Sign-inversion of magntovortical charge from gauge invariant thermodynamics

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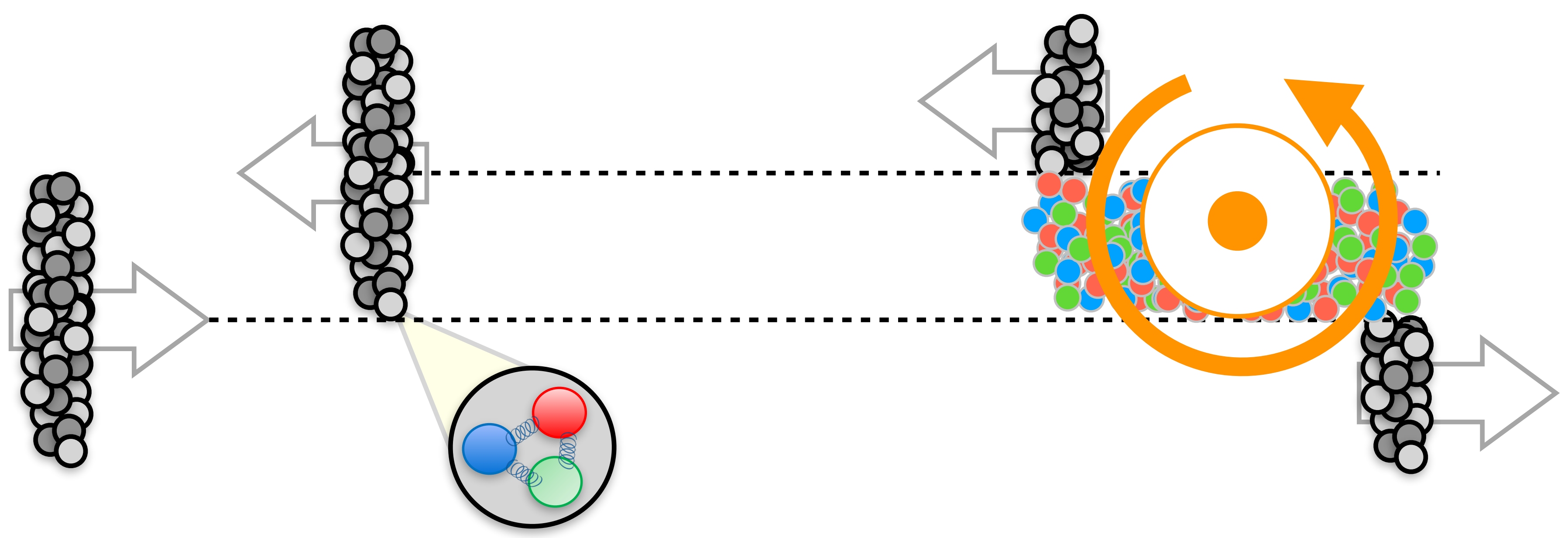
K. Fukushima, K. Hattori and K. Mameda, arXiv:2409.18652

QCD Matter under Rotation

heavy-ion collision

$\vec{\Omega}$

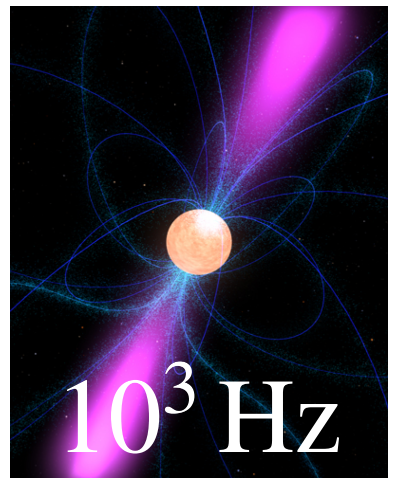


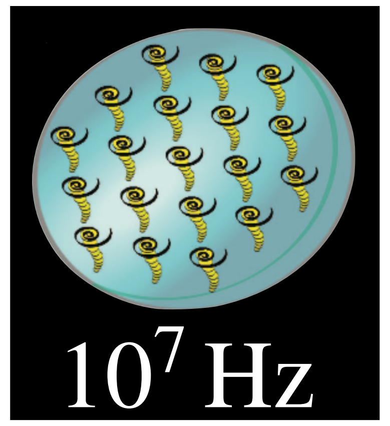
The Fastest Fluid

by Sylvia Morrow

Superhot material spins at an incredible rate.

