Holographic study of deconfinement phase transition and heavy quarkonium under rotation

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Content

- Background
- Polyakov loop and string tension
- Drag force and heavy quark potential
- Conclusion

[1] J.-X. Chen, S. Wang, D.-F. Hou, and H.-C. Ren, arXiv: 2410.04763 [hep-ph].
[2] J.-X. Chen, D.-F. Hou, and H.-C. Ren, JHEP 03 (2024) 171, arXiv:2308.08126 [hep-ph].

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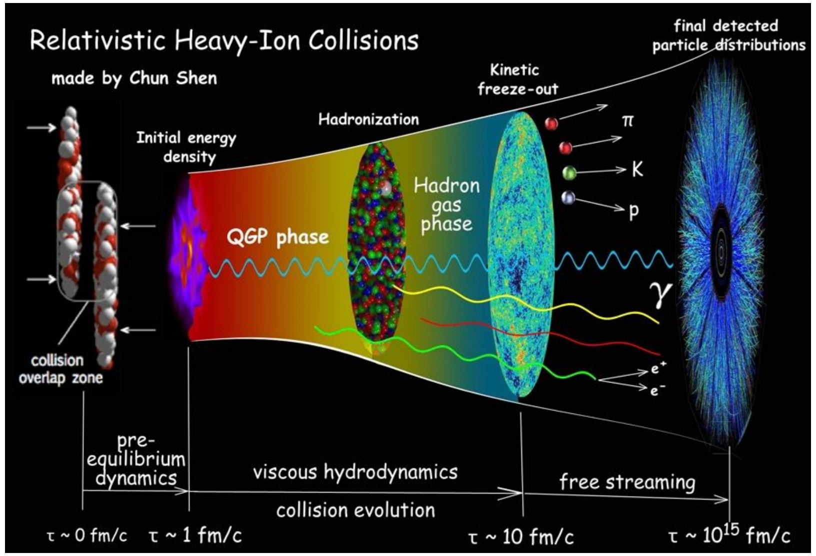
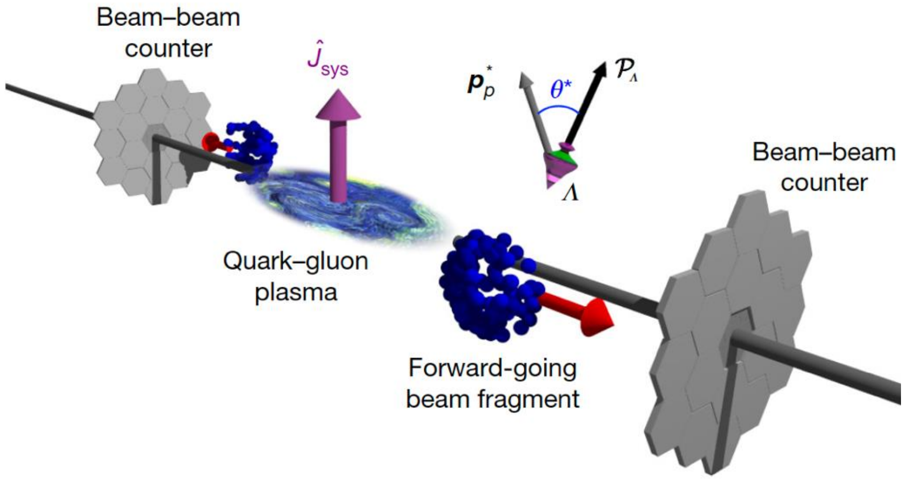
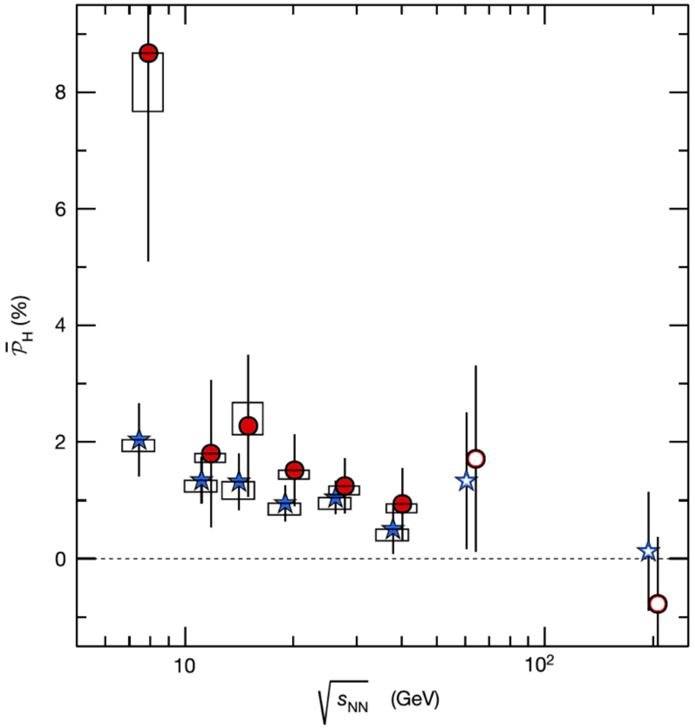
Motivation







