West Lake Workshop on Nuclear Physics 2024

Clustering structure of light nuclei

- Introduction (^{12}C)
- Container picture
- 5α condensate
- Summary

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2024-10-19

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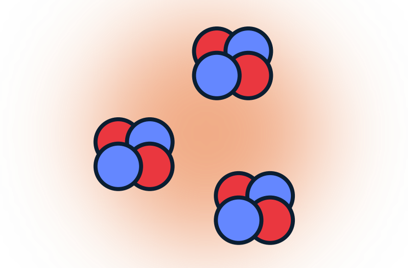
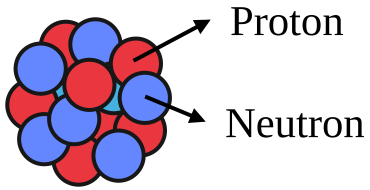
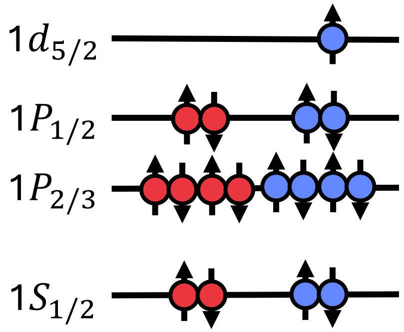


Nuclear Cluster Physics

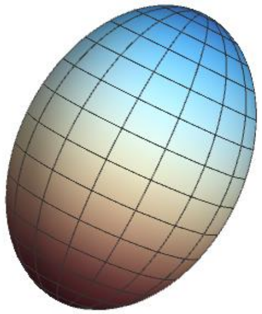
Nuclear many-body problem



$$\boldsymbol{H}\,|\boldsymbol{\Psi}\rangle = \mathbf{E}\,|\boldsymbol{\Psi}\rangle$$

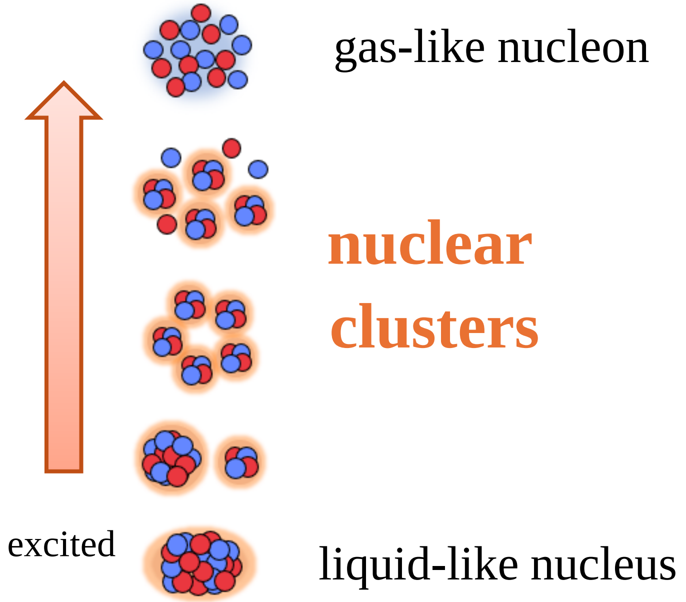


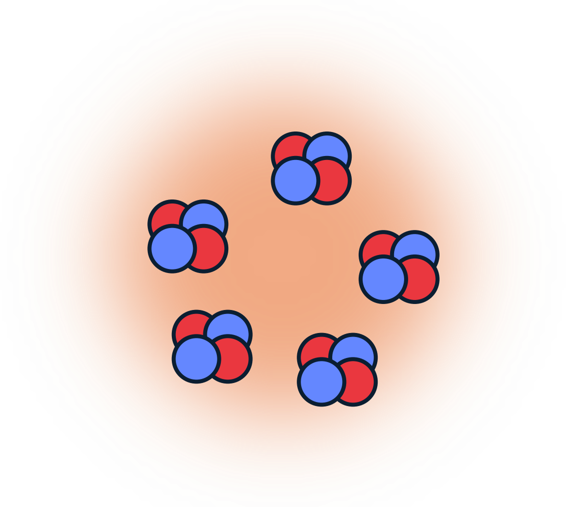
single-particle shell model



collective model

A "phase transition" can occur ,



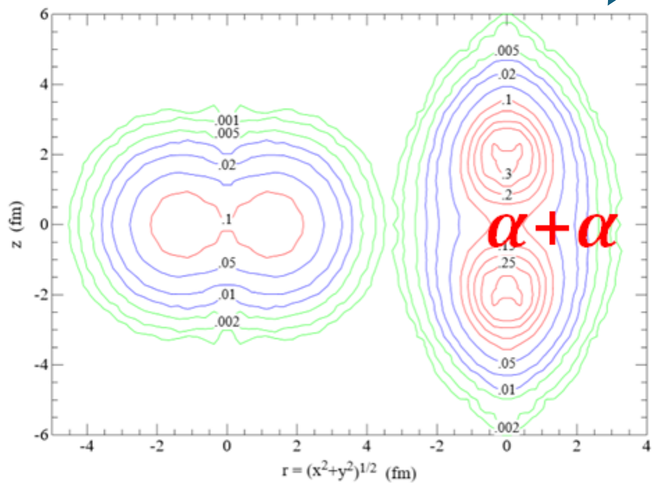


nuclear **cluster** structure

Ikeda diagram for light nculei

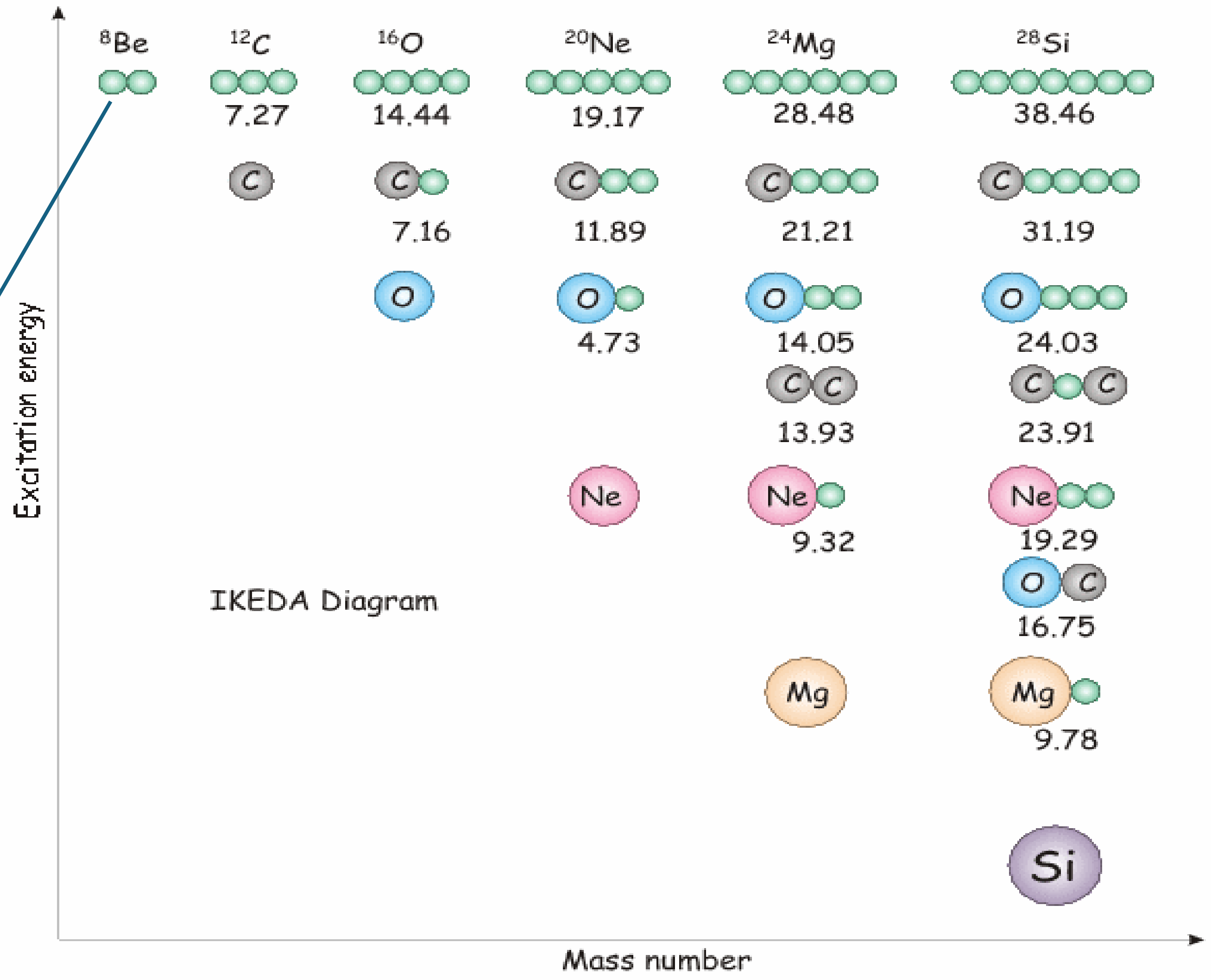
$$H = T - T_G + \sum_{i>j}^{A} V_{ij}$$

$$\Psi = \mathcal{A}\{\chi(\xi_1, \ldots, \xi_{n-1})\phi_1 \ldots \phi_n\}$$

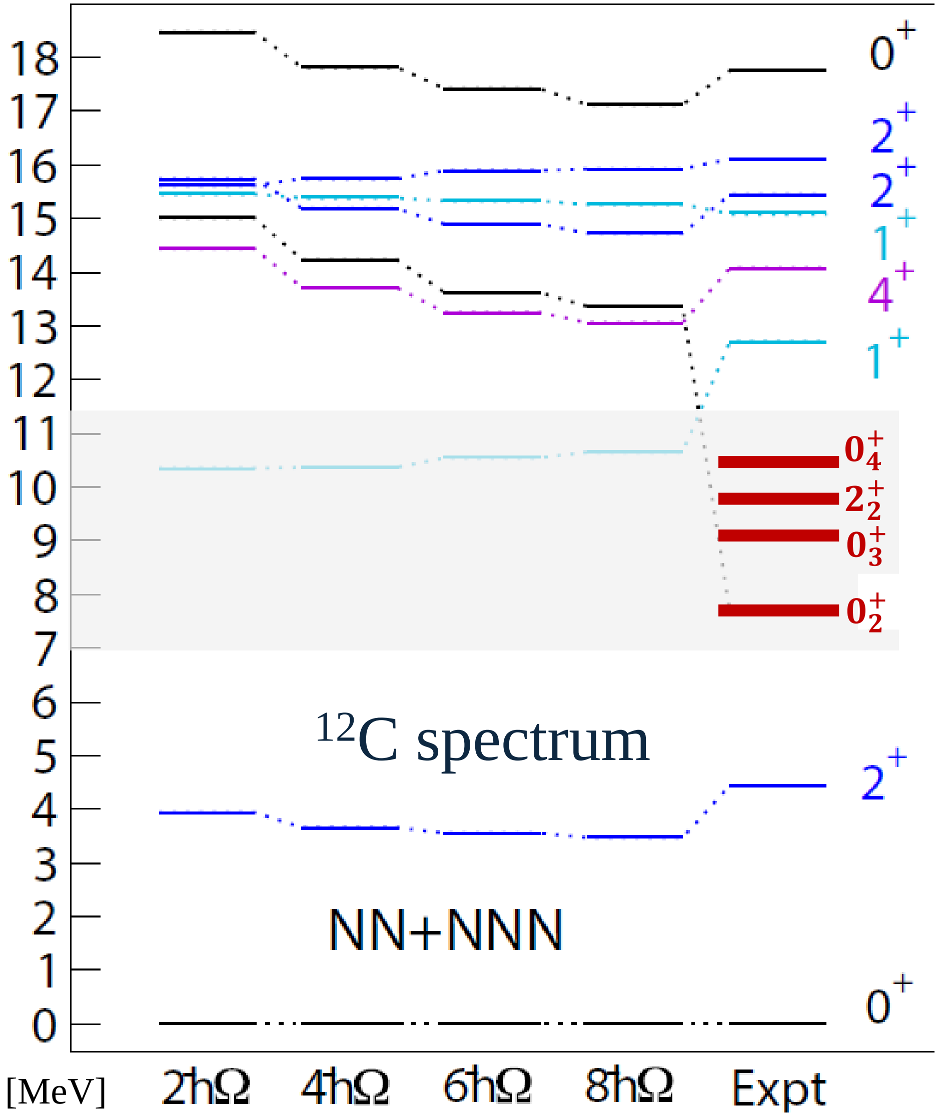


Monte Carlo for the ground state density of ^{8}Be

R. B. Wiringa, *et al.*, PRC **62**, 014001(2000)



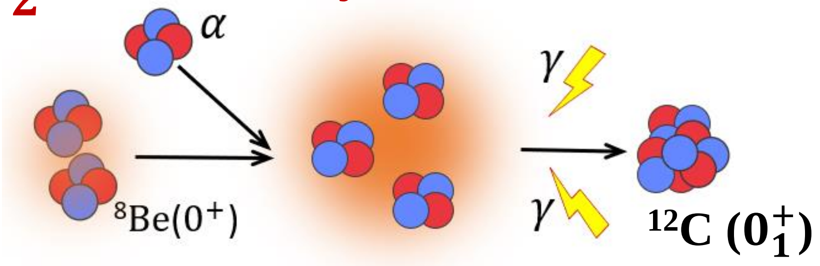
Cluster states of ^{12}C



Recent No-Core-Shell-Model calculations

V.Somà,P.Navrátil, *et al.* PRC,101,014318 (2020)

0_2^+ Hoyle State



The 3α gas-like state **&** Bose-Einstein Condensate.

Rev. Mod. Phys. **89**, 011002 (2017)

$0_{3,4}^+$

Two broad resonance states with large decay width

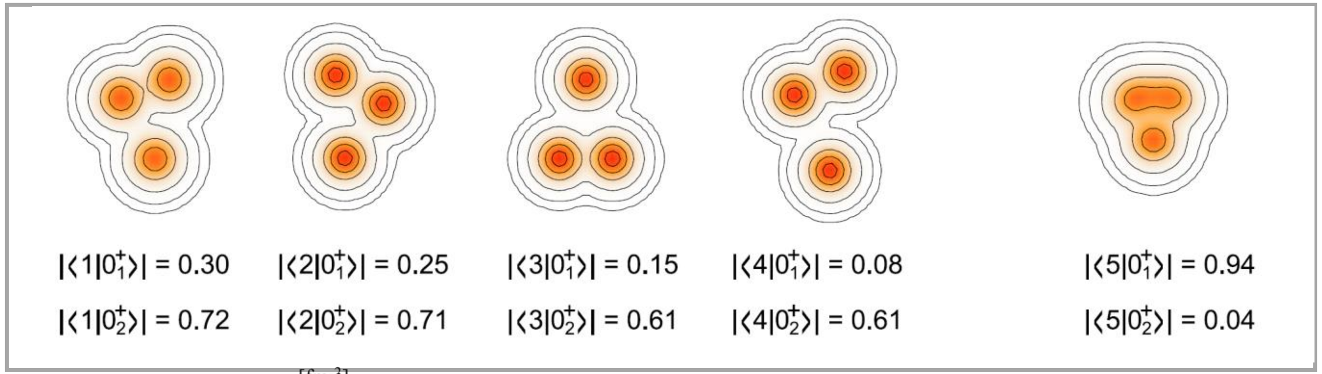
Phys. Rev. C **84**, 054308 (2011)

2_2^+

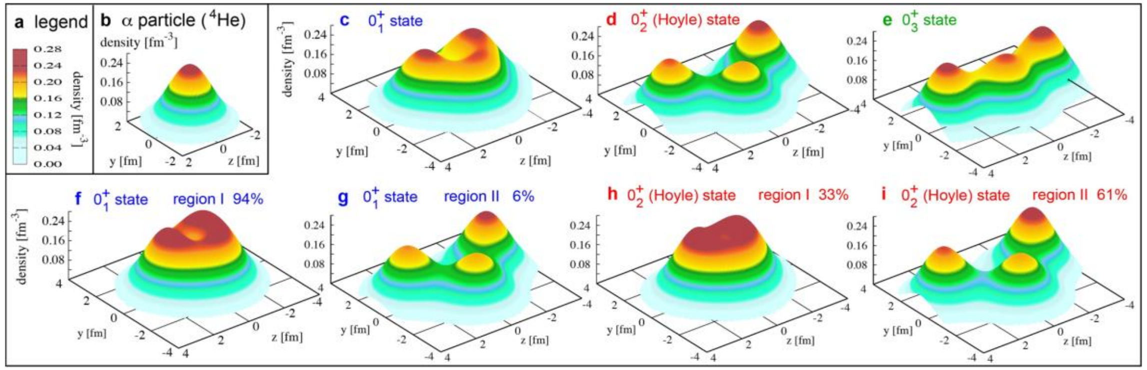
Long puzzle and it now has been confirmed for its existence.