10^{22} Hz $\ggg$





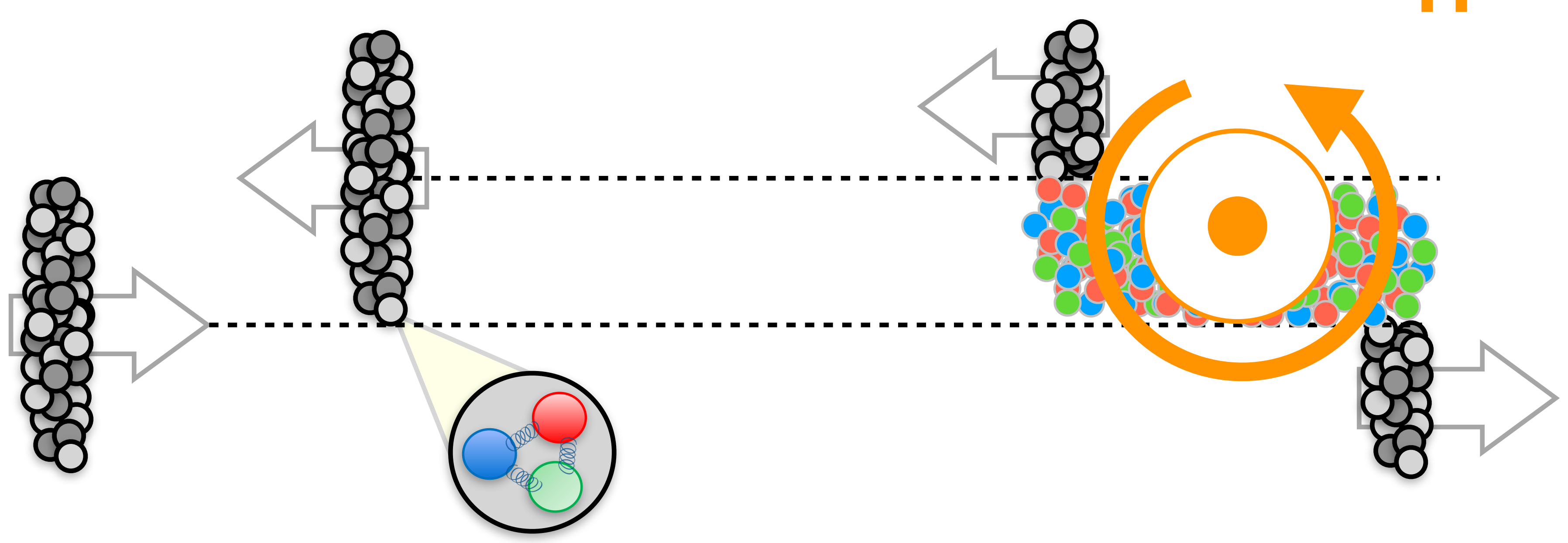
superfluid ^{4}He

✓ $\vec{\Omega}$ is the source of angular momentum

QCD Matter under Rotation and Magnetic field

heavy-ion collision

$\vec{\Omega} \parallel \vec{B}$



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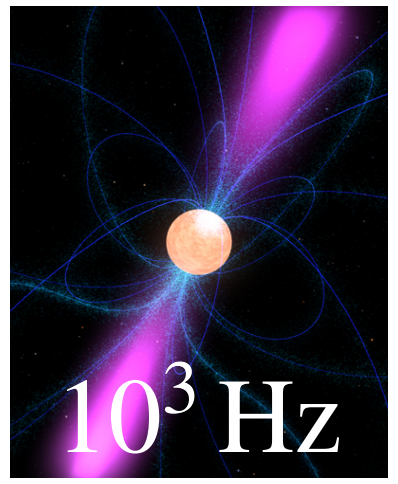
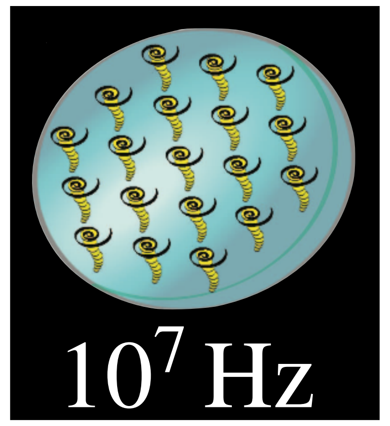
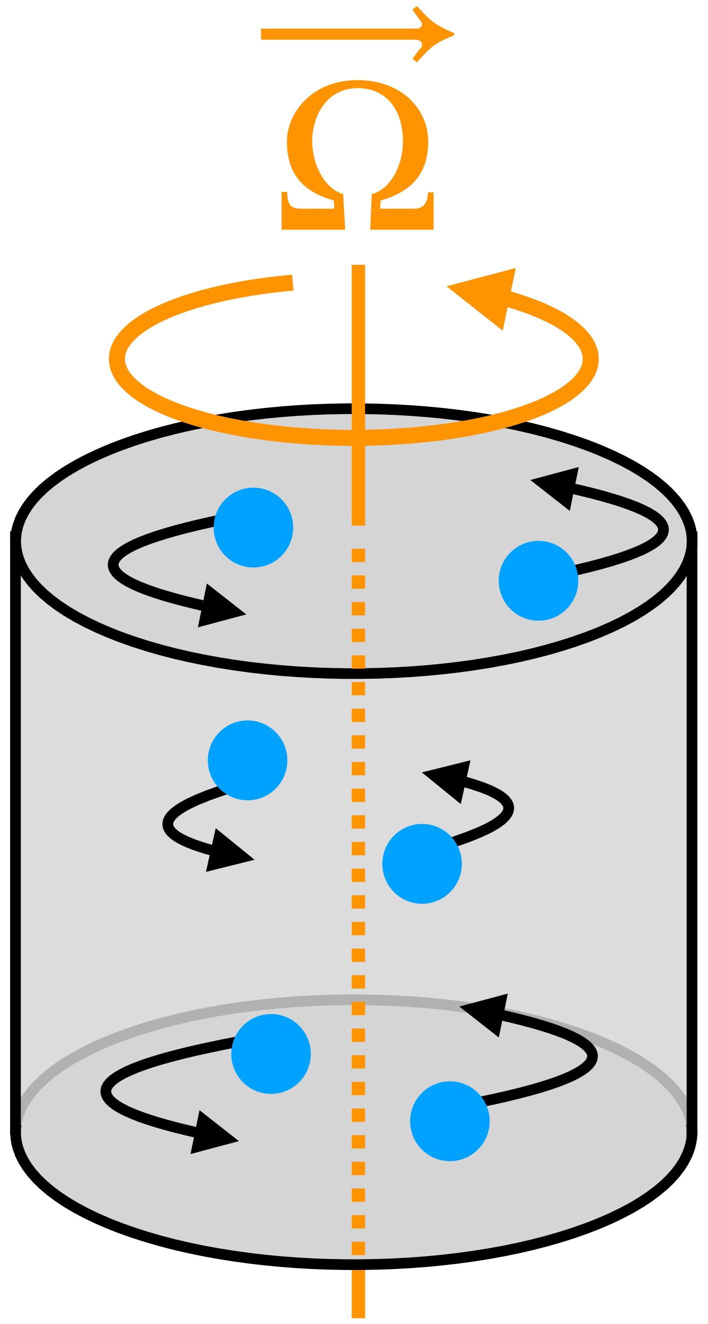




superfluid ^{4}He

- ✔ $\vec{\Omega}$ is the source of angular momentum
- ✔ $\vec{\Omega} \parallel \vec{B}$ is as crucial as either $\vec{\Omega}$ or $\vec{B}$

