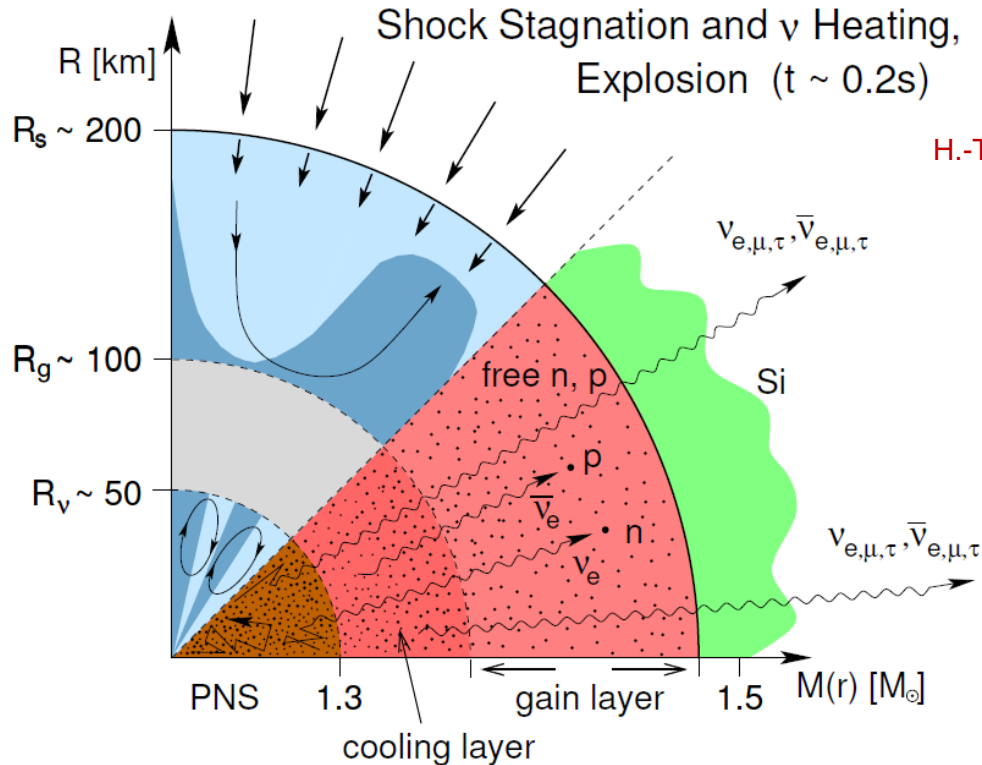
A. Vilenkin, *Astrophys. J.* 451, 700 (1995)

M. Kaminski et al., *PLB* 760, 170 (2016)

K. Fukushima, C. Yu, arXiv: 2401.04568

Shock revival by neutrino heating

- Heating by non-equilibrium neutrinos :



H.-Th. Janka et al., astro-ph/0612072

❖ How to obtain chiral effects from non-equilibrium neutrinos in the gain region?

💡 We can solve the chiral kinetic equation for left-handed electrons near equilibrium with neutrino radiation.

Effective CME from neutrinos out of equilibrium

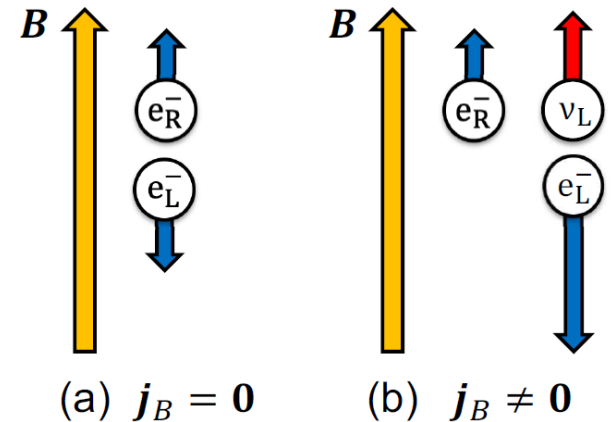
- Effective CME from neutrino radiation (**far from equilibrium**) : N. Yamamoto, DY, PRL 131, 012701 (2023)

$$j_B^\mu(x) \approx e^2 \int \frac{d^4q}{(2\pi)^3} \frac{\delta(q^2)}{q_0} (B^\mu q \cdot \partial_q - q \cdot B \partial_q^\mu) \delta f_W^{(e)} = \xi_B B^\mu,$$