Strong vorticity fields exist in relativistic heavy-ion collisions $\omega = (9 \mp 1) \times 10^{21} s^{-1}$

[1] L. Adamczyk et al. (STAR), Nature 548, 62 (2017), arXiv:1701.06657 [nucl-ex]

Polyakov loop and string tension

Lattice:

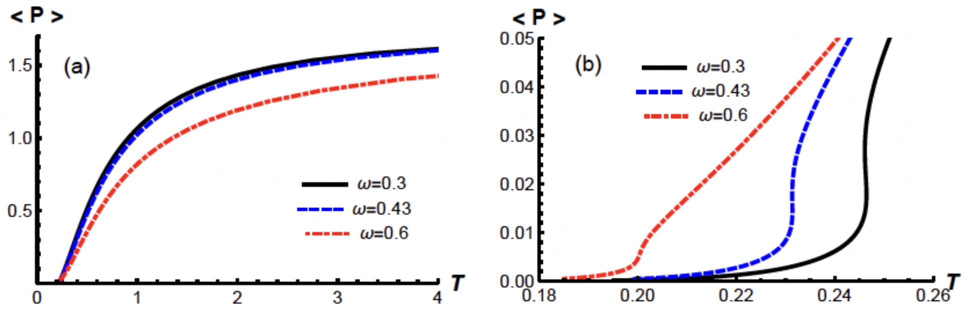
V. V. Braguta, A. Y. Kotov, D. D. Kuznedelev, et al, arXiv:2102.05084 [hep-lat].

$$T_c(\Omega)/T_c(0) = 1 + C_2\Omega^2 \text{ with } C_2 > 0$$

Angular velocity Ω ↑ Temperature T ↑

Holography:

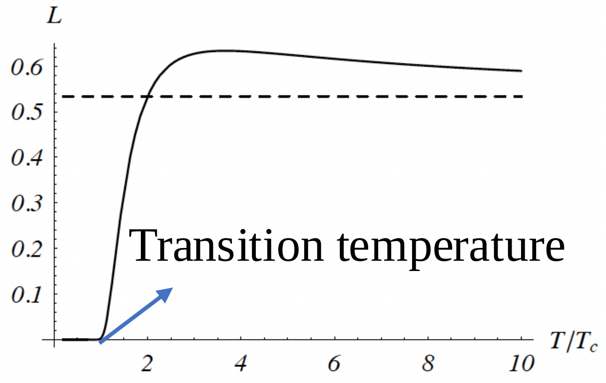
X. Chen, L. Zhang, D. Li, D. Hou, and M. Huang, (2020), arXiv:2010.14478 [hep-ph].



Angular velocity Ω ↑ Temperature T ↓

O. Andreev, Phys. Rev. Lett. 102 (2009) 212001, arXiv:0903.4375 [hep-ph].

$T < T_c, L \simeq 0$	Confined phase
$T > T_c, L > 0$	Deconfined phase



Expectation value of the Polyakov loop versus temperature

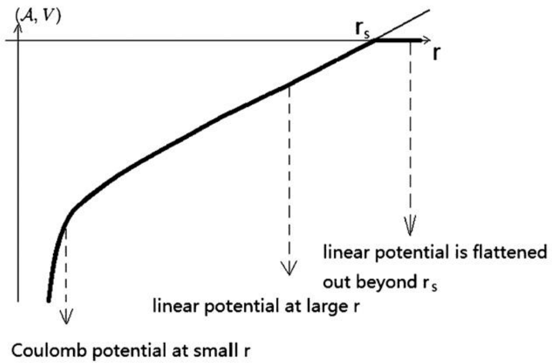
O. Andreev and V. I. Zakharov, Phys. Rev. D 74 (2006) 025023, arXiv:hep-ph/0604204.