Early Attempt : Thermodynamics

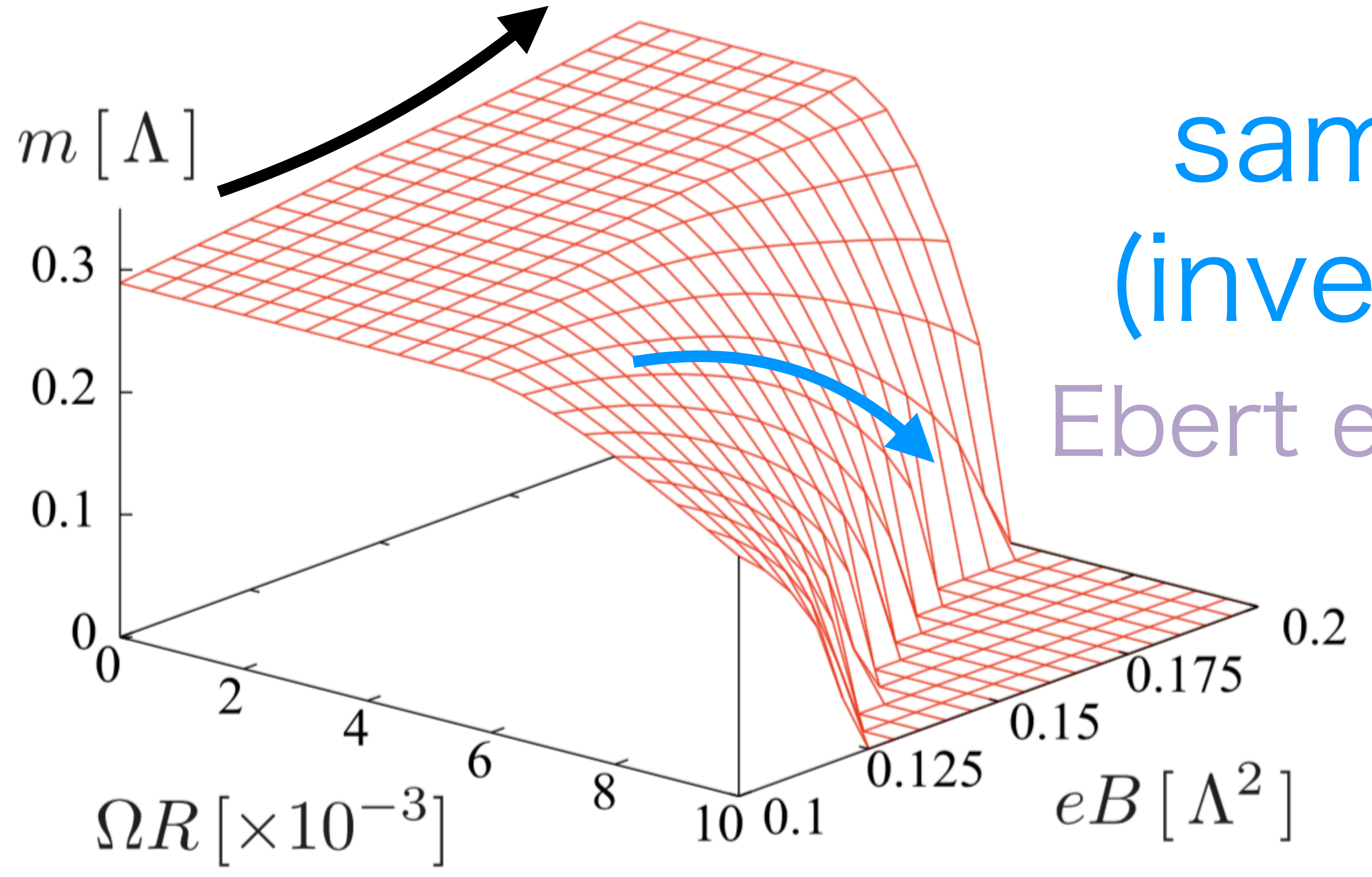


Landau-Lifshitz (1958)
Vilenkin (1979,1980)

$$Z = \mathrm{tr}\exp[-\beta(H - \Omega\mathcal{J})] \longleftrightarrow H - \mu N$$

Chen-Fukushima-Huang-Mameda (2016)

NJL model under $\vec{\Omega} \parallel \vec{B}$



same as μ
(inverse MC)
Ebert et al. (2016)

Early Attempt : Induced Charge

Kubo formula $$\rho = \frac{eB\Omega}{4\pi^2}$$ Hattori-Yin (2016)

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vorticity coupling $E = E_0 - \Omega J$

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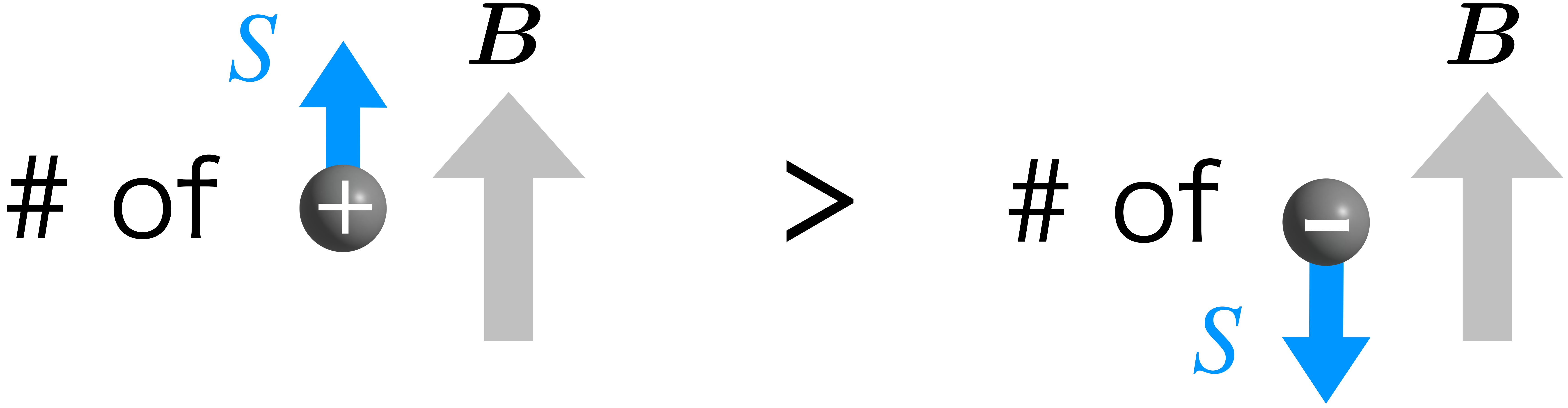


vorticity coupling $$E = E_0 - \Omega J$$

Puzzle on Magnetovortical Charge

Kubo formula

$$\rho = \frac{eB\Omega}{4\pi^2}$$

Hattori-Yin (2016)

Partition function

$$\rho = \frac{eB\Omega}{4\pi^2} + (\text{divergence w.r.t. AM})$$

Chen-Fukushima-Huang-Mameda (2016)

Ebihara-Fukushima-Mameda (2017)

Puzzle on Magnetovortical Charge

Kubo formula

$$\rho = \frac{eB\Omega}{4\pi^2}$$

Hattori-Yin (2016)

Partition function

Answer?

$$\rho = \frac{eB\Omega}{4\pi^2} + (\text{divergence w.r.t. AM})$$

Chen-Fukushima-Huang-Mameda (2016)
Ebihara-Fukushima-Mameda (2017)

Finial Answer

Both of Chen-Fukushima-Huang-Mameda (2016) Hattori-Yin (2016) are incorrect

small eB

$$\rho = \frac{eB\Omega}{4\pi^2}$$

strong eB

$$\rho = -\frac{eB\Omega}{4\pi^2}$$

I will convince you!