$\Delta t \nearrow \Rightarrow \xi_B \nearrow$
(neutrino emission time)

$$\delta f_W^{(e)}(q, x) = -\frac{1}{q_0} \int_0^{\Delta t} dt'_0 F_W(q, x')|_c,$$

$$F_W \approx \frac{(q \cdot u)^3}{\pi} (g_V^2 + 3g_A^2) G_F^2 (n_p - n_n) \left[\frac{\bar{f}^{(e)}(1 - f^{(\nu)})}{1 - e^{\beta(\mu_n - \mu_p)}} + \frac{(1 - \bar{f}^{(e)})f^{(\nu)}}{1 - e^{\beta(\mu_p - \mu_n)}} \right].$$



- Approximate upper bounds (in the gain region) :

$$\xi_B^{\text{tot}} \approx -0.5 \text{ MeV} \text{ for } \Delta t = 0.1 \text{ s}$$

$$\square \text{ Kick velocity : } v_{\text{kick}} \sim \frac{|T_{B,\text{tot}}^{i0}|}{\rho_{\text{core}}} \approx \left(\frac{eB}{10^{13-14} \text{ G}} \right) \text{ km/s. } \left. \begin{array}{l} B \sim 10^{15-16} \text{ G} \\ \text{for} \\ v_{\text{kick}} \sim 10^2 \text{ km/s} \end{array} \right\}$$

see also A. Vilenkin, Astrophys. J. 451, 700 (1995).

Chiral plasma instability

- Chiral plasma instability (CPI) :

M. Joyce, M. E. Shaposhnikov, PRL 79, 1193 (1997)

Y. Akamatsu, N. Yamamoto, PRL 111, 052002 (2013)

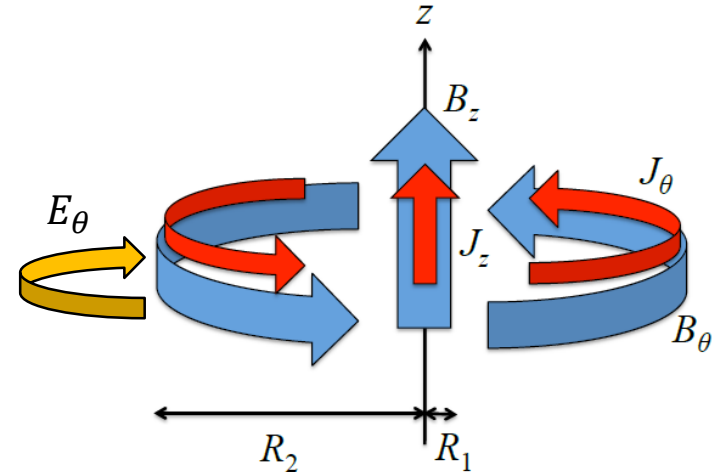
- Anomalous Maxwell's eq. :

$$\partial_t \mathbf{B} = -\nabla \times \mathbf{E}, \quad \nabla \times \mathbf{B} = \eta^{-1} \mathbf{E} + \boxed{\xi_B \mathbf{B}}$$

CME

$$\Rightarrow \frac{\partial \mathbf{B}}{\partial t} = \boxed{\eta \nabla^2 \mathbf{B}} + \boxed{\eta \nabla \times (\xi_B \mathbf{B})}$$

diffusion CME (instability)



Y. Akamatsu, N. Yamamoto, PRD 90, 125031 (2014)

- Unstable modes at long wavelength : $\delta \mathbf{B} \propto e^{\sigma t + i \mathbf{k} \cdot \mathbf{x}}$

$$\sigma = \eta k (\xi_B - k) \quad (\sigma > 0 \text{ for small } k, \text{ long wavelength})$$