In large distance limit, the string tension is

$$\kappa = \frac{V}{r}$$

V → Heavy quark potential

r → Distance between the quark and anti-quark



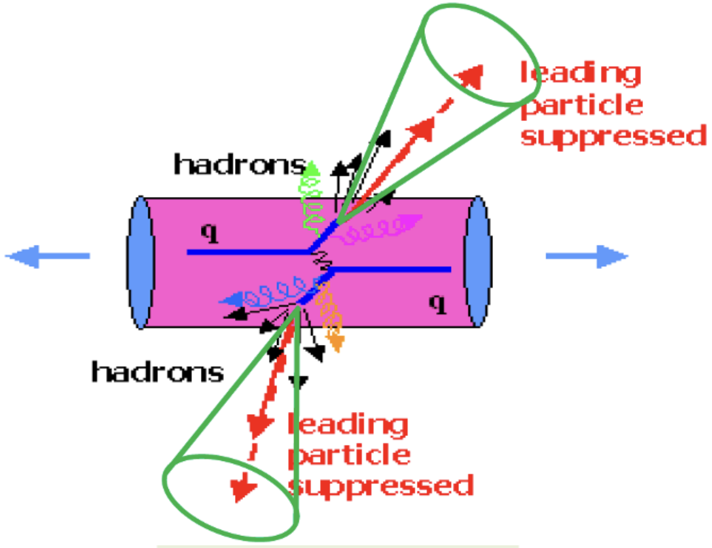
Drag force and heavy quark potential

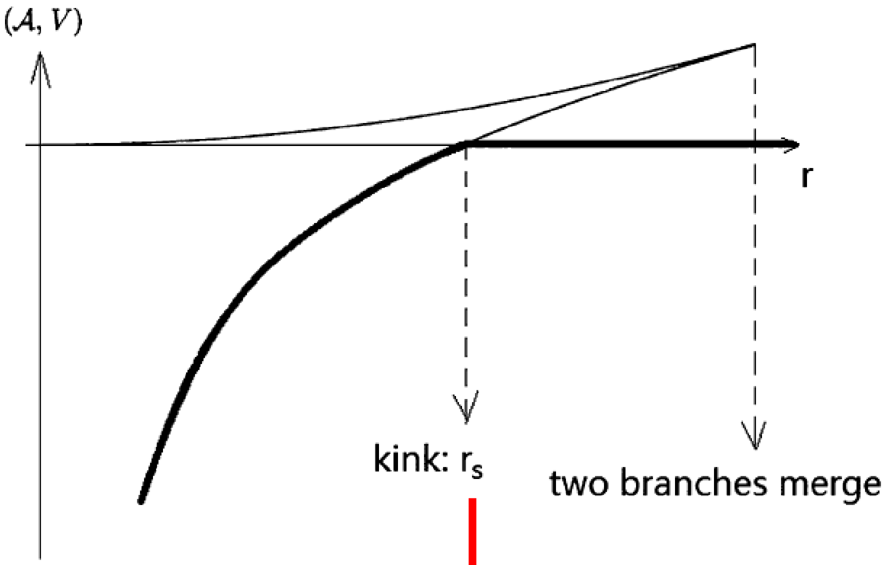
C.P. Herzog et al., JHEP 07 (2006) 013 [hep-th/0605158]

S.J. Rey, S. Theisen, J.T. Yee, Nucl. Phys. B 527 (1998) 171–186, arXiv :hep -th /9803135.

When a heavy quark traverses a hot QGP, its momentum and energy are dissipated through friction with medium. Such a friction is quantified by a drag force.

$$\frac{dp_\mu}{dt} = -F_\mu^{drag}$$





Screening length

AdS/CFT duality

The Large N limit of superconformal field theories and supergravity

Juan Martin Maldacena (Harvard U.) (Nov, 1997)

Published in: *Int.J.Theor.Phys.* 38 (1999) 1113-1133 (reprint), *Adv.Theor.Math.Phys.* 2 (1998) 231-252 • e-Print: hep-th/9711200 [hep-th]

pdf DOI cite claim reference search 20,112 citations

The strongly coupled gauge theory in four-dimensional space-time is dual to the weakly coupled string theory on $AdS_5 \times S^5$.

Several methods

- Local Lorentz transformation

$$t \to \frac{1}{\sqrt{1-(\omega l_0)^2}}(t+\omega l_0^2 \phi)$$

$$\phi \to \frac{1}{\sqrt{1-(\omega l_0)^2}}(\phi+\omega t)$$

This method can only describe a small neighbourhood around l_0.

Period $T = 2\pi\sqrt{1-(\omega l_0)^2} \leq 2\pi$

Phase diagram, free energy, entropy and so on.

- Global transformation $\phi \to \phi + \omega t$

The angular velocity of QGP $\omega = 1 \times 10^{22} s^{-1} = 0.007 GeV$

radius $8fm = 40.5 GeV^{-1}$ the linear velocity $v \simeq 0.3$

Lattice QCD, effective field theories, and so on.

- Kerr-AdS5, rotating string

[1] Y.-Q. Zhao, S. He, D. Hou, et, al, JHEP 04, 115 (2023), arXiv:2212.14662 [hep-ph].
[2] X. Chen, L. Zhang, D. Li, D. Hou, and M. Huang, (2021), arXiv:2010.14478 [hep-ph].

- Background
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J.-X. Chen, S. Wang, D.-F. Hou, and H.-C. Ren, arXiv: 2410.04763 [hep-ph].