Choice of Angular Momenta

$$Z = \mathrm{tr}\left[\mathrm{e}^{-\beta(H-\Omega\mathcal{J})}\right] = \det\left[-\mathrm{i}\gamma^\mu D_\mu + m - \gamma^0\Omega(L+S)\right]$$

Chen-Fukushima-Huang-Mameda (2016)

$$L_{\mathrm{can}} = xp_y - yp_x$$

conserved AM

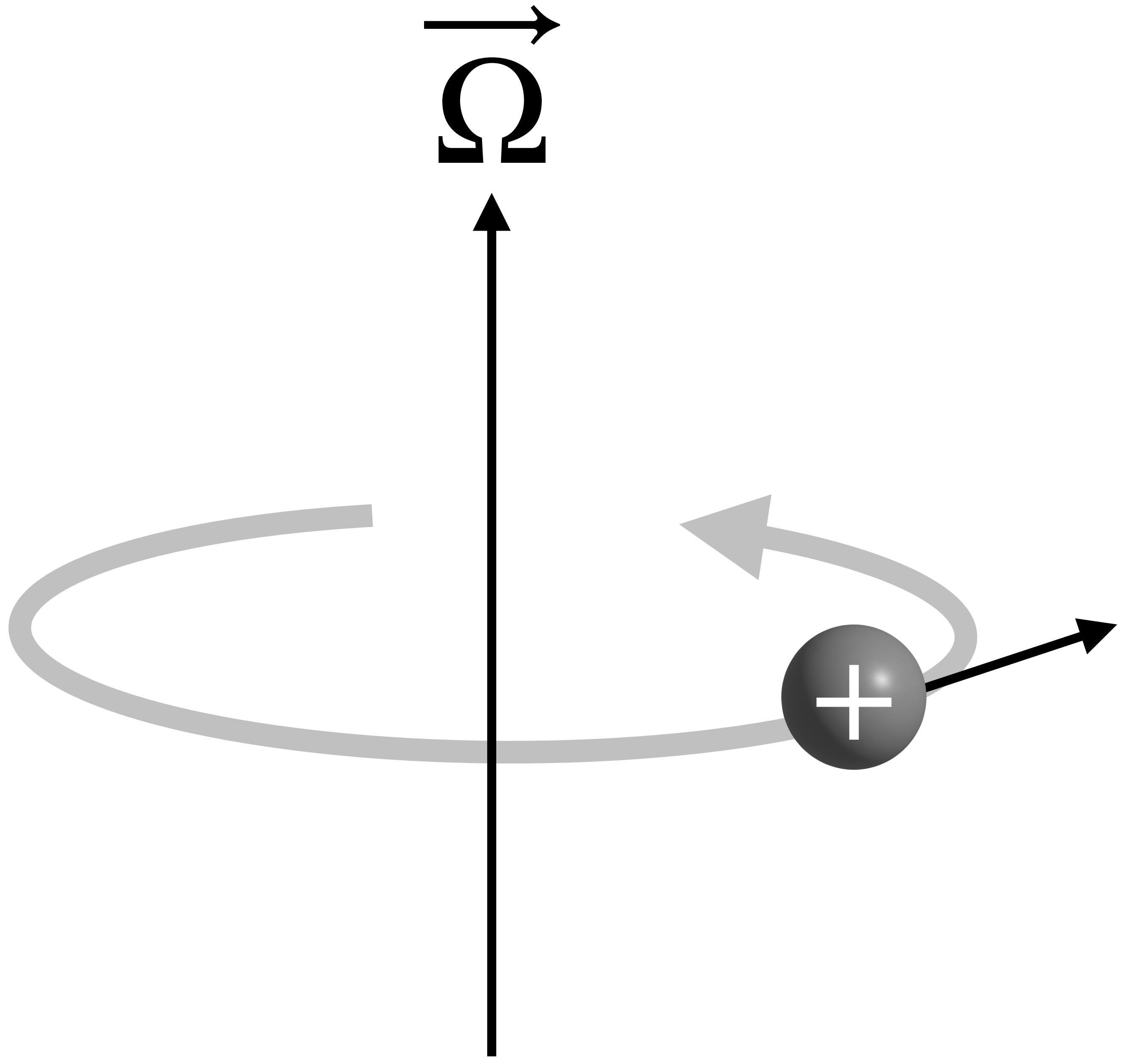
Fukushima-Hattori-Mameda (2024)