Phys. Rev. Lett. **110**, 152502 (2013)

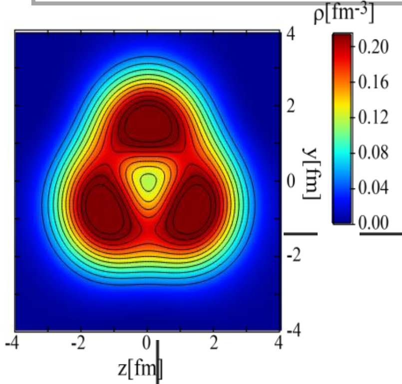
Structure of the ^{12}C



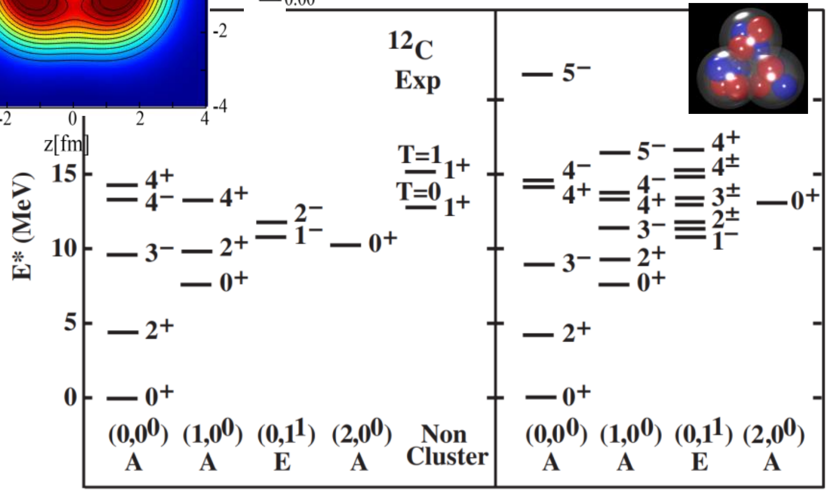
M. Chernykh, et al., PRL 98, 032501 (2007)



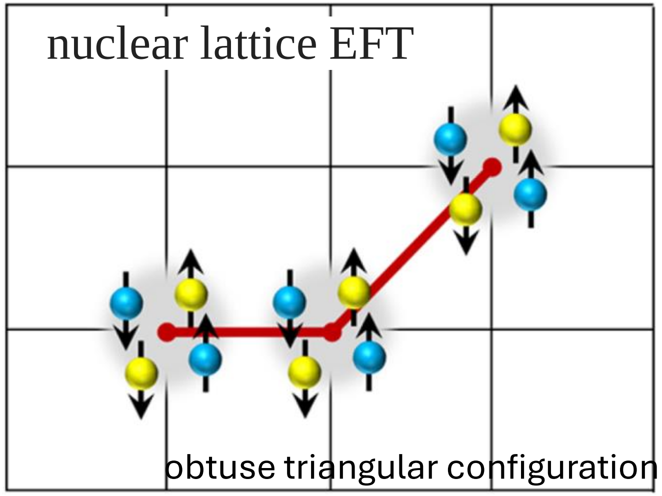
Otsuka, et al., Nat Commun 13, 2234 (2022)



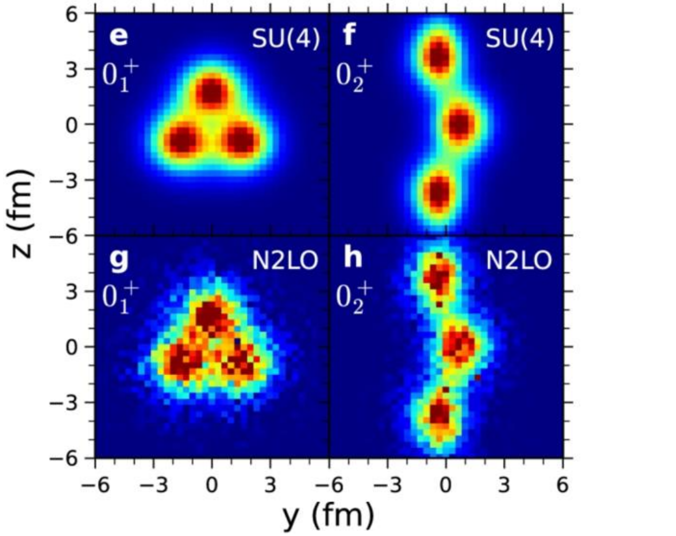
M.Kimura



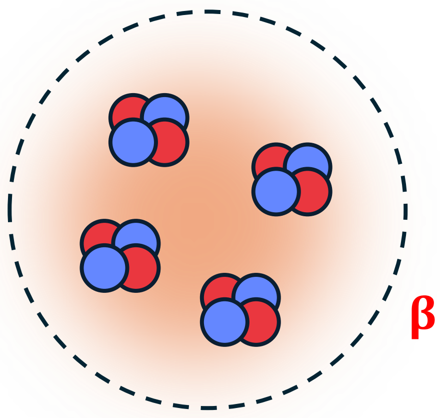
D J Marín-Lámbarri,et al., PRL 113, 012502 (2014)



E.Epelbaum,et al,,PRL 109,252501 (2012)



S.Shen, et al., *Nat Commun* **14**, 2777 (2023)



Alpha Cluster Condensation in ^{12}C and ^{16}O

A. Tohsaki,[1] H. Horiuchi,[2] P. Schuck,[3] and G. Röpke[4]

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(Received 29 June 2001; published 17 October 2001)

THSR wave function

A new α-cluster wave function is proposed which is of the α-particle condensate type. Applications to ^{12}C and ^{16}O show that states of low density close to the 3 and 4 α-particle thresholds in both nuclei are possibly of this kind. It is conjectured that all self-conjugate $4n$ nuclei may show similar features.

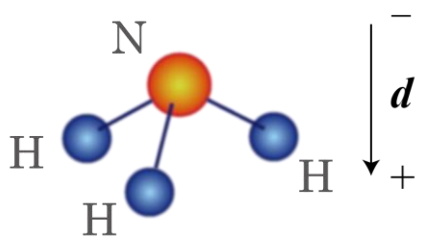
$$\Phi^{THSR}(\beta) = \int d^3R_1 \ldots d^3R_n \mathrm{Exp}\left[-\frac{R_1^2 + \ldots + R_n^2}{\beta^2}\right]\Phi^{Brink}(R_1, \ldots, R_n)$$

$$\propto \phi_G \mathcal{A}\left\{\prod_{i=1}^{n}\left[\mathrm{Exp}\left(-\frac{2(X_i - X_G)^2}{B^2}\right)\phi(\alpha_i)\right]\right\}$$

$$\phi(\alpha) \propto \exp\left[-\sum_{1\le i<j\le 4}(r_i - r_j)^2/(8b^2)\right] \qquad B^2 = b^2 + 2\beta^2$$

β can be considered as the **size parameter of the nucleus**

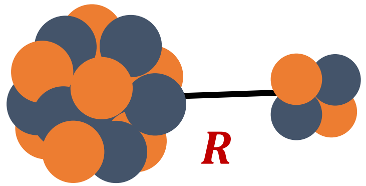
Nonlocalized clustering



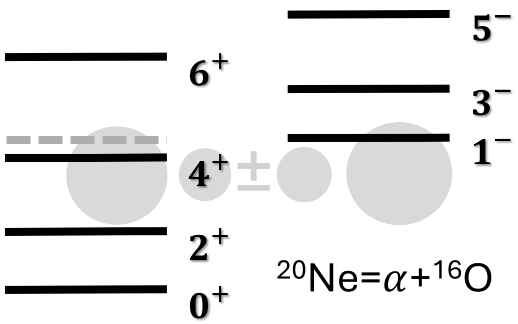
from M.Kimura

Clusters make the localized motion confined by the inter-cluster distance parameter *R*.

$$\mathcal{A}\{\exp[-\frac{8\,(\boldsymbol{r}-\boldsymbol{R})^2}{5\boldsymbol{b}^2}]\phi(\alpha)\phi(^{16}\text{O})\}$$



Brink cluster model



Inversion doublet rotational bands in ^{20}Ne

H.Horiuchi and K.Ikeda,PTP**40**,277(1968)

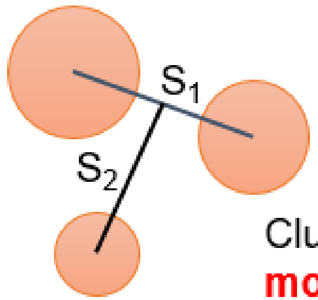
Single THSR wave function
≈ Superposed Brink wave functions

$$\mathcal{A}\{\exp[-\frac{8\,r^2}{5\boldsymbol{B}^2}]\phi(\alpha)\phi(^{16}\text{O})\}$$

Clusters make the nonlocalized motion in a container whose size is described by parameter $\boldsymbol{\beta}$ ($\boldsymbol{B}^2 = \boldsymbol{b}^2 + 2\boldsymbol{\beta}^2$)

Container picture

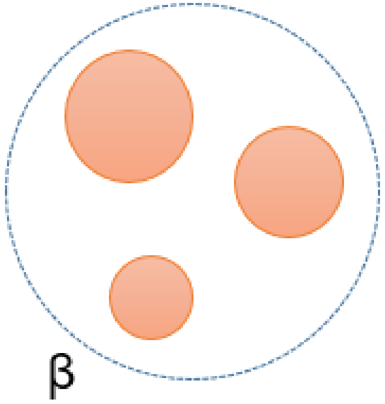
Container picture for various cluster systems



S_1 and S_2: inter-cluster distance parameters.

Clusters make the **localised motions** around the fixed positions.

Traditional cluster model



β: the size of the nucleus parameter.

Clusters move around in the **cluster-type mean field**.

Container model

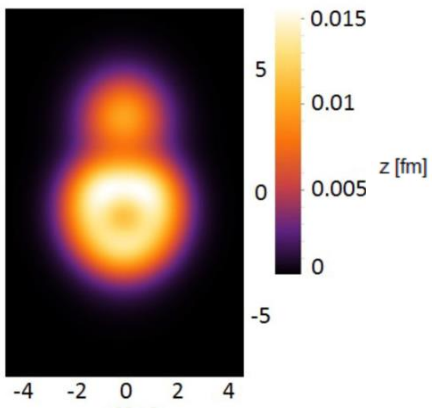
B. Zhou, Y. Funaki, H. Horiuchi, Z. Ren,et al.,PRL. **110**, 262501 (2013) PRC89, 034319 (2014). Front. Phys. 15, 14401 (2020)

Y. Funaki et al. / Progress in Particle and Nuclear Physics 82 (2015) 78–132 113

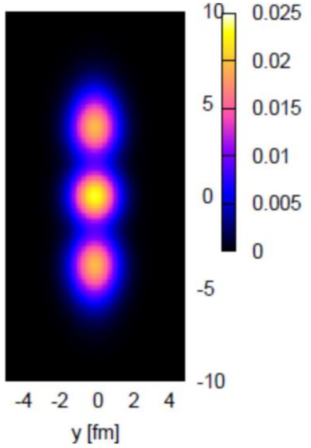
Table 5
The maximum squared overlaps between the single THSR wave functions and RGM/GCM wave functions. The corresponding $\boldsymbol{\beta}$ values, where $(\beta_x = \beta_y, \beta_z) = (\beta_\perp, \beta_z)$, are shown in parentheses in a unit of fm.

	Max.$(\beta_\perp, \beta_z)$					
	^{8}Be	^{12}C	^{20}Ne	3αLCS	4αLCS	$^{9}_{\Lambda}$Be
0^+	1.000(1.8, 7.8)	0_1^+:0.93(1.5, 1.5) (0_1^+:0.978)[a] 0_2^+:0.993(5.3, 1.5)	0.993(0.9, 2.5)	0.987(0.1, 5.1)	0.944(0.1, 8.2)	0.995(1.6, 3.0)
2^+			0.988(0.0, 2.2)	0.989(0.1, 5.4)	0.942(0.1, 8.4)	0.994(0.1, 3.0)
4^+			0.978(0.0, 1.8)	0.981(0.1, 6.6)	0.931(0.1, 9.0)	0.977(0.1, 2.1)
			1.000(3.7, 1.4)			
3^-			0.999(3.7, 0.0)			

[a] The value by the use of the extended version of the THSR wave function, with the parameter values $(\beta_{1\perp}, \beta_{1z}, \beta_{2\perp}, \beta_{2z}) = (0.1, 2.3, 2.8, 0.1)$, which we will discuss in Section 3.9.3.

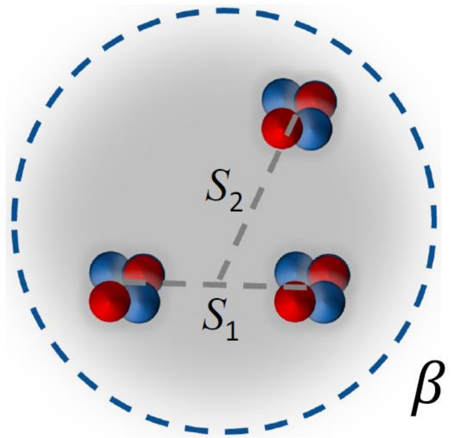


two-body



linear-chain

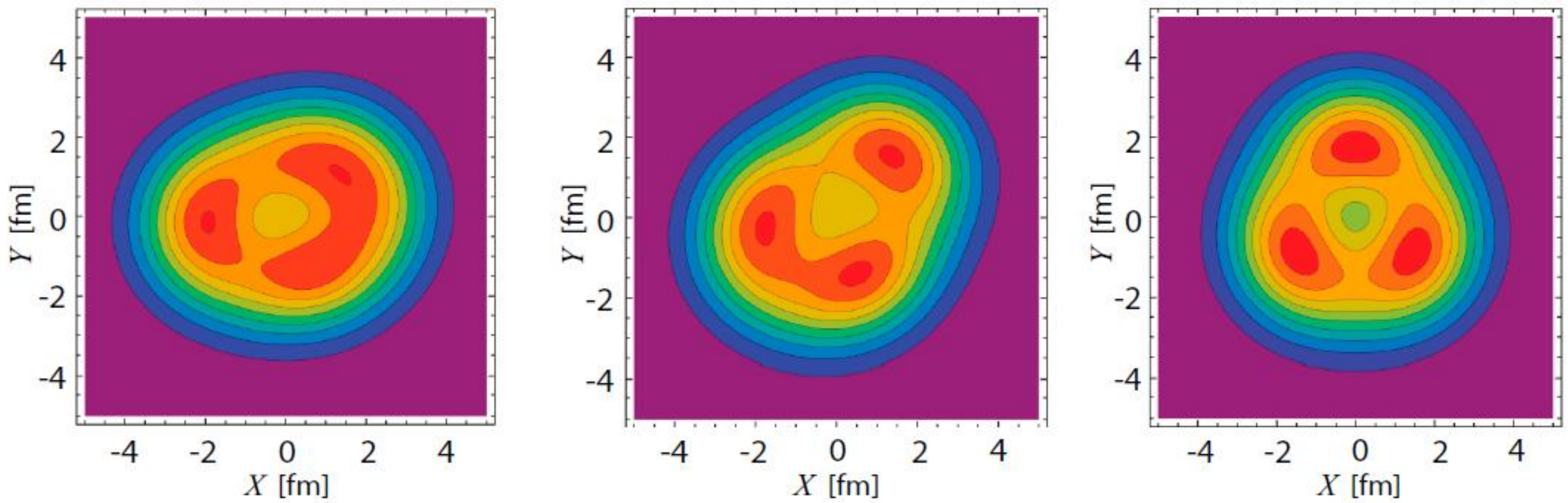
Nonlocalized cluster motion of 3α clusters in ^{12}C



We really obtained the single high-accuracy THSR-type wave functions for 3^- and 4^- states,

$$\propto \mathcal{A}\{\exp[-\frac{(\boldsymbol{\xi}_1-\boldsymbol{S}_1)^2}{b^2+2\boldsymbol{\beta}^2}-\frac{(\boldsymbol{\xi}_2-\boldsymbol{S}_2)^2}{3/4\ (b^2+2\boldsymbol{\beta}^2)}]\phi(\alpha_1)\phi(\alpha_2)\phi(\alpha_3)\}$$

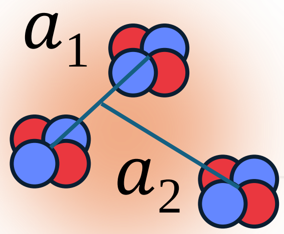
Size parameters β obtained by variational calculations.