Background geometry

Minkowski background metric in cylindrical coordinates

$$ds^2 = \frac{R^2}{w^2} h \left(-f dt^2 + dl^2 + l^2 d\phi^2 + dz^2 + \frac{1}{f} dw^2 \right)$$

$$h = e^{\frac{1}{2}cw^2}, f = 1 - \frac{w^4}{w_t^4}$$

Azimuth angle

Warp factor generates confinement

Global rotation

$\phi \rightarrow \phi + \omega t$

$$ds^2 = \frac{R^2}{w^2} h[-(f - \omega^2 l^2)dt^2 + l^2 d\phi^2 + 2\omega l^2 dt d\phi + dl^2 + dz^2 + \frac{1}{f} dw^2]$$

Hawking temperature

$$T = \frac{1}{\pi w_t}$$

Background geometry

Minkowski background metric in cylindrical coordinates

$$ds^2 = \frac{R^2}{w^2} h \left(-f dt^2 + dl^2 + l^2 d\phi^2 + dz^2 + \frac{1}{f} dw^2 \right)$$

Local rotation

$$t \rightarrow \frac{1}{\sqrt{1-(\omega l_0)^2}} (t + \omega l_0^2 \phi)$$

$$\phi \rightarrow \frac{1}{\sqrt{1-(\omega l_0)^2}} (\phi + \omega t)$$

$$ds^2 = \frac{R^2}{w^2} h \{ \frac{1}{1-(\omega l_0)^2} [(-f + \omega^2 l^2) dt^2 + (l^2 - \omega^2 l_0^4 f) d\phi^2 + 2\omega (l^2 - l_0^2 f) dt d\phi] + dl^2 + dz^2 + \frac{1}{f} dw^2 \}$$

Hawking temperature $T = \frac{\sqrt{1-(\omega l_0)^2}}{\pi w_t}$

Euler-Lagrange equations

Taking $\sigma^\alpha = (t, w)$ as the string world-sheet coordinates, and assume

$$z = z(t,w), \phi = \phi(t,w), l = l(t,w)$$

Nambu-Goto action $S = -\frac{1}{2\pi\alpha'}\int dtdw\sqrt{-g}$

$$g_{\alpha\beta} = G_{\mu\nu}\frac{\partial X^\mu}{\partial\sigma^\alpha}\frac{\partial X^\nu}{\partial\sigma^\beta}$$

Determinant of the induced metric

Metric of target space

Euler-Lagrange equations

$$\frac{d}{dt}\left(\frac{\partial\sqrt{-g}}{\partial\dot{z}}\right) + \frac{d}{dw}\left(\frac{\partial\sqrt{-g}}{\partial z'}\right) - \frac{\partial\sqrt{-g}}{\partial z} = 0$$

$$\frac{d}{dt}\left(\frac{\partial\sqrt{-g}}{\partial\dot{\phi}}\right) + \frac{d}{dw}\left(\frac{\partial\sqrt{-g}}{\partial \phi'}\right) - \frac{\partial\sqrt{-g}}{\partial \phi} = 0$$

$$\frac{d}{dt}\left(\frac{\partial\sqrt{-g}}{\partial\dot{l}}\right) + \frac{d}{dw}\left(\frac{\partial\sqrt{-g}}{\partial l'}\right) - \frac{\partial\sqrt{-g}}{\partial l} = 0$$

Solution ⟷ Shape of string

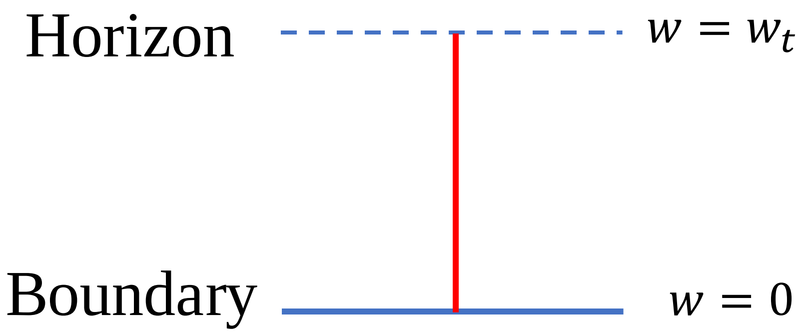
Polyakov loop

Expectation value of Polyakov loop $L = e^{-S_q}$ → Nambu-Goto action of a single quark with an imaginary time

