Positivity at (future) colliders

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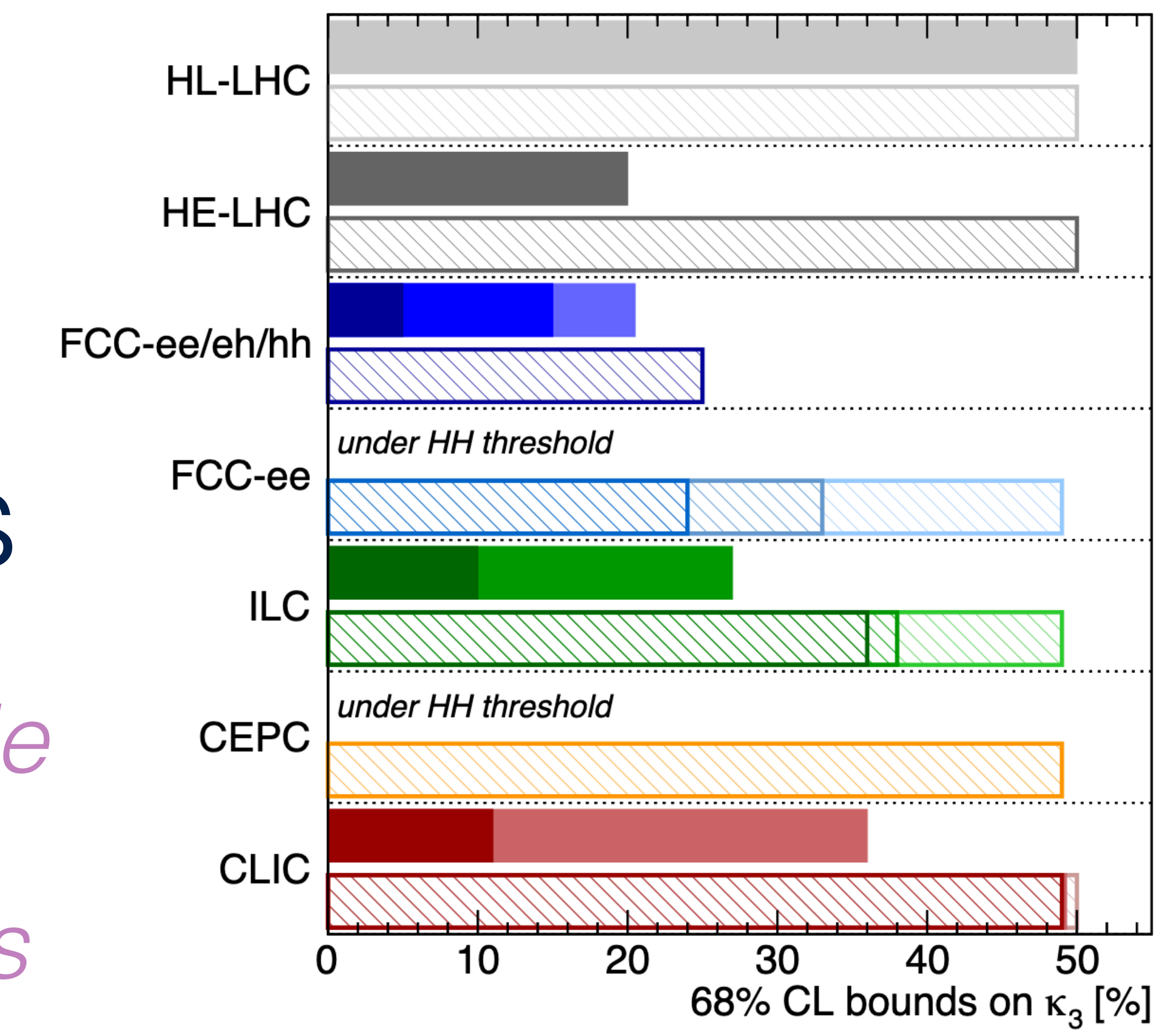
Science and Technology Facilities Council

University of Southampton

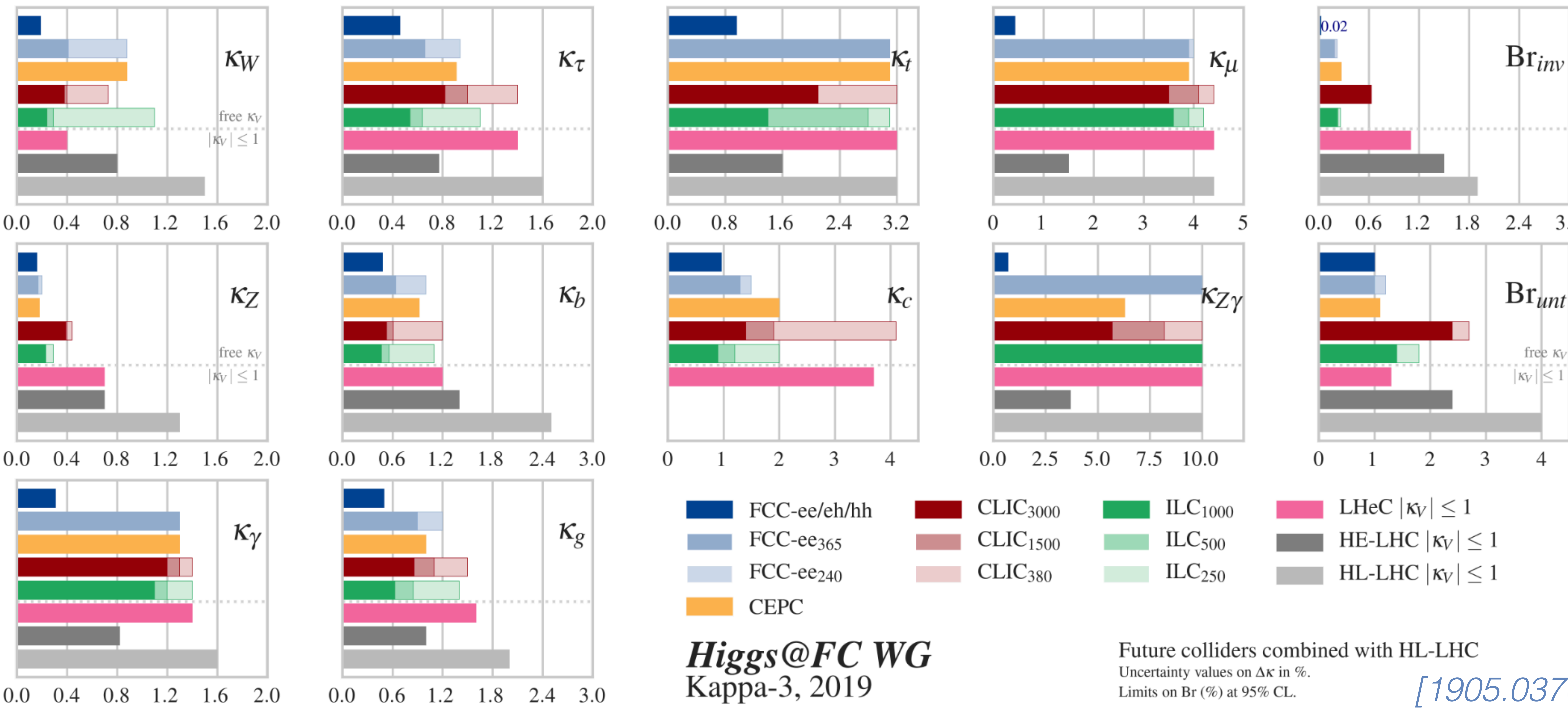
The indirect way



Precision measurements to probe new physics

dynamics of EWSB *Higgs potential/self-coupling* *flavor puzzle*

baryon asymmetry *dark sector* *new forces/matter/Higgses*



[1905.03764]

Modern approach: SMEFT=SM v2.0

$$\mathcal{L} = \mathcal{L}_{\text{SM}} + \sum_{i,D} \frac{c_i^{(D)} \mathcal{O}_i^{(D)}}{\Lambda^{D-4}}$$

BSM particle masses M ⇔ *Generic new physics scale Λ*

Low energy limit of $\mathscr{A}_{BSM}$ ⇔ *Tower of operators $\mathscr{O}_i^{(D)}$*

$\mathscr{O}_i^{(D)} \supset$ SM: $W^{\pm}$, Z, H, t, … — *Low energy (SM) fields & symmetries*

Model parameters $\{g^i_{\text{BSM}}, M_k\}$ ⇔ *Wilson coefficients $\frac{c_j^{(D)}}{\Lambda^{D-4}}(g^i_{\text{BSM}}, M_k)$*

measure g_i : new physics model parameters

"Matching"

measure c_i : coupling strengths of new BSM interactions

Operators ⇔ amplitudes

$$\mathcal{A}^{n}_{\rm BSM}(E,M) \overset{E \ll M}{\sim} E^{4-n}\left(a_0 + a_1\frac{E}{M} + a_2\frac{E^2}{M^2} + \cdots\right), \quad a_i(C_j)$$

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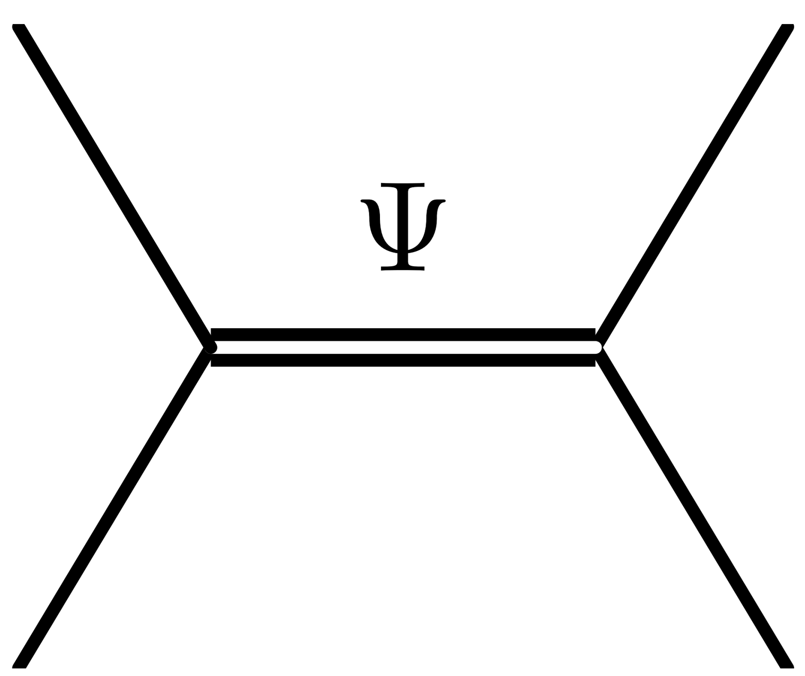

$$\frac{g^2}{p^2 - M^2} \approx -\frac{g^2}{M^2}\left[1 + \frac{p^2}{M^2} + \frac{p^4}{M^4} + \cdots\right]$$

