Hamiltonian lattice gauge theory: Application to nonequilibrium and dense QCD matter

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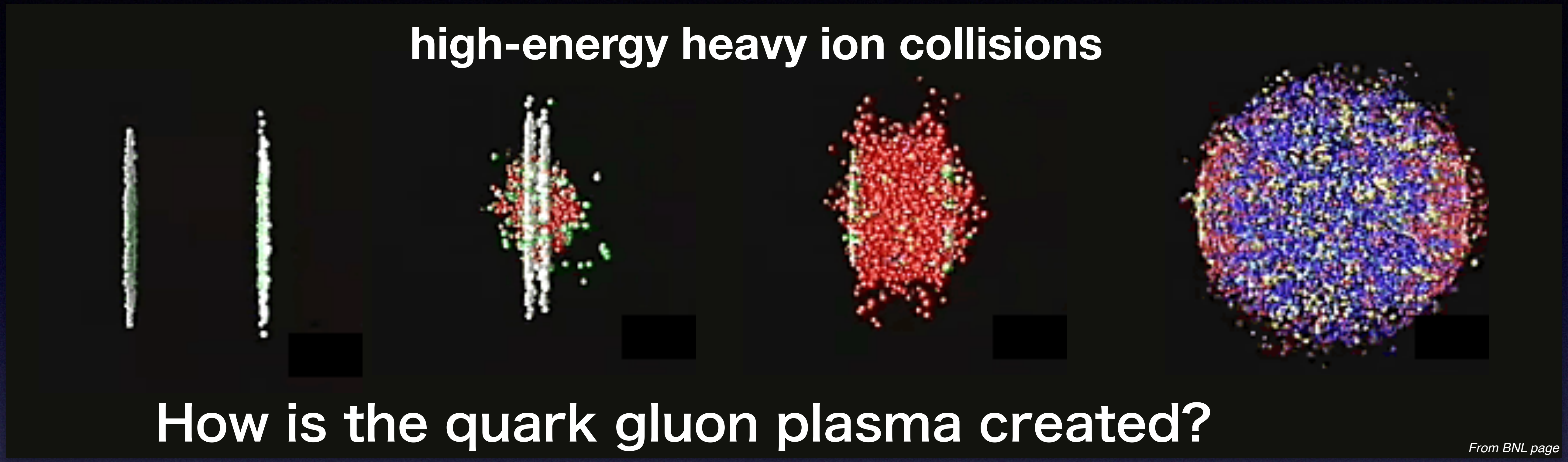
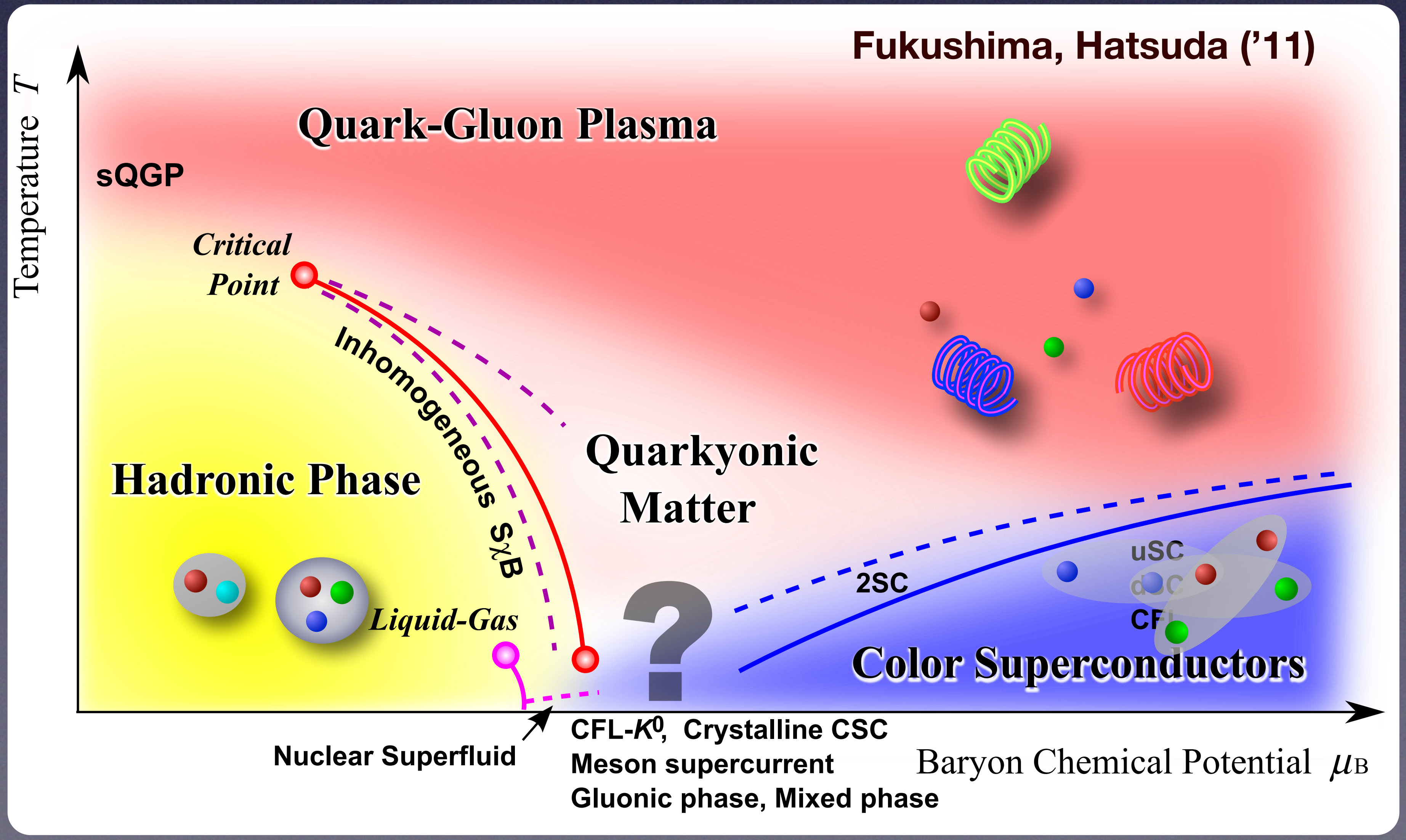
Based on
Hayata, YH, PRD 103 (2021) , 094502, JHEP 09 (2023) 123; JHEP 09 (2023) 126
Hayata, YH, Kikuchi PRD 104 (2021) 7, 074518
Hayata, YH, Nishimura, 2311.11643

Big Challenges in QCD

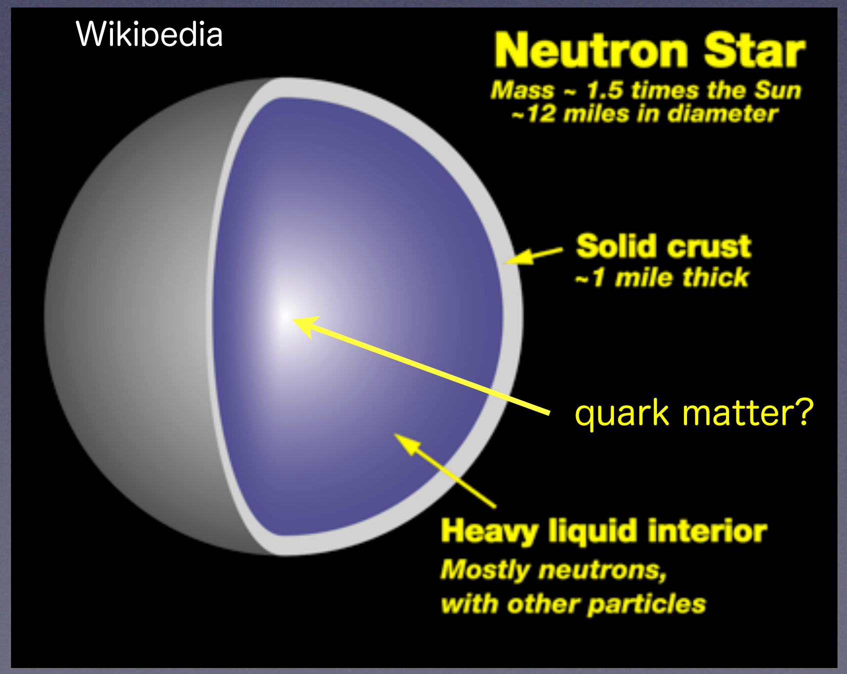
Manybody dynamics of QCD



Dense QCD



What phases are realized in the interior of a neutron star?



Difficulty

Sign problem: Difficulties in first-principles calculations based on importance sampling

$$\langle O \rangle = \int \mathcal{D}A \det(D+m) e^{iS} O$$

In real-time, finite-density problems, the weight is complex

$$\not\approx \frac{1}{N} \sum_j O_j$$

Hamiltonian approach

Directly solve Schrödinger equation!

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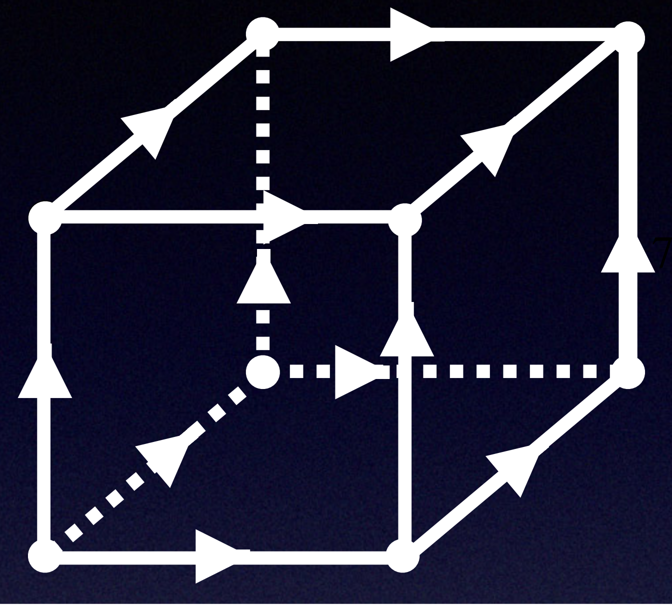
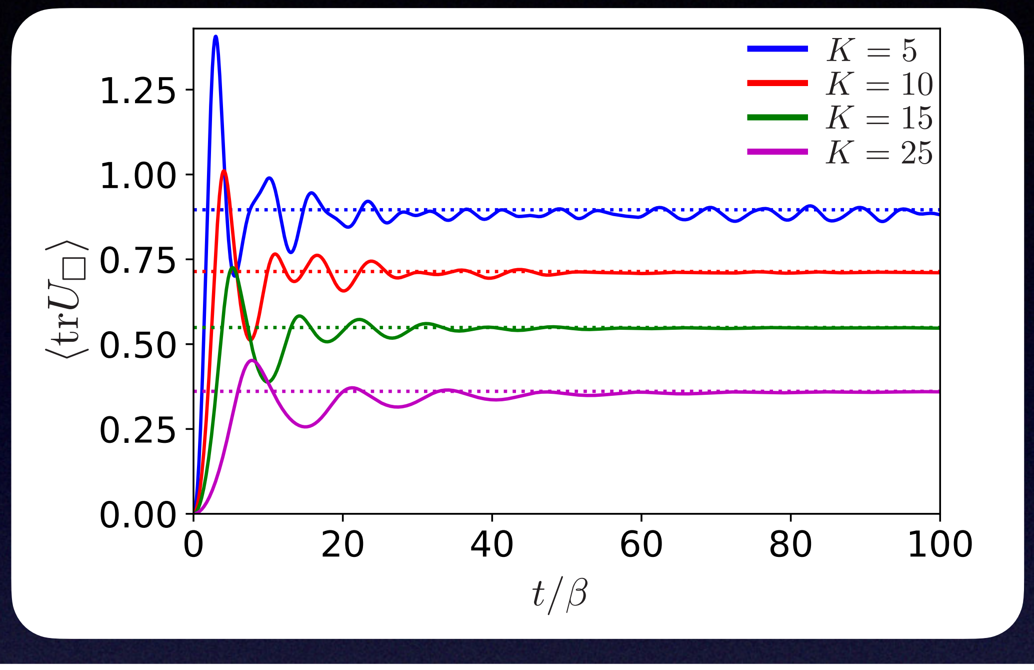
Directly solve Schrödinger equation!

Smaller systems can be simulated directly





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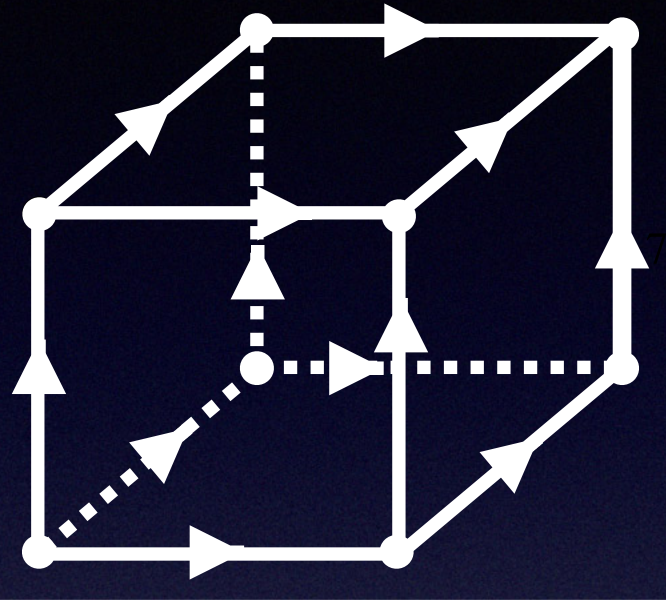
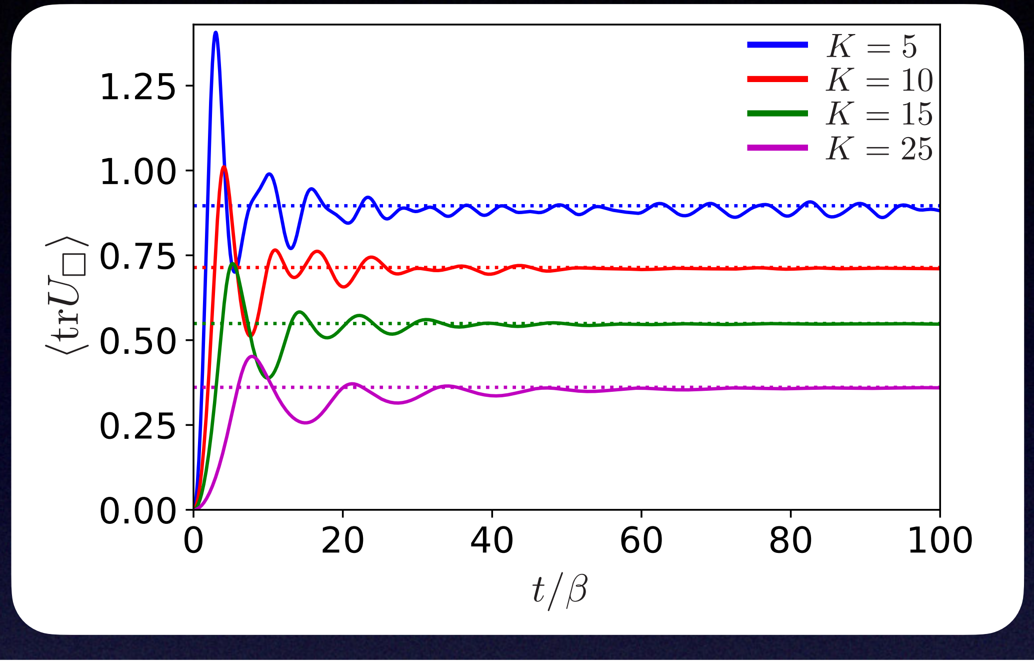
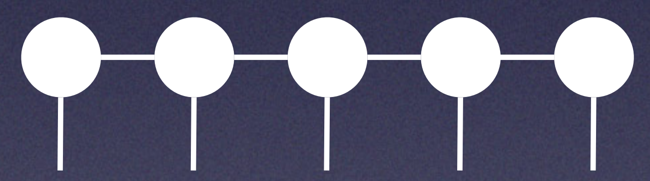


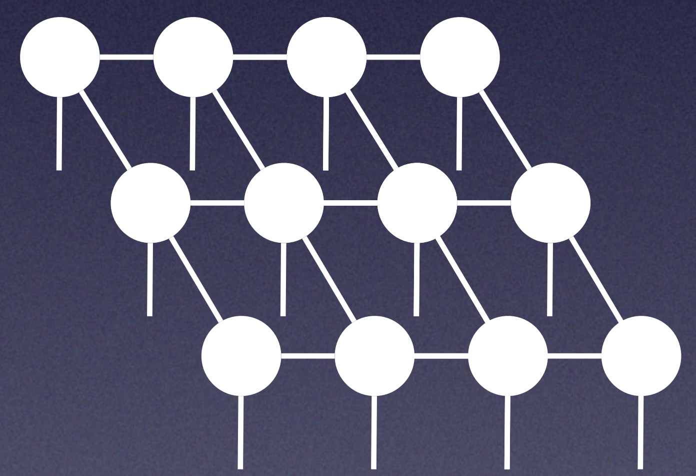


Tensor Networks

MPS



PEPS



Hamiltonian approach

Directly solve Schrödinger equation!

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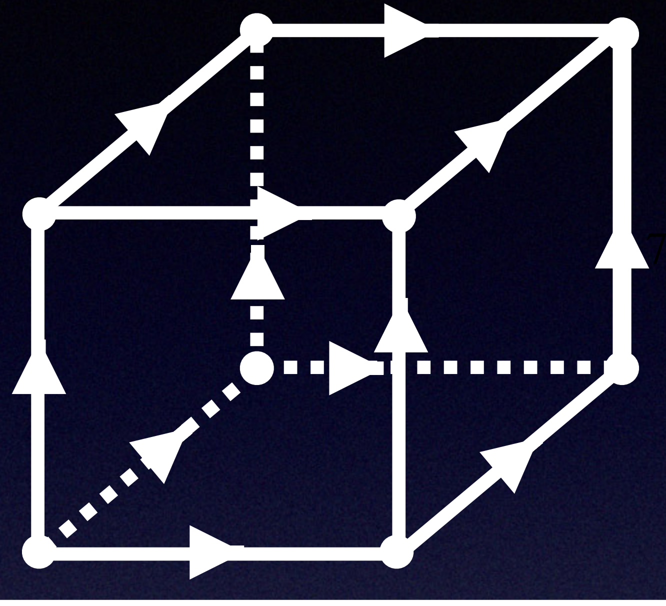

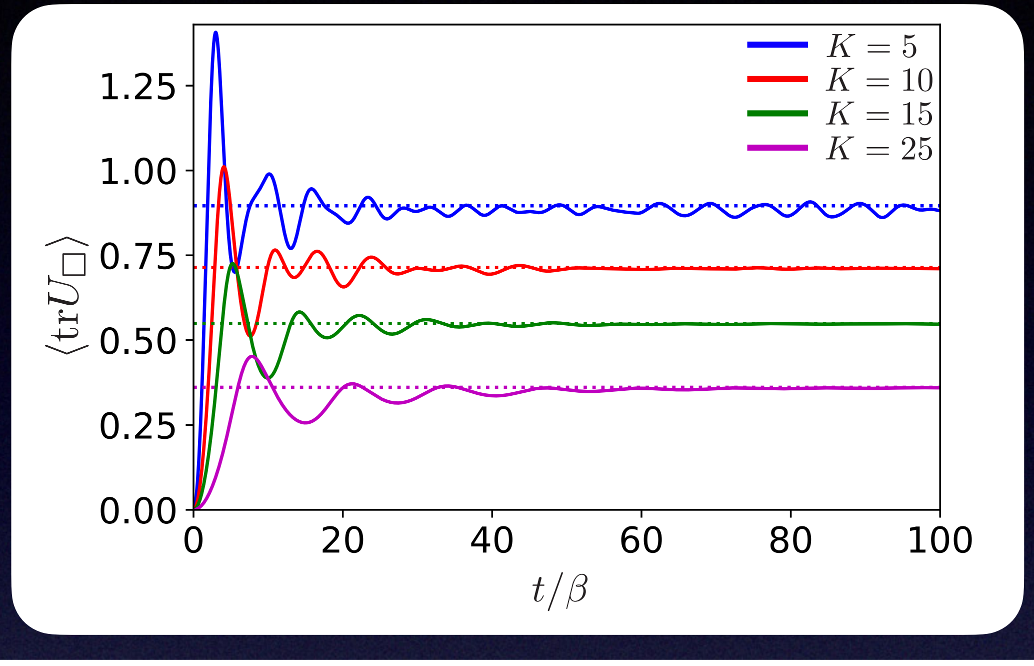

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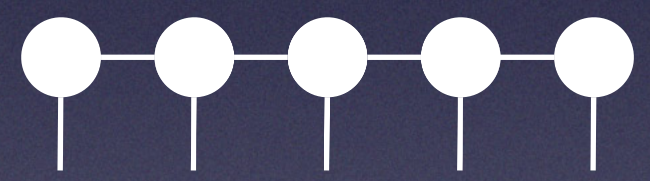

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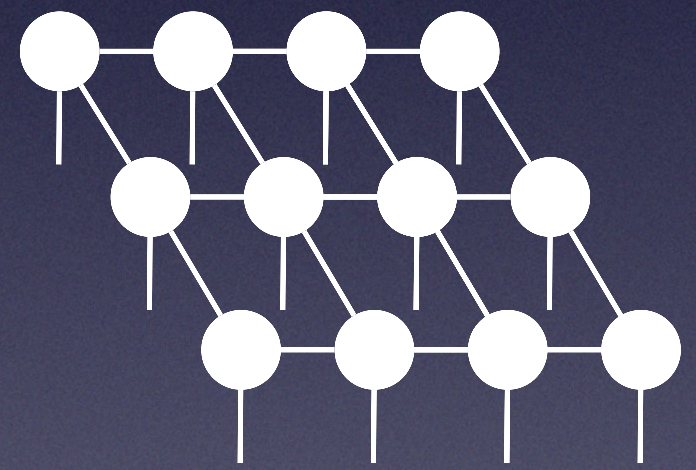
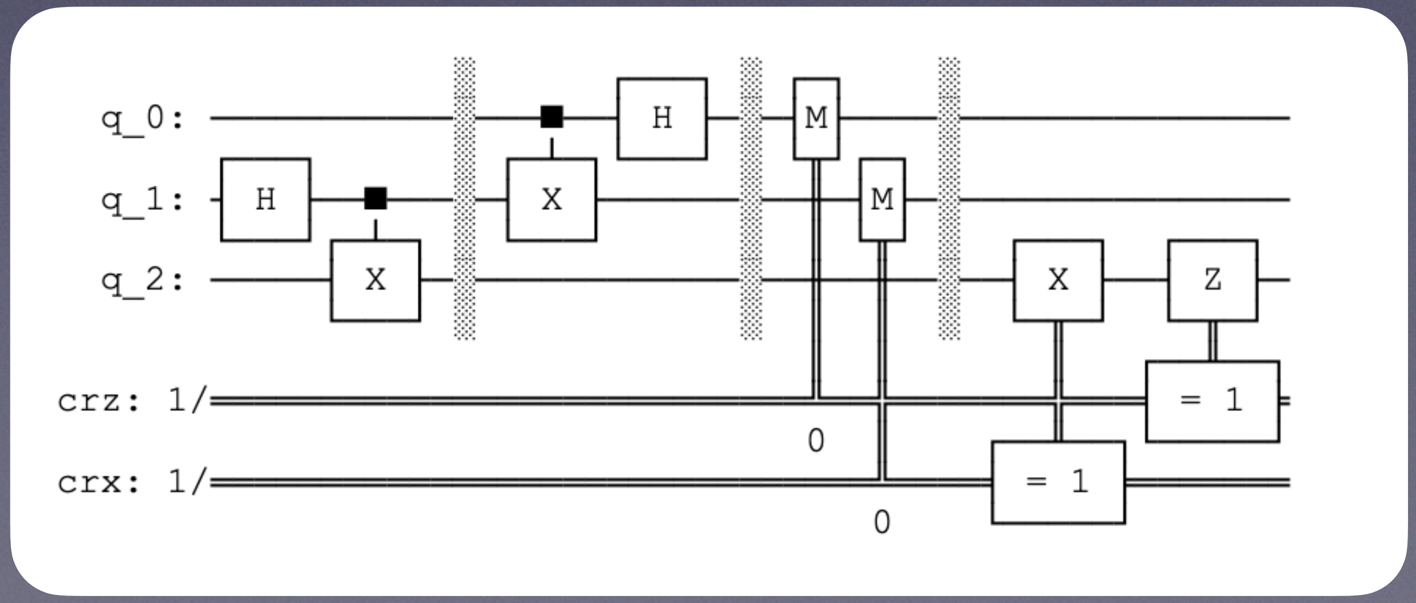


Quantum simulation



Difficulty in Hamiltonian gauge theory

Infinite degrees of freedom

Link variable is continuous (regularization required)

$U \in \mathrm{SU}(N)$

continuous

What approximation is compatible with gauge symmetry?

Large gauge redundancy

$$\dim \mathscr{H}_{\mathrm{phys}} \ll \dim \mathscr{H}_{\mathrm{total}}$$

need to solve Gauss law constraint

Outline

- **Formalism**
  - Kogut-Susskind Hamiltonian formalism
- **Application**
  - Confinement-deconfinement phase transition in mean field approximation (JHEP 09 (2023) 123)
  - Thermalization on a small lattice (Phys. Rev. D 103, 094502(2021))
  - QCD_2 at finite density (2311.11643)
  - Quantum scar (JHEP 09 (2023) 126)
  - Scrambling (Phys. Rev. D 104 (2021) 7, 074518)
- **Summary**

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- **Summary**

Kogut-Susskind Hamiltonian formalism

Kogut, Susskind ('75)

$SU(N)$ gauge theory (Temporal gauge $A_0 = 0$)

Commutation relation $[A_n^i(x), E_{mj}(x')] = i\delta_{nm}\delta_j^i\delta(x - x')$

Gauge field — Electric field

Hamiltonian $H = \int d^3x\left(\frac{g^2}{2}E^2(x) + \frac{1}{2g^2}B^2(x)\right)$

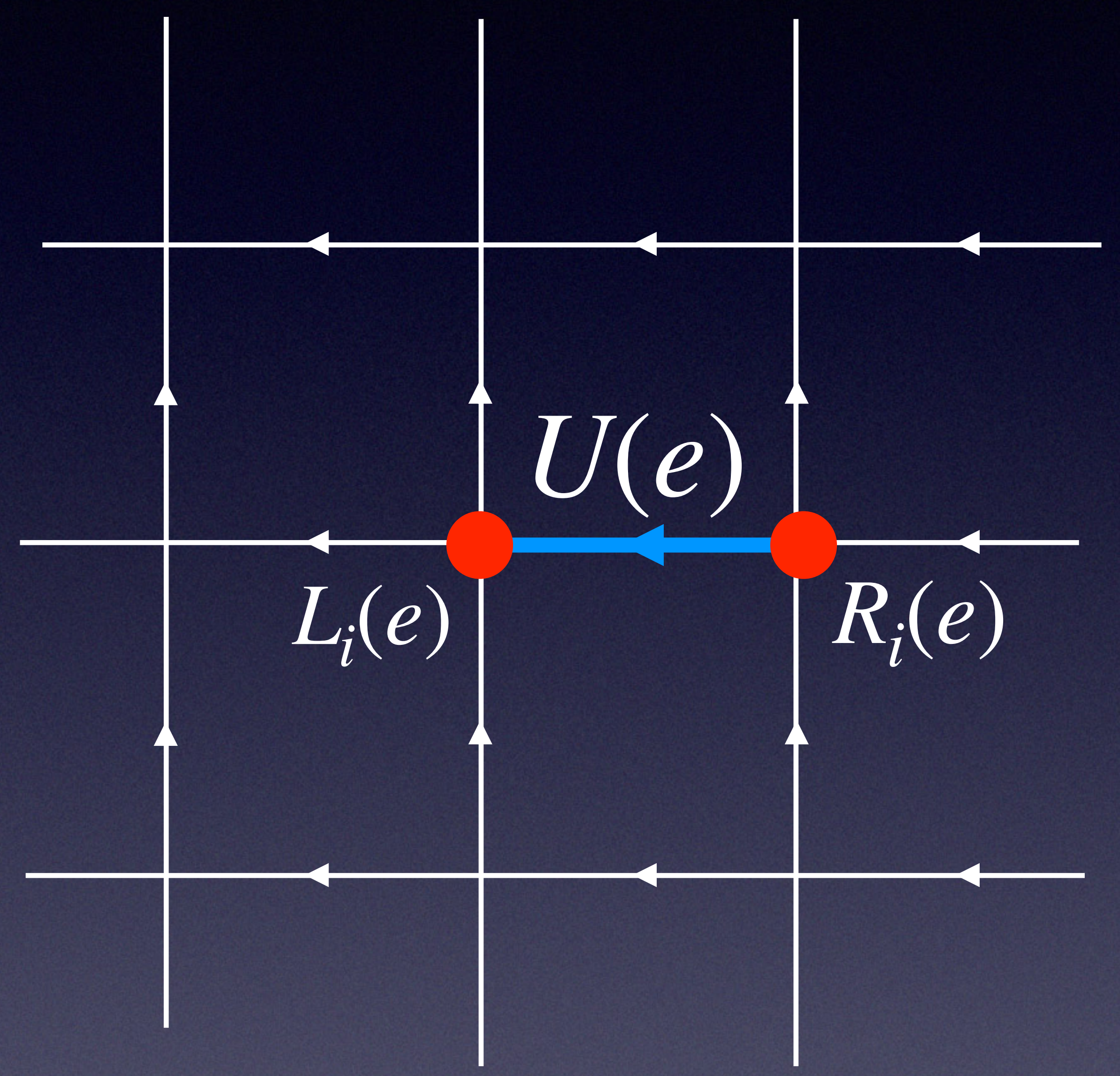
Magnetic field $B_l^i = \frac{1}{2}\epsilon_{lnm}(\partial_m A_n^i - \partial_m A_n^i + f_{jk}^i A_m^j A_n^k)$

Gauss law constraint $(\boldsymbol{D} \cdot \boldsymbol{E})^i \,|\,\Psi_{\text{phys}}\rangle = 0$

Kogut-Susskind Hamiltonian formalism

Kogut, Susskind, Phys. Rev. D 11, 395 (1975)

Time is continuous, space is discretized



$e^{i\int A} \to U(e)$:Link variable $\in \mathrm{SU}(N)$ on edge e

$L_i(e), R_i(e)$:Left and right electric fields $\in \mathrm{su}(N)$

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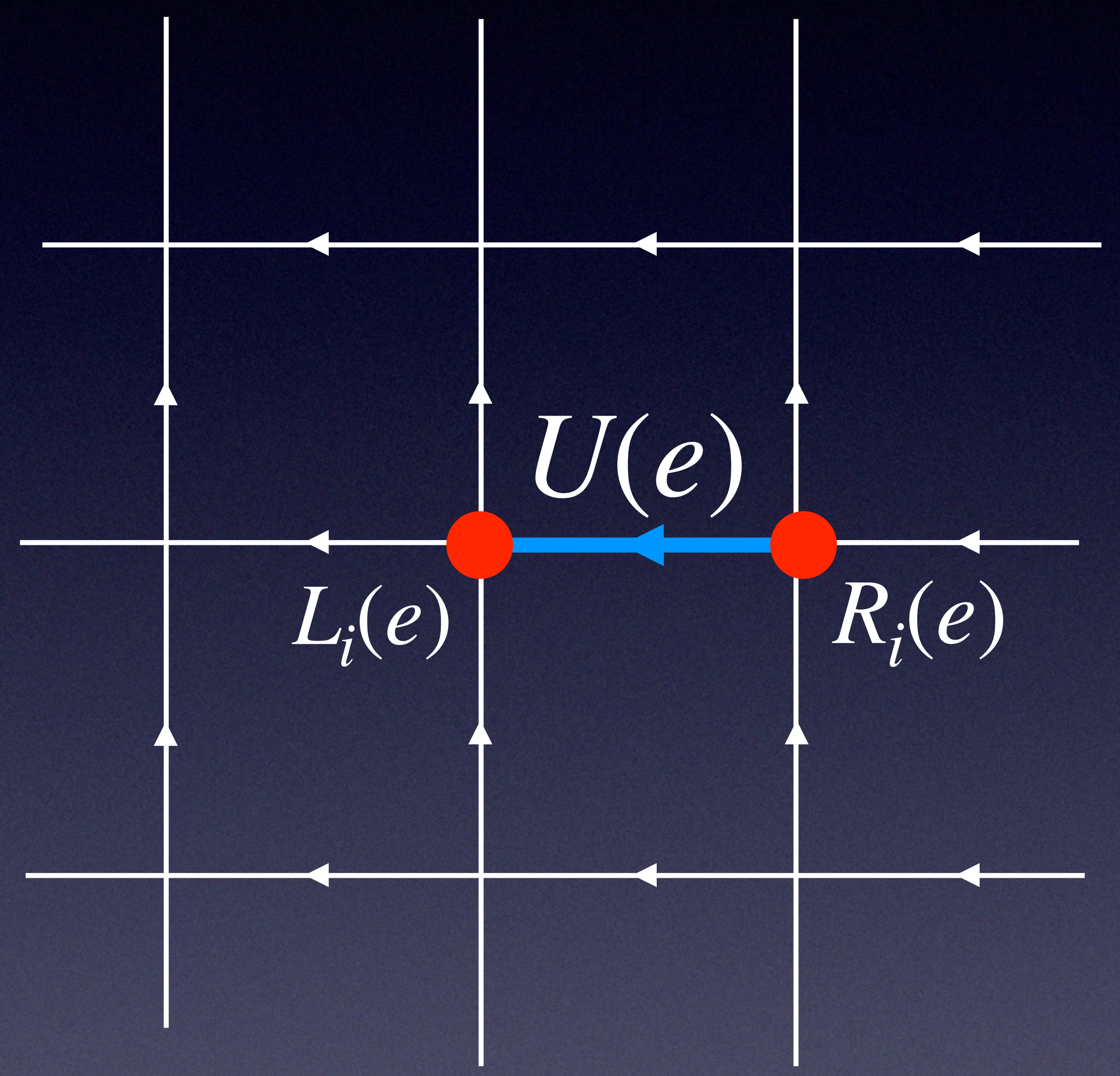

$e^{i\int A} \to U(e)$:Link variable $\in \mathrm{SU}(N)$ on edge e

$L_i(e), R_i(e)$:Left and right electric fields $\in \mathrm{su}(N)$

$L_i(e)$ and $R_i(e)$ are not independent

$$[U_{\mathrm{adj}}(e)]_i^j L_j(e) = R_i(e) \quad \Rightarrow \quad R_i^2(e) = L_i^2(e) =: E_i^2(e)$$

Commutation relation

$$[A_n^i(x), E_{mj}(x')]$$
$$= i\delta_{nm}\delta_j^i\delta(x - x')$$

Commutation relation

$$[A^i_n(x), E_{mj}(x')] = i\delta_{nm}\delta^i_j\delta(x-x')$$

$$[R_i(e), U(e')] = U(e)T_i\delta_{e,e'}$$
$$[L_i(e), U(e')] = T_iU(e)\delta_{e,e'}$$
$$[L_i(e), L_j(e')] = -\mathrm{i}f^k{}_{ij}L_k(e)\delta_{e,e'}$$
$$[R_i(e), R_j(e')] = \mathrm{i}f^k{}_{ij}R_k(e)\delta_{e,e'}$$

Commutation relation

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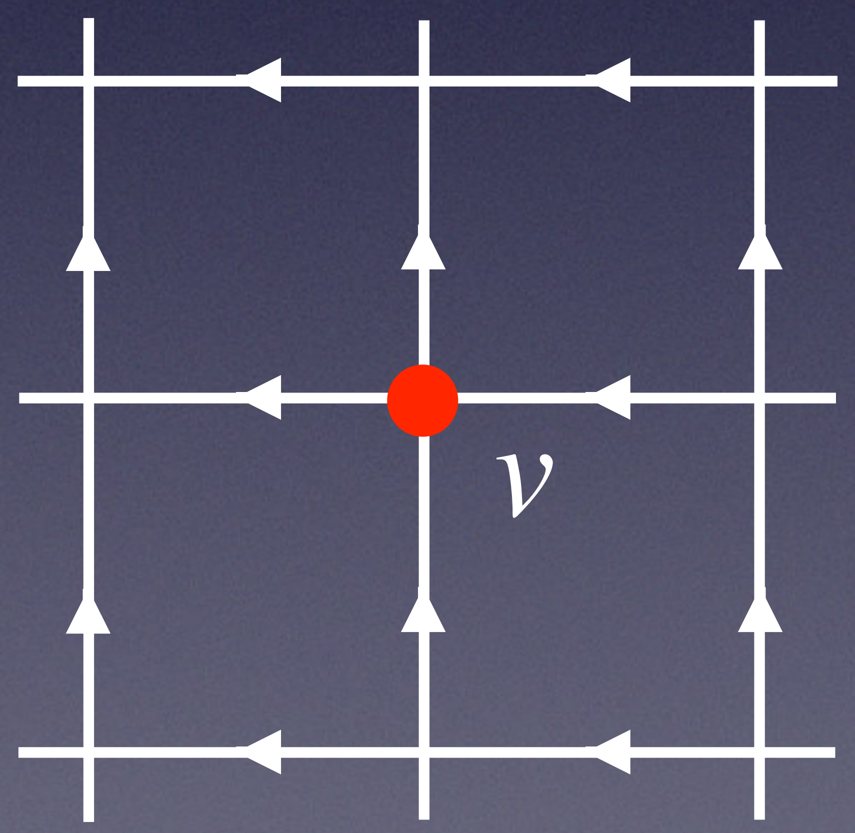


Gauss law constraint $(D \cdot E)^i\,|\Psi_{\mathrm{phys}}\rangle = 0$

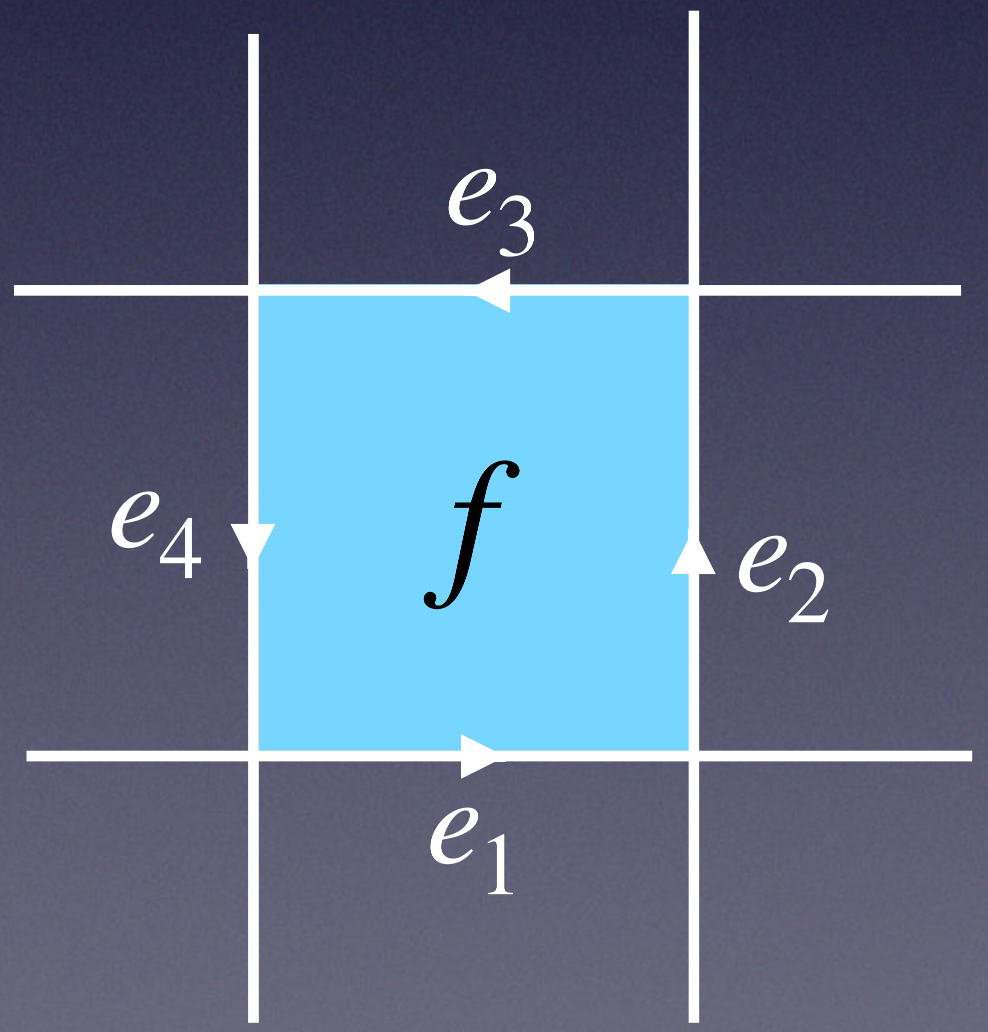
$$\Rightarrow \left(\sum_{e\in C_1|\mathrm{s}(e)=v} R_i(e) - \sum_{e\in C_1|\mathrm{t}(e)=v} L_i(e)\right)|\Psi_{\mathrm{phys}}\rangle = 0$$

C_1**:set of edges,** s,t**: source and target functions**

Hamiltonian

$$H = \frac{1}{2}\sum_{e\in C_1}(E(e))^2 - \frac{K}{2}\sum_{f\in C_2}(\mathrm{tr}U(f) + \mathrm{tr}U^\dagger(f))$$

C_2:set of faces



$$U(f) := U(e_4)U(e_3)U(e_2)U(e_1)$$

Regularization:

$SU(3) \to SU(3)_k$ **:Quantum deformation**

which preserve properties of gauge symmetry

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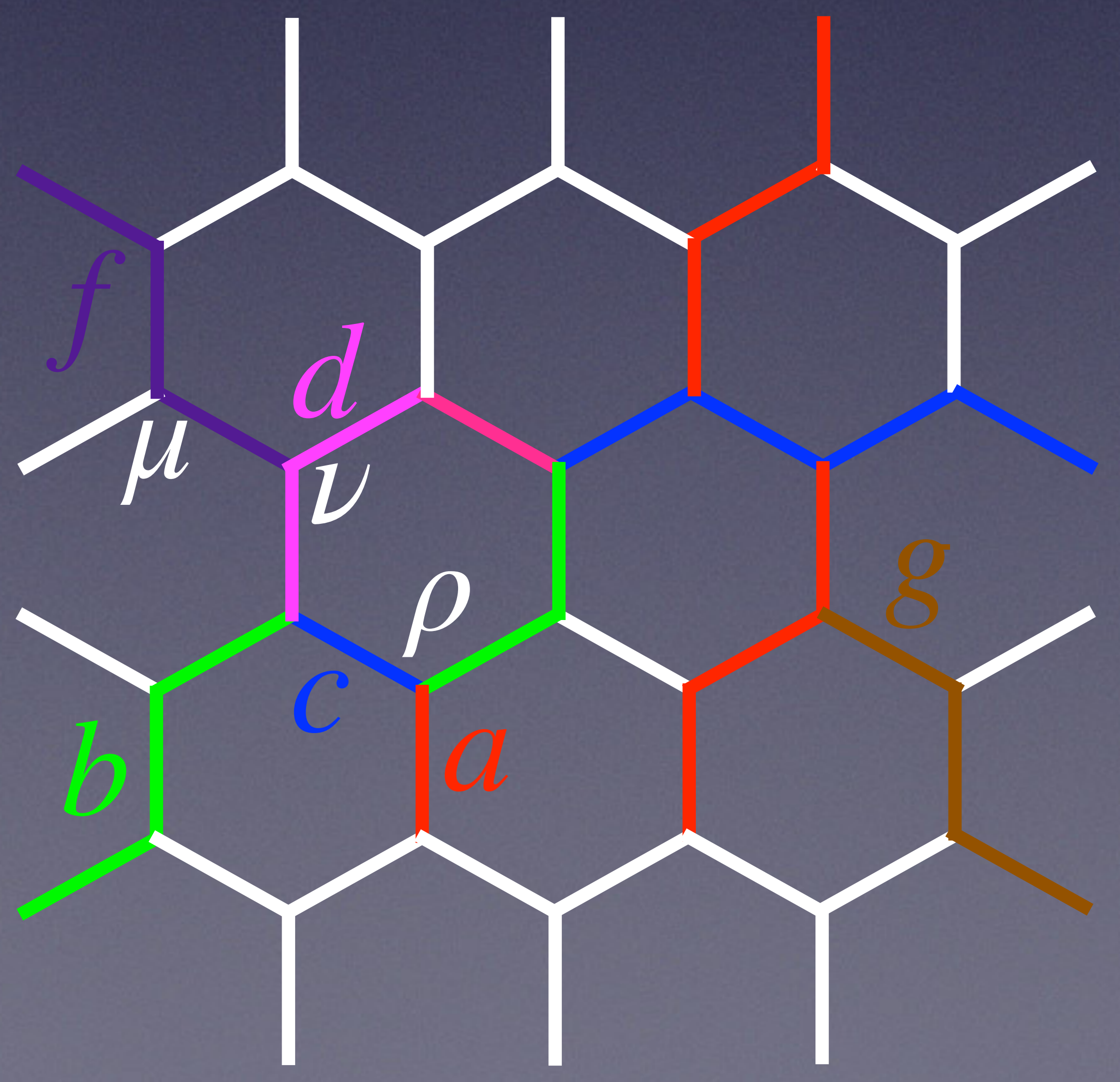
$SU(3) \to SU(3)_k$:Quantum deformation

which preserve properties of gauge symmetry

Solving Gauss law constraint:

Physical states are network of Wilson lines

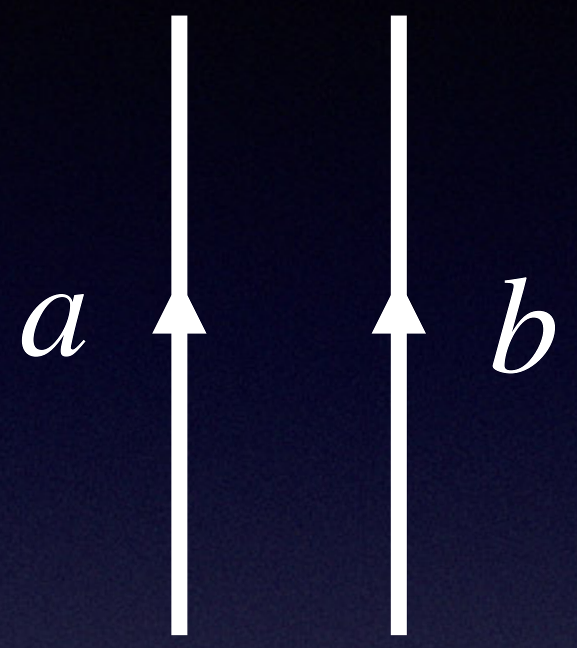


$a, b, c, \cdots$: representation of Wilson lines, e.g., fundamental, adjoint,…

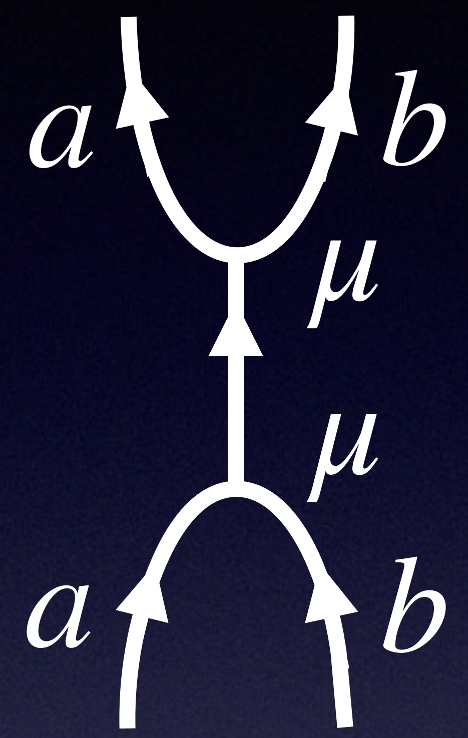
$\mu, \nu, \rho, \cdots \in N^c_{ab}$: Multiplicity index

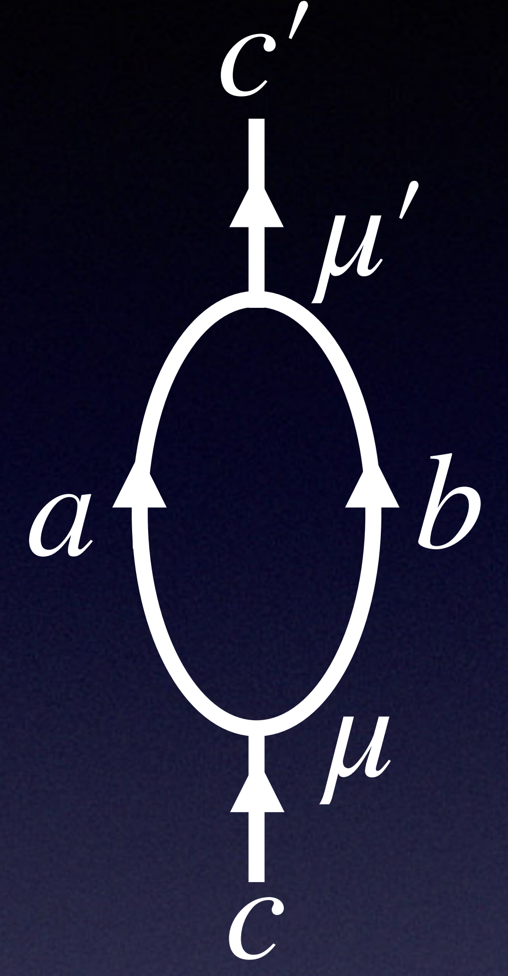
with fusion rule $a \times b = \sum_c N^c_{ab}$

Algebra of Wilson lines for $\mathrm{SU}(3)_k$

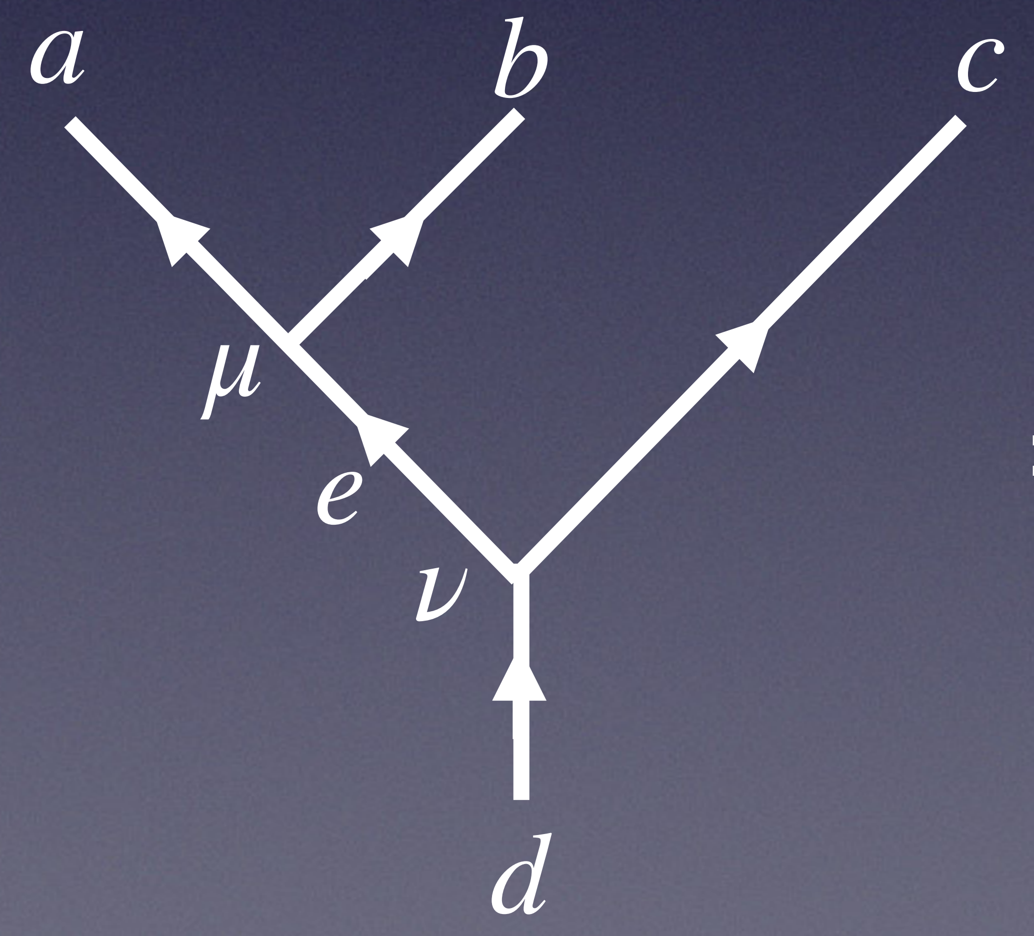


$$= \sum_{c,\mu} \sqrt{\frac{d_c}{d_a d_b}}$$

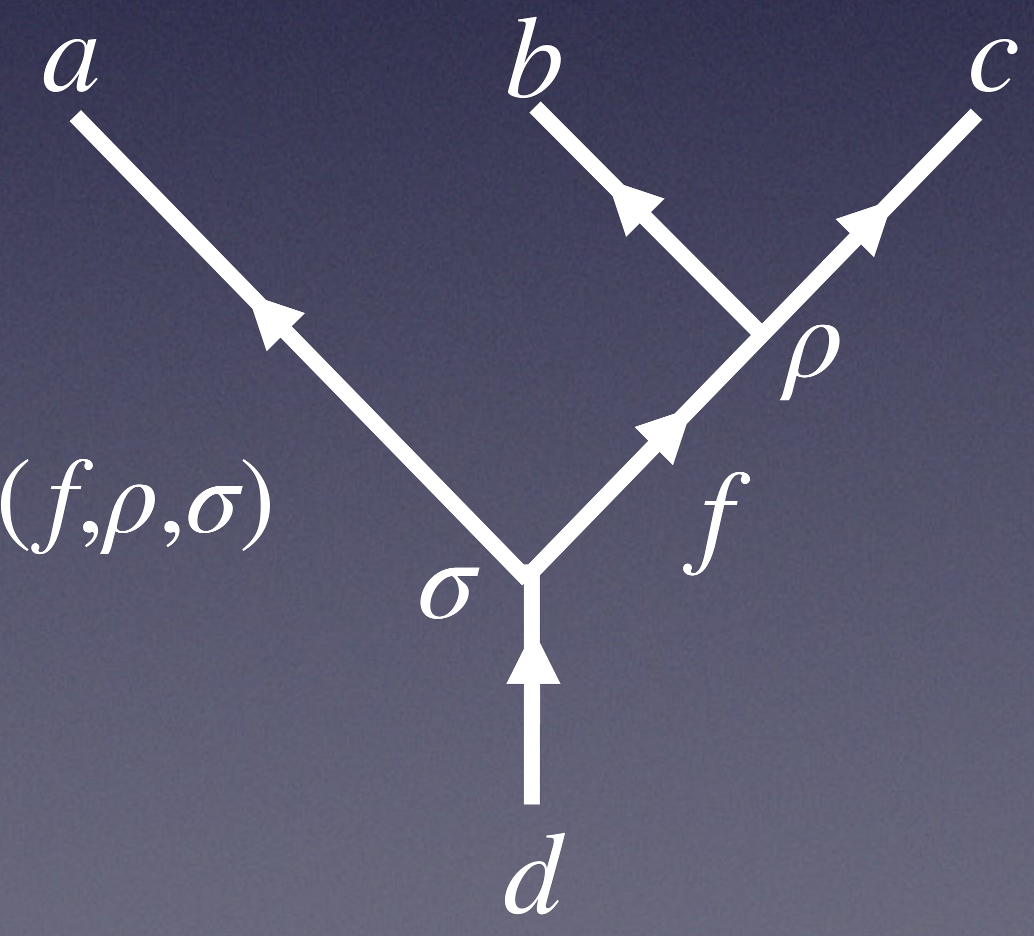




$$= \delta_c^{c'} \delta_\mu^{\mu'} \sqrt{\frac{d_a d_b}{d_c}}$$



$$= \sum_{f,\rho,\sigma} [F_d^{abc}]_{(e,\mu,\nu)(f,\rho,\sigma)}$$



+consistency condition

$\mathrm{SU}(3)_k$ Yang-Mills theory

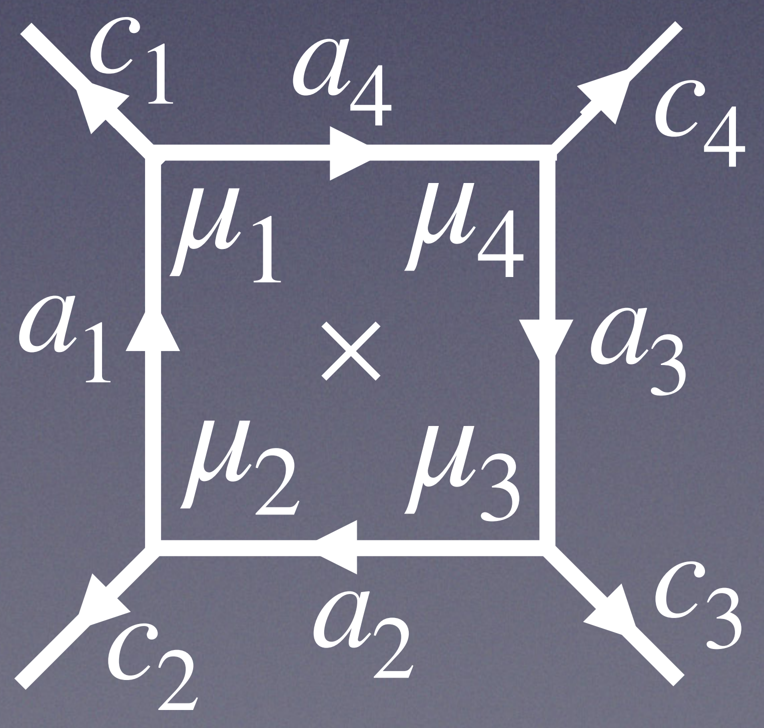
Hamiltonian $$H = \frac{1}{2}\sum_{e\in C_1}(E(e))^2 - \frac{K}{2}\sum_{f\in C_2}(\mathrm{tr}U(f) + \mathrm{tr}U^\dagger(f))$$

Action on a state

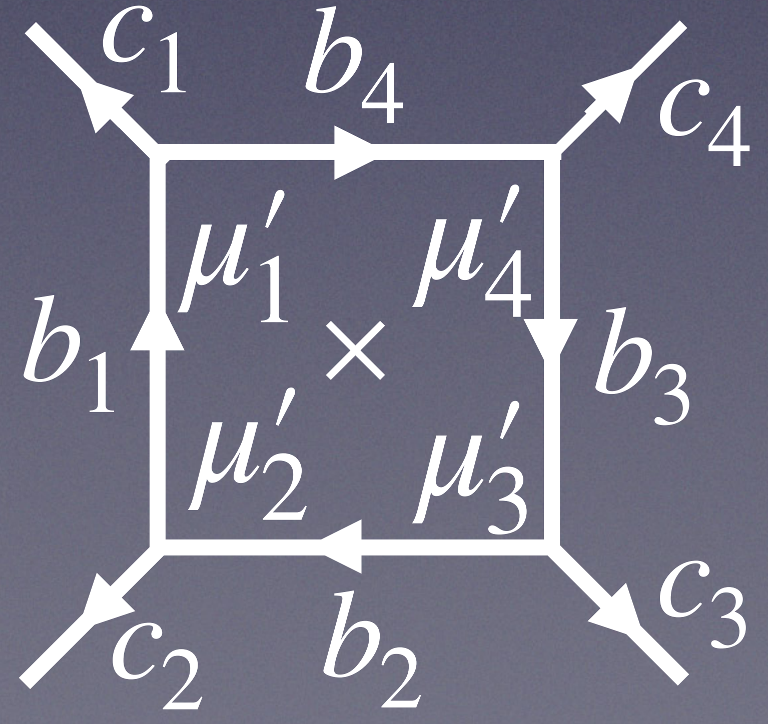
Electric fields E_i^2  a $= C_2(a)$  a

Casimir

Wilson loop

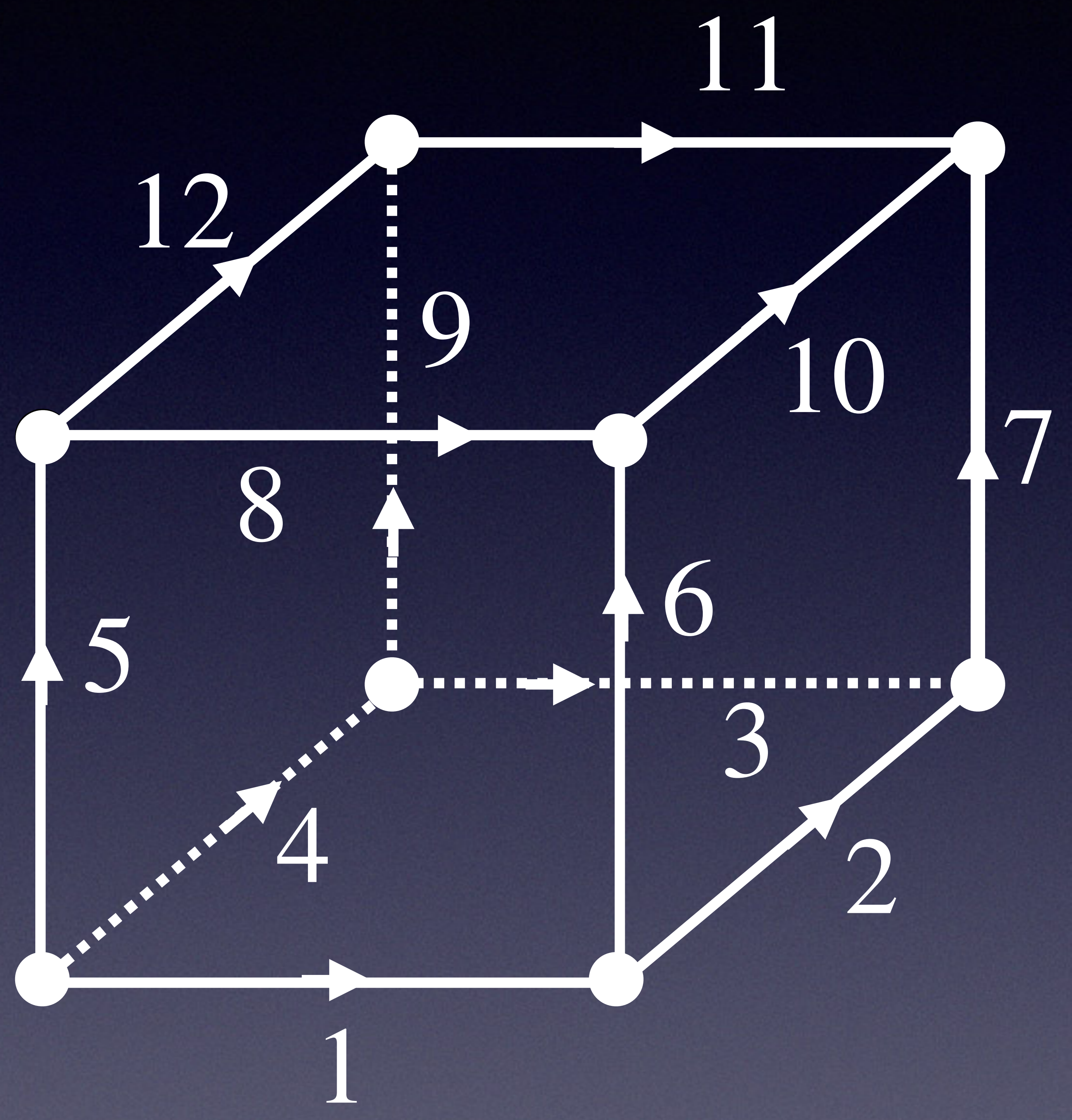
$\mathrm{tr}U$ 

$$= \prod_{i=1}^{4}\sum_{b_i}[F_{b_i}^{c_i a_{i-1}\frac{1}{2}}]_{(a_i,\mu_i,\mu'_{i+1})(b_{i-1},\mu_{i-1},\mu'_i)}$$



Thermalization on a small lattice

Small lattice system

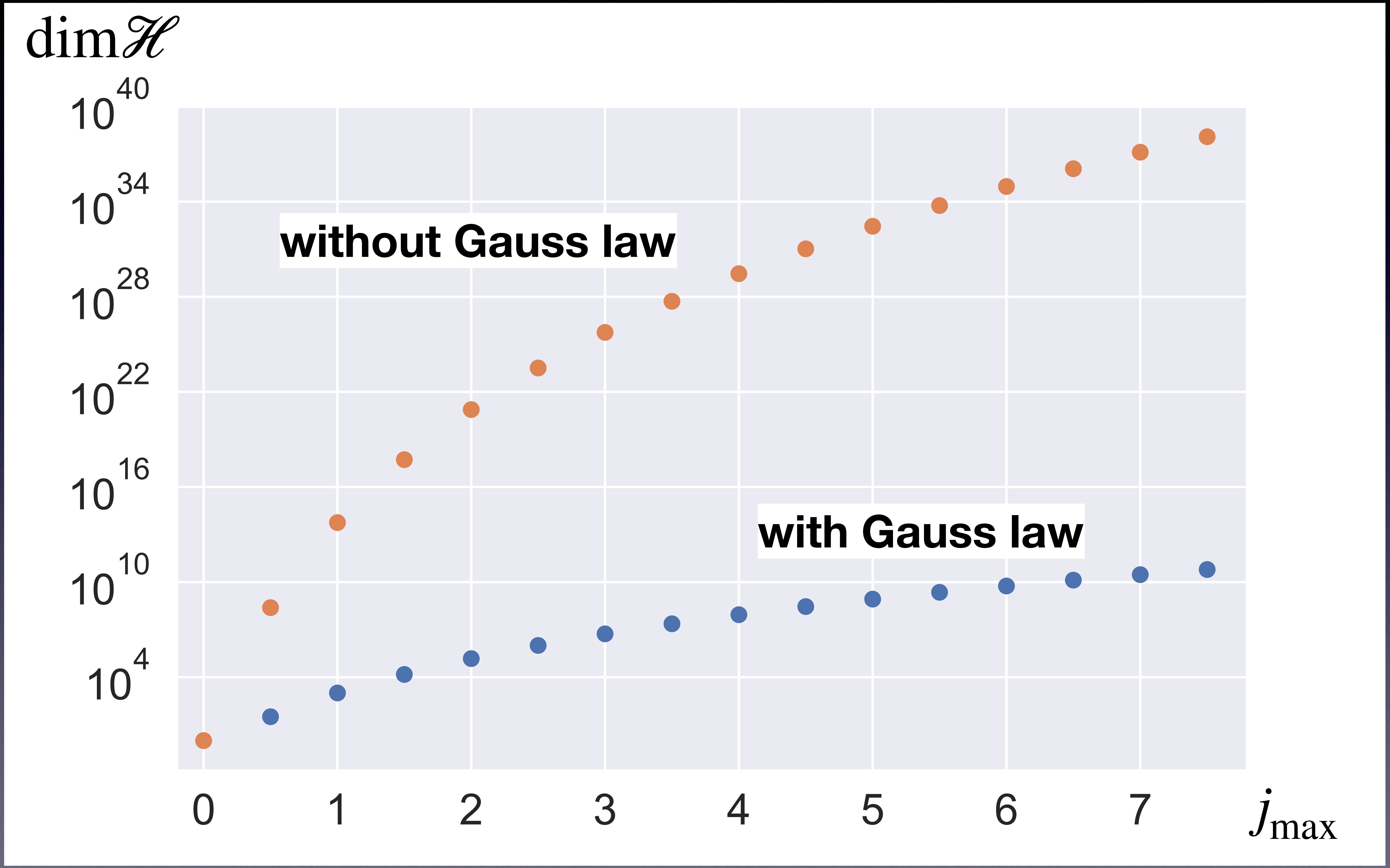


Basis

$$|j_1,\cdots,j_{12}\rangle = |j_1,j_2,j_6\rangle\,|j_2,j_3,j_7\rangle\,|j_3,j_4,j_8\rangle\,|j_1,j_4,j_5\rangle$$
$$|j_6,j_9,j_{10}\rangle\,|j_7,j_{10},j_{11}\rangle\,|j_8,j_{11},j_{12}\rangle\,|j_5,j_9,j_{12}\rangle$$

Cutoff $j_i \leq j_{\max} = k/2$

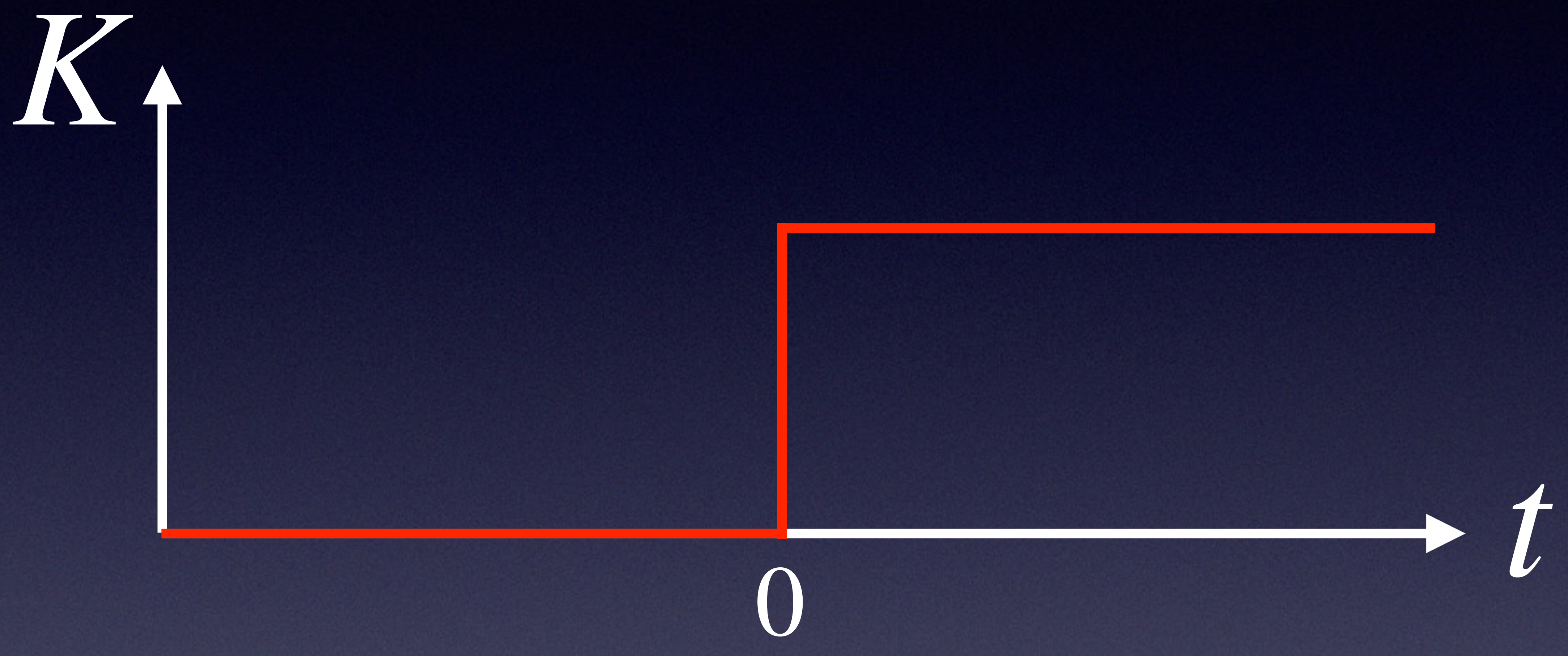
Dimension of Hilbert space



We employ $j_{\max} = 4$: $\dim\mathscr{H} = 87{,}426{,}119$

Setup

In order to mimic heavy ion collision experiments, the interaction quenching

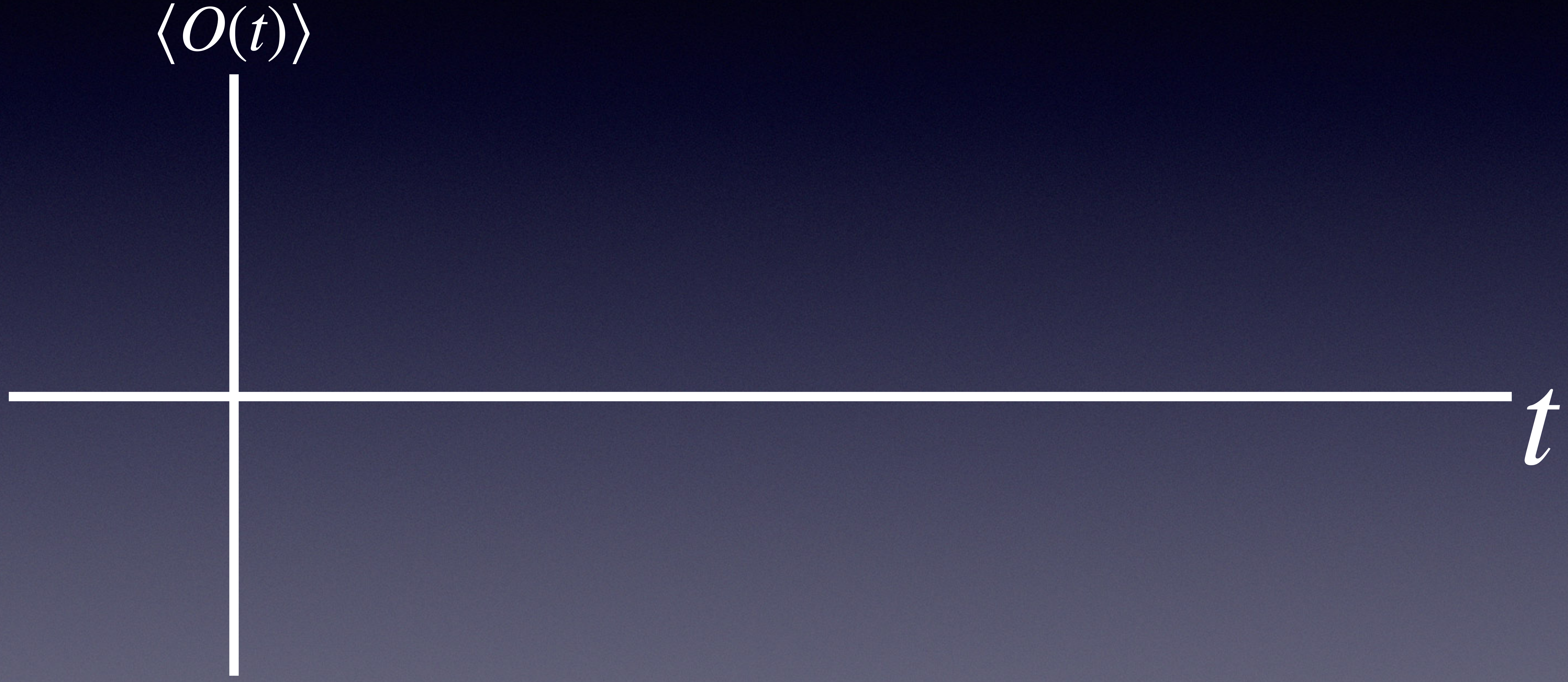


$$t < 0 \quad |\mathrm{Vac}\rangle_{K=0}$$

$$t \geq 0 \quad |\Psi(t)\rangle = e^{-\mathrm{i}Ht}|\mathrm{Vac}\rangle_{K=0}$$

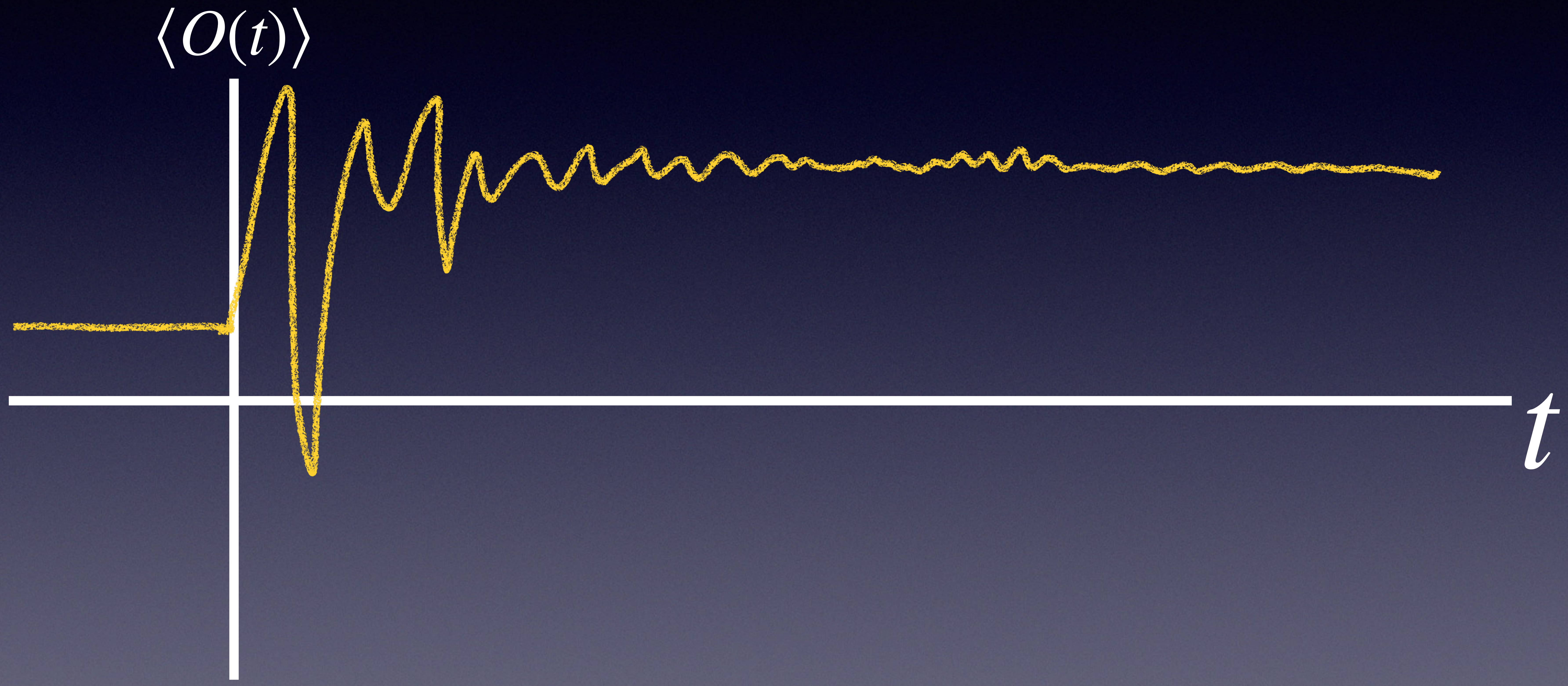
Expected behavior

for an operator O $\langle O(t) \rangle := \langle \Psi(t) | O | \Psi(t) \rangle$



Expected behavior

for an operator O $\langle O(t) \rangle := \langle \Psi(t) \,|\, O \,|\, \Psi(t) \rangle$



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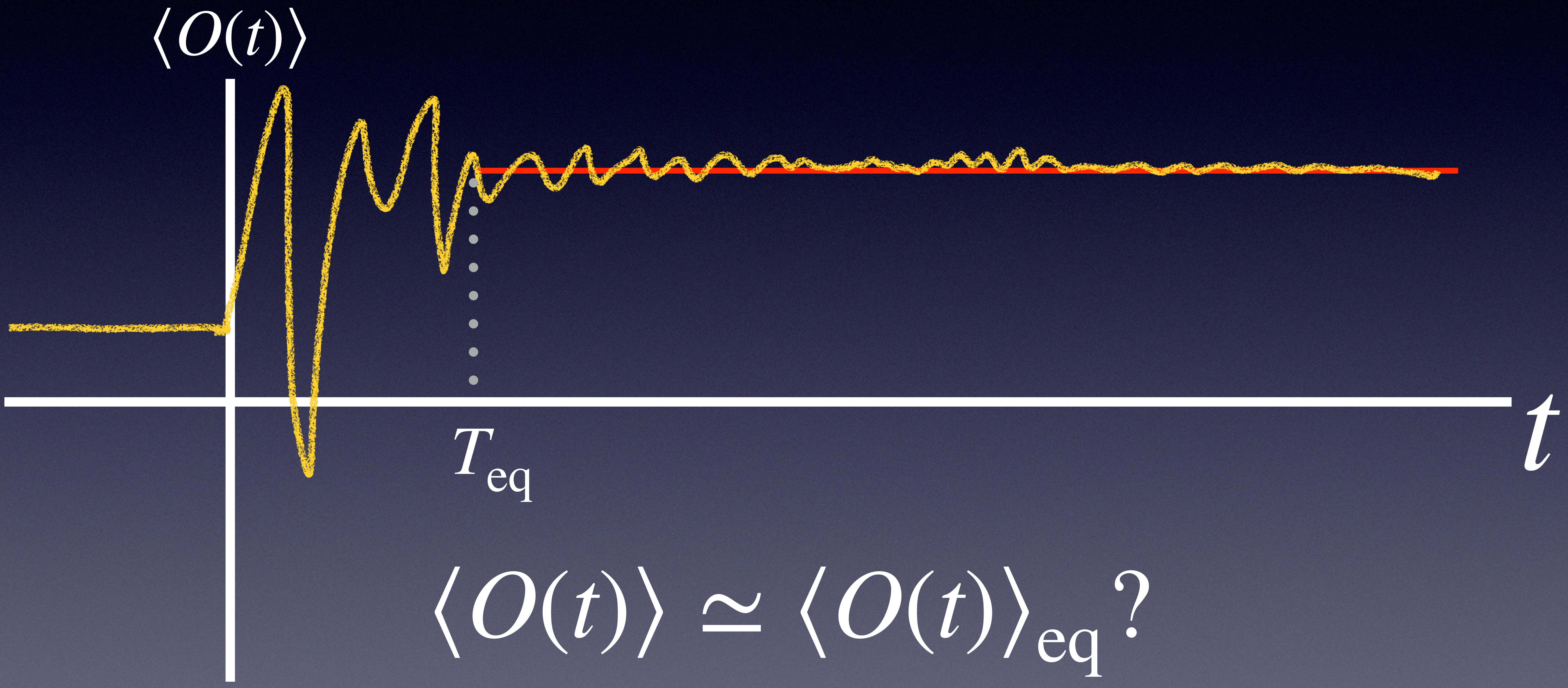


$\langle O(t)\rangle \simeq \langle O(t)\rangle_{\rm eq}$?

Temperature and Canonical Ensemble

Energy is fixed by an initial condition

$$E = \langle H \rangle = \langle \Psi(t) \,|\, H \,|\, \Psi(t) \rangle$$

(Independent of time)

For a given energy,
a canonical distribution that reproduces
the expected value can be defined

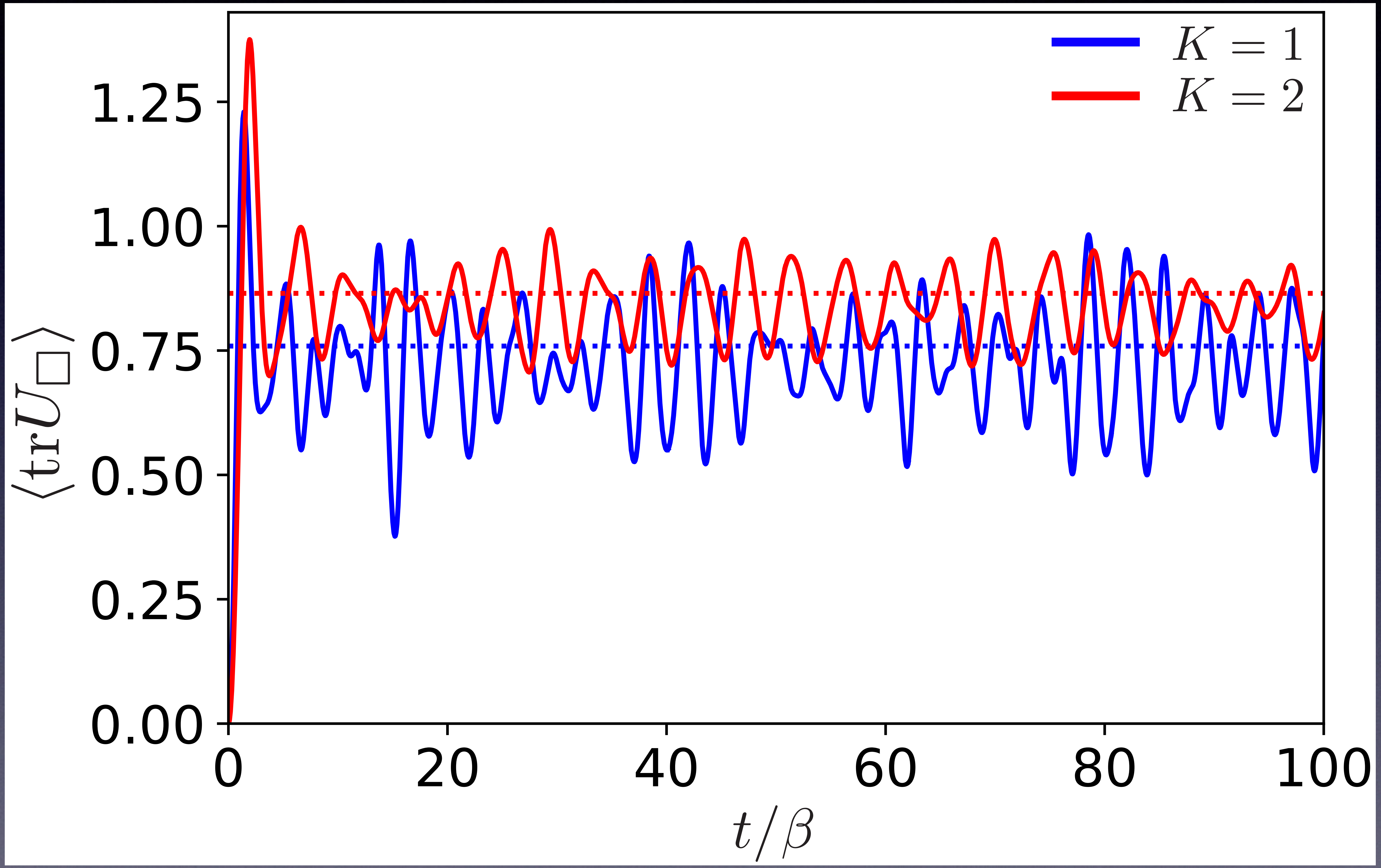
$$E = \langle H \rangle_{\mathrm{eq}} := \mathrm{tr} \rho_{\mathrm{eq}} H \quad \textbf{with} \quad \rho_{\mathrm{eq}} = \frac{e^{-\beta H}}{\mathrm{tr} e^{-\beta H}}$$

Numerical results

Expected value of Wilson loop

Strong coupling (low temperature $T < E_1$)

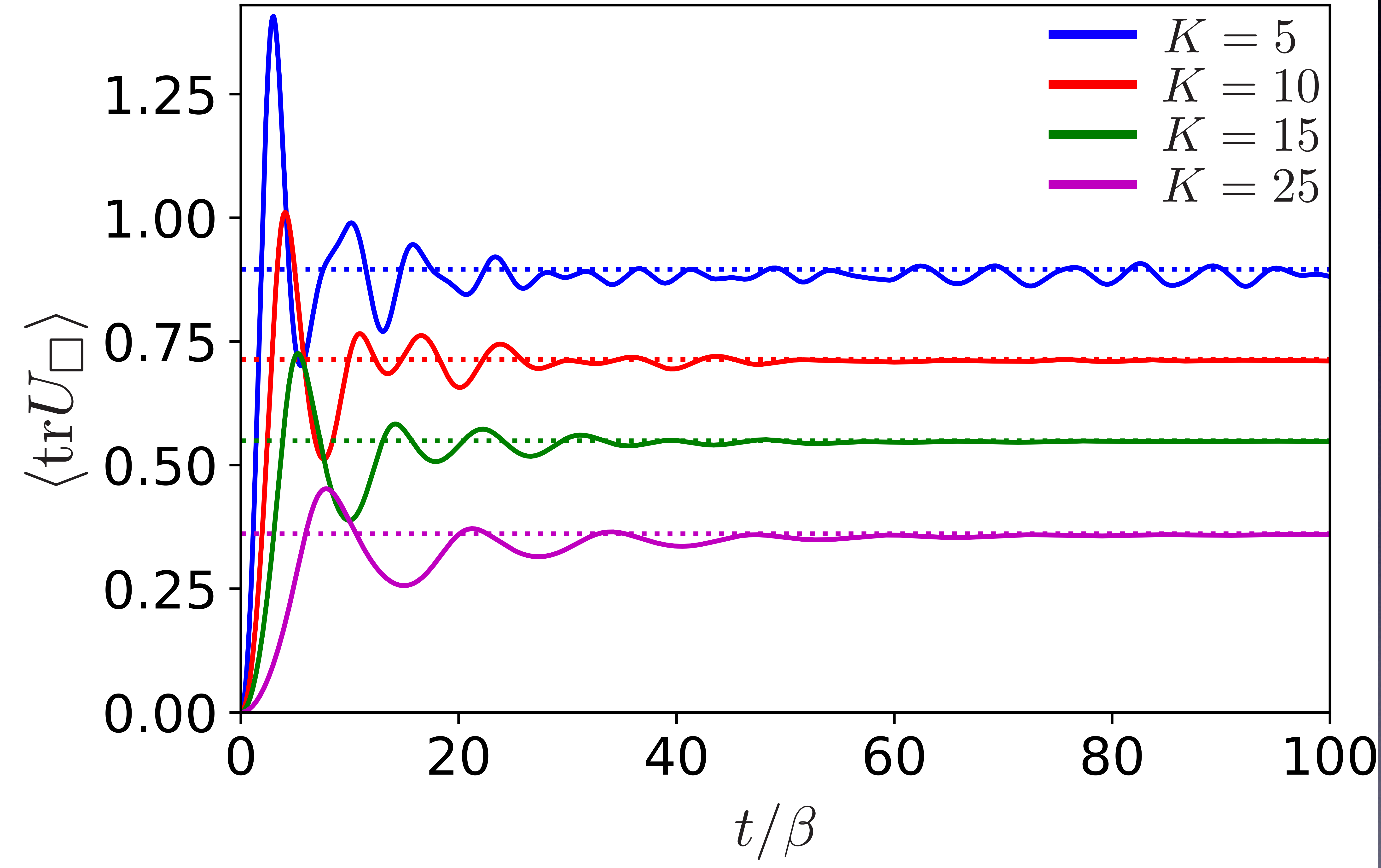
first excited energy



Fluctuations are not small.