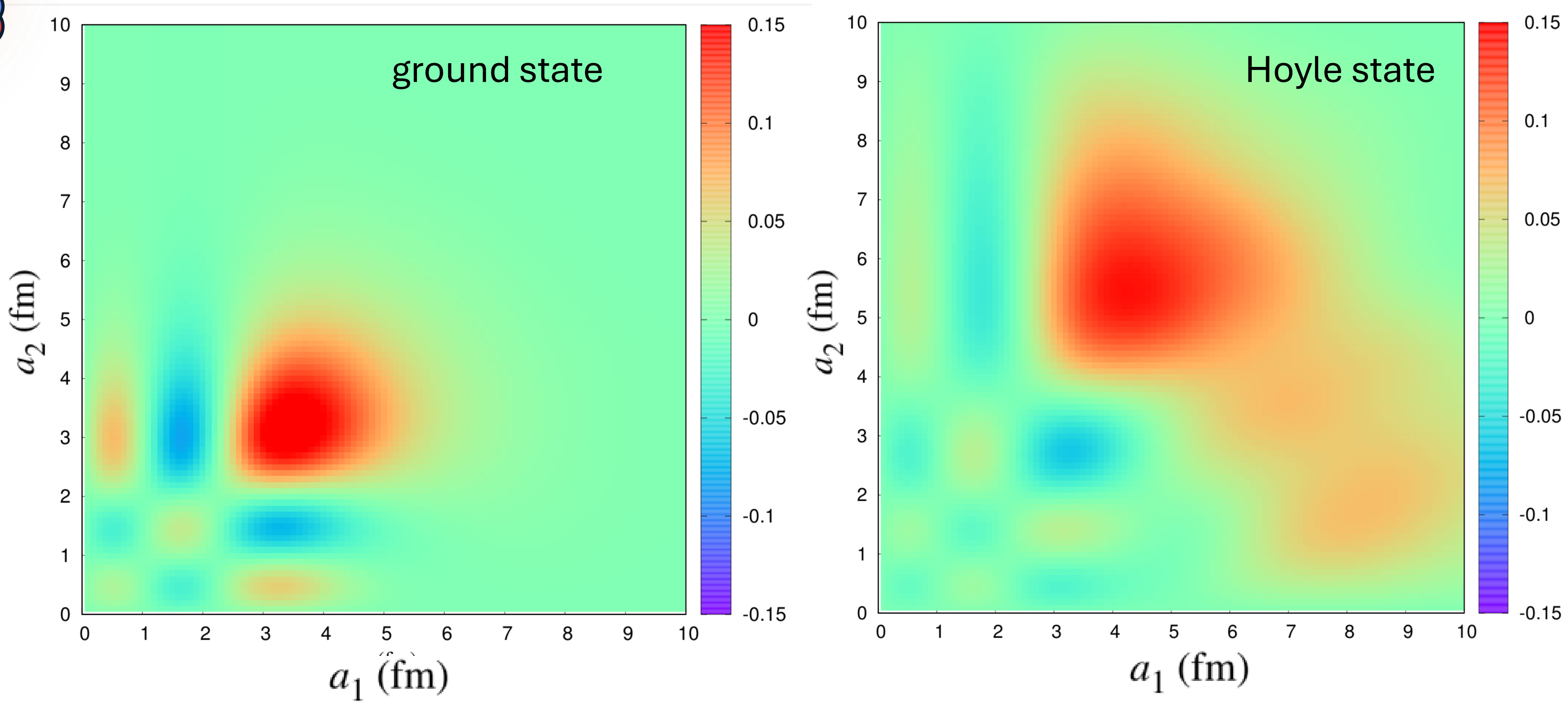
$|\langle\Phi^{3^-}(3/2,3/2,1/2)|\Phi^{3^-}_{\rm GCM}\rangle|^2=0.94$ $|\langle\Phi^{3^-}(1,3/2,3/2)|\Phi^{3^-}_{\rm GCM}\rangle|^2=0.93$ $|\langle\Phi^{3^-}(3/2,0,3/2)|\Phi^{3^-}_{\rm GCM}\rangle|^2=0.94$

$|\langle\Phi^{4^-}(3/2,3/2,1/2)|\Phi^{4^-}_{\rm GCM}\rangle|^2=0.92$ $|\langle\Phi^{4^-}(1,3/2,3/2)|\Phi^{4^-}_{\rm GCM}\rangle|^2=0.92$ $|\langle\Phi^{4^-}(3/2,0,3/2)|\Phi^{4^-}_{\rm GCM}\rangle|^2=0.92$

B. Zhou, et al., Phys. Rev. C **99**, 051303(R) (2019).



Ground state and Hoyle state

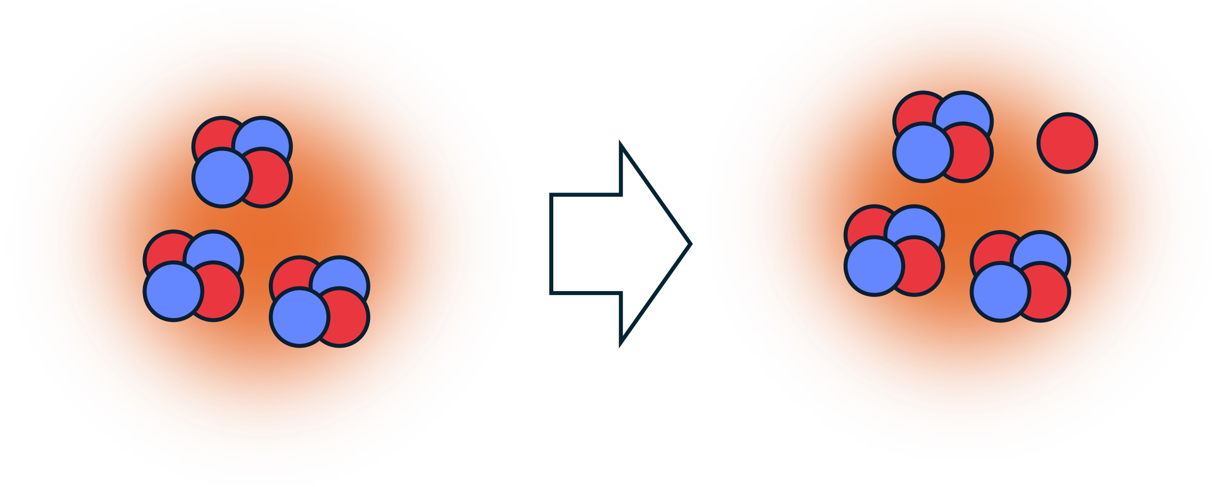


$[\alpha \otimes [\alpha \otimes \alpha]_0]_0 \otimes [0 \otimes 0]_0$

preliminary result

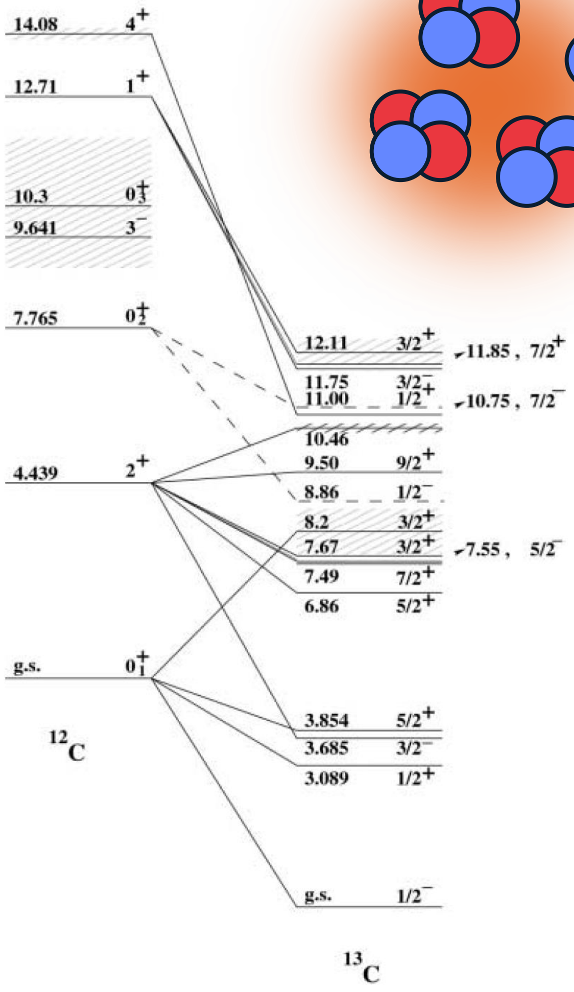
$$\mathcal{Y}_c^{J\pi}(a_1,a_2)=\sqrt{\frac{A!}{C_1!C_2!C_3!}}\left\langle \frac{\delta(r_1-a_1)\delta(r_2-a_2)}{r_1^2 r_2^2}\left[[Y_{l_1}(\hat{r}_1)\otimes Y_{l_2}(\hat{r}_2)]_L \otimes \left[\Phi_{C_1}^{j_1\pi_1}\otimes\left[\Phi_{C_2}^{j_2\pi_2}\otimes\Phi_{C_3}^{j_3\pi_3}\right]_{j_{23}}\right]_{j_{123}}\right]_{JM}\right|\left.\Psi_M^{J\pi}\right\rangle$$

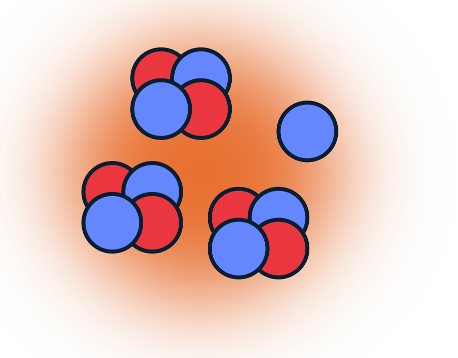
De-ye Tao et al.

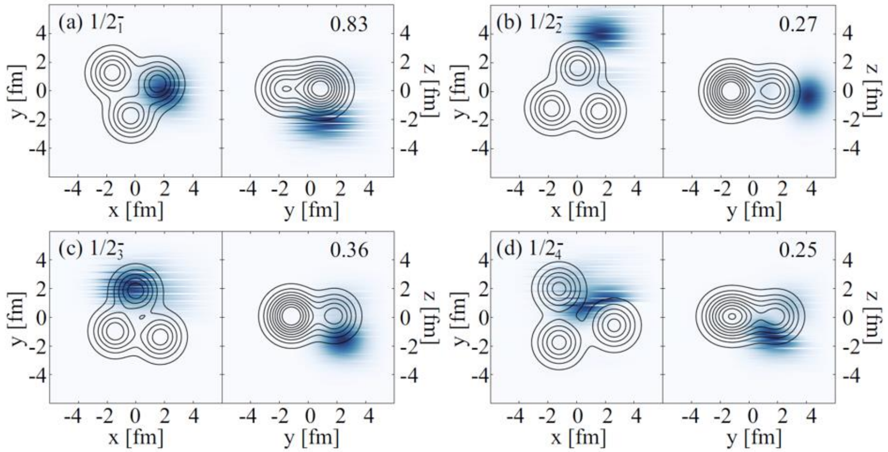


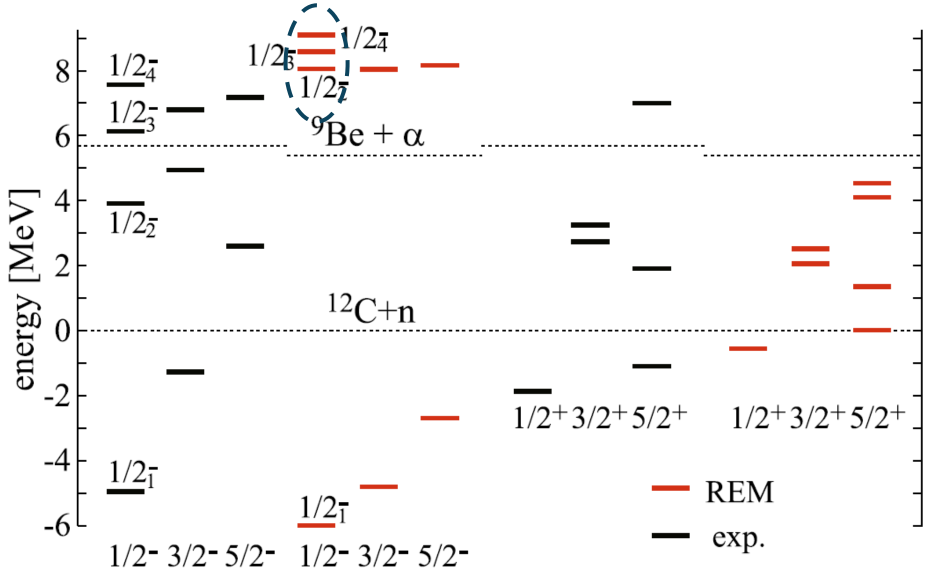
Clustering structure of $3\alpha+p$ in ^{13}N

Search for the Hoyle-analog state in ^{13}C







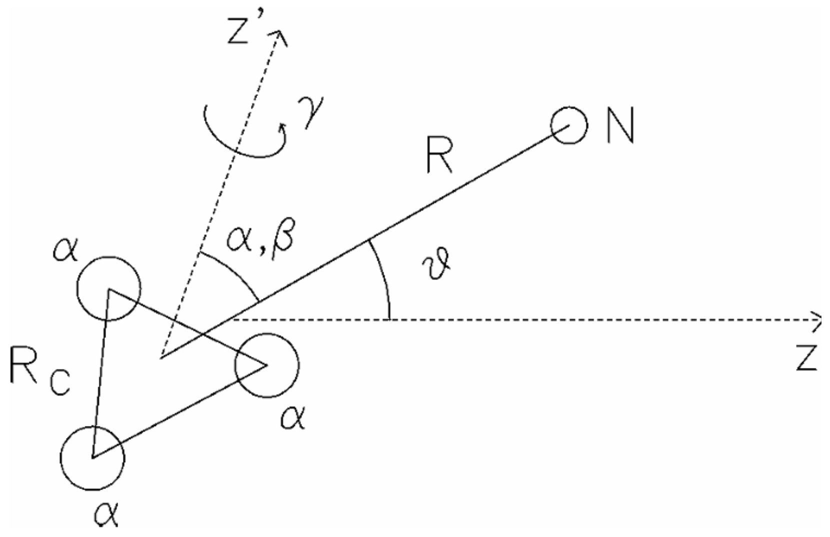


^{12}C(states) + single particle states ⇒ ^{13}C (states)

M. Milin and W. von Oertzen, Eur Phys J A **14**, 295 (2002).

S. Shin, et al., Phys. Rev. C **103**, 054313 (2021). arXiv:2404.09712

Multicluster study of the $^{12}C+n$ and $^{12}C+p$ systems



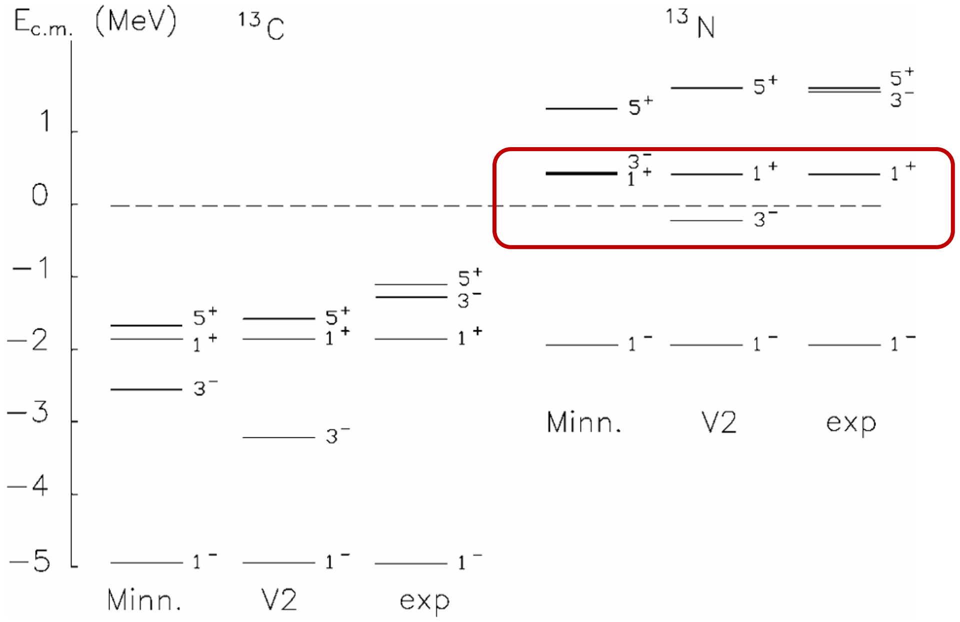


FIG. 3. Energy spectra of ^{13}C and ^{13}N. The states are labeled by $2J$. Experimental data are taken from Ref. [1].

^{13}N	R_C=0.4	R_C=1.4	R_C=2.7	R_C=4.0	Mixed	Expt.[a]
$B(E1,1/2^+\rightarrow1/2^-)$	0.21	0.2	0.11	0.02	0.136	0.10±0.01
	0.18	0.18	0.071	6.5×10^{-4}	0.128	
$B(E1,3/2^-\rightarrow1/2^+)$	0.56	0.31	0.088	0.031	0.144	0.1
	0.44	0.25	0.052	6.9×10^{-3}	0.136	
$\Gamma_p(1/2^+)$ (keV)	35.6	39.9	43.1	38.4	40.2	31.7±0.8
	31.2	32.8	41.7	36.1	34.2	
$\theta^2(3/2^-)$ (%)	6.1	6.2	7.6	10.2	7.3	2.9±0.2
	4.1	4.6	5.4	7.2	4.8	

[a]Reference [1].