$$L_{\mathrm{kin}} = x\Pi_y - y\Pi_x$$

$$\Pi_i = p_i - eA_i$$

gauge invariant AM

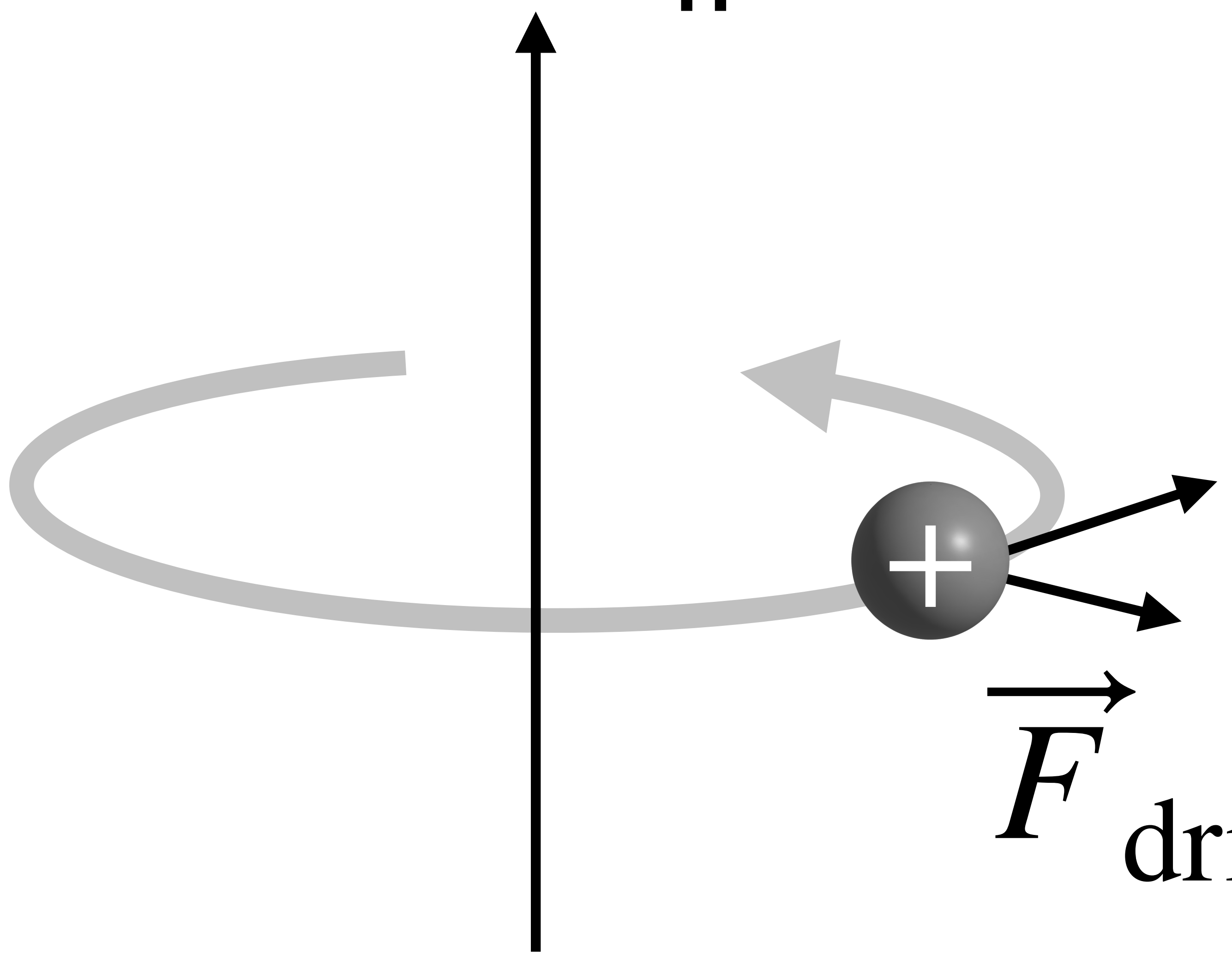
Classical Argument



$\vec{v} = \Omega r \hat{\theta}$

Classical Argument

$$\vec{\Omega} \parallel \vec{B}$$



$$\vec{v} = \Omega r \hat{\theta}$$

$$\vec{F}_{\text{drift}} = eB\Omega\vec{r}$$

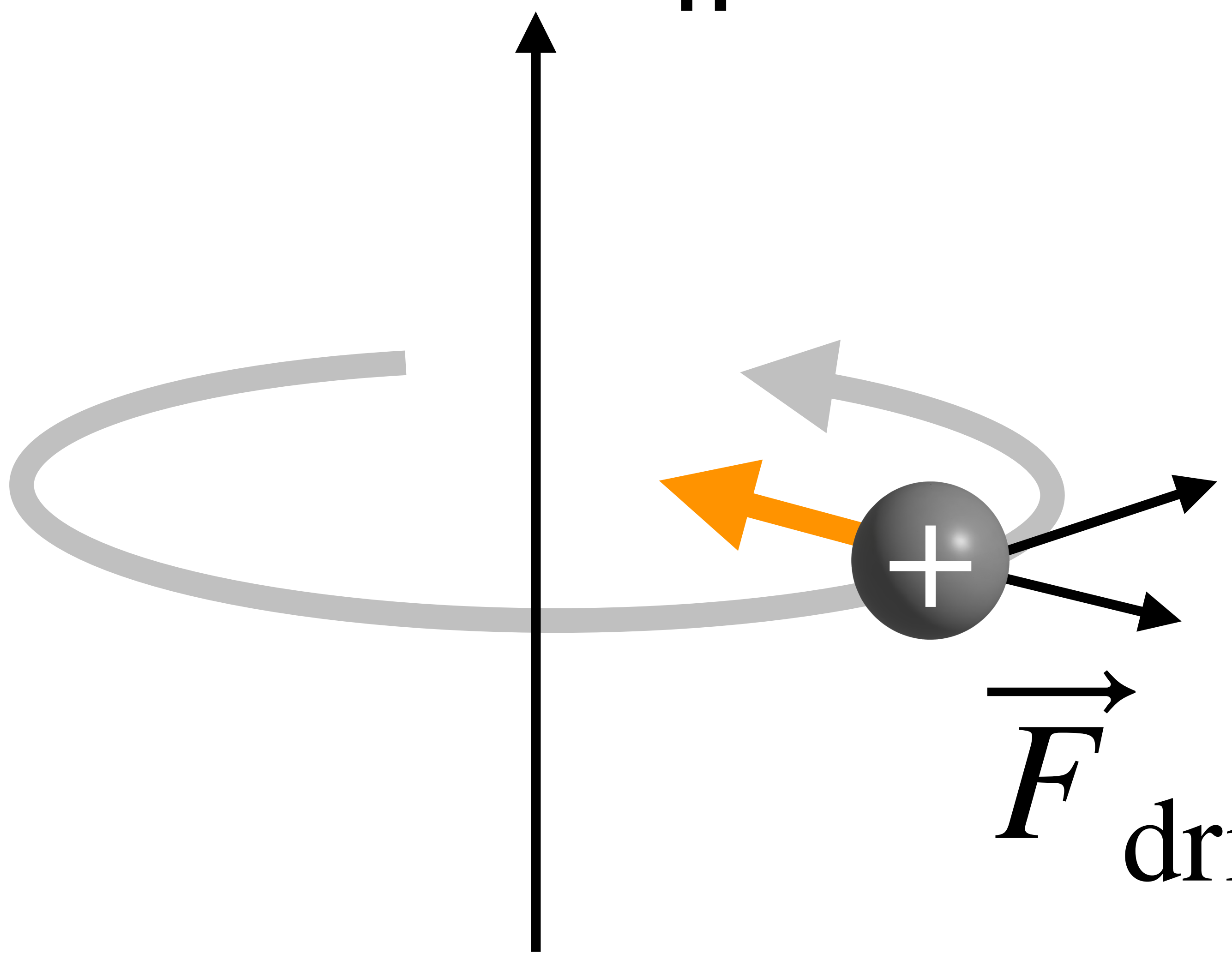
no longer circular

$$H - \Omega L_{\text{can}}$$

unstable

Classical Argument

$$\vec{\Omega} \parallel \vec{B}$$



$$\vec{v} = \Omega r \hat{\theta}$$

$$\vec{F}_{\mathrm{drift}} = eB\Omega\vec{r}$$

$$e\vec{E} = -eB\Omega\vec{r}$$

$$= -\vec{\nabla}[\Omega(L_{\mathrm{can}} - L_{\mathrm{kin}})]$$

$$H + \Omega(L_{\mathrm{can}} - L_{\mathrm{kin}}) - \Omega L_{\mathrm{can}} = H - \Omega L_{\mathrm{kin}}$$ stable

cf. Buzzegoli (2020)

gauge invariance ⟷ thermodynamic stability

Almost Solved?