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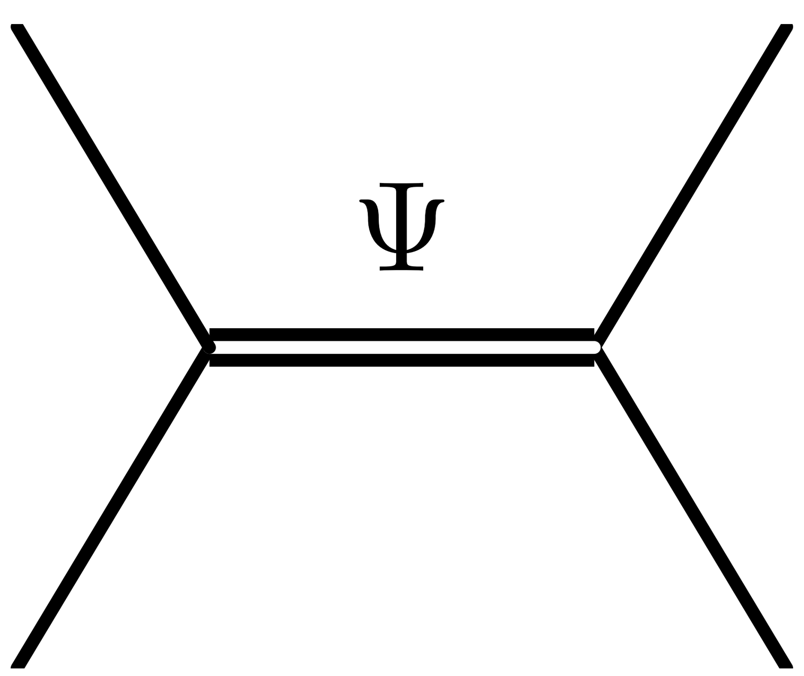

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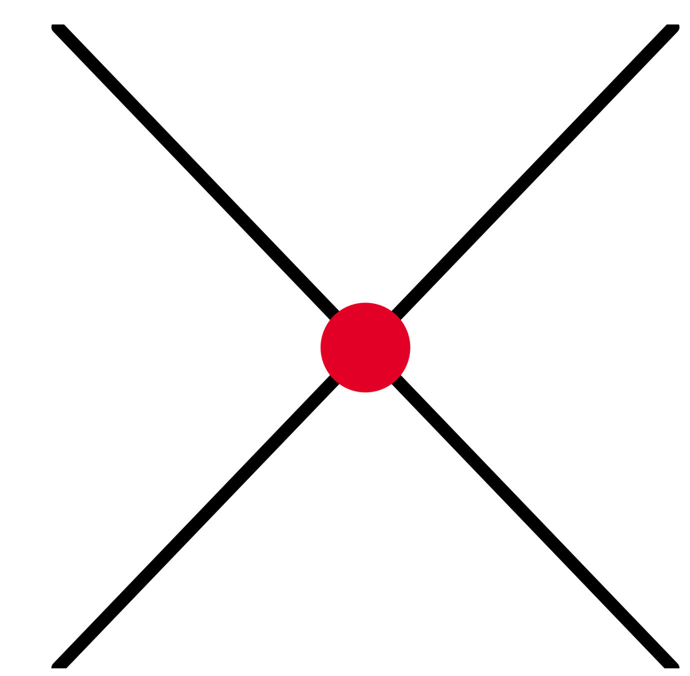

$$\mathscr{L} \sim g\Psi\left(\phi^\dagger\phi\right) \approx \frac{g^2}{M^2}(\phi^\dagger\phi)^2 + \frac{g^2}{M^4}\phi^\dagger\phi\,\partial^\mu\phi^\dagger\partial_\mu\phi + \frac{g^2}{M^6}\partial^\mu\phi^\dagger\partial_\mu\phi\partial^\nu\phi^\dagger\partial_\nu\phi + \cdots$$

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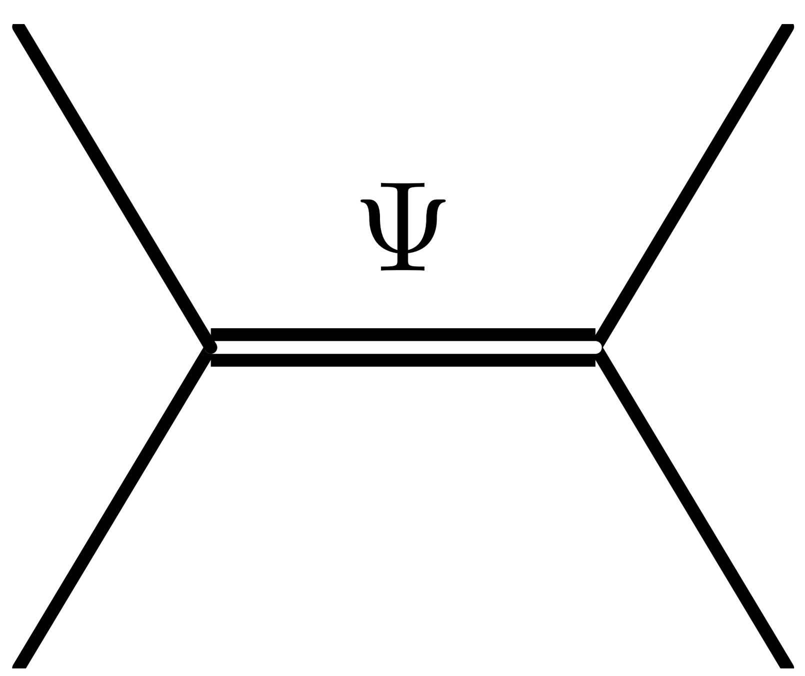

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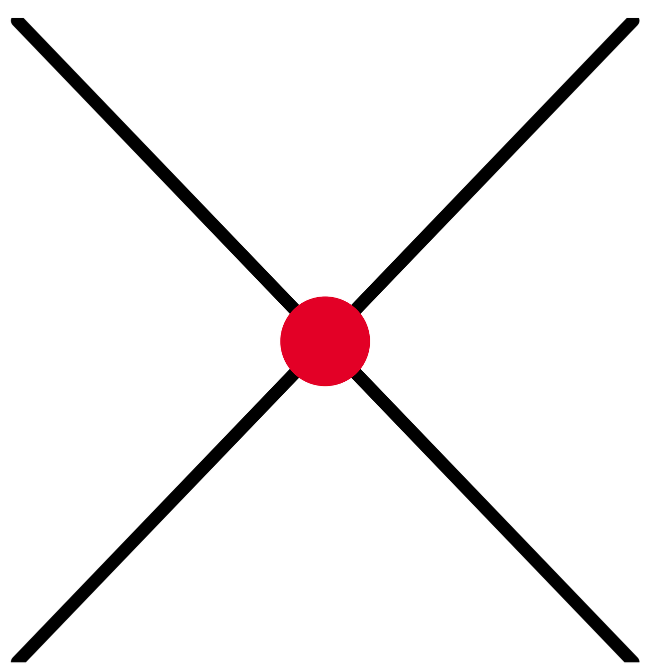

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$ab \to cd$

	s	t	st	s^2	$\cdots$
$\{\mathcal{O}_i\} : C_i \;\Leftrightarrow\; \{\mathcal{A}_j\} :$	a_j^s	a_j^t	a_j^{st}	$a_j^{s^2}$	$\cdots$

Theoretical constraints in the IR

$$\mathscr{A}_{2\to2} = a_0 + a_1^s s + a_1^t t + a_1^u u + a_2^s s^2 + a_2^t t^2 + a_2^u u^2 + a_1^{st} st + \cdots$$

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$$\{C_i(g, M)\} \Rightarrow \{a_j(C_i)\}$$

- Particle content, operator dimension
- Symmetries: gauge, flavor, custodial, CP, $\mathbb{Z}_2$,…
- Linear vs non-linear EWSB

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- Unitarity $\Rightarrow$ $c_i \frac{s^n}{\Lambda^{2n}} \lesssim 8\pi$ *Signal breakdown of the EFT: new resonances*

Theoretical constraints from the UV

What is UV?
assume QFT?
local, causal, unitary?

$$p^2 \ll M_{UV}^2$$

$$\mathscr{A}_{UV} \to \mathscr{A}_{2\to 2}$$

Imprints on the EFT
patterns? restrictions?