M. Dufour and P. Descouvemont, Phys. Rev. C **56**, 1831 (1997)

PHYSICAL REVIEW C **109**, 054308 (2024)

Cluster structure of $3\alpha + p$ states in ^{13}N

J. Bishop,[1,2] G. V. Rogachev,[1,3,4] S. Ahn,[5] M. Barbui,[1] S. M. Cha,[5] E. Harris,[1,3] C. Hunt,[1,3] C. H. Kim,[6] D. Kim,[5] S. H. Kim,[6] E. Koshchiy,[1] Z. Luo,[1,3] C. Park,[5] C. E. Parker,[1] E. C. Pollacco,[7] B. T. Roeder,[1] M. Roosa,[1,3] A. Saastamoinen,[1] and D. P. Scriven[1,3]

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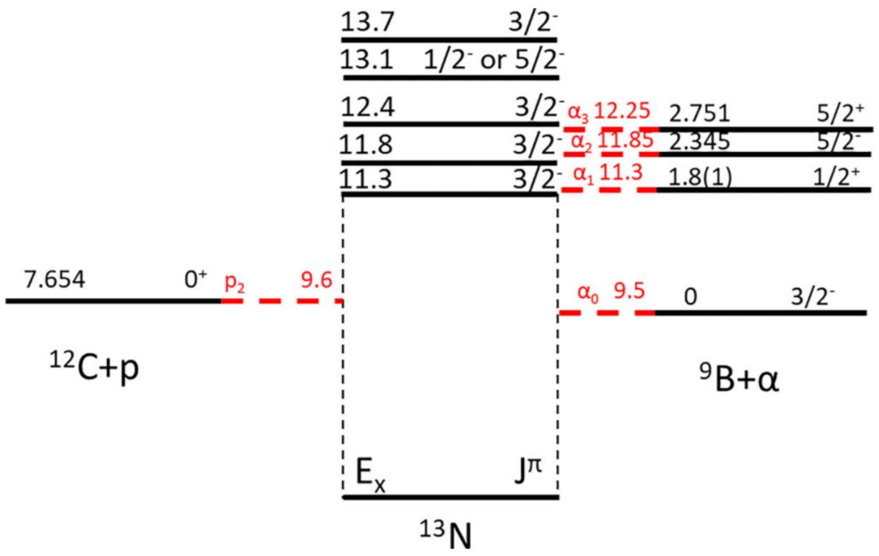
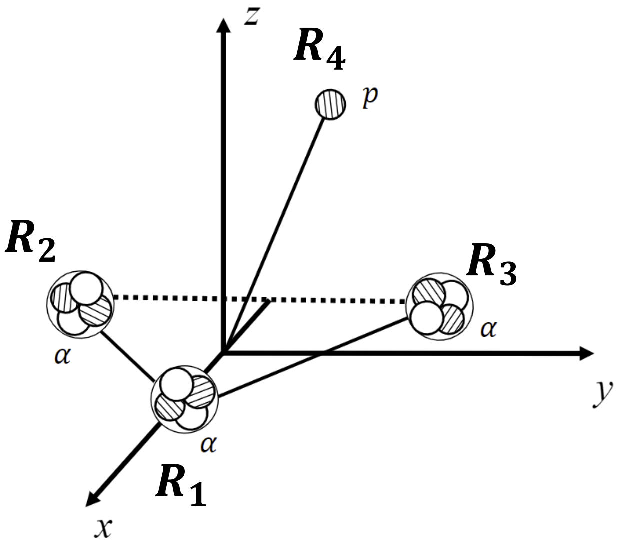


State		Counts					
E_x	J^π	α_0	α_1	α_3	p_0 [13]	p_1 [13]	p_2
11.3(1)	$(3/2^-)$	18(4.4)	0	0	6(2.6)	<3	7(2.8)
11.8(1)	$(3/2^-)$	<1.8	0	0	28(14)	<4	4(2.2)
12.4(1)	$(3/2^-)$	22(4.8)	4(2.2)	0	<3	<10	5(2.5)
13.1	$(1/2^-)$ $(5/2^-)$	0	3(2)	5(2.5)	21(6)	<10	0
13.7(1)	$(3/2^-)$	1(1.4)	3(2)	4(2.2)	<3	<10	6(2.7)

Conclusions: These states are seen to have a $[^9\mathrm{B(g.s)} \otimes \alpha / p + {}^{12}\mathrm{C}(0_2^+)]$, $[^9\mathrm{B}(\frac{1}{2}^+) \otimes \alpha]$, $[^9\mathrm{B}(\frac{5}{2}^+) \otimes \alpha]$, and $[^9\mathrm{B}(\frac{5}{2}^+) \otimes \alpha]$ structure, respectively. A previously seen state at 11.8 MeV was also determined to have a $[p + {}^{12}\mathrm{C(g.s.)}/ p + {}^{12}\mathrm{C}(0_2^+)]$ structure. The overall magnitude of the clustering is not able to be extracted, however,

Nuclear cluster model for $3\alpha+p$ in ^{13}N

The four-cluster （$3\alpha+p$） wave function,



$$\Phi(\boldsymbol{R}_1,\boldsymbol{R}_2,\boldsymbol{R}_3,\boldsymbol{R}_4)=\mathcal{A}\{\Phi_\alpha(\boldsymbol{R}_1)\Phi_\alpha(\boldsymbol{R}_2)\Phi_\alpha(\boldsymbol{R}_3)\Phi_p(\boldsymbol{R}_4)\}$$

$$\Phi_\alpha(\boldsymbol{R})=\mathcal{A}\{\prod_{i=1}^{4}\phi(\boldsymbol{R},\boldsymbol{r}_i)\chi_i\tau_i\} \qquad \phi(\boldsymbol{R},\boldsymbol{r}_i)=(\frac{1}{\pi b^2})^{3/4}e^{-\frac{(\boldsymbol{r}_i-\boldsymbol{R})^2}{2b^2}}$$

$$\Psi_M^{J\pi}=\sum_{\{R\}K}f_{\{R\}K}\Phi_{MK}^{J\pi}(\{R\})$$

$$\Phi_{MK}^{J\pi}(\{R\})=P_{MK}^{J}P^{\pi}\Phi(\{R\})$$

intrinsic wave function

The Hamiltonian of the system ,

$$\hat{H}=\sum_{i=1}^{13}t_i-t_{\text{c.m.}}+\sum_{i<j}^{13}v_{ij}^{NN}+\sum_{i<j}^{13}v_{ij}^{C},$$

$$V_{ij}^{NN}=\sum_{n=1}^{2}V_n e^{-r_{ij}^2/a_n^2}(W+BP_\sigma-HP_\tau-MP_\sigma P_\tau)$$

$$+\sum_{n=1}^{2}w_n e^{-b_n r_{ij}^2}P(^3O)\boldsymbol{L}\cdot\boldsymbol{S},$$

Energy spectra of ^{13}N

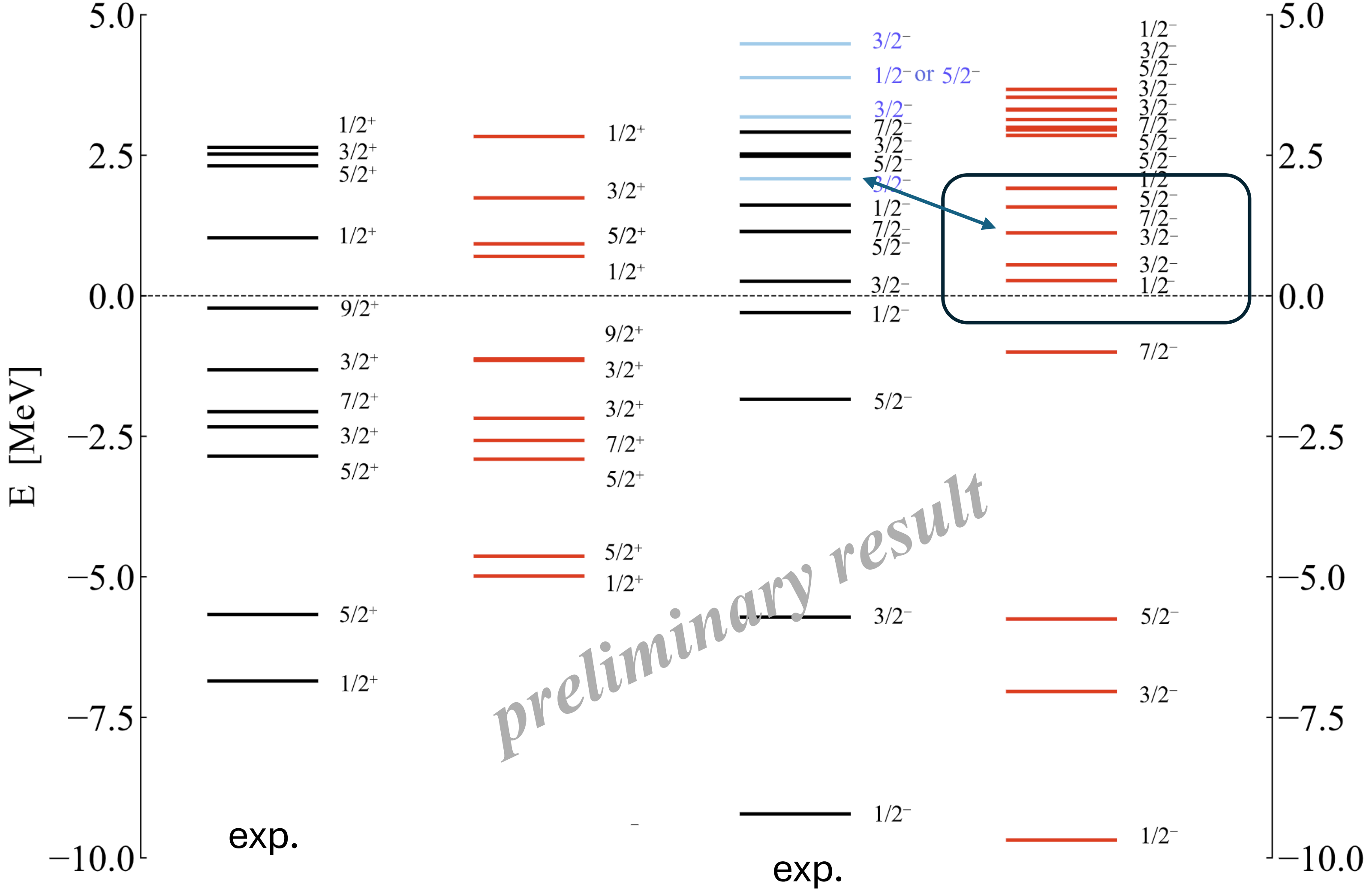
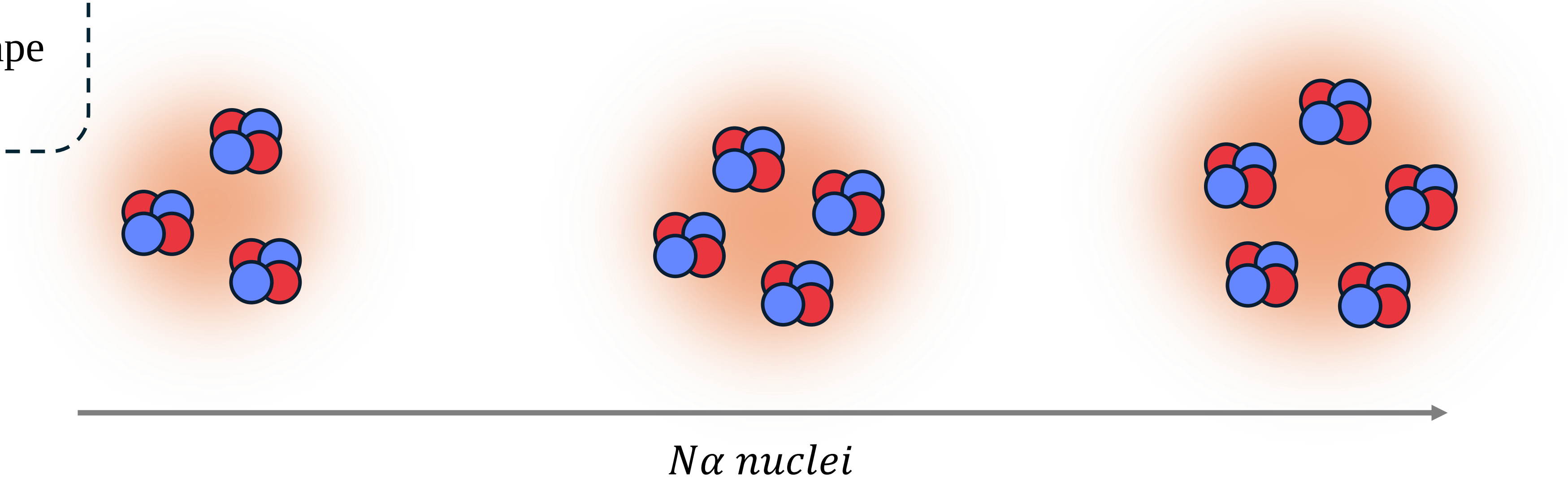


FIG. 2. (Color online) The calculated and experimental energy spectra of ^{13}N.

gas-like cluster state

no-geometry shape

excited states

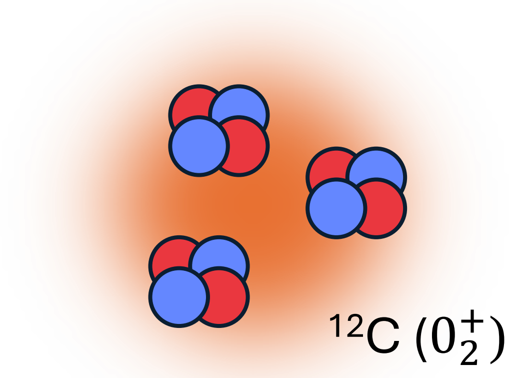


Search for the 5α condensate state

3α condensate	4α condensate	5α condensate
(Hoyle state)	(0_6^+ state)	(?)
2001 (THSR)	2008~ (OCM,THSR)	2019~

study of alpha condensate in finite nuclei

Hoyle states of ^{12}C



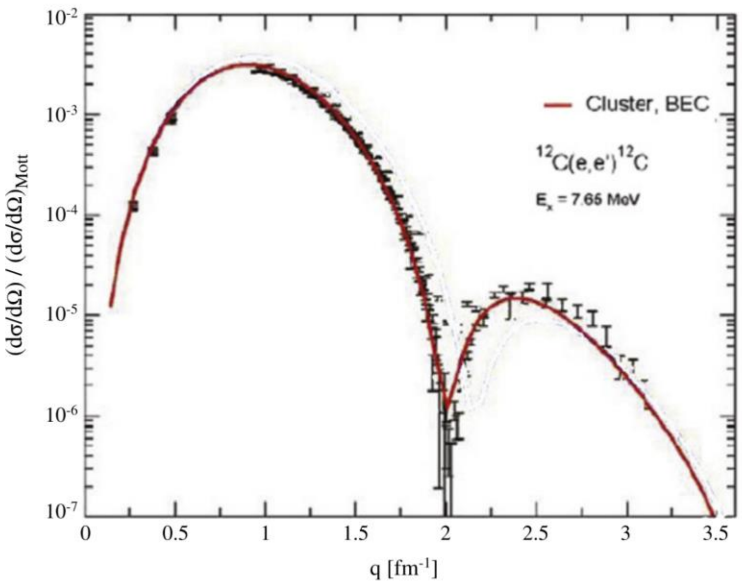
3α Bose-Einstein state

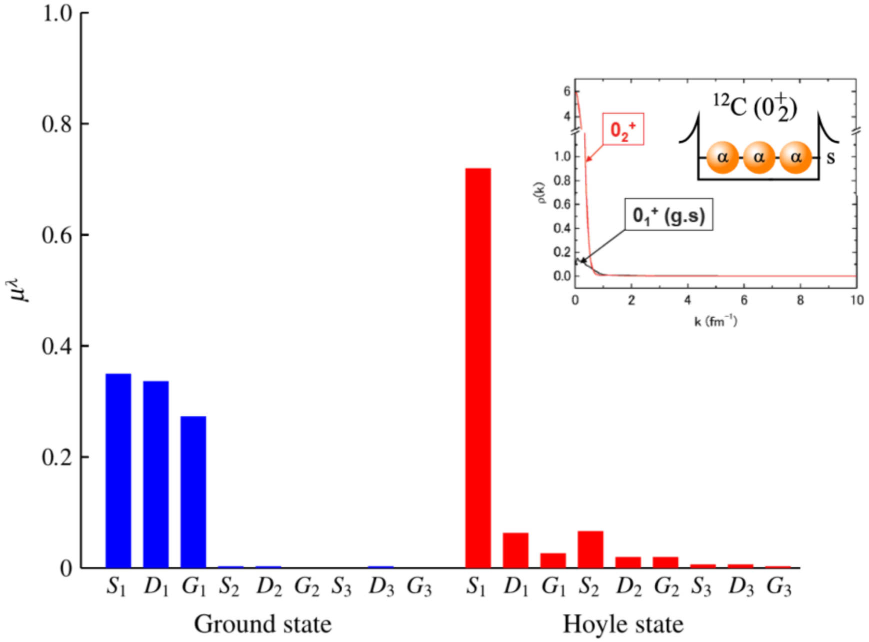
$$\Psi_{3\alpha}^{\mathrm{THSR}} = \mathcal{A}\left\{\exp\left[-\frac{2}{B^2}(\boldsymbol{X}_1^2+\boldsymbol{X}_2^2+\boldsymbol{X}_3^2)\right]\phi(\alpha_1)\phi(\alpha_2)\phi(\alpha_3)\right\}$$
$$= \exp\left(-\frac{6}{B^2}\boldsymbol{\xi}_3^2\right)\mathcal{A}\left\{\exp\left(-\frac{4}{3B^2}\boldsymbol{\xi}_1^2-\frac{1}{B^2}\boldsymbol{\xi}_2^2\right)\phi(\alpha_1)\phi(\alpha_2)\phi(\alpha_3)\right\},$$

$$\boldsymbol{\xi}_1 = \boldsymbol{X}_1 - \frac{1}{2}(\boldsymbol{X}_2+\boldsymbol{X}_3), \qquad \boldsymbol{\xi}_2 = \boldsymbol{X}_2 - \boldsymbol{X}_3, \qquad \boldsymbol{\xi}_3 = \frac{1}{3}(\boldsymbol{X}_1+\boldsymbol{X}_2+\boldsymbol{X}_3)$$