- Helicity conservation :

$$\frac{dH_{\text{tot}}}{dt} = 0,$$

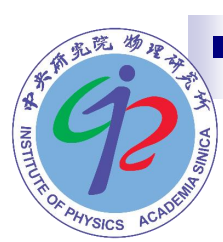
$$H_{\text{tot}} \equiv N_5 + \frac{H_{\text{mag}}}{4\pi^2},$$

$$N_5 \equiv \int d^3x n_5, \quad H_{\text{mag}} \equiv \int d^3x \mathbf{A} \cdot \mathbf{B}.$$

(linking of magnetic fluxes)

exchange btw the (effective) axial charge & magnetic helicity

Y. Hirono, D. Kharzeev, Y. Yin, PRD 92, 125031 (2015)

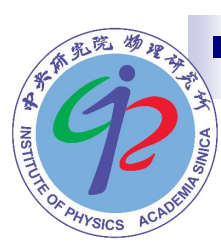


Chiral magnetohydrodynamics

■ Chiral magnetohydrodynamics (ChMHD) equations:

$$\left. \begin{aligned} \partial_\mu J^\mu &= 0 \\ \partial_\mu T^{\mu\nu} &= F^{\nu\lambda} J_\lambda \\ \text{energy-momentum \& charge conservation} \\ \partial_\nu F^{\nu\mu} &= J^\mu \\ \text{Maxwell + CME} \\ \partial_\mu J_5^\mu &= \frac{\mathbf{E} \cdot \mathbf{B}}{2\pi^2} \\ \text{helicity conservation} \end{aligned} \right\} \begin{aligned} \frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) &= 0, \quad (17) \\ \frac{\partial}{\partial t} (\rho \mathbf{v}) + \nabla \cdot \left[\rho \mathbf{v} \mathbf{v} - \mathbf{B} \mathbf{B} + \left(P + \frac{B^2}{2} \right) \mathbf{I} \right] &= \mathbf{S}, \quad (18) \\ \frac{\partial}{\partial t} \left(\frac{1}{2} \rho v^2 + \frac{1}{\Gamma - 1} P + \frac{B^2}{2} \right) + \nabla \cdot \left[\left(\frac{1}{2} \rho v^2 + \frac{\Gamma}{\Gamma - 1} P \right) \mathbf{v} \right. \\ &\quad \left. + \mathbf{E} \times \mathbf{B} \right] = -\mathbf{S} \cdot \mathbf{v} - \Delta \mathbf{J} \cdot \mathbf{E}, \quad (19) \\ \frac{\partial \mathbf{B}}{\partial t} &= \nabla \times (\mathbf{v} \times \mathbf{B}) + \eta \nabla^2 \mathbf{B} + \eta \nabla \times (\xi_B \mathbf{B}), \quad (20) \\ \frac{\partial n_{5,\text{eff}}}{\partial t} &= \frac{1}{2\pi^2} \mathbf{E} \cdot \mathbf{B}, \quad (21) \end{aligned}$$

$$\mathbf{S} = \rho \nu \nabla^2 \mathbf{v} + \frac{1}{3} \rho \nu \nabla (\nabla \cdot \mathbf{v}) \quad (\text{viscous correction})$$