without rotation

Solution ansatz

$$\phi = const., l = const., z = const.$$



with rotation

Solution ansatz for small ω

$$l = l_0 + \omega^2 l_1(w)$$

$$\phi = \phi_0 + \omega \phi_1(w)$$

$$z = z_0 + \omega^2 z_1(w)$$

Solve EoM →

Solution

$$l = l_0$$

$$\phi_1' = -\frac{w^2 h(w_t)}{w_t^2 h f}$$

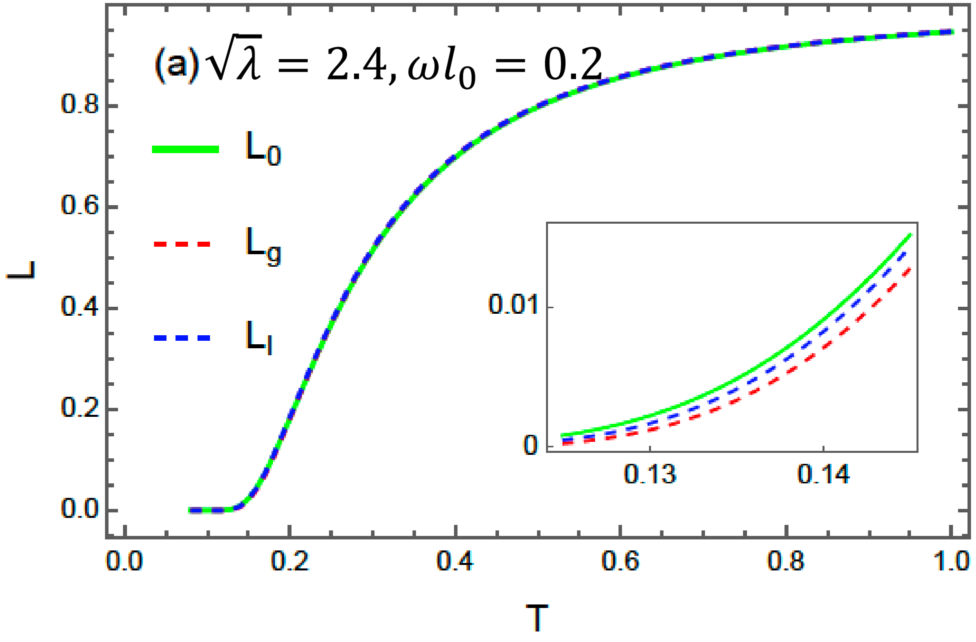
$$z = z_0$$

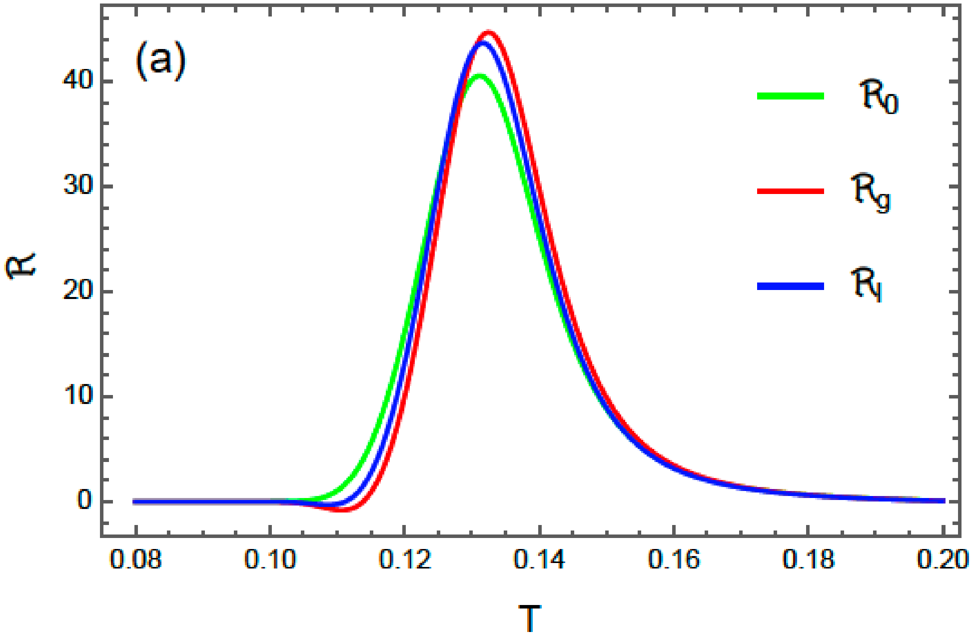
Polyakov loop

$$L = L_0 + \omega^2 l_0^2 L_1$$

O. Andreev, Phys. Rev. Lett. 102 (2009) 212001, arXiv:0903.4375 [hep-ph].

Curvature $$\mathcal{R} = \frac{\left|\frac{d^2L}{dT^2}\right|}{\left[1+\left(\frac{dL}{dT}\right)^2\right]^{\frac{3}{2}}} \simeq \mathcal{R}_0 + \omega^2 l_0^2 \mathcal{R}_1$$





Transition temperature ↑

[1] V. V. Braguta, A. Y. Kotov, D. D. Kuznedelev, et al, Phys.Rev. D 103 no. 9, (2021) 094515, arXiv:2102.05084 [hep-lat].
[2] Y. Chen, X. Chen, D. Li, and M. Huang, arXiv:2405.06386 [hep-ph].
[3] F. Sun, J. Shao, R. Wen, K. Xu, and M. Huang, Phys. Rev. D 109 no. 11, (2024) 116017, arXiv:2402.16595 [hep-ph].

Polyakov loop

Transition temperature without rotation

$\sqrt{\lambda}$	$T_d^{(0)}$	δT_g	δT_l	C_g	C_l
0.94	104.21	5.28	3.92	0.63	0.47
1.44	114.18	3.29	2.00	0.36	0.22
2.4	131.18	1.71	0.57	0.16	0.05

Unit is MeV

$\sqrt{\lambda}$ ↑ $T_d^{(0)}$ ↑ δT ↓

$$\frac{\delta T}{T} = C v^2$$ Linear velocity on the boundary

$$C = -\frac{1}{2T_d^{(0)}} \frac{\frac{d\mathcal{R}_1}{dT}}{\frac{d^2\mathcal{R}_0}{dT^2}} \Big|_{T=T_d^{(0)}}$$

V. V. Braguta, A. Y. Kotov, D. D. Kuznedelev, et al, Phys.Rev. D 103 no. 9, (2021) 094515, arXiv:2102.05084 [hep-lat].

Coefficient of lattice 0.7

[1] O. Andreev, Phys. Rev. Lett. 102 (2009) 212001, arXiv:0903.4375 [hep-ph].
[2] O. Andreev and V. I. Zakharov, JHEP 04 (2007) 100, arXiv:hep-ph/0611304.