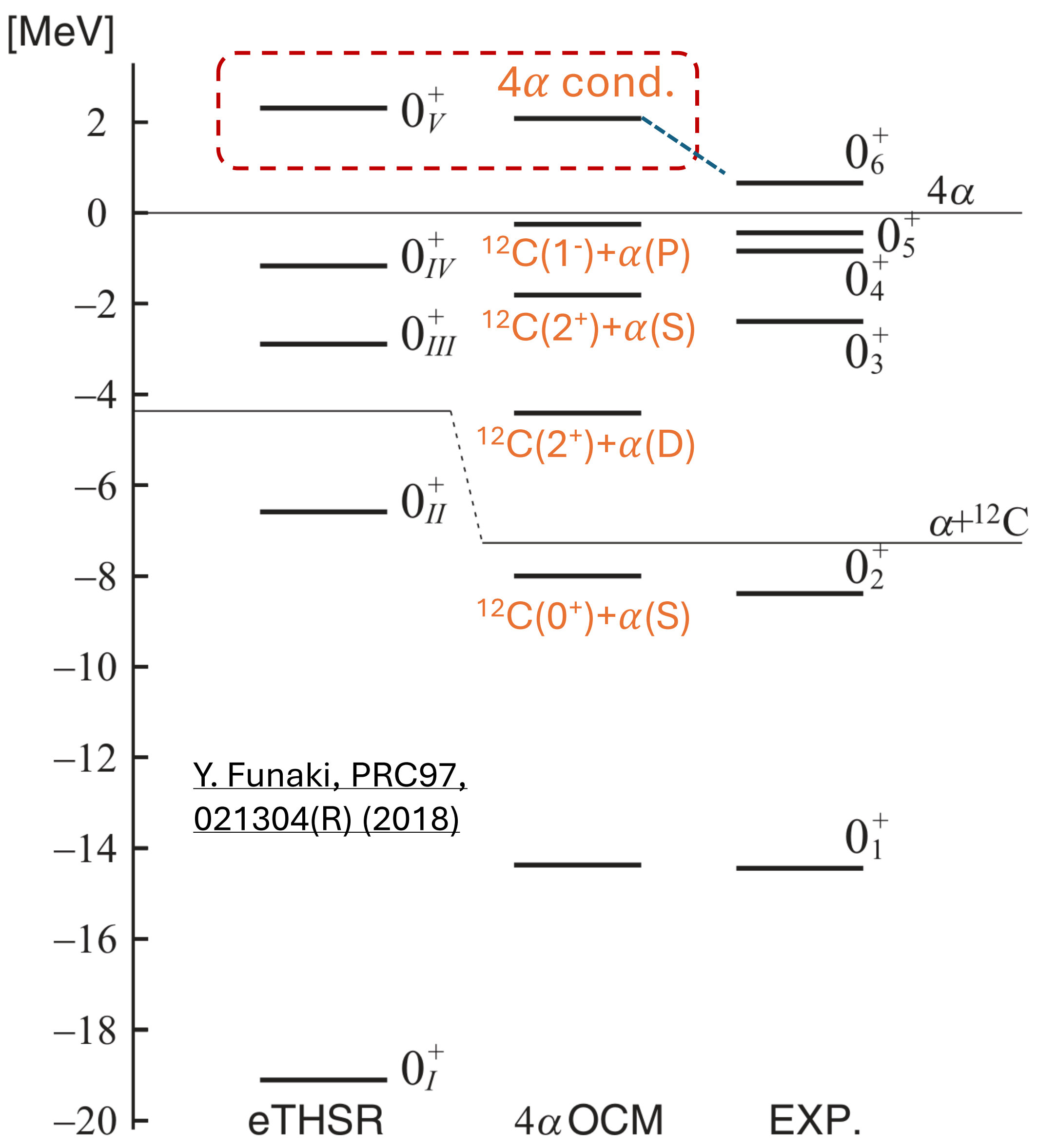
THSR, PRL 87, 192501 (2001)

Y. Funaki et al. / Progress in Particle and Nuclear Physics 82 (2015) 78–132

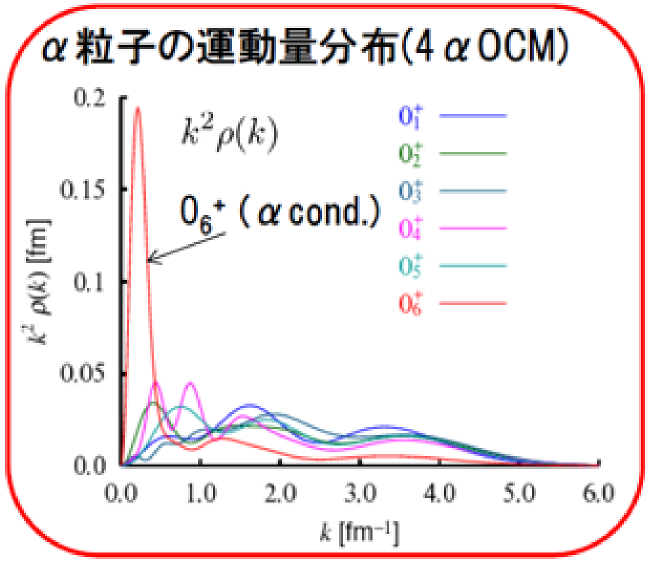




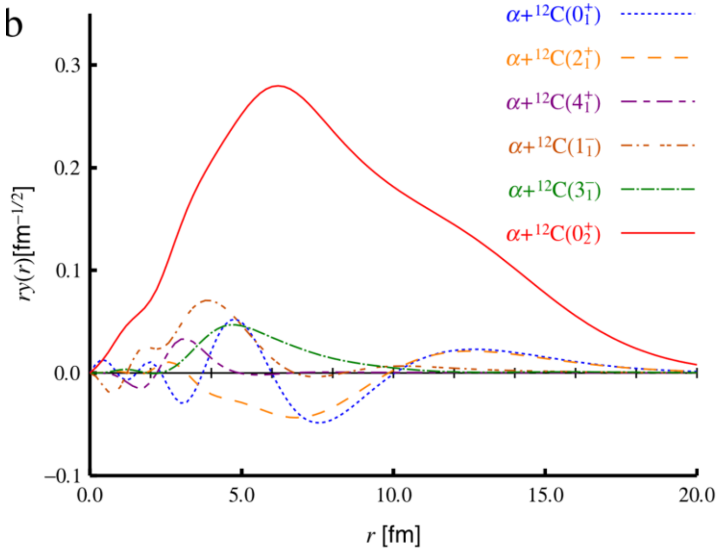
Alpha condensate in ^{16}O



Y. Funaki, PRC97, 021304(R) (2018)



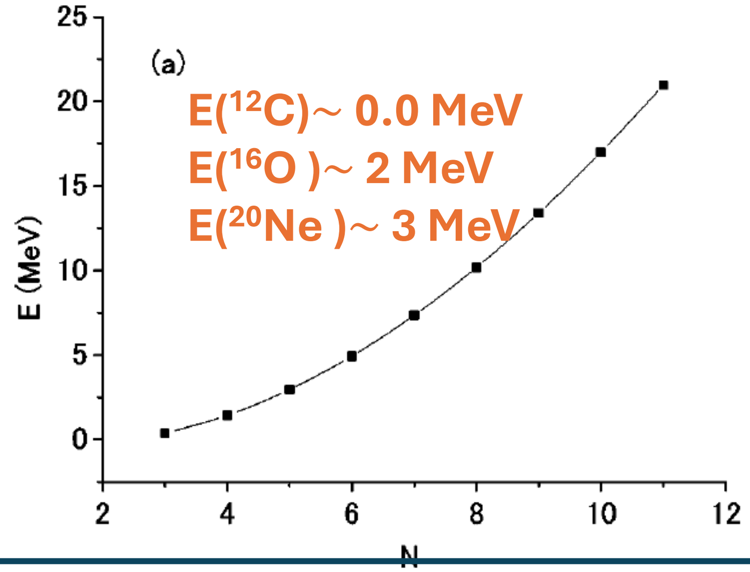
4αOCM
Y. F. et al., PRL **101**, 082502 (2008).
4αTHSR
Y. F. et al., PRC **82**, 024312 (2010).



Multi-alpha condensation

Dilute multi-α cluster condensed states with spherical and axially deformed shapes are studied with the Gross-Pitaevskii equation and Hill-Wheeler equation where the α cluster is treated as a structureless boson, **it is predicted to exist in heavier self-conjugate 4*N* nuclei up to *N*=10.**

T. Yamada and P. Schuck, Phys. Rev. C 69, 024309 (2004).



Some candidates for α condensate were found from experiments for ^{12}C and ^{16}O.

Rev. Mod. Phys. **89**, 011002 (2017).

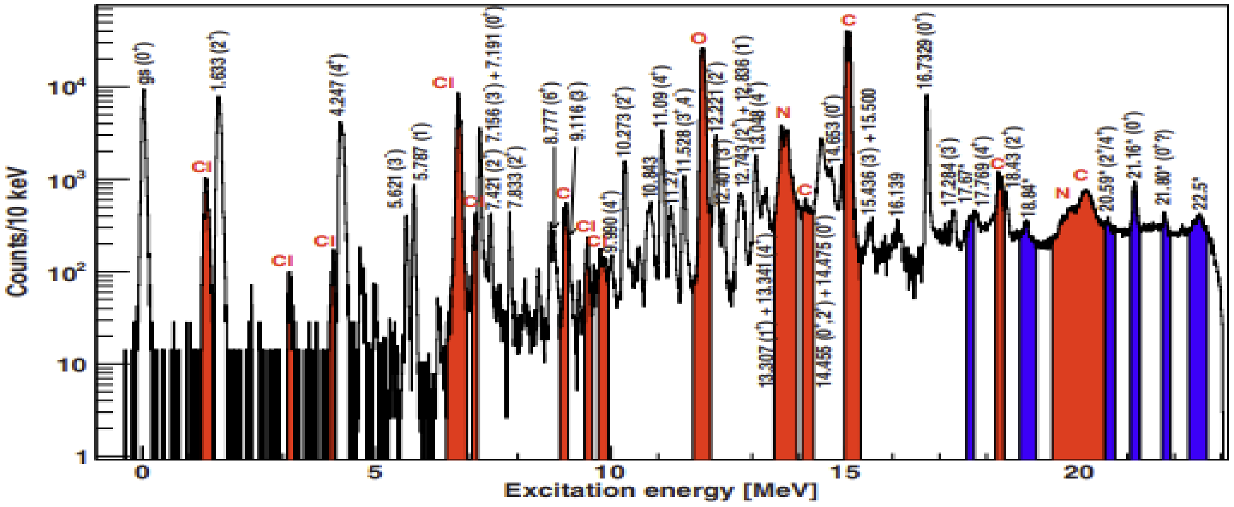
No experimental signatures for α condensation were observed

Phys. Rev. C **100**, 034320 (2019)

An experimental way of testing Bose-Einstein condensation of clusters in the atomic nucleus is reported. The enhancement of cluster emission and the multiplicity partition of possible emitted clusters could be direct signatures for the condensed states.

PRL **96,** 192502 (2006)

Recent experiment for 5α condensation

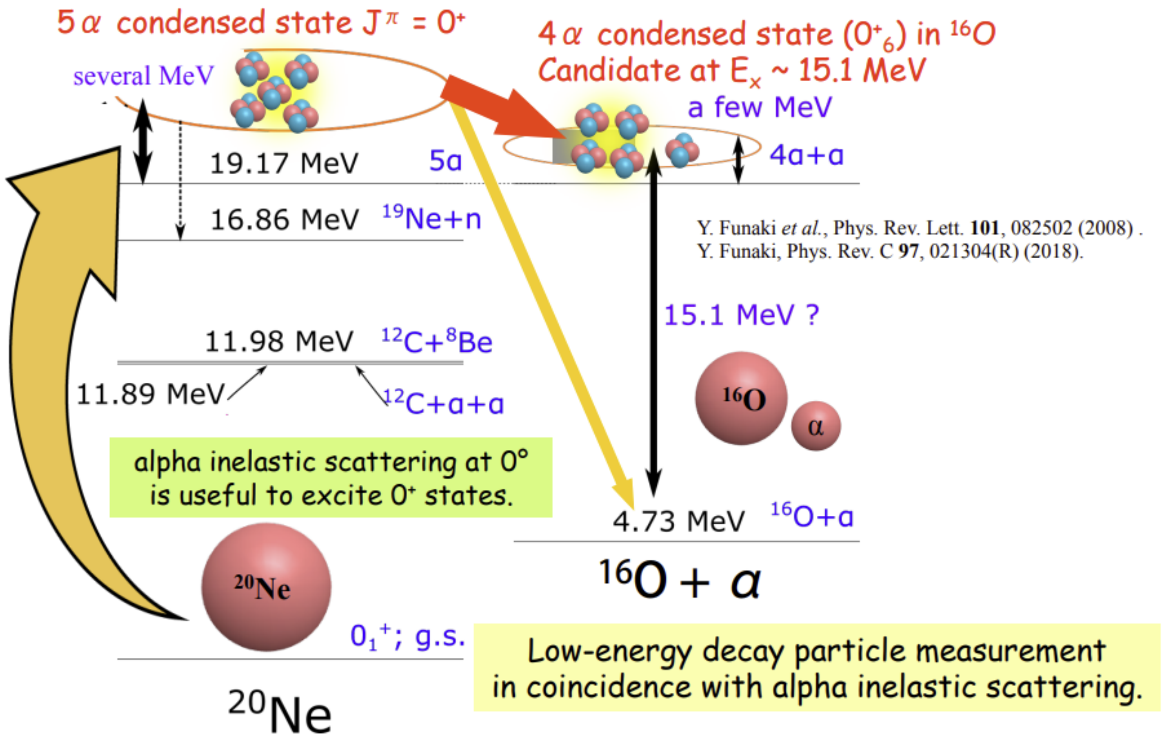


E_x calculated MeV	Model	E_x measured [MeV]
6.05	$2\hbar\omega$	6.725
6.70	USDB	7.191
12.58	$2\hbar\omega$	13.642
14.43	USDB	14.455/14.475
14.67	USDB	14.653
16.76	USDB	16.73
18.06	USDB	17.90
19.02	USDB	18.840(56)
20.47	$2\hbar\omega$	21.160(53)
22.14	5-α [31]	22.500(52)

PHYSICAL REVIEW C 91, 034317 (2015)

Spectroscopy of narrow, high-lying, low-spin states in ^{20}Ne

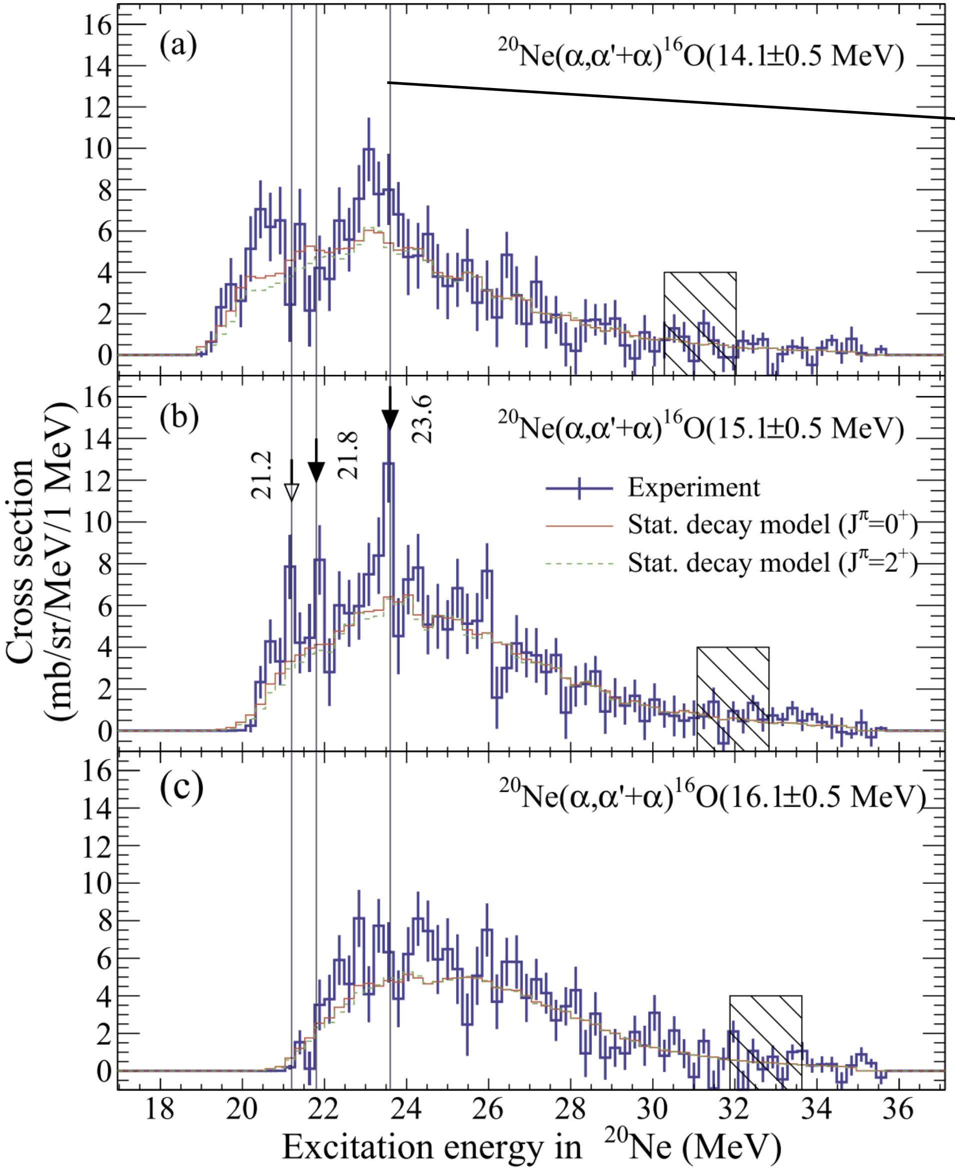
J. A. Swartz,[1,2,*] B. A. Brown,[3,4] P. Papka,[1,2] F. D. Smit,[2] R. Neveling,[2] E. Z. Buthelezi,[2] S. V. Förtsch,[2] M. Freer,[5] Tz. Kokalova,[5] J. P. Mira,[1,2] F. Nemulodi,[1,2] J. N. Orce,[6] W. A. Richter,[2,6] and G. F. Steyn[2]



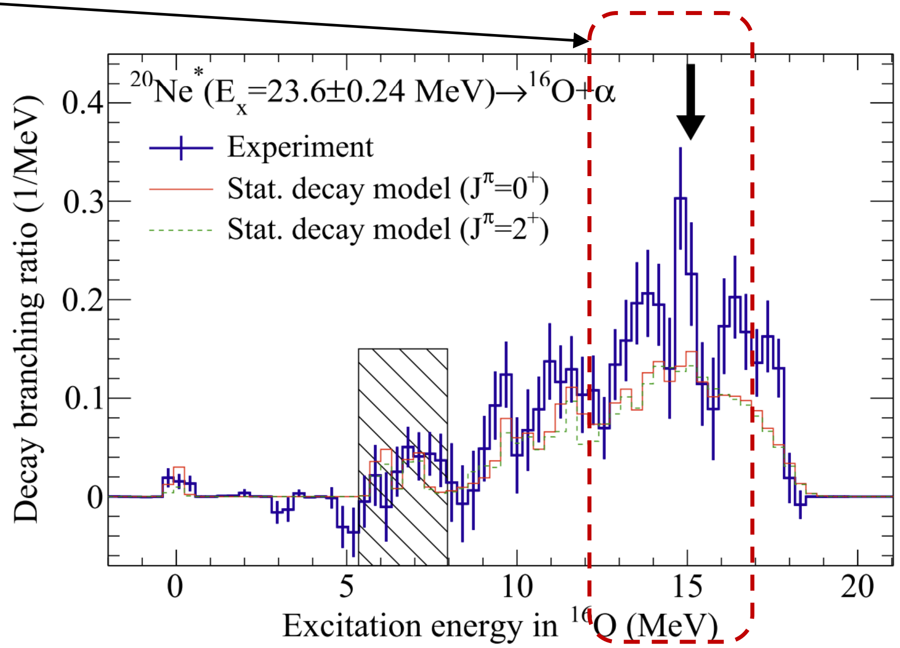
from Kawabata

The state at ***Ex=22.5 MeV***, which could not be interpreted by the shell-model calculations, is a tentative candidate for the 5α cluster state.