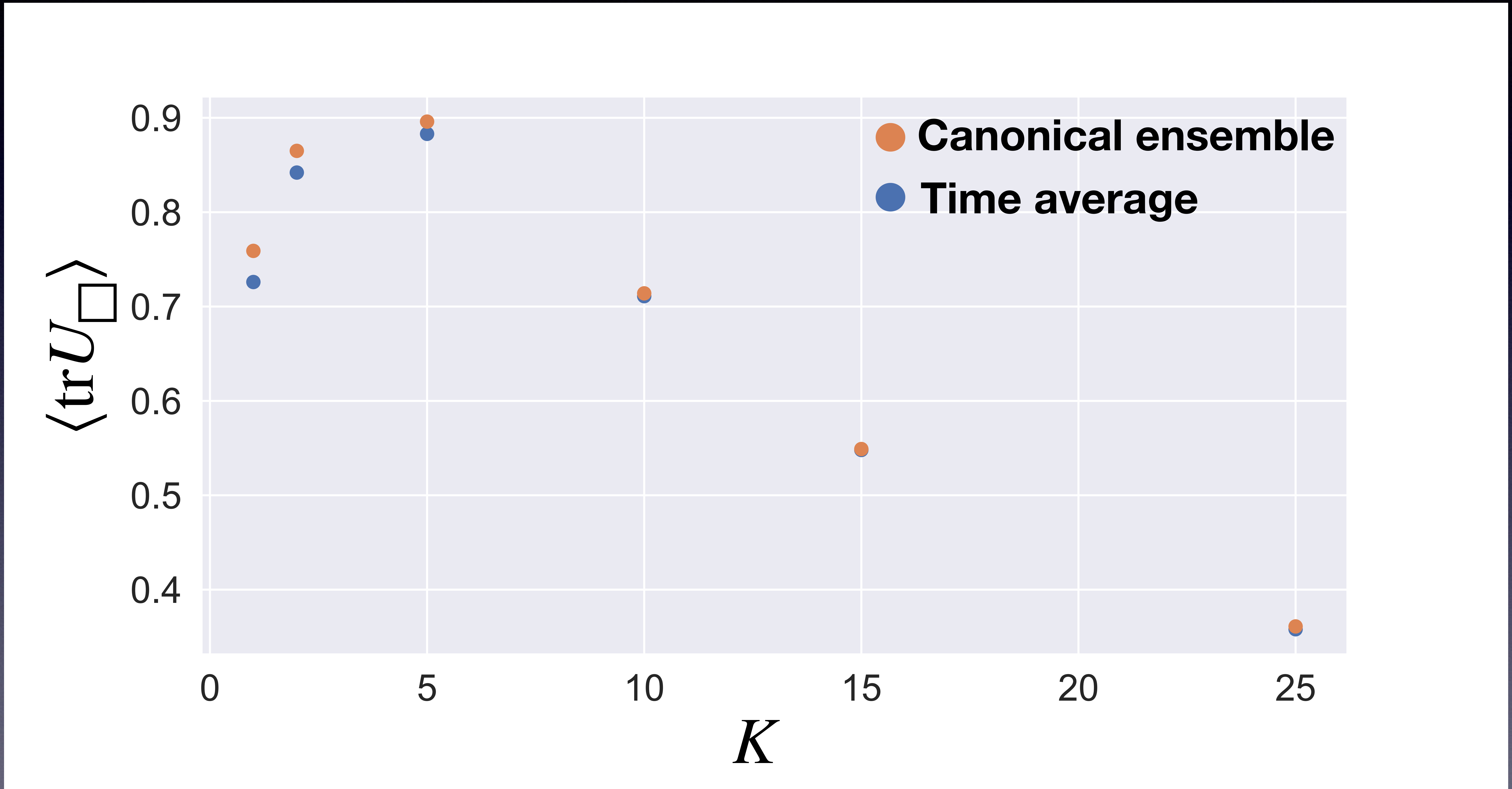
Expected value of Wilson loop

Weak coupling (high temperature $T > E_1$)



Steady state observed

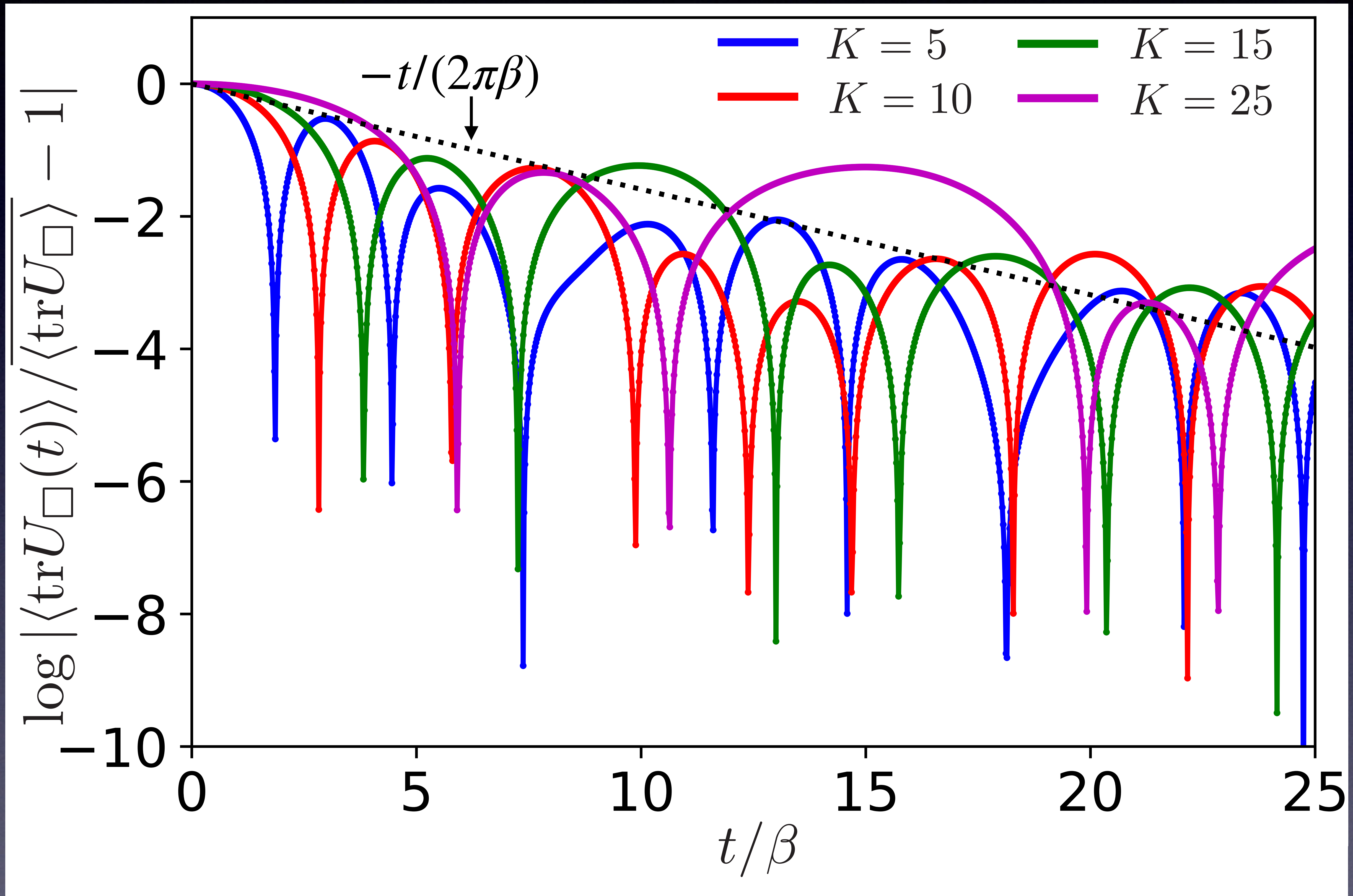
Long-time average vs canonical ensemble



Difference is less than 1% for $K > 5$

Relaxation to equilibrium

$$\langle \mathrm{tr} U_{\square}(t) \rangle - \overline{\langle \mathrm{tr} U_{\square} \rangle} \sim e^{-t/\tau_{\mathrm{eq}}}$$

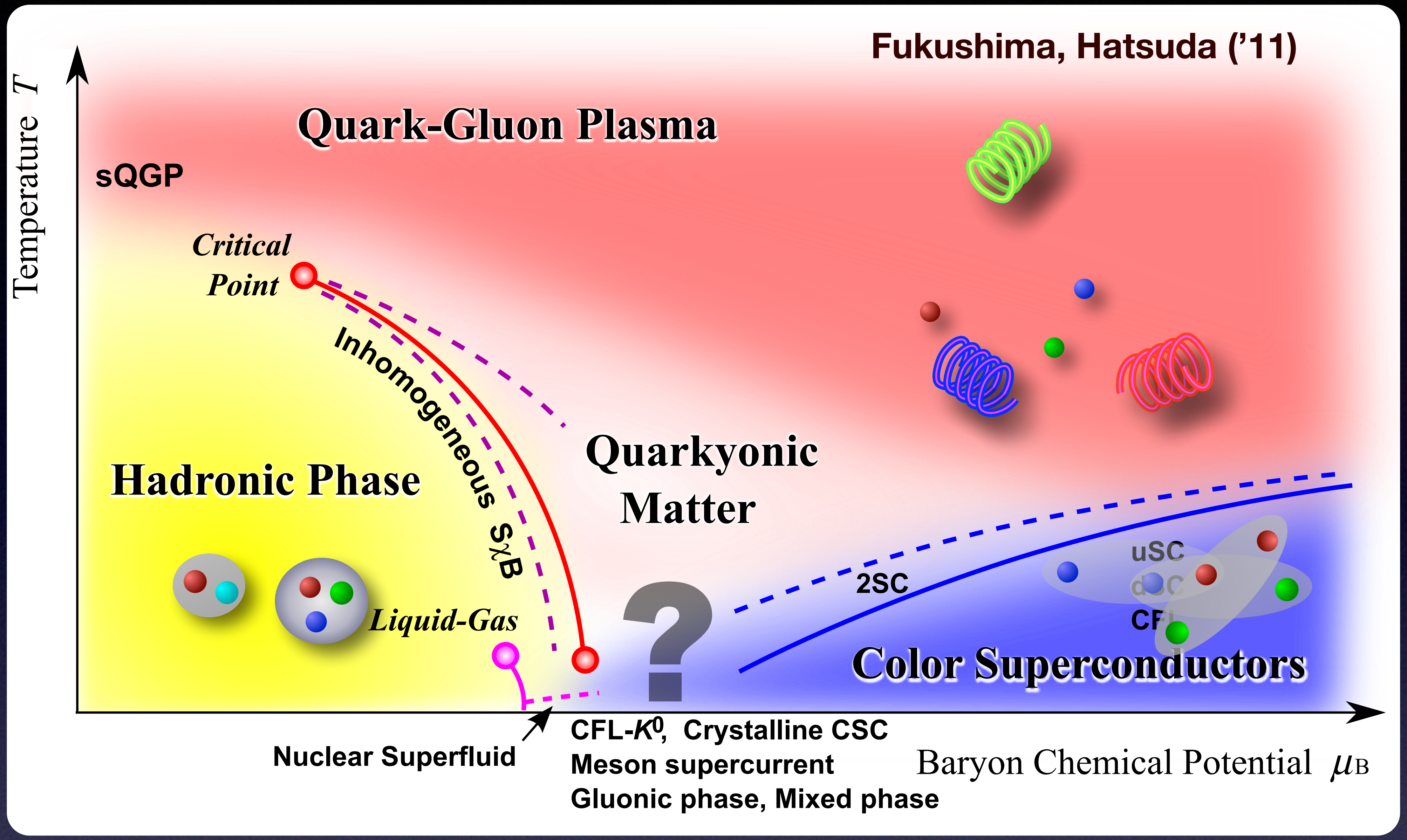


Close to Boltzmann time $2\pi\beta$.

Goldstein, Hara, Tasaki, New J. Phys. 17 (2015) 045002

QCD_2 at finite density

QCD at finite density



- What is the equation of state for QCD at finite density?
- How does the quark distribution function change when transitioning from baryonic matter to quark matter?
  - What kind of phase is realized? An inhomogenous phase?

QCD_2

QCD_2

Properties of (1+1) dimensions

- Gauge fields are nondynamical
- Hilbert space is finite dimensional in Open Boundary Condition(OBC)

(dimensionless)QCD_2 Hamiltonian

$$J = \frac{ag_0}{2}, w = \frac{1}{2g_0 a}, m = m_0/g_0$$

We use $g_0 = 1$ unit

$$H/g_0 = J \sum_{n=1}^{N-1} E_i^2(n)$$ **Electric field term**

$$+w \sum_{n=1}^{N-1} \left(\chi^\dagger(n+1)U(n)\chi(n) + \chi^\dagger(n)U^\dagger(n)\chi(n+1) \right)$$

Hopping term

$$+m \sum_{n=1}^{N} (-1)^n \chi^\dagger(n)\chi(n)$$ **Mass term**

Elimination of Link variables U

Sala, Shi, Kühn, Bañuls, Demler, Cirac, Phys. Rev. D 98, 034505 (2018)
Atas, Zhang, Lewis, Jahanpour, Haase , Muschik , Nature Commun. 12, 6499 (2021)

$$\Theta\chi(n)\Theta^\dagger := U(n-1)U(n-2)\cdots U(1)\chi(n)$$

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Atas, Zhang, Lewis, Jahanpour, Haase , Muschik , Nature Commun. 12, 6499 (2021)

$$\Theta\chi(n)\Theta^\dagger := U(n-1)U(n-2)\cdots U(1)\chi(n)$$

$$\Theta H\Theta^\dagger = J\sum_{n=1}^{N-1}\Big(\sum_{m=1}^{n}\chi^\dagger(m)T_i\chi(m)\Big)^2$$ **Electric fields term**

$$+w\sum_{n=1}^{N-1}\Big(\chi^\dagger(n+1)\chi(n)+\chi^\dagger(n)\chi(n+1)\Big)$$

Hopping term

$$+m\sum_{n=1}^{N}(-1)^n\chi^\dagger(n)\chi(n)$$ **mass term**

As a variational ansatz of wave function

- We employ a matrix product state

$$|\psi\rangle = \sum_{\{n_i\}} |n_1\rangle \cdots |n_N\rangle \mathrm{tr} M_1^{n_1} \cdots M_N^{n_N}$$

$[M_i^{n_i}]_{ij}$: $D \times D$ matrix

- Optimize the wave function by density matrix renormalization group technique

$$E = \min_{\psi} \langle \psi | H | \psi \rangle$$

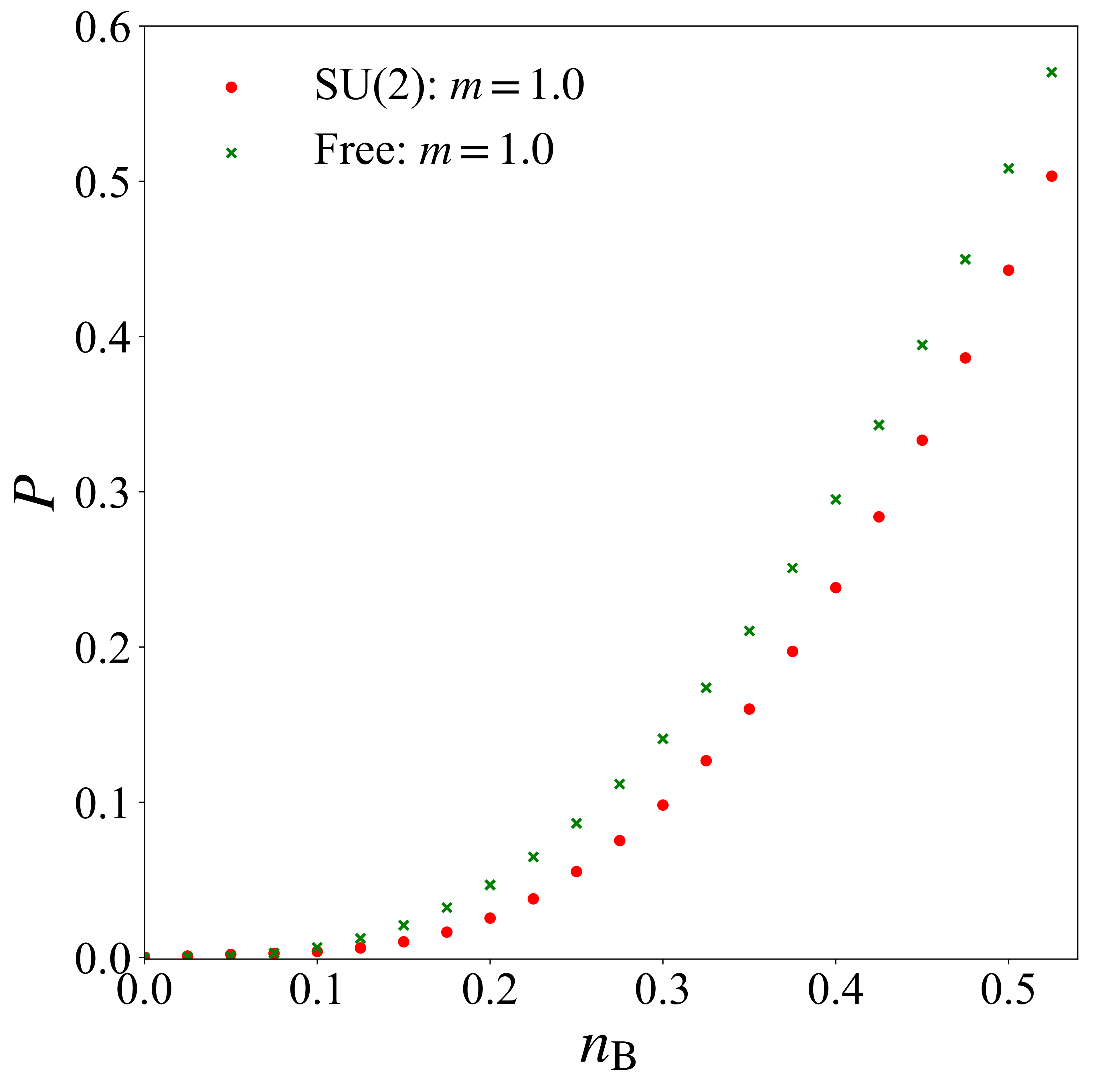
We employ iTensor

Numerical results

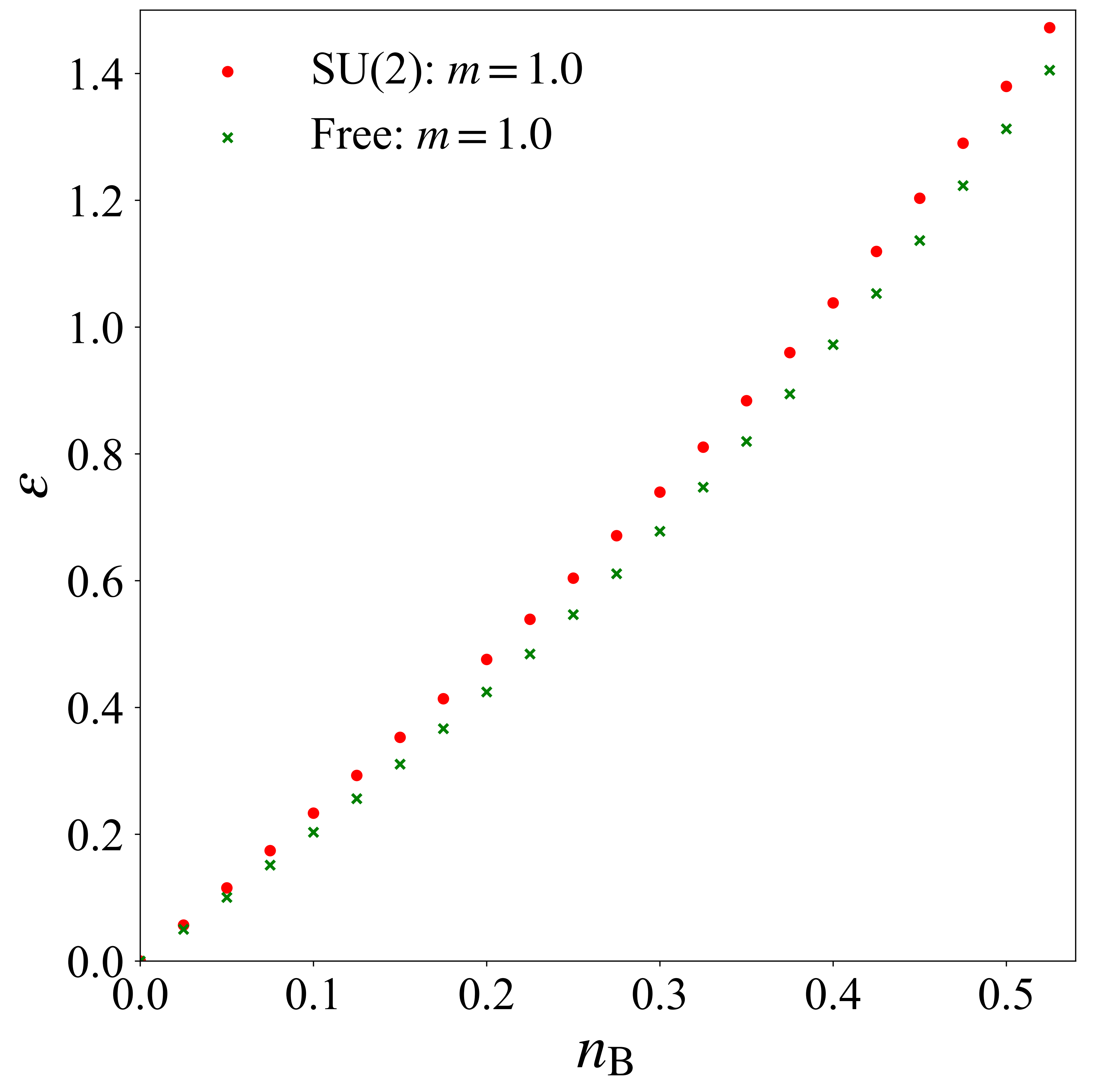
Color SU(2), 1 flavor, vacuum

$J = 1/8 \;\; w = 2 \;\; V = 40 \;\; \dim\mathscr{H} = 2^{320}$

Pressure

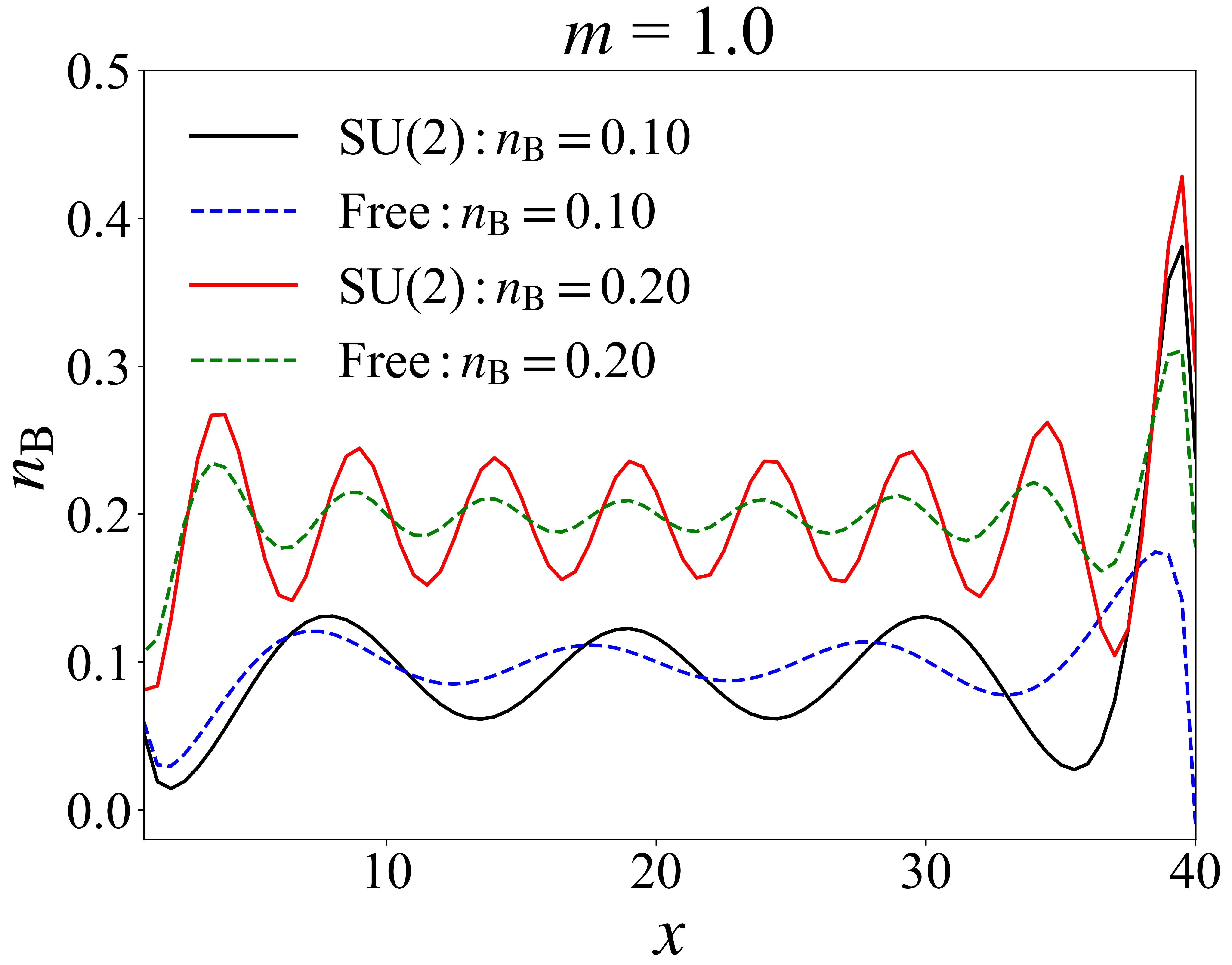


Energy density



Inhomogeneous phase (density wave)

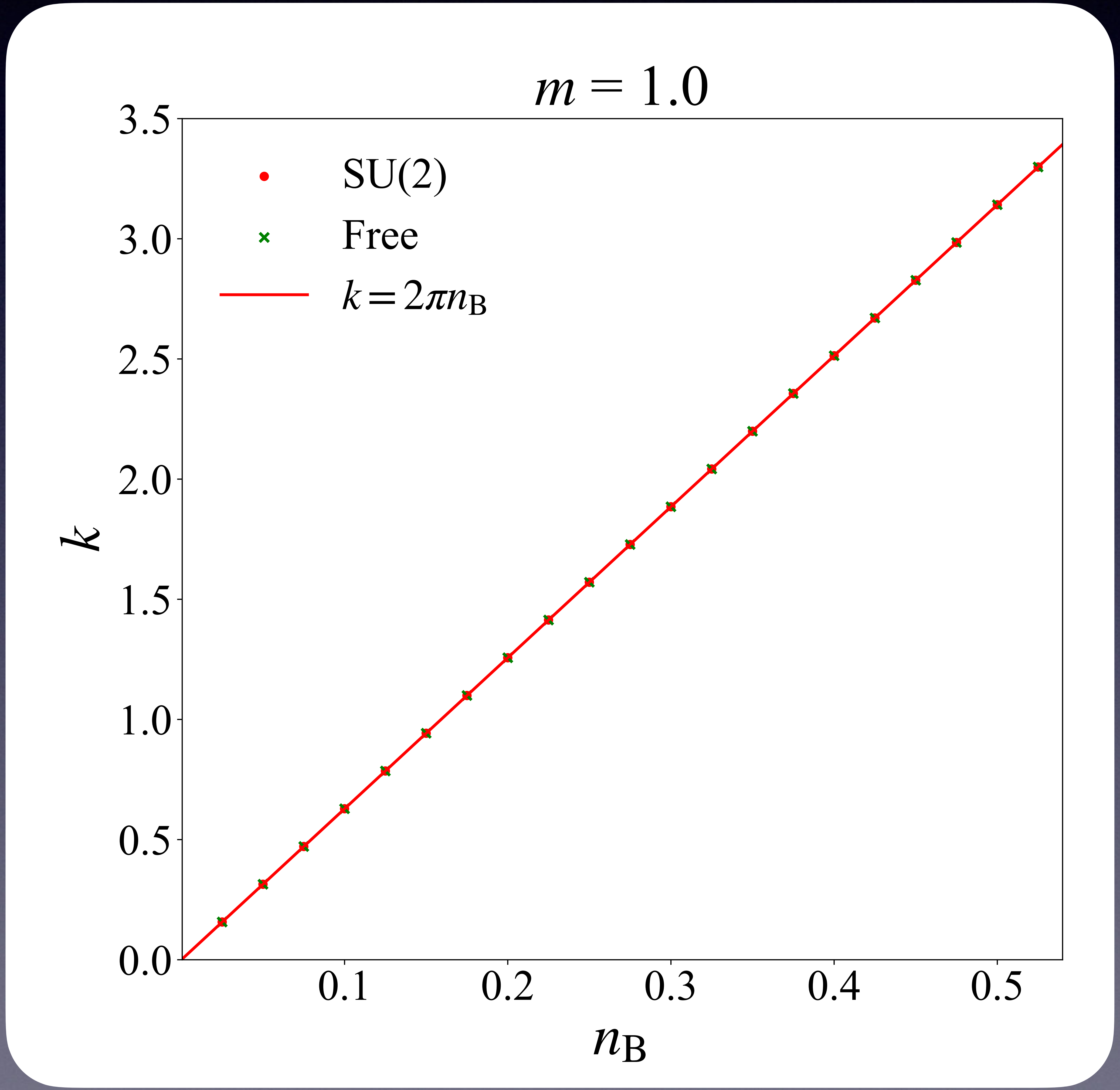
$J = 1/8 \;\; w = 2 \;\; V = 40 \;\; \dim\mathscr{H} = 2^{320}$



Wave number dependence

$J = 1/8 \quad w = 2 \quad V = 40 \quad \dim\mathscr{H} = 2^{320}$

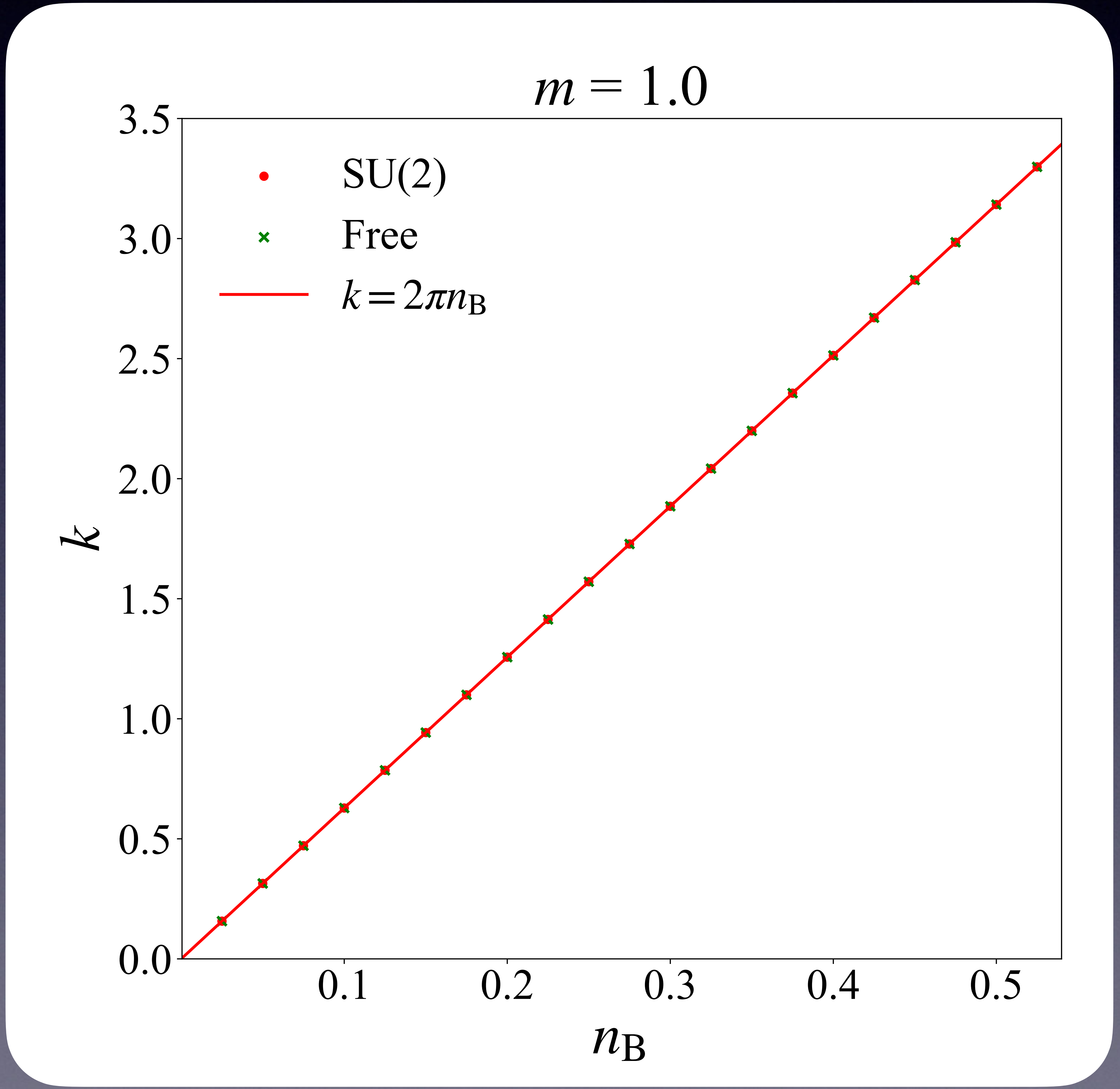
Wave number dependence



Wave number dependence

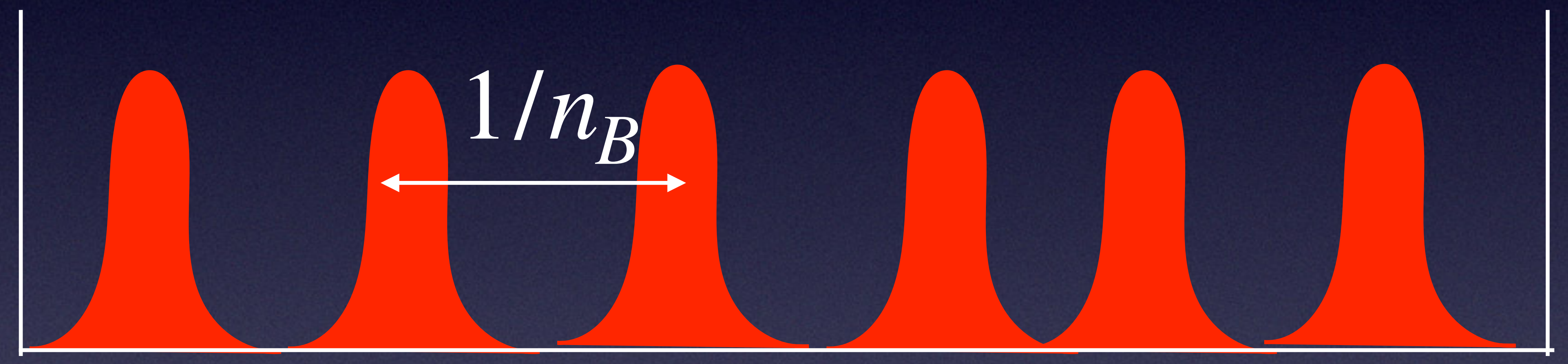
$$J = 1/8 \quad w = 2 \quad V = 40 \quad \dim\mathscr{H} = 2^{320}$$

Wave number dependence



Hadronic picture

If hadron interactions are repulsive

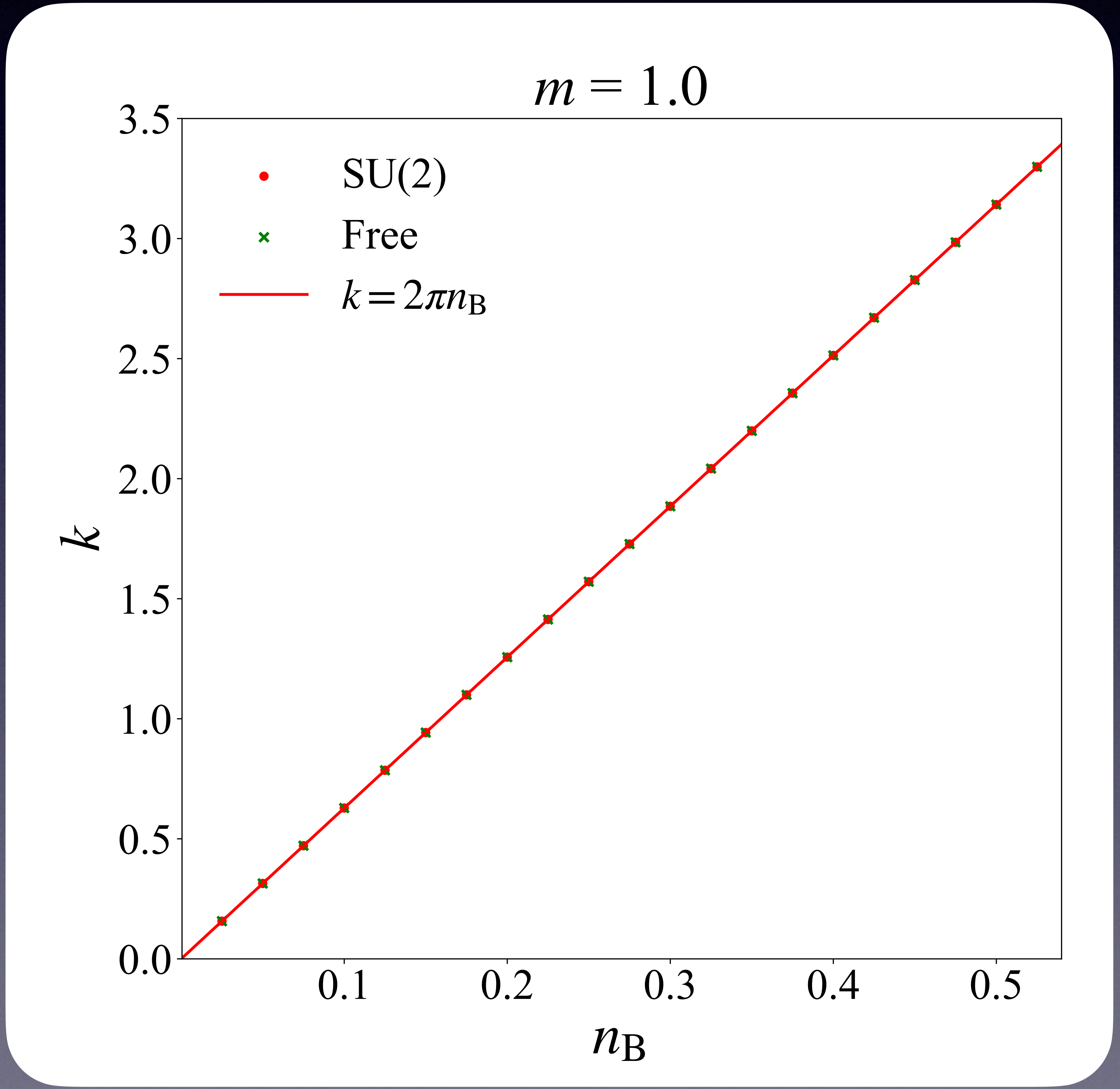


distance $1/n_B \Rightarrow k = 2\pi n_B$

Wave number dependence

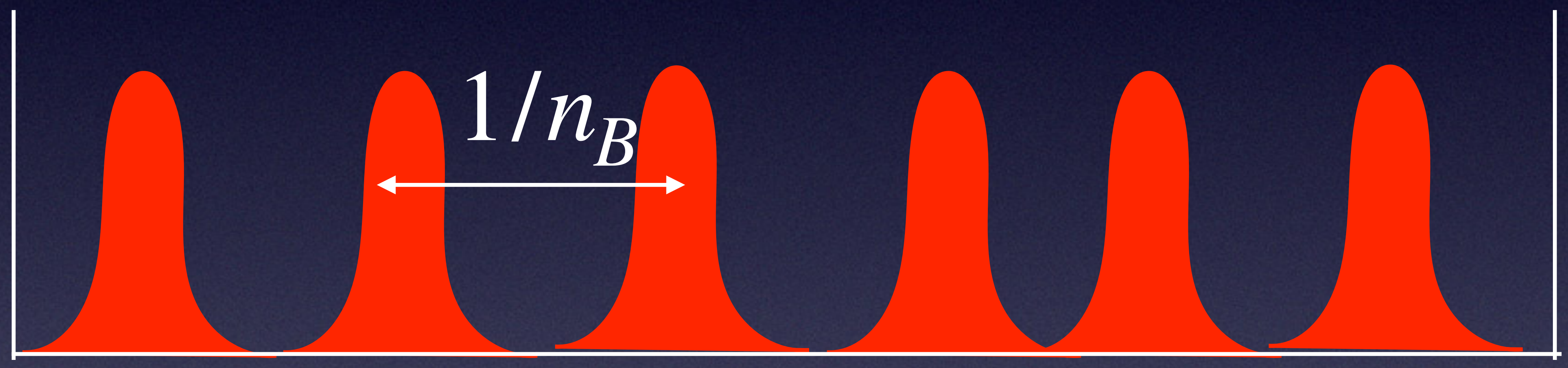
$J = 1/8 \;\; w = 2 \;\; V = 40 \;\; \dim\mathscr{H} = 2^{320}$

Wave number dependence



Hadronic picture

If hadron interactions are repulsive



distance $1/n_B \Rightarrow k = 2\pi n_B$

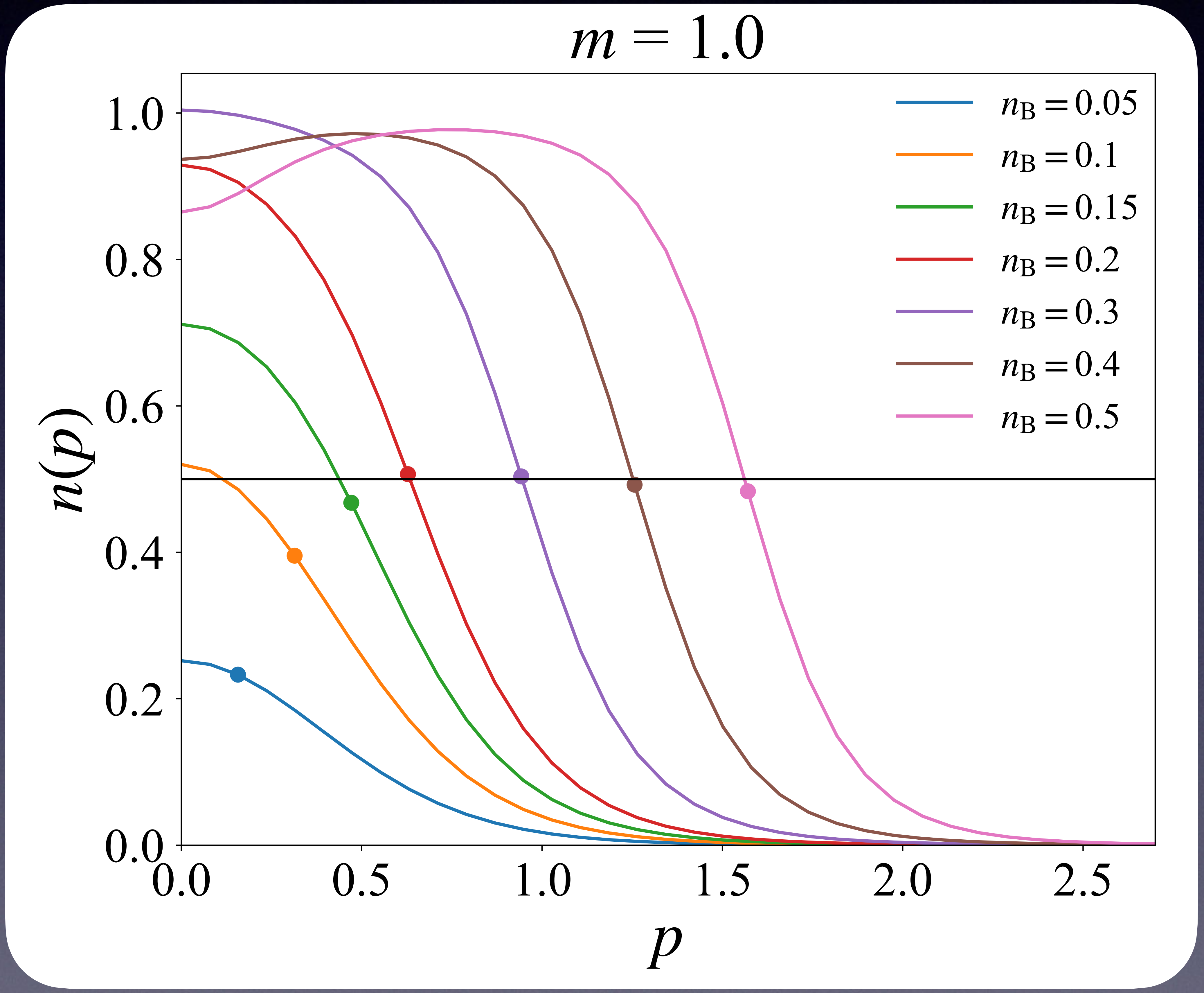
Quark picture

If interactions between quarks
Fermi surface is unstable

$\Rightarrow$density wave $k = 2p_\mathrm{F} = 2\pi n_\mathrm{B}$

Quark distribution function

$J = 1/8 \quad w = 2 \quad V = 60 \qquad \dim\mathcal{H} = 2^{480}$



- **Low density**
  No Fermi sea
- **High density**
  Fermi-sea
  +BCS like pairing
  (density wave)