String tension

String tension for a quarkonium parallel to the rotation axis

$$\kappa_{g\parallel} = \underline{\frac{\sqrt{\lambda}\pi T^2}{2b}\sqrt{1-b^2}e^{\frac{3\sqrt{3}bT_1^2}{2T^2}}}\left[1 - \underline{\frac{1}{2(1-b^2)}\omega^2 l_0^2}\right] \qquad b = \frac{2}{\sqrt{3}}\sin\left(\frac{1}{3}\sin^{-1}\frac{T^2}{T_1^2}\right) \qquad T_1 = \frac{1}{\pi}\sqrt{\frac{c}{\sqrt{27}}}$$

< 0

String tension for a quarkonium symmetric with respect to the rotation axis

$$\kappa_{g\perp} = \underline{\frac{\sqrt{\lambda}\pi T^2}{2b}\sqrt{1-b^2}e^{\frac{3\sqrt{3}bT_1^2}{2T^2}}}\left[1 - \underline{\frac{1}{24(1-b^2)}\omega^2 r^2}\right]$$

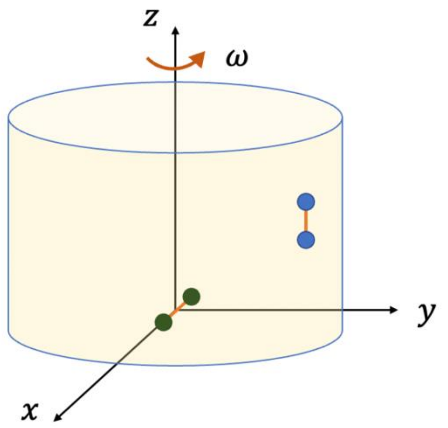
< 0

O. Andreev and V. I. Zakharov, JHEP 04 (2007) 100, arXiv:hep-ph/0611304.

String tension in local rotating background

$$\kappa_l = \underline{\frac{\sqrt{\lambda}\pi T^2}{2b}\sqrt{1-b^2}e^{\frac{3\sqrt{3}bT_1^2}{2T^2}}}\left\{1 - \underline{\left[\frac{b^2}{2(1-b^2)} + \frac{3\sqrt{3}bT_1^2}{2T^2} - 1\right]\omega^2 l_0^2}\right\}$$

< 0



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J.-X. Chen, D.-F. Hou, and H.-C. Ren, JHEP 03 (2024) 171, arXiv:2308.08126 [hep-ph].

Background geometry

The Schwarzschild metric in cylindrical coordinates, (t, l, ϕ, z, r) are coordinates of AdS5.

$$ds^2 = -\frac{r^2}{R^2} f(r) dt^2 + \frac{r^2}{R^2}(dl^2 + l^2 d\phi^2 + dz^2) + \frac{1}{f(r)} \frac{R^2}{r^2} dr^2$$

Azimuth angle

Global rotation
$\phi \to \phi + \omega t$

$$ds^2 = \frac{r^2}{R^2}[-(f(r) - \omega^2 l^2) dt^2 + l^2 d\phi^2 + 2\omega l^2 dt d\phi + dl^2 + dz^2] + \frac{1}{f(r)} \frac{R^2}{r^2} dr^2$$

Hawking temperature

$$T = \frac{r_t}{\pi R^2}$$

Drag force

Components of drag force

Radial
$$F_l = -\omega^2 l_0 \frac{1}{\sqrt{1-v^2}}\left[\frac{T\sqrt{\lambda}}{2(1-v^2)^{\frac{1}{4}}} - m_{rest}\right] \rightarrow \text{Centrifugal force}$$

Azimuthal
$$F_\phi = -\frac{\pi\sqrt{\lambda}T^2}{2}\omega l_0^2 \frac{1}{\sqrt{1-v^2}}$$

Longitudinal
$$F_z = -\frac{\pi\sqrt{\lambda}T^2}{2}\frac{v}{\sqrt{1-v^2}}\left(1+\underline{\frac{\omega^2 l_0^2}{2}\frac{1}{1-v^2}}\right)$$