Local ChMHD simulations

- ChMHD : energy-momentum & charge conservation
+ helicity conservation

J. Matsumoto, N. Yamamoto, DY, PRD 105, 123029 (2022)
see also Y. Masada et al., PRD 98, 083018 (2018)

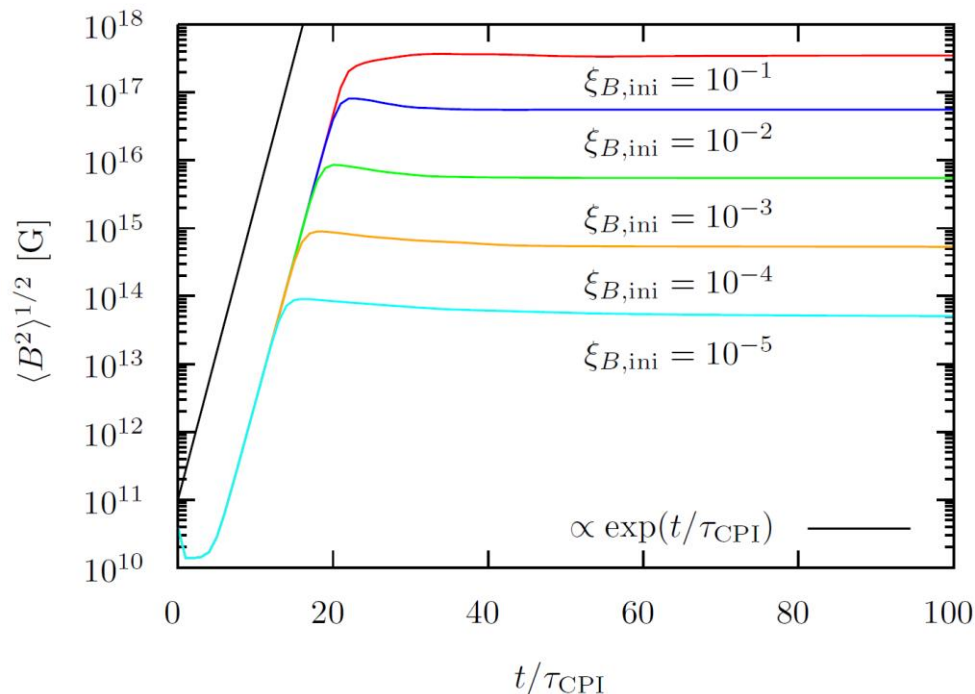
- Local simulations :

resistivity : $\eta = 1$ viscosity : $\nu = 0.01$

in the units of 100 MeV = 1

TABLE I. Summary of the simulation runs.

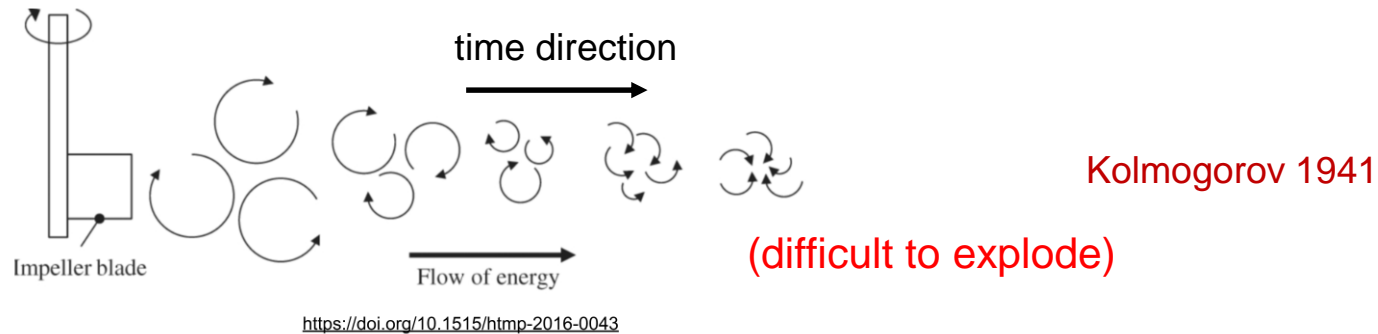
Name	L	$\xi_{B,ini}$	τ_{CPI}
Model 1	8×10^2	10^{-1}	4×10^2
Model 2	8×10^3	10^{-2}	4×10^4
Model 3	8×10^4	10^{-3}	4×10^6
Model 4	8×10^5	10^{-4}	4×10^8
Model 5	8×10^6	10^{-5}	4×10^{10}