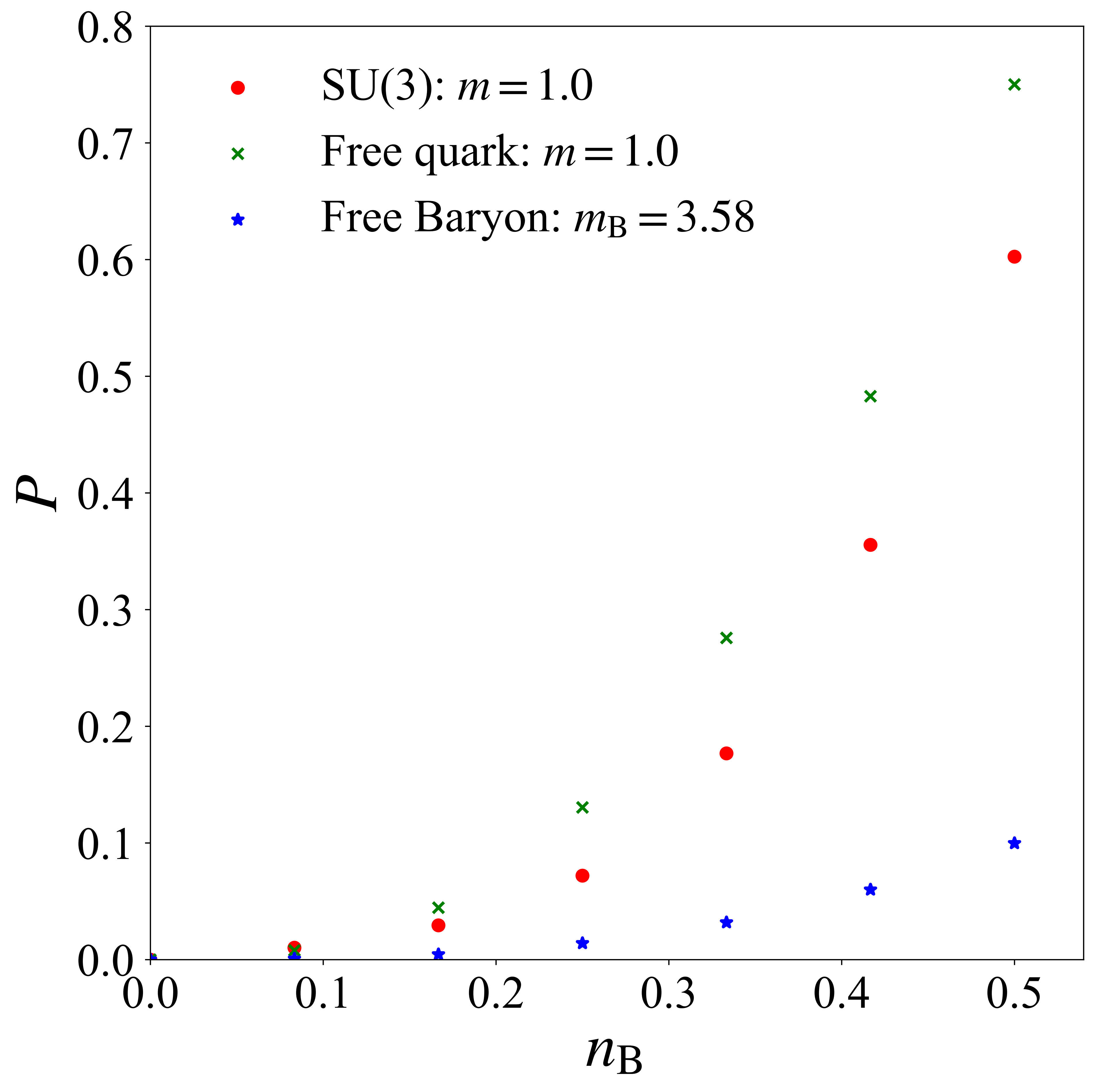
baryon quark transition around $n_B \sim 0.2$

SU(3) QCD with $N_f = 1$

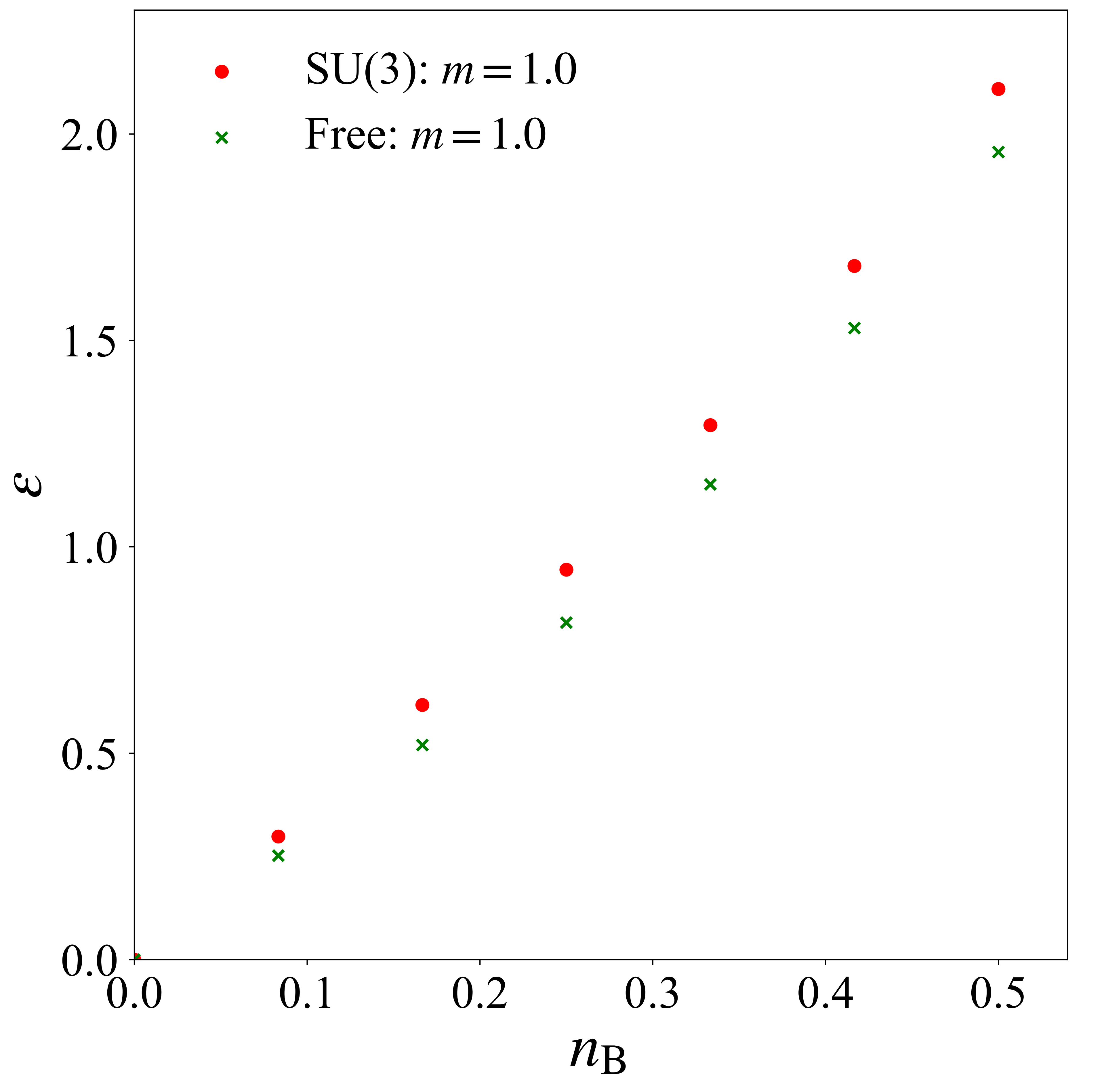
Color SU(3), 1 flavor

$$J = 1/8 \;\; w = 2 \;\; V = 12 \;\; \dim\mathscr{H} = 2^{144}$$

Pressure



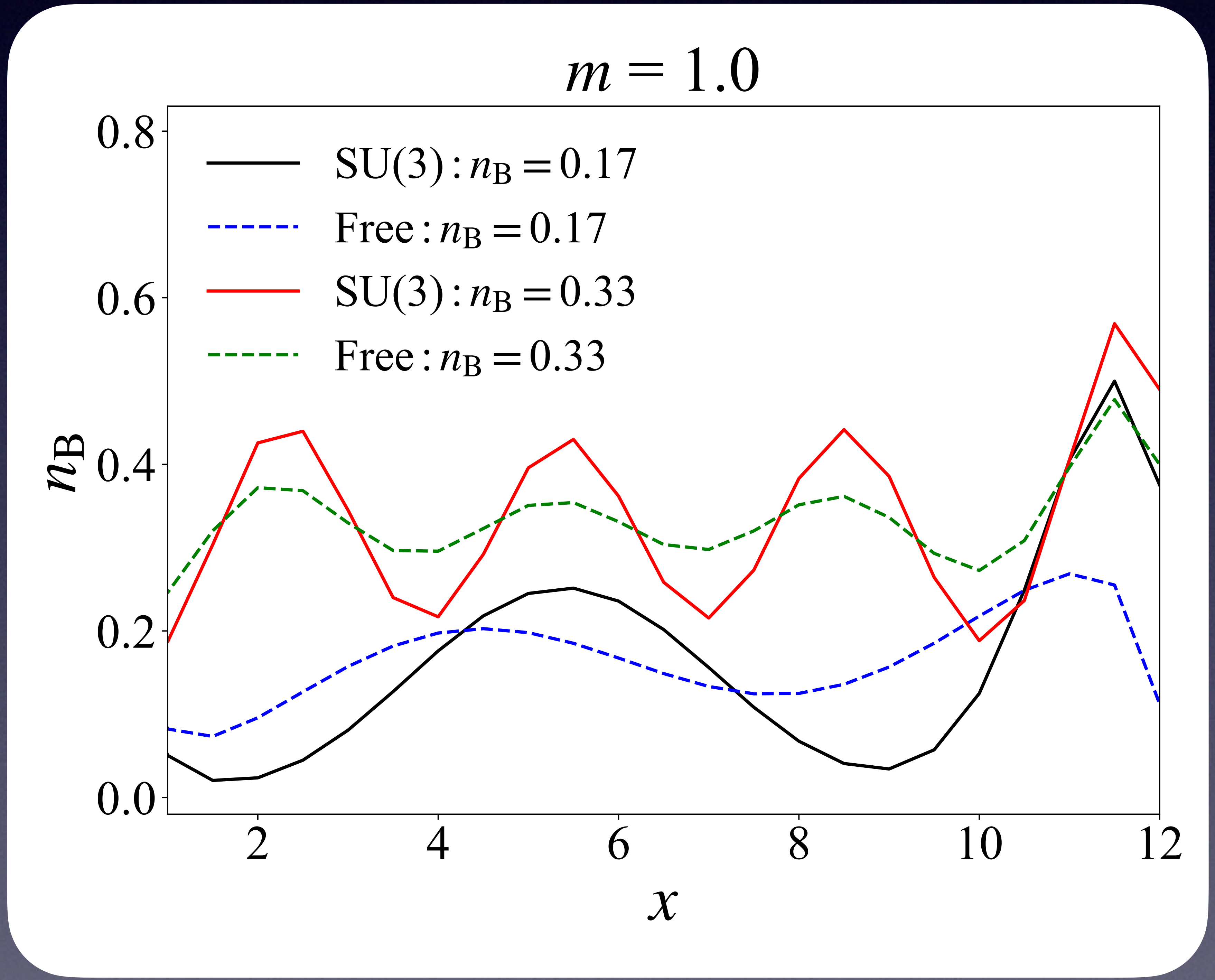
Energy density



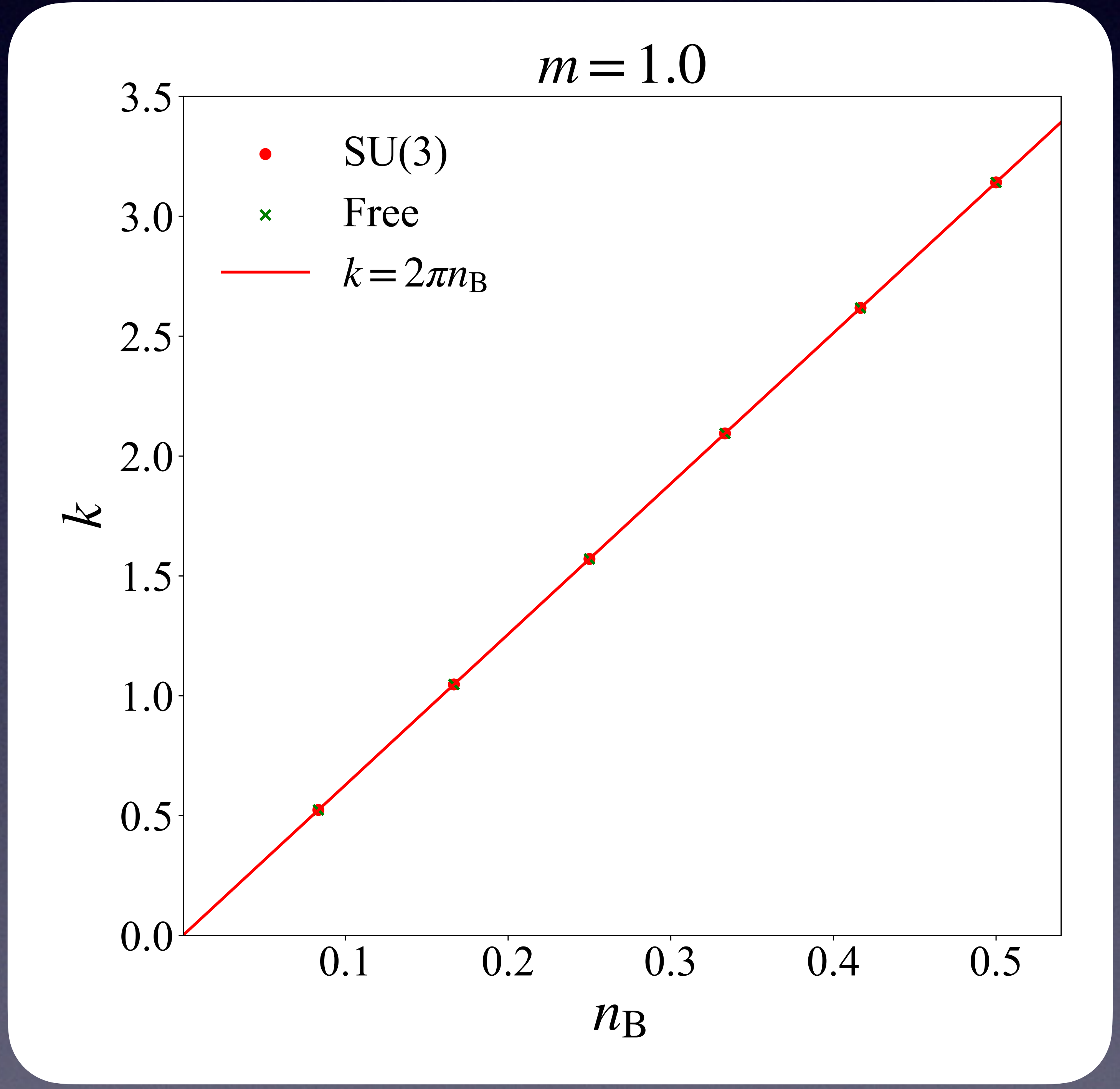
Color SU(3), 1 flavor

$$J = 1/8 \quad w = 2 \quad V = 12 \quad \dim\mathscr{H} = 2^{144}$$

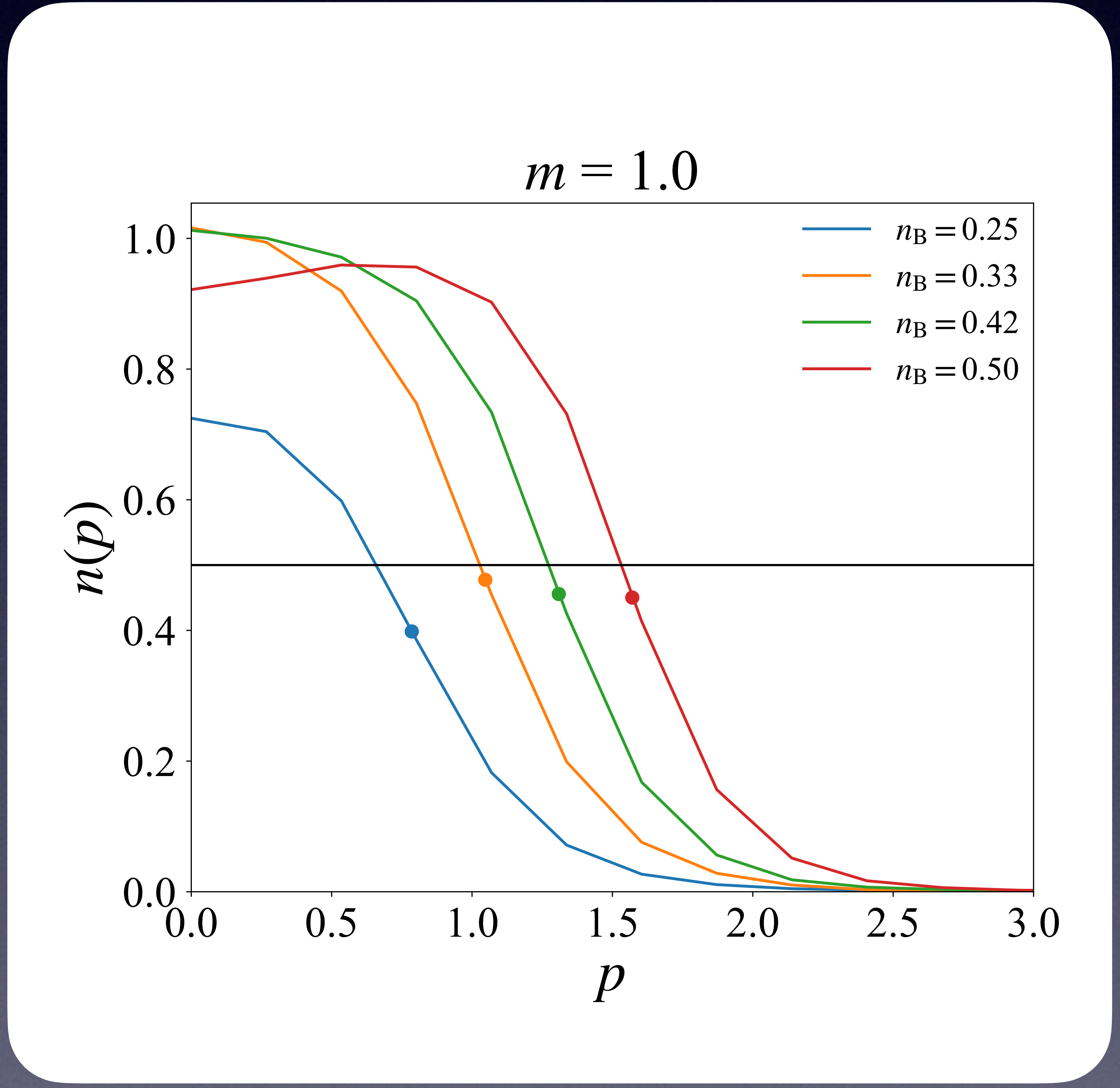
density wave



Wave number dependence



Quark distribution



Baryon quark transition around $n_B = 0.3$?

Summary

- **Formalism**

  Kogut-Susskind Hamiltonian formalism

- **Application**

  Thermalization of Yang-Mills theory in (3+1)-dimensional small systems

  Relaxation time of thermalization

  $\tau_{\mathrm{eq}} \sim 2\pi/T$ Boltzmann time

  QCD_2 **at finite density**

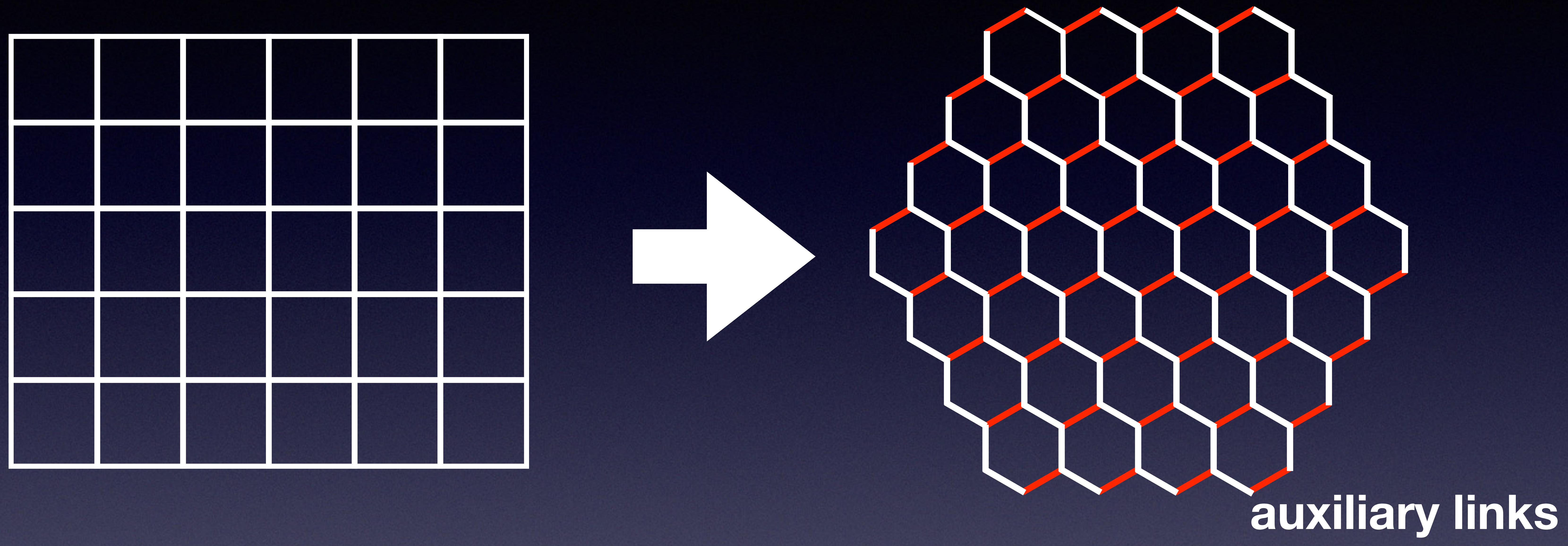
  baryon quark transition, inhomogeneous phase

Outlook

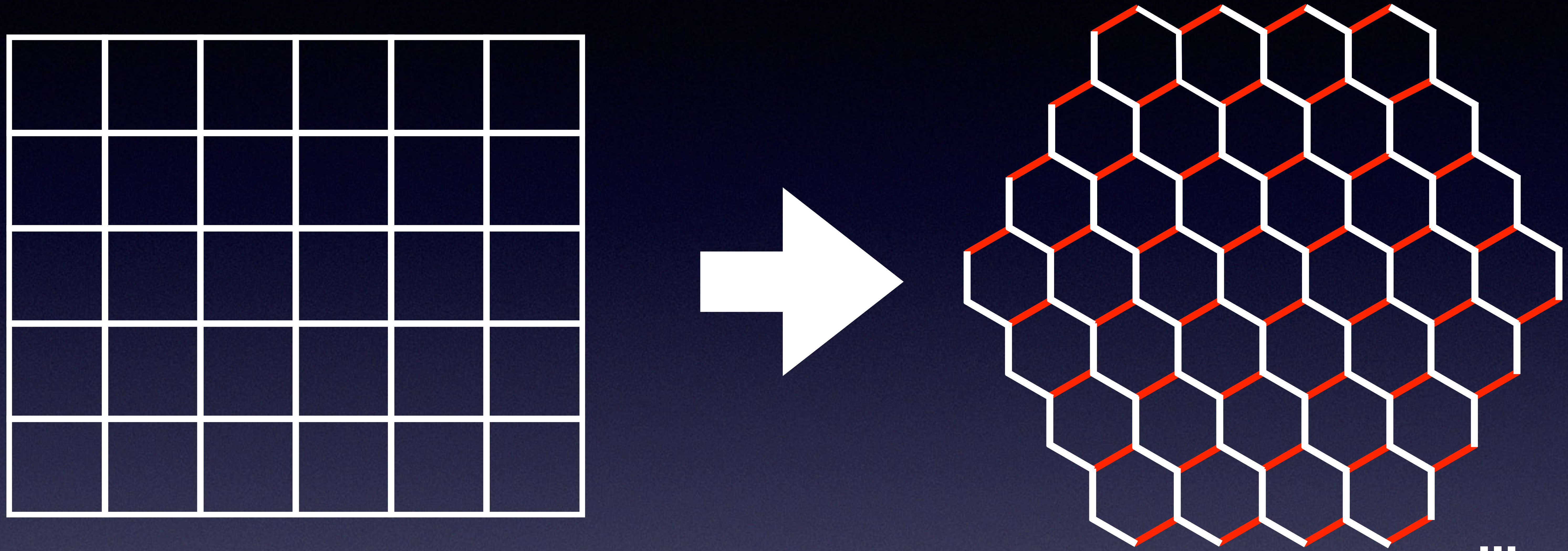
- Large dimension
- Large volume
- Quantum simulation
- Calculation of entanglement entropy(EE), negativity(NE) etc.

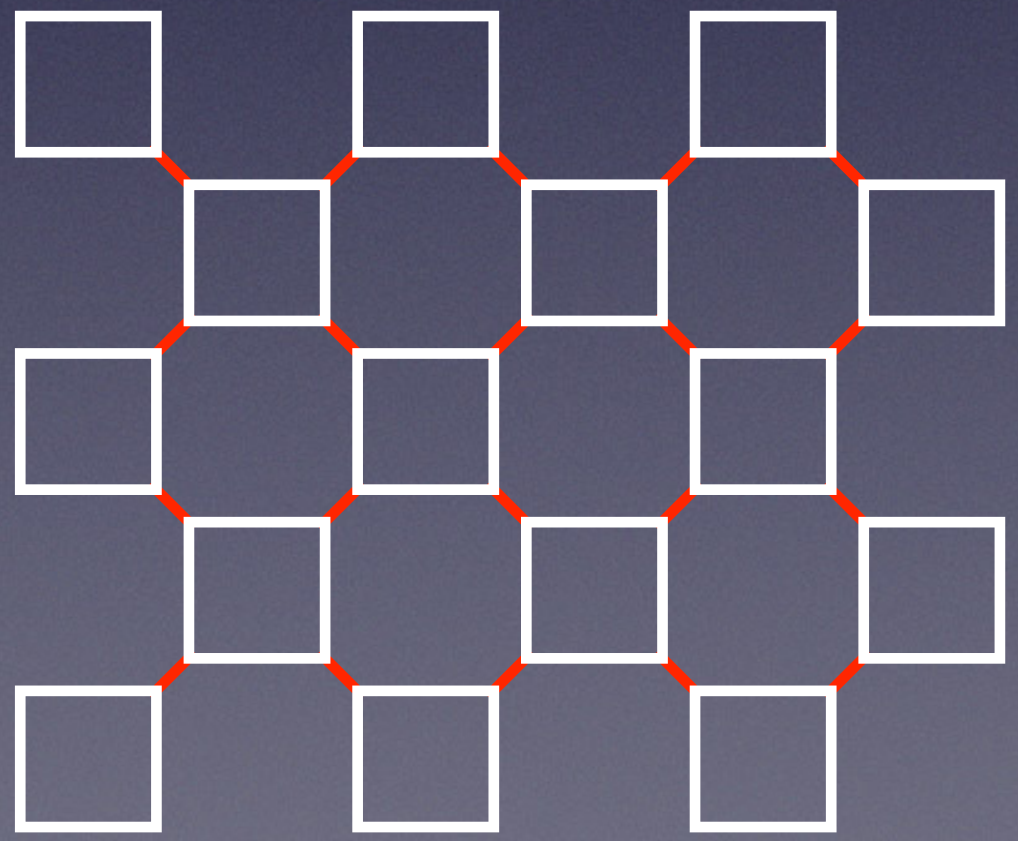
Backup

How to treat square lattice



How to treat square lattice





By the composition rule of the network
Matrix elements do not depend
on the inclusion of auxiliary links

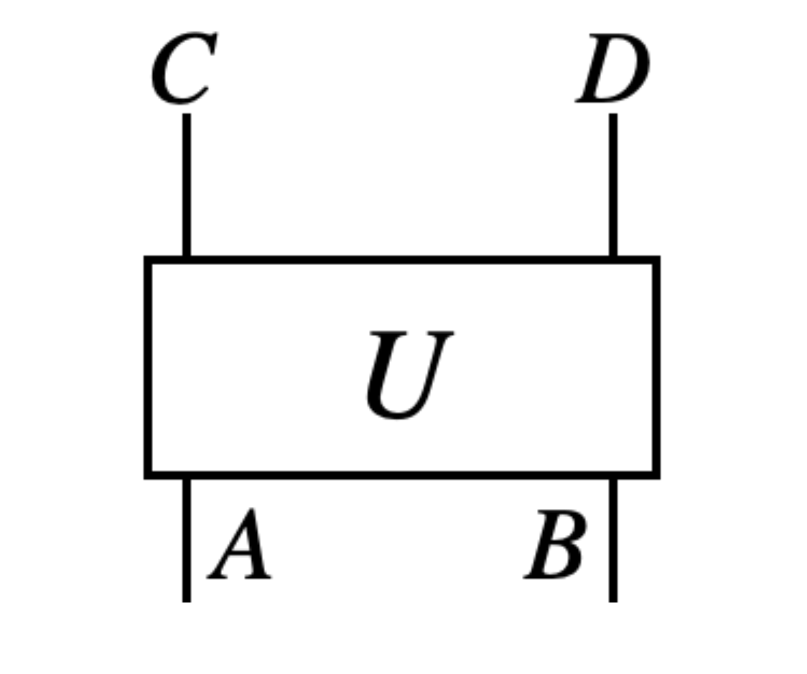
Scrambling in Yang-Mills theory

Average OTOC

$$\langle\overline{\mathrm{OTOC}}\rangle = \int_{\mathrm{Haar}} dO_A dO_D \mathrm{Tr}[O_A O_D(t) O_A^\dagger O_D^\dagger(t)]$$

Used as an indicator of chaos and scrambling

Time evolution



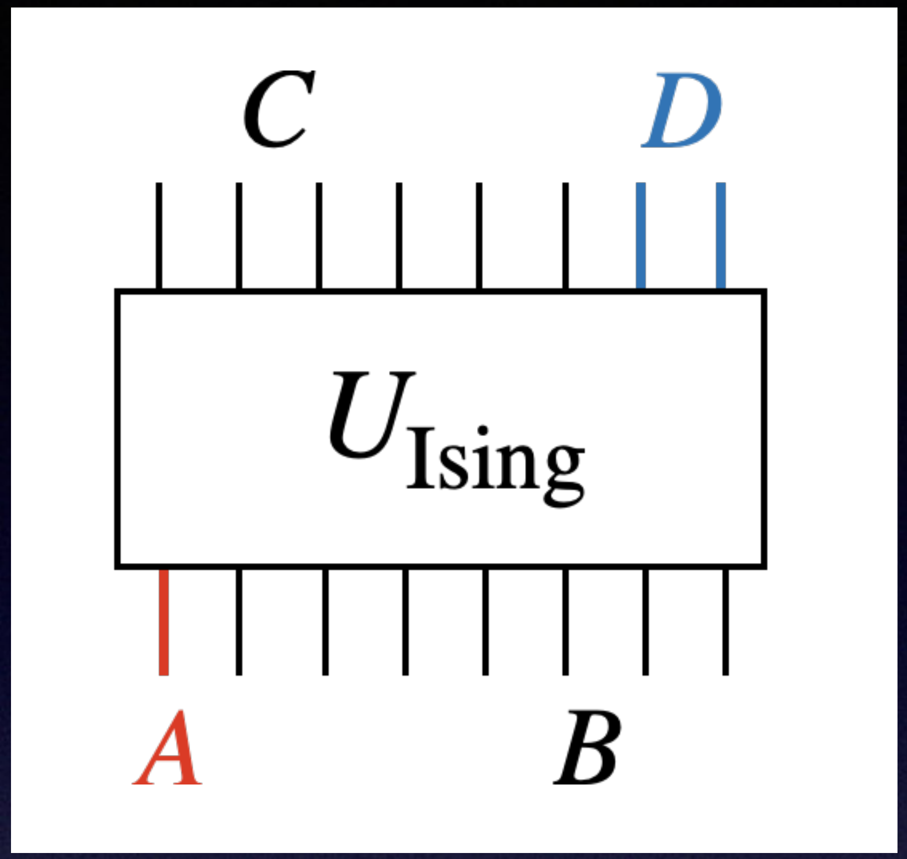
O_A, **e.g., unitary operator on** A

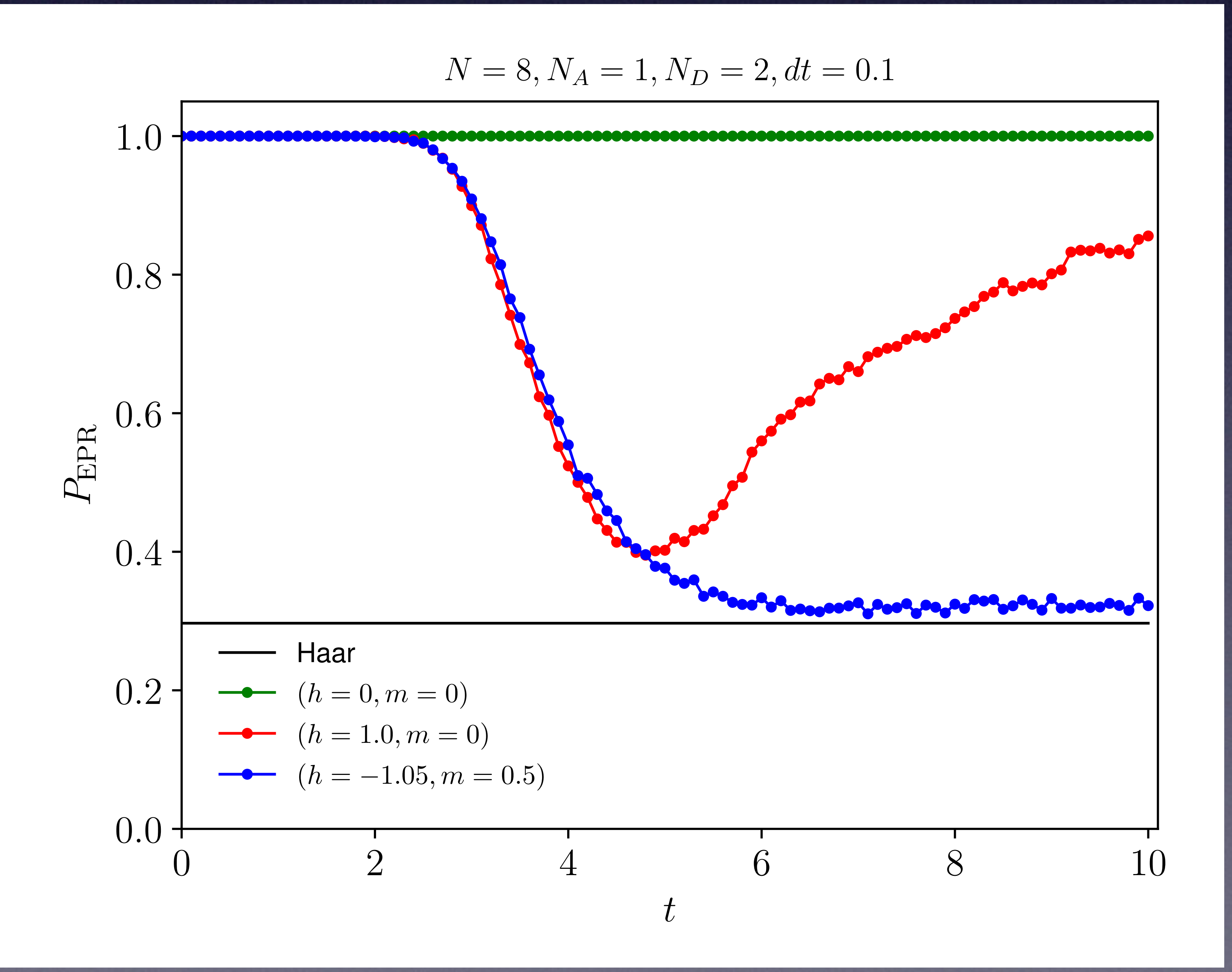
small OTOC≈scrambling

Does Yang-Mills theory exhibit scrambling?

example: transverse Ising

$$H_{\text{Ising}} = -\sum_{i=1}^{N-1} Z_i Z_{i+1} - h\sum_{i=1}^{N} X_i - m\sum_{i=1}^{N} Z_i$$





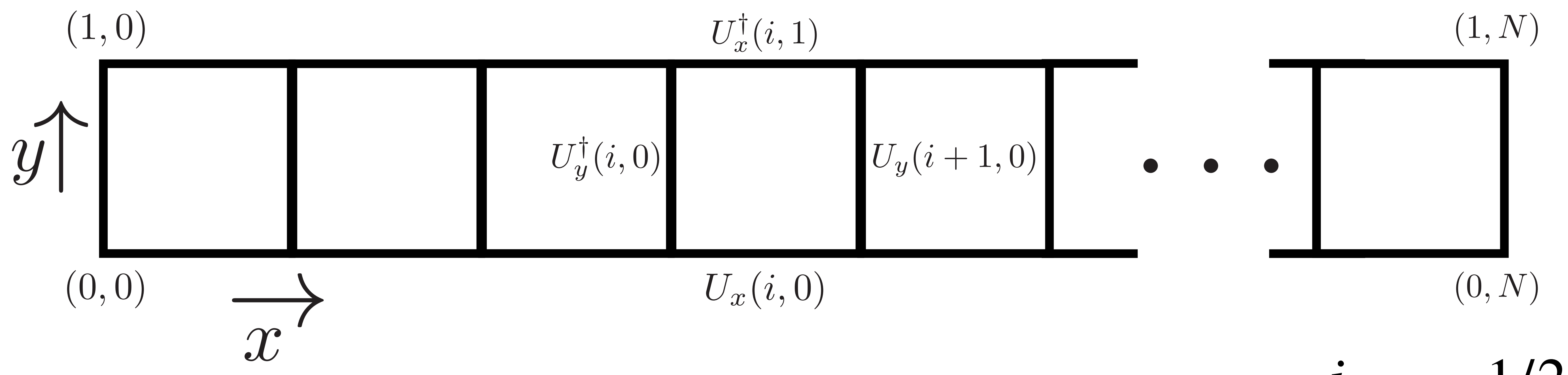
h=0: classical ising

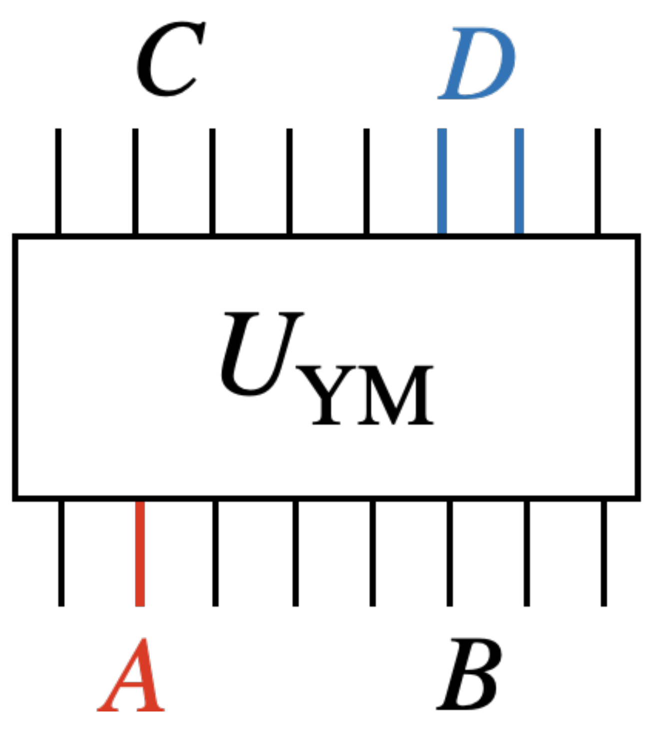
h=1:critical

h=-1.05 chaos

Haar：time evolution is Haar random

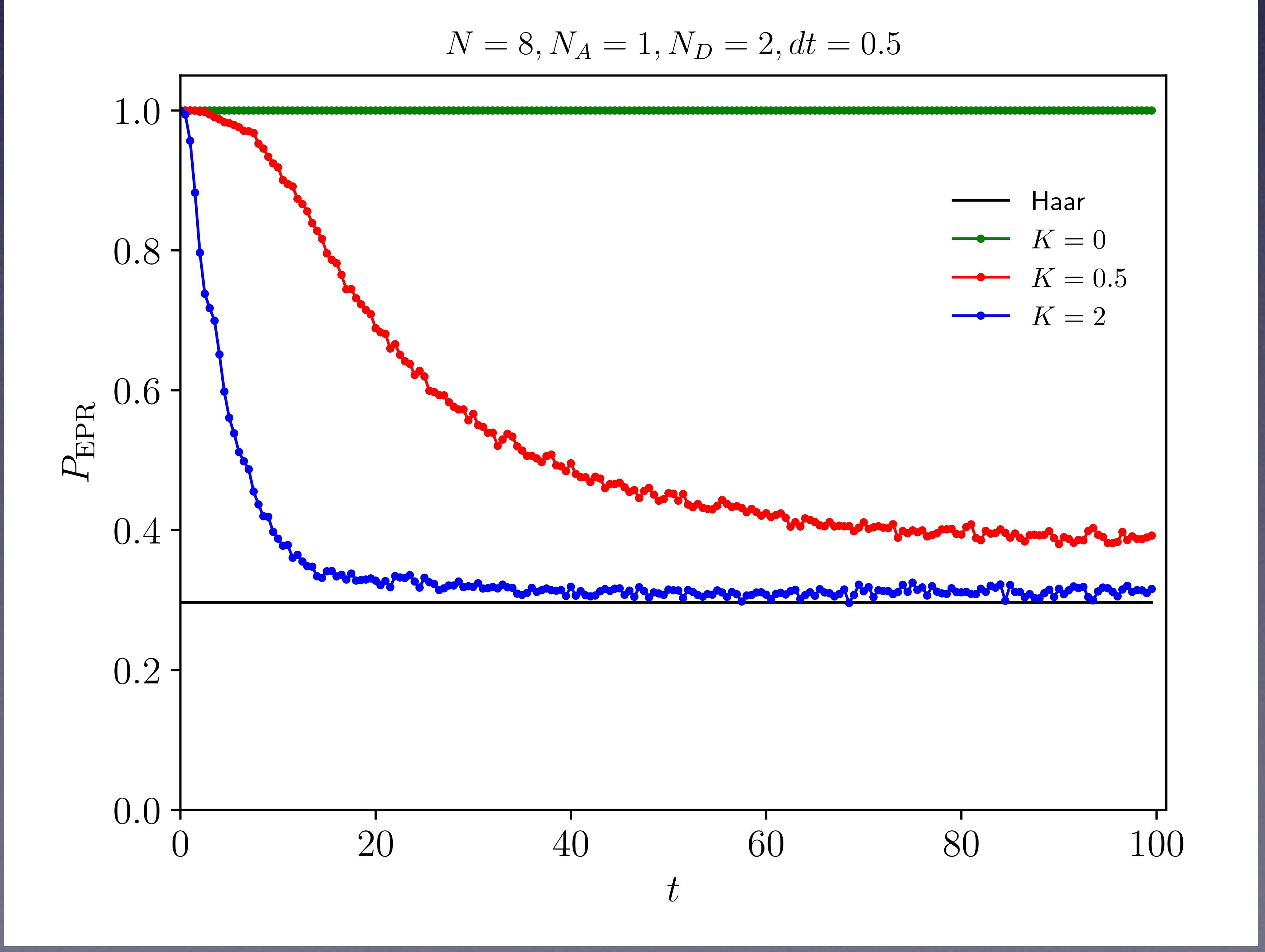
Yang-Mills on a ladder





$j_{\text{max}} = 1/2$

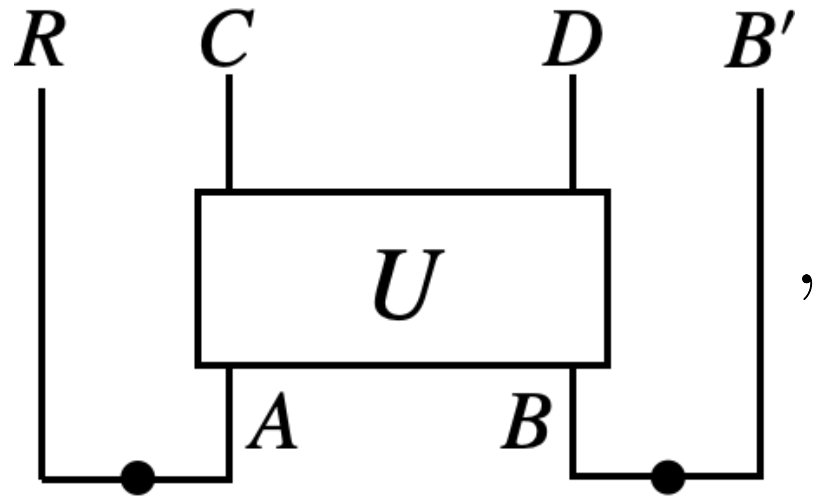
$$H = \sum_{i=0}^{N} \frac{3}{16}(3 - 2Z_i - Z_{i-1}Z_i) - K \sum_{i=0}^{N-1} \frac{1}{16} X_i(1 + 3Z_{i-1})(1 + 3Z_{i+1})$$



scrambling at weak coupling

Hayden-Preskill protocol

$$|\Psi\rangle = (I_R \otimes U_{AB} \otimes I_{B'})(|\mathrm{EPR}\rangle_{RA} \otimes |\mathrm{EPR}\rangle_{BB'}) =$$

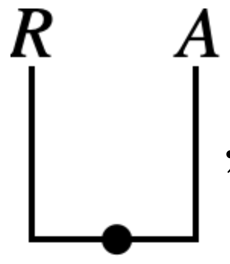


,

$d_A \ll d_B \quad d_D \ll d_C$

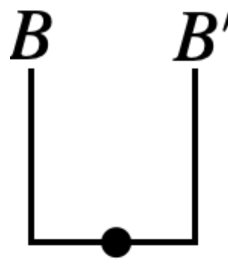
where

$$|\mathrm{EPR}\rangle_{RA} = \frac{1}{\sqrt{d_A}} \sum_{i=1}^{d_A} |i\rangle_R \otimes |i\rangle_A =$$



,

$$|\mathrm{EPR}\rangle_{BB'} = \frac{1}{\sqrt{d_B}} \sum_{i=1}^{d_B} |i\rangle_B \otimes |i\rangle_{B'} =$$