$\propto \omega^2$

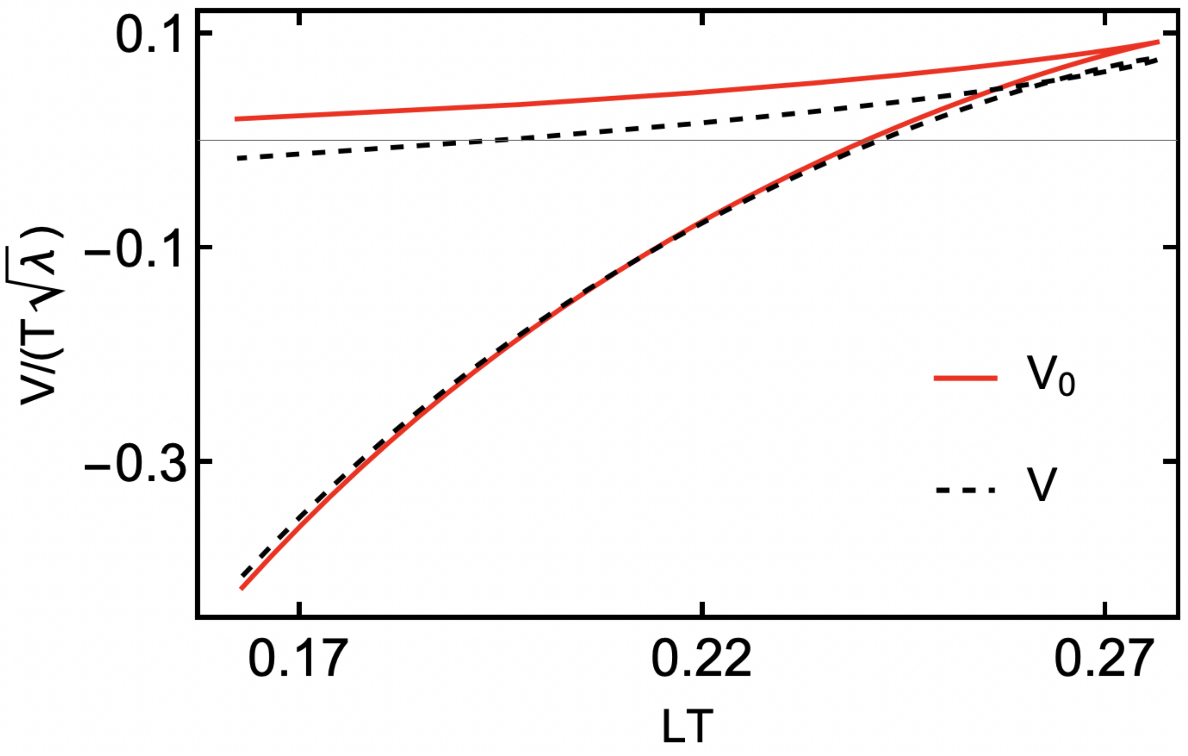
$\omega = 0$ $\Rightarrow$ $F_l = 0$, $F_\phi = 0$, $F_z = -\frac{\pi\sqrt{\lambda}T^2}{2}\frac{v}{\sqrt{1-v^2}}$

[1] *S. S. Gubser, Phys. Rev. D 74, 126005 (2006), arXiv:hep-th/0605182.*

[2] *C. P. Herzog, A. Karch, P. Kovtun, C. Kozcaz, and L. G. Ya e, JHEP 07, 013 (2006),arXiv:hep-th/0605158.*