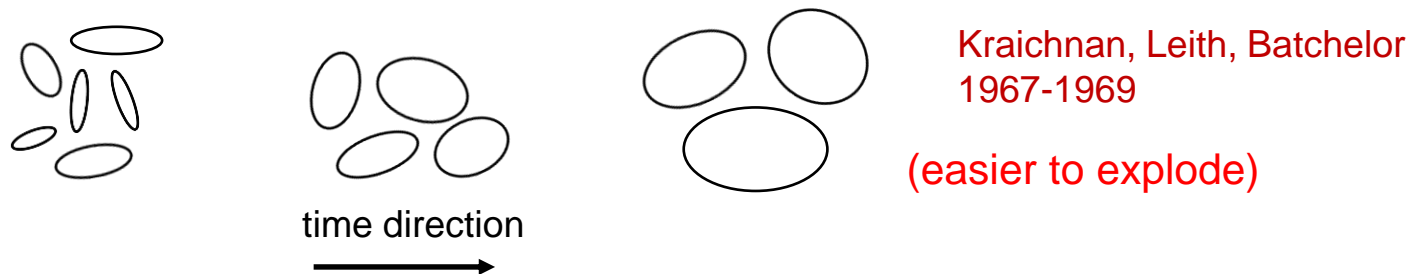
$$B_{CPI} \sim \mu_{5,eff}^2$$

Direct & inverse energy cascades

- Supernova evolution is sensitive to the turbulence.
- Turbulence in 3D : direct energy cascade



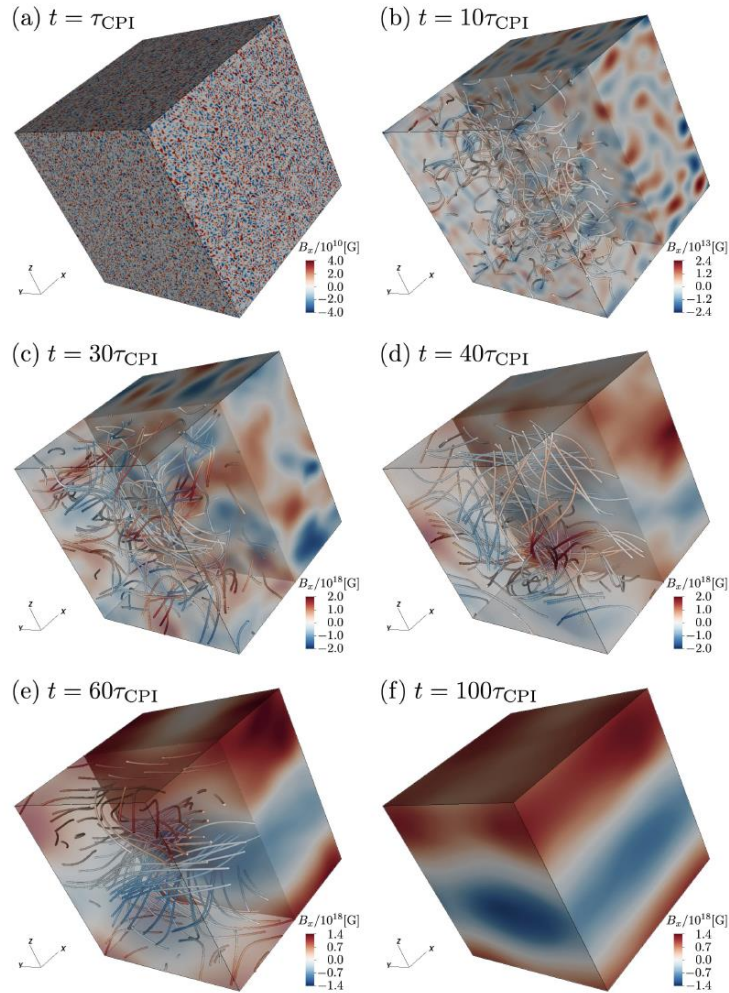
- Turbulence in 2D : inverse energy cascade