$$\mathcal{J} = \int_{\boldsymbol{x}} \psi^\dagger (L + S)\psi$$

$$L = x\Pi_y - y\Pi_x$$

gauge invariant AM

free Dirac fermion under B

$$Z = \mathrm{tr}\left[\mathrm{e}^{-\beta(H - \Omega\mathcal{J})}\right]$$

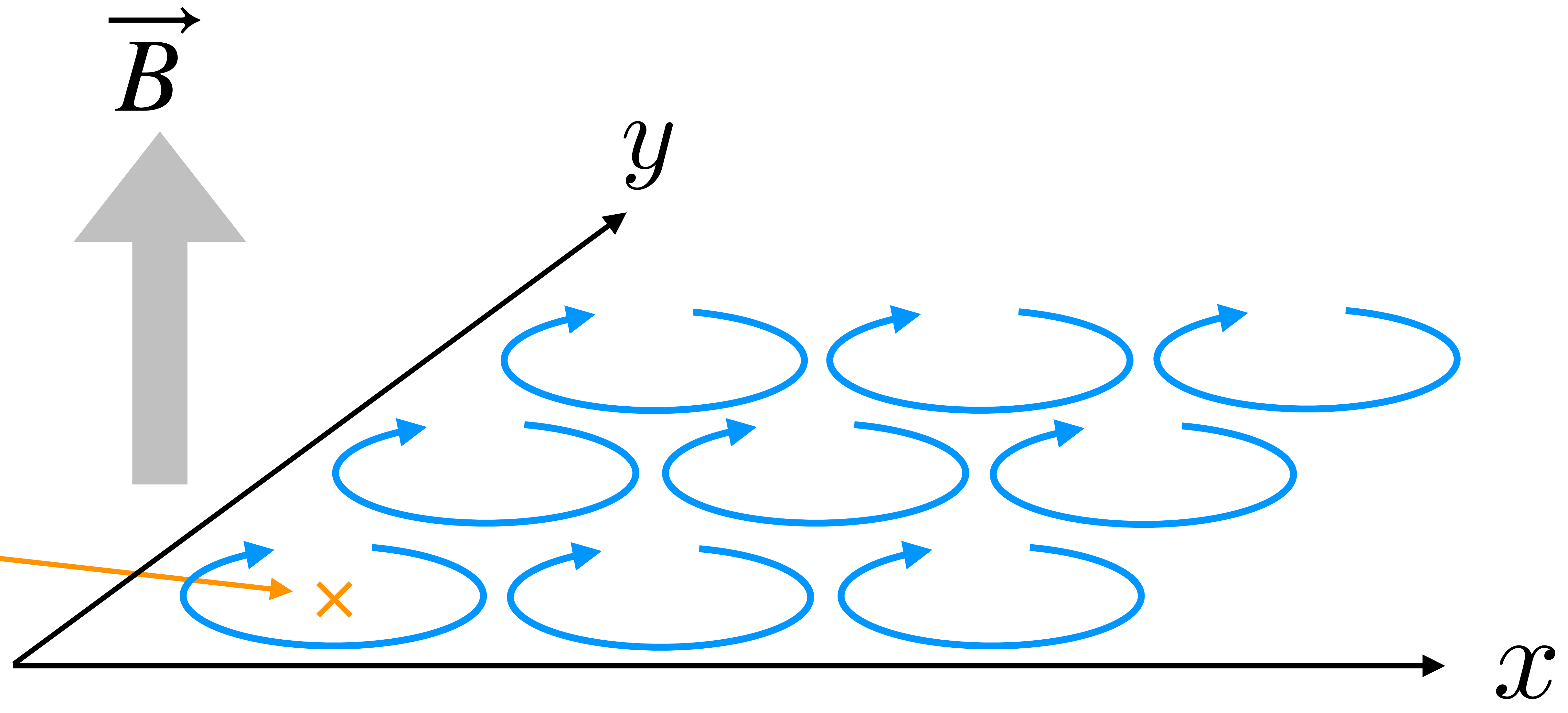
$$= \det\left[-\mathrm{i}\gamma^\mu D_\mu + m - \gamma^0\Omega(L + S)\right]$$

How to diagonalize this?

Landau Level Basis

kinetic momentum $\vec{\Pi} = \vec{p} - e\vec{A}$

guiding center $\vec{X}$



$$a = \frac{1}{\sqrt{2eB}}(\Pi_x + \mathrm{i}\Pi_y) \qquad b = \sqrt{\frac{eB}{2}}(X - \mathrm{i}Y)$$

Landau level basis $|\, n\,, m\,\rangle \propto (a^\dagger)^n\,(b^\dagger)^m\,|0,0\rangle$