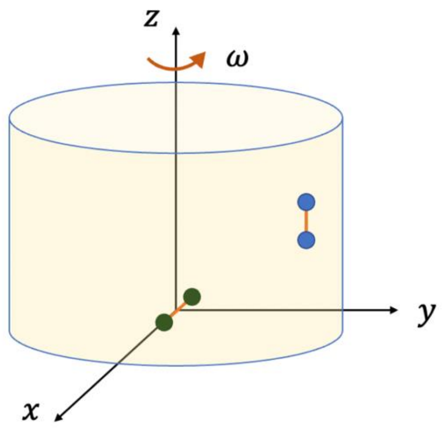
Heavy quark potential $V_{\parallel}$

$$V(L) = V_0(L) + \omega^2 l_0^2 V_1(L) + O(\omega^4)$$



$$\omega l_0 = 0.24$$

1. Binding force ↓
2. Force range ↑

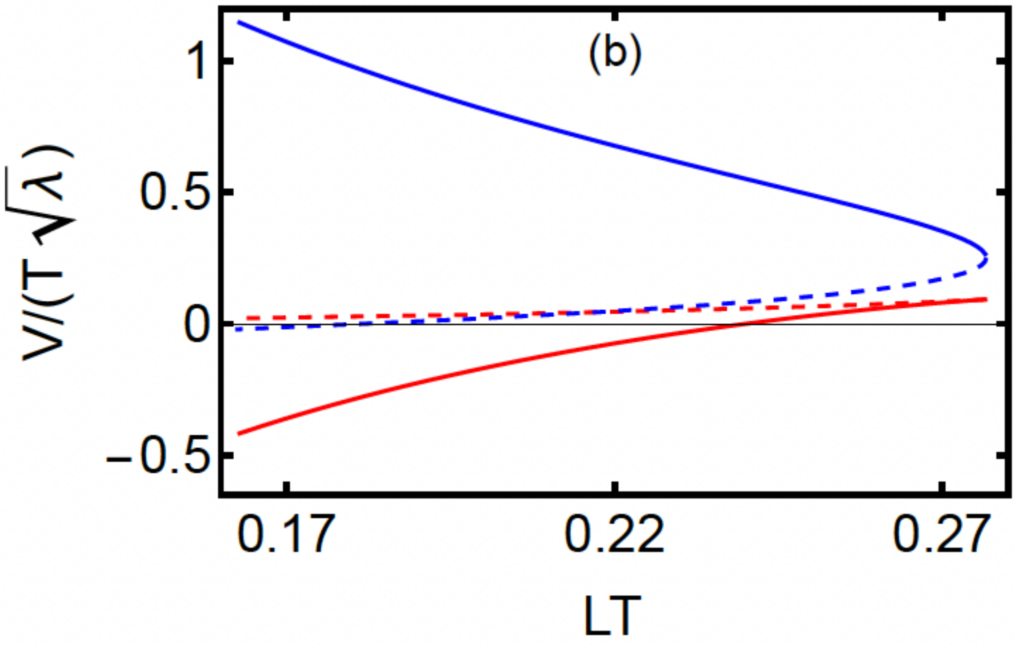


$$V_0(L_0) = 0$$

$$\delta L = \frac{|V_1(L_0)|}{V_0'(L_0)} \omega^2 l_0^2$$

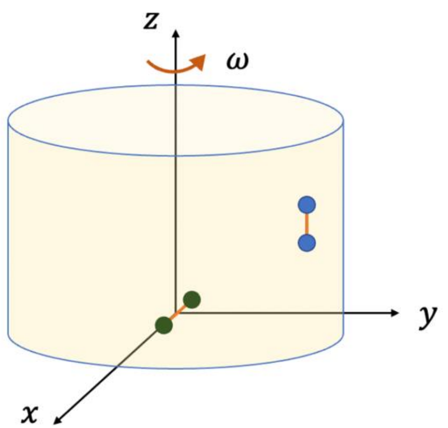
Heavy quark potential $V_\perp$

$$V(L) = V_0(L) + \frac{1}{4}\omega^2 L^2 V_1(L) + O(\omega^4)$$



V_0 V_1

1. Potential ↑
2. Force range ↓



$$V_0(L_0) = 0$$

$$\delta L = -\frac{V_1(L_0)}{4V_0'(L_0)}\omega^2 L_0^2$$

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Conclusion

- The deconfinement phase transition temperature increases with increasing angular velocity.
- The string tension decreases with the increasing angular velocity in all cases examined.
- The rotation enhances the drag force along the direction of motion and thereby cause additional energy loss.
- For a quarkonium parallel to the rotation axis, the rotation reduces the binding force but increases the force range.
- For a quarkonium symmetric with respect to the rotation axis, the potential is weakened with reduced magnitude and range.

Thanks for your attention

String tension

In large distance limit $\kappa = \frac{V}{r}$

Heavy quark potential $V = -\frac{S}{\mathcal{T}}$ → Nambu-Goto action

Taking $\sigma^{\alpha} = (t, w)$ as the string world-sheet coordinates

without rotation

Solution ansatz

$$\phi = const., l = const., z = z(w).$$

with rotation

Solution

$$z(w) = z_0(w) + \underbrace{\xi(w)}_{O(\omega^2)}$$

Solution

Euler-Lagrange equations

$$\frac{d}{dt}\left(\frac{\partial\sqrt{-g}}{\partial \dot{z}}\right)+\frac{d}{dr}\left(\frac{\partial\sqrt{-g}}{\partial z'}\right)-\frac{\partial\sqrt{-g}}{\partial z}=0$$

$$\frac{d}{dt}\left(\frac{\partial\sqrt{-g}}{\partial \dot{\phi}}\right)+\frac{d}{dr}\left(\frac{\partial\sqrt{-g}}{\partial \phi'}\right)-\frac{\partial\sqrt{-g}}{\partial \phi}=0$$

$$\frac{d}{dt}\left(\frac{\partial\sqrt{-g}}{\partial \dot{l}}\right)+\frac{d}{dr}\left(\frac{\partial\sqrt{-g}}{\partial l'}\right)-\frac{\partial\sqrt{-g}}{\partial l}=0$$

Static gauge for small ω

$$z = vt + \xi_0(r) + \omega^2\xi_1(r)$$

$$l = l_0 + \omega^2 l_1(r)$$

$$\phi = \phi_0 + \omega\phi_1(r)$$

Solution without rotation ($\xi_0(r)$)

$$\xi_1' = 0$$

$$\phi_1' = \frac{R^2 r_t^2}{r^4 f}$$

$$l_1' = \frac{R^4}{r^4}\frac{r_c - f}{f - v^2} l_0$$

I. Y. Aref'eva, A. A. Golubtsova, and E. Gourgoulhon, JHEP 04, 169 (2021), arXiv:2004.12984[hep-th].