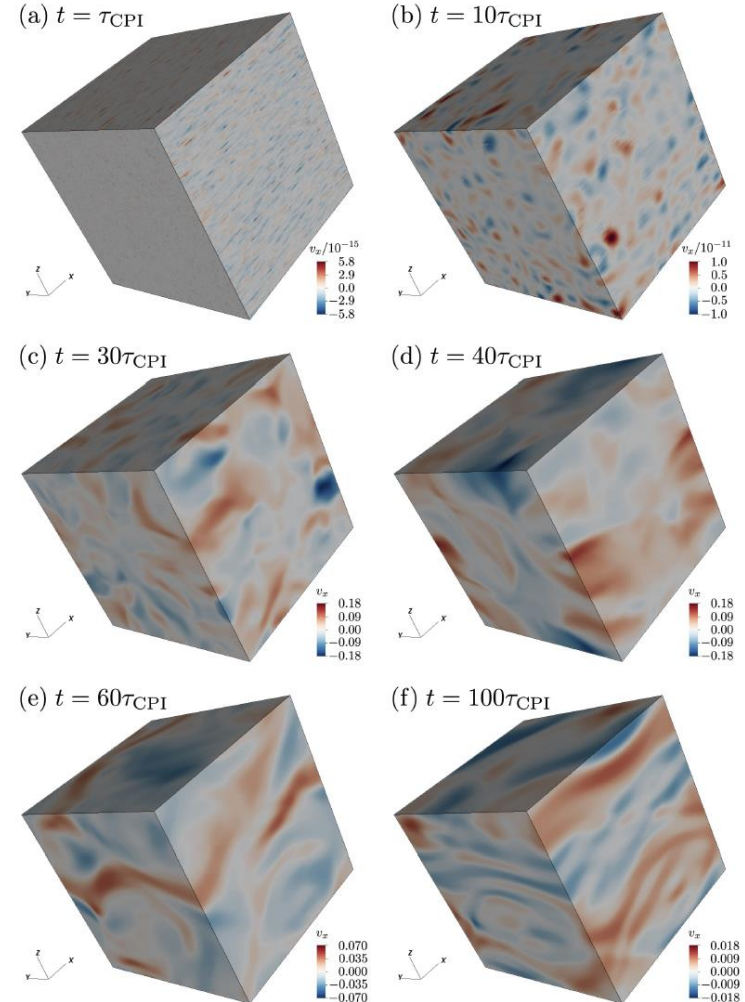
- Chiral effects in 3D : inverse cascade

Inverse cascade

- Inverse cascade : opposite to the direct energy cascade for turbulence in 3D



magnetic field



fluid velocity

Neutrino spin Hall effect

- We previously focus on just the collisional effects for neutrinos.

- The medium effect could also modify the onshell condition :

e.g. $q^2 = 0 \rightarrow q^2 = 2q \cdot \bar{\Sigma} = m_{\text{th}}^2 \implies \partial_\mu \bar{\Sigma}_\nu \sim \boxed{F_{\mu\nu}^{\text{eff}}}$ J. Blaizot, E. Iancu, Phys.Rept. 359 (2002) 355-528

Effective EM fields

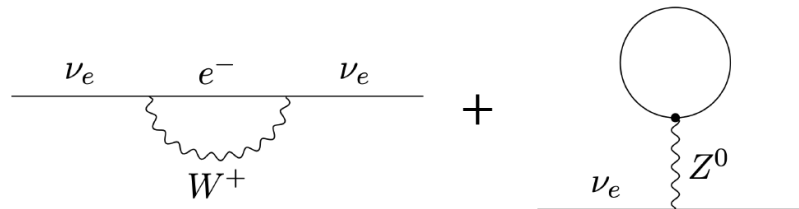
- For chiral transport : an effective spin Hall effect can be induced.

(The CKT is accordingly modified.)

N. Yamamoto, DY, PRD 109, 056010 (2024)

S. Fang, S. Pu, DY, PRD 109, 034034 (2024)

- Neutrino self-energy :



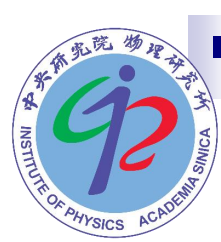
$$\bar{\Sigma}_L^\mu = V n^\mu + \mathcal{O}(\hbar), \quad V = \frac{G_F}{\sqrt{2}} [(1 + 4 \sin^2 \theta_W) N_e - N_n + (1 - 4 \sin^2 \theta_W) N_p].$$

D. Notzold and G. Raffelt, Nucl. Phys. B 307 (1988) 924

\implies **neutrino spin Hall effect :** $J_{\text{SHE}}^\mu = \hbar \epsilon^{\mu\nu\rho\sigma} \ell_\nu n_\sigma (\partial_\rho V),$

$$\ell_\nu = \int \frac{d^4 q}{(2\pi)^3} \frac{\delta(q_0 - |\mathbf{q}|)}{2q_0} \partial_{q\nu} f^{(\nu)}(q).$$