Recent experiment for 5α condensation



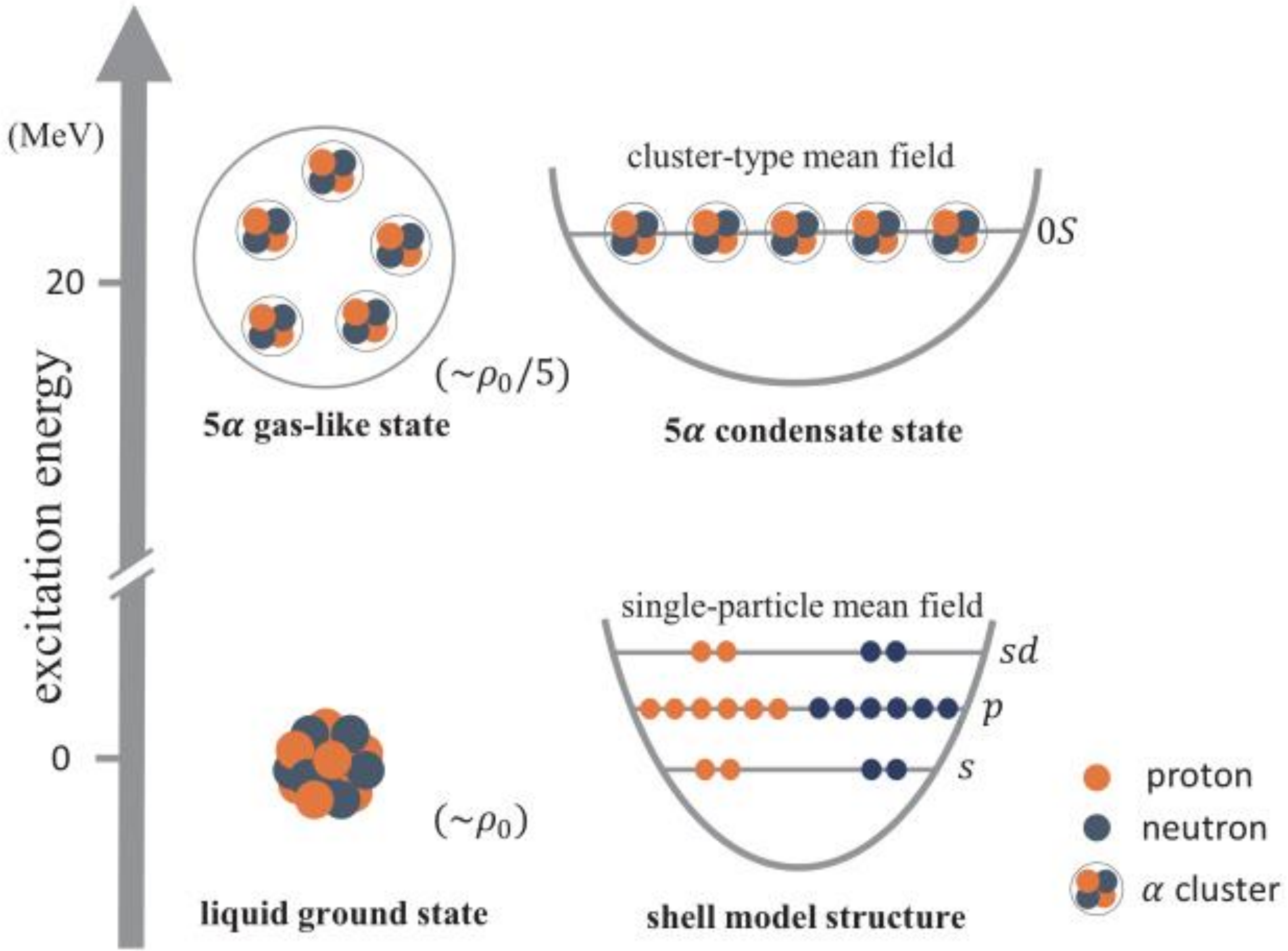
23.6-MeV state enhances in the decay to the ^{16}O (0_6^+) +α channel

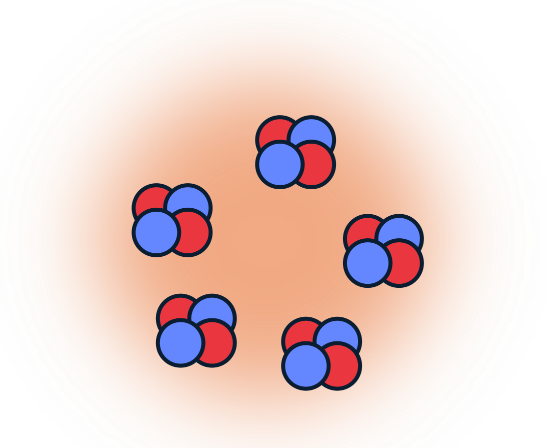


- ❑ 3.3 MeV above the 5α threshold
- ❑ strongly coupled to ^{16}O(0_6^+) state

S. Adachi et al., *Candidates for the 5α condensed state in ^{20}Ne*, PLB **819**,136411 (2021).

Search for the 5alpha condensate state

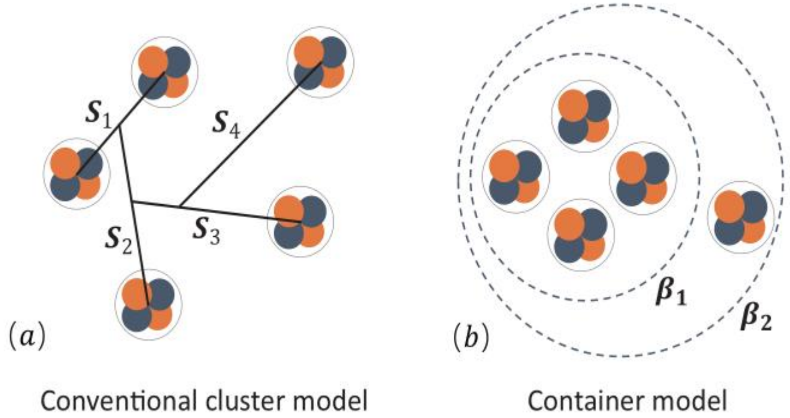




5α BEC state ?

The 5alpha microscopic wave function

To solve the configurations problem:



| **Schematic illustrations of two distinct microscopic cluster models. a**, The conventional cluster model of Φ^{B}, in which the inter-cluster variables $\{S_i\}$ are the Jacobi coordinates of $\{R_i\}$. **b**, Container picture for $4\alpha + \alpha$ cluster structure of ^{20}Ne. The β_1 is the size variable for the description of 4α and β_2 for the description of the relative motion between 4α and α clusters.

$$\Psi(\beta_1,\beta_2) = \int d^3R_1 d^3R_2 d^3R_3 d^3R_4 d^3R_5 \,\mathrm{Exp}\left[-\frac{1/2S_1^2+2/3S_2^2+3/4S_3^2}{\beta_1^2} - \frac{4/5S_4^2}{\beta_2^2}\right]\Phi^{\mathrm{B}}(R_1,R_2,R_3,R_4,R_5) \tag{1}$$

$$= n_0\,\mathscr{A}\left\{\mathrm{Exp}\left[-\frac{2\xi_1^2+8/3\xi_2^2+3\xi_3^2}{2(b^2+2\beta_1^2)}\right]\mathrm{Exp}\left[-\frac{16/5\xi_4^2}{2(b^2+2\beta_2^2)}\right]\prod_{i=1}^{5}\varphi_i^{\mathrm{int}}(\alpha)\right\},$$

where the conventional Brink cluster wave function Φ^{B},

$$\Phi^{\mathrm{B}}(R_1,R_2,R_3,R_4,R_5) = \frac{1}{\sqrt{20!}}\mathscr{A}[\phi_1(R_1)\ldots\phi_5(R_2)\ldots\phi_{20}(R_5)],$$

$$\propto \phi_g\,\mathscr{A}\left\{\mathrm{Exp}\left[-\frac{2(\xi_1-S_1)^2+8/3(\xi_2-S_2)^2+3(\xi_3-S_3)^2}{2b^2}\right]\mathrm{Exp}\left[-\frac{16/5(\xi_4-S_4)^2}{2b^2}\right]\prod_{i=1}^{5}\varphi_i^{\mathrm{int}}(\alpha)\right\}, \tag{4}$$

with the single-nucleon wave function,

$$\phi_i(R_k) = \left(\frac{1}{\pi b^2}\right)^{\frac{3}{4}} e^{-\frac{1}{2b^2}(r_i-R_k)^2}\chi_i\tau_i.$$

Three-body effective interaction

To solve the interaction problem:

The Hamiltonian for ^{20}Ne in this work can be written as:

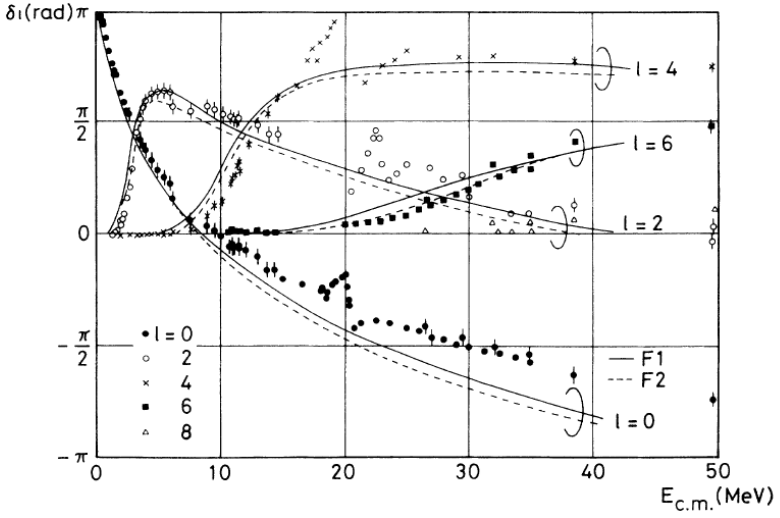
$$\mathcal{H} = -\frac{\hbar^2}{2M}\sum_i \nabla_i^2 - T_G + \sum_{i<j} V_{ij}^C + \sum_{i<j} V_{ij}^{(2)} + \sum_{i<j<k} V_{ijk}^{(3)},$$

The effective nucleon-nucleon potential part is taken a Gaussian form, which is expressed as:

$$V_{ij}^{(2)} = \sum_n v_n^{(2)} \exp\left\{-\left(\frac{r_{ij}}{r_n^{(2)}}\right)^2\right\}(W_n^{(2)} + M_n^{(2)} P_{ij})$$

and

$$V_{ijk}^{(3)} = \sum_n v_n^{(3)} \exp\left\{-\left(\frac{r_{ij}}{r_n^{(3)}}\right)^2 - \left(\frac{r_{jk}}{r_n^{(3)}}\right)^2\right\}$$
$$\times (W_n^{(3)} + M_n^{(3)} P_{ij})(W_n^{(3)} + M_n^{(3)} P_{jk}),$$



Tohsaki F1 three-body interaction was used.

A. Tohsaki, Phys. Rev. C **49**, 1814 (1994).

Radius-Constraint Method + Stabilization Method

To solve the resonance problem:

Radius-Constraint Method,

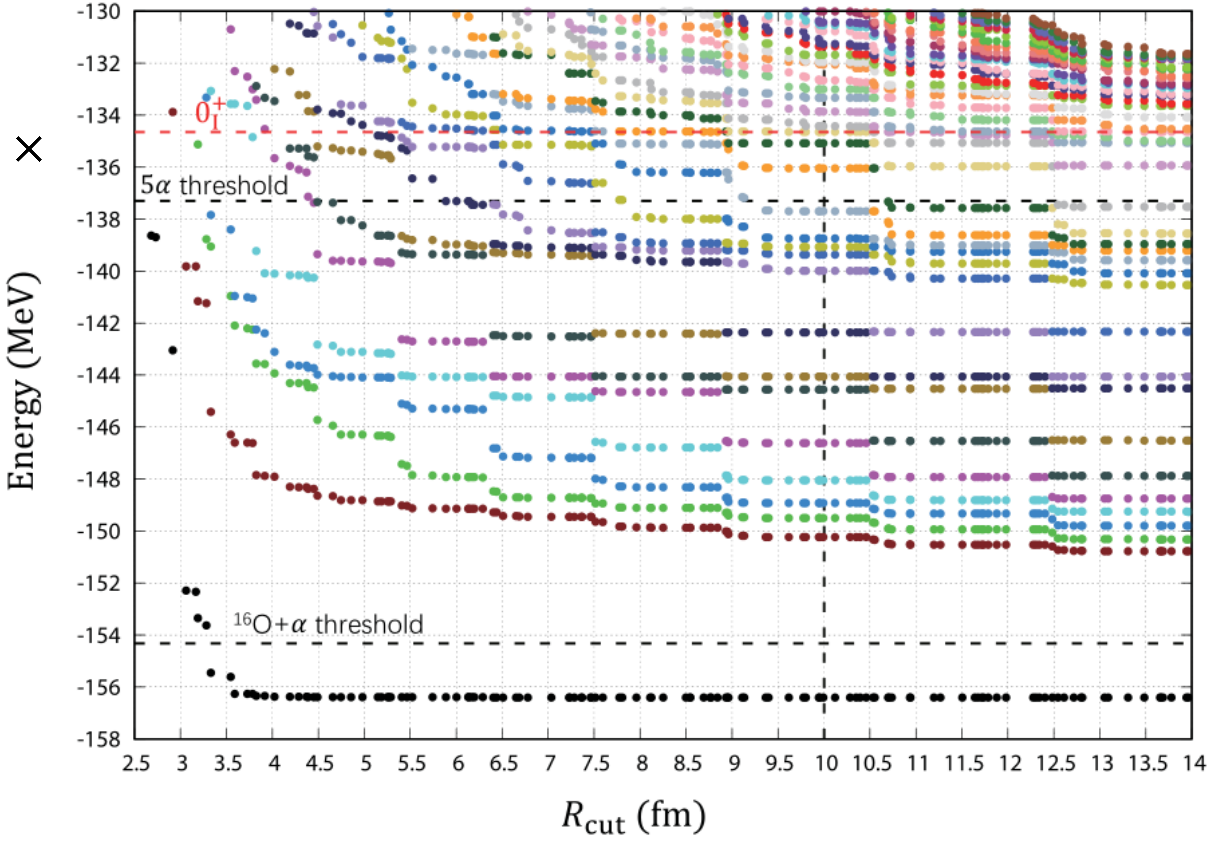
$$\sum_{\beta_1^{'},\beta_2^{'}} \left\langle \widehat{\Phi}^{0^+}_{4\alpha+\alpha}(\beta_1,\beta_2) \middle| \sum_{i=1} \frac{1}{20}(r_i - X_G)^2 \middle| \widehat{\Phi}^{0^+}_{4\alpha+\alpha}(\beta_1^{'},\beta_2^{'}) \right\rangle \times g^{(\gamma)}\left(\beta_1^{'},\beta_2^{'}\right)$$

$$= \left\{R^{(\gamma)}\right\} g^{(\gamma)}\left(\beta_1,\beta_2\right) \left\langle \widehat{\Phi}^{0^+}_{4\alpha+\alpha}(\beta_1,\beta_2) \middle| \widehat{\Phi}^{0^+}_{4\alpha+\alpha}(\beta_1^{'},\beta_2^{'}) \right\rangle$$

$$\Psi^{0^+}_{GCM} = \sum_{\beta_1,\beta_2} g^{(\gamma)}\left(\beta_1,\beta_2\right) \widehat{\Phi}^{0^+}_{4\alpha+\alpha}(\beta_1^{'},\beta_2^{'})$$

Here ,$R^{(\gamma)} \leq R_{cut}$ and R_{cut} is the radius cut-off parameter.