kinetic energy

$$\vec{\Pi}^2 = eB(2a^\dagger a + 1)$$

distance from origin

$$\vec{X}^2 = (2b^\dagger b + 1)/eB$$

Angular Momentum

$$L = x\Pi_y - y\Pi_x = -\left(2a^\dagger a + 1\right) + [\text{off-diagonal}]$$

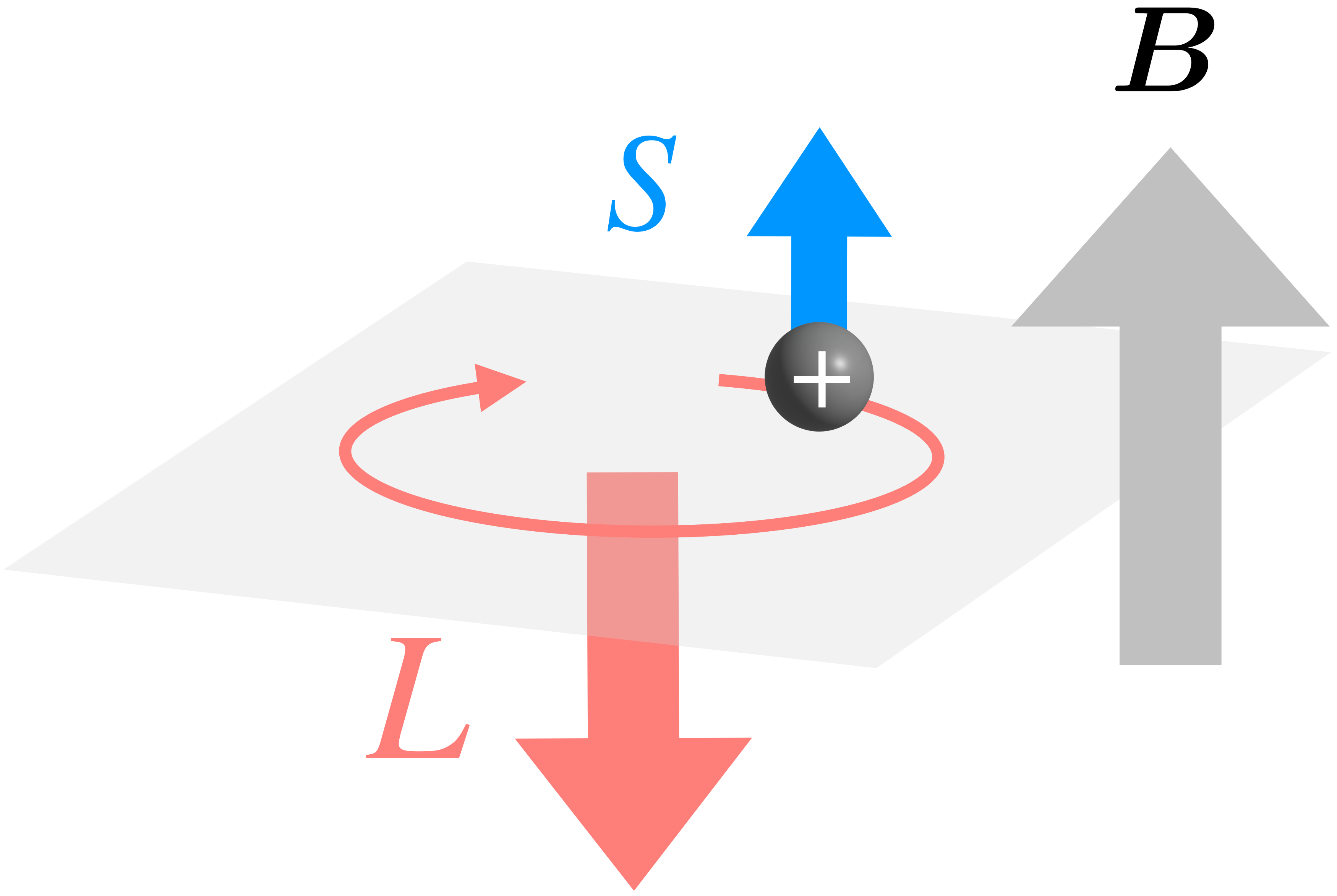
Angular Momentum

$$L = x\Pi_y - y\Pi_x = -\left(2a^\dagger a + 1\right) + [\text{off-diagonal}]$$

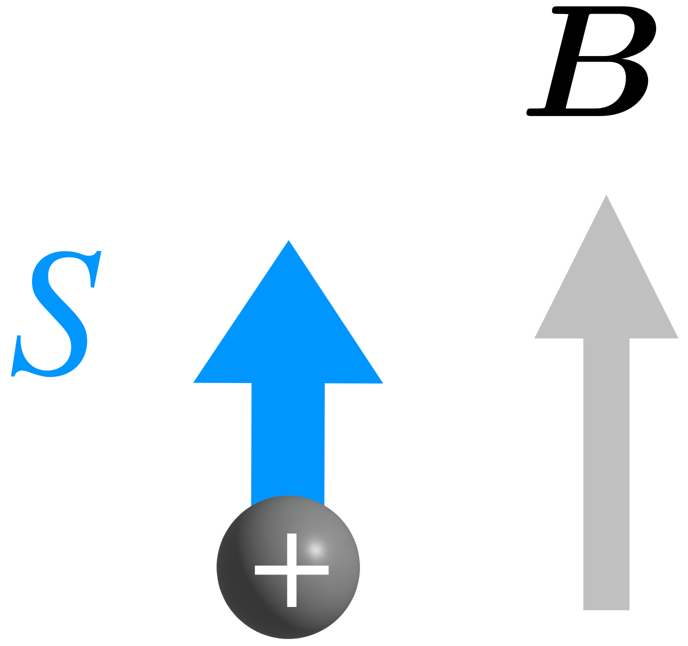
strong eB

$$\langle J\rangle_{\mathrm{LLL}} = \langle S\rangle_{\mathrm{LLL}} + \langle L\rangle_{\mathrm{LLL}} = -1/2$$

$\langle S\rangle_{\mathrm{LLL}}$: $+1/2$ $\langle L\rangle_{\mathrm{LLL}}$: -1



cf. small eB $\langle J\rangle = \langle S\rangle = +1/2$



Partition Function under Strong B

Fukushima-Hattori-Mameda (2024)

$$Z = \det\left[-\mathrm{i}\gamma^\mu D_\mu + m - \gamma^0\Omega(L+S)\right]$$

Not calculable analytically, except for the LLL limit

Partition Function under Strong B

Fukushima-Hattori-Mameda (2024)

$$Z = \det\left[-\mathrm{i}\gamma^\mu D_\mu + m - \gamma^0 \underline{\Omega(L+S)}\right]_{\nu = -\Omega/2 \text{ (LLL)}}$$

Not calculable analytically, except for the LLL limit

$$P_{\mathrm{LLL}} = \frac{eB}{2\pi}\int \frac{\mathrm{d}p_z}{2\pi}\left[\epsilon + T\sum_{\eta=\pm}\ln\left(1+\mathrm{e}^{-\beta(\epsilon-\eta\nu)}\right)\right]$$

massless limit $\rho = \frac{\partial P_{\mathrm{LLL}}}{\partial \nu} = -\frac{eB\Omega}{4\pi^2}$ (T-independent)

It Should Be Negative

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vorticity coupling $E = E_0 - \Omega J$

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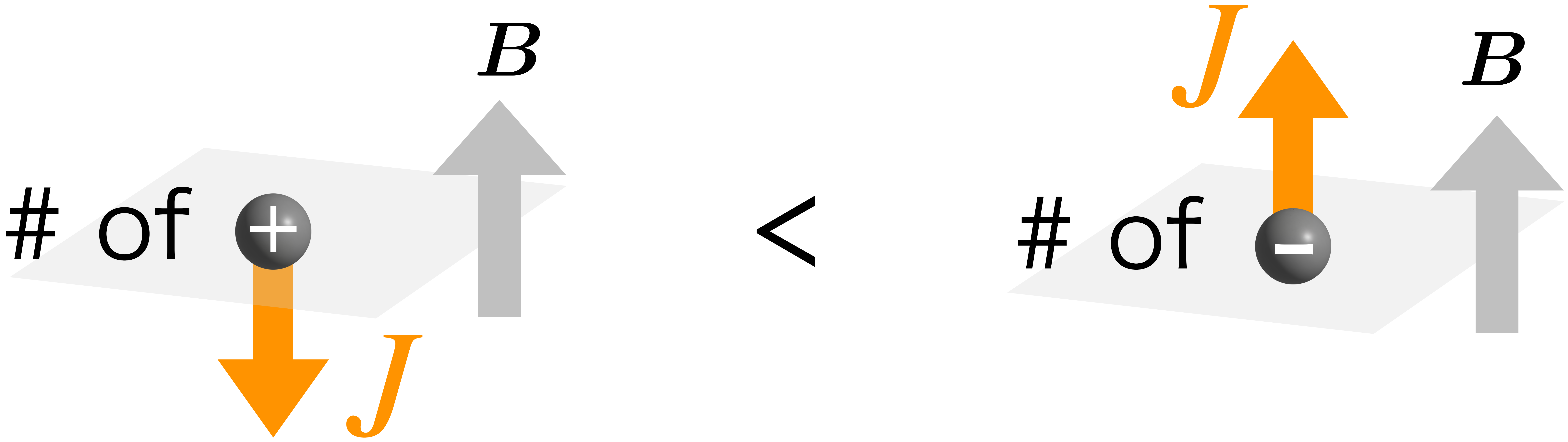