(anisotropic neutrino distribution)



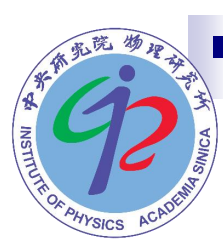
Summary & outlook

❖ Summary

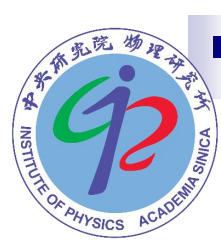
- ✓ The chiral effect stemming from “parity violation” should be included for leptonic transport with the weak interaction.
- ✓ Chiral effects with magnetic fields : “effective CME” induced by neutrino radiation, chiral plasma instability, helicity conservation, inverse cascade.
- ✓ Relevant to pulsar kicks, magnetars, and explosion dynamics of CCSN.

❖ Outlook

- ❑ Ultimate goal : global simulations for chiral radiation hydro.
- ❑ Chiral effects from vorticity (chiral vortical effect), temperature/chemical-potential gradients (spin-Hall effect), etc.
- ❑ Other applications of chiral effects to dense astrophysical systems?



Thank you!



Chiral kinetic theory from QFT

- Wigner functions : $\dot{S}_L^<(q, x) \equiv \int_y e^{-\frac{iq \cdot y}{\hbar}} \langle \psi_L^\dagger(x, y/2) \psi_L(x, -y/2) \rangle \equiv \sigma^\mu \mathcal{L}_\mu^<(q, x)$

see e.g. Y. Hidaka, S. Pu, DY, PRD 95, 091901 (2017) reviews : Y. Hidaka S. Pu, Q, Wang, DY, PPNP 127 (2022) 103989
K. Kamada, N. Yamamoto, DY, PPNP 129 (2023) 104016

- Perturbative solution up to $\mathcal{O}(\hbar)$: ($\sim \partial/q$: gradient expansion)

$$\mathcal{L}^<\mu = 2\pi \left[\delta(q^2) (q^\mu - \hbar S_{(n)}^{\mu\nu} \mathcal{D}_\nu) - \hbar \tilde{F}^{\mu\nu} q_\nu \delta'(q^2) \right] f_L,$$

$$\mathcal{D}_\mu \mathcal{L}_\nu^< \equiv (\nabla_\mu - \Gamma_{\mu\rho}^\lambda q^\rho \partial_{q^\lambda} + F_{\rho\mu} \partial_q^\rho) \mathcal{L}_\nu^< - \Sigma_\mu^< \mathcal{L}_\nu^> + \Sigma_\mu^> \mathcal{L}_\nu^<, \quad S_{(n)}^{\mu\nu} = \frac{\epsilon^{\mu\nu\alpha\beta} q_\alpha n_\beta}{2q \cdot n}.$$

- Chiral kinetic equation :

$$0 = \delta(q^2 - \hbar F_{\alpha\beta} S_{(n)}^{\alpha\beta}) \left\{ \left[q \cdot \mathcal{D} - \hbar \left(\frac{S_{(n)}^{\mu\nu} F_{\mu\rho} n^\rho}{q \cdot n} + (D_\mu S_{(n)}^{\mu\nu}) \right) \mathcal{D}_\nu - \hbar S_{(n)}^{\mu\nu} (\nabla_\mu F_\nu^\lambda - q^\rho R_{\rho\mu\nu}^\lambda) \partial_{q^\lambda} \right] f_L \right. \\ \left. - \hbar \frac{\epsilon^{\mu\nu\alpha\beta} n_\nu}{2q \cdot n} ((1 - f_L) \Delta_\alpha^> \Sigma_\beta^< - f_L \Delta_\alpha^< \Sigma_\beta^>) \right\}.$$

- Current and EM tensor : $J^\mu = 2 \int_q \mathcal{L}^<\mu, \quad T^{\mu\nu} = \int_q (\mathcal{L}^<\mu q^\nu + \mathcal{L}^<\nu q^\mu).$