,

mutual information

$$I(R,C) := S(R) + S(C) - S(RC)$$

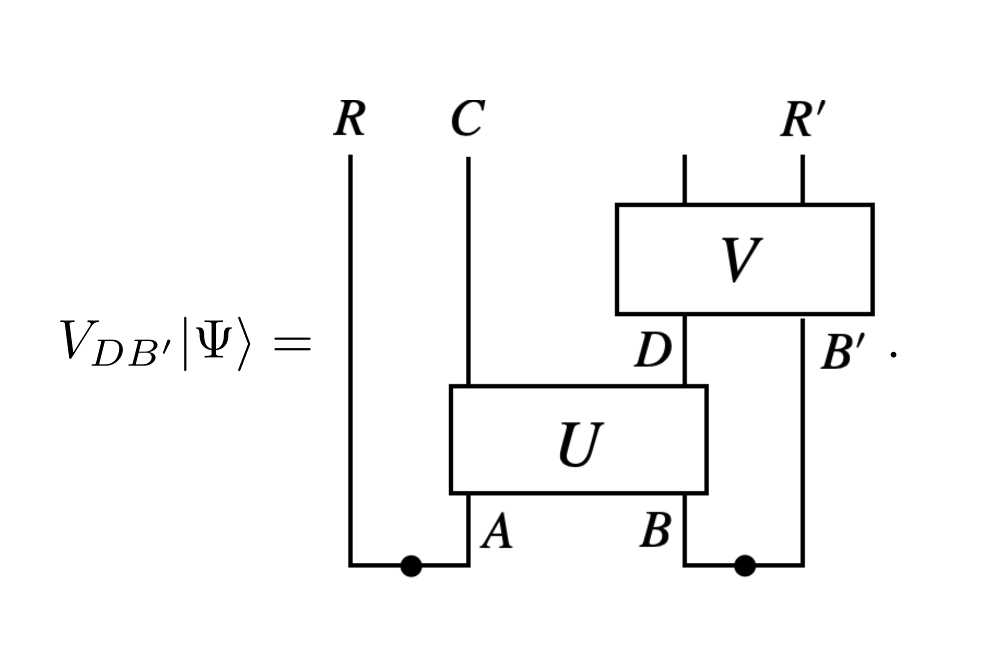
$S(R)$:von Neumann entropy of reduced density operator

Tripartite information

$$-I(R,C,D) := I(R,CD) - I(R,C) - I(R,D)$$

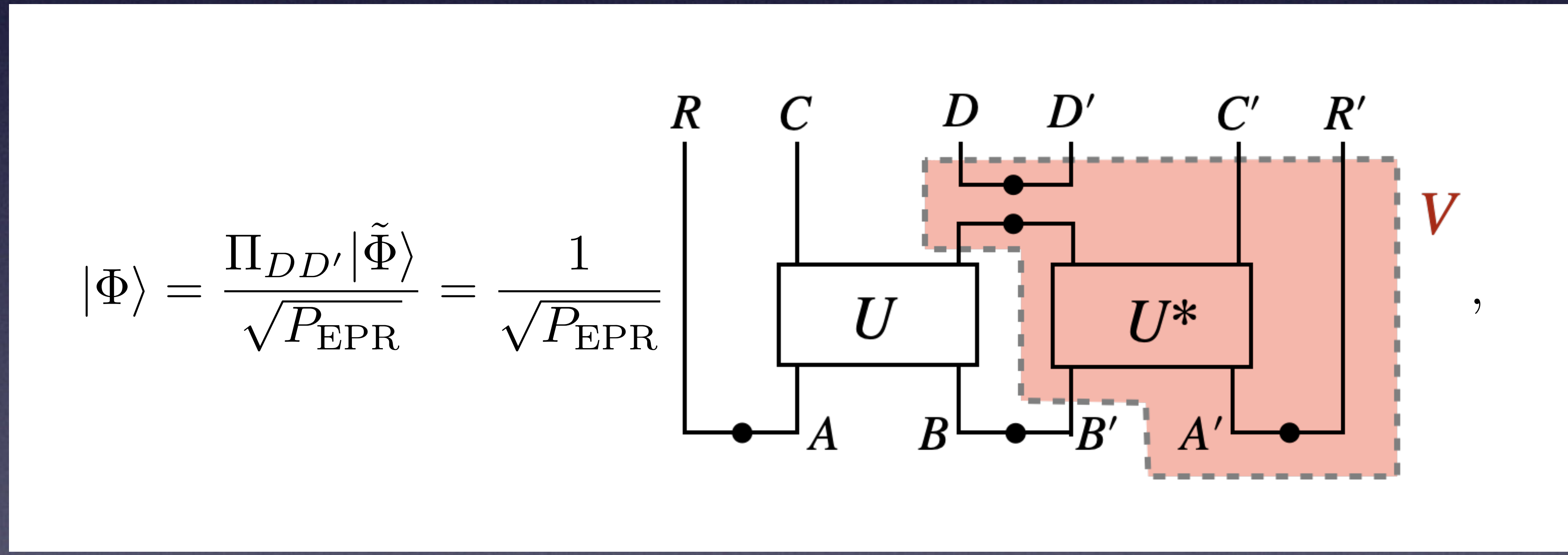
If this takes minimum = scambling

Hayden-Preskill protokol



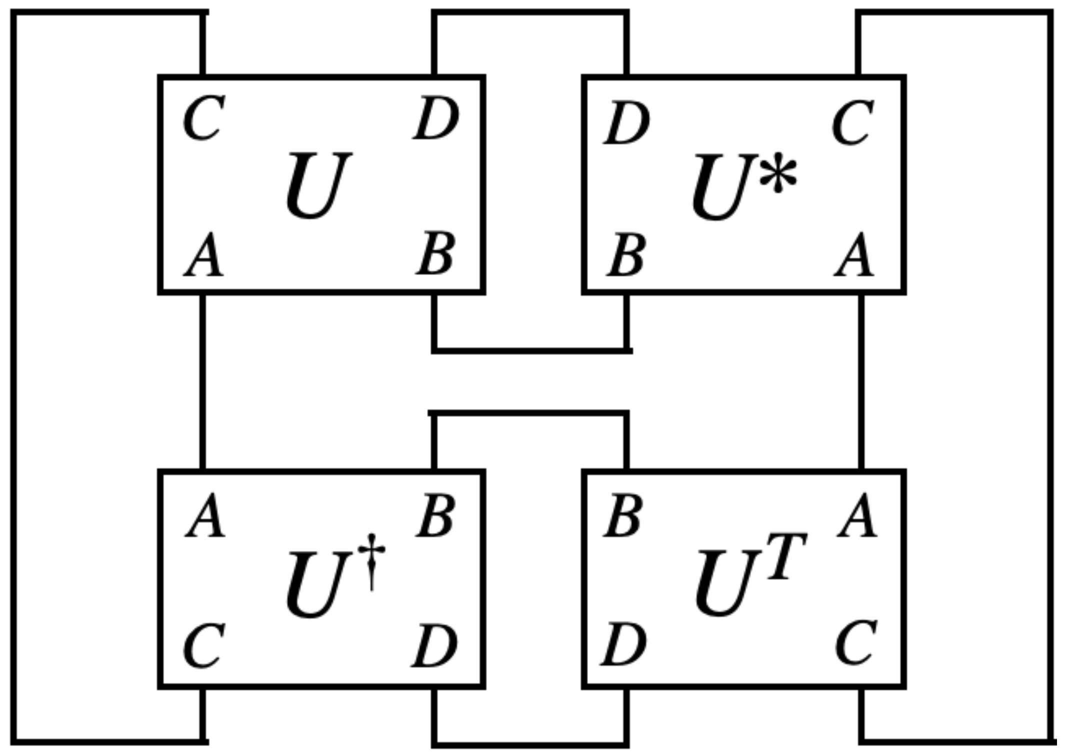
If U shows scambling dynamics, an V exists, and information from R' to R can be exstucted.

Yoshida-Kitaev



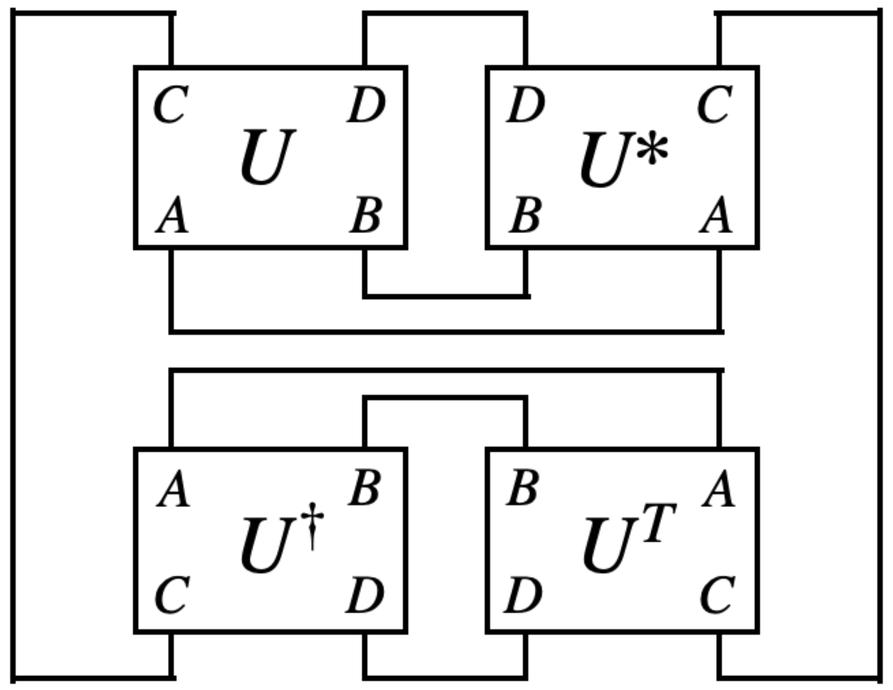
construct explicit V

$$P_{\mathrm{EPR}} = \mathrm{Tr}[\Pi_{DD'}|\tilde{\Phi}\rangle\langle\tilde{\Phi}|] = \frac{1}{d_A^2 d_B d_D}$$



,

$$F_{\mathrm{EPR}} = \mathrm{Tr}[\Pi_{RR'}|\Phi\rangle\langle\Phi|] = \frac{1}{d_A^3 d_B d_D P_{\mathrm{EPR}}}$$

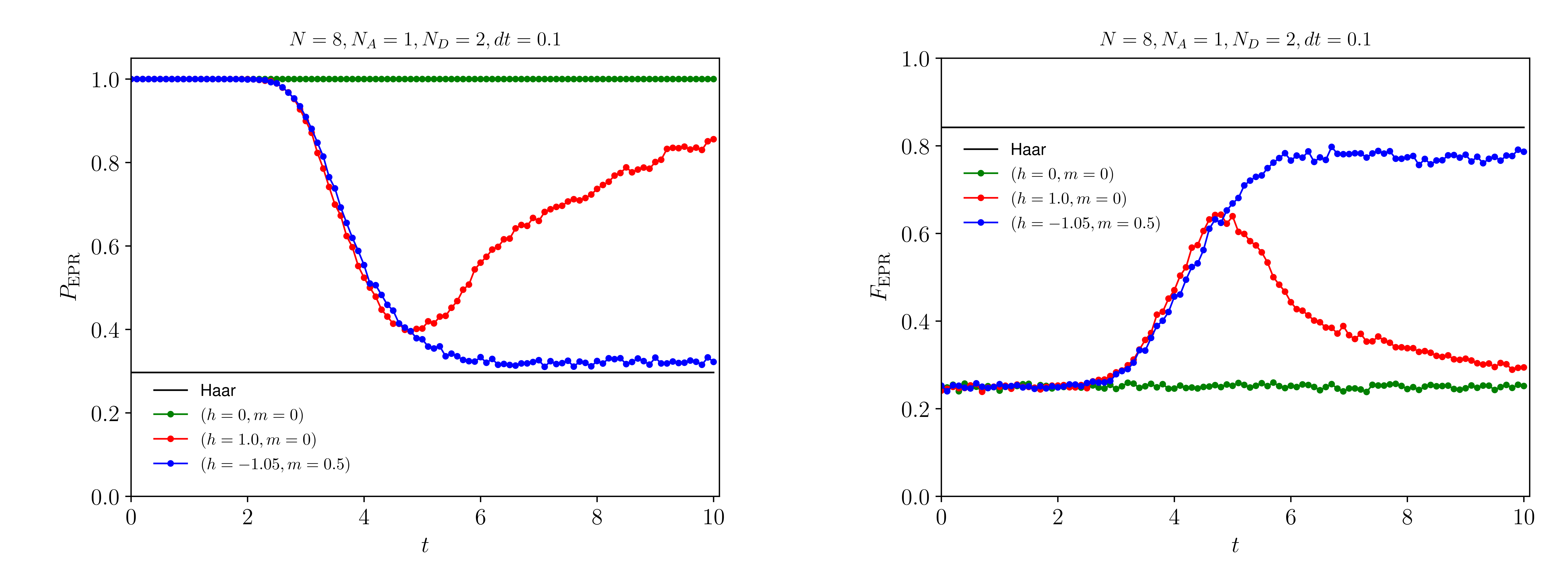


$$= \frac{1}{d_A^2 P_{\mathrm{EPR}}}.$$

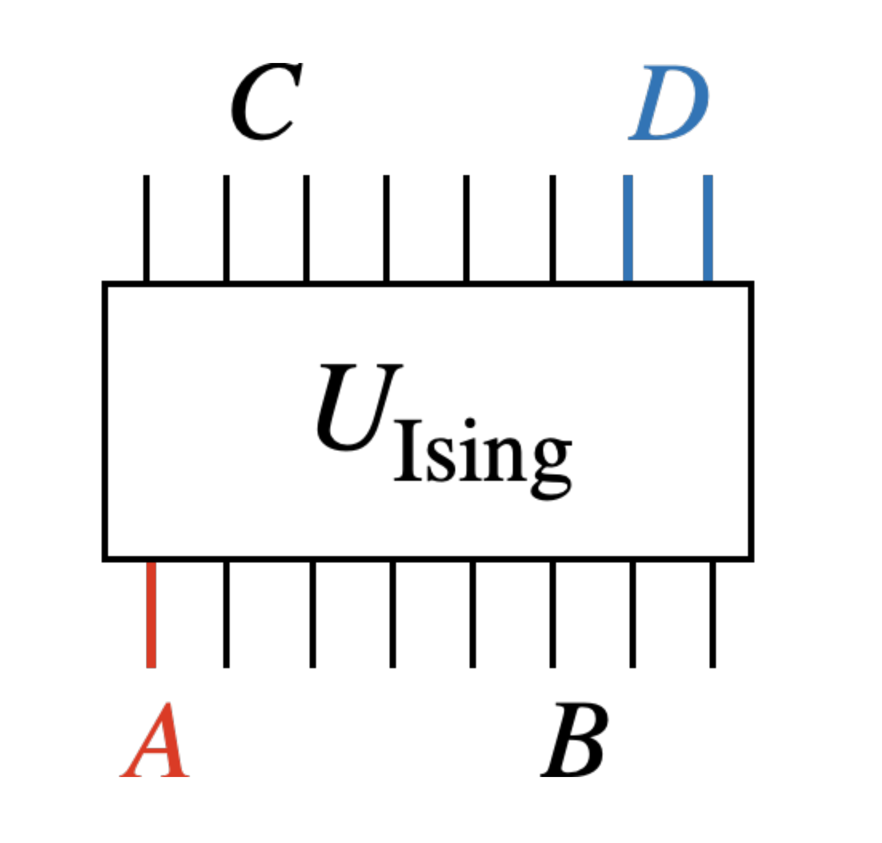
$$P_{\mathrm{EPR}} = \overline{\langle \mathrm{OTOC} \rangle} := \int_{\mathrm{Haar}} \mathrm{d}O_A \mathrm{d}O_D \, \mathrm{Tr}[O_A O_D(t) O_A^\dagger O_D^\dagger(t)].$$

$$F_{\mathrm{EPR}} = \frac{1}{d_A^2 \overline{\langle \mathrm{OTOC} \rangle}}.$$

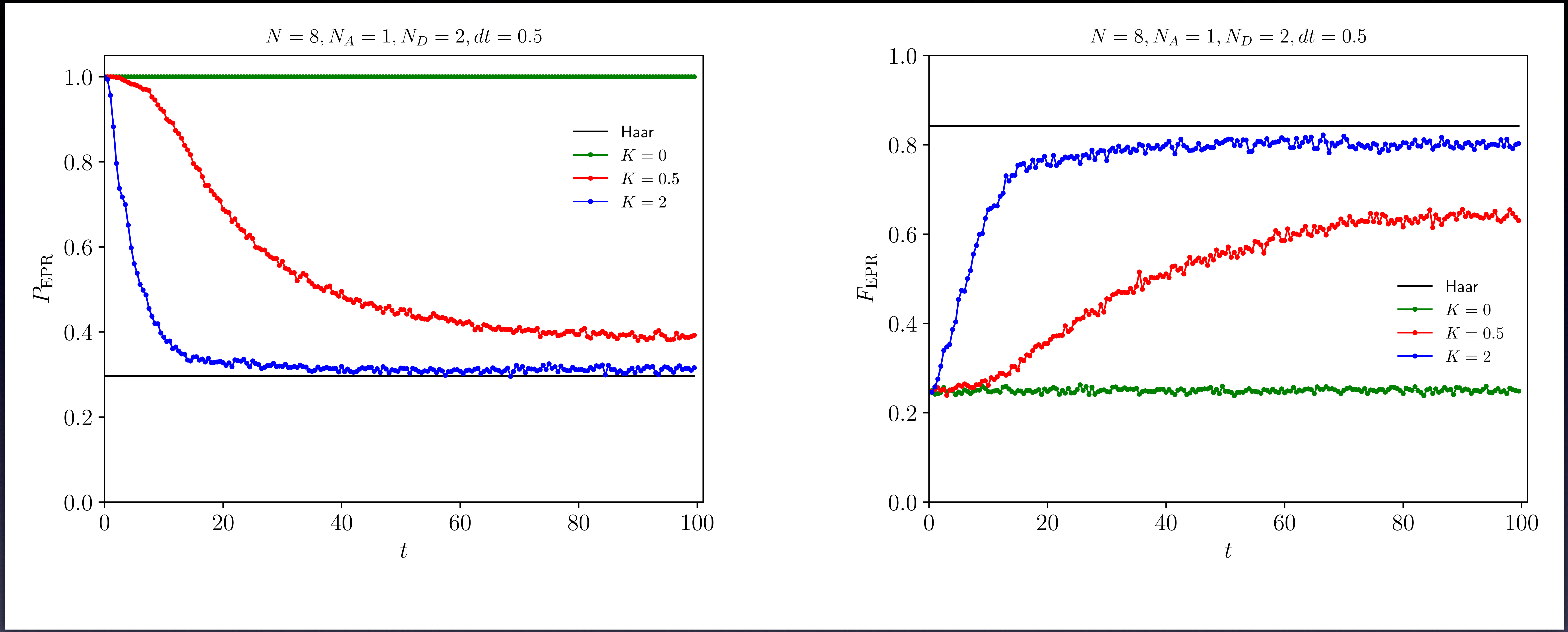
Ising model



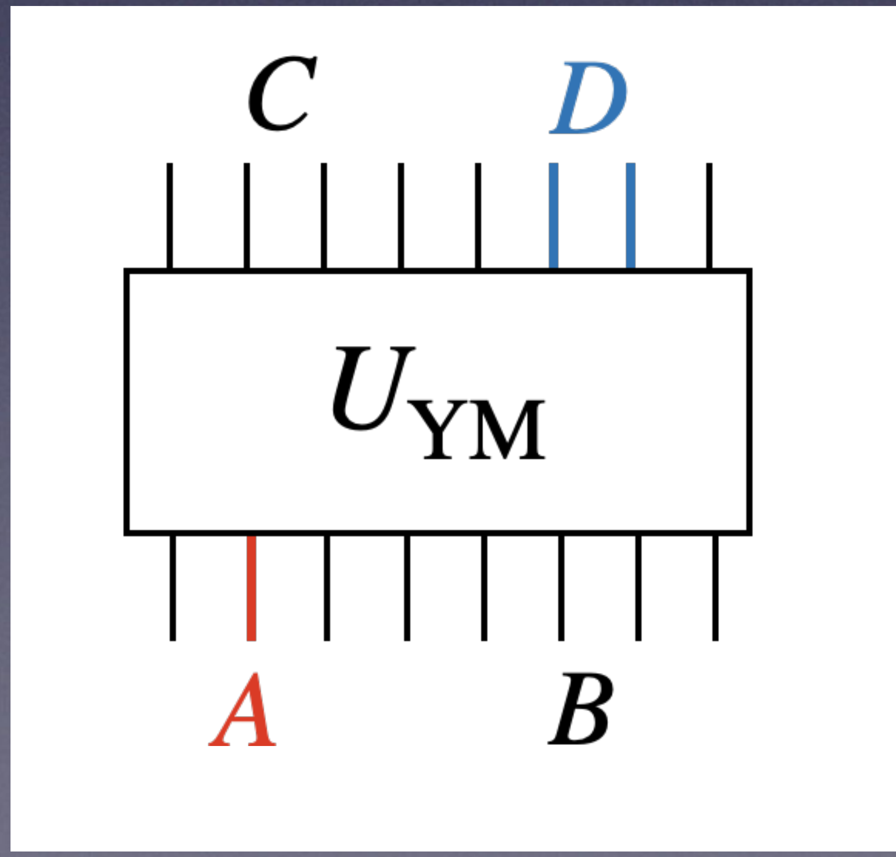
$$H_{\text{Ising}} = -\sum_{i=1}^{N-1} Z_i Z_{i+1} - h\sum_{i=1}^{N} X_i - m\sum_{i=1}^{N} Z_i$$



Yang-Mills



$$H = \sum_{i=0}^{N} \frac{3}{16}(3 - 2Z_i - Z_{i-1}Z_i) - K \sum_{i=0}^{N-1} \frac{1}{16} X_i(1 + 3Z_{i-1})(1 + 3Z_{i+1})$$



Haar random unitary

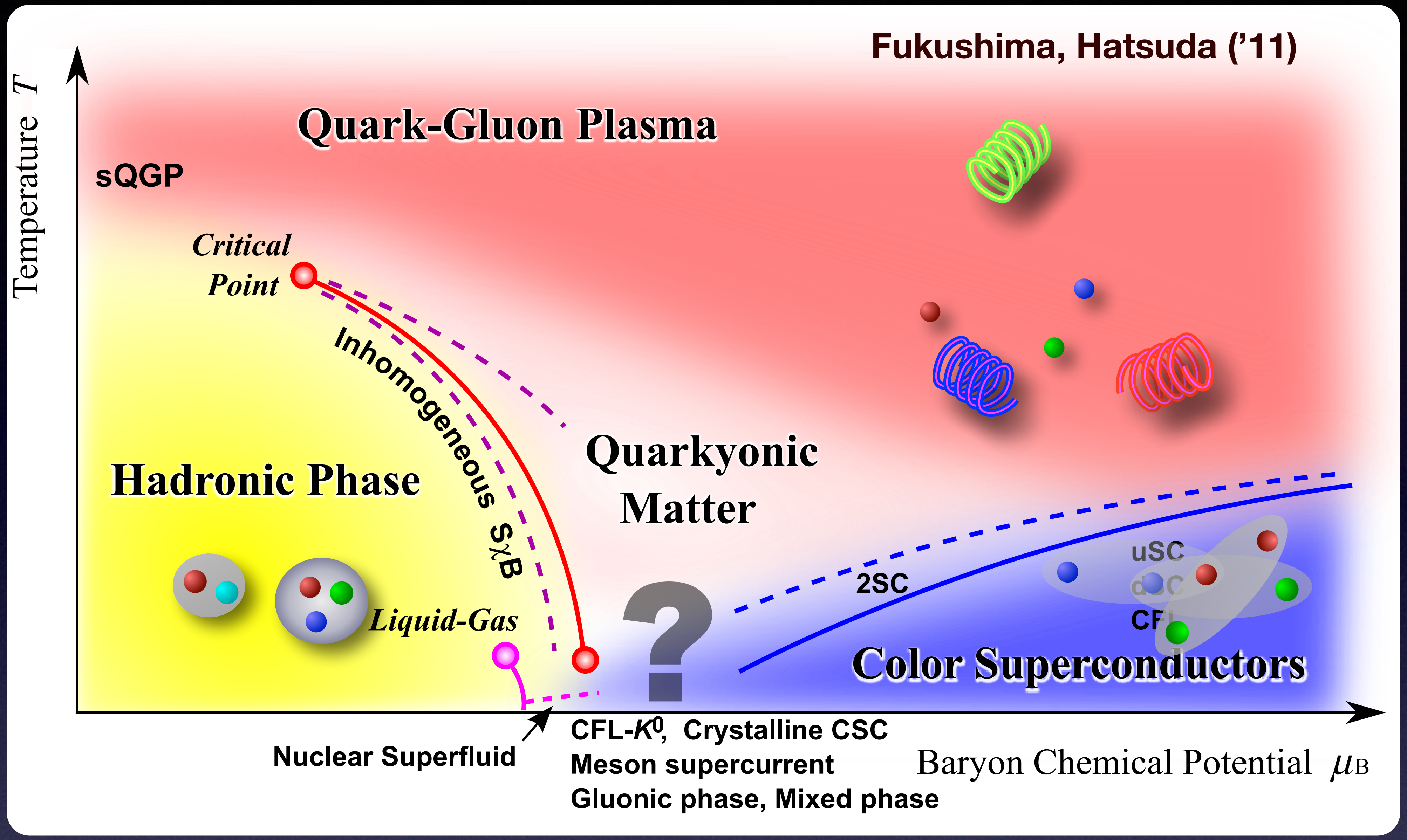
$$\int \mathrm{d}U = 1, \qquad \int \mathrm{d}f(U) = \int \mathrm{d}f(UV) = \int \mathrm{d}f(VU)$$

$$\int \mathrm{d}U \; U_{i_1 j_1} U^*_{i_2 j_2} = \frac{\delta_{i_1 i_2} \delta_{j_2 j_1}}{d},$$

$$\int \mathrm{d}U \; U_{i_1 j_1} U_{i_2 j_2} U^*_{i_3 j_3} U^*_{i_4 j_4} = \frac{\delta_{i_1 i_3} \delta_{i_2 i_4} \delta_{j_1 j_3} \delta_{j_2 j_4} + \delta_{i_1 i_4} \delta_{i_2 i_3} \delta_{j_1 j_4} \delta_{j_2 j_3}}{d^2 - 1} - \frac{\delta_{i_1 i_3} \delta_{i_2 i_4} \delta_{j_1 j_4} \delta_{j_2 j_3} + \delta_{i_1 i_4} \delta_{i_2 i_3} \delta_{j_1 j_3} \delta_{j_2 j_4}}{d(d^2 - 1)}$$

Finite density
for (1+1)-dimensional QCD

Finite density calculations for QCD



- What is the finite density equation of state for QCD?
  - How does the distribution function of quarks change when baryonic matter changes to quark matter?
- What kind of phase is realized? inhomogeneous phase?

Finite density calculations for (1+1)-dimensional QCD

Finite density calculations for (1+1)-dimensional QCD

Properties of (1+1) dimension

- Gauge field is not dynamical.
- Gauge field can be removed by unitary transformation
- finite degrees of freedom on open boundary Conditions(OBC)

QCD_2 Hamiltonian

$$H = \frac{g^2}{2}\sum_{n=1}^{N-1} E_i^2(n)$$ **Electric term**

$$+\epsilon \sum_{n=1}^{N-1}\left(\chi^\dagger(n+1)U(n)\chi(n) + \chi^\dagger(n)U^\dagger(n)\chi(n+1)\right)$$

Hopping term

$$+m\sum_{n=1}^{N}(-1)^n\chi^\dagger(n)\chi(n)$$ **Mass term**

QCD_2 Hamiltonian

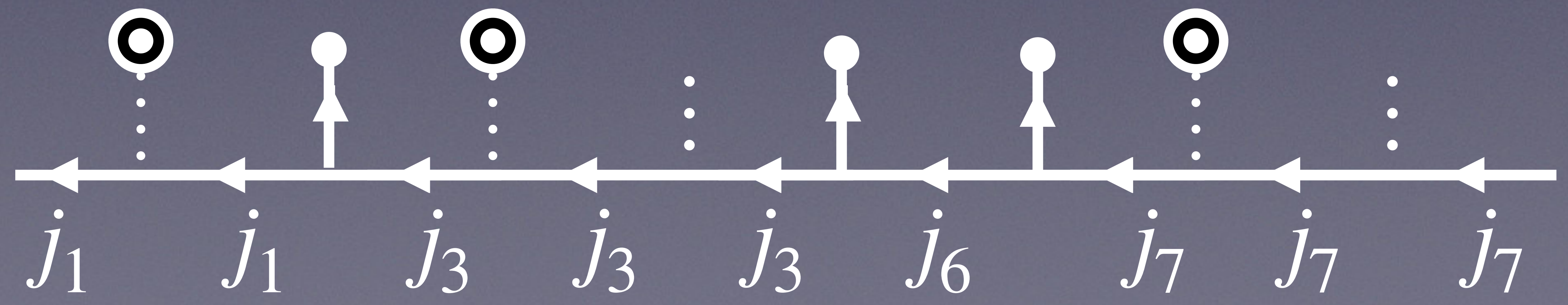
$$H = \frac{g^2}{2}\sum_{n=1}^{N-1} E_i^2(n)$$ **Electric term**

$$+\epsilon \sum_{n=1}^{N-1}\left(\chi^\dagger(n+1)U(n)\chi(n) + \chi^\dagger(n)U^\dagger(n)\chi(n+1)\right)$$

Hopping term

$$+m\sum_{n=1}^{N}(-1)^n\chi^\dagger(n)\chi(n)$$ **Mass term**

Physical state SU(2)



Eliminate U with unitary transformation

Sala, Shi, Kühn, Bañuls, Demler, Cirac, Phys. Rev. D 98, 034505 (2018)
Atas, Zhang, Lewis, Jahanpour, Haase , Muschik , Nature Commun. 12, 6499 (2021)

$$\Theta\chi(n)\Theta^\dagger := U(n-1)U(n-2)\cdots U(1)\chi(n)$$

Eliminate U with unitary transformation

Sala, Shi, Kühn, Bañuls, Demler, Cirac, Phys. Rev. D 98, 034505 (2018)
Atas, Zhang, Lewis, Jahanpour, Haase , Muschik , Nature Commun. 12, 6499 (2021)

$$\Theta\chi(n)\Theta^\dagger := U(n-1)U(n-2)\cdots U(1)\chi(n)$$

$$\Theta H \Theta^\dagger = \frac{g^2}{2}\sum_{n=1}^{N-1}\Big(\sum_{m=1}^{n}\chi^\dagger(m)T_i\chi(m)\Big)^2$$ **electric**

$$+\epsilon\sum_{n=1}^{N-1}\Big(\chi^\dagger(n+1)\chi(n)+\chi^\dagger(n)\chi(n+1)\Big)$$

Hopping

$$+m\sum_{n=1}^{N}(-1)^n\chi^\dagger(n)\chi(n)$$ **Mass**

- **Matrix product state**

$$|\psi\rangle = \sum_{\{n_i\}} |n_1\rangle \cdots |n_N\rangle \mathrm{tr} M_1^{n_1} \cdots M_N^{n_N}$$

$[M_i^{n_i}]_{ij}$ **:** $D \times D$ **matrix**

- **Density matrix renormalization group**

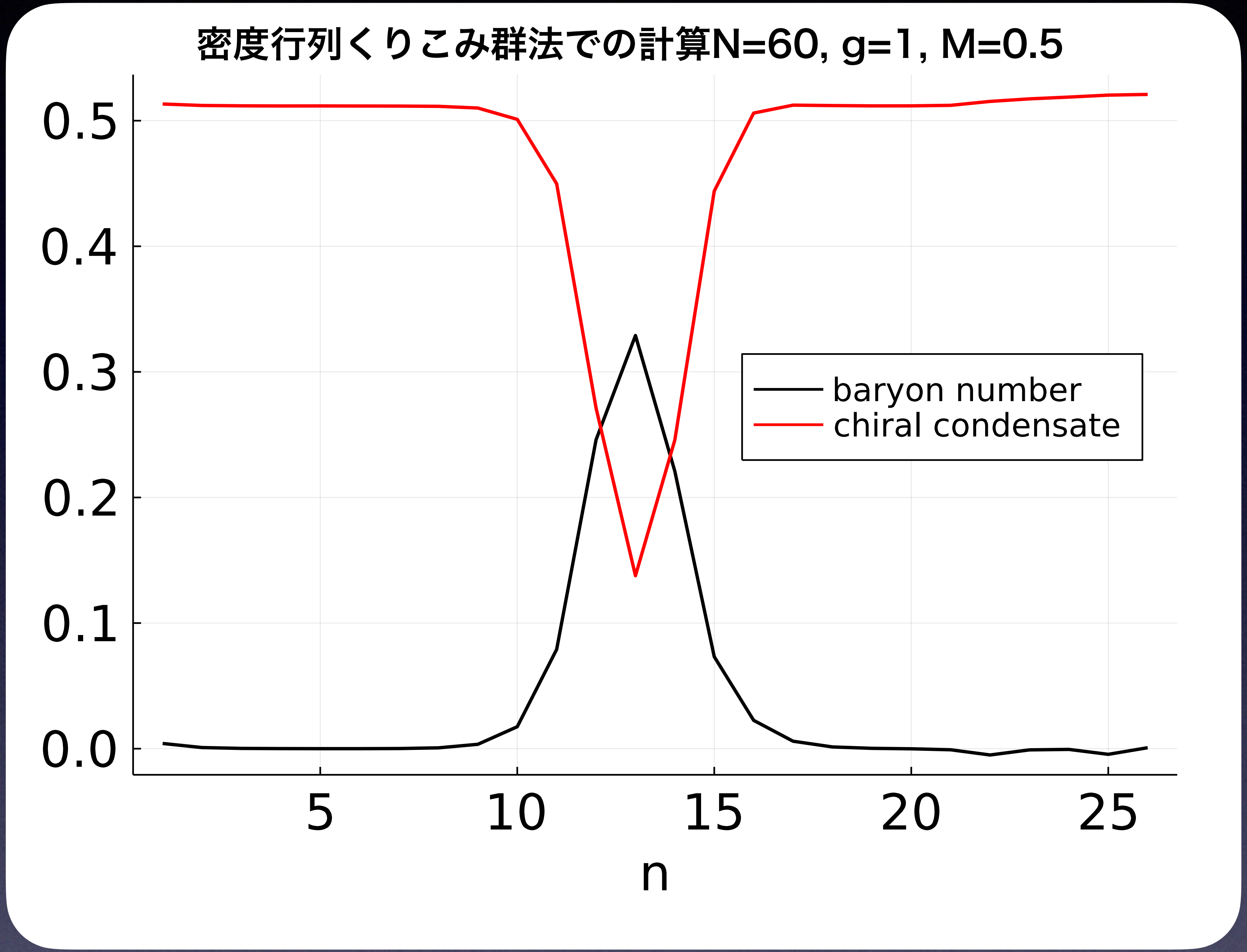
$$E = \min_{\psi} \langle \psi | H | \psi \rangle$$

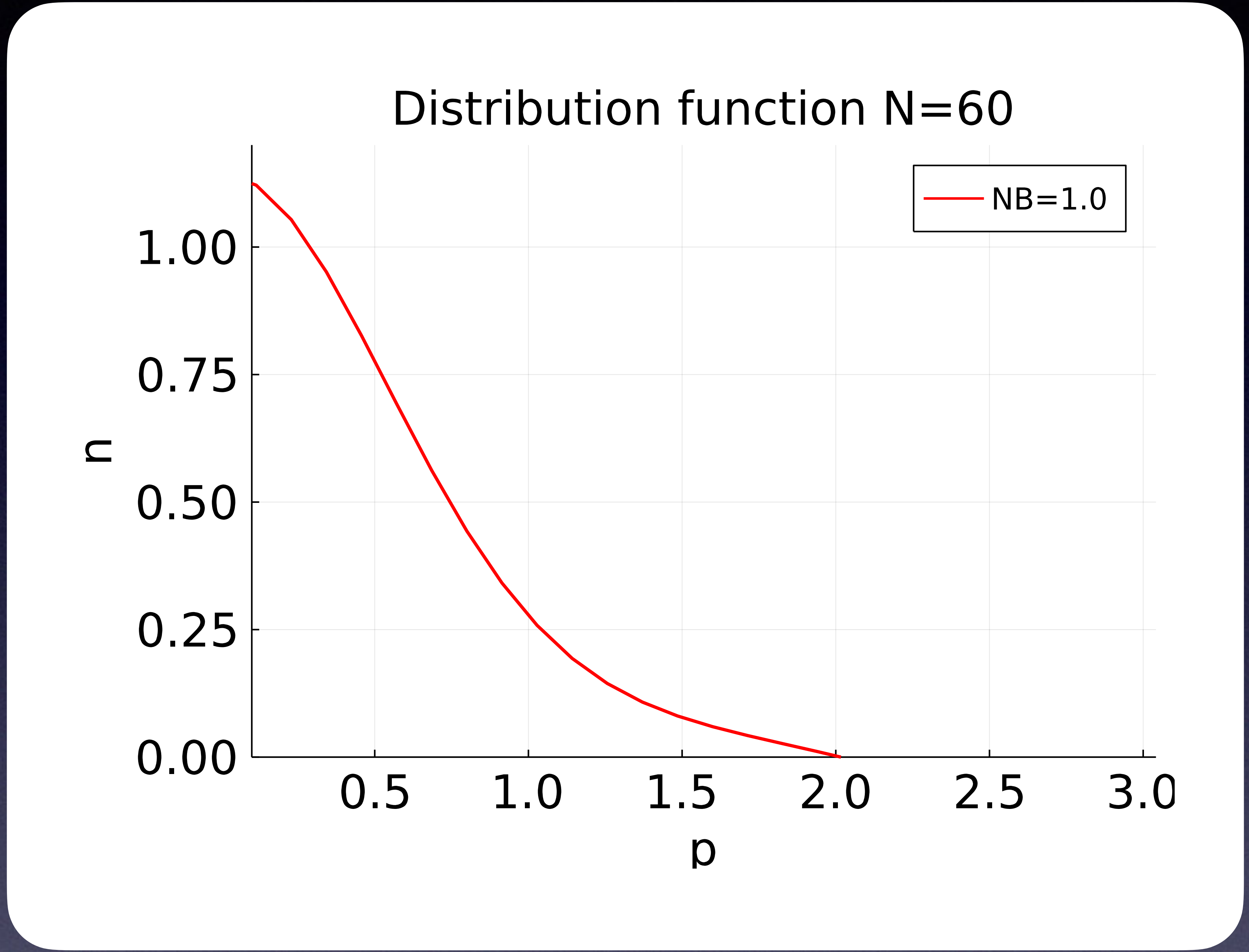
Use iTensor library

Numerical results

カラーSU(2), 1フレーバー,真空中のバリオン

Hayata, YH, Nishimura (2022)

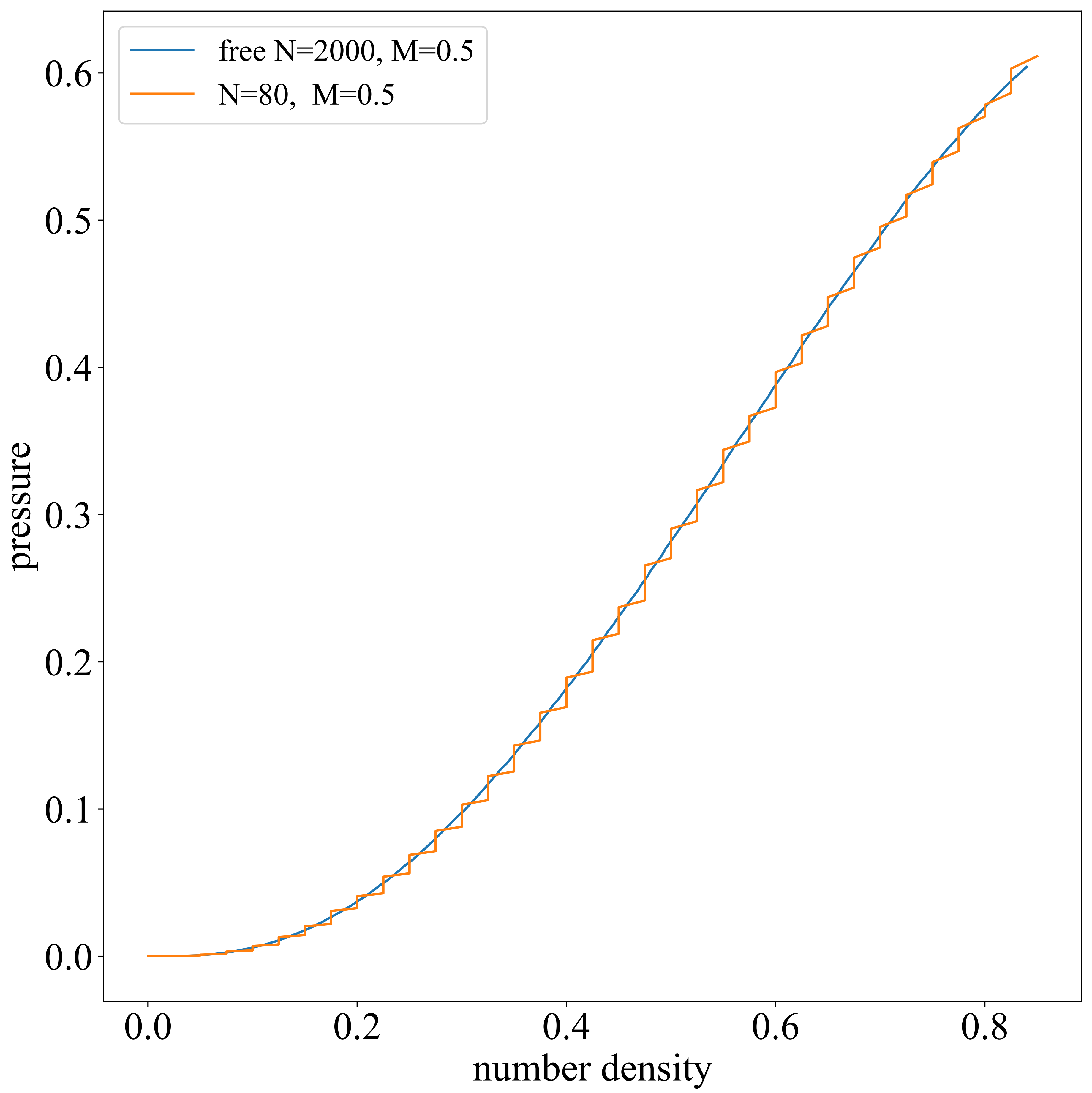




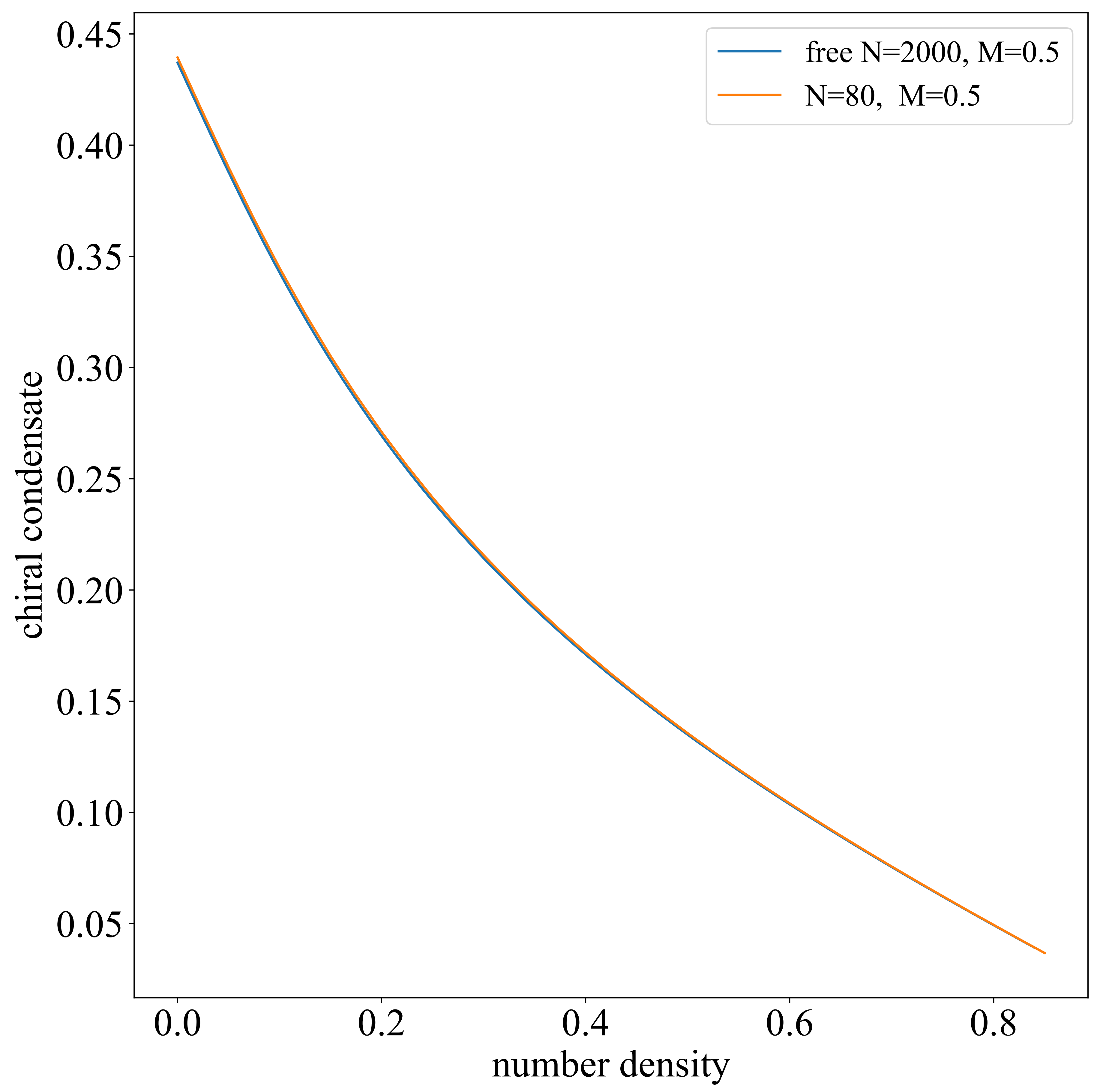
核子中は, カイラル対称性が部分的に回復

熱力学量(Free theory g=0)