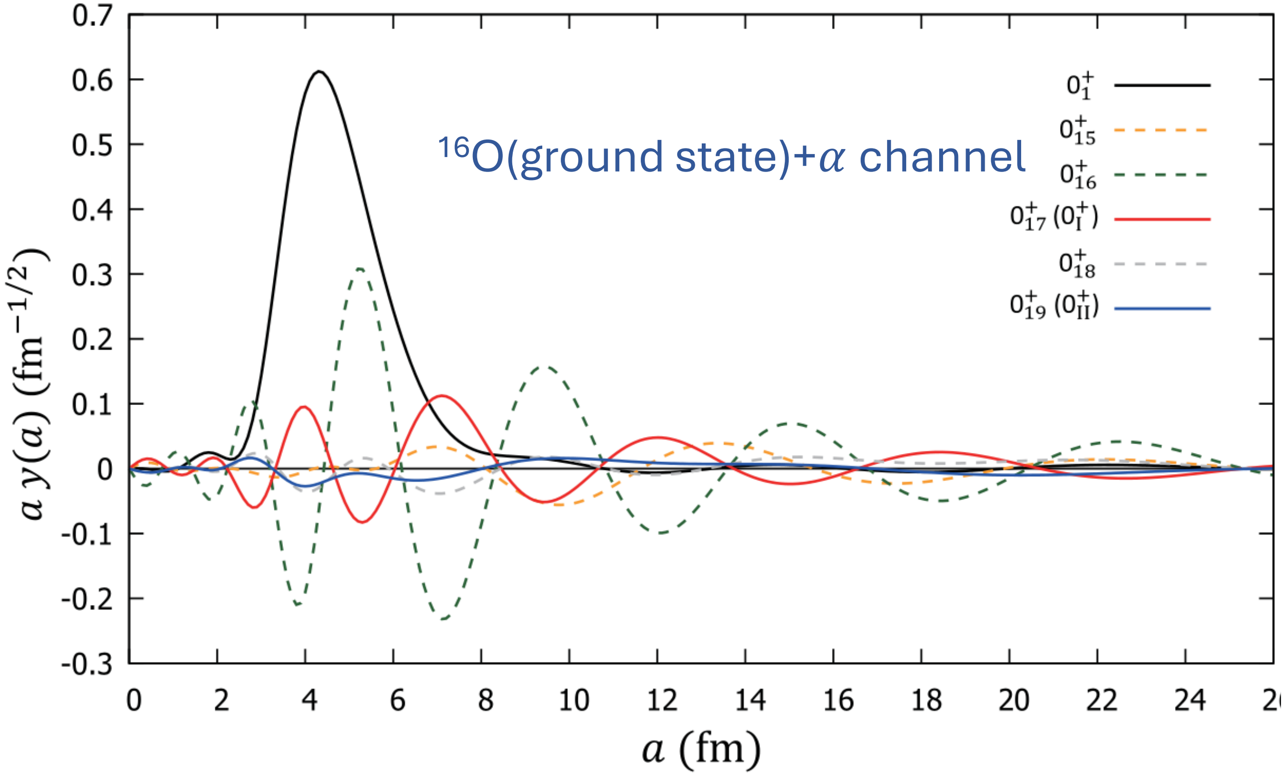
Stabilization Method

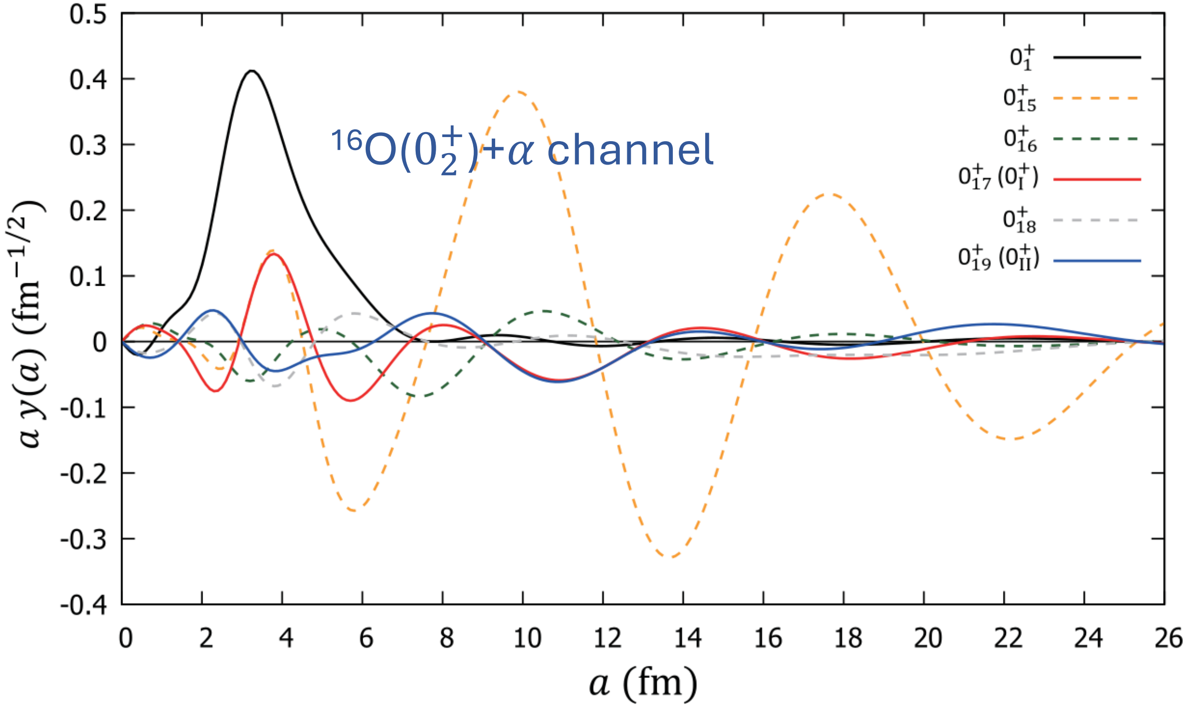


Above the 5alpha threshold: $\mathbf{0^+_{15}} \sim \mathbf{0^+_{19}}$

Five 0^+ states above the threshold

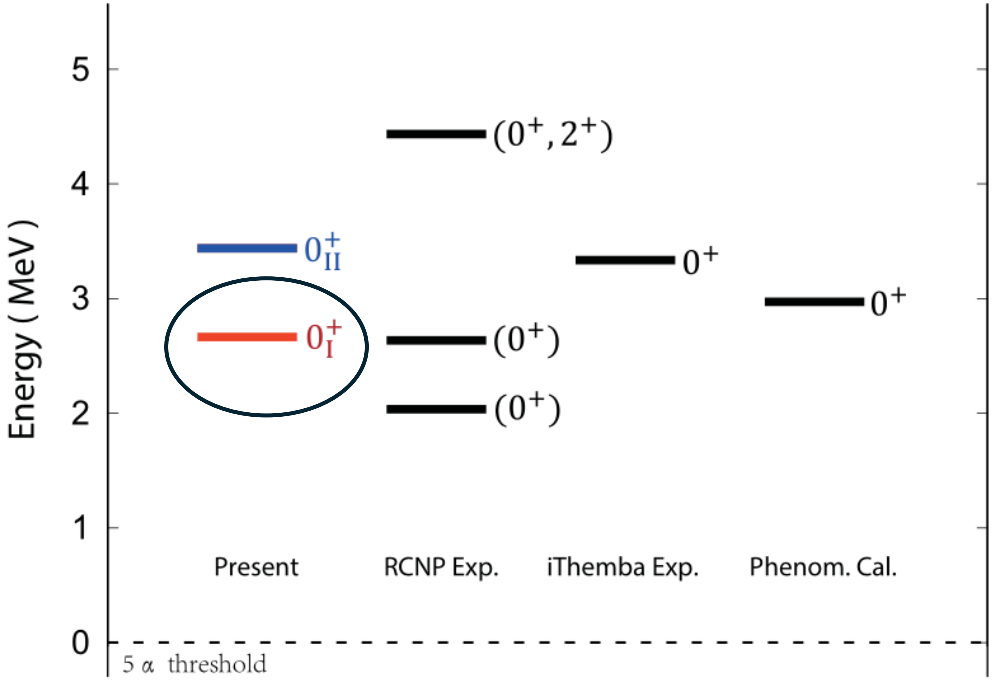
To solve the resonance problem



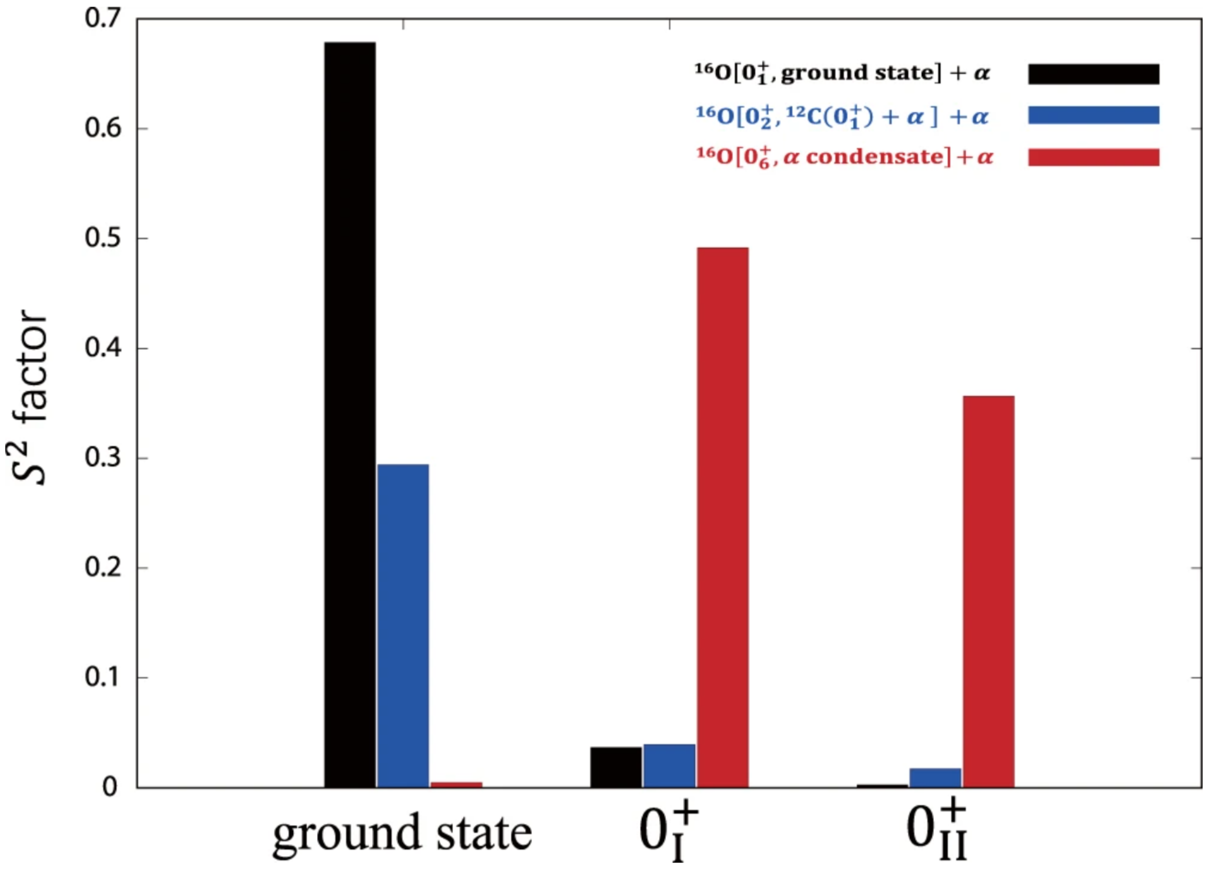


$$y(a) = \sqrt{\frac{20!}{4!16!}} \left\langle \left[\left[\Psi^{0_s^+}_{\mathrm{gcm}}(^{16}\mathrm{O})\varphi_5(\alpha)\right]_{0^+} Y_{00}(\hat{\xi}_4)\right]_{0^+} \frac{\delta(\xi_4 - a)}{\xi_4^2} \middle| \Psi^{0_\lambda^+}_{\mathrm{gcm}}(^{20}\mathrm{Ne})\right\rangle$$

Five 0^+ states above the threshold



Two 0^+ states around 3 MeV are found.

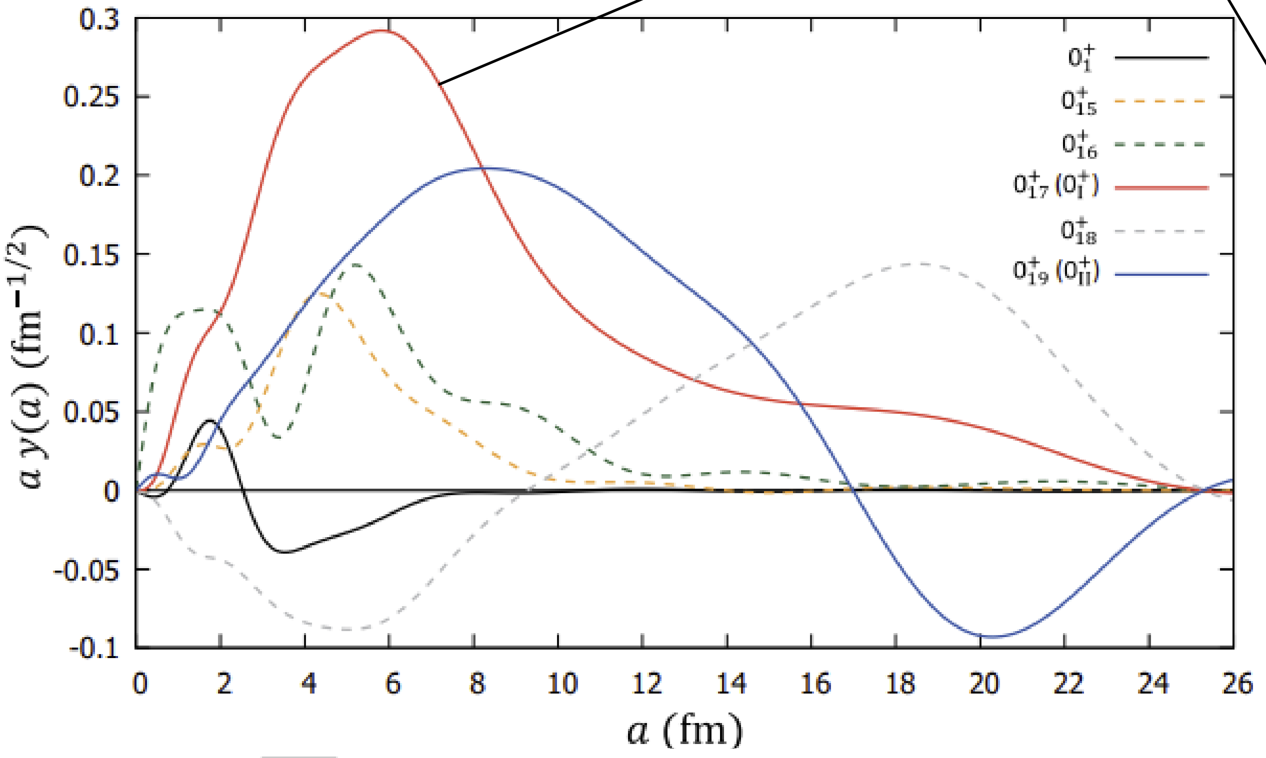


B. Zhou, Y. Funaki, H. Horiuchi, Y-G Ma, G. Röpke, P. Schuck, A. Tohsaki & T. Yamada, *Nat. Commun.* **14**, 8206 (2023).

The 5alpha condensate state

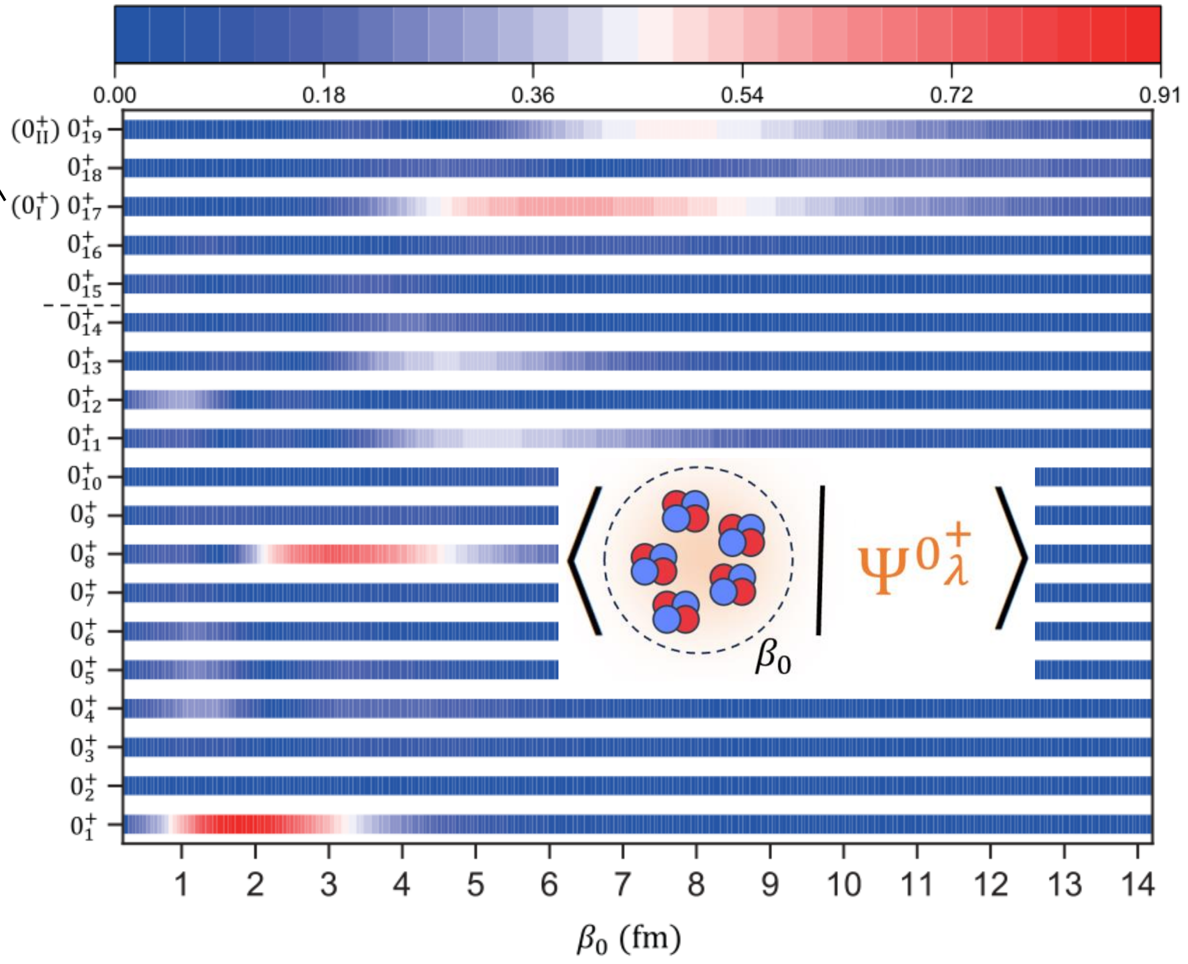
larger amplitude

Reduced width amplitude



$$y(a) = \sqrt{\frac{20!}{4!16!}} \left\langle \left[\left[\Psi^{0_s^+}_{\mathrm{gcm}}(^{16}\mathrm{O}) \varphi_5(\alpha) \right]_{0^+} Y_{00}(\hat{\xi}_4) \right]_{0^+} \frac{\delta(\xi_4 - a)}{\xi_4^2} \middle| \Psi^{0_\lambda^+}_{\mathrm{gcm}}(^{20}\mathrm{Ne}) \right\rangle$$