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2b) Amplitudes have rules: can dictate $\mathscr{L}_{EFT}$

- Unitarity, locality, causality in the UV
- At fixed t, $\mathscr{A}(s,t)$ is analytic in the complex s plane

Up to poles & branch cuts on real line

- Define 'subtracted' amplitude: $M_{ijkl}(s,t) = \mathscr{A}_{ijkl}(s,t)$ *- low energy discontinuities*

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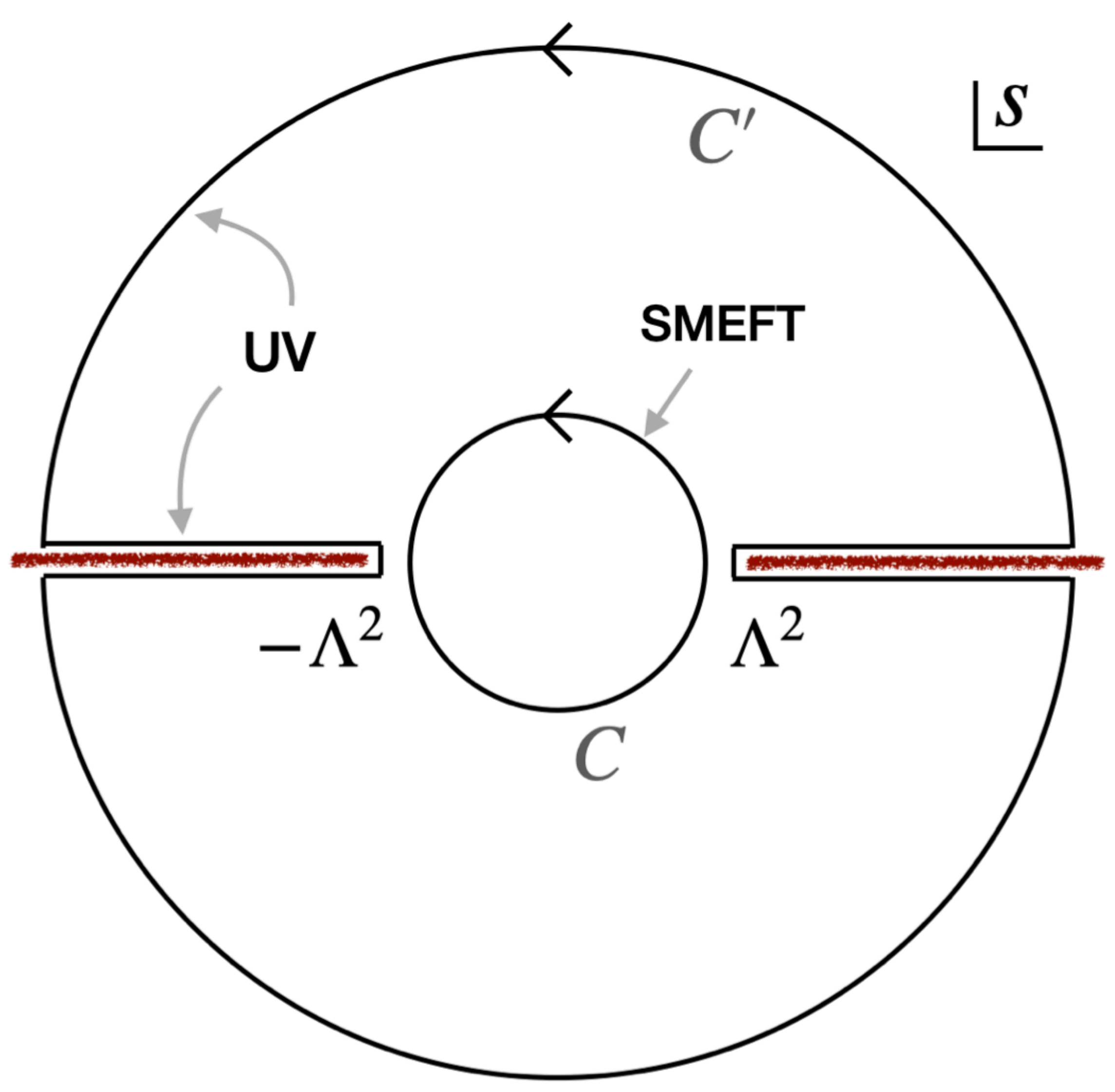

$$\frac{1}{2}\frac{d^2 M_{ijkl}(s)}{ds^2} = \oint_C \frac{d\mu}{2\pi i}\frac{M_{ijkl}(\mu)}{(\mu - s)^3}$$

Cauchy's integral formula avoiding UV branch cuts

$$M \lesssim s \log^2 s, \;\; s \to \infty$$

[Froissart; Phys. Rev. 123 (1961) 1053-1057]

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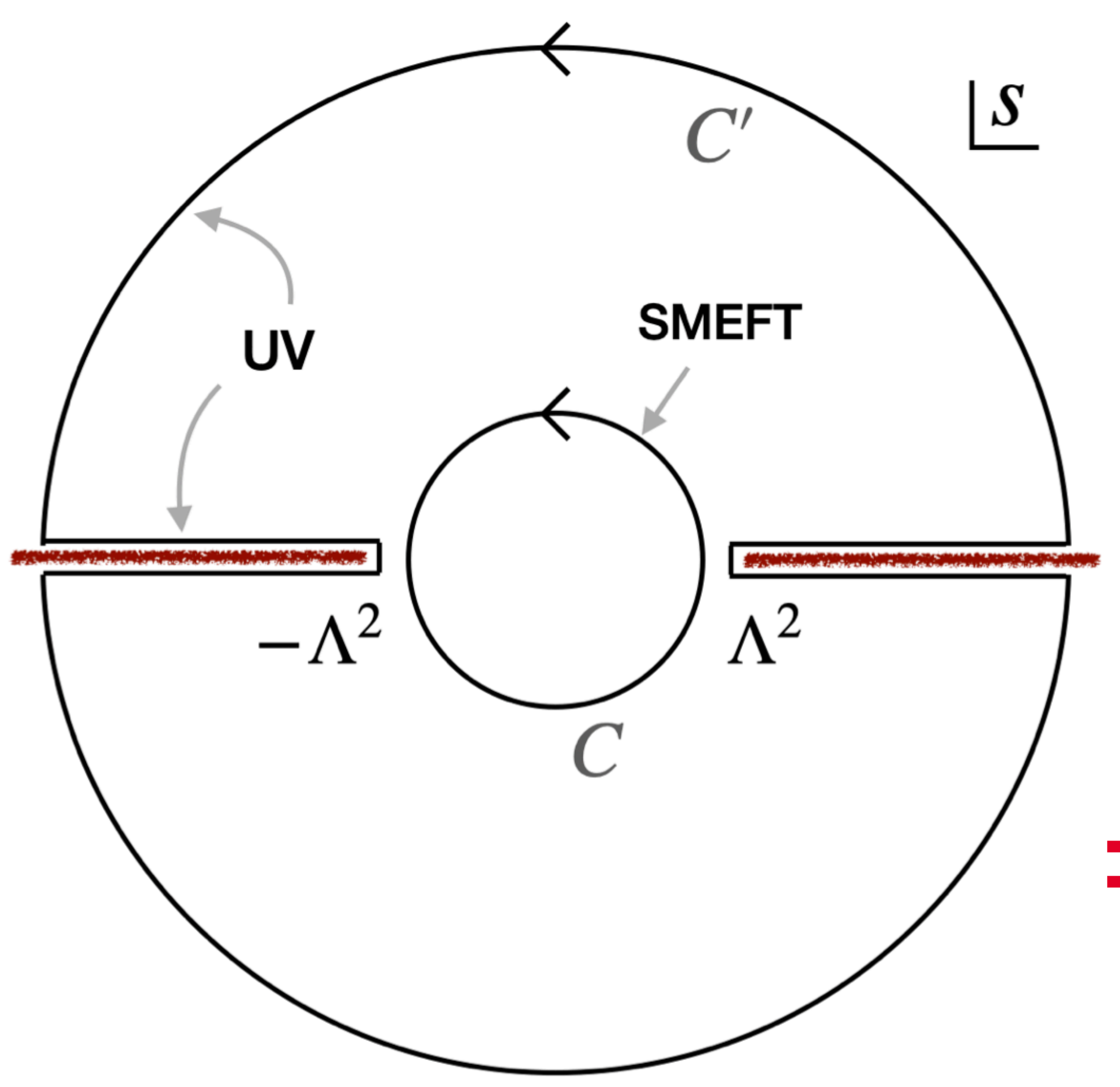

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$$\Rightarrow \quad \frac{1}{2}\frac{d^2 M_{ijkl}(s)}{ds^2} = \int_{-\infty}^{\infty} \frac{d\mu}{2\pi i}\frac{\mathrm{Disc}[M_{ijkl}(\mu)]}{(\mu - s)^3}$$

IR UV

"Dispersion relation"

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$$\frac{1}{2}\frac{d^2 M_{ijkl}(0)}{ds^2} = \sum_X \int d\Pi_X \int_{\Lambda^2}^{\infty} \frac{d\mu}{2\pi\mu^3}\left(m_{ij}m^*_{kl} + m_{i\tilde{l}}m^*_{k\tilde{j}}\right) \qquad m_{ij} \equiv M_{ij\to X}(\mu)$$

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Elastic ($ij = kl$):

$$\frac{1}{2}\frac{d^2 M_{ijij}(0)}{ds^2} = \sum_X \int d\Pi_X \int_{\Lambda^2}^{\infty} \frac{d\mu}{2\pi\mu^3}\left(|m_{ij}|^2 + |m_{i\tilde{j}}|^2\right) \geq 0$$

$$= \sum_i b_i C_i^{(8)} \geq 0$$

"Positivity"

[Zhang; 2112.11665]

Positivity

Not all EFTs are created equal!

$$\sum_i b_i C_i^{(8)} \geq 0$$

Finding optimal bounds is a solved (numerical) problem

Vector boson scattering

[Bi, Zhang, Zhou; 1902.08977]

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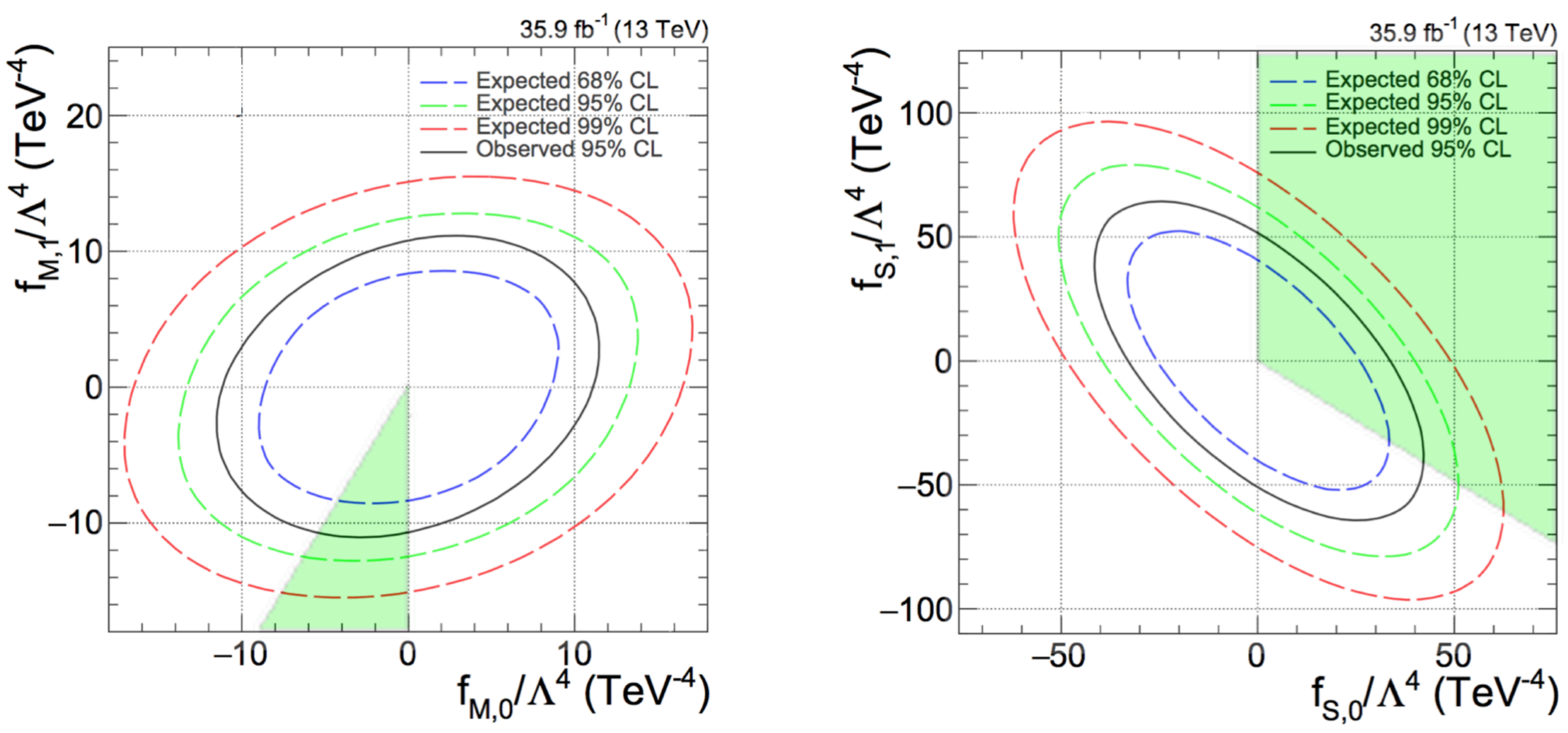