圧力



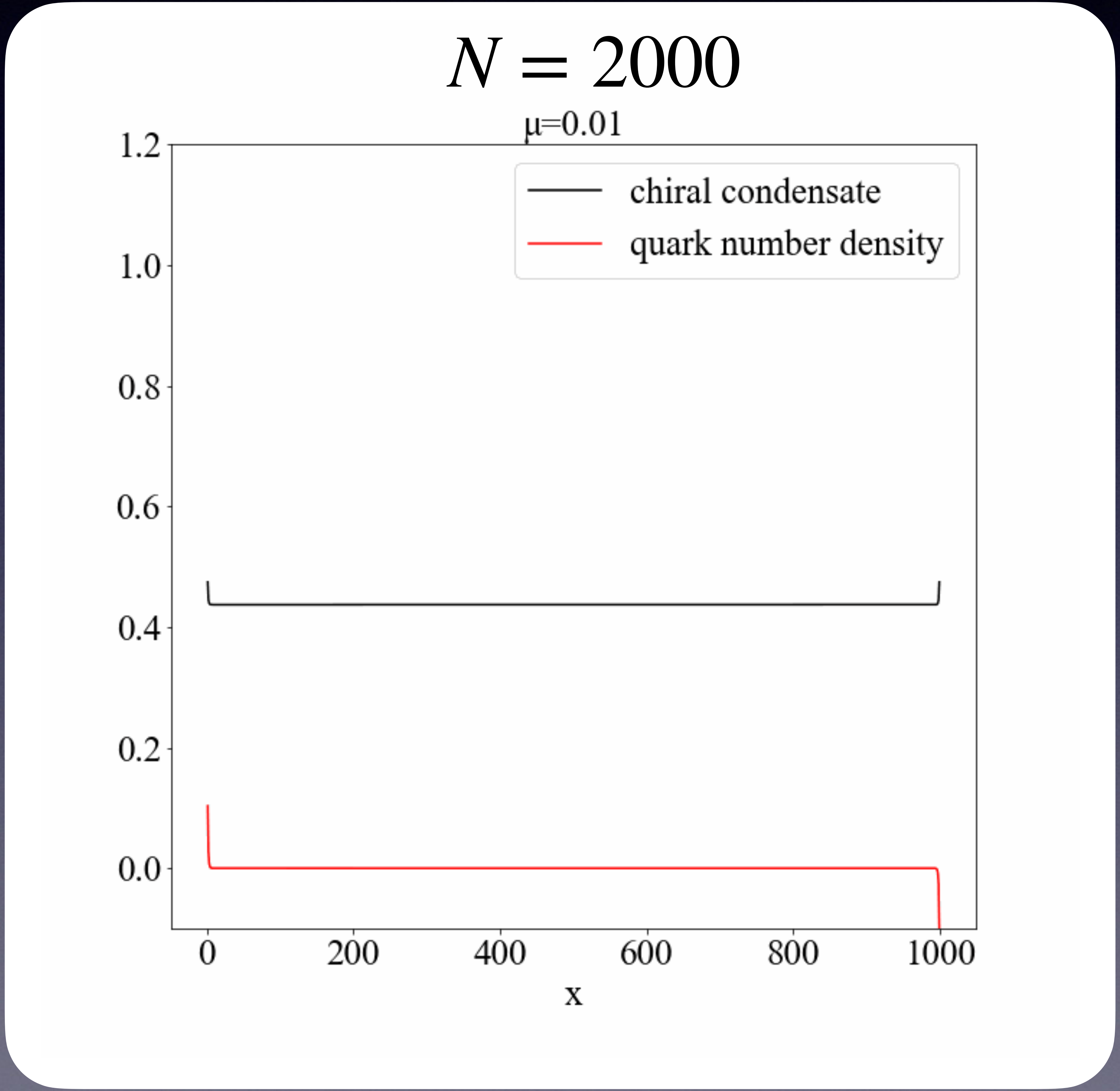
カイラル凝縮



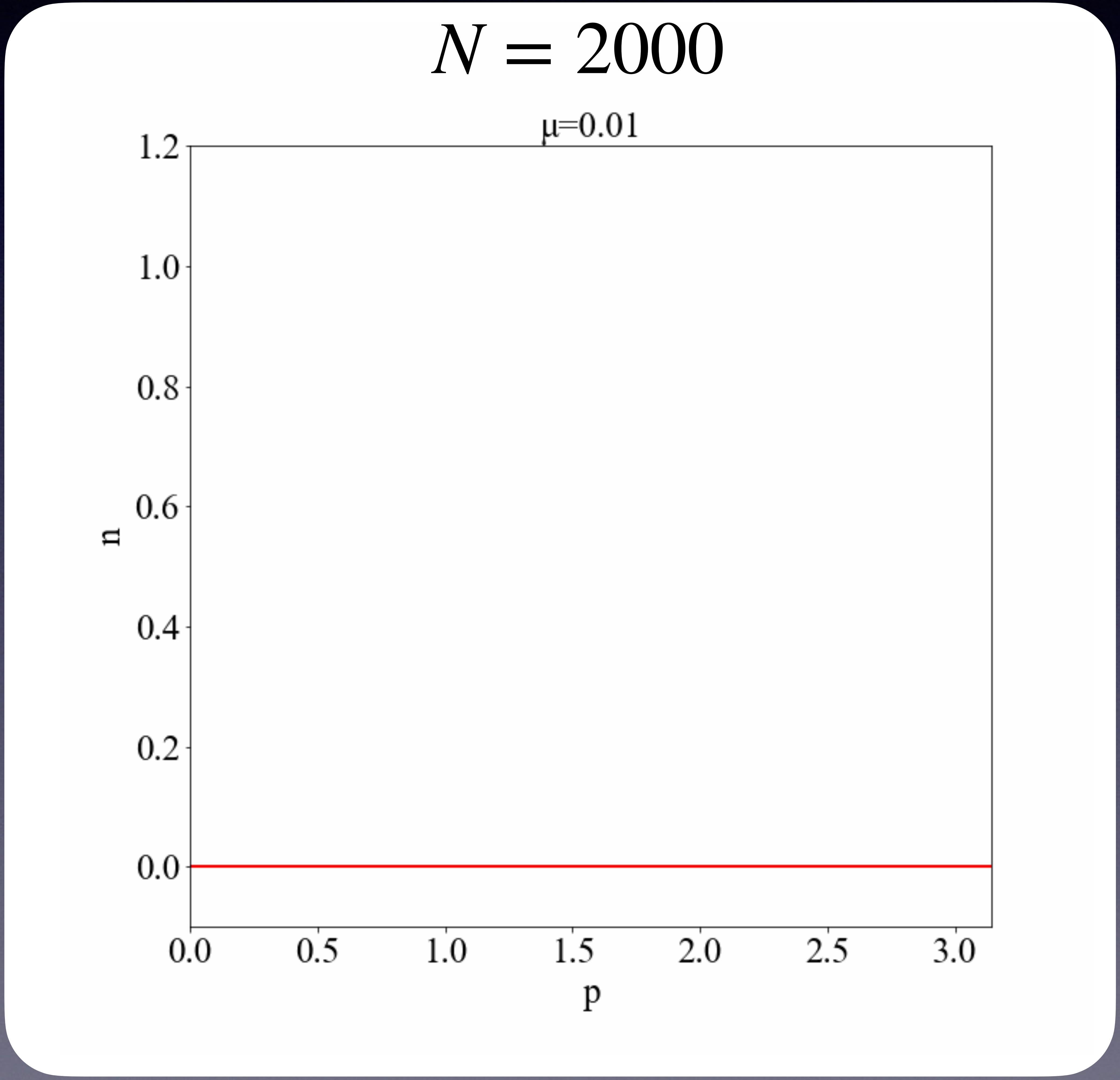
熱力学量(Free theory g=0)

Hayata, YH, Nishimura (2022)

クォーク数密度，カイラル凝縮



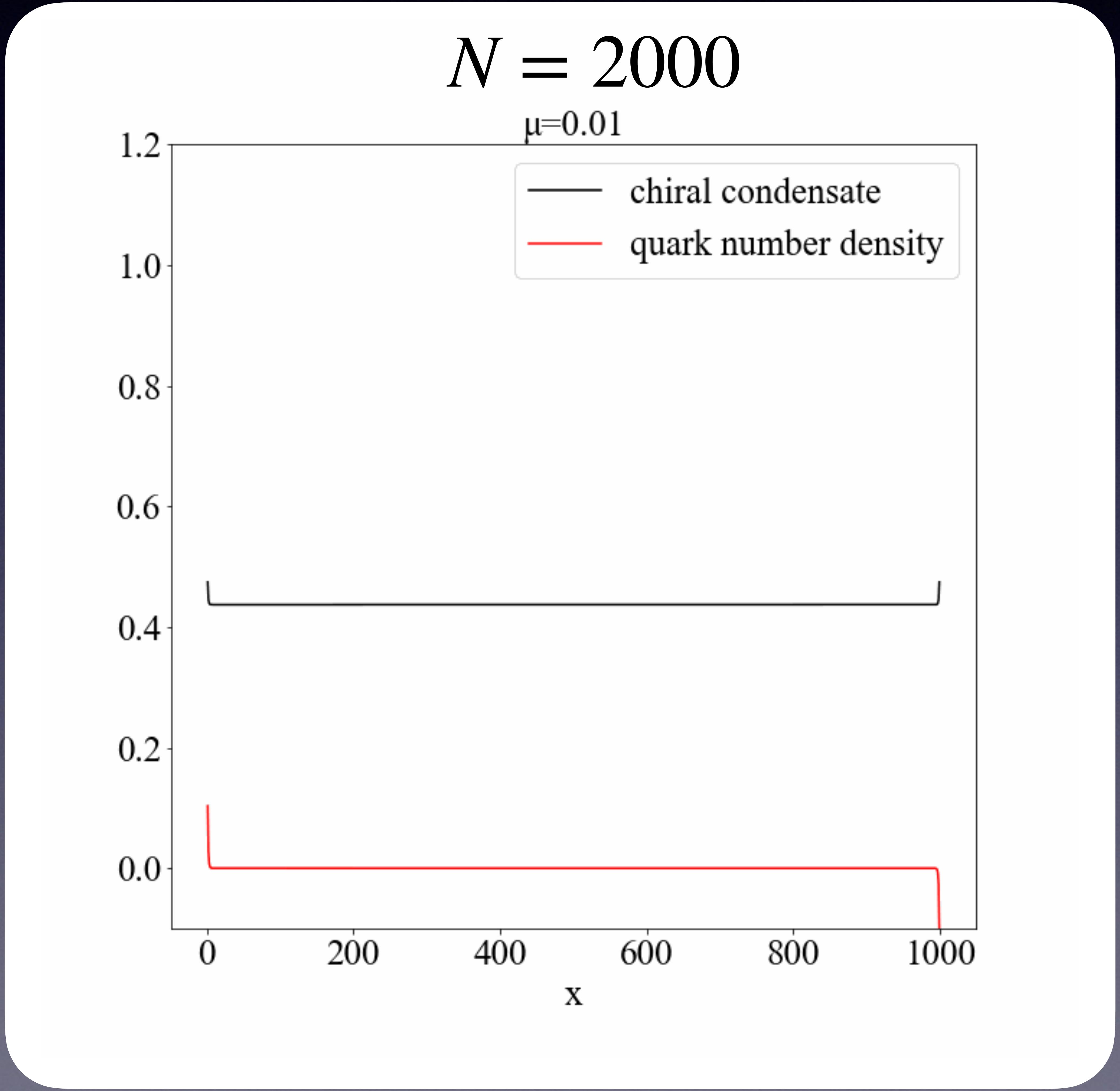
クォーク分布関数



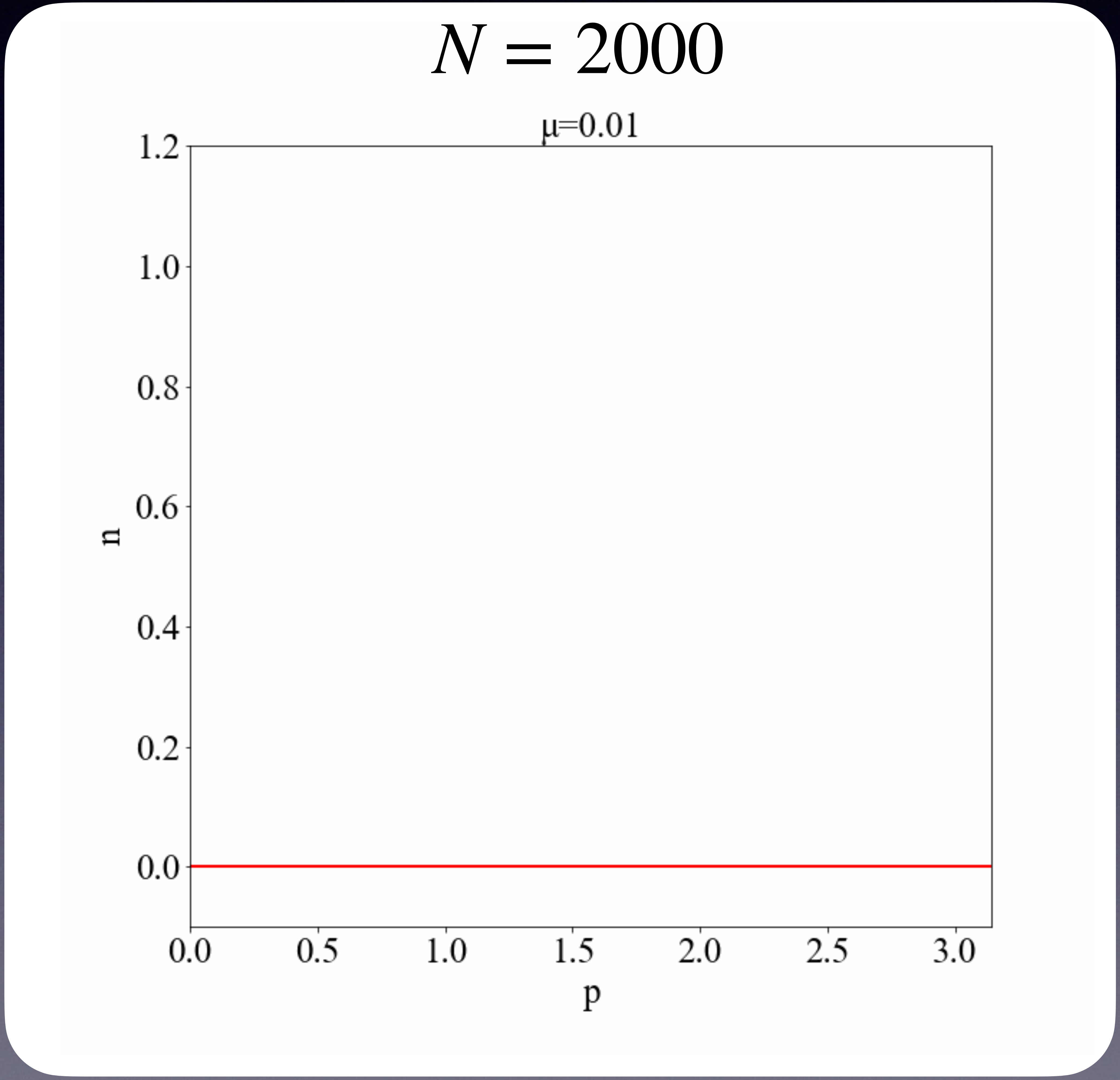
熱力学量(Free theory g=0)

Hayata, YH, Nishimura (2022)

クォーク数密度，カイラル凝縮



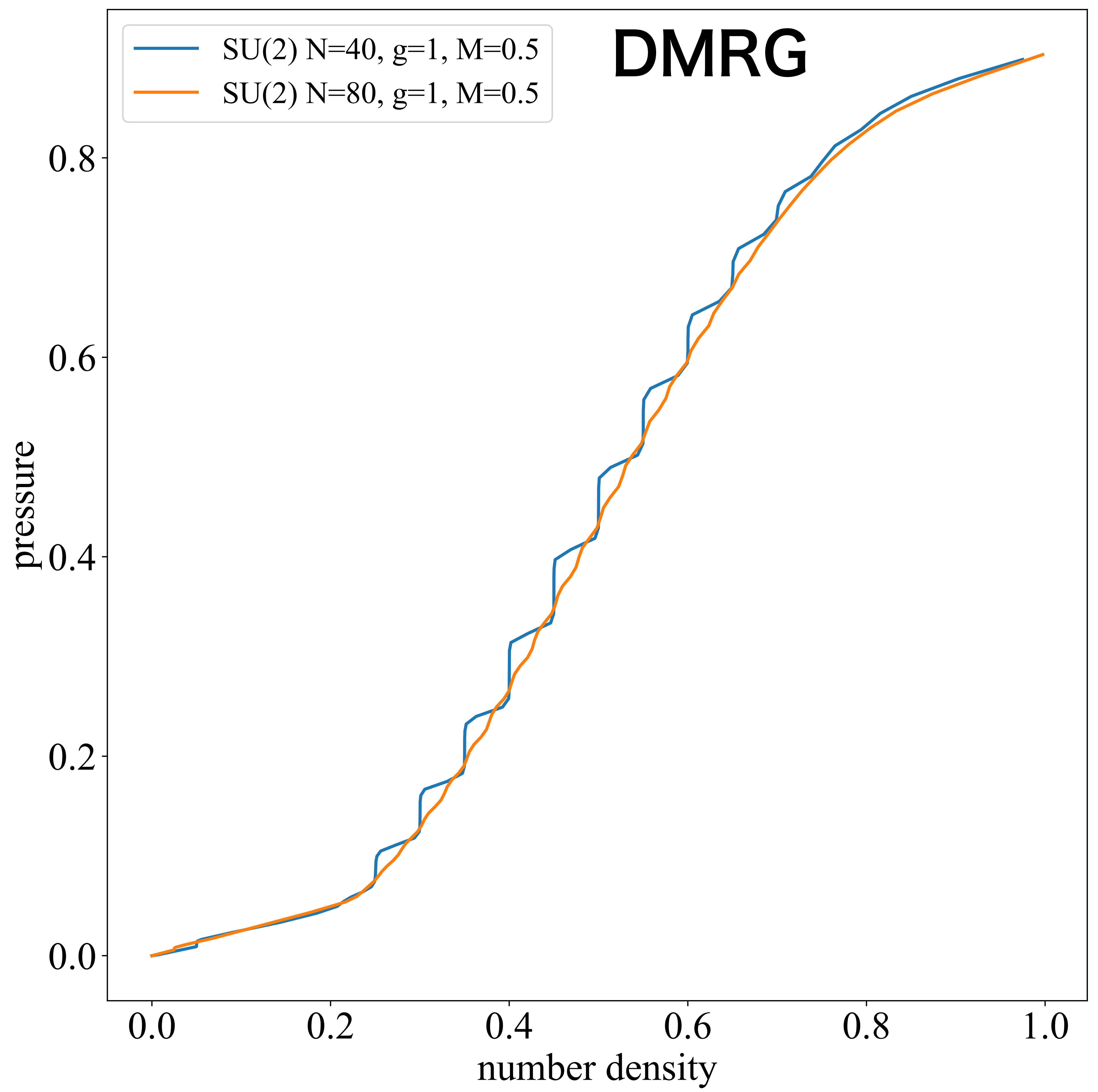
クォーク分布関数



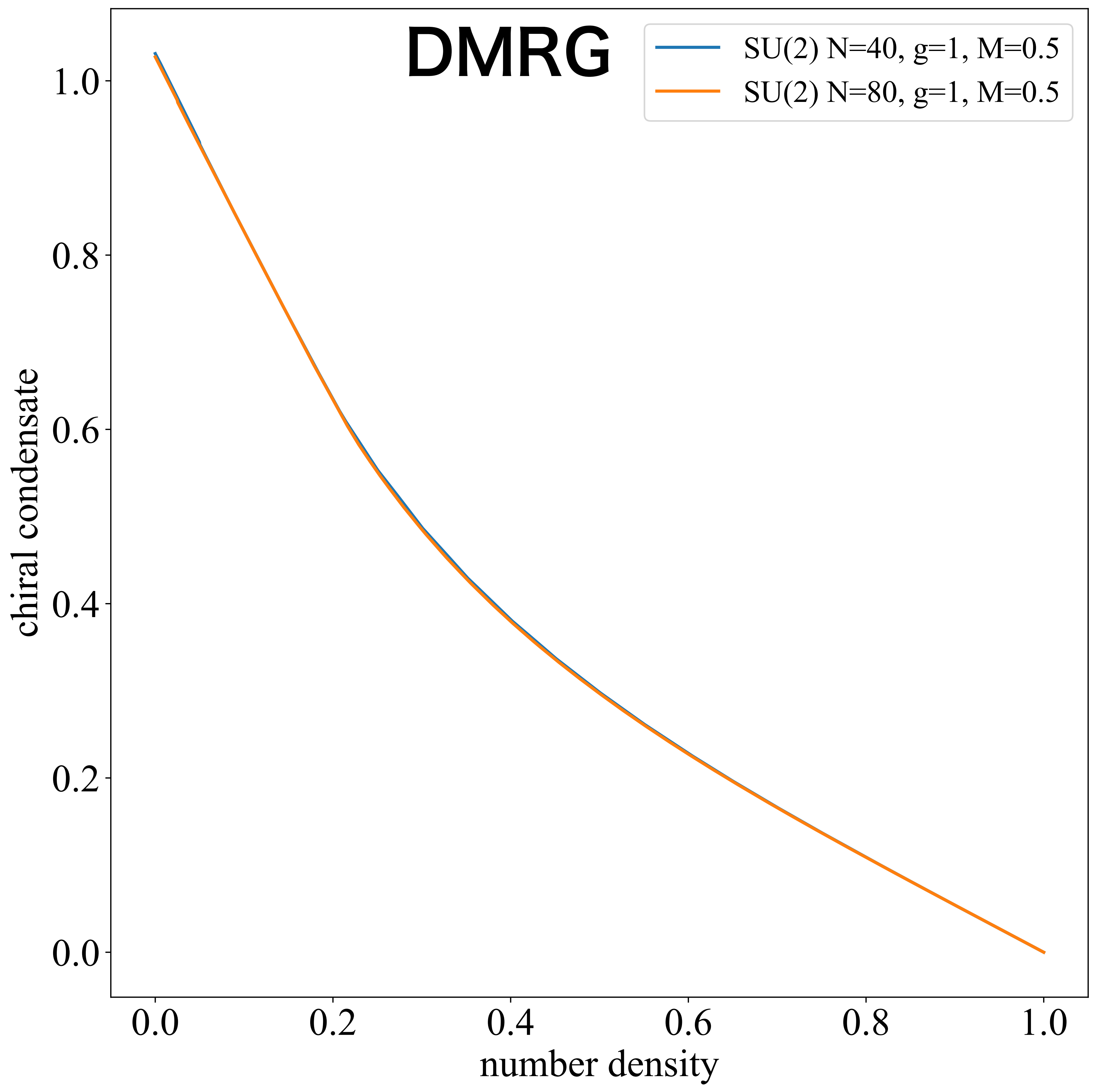
熱力学量 SU(2)

Hayata, YH, Nishimura (2022)

圧力



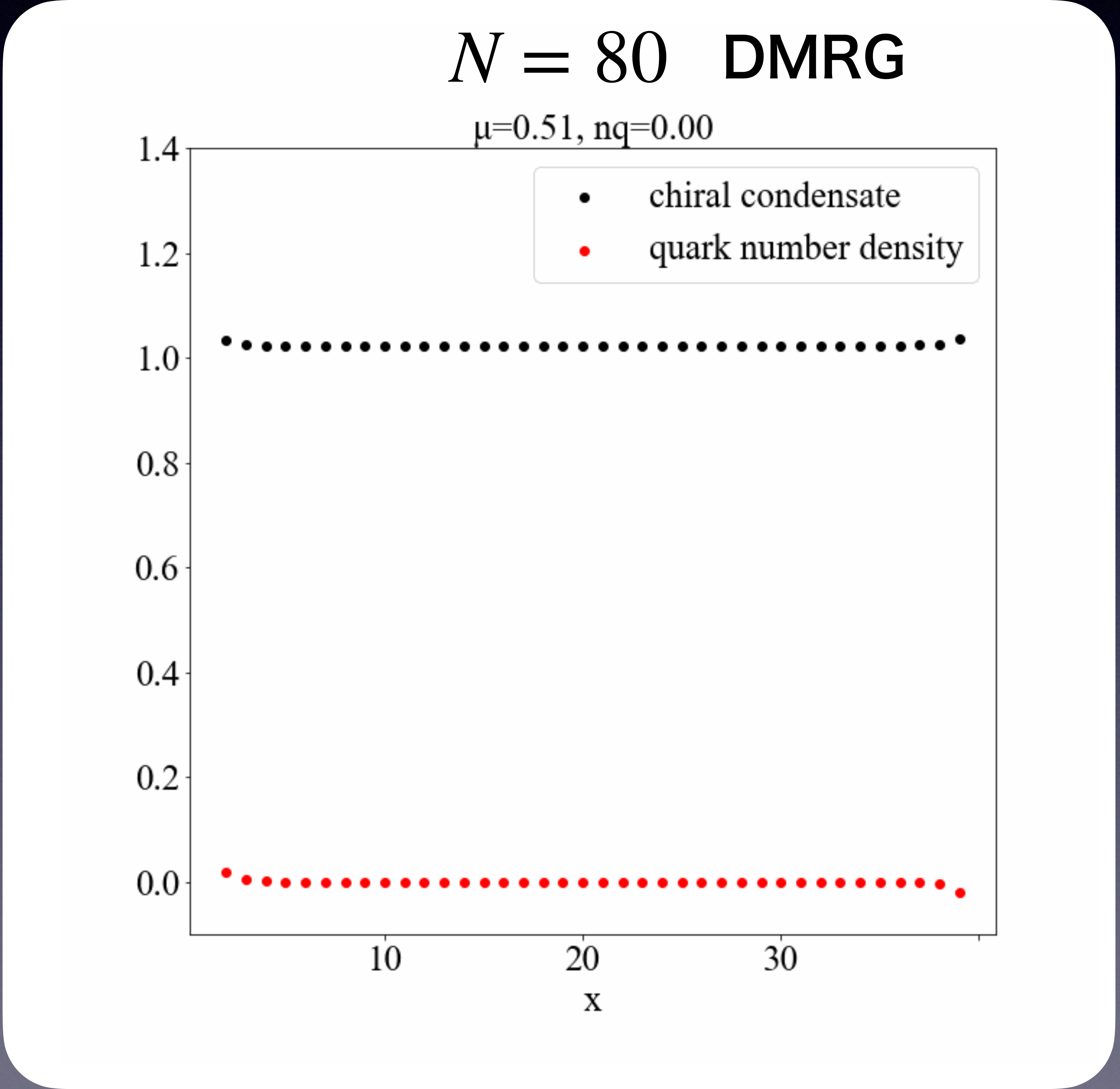
カイラル凝縮



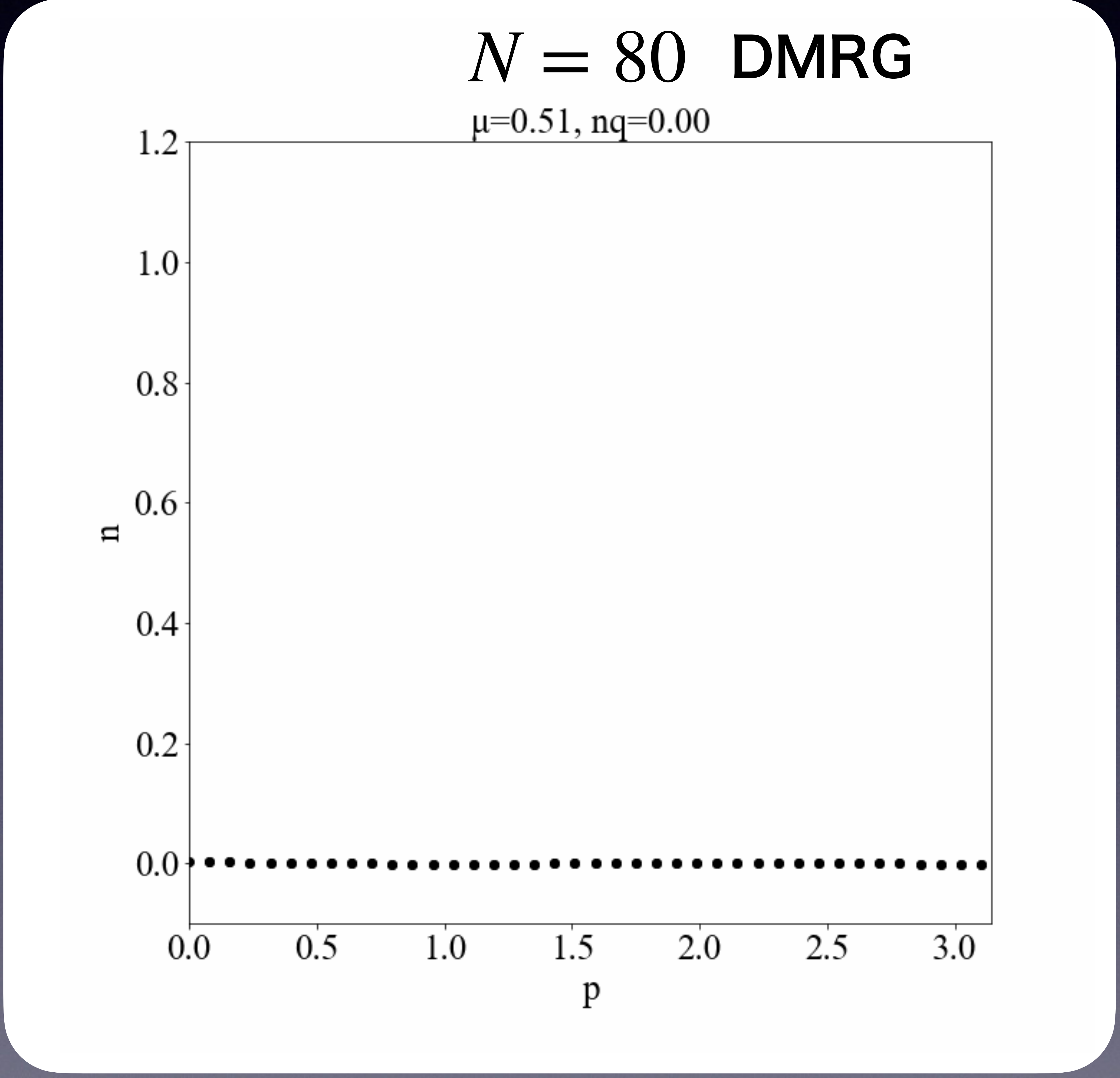
熱力学量 SU(2)

Hayata, YH, Nishimura (2022)

クォーク数密度，カイラル凝縮



クォーク分布関数

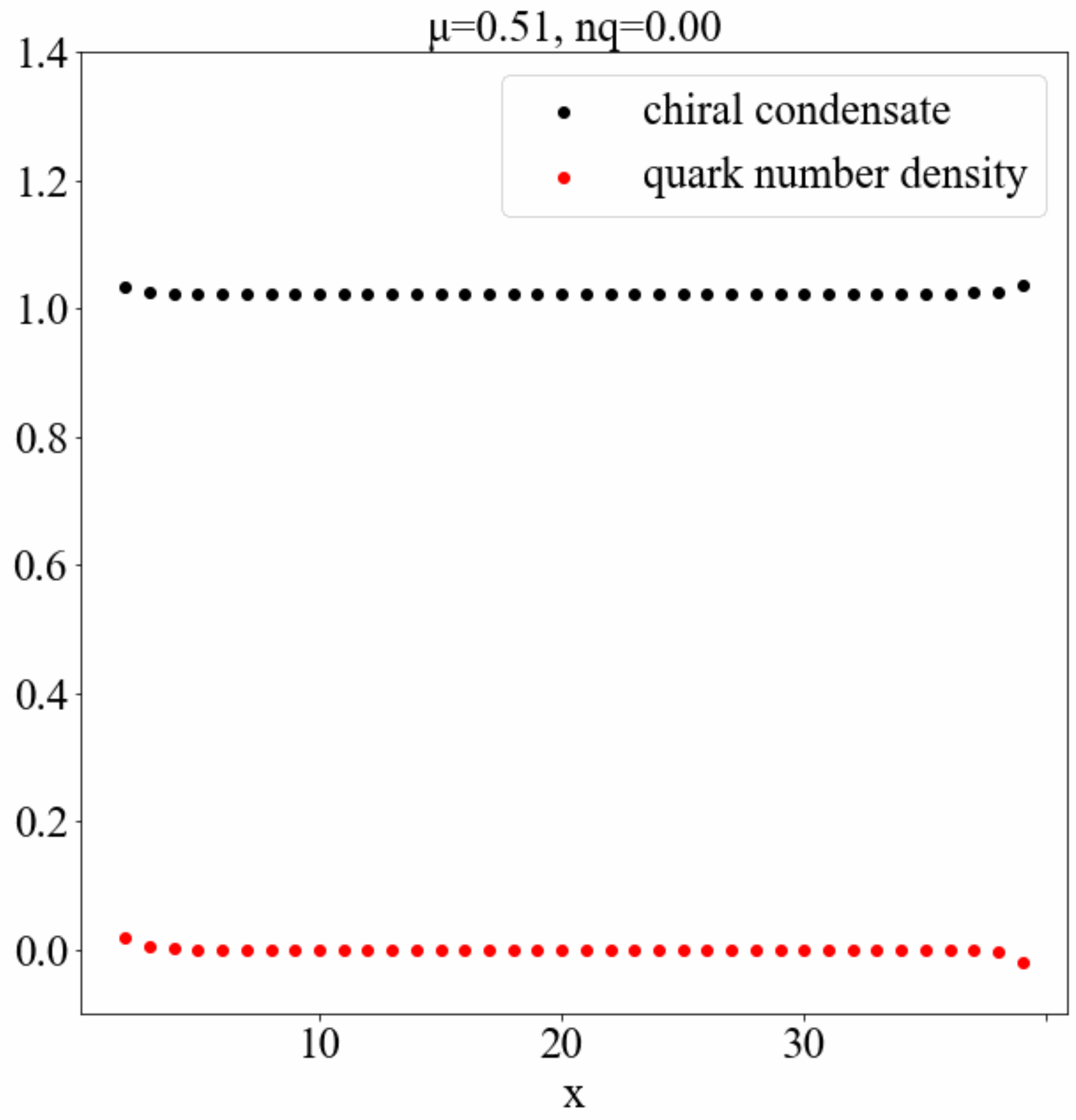


熱力学量 SU(2)

Hayata, YH, Nishimura (2022)

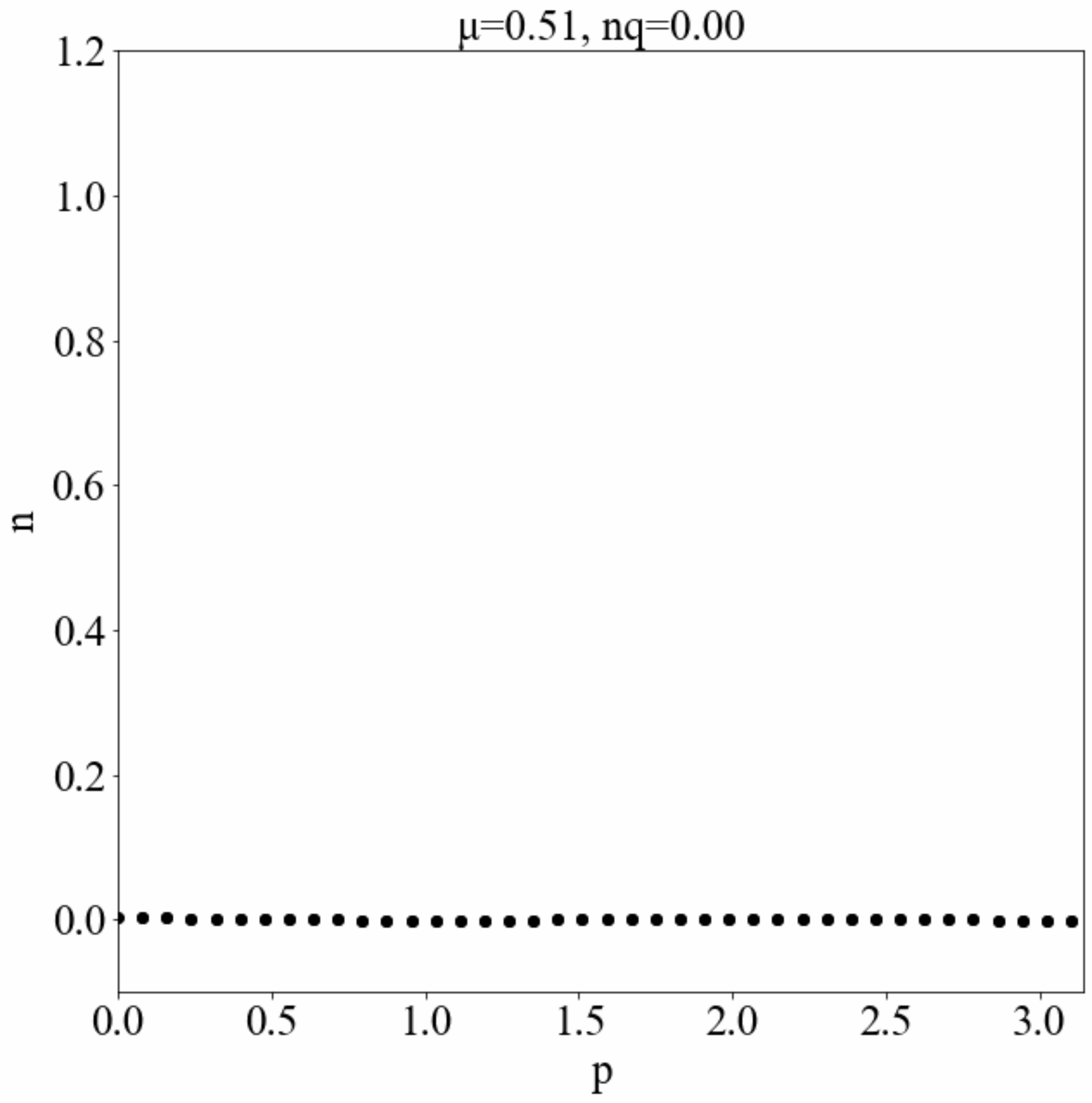
クォーク数密度，カイラル凝縮

$N = 80$ DMRG



クォーク分布関数

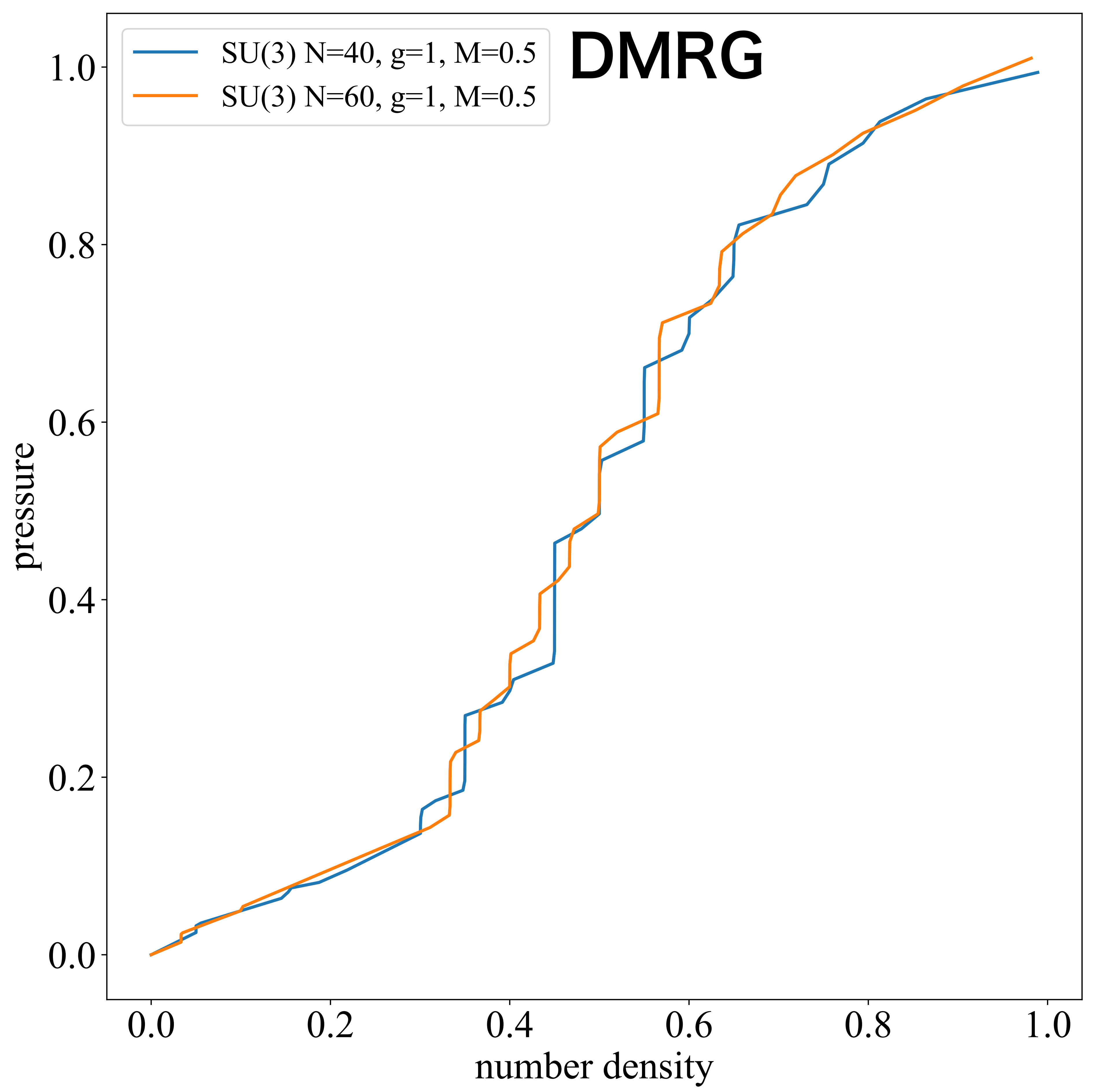
$N = 80$ DMRG



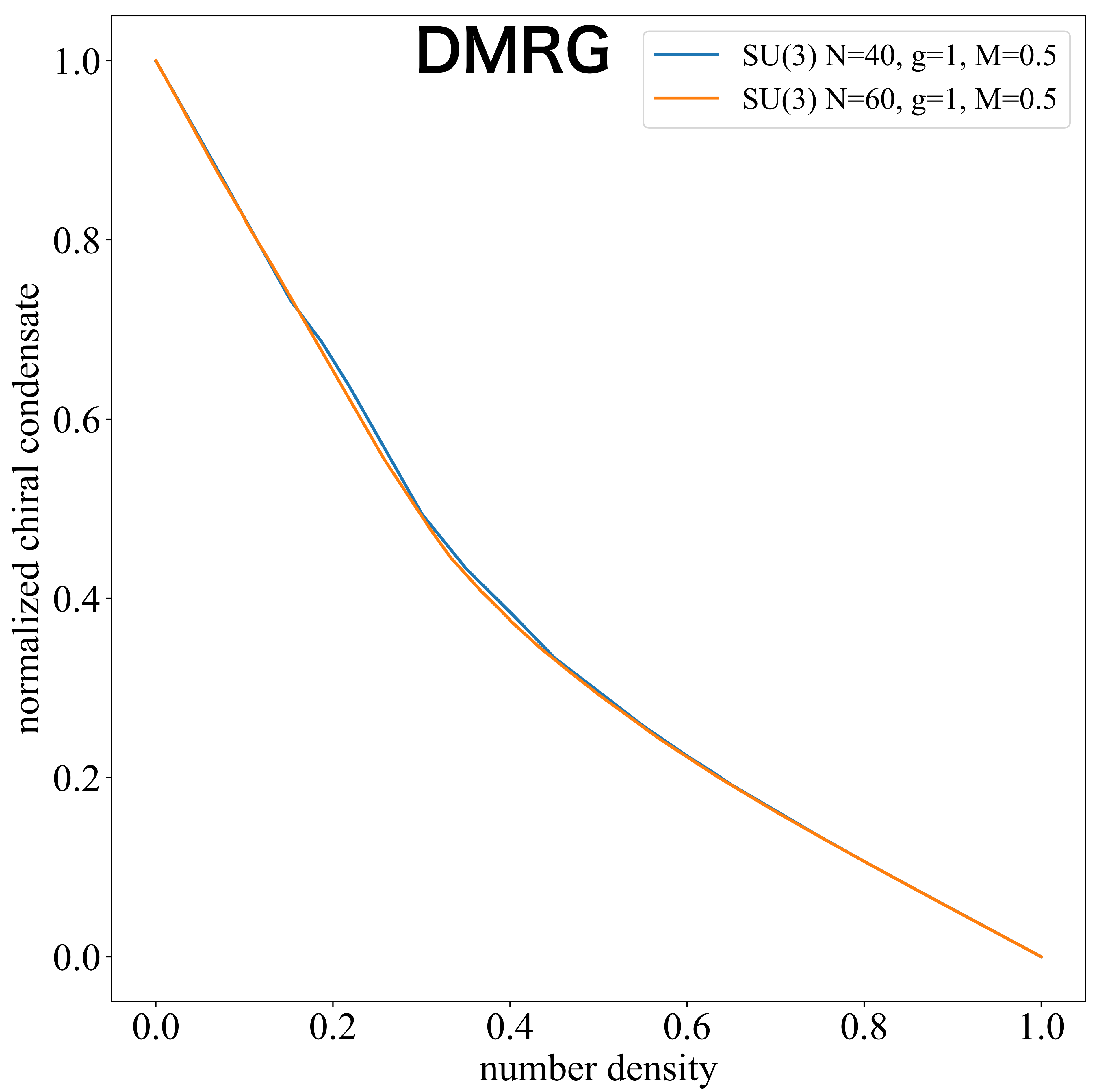
熱力学量 SU(3)