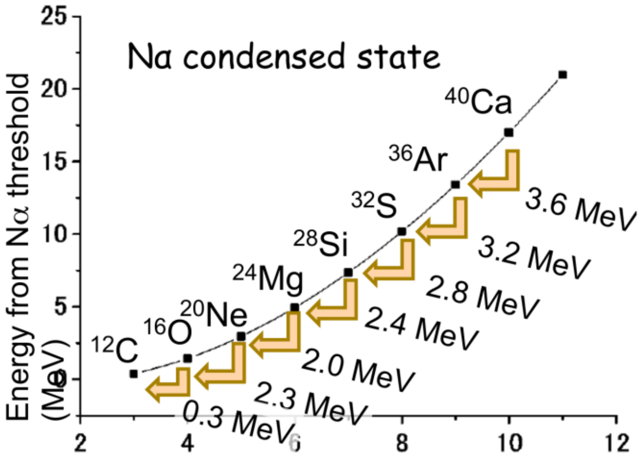
Simple way to confirm condensate state



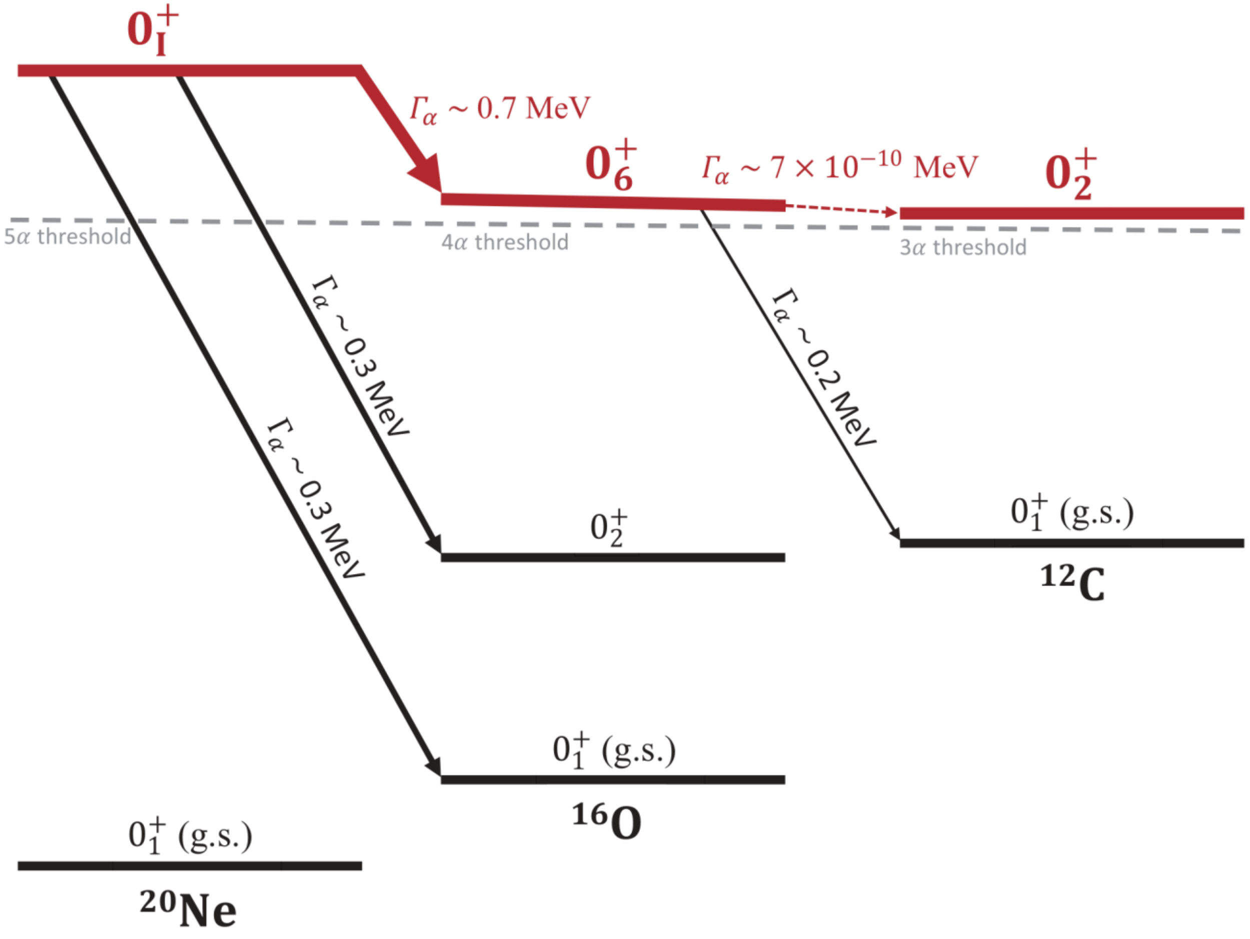
B. Zhou, Y. Funaki, H. Horiuchi, Y-G Ma, G. Röpke, P. Schuck, A. Tohsaki & T. Yamada, *Nat. Commun.* **14**, 8206 (2023).

The decay scheme and connections

Exotic clustering structure ?



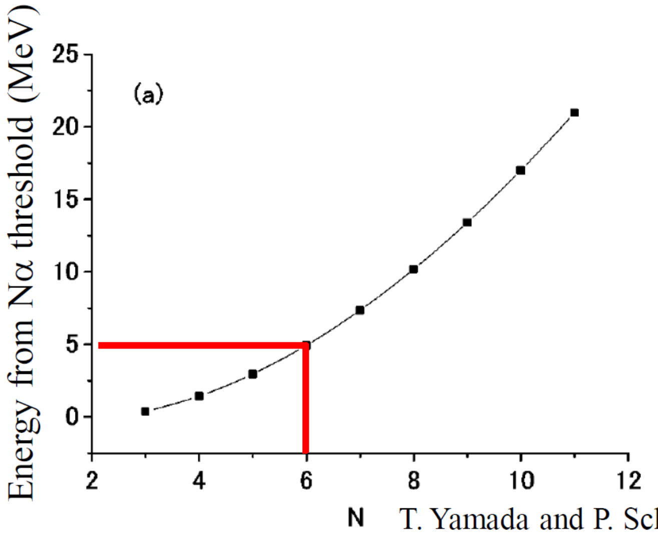
T. Yamada and P. Schuck, PRC 69, 024309 (2004).



B. Zhou, Y. Funaki, H. Horiuchi, Y-G Ma, G. Röpke, P. Schuck, A. Tohsaki & T. Yamada, *Nat. Commun.* **14**, 8206 (2023).

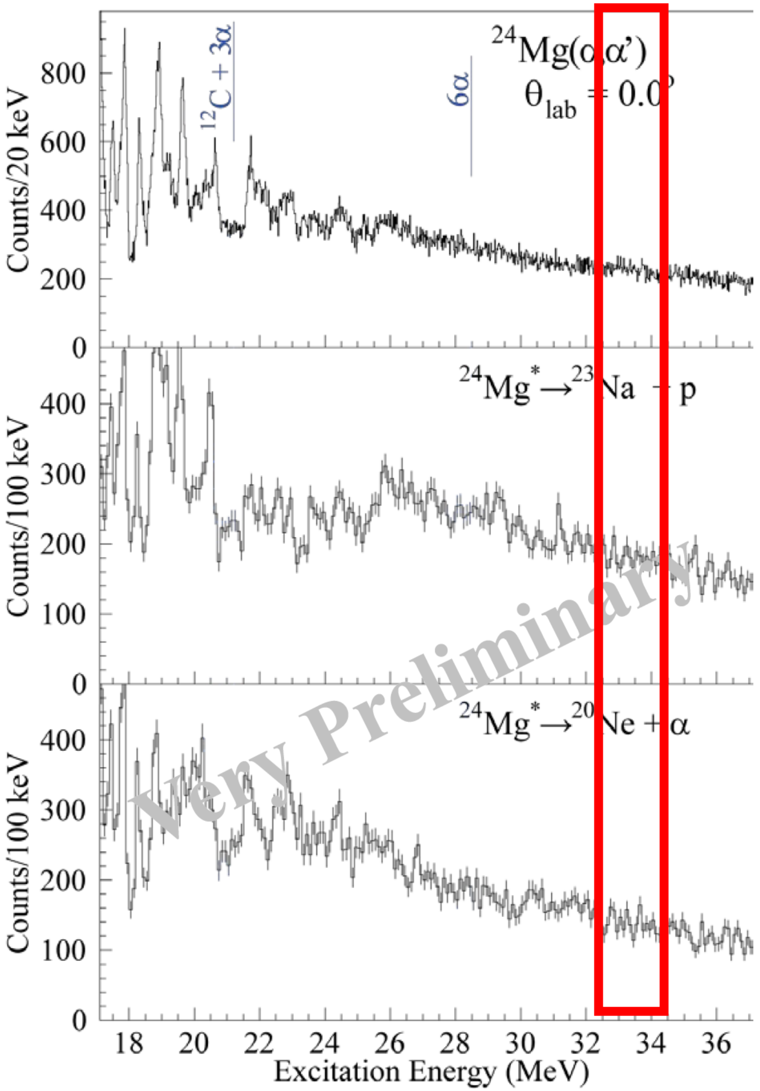
The 6α clustering structure probed by Inelastic Scattering

6α condensed state was searched for in the highly excited region.



T. Yamada and P. Schuck,
Phys. Rev. C **69**, 024309 (2004).

- 6α condensed state is expected at 5 MeV above the 6α threshold.
  - $E_x \sim 28.5 + 5 = 33.5$ MeV
- No significant structure suggesting the 6α condensed state.
  - Several small structures indistinguishable from the statistical fluctuation. → Need more statistics.



PLB,848 (2024)

The 6α THSR calculations

A. Tohsaki et al. / Nuclear Physics A738 (2004) 259–263 261

Table 1
The independent number of permutations for each kernel. Here, the case of the norm kernel for ^{24}Mg is added. The final row shows a full number of permuations without any reduction for the norm kernel.

	^{8}Be(2α)	^{12}C(3α)	^{16}O(4α)	^{20}Ne(5α)	^{24}Mg(6α)
norm	3	9	35	185	1614
kinetic	7	34	242	2546	
two-body	9	58	669	10912	
three-body	40	366	6773	156617	
$(n!)^4$	16	1296	3.32×10^5	2.07×10^8	2.79×10^{11}

$$\left\langle \Psi_{n\alpha}^{\mathrm{THSR}}(\beta)\left|\mathcal{O}\right|\Psi_{n\alpha}^{\mathrm{THSR}}(\beta')\right\rangle = \sum_{p=0}^{m_p^{(1)}-1} W_p^{(1)} I_p^{(1)} = (a_0 a_0')^{-3n/2} \sum_{l=0} t^l \sum_{m=n_p} \gamma_l^{(1)} x^m$$

Neutron Pairs Condense in Excited Helium-8

Yoshiko Kanada-En'yo originally proposed the dineutron condensate of ^{8}He (Phys. Rev. C **88**, 034321 (2013))

PHYSICAL REVIEW LETTERS **131**, 242501 (2023)

Editors' Suggestion   Featured in Physics

Observation of the Exotic 0_2^+ Cluster State in ^{8}He

Z. H. Yang,[1,2,*,†] Y. L. Ye,[1,*,‡] B. Zhou,[3,4,5] H. Baba,[2] R. J. Chen,[6] Y. C. Ge,[1] B. S. Hu,[1] H. Hua,[1] D. X. Jiang,[1] M. Kimura,[2,5,7] C. Li,[2] K. A. Li,[6] J. G. Li,[1] Q. T. Li,[1] X. Q. Li,[1] Z. H. Li,[1] J. L. Lou,[1] M. Nishimura,[2] H. Otsu,[2] D. Y. Pang,[8] W. L. Pu,[1] R. Qiao,[1] S. Sakaguchi,[2,9] H. Sakurai,[2] Y. Satou,[10] Y. Togano,[2] K. Tshoo,[10] H. Wang,[2,11] S. Wang,[2] K. Wei,[1] J. Xiao,[1] F. R. Xu,[1] X. F. Yang,[1] K. Yoneda,[2] H. B. You,[1] and T. Zheng[1]

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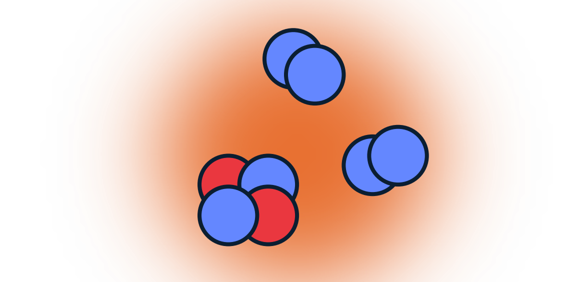
[3]*Key Laboratory of Nuclear Physics and Ion-beam Application (MOE), Institute of Modern Physics, Fudan University, Shanghai 200433, China*
[4]*Shanghai Research Center for Theoretical Nuclear Physics, NSFC and Fudan University, Shanghai 200438, China*
[5]*Department of Physics, Hokkaido University, 060-0810 Sapporo, Japan*
[6]*Institute of Modern Physics, Chinese Academy of Science, Lanzhou 730000, China*



0_2^+ state

$$\Phi(\boldsymbol{B}, b_n) \propto \mathcal{A}\left\{\exp\left[-\frac{4\xi_1^2}{3\boldsymbol{B}^2} - \frac{3\xi_2^2}{2\boldsymbol{B}^2}\right] \times \phi_\alpha(b_\alpha)\phi_{{}^2_1n}(b_n)\phi_{{}^2_2n}(b_n)\right\},$$

(new trial wave function)

PTEP   Prog. Theor. Exp. Phys. **2018**, 041D01 (10 pages)
DOI: 10.1093/ptep/pty034

Letter

New trial wave function for the nuclear cluster structure of nuclei

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Received December 5, 2017; Revised February 21, 2018; Accepted March 2, 2018; Published April 16, 2018

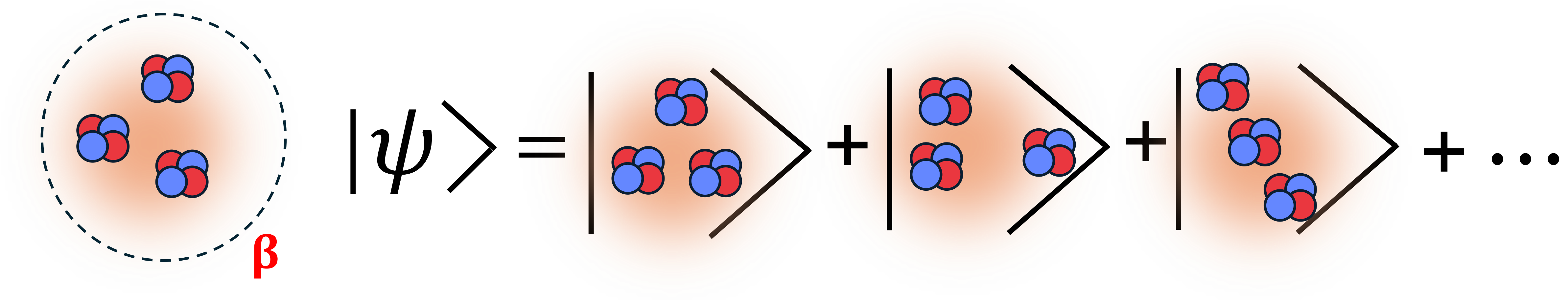
A new trial wave function is proposed for nuclear cluster physics, in which an exact solution to the long-standing center-of-mass problem is given. In the new approach, the widths of the

$$\Psi(\mathbf{r}) = \Phi_{\mathrm{g}}(\mathbf{r}_g)\Phi_{\mathrm{int}}(\mathbf{r}_i - \mathbf{r}_j)$$

$$\begin{aligned}
\Psi_{\mathrm{new}} &= \widehat{L}_{n-1}(\boldsymbol{\beta})\widehat{G}_n(\beta_0)\widehat{D}(\boldsymbol{Z})\Phi_0(\boldsymbol{r}) \\
&= \int d^3\widetilde{T}_1 \cdots d^3\widetilde{T}_{n-1} \exp\left[-\sum_{i=1}^{n-1}\frac{\widetilde{T}_i^2}{\beta_i^2}\right] \int d^3R_1 \cdots d^3R_n \exp\left[-\sum_{i=1}^{n}\left(\frac{A_i}{\beta_0^2 - 2b_i^2}\right)(\boldsymbol{R}_i - \boldsymbol{Z}_i - \boldsymbol{T}_i)^2\right]\Phi_0(\boldsymbol{r} - \boldsymbol{R}) \\
&= n_0 \exp\left[-\frac{A}{\beta_0^2}X_g^2\right] \mathcal{A}\left\{\prod_{i=1}^{n-1}\exp\left[-\frac{1}{2B_i^2}(\boldsymbol{\xi}_i - \boldsymbol{S}_i)^2\right]\prod_{i=1}^{n}\phi_i^{\mathrm{int}}(b_i)\right\}.
\end{aligned}$$

a tool for studying the cluster correlations

Nuclear clustering structure $\Longleftrightarrow$ High-energy nuclear collisions



light nuclei, ground and excited states, superposed many clustering configurations

Summary and Prospect

- There are rich clustering structure in light nuclei. The 5α condensate problem.
- Search for the novel clustering states in Nα+X system. （neutron correlations…）

  (Multi-cluster problem, Resonance problem, Reaction problem.)