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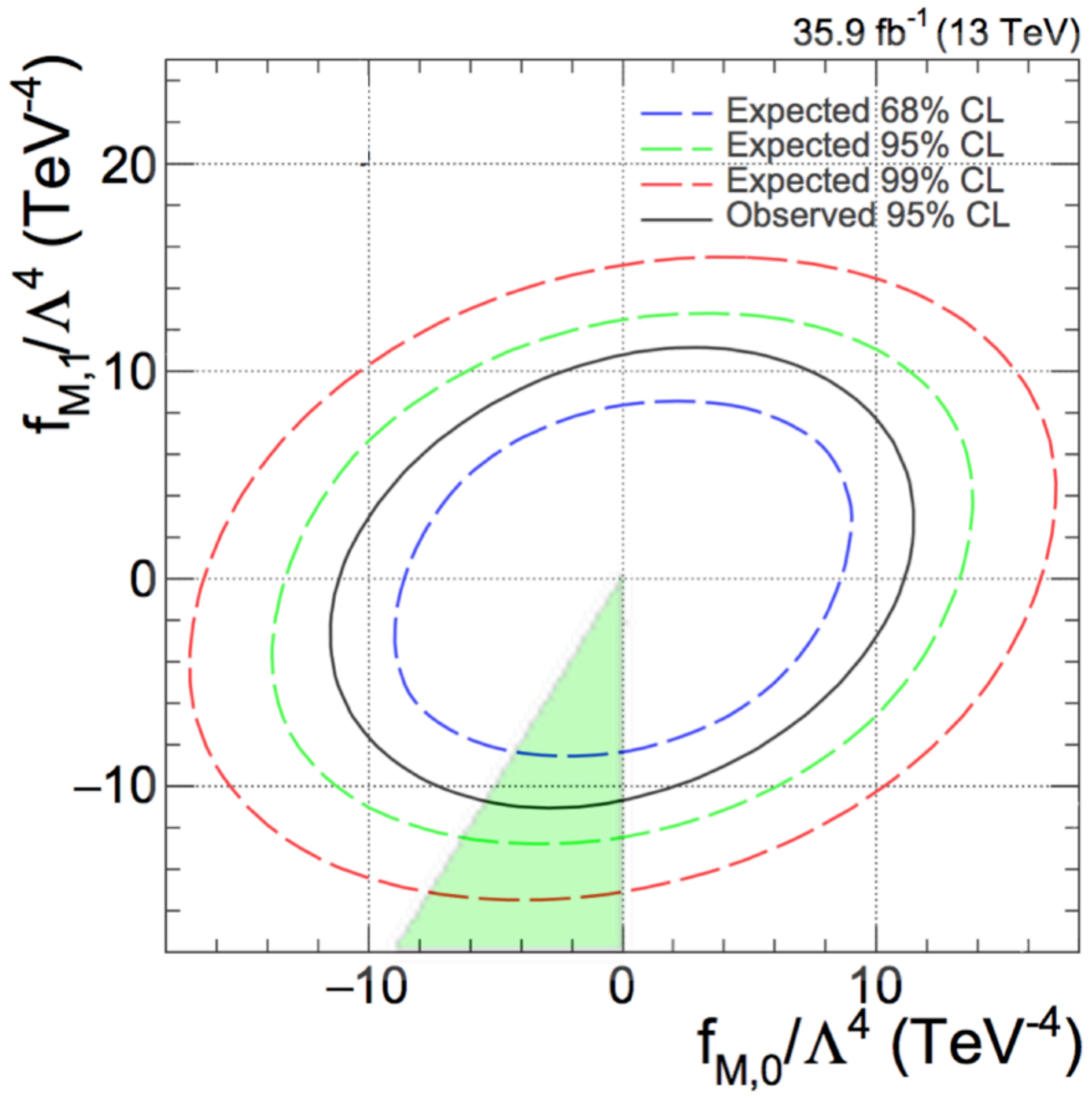

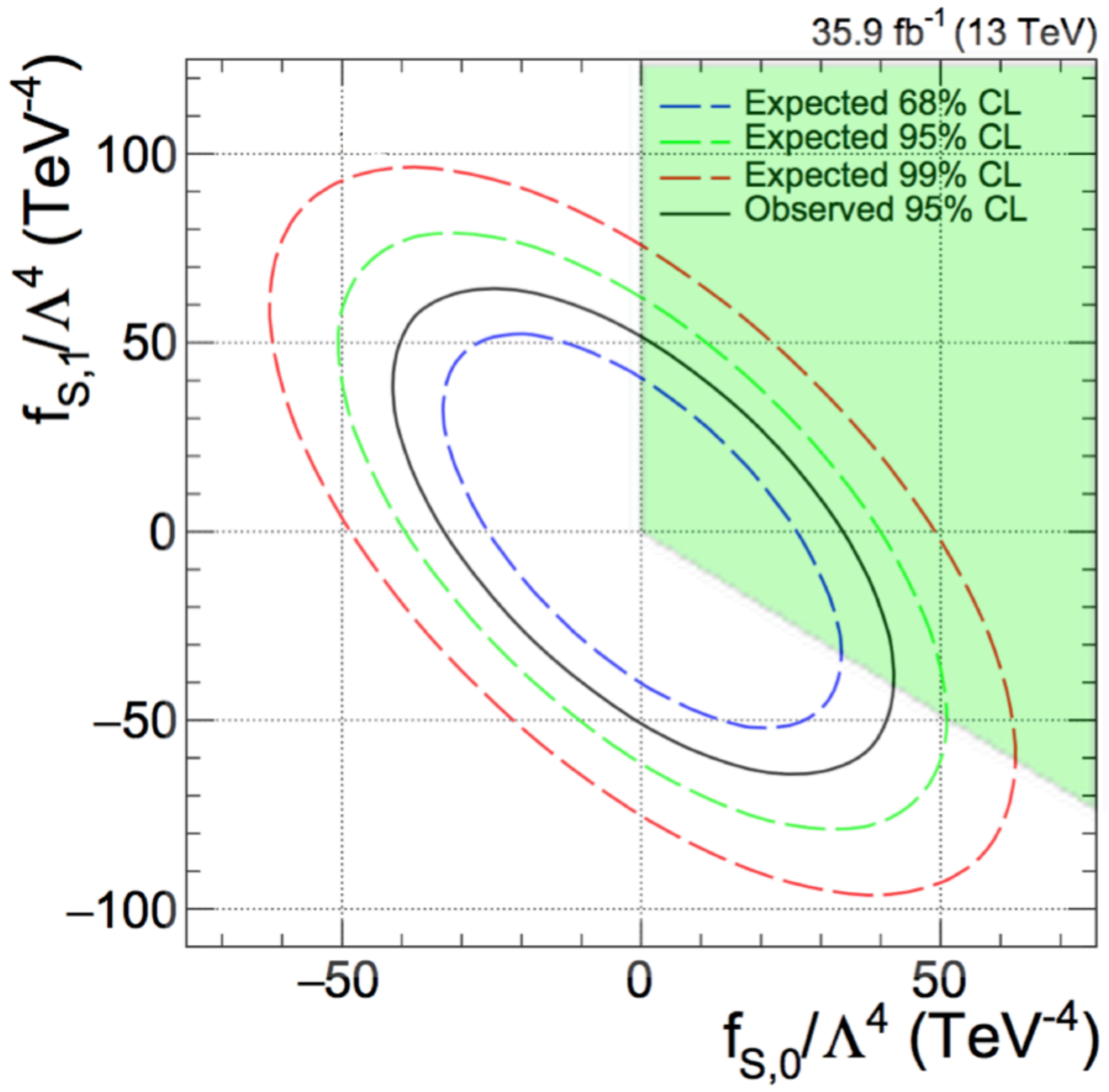

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- Top down: **predict**
- Positivity: **rule out**
- Bottom-up: **agnostic**