Hayata, YH, Nishimura (2022)

圧力



カイラル凝縮

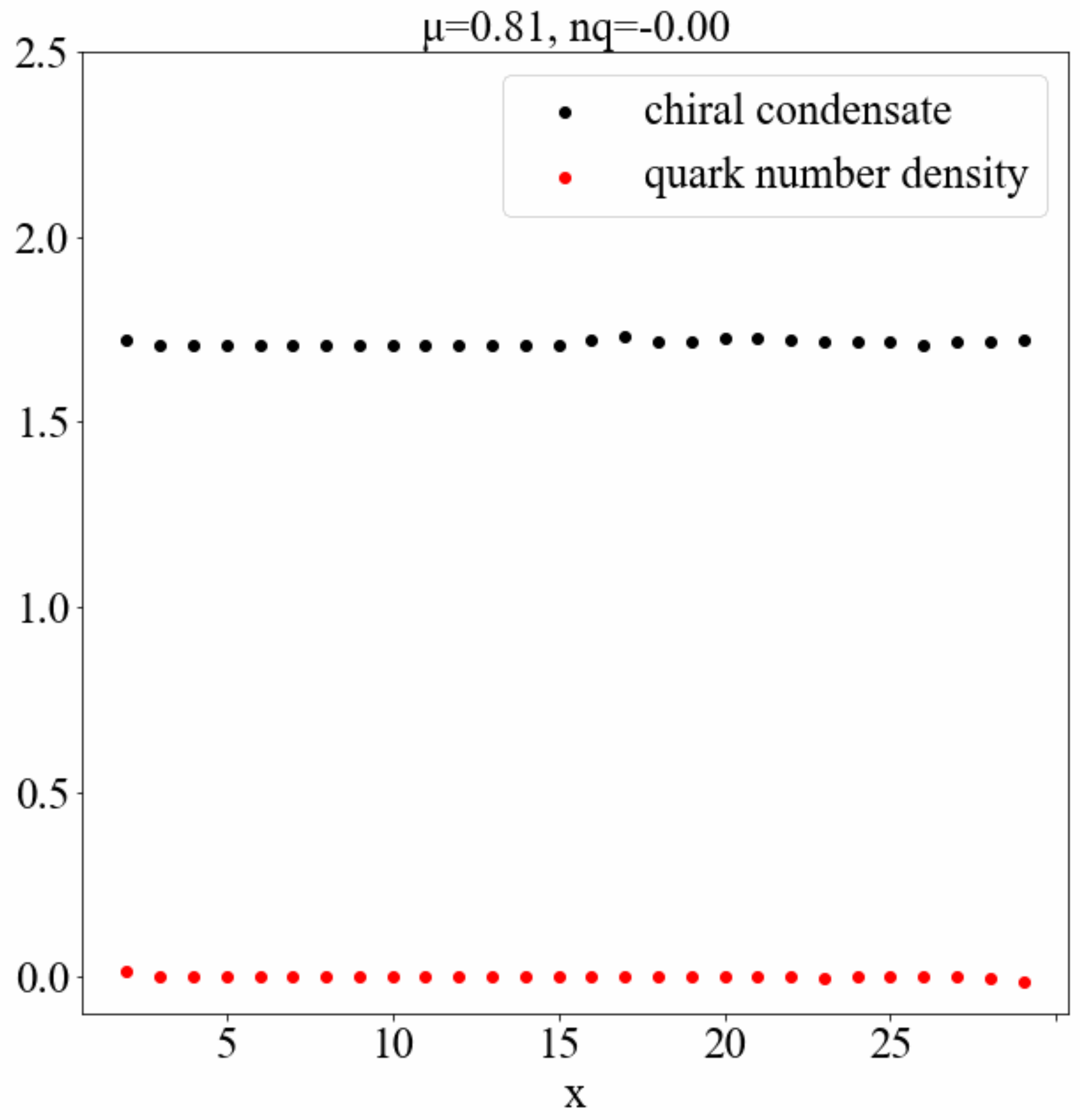


熱力学量 SU(3)

Hayata, YH, Nishimura (2022)

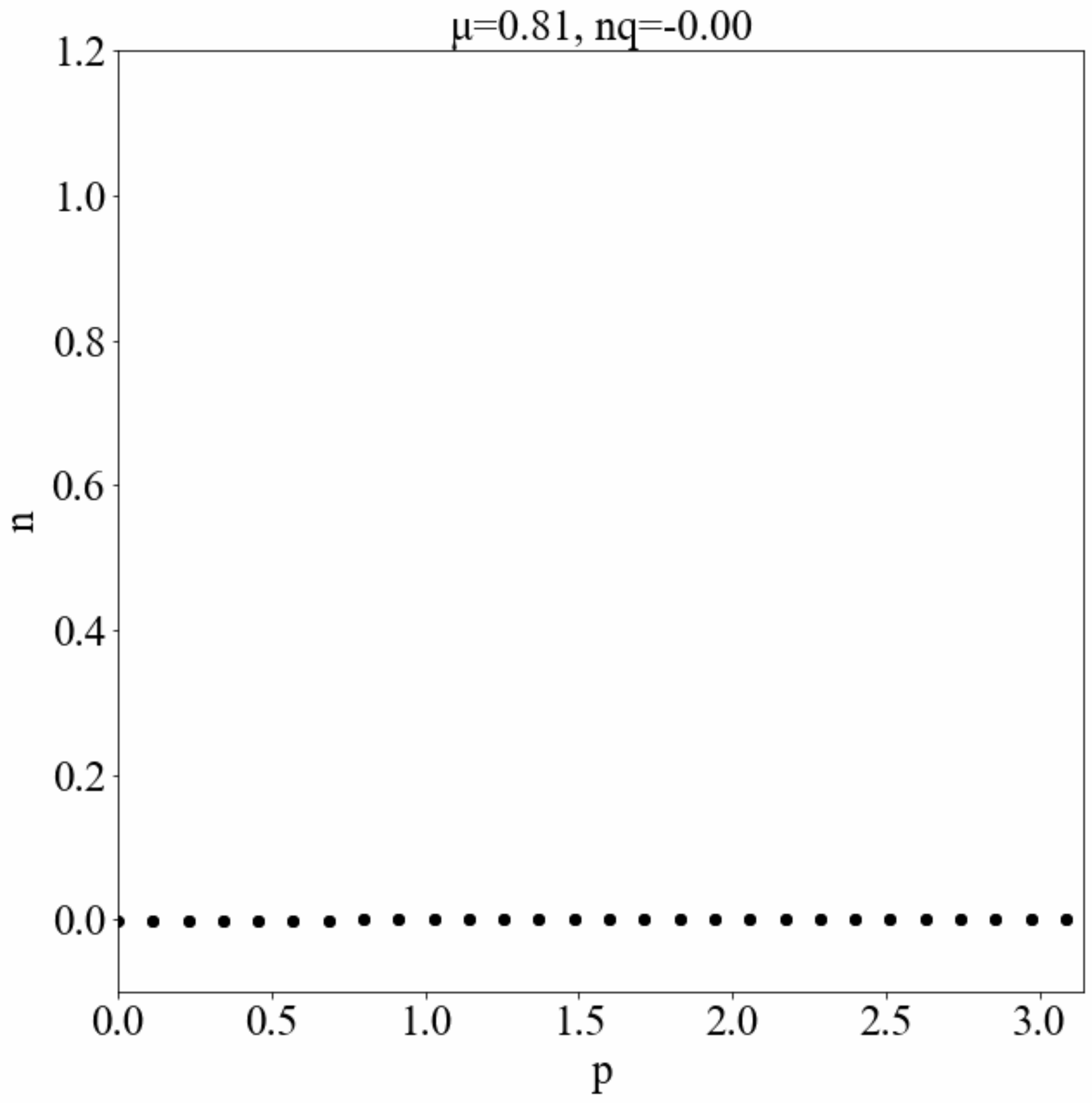
クォーク数密度，カイラル凝縮

$N = 60$ DMRG



クォーク分布関数

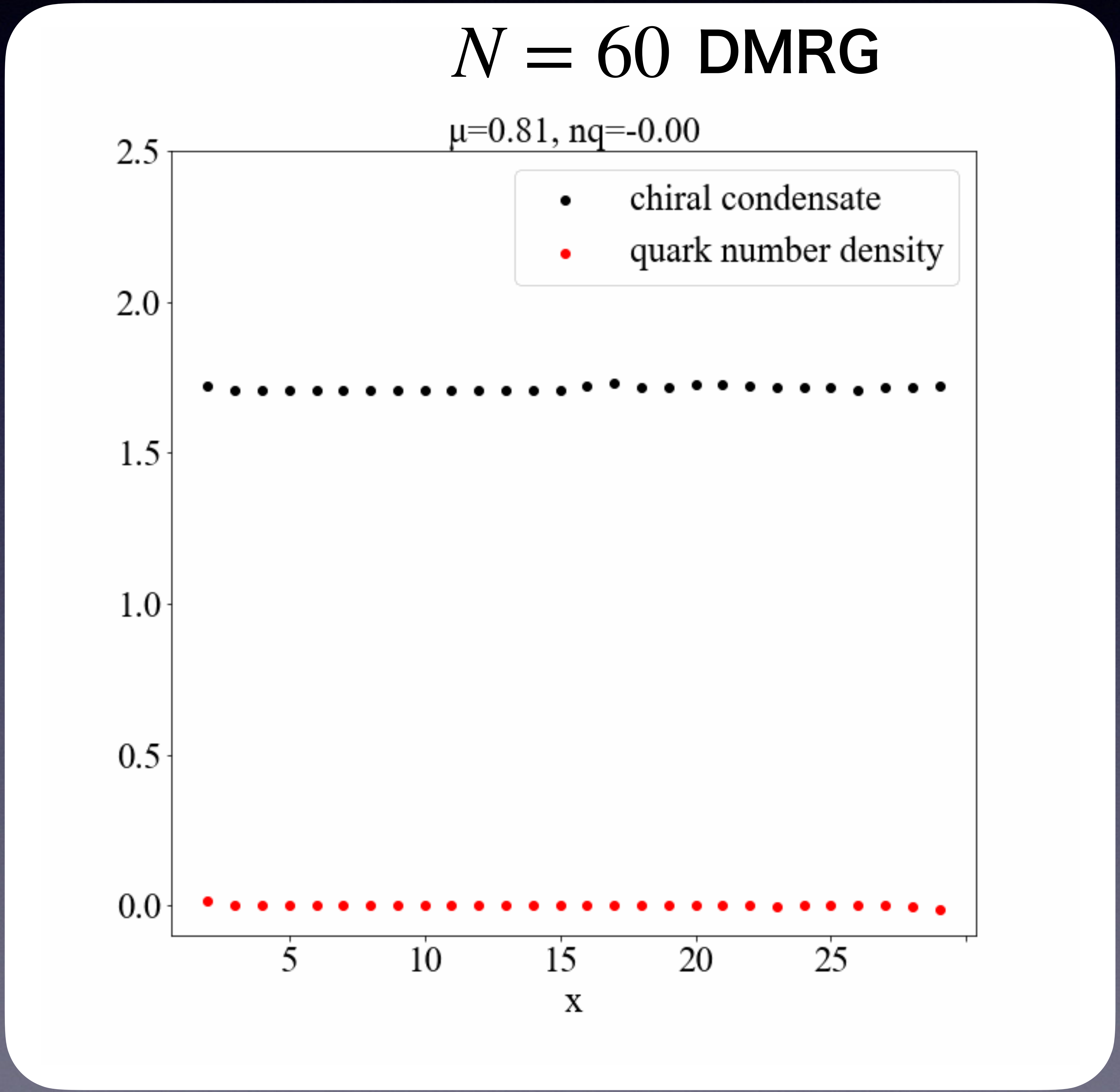
$N = 60$ DMRG



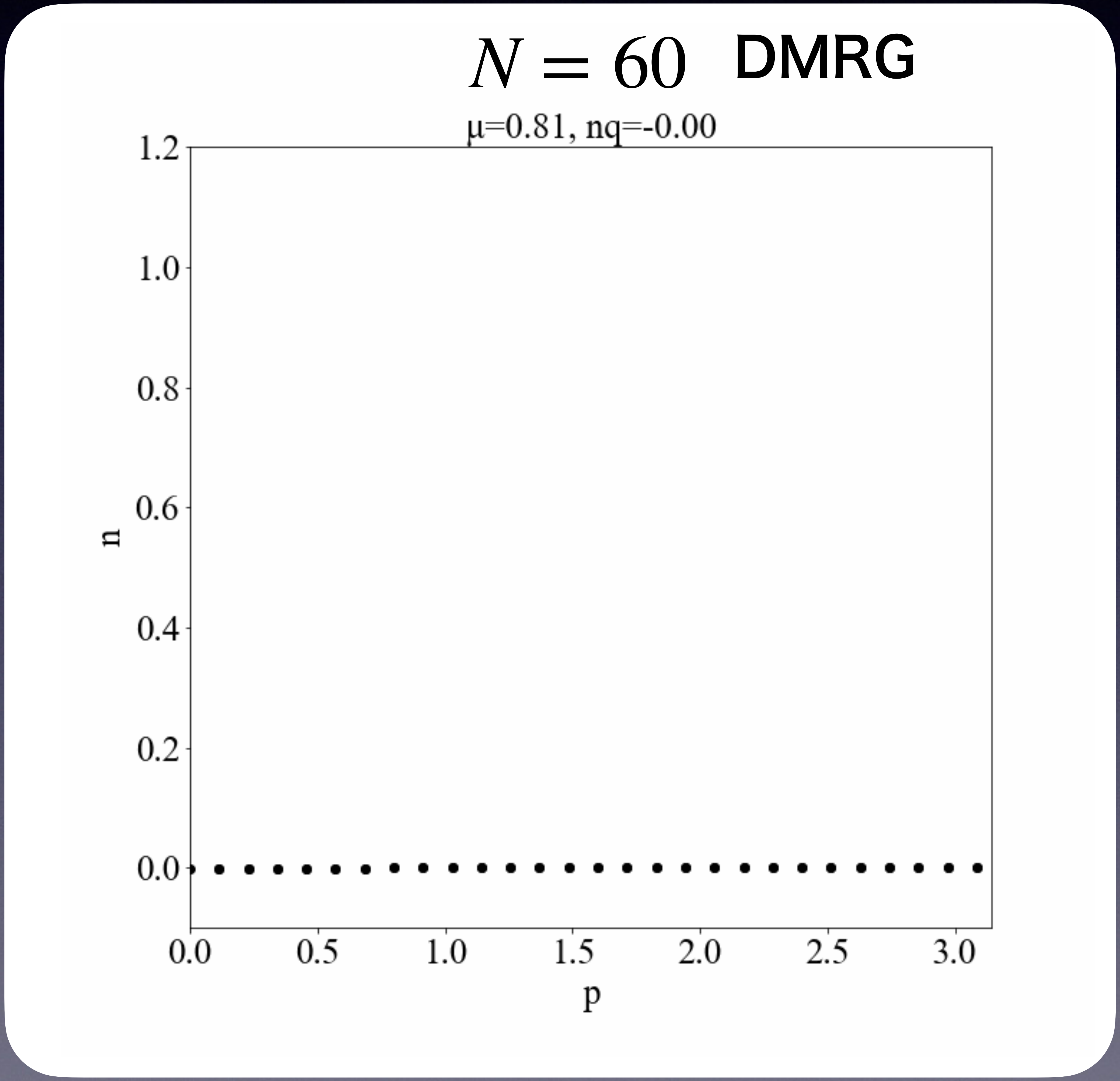
熱力学量 SU(3)

Hayata, YH, Nishimura (2022)

クォーク数密度，カイラル凝縮



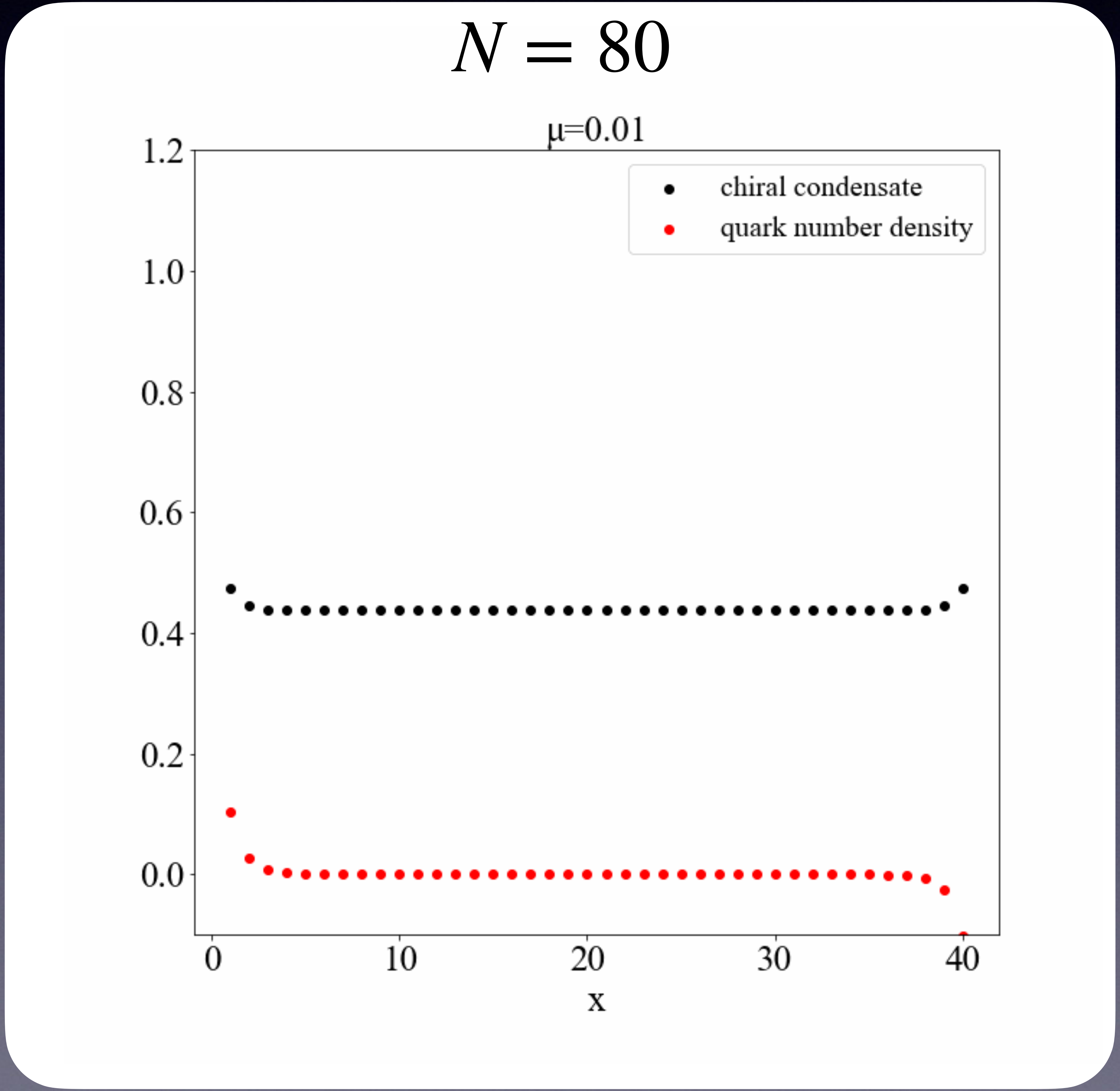
クォーク分布関数



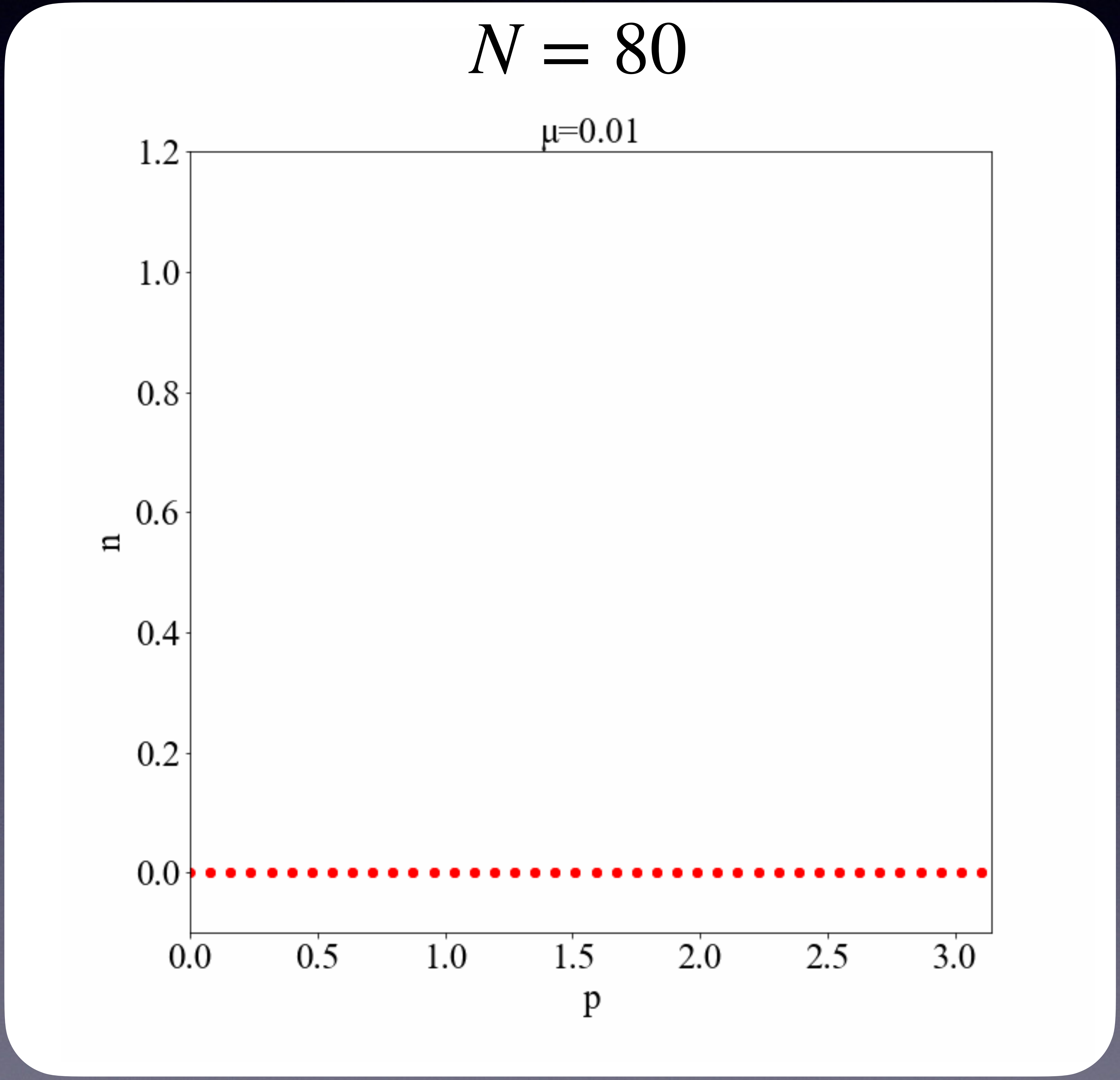
熱力学量(Free theory g=0)

Hayata, YH, Nishimura (2022)

クォーク数密度，カイラル凝縮



クォーク分布関数

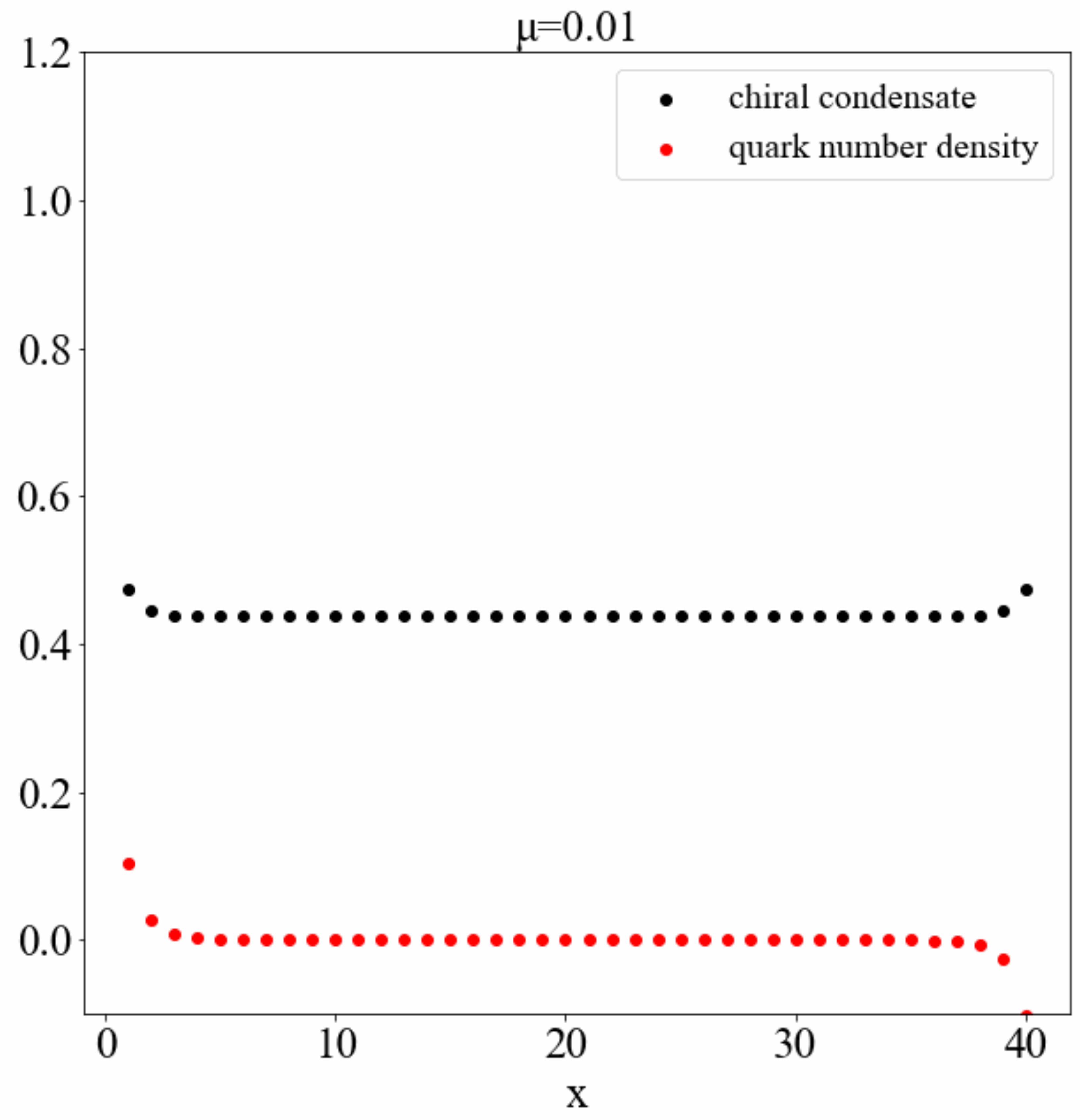


熱力学量(Free theory g=0)

Hayata, YH, Nishimura (2022)

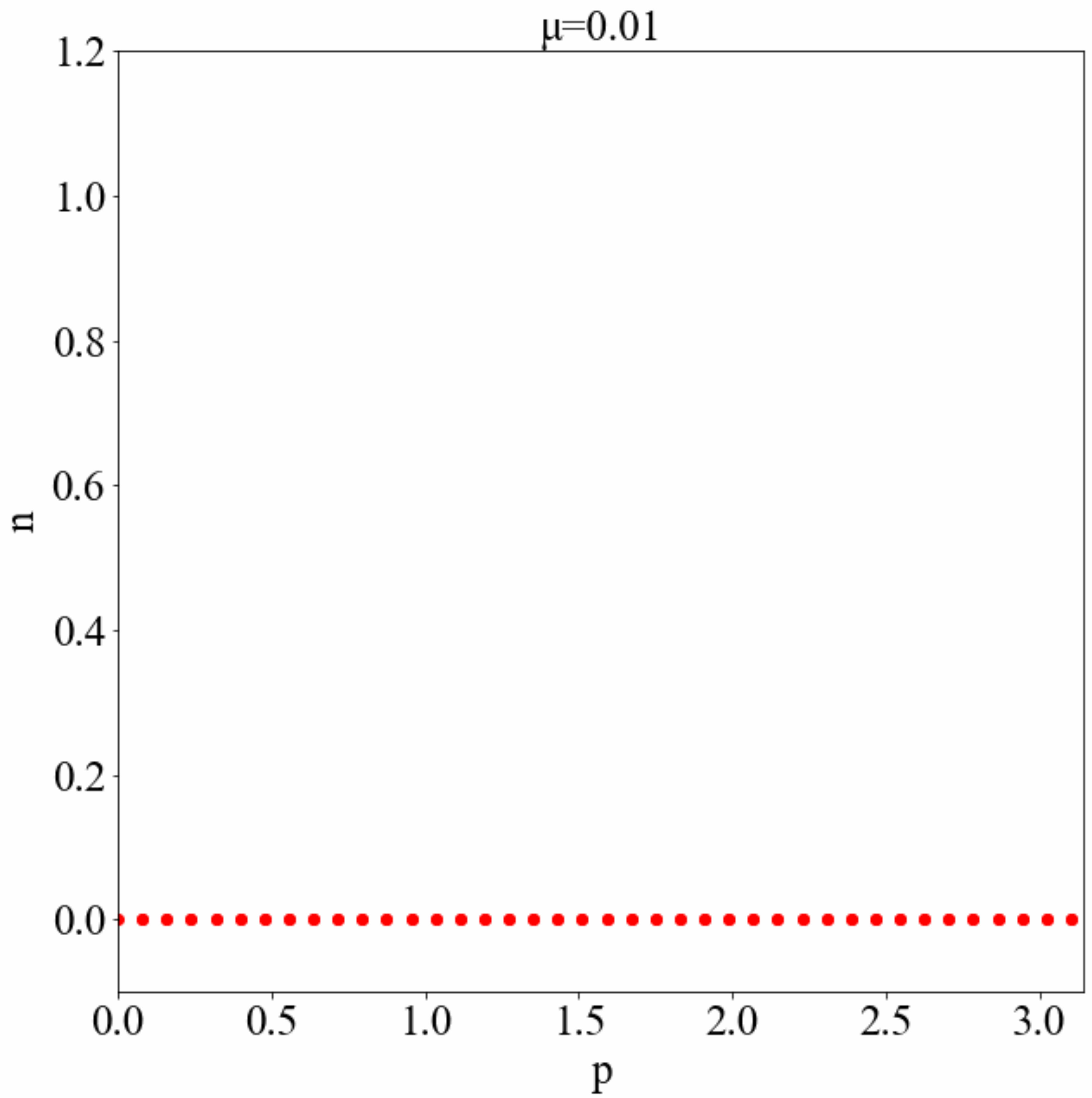
クォーク数密度，カイラル凝縮

$N = 80$



クォーク分布関数

$N = 80$



熱力学量(Free theory g=0)

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クォーク数密度，カイラル凝縮

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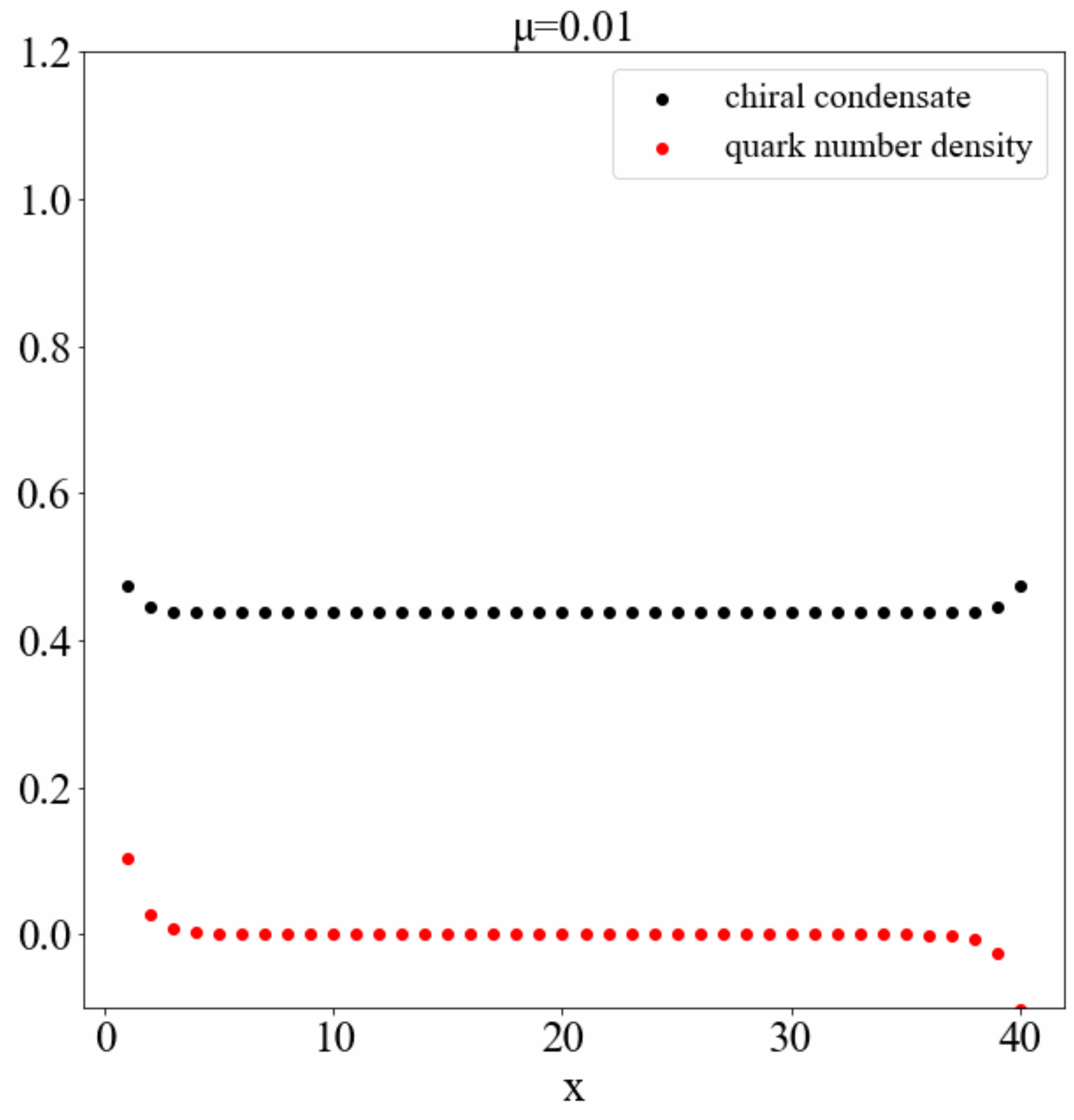
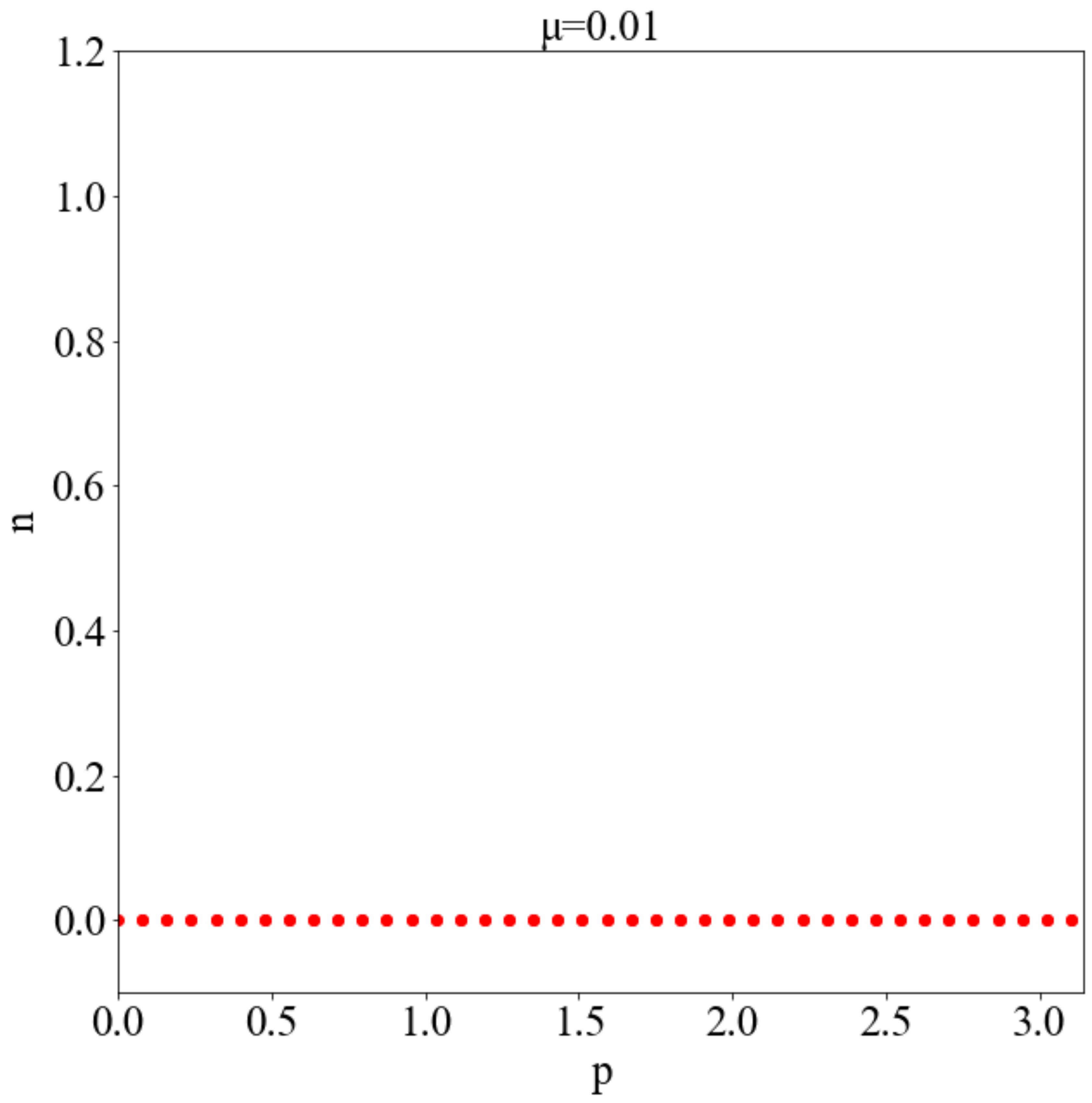


クォーク分布関数

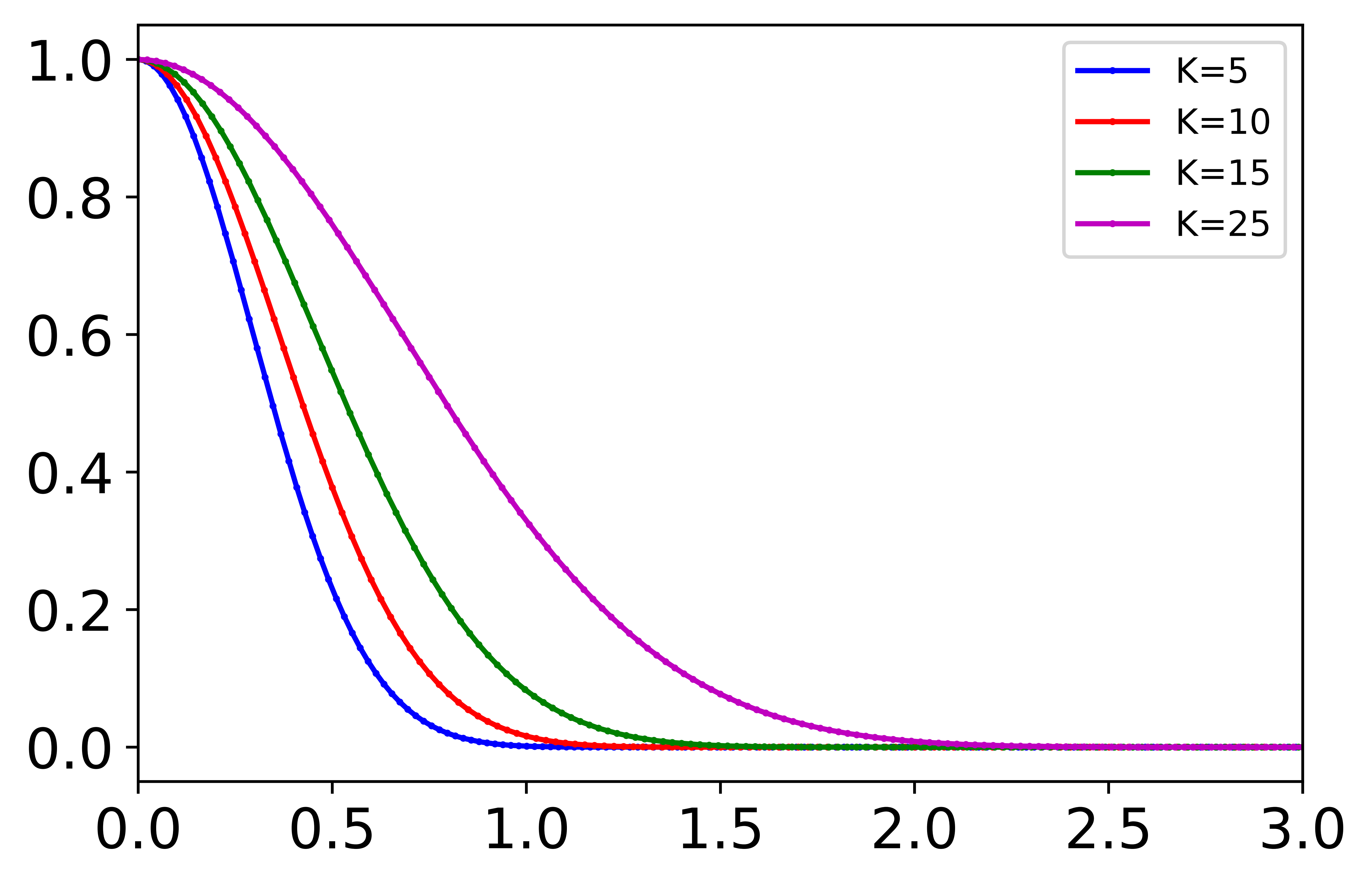
$N = 80$



Thermalization

Vacuum persistency probability (Loschmidt echo)

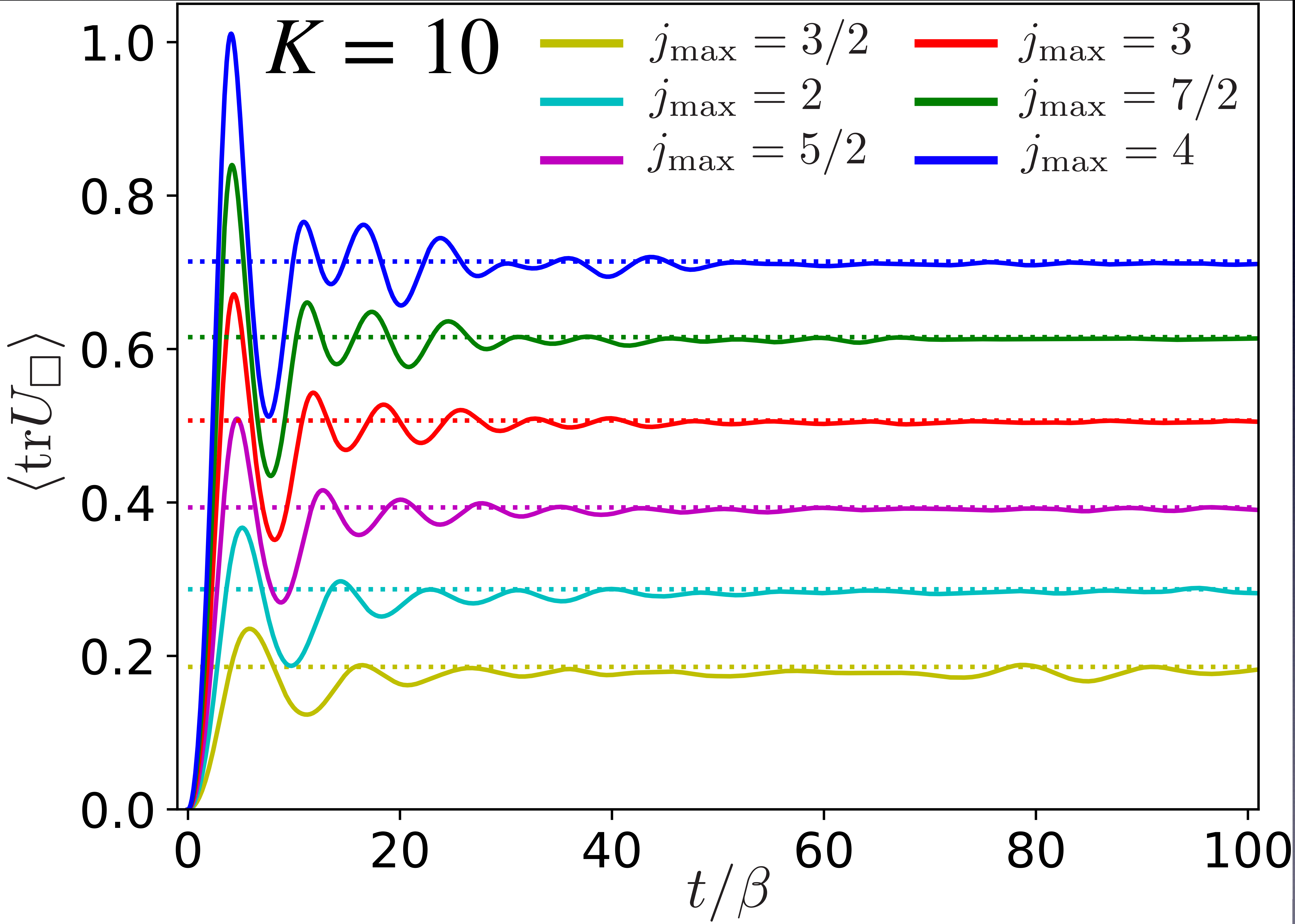
$$P_{\mathrm{vac}} := |\langle \Psi(0) | \Psi(t) \rangle|^2$$