vorticity coupling $E = E_0 - \Omega J$

Comparisons

Fukushima-Hattori-Mameda (2024)
partition function (LLL)

$$\rho = \frac{eB\Omega}{4\pi^2} - \frac{eB\Omega}{2\pi^2}$$

spin orbital

Ebihara-Fukushima-Mameda (2017)
partition function (LLL)
incorrect

$$\rho = \frac{eB\Omega}{4\pi^2} + (\text{divergence w.r.t. AM})$$

due to $\vec{F}_{\text{drift}} = eB\Omega\vec{r}$

Hattori-Yin (2016)
Kubo formula (LLL)
incorrect

$$\rho = \frac{eB\Omega}{4\pi^2}$$

sign-mistake

Yang et. al (2020) Mameda(2023)
chiral kinetic theory
correct

$$\rho = \frac{eB\Omega}{4\pi^2}$$

no Landau level formed by weak B

Relation to Chiral Anomaly

charge $$\rho = \frac{\partial P_{\mathrm{LLL}}}{\partial \mu} = \frac{eB\Omega}{4\pi^2} - \frac{eB\Omega}{2\pi^2} + \frac{eB\mu}{2\pi^2}$$

angular momentum $$J = \frac{\partial P_{\mathrm{LLL}}}{\partial \Omega} = \frac{eB\mu}{4\pi^2} - \frac{eB\mu}{2\pi^2} + \frac{eB\Omega}{8\pi^2}$$

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$$\frac{\partial \rho}{\partial \Omega} = \frac{\partial J}{\partial \mu} = \frac{\partial^2 P_{\rm LLL}}{\partial \mu\, \partial \Omega}$$ →

same coefficients shared

$$\frac{eB}{4\pi^2} \qquad -\frac{eB}{2\pi^2}$$

Relation to Chiral Anomaly

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$$= S = j^5_{\rm CSE}/2$$

$$\frac{\partial \rho}{\partial \Omega} = \frac{\partial J}{\partial \mu} = \frac{\partial^2 P_{\rm LLL}}{\partial \mu\, \partial \Omega}$$ → Since $j^5_{\rm CSE}$ is anomaly-related, so is ρ

cf. Yang-Yamamoto (2021)

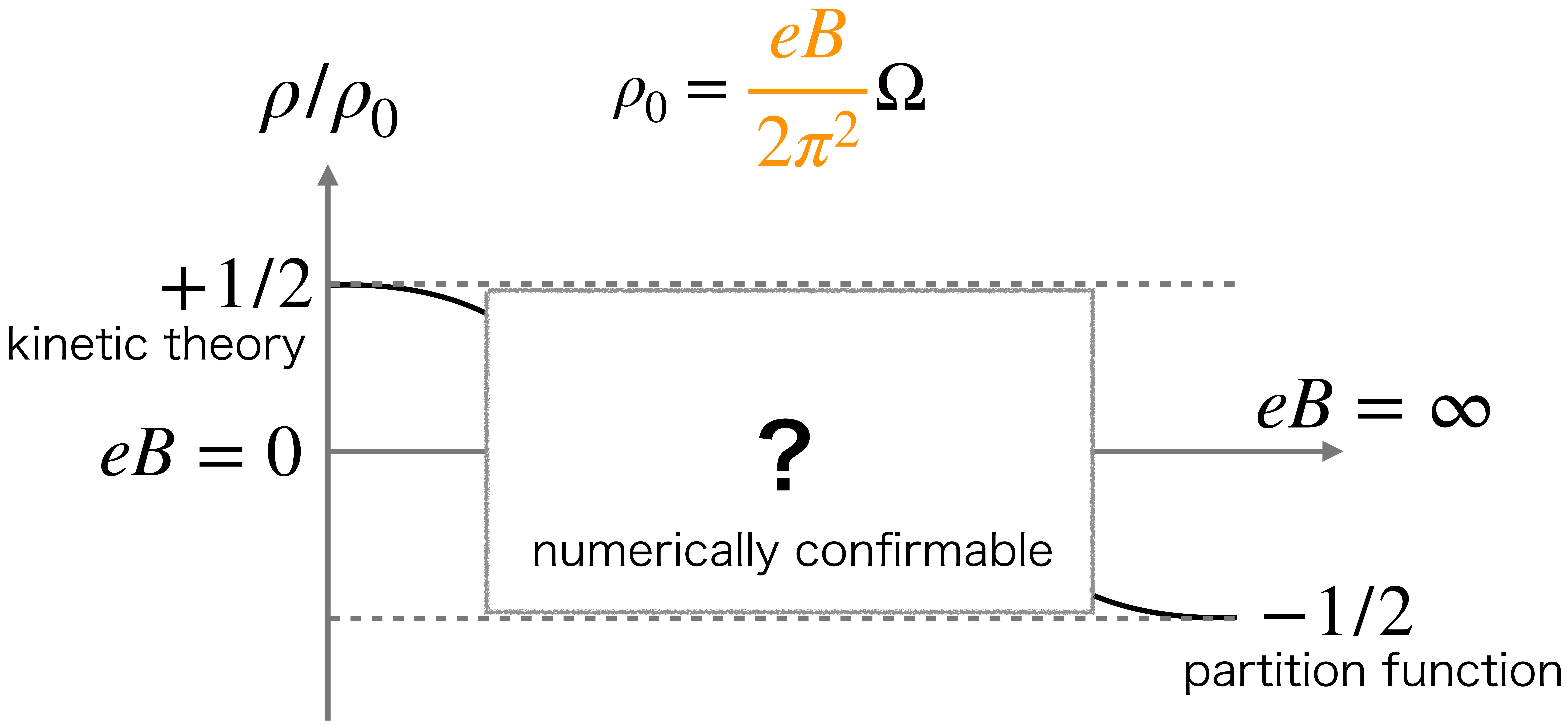
Summary

- ✓ reformulate gauge-invariant and stable thermodynamics
- ✓ Magnetovortical charge sign-inverted by cyclotron motion
- ✓ The charge is anomaly-related
- ✓ applicability to

  HIC : spin polarization under strong B
  cold atoms : quantum simulator
  (nonrelativistic Hamiltonian can be diagonalized)

- ✓ Sign-inversion should takes place at a certain B

Charge Density



Total Angular Momentum