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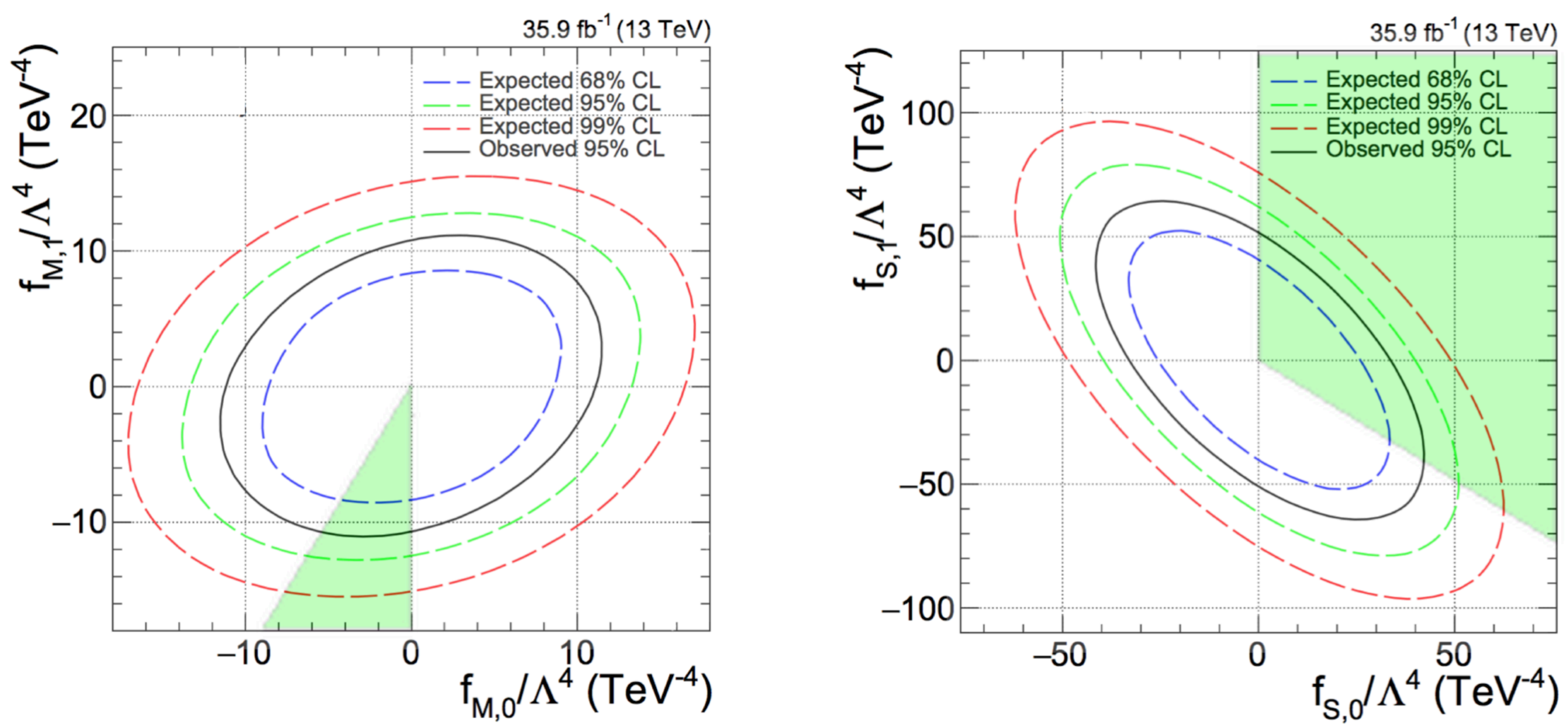

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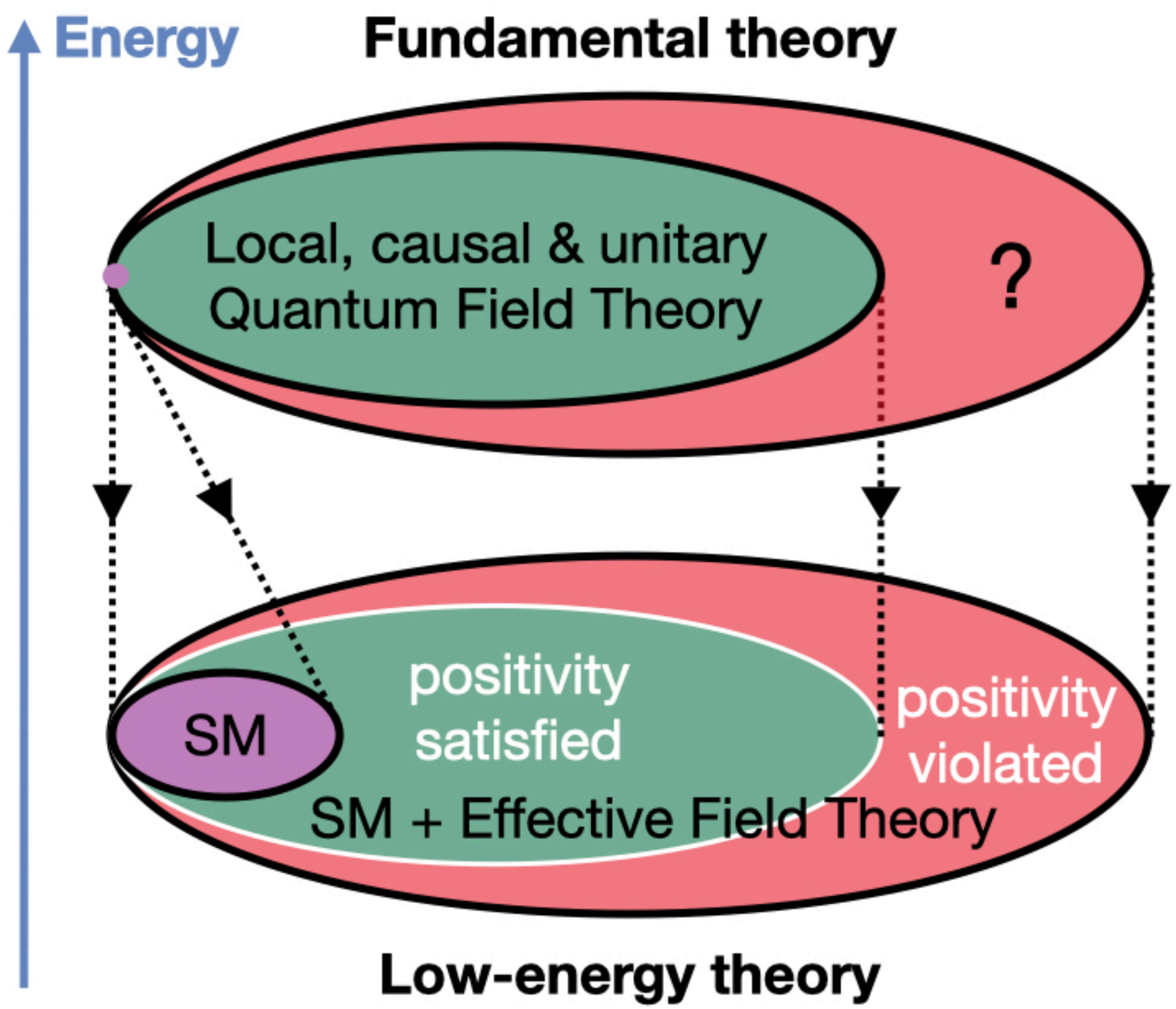
98% of 18D parameter space ruled out by positivity

Positivity

Recent Snowmass review: [de Rham et al.; arXiv:2203.06805]
[Pham & Troung; PRD 31 (1985 3027)]
[Anathanarayan et al.; PRD 51 (1995) 1093-1100]
[Adams et al.; JHEP 10 (2006) 014]



Staying positive

How can we use this information?

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Positivity as a **theoretical prior**

$$P(C_i|\vec{x}) \propto \int_{C_i} \pi_{flat}(C_i) \cdot L(\vec{x}|C_i)$$

$$\Downarrow$$

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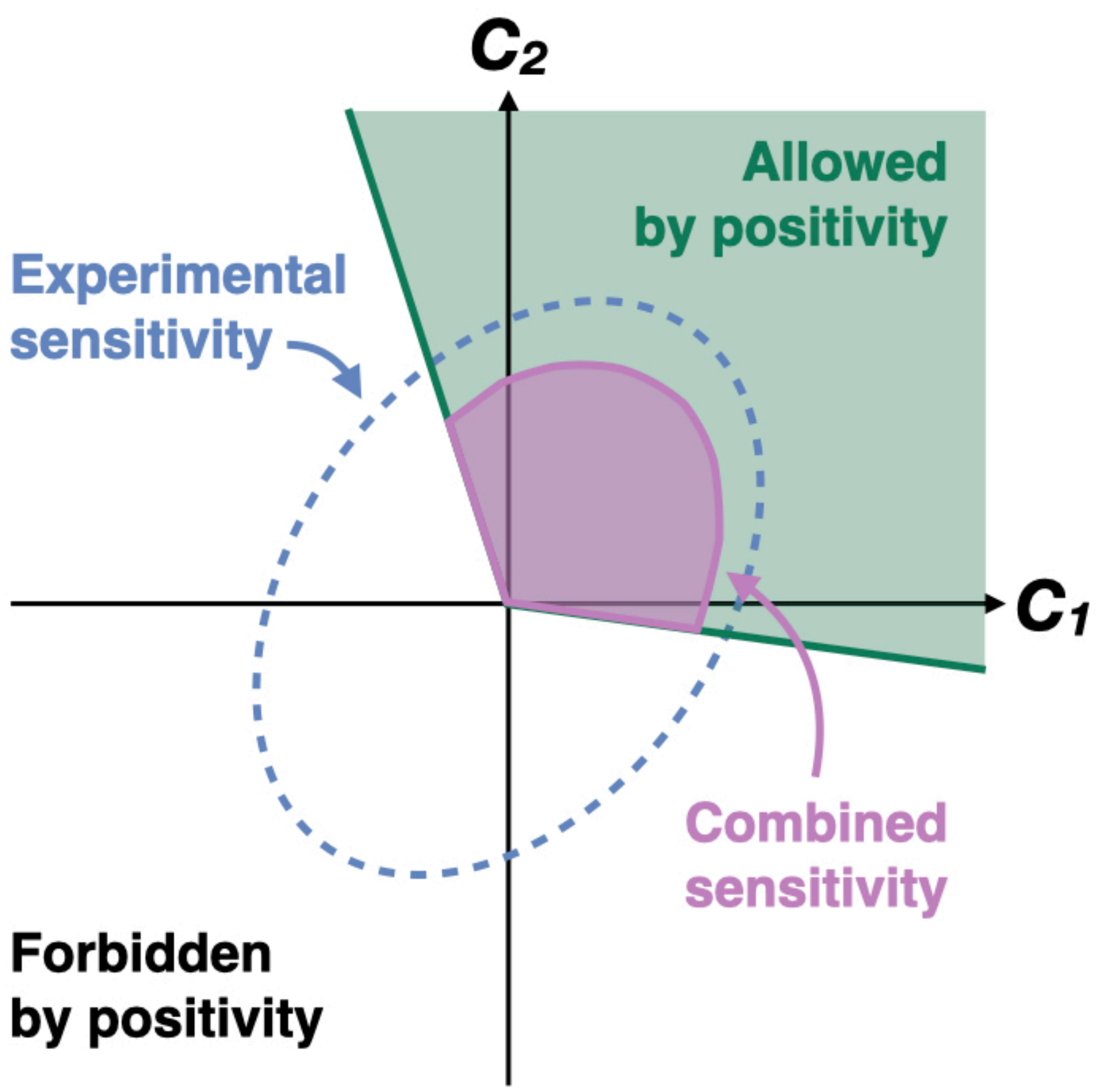

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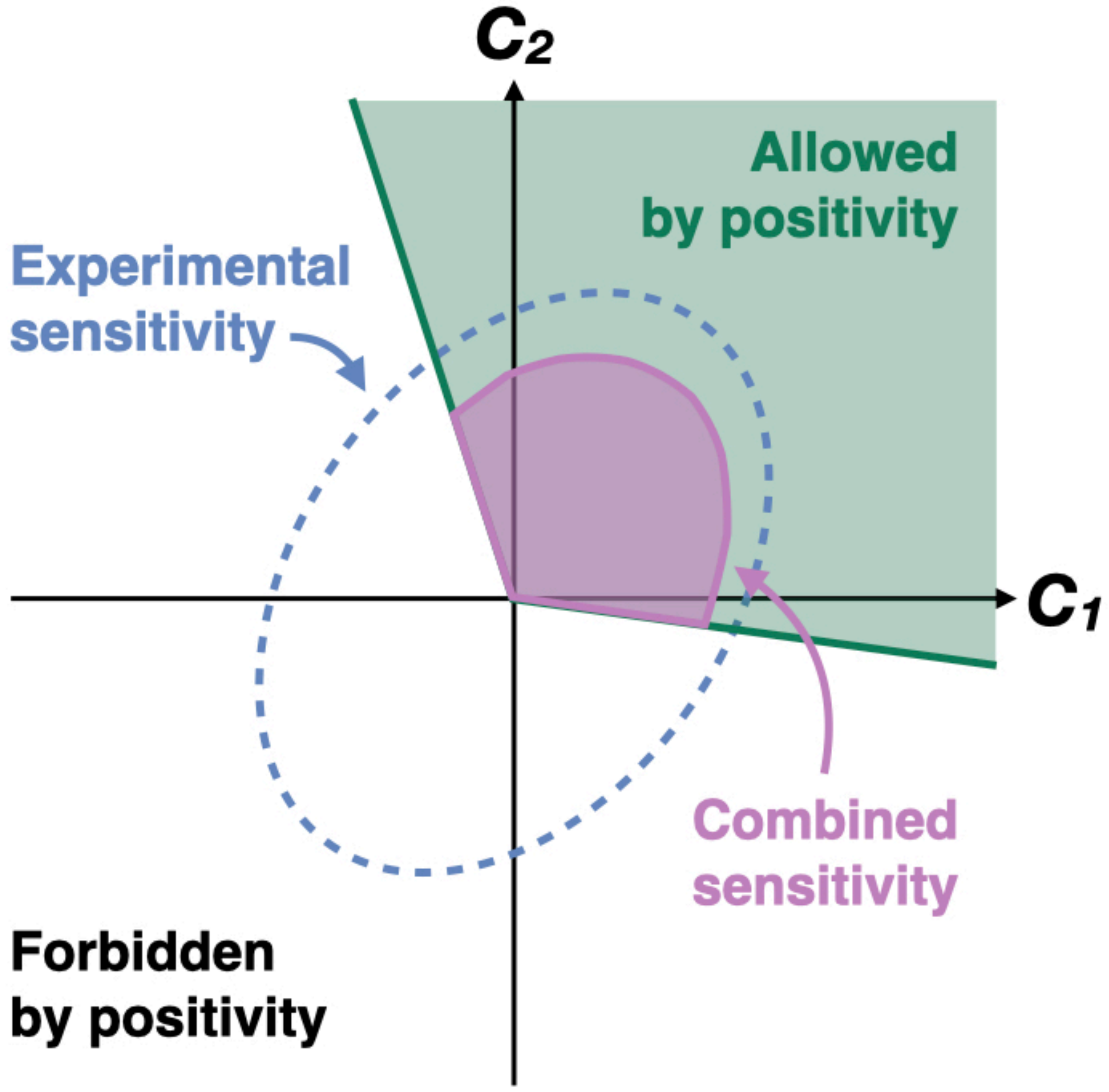

Search for **positivity violation**

"Test fundamental principles of QFT in the UV"

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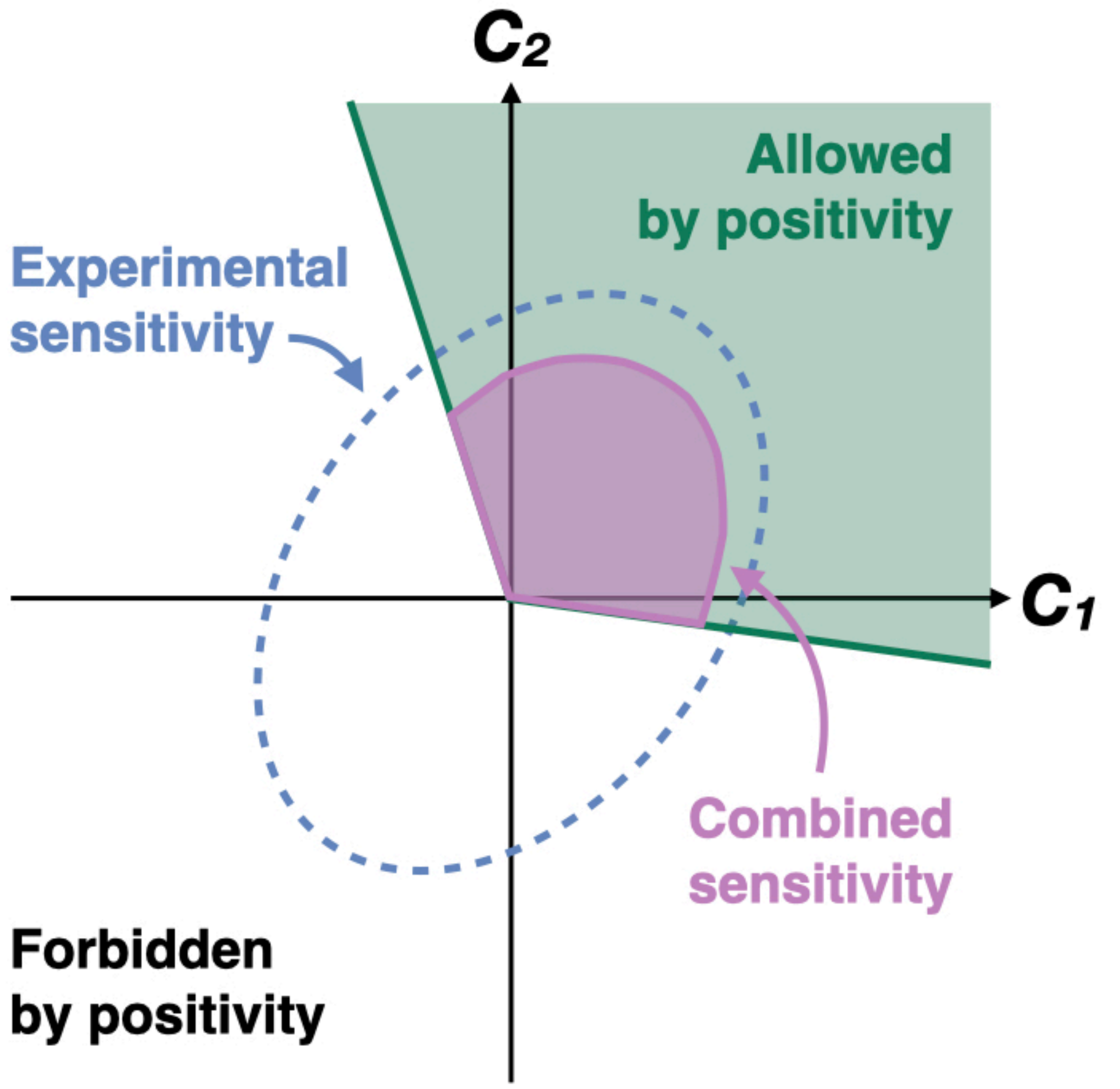

Search for **positivity violation**

"Test fundamental principles of QFT in the UV"

- What kind of exotic UV theory?
- Something revolutionary!

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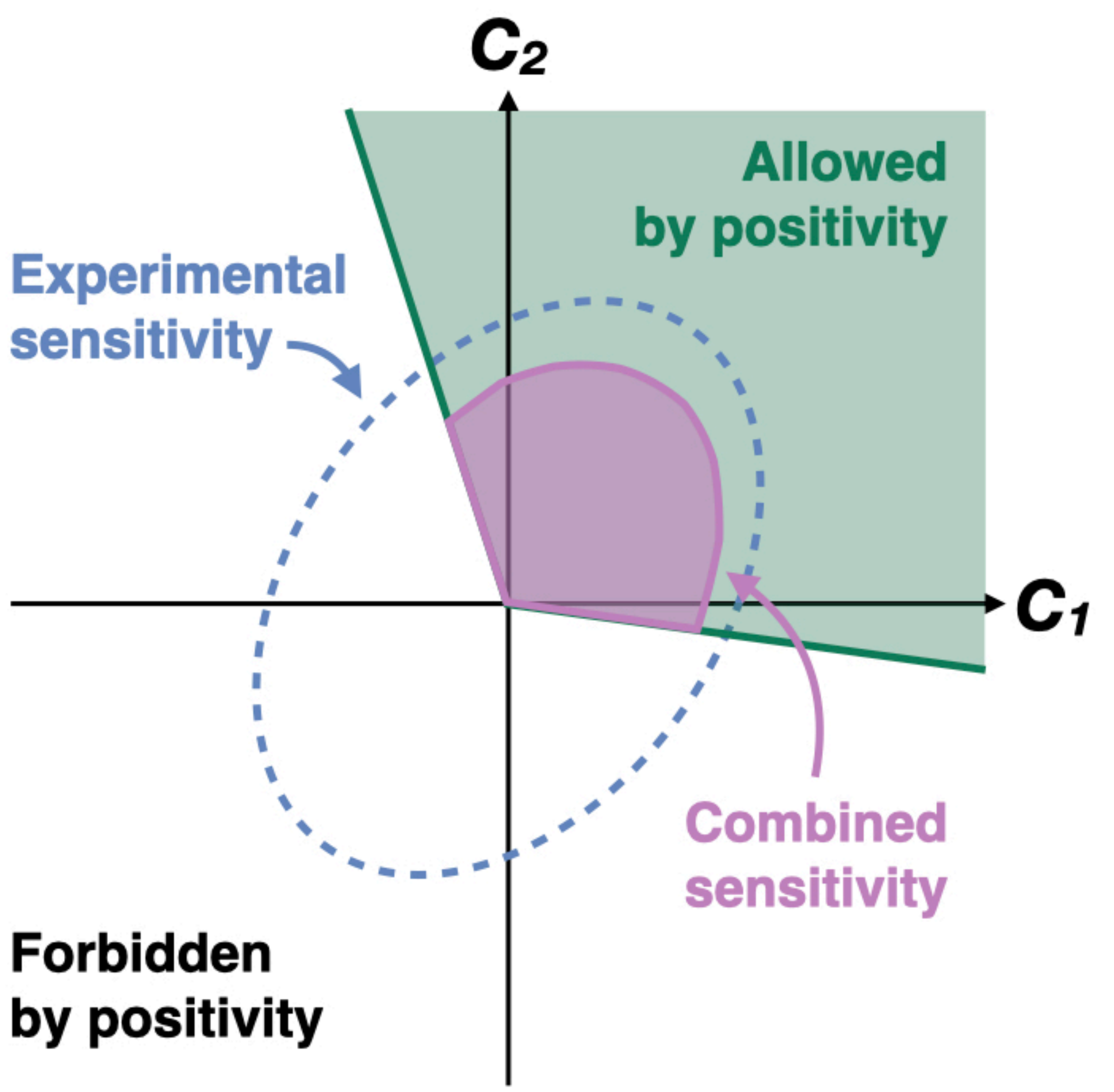

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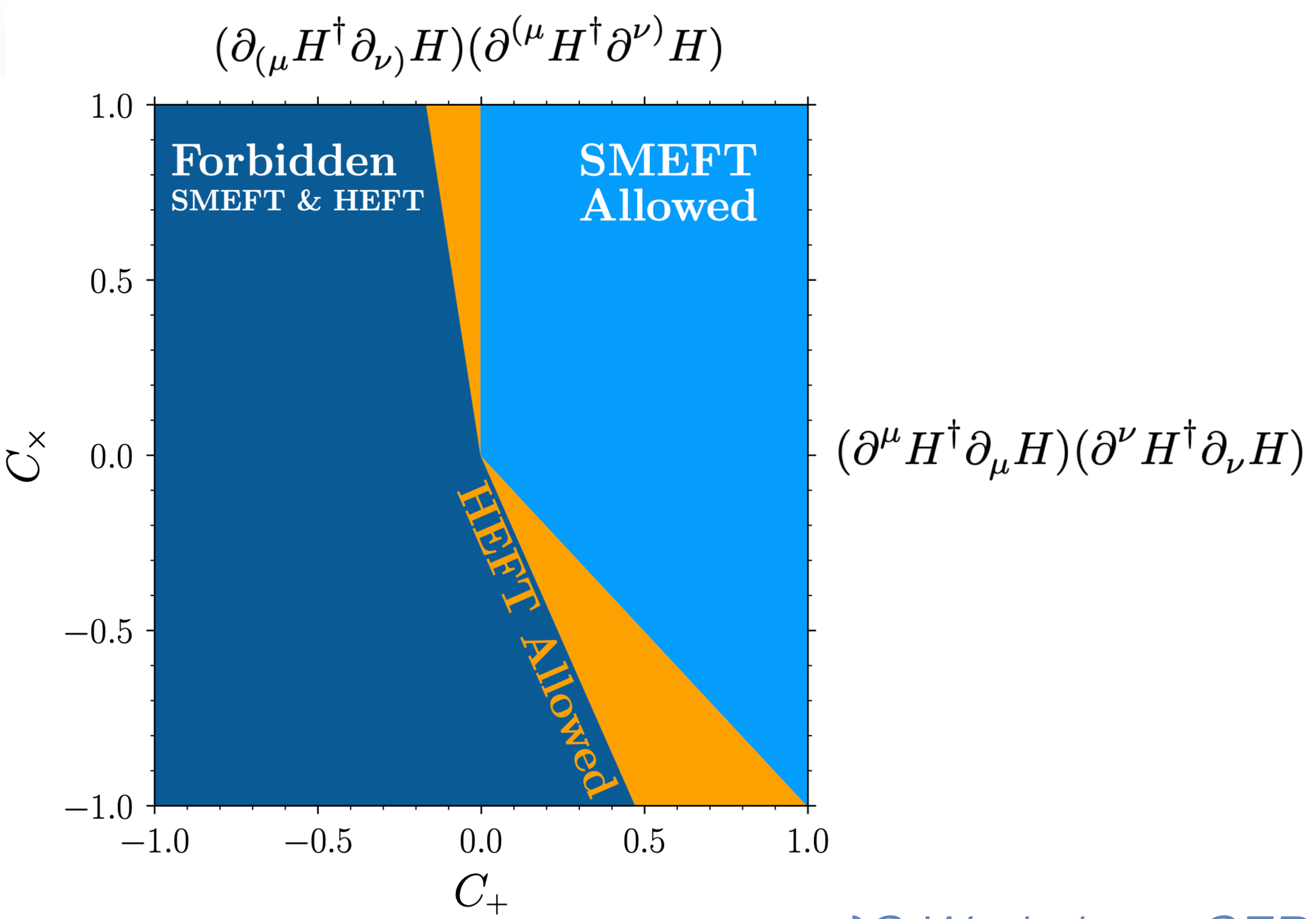
- Something revolutionary!
- More down to earth: **HEFT vs SMEFT**

[Remmen & Rodd; 2412.07827]



Probing positivity

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Dimension-8 operators

$$(\partial\phi)^4$$

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$$(\partial\phi)^4$$

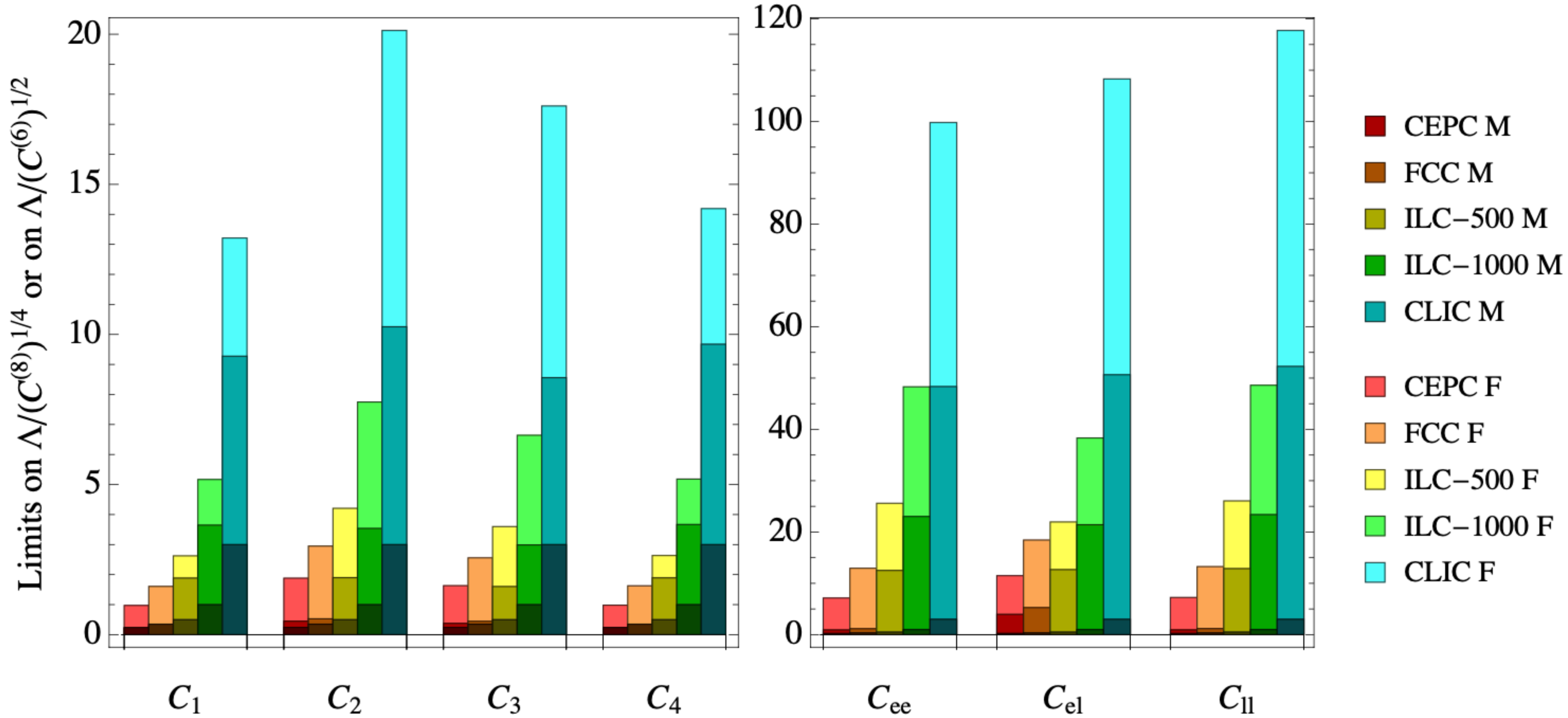
e.g. $e^+e^- \to e^+e^-$ at future colliders [Fuks et al.; 2009.02212]

Dim-6

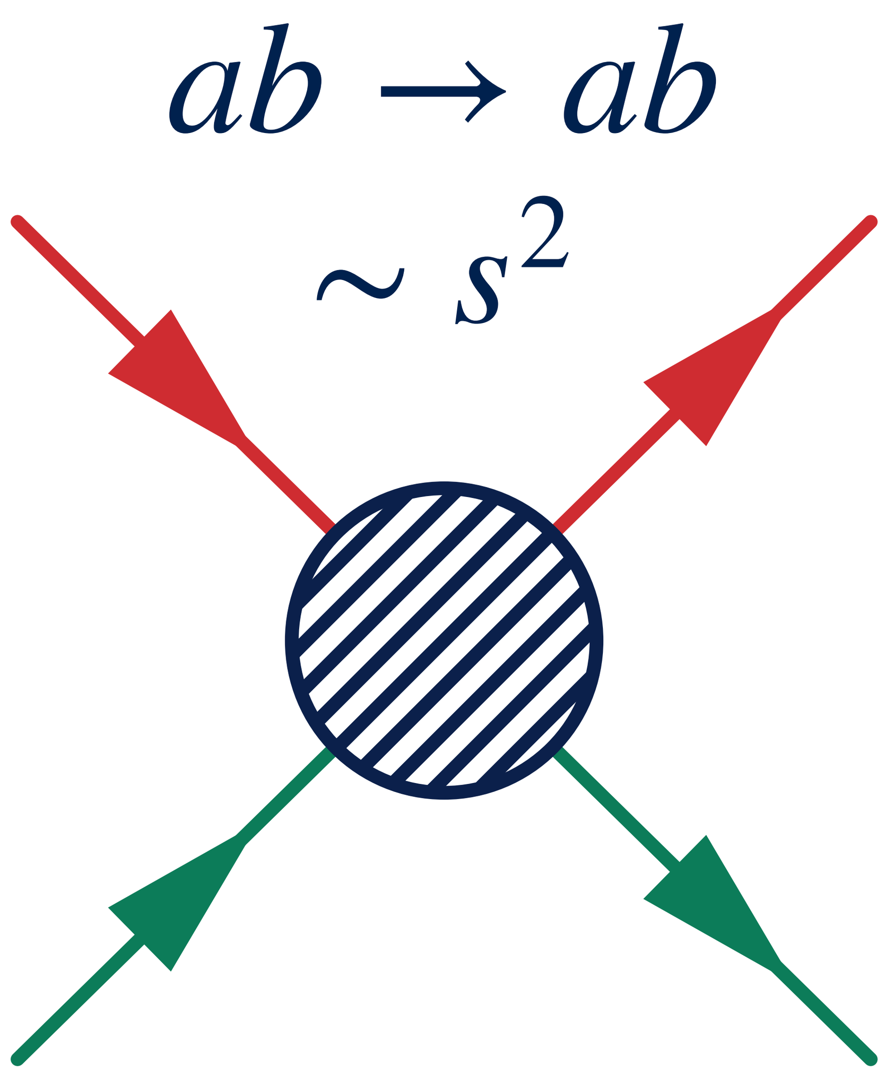
$$O_{ee} = (\bar{e}\gamma^\mu e)\,(\bar{e}\gamma_\mu e)\ ,$$
$$O_{el} = (\bar{e}\gamma^\mu e)\,(\bar{l}\gamma_\mu l)\ ,$$
$$O_{ll} = (\bar{l}\gamma^\mu l)\,(\bar{l}\gamma_\mu l)\ ,$$

Dim-8

$$O_1 = \partial^\alpha(\bar{e}\gamma^\mu e)\partial_\alpha(\bar{e}\gamma_\mu e)\ ,$$
$$O_2 = \partial^\alpha(\bar{e}\gamma^\mu e)\partial_\alpha(\bar{l}\gamma_\mu l)\ ,$$
$$O_3 = D^\alpha(\bar{e}l)\ D_\alpha(\bar{l}e),$$
$$O_4 = \partial^\alpha(\bar{l}\gamma^\mu l)\ \partial_\alpha(\bar{l}\gamma_\mu l)\ ,$$
$$O_5 = D^\alpha(\bar{l}\gamma^\mu\tau^I l)\ D_\alpha(\bar{l}\gamma_\mu\tau^I l)\ ,$$



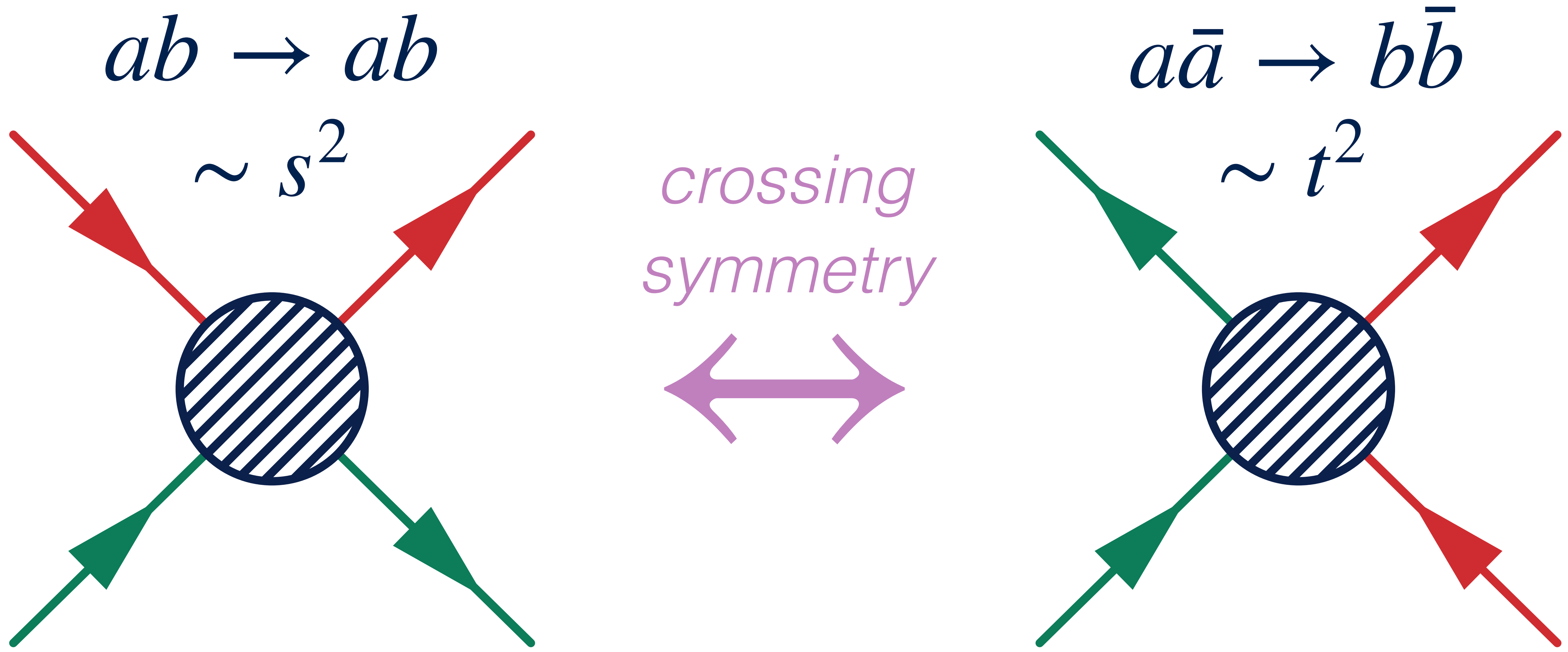
Angular distributions



positivity bounds on elastic scattering

Angular distributions

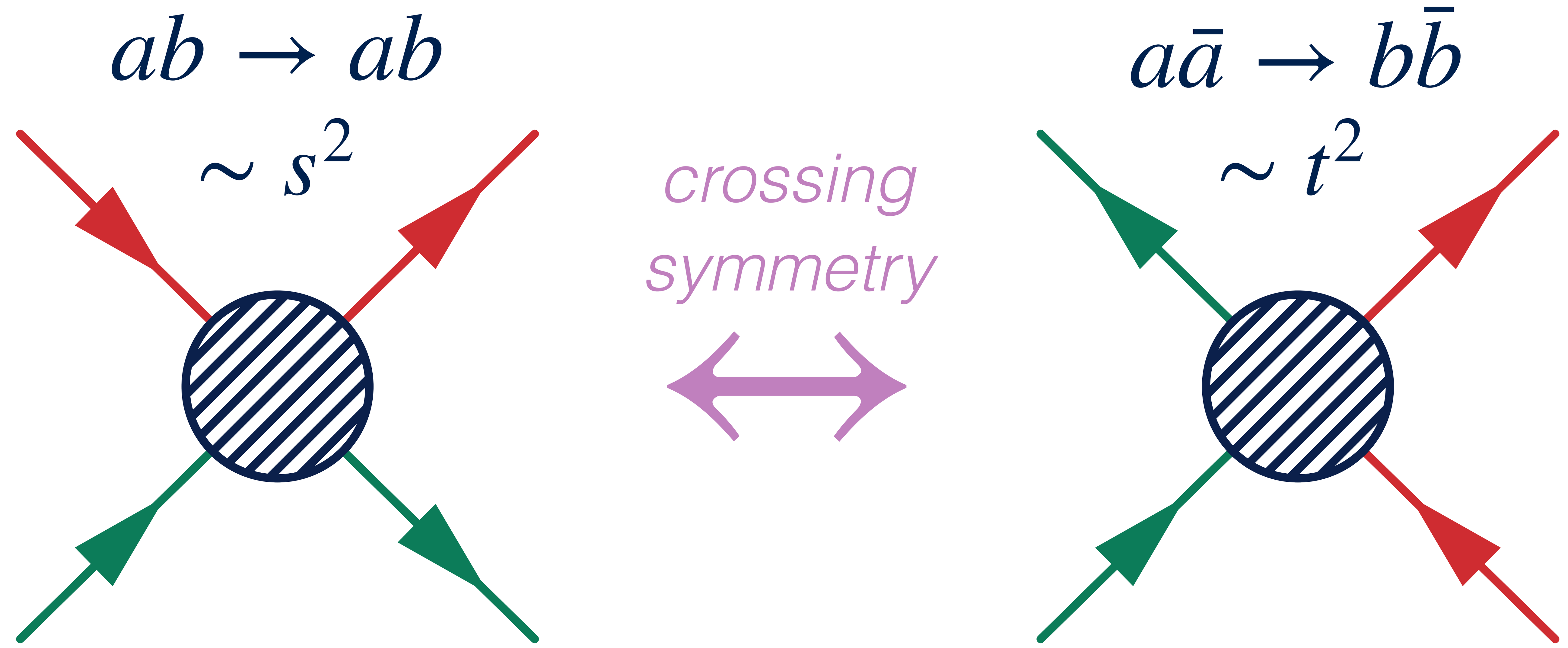
positivity bounds on elastic scattering



New angular dependence

Angular distributions