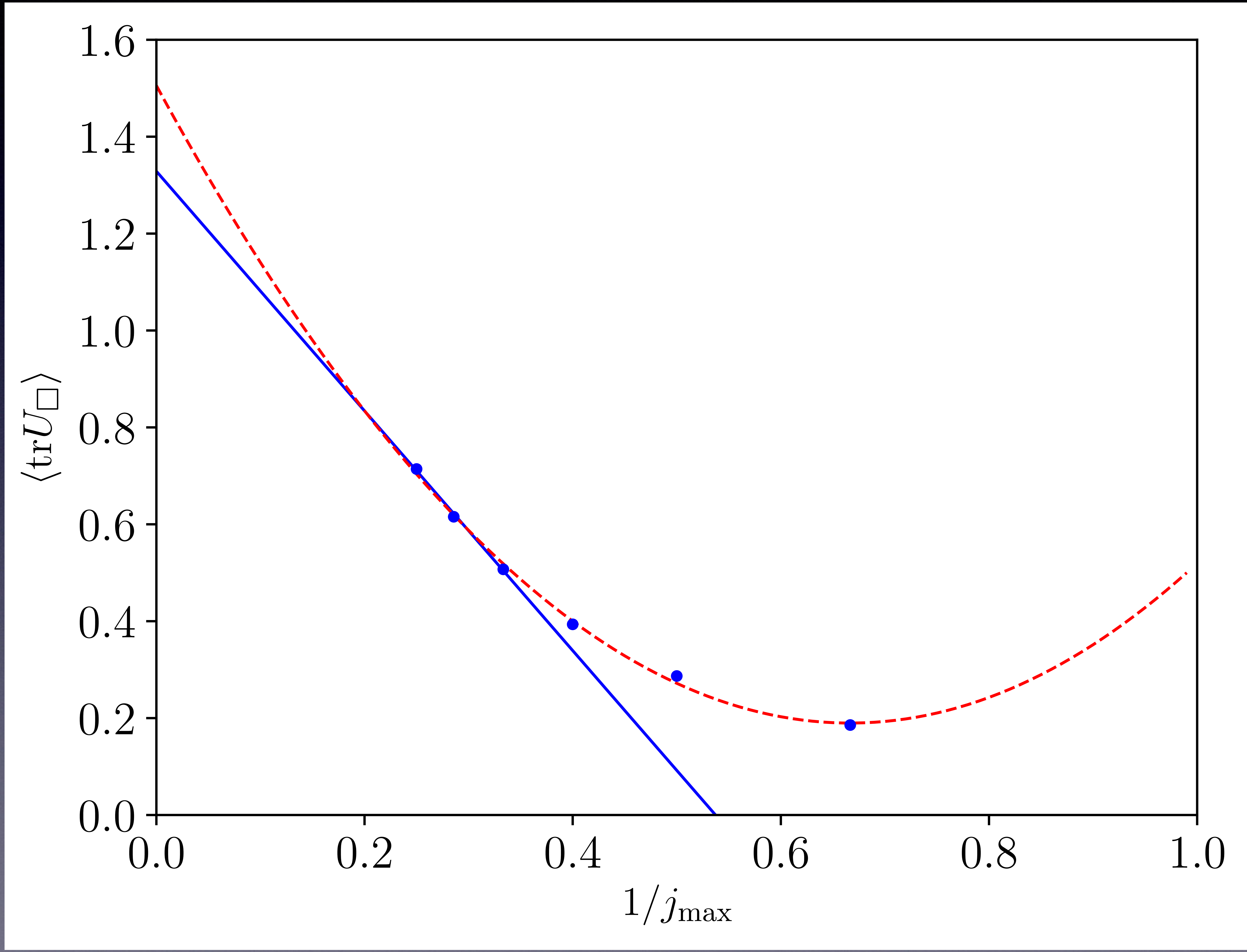
P_{vac}

t/β

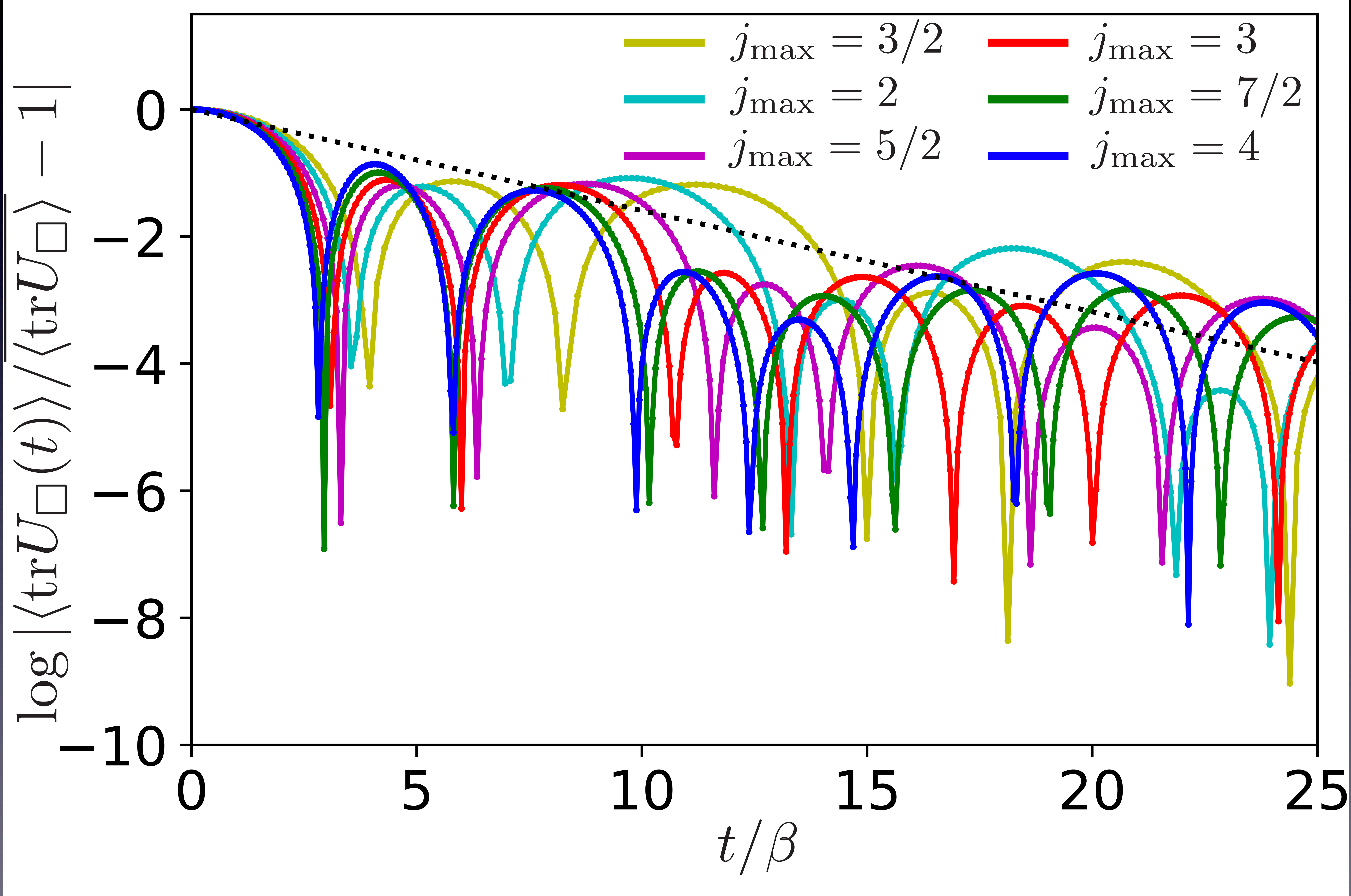
jmax dependence



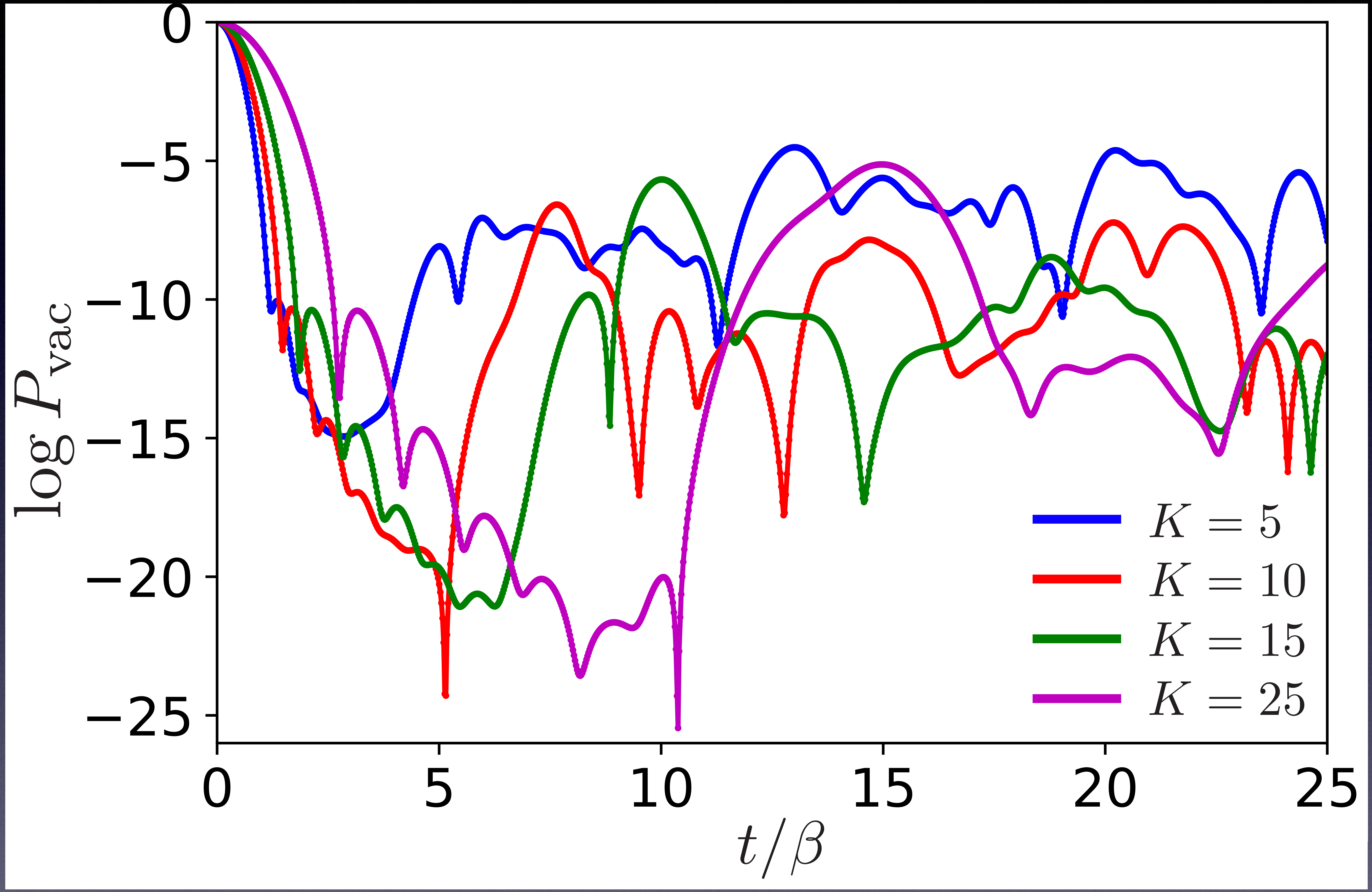
Extrapolation



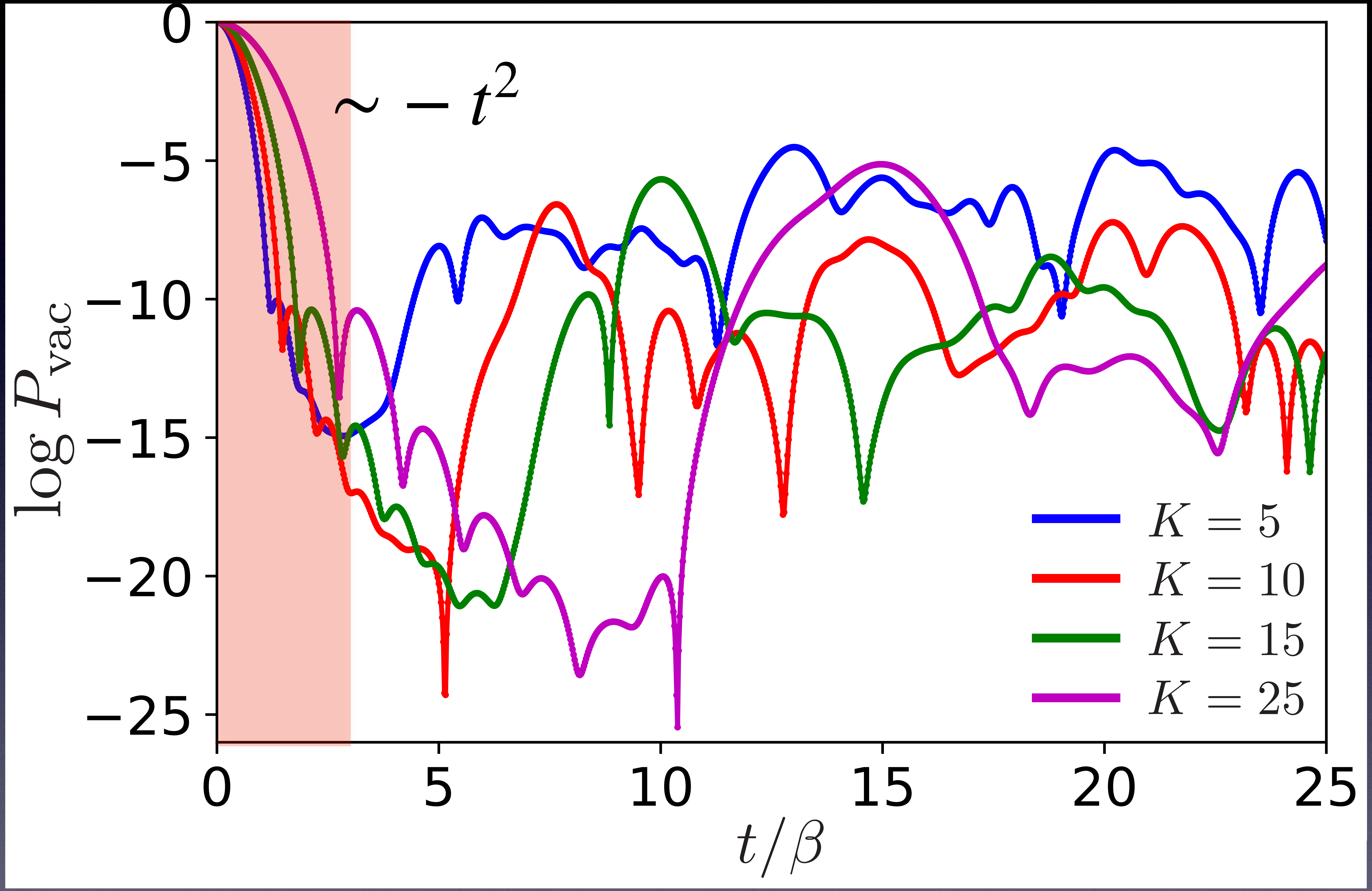
$j_{\max}$ dependence for relaxation time



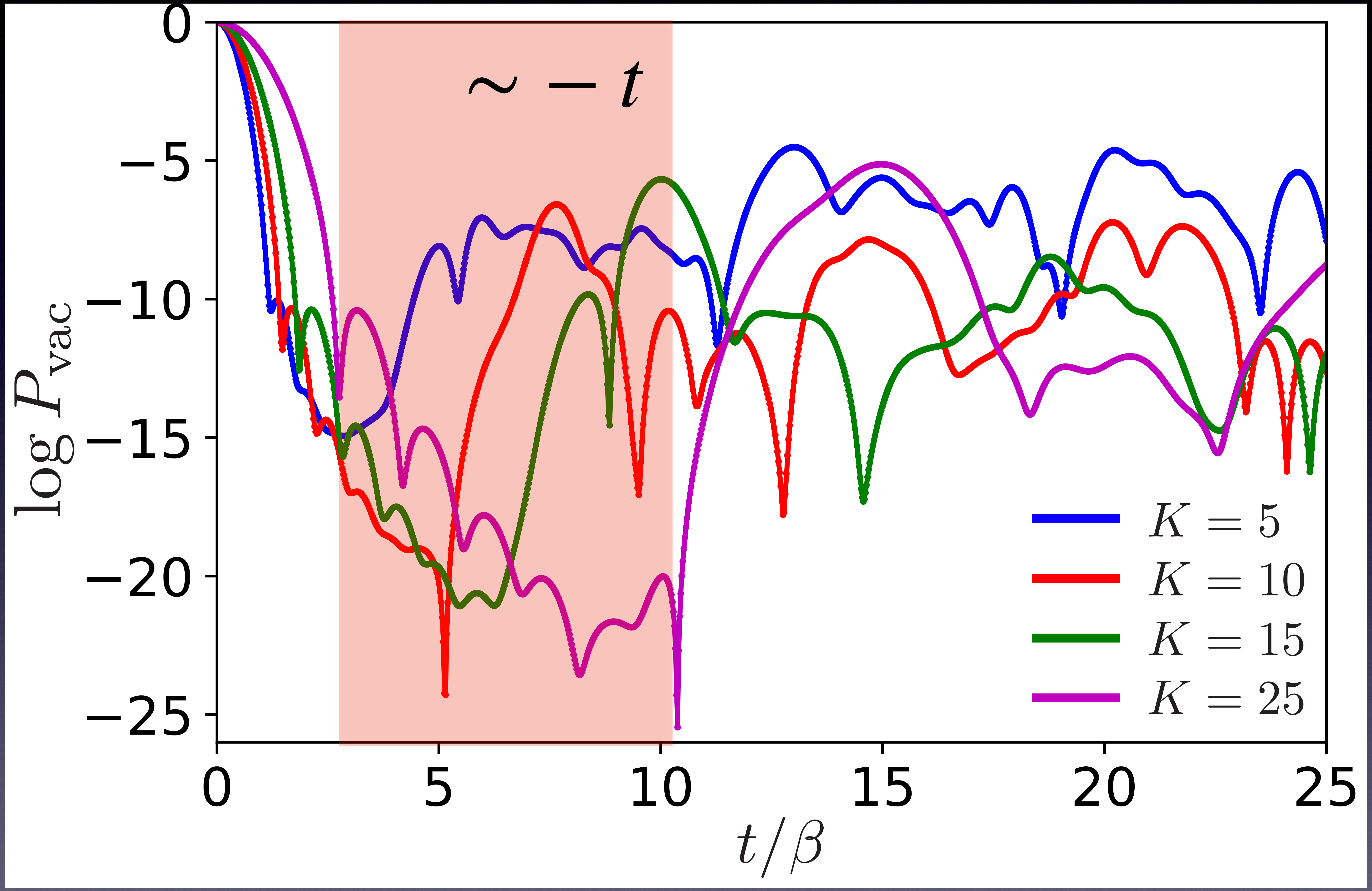
log of Loschmidt echo



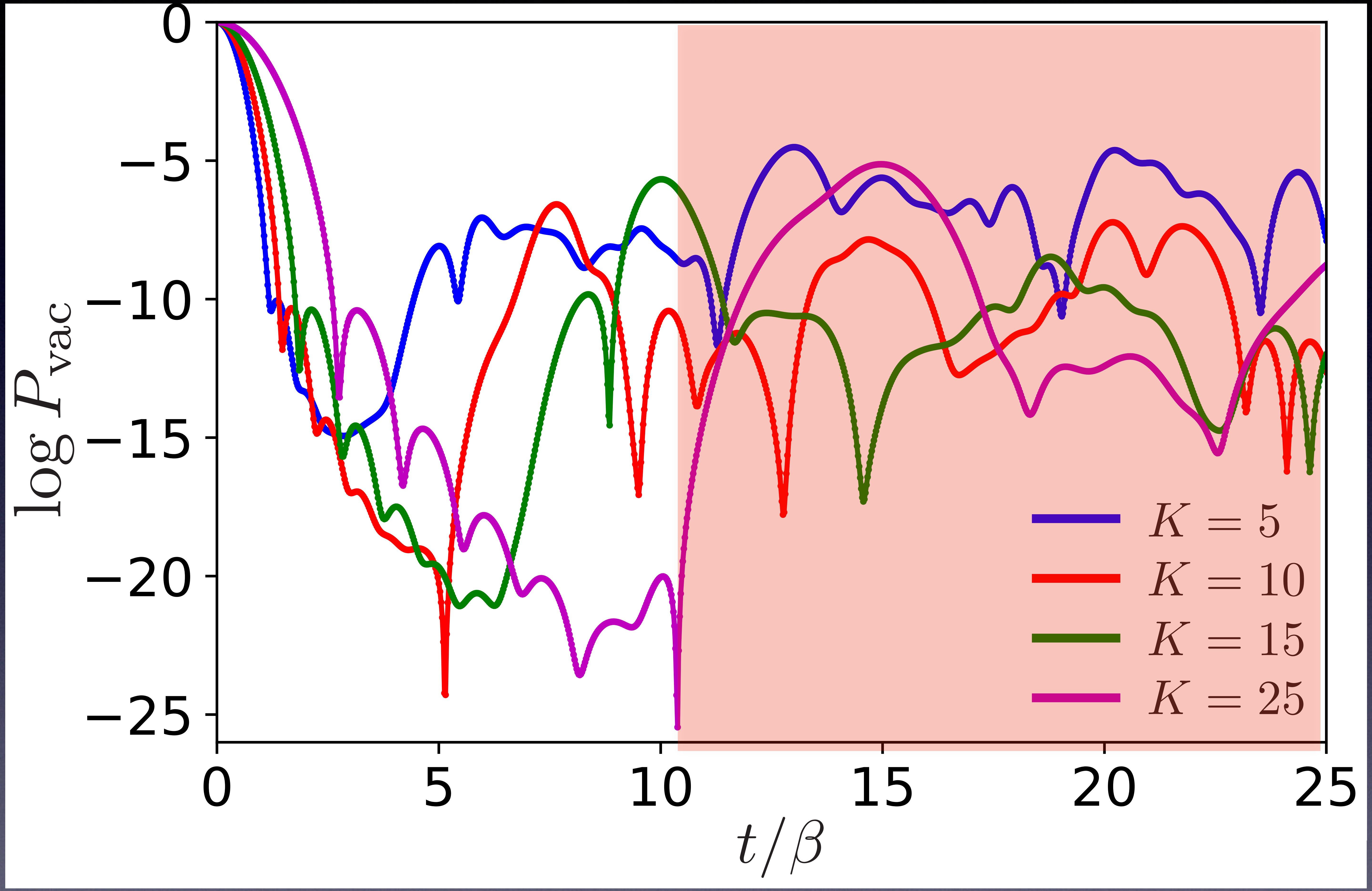
log of Loschmidt echo



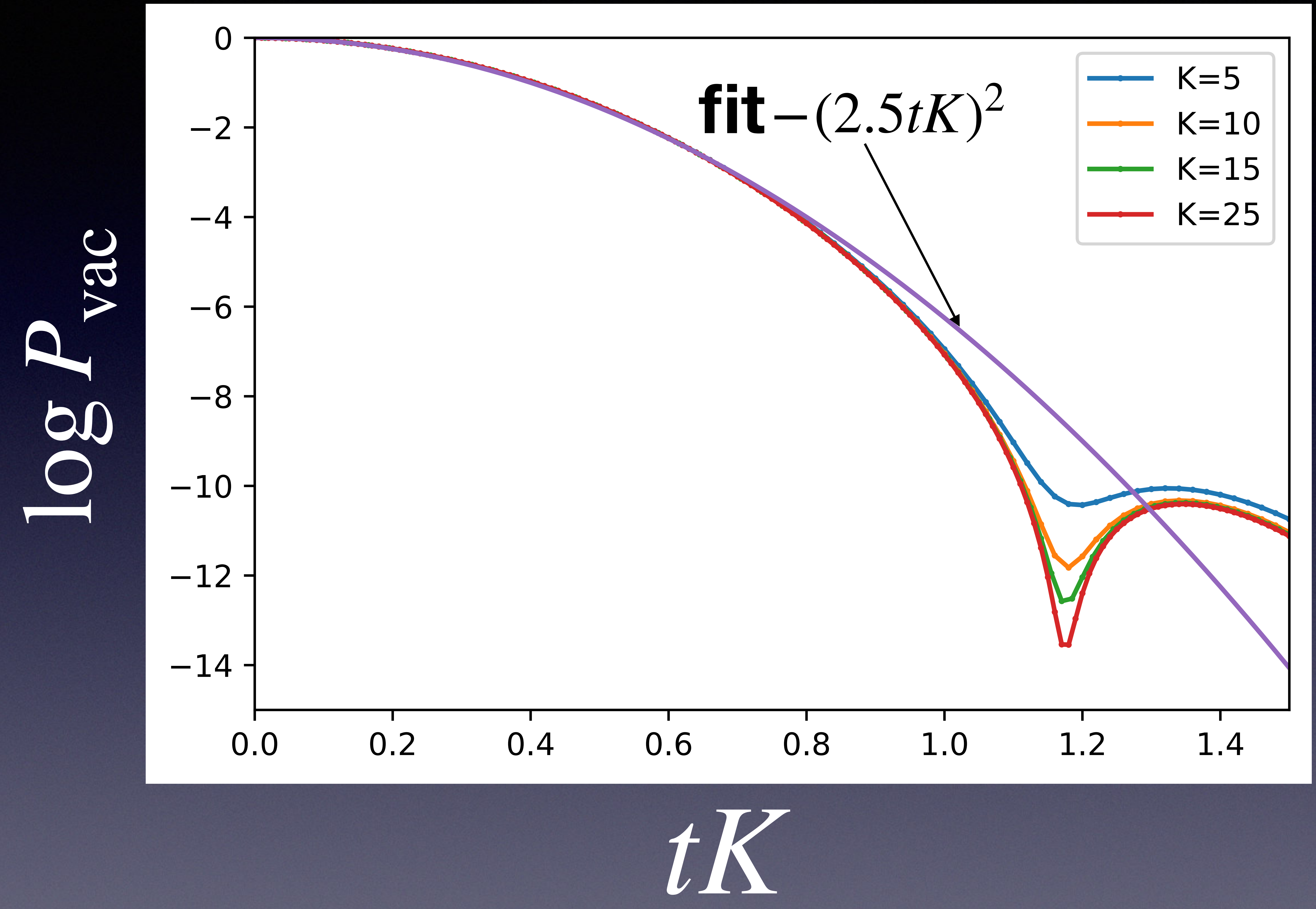
log of Loschmidt echo



log of Loschmidt echo

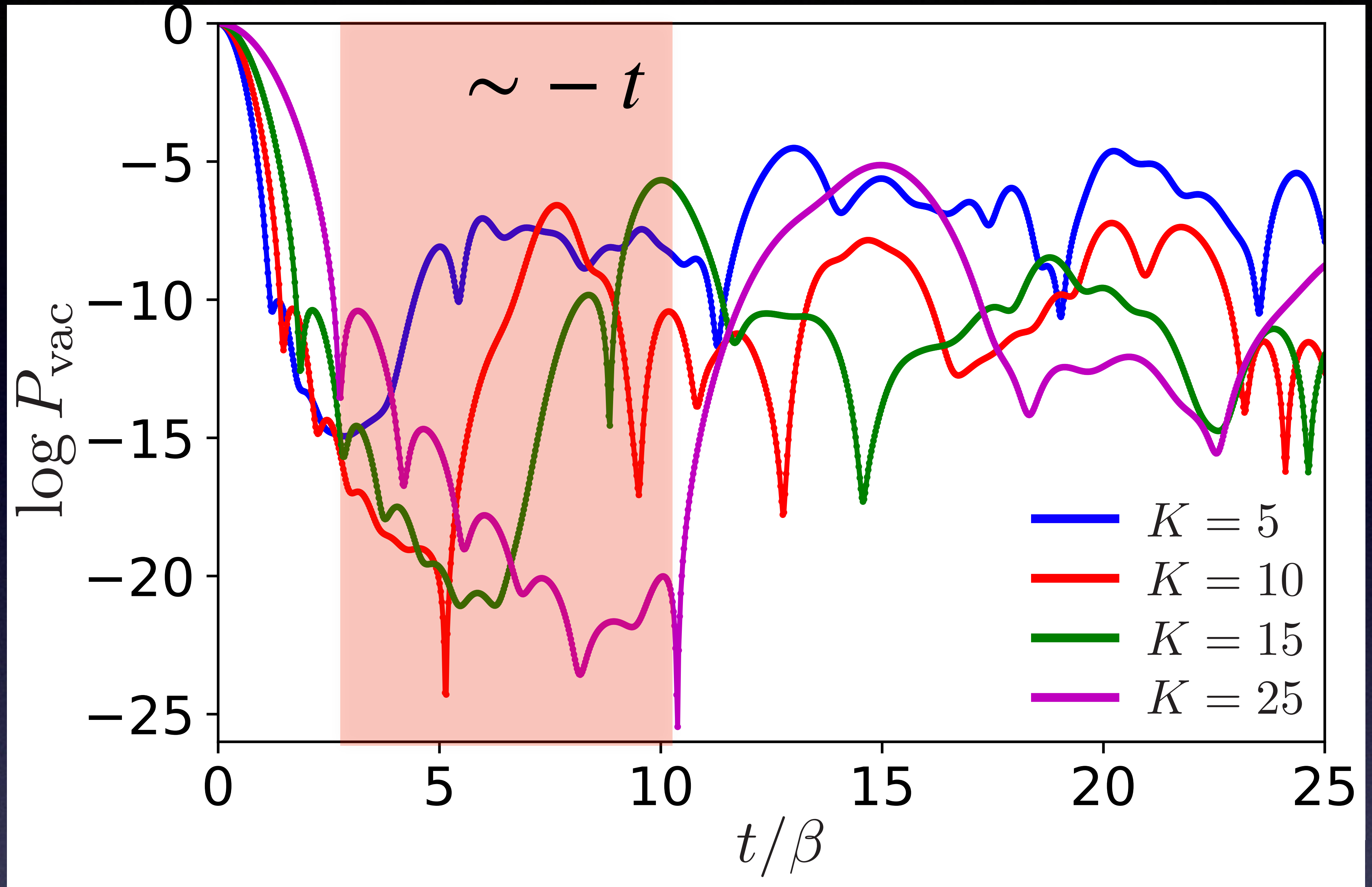


early time behavior



$\log P_{\rm vac} \sim -(tK)^2$ **perturbative**

Intermediate time



$$\log P_{\rm vac} \sim -t/\tau_{\rm LE}$$

$$\tau_{LE} = \begin{cases} 0.23 \text{ for } K=10 \\ 0.28 \text{ for } K=15 \\ 0.40 \text{ for } K=25 \end{cases}$$

Estimate slope from maxima

$$\tau_{\rm LE} \sim 0.3\beta \sim 1/(\pi T)$$

Chaotic behavior?

$\mathrm{SU}(3)_k$

$\mathrm{SU}(3)_k$ fusion coefficients

Begin, Walton, Mod. Phys. Lett. A 7 (1992) 3255

$$N_{ab}^{c} = (k_0^{\max} - k_0^{\min} + 1)\delta_{ab}^{c}$$

$$\begin{aligned}
k_0^{\min} &= \max(p_a + q_a, p_b + q_b, p_c + q_c, \mathcal{A} - \min(p_a, p_b, q_c), \mathcal{B} - \min(q_a, q_b, p_c)),\\
k_0^{\max} &= \min(\mathcal{A}, \mathcal{B}),\\
\mathcal{A} &= \frac{1}{3}(2(p_a + p_b + q_c) + q_a + q_b + p_c),\\
\mathcal{B} &= \frac{1}{3}(p_a + p_b + q_c + 2(q_a + q_b + p_c)),\\
\delta_{ab}^{c} &= \begin{cases} 1 & \text{if } k_0^{\max} > k_0^{\min} \text{ and } \mathcal{A}, \mathcal{B} \in \mathbb{Z}_+ \\ 0 & \text{otherwise} \end{cases}
\end{aligned}$$

Quantum dimension

e.g., Coquereaux, Hammaoui, Schieber, Tahri, J. Geom. Phys. 57 (2006) 269

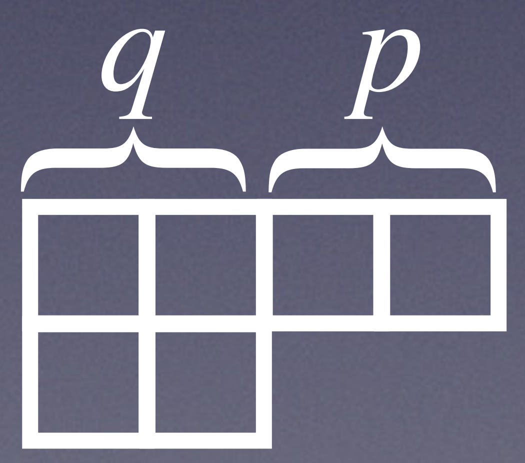
$$d_a = \frac{1}{[2]}[p_a + 1][q_a + 1][p_a + q_a + 2]$$

Casimir invariant

e.g., Bonatsos, Daskaloyannis, Prog. Part. Nucl. Phys. 43 (1999) 537

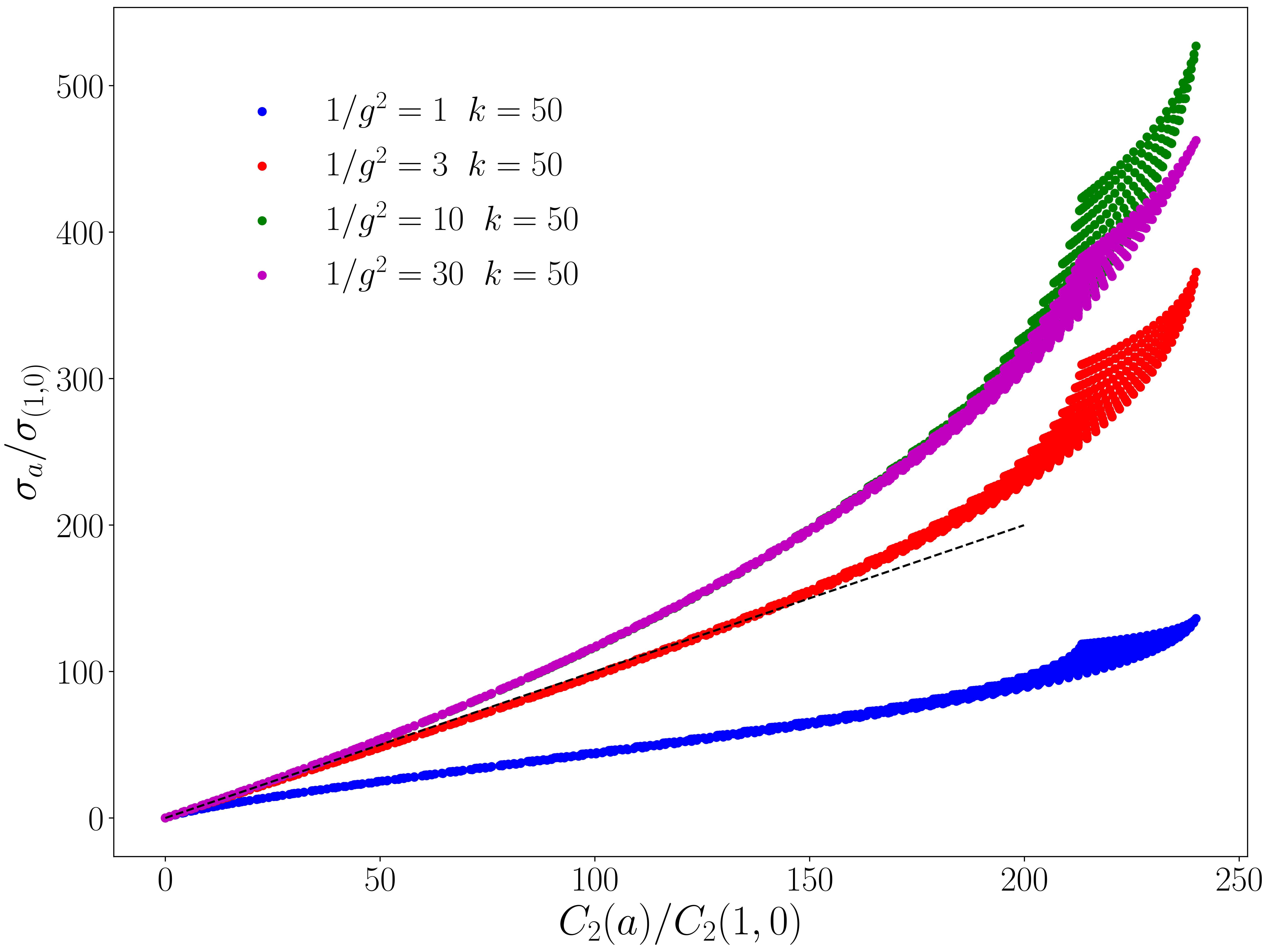
$$C_2(a) = \frac{1}{2}\left(\left[\frac{p_a}{3} - \frac{q_a}{3}\right]^2 + \left[\frac{2p_a}{3} + \frac{q_a}{3} + 1\right]^2 + \left[\frac{p_a}{3} + \frac{2q_a}{3} + 1\right]^2 - 2\right)$$

(p, q)

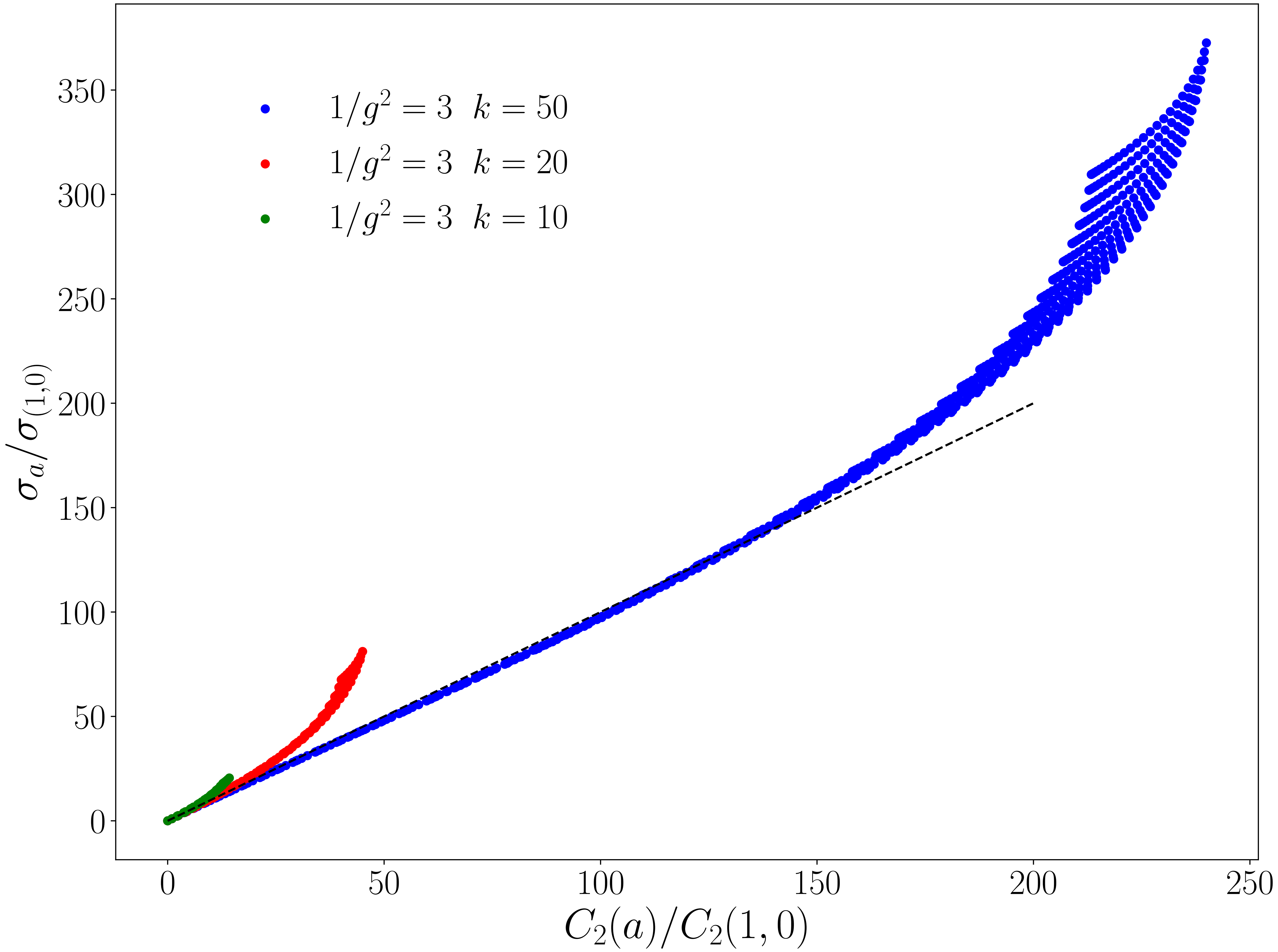


$$[n] := \frac{q^{\frac{n}{2}} - q^{-\frac{n}{2}}}{q^{\frac{1}{2}} - q^{-\frac{1}{2}}} = \frac{\sin\left(\frac{\pi}{k+3}n\right)}{\sin\left(\frac{\pi}{k+3}\right)} \qquad q = \exp\left(\mathrm{i}\frac{2\pi}{3+k}\right)$$

Casimir scaling

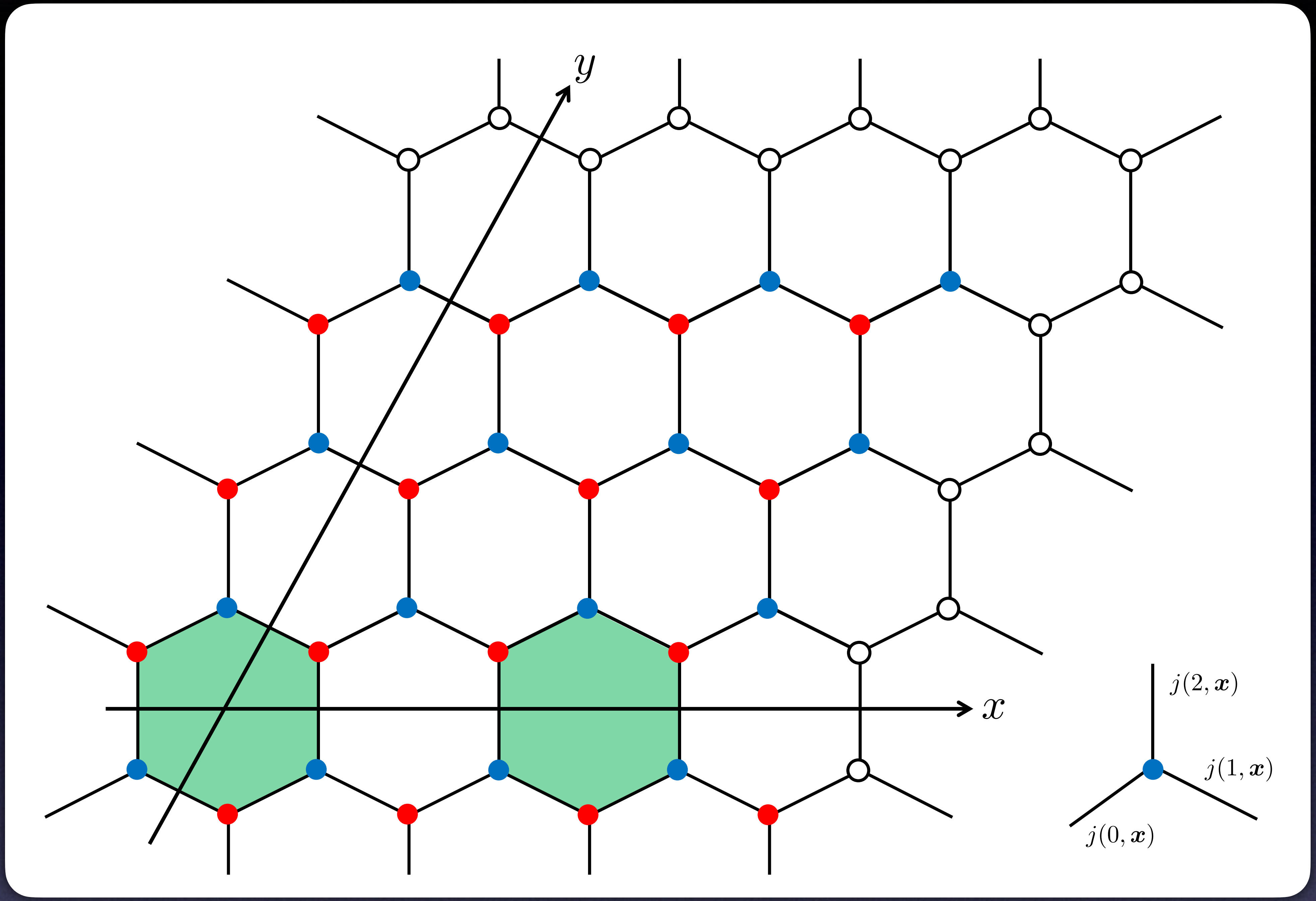


Casimir scaling



Quantum scar

Honeycomb Lattice



Model	$(N_x, N_y, j_{\max})$	Total	Gauss law	Winding	Scars
Yang-Mills	$(4, 4, \frac{1}{2})$	2^{48}	131072	32768	3
	$(4, 2, 1)$	3^{24}	131328	33024	29
Levin-Wen	$(4, 2, \frac{3}{2})$	2^{24}	29375	-	0
	$(4, 2, 1)$	3^{24}	131328	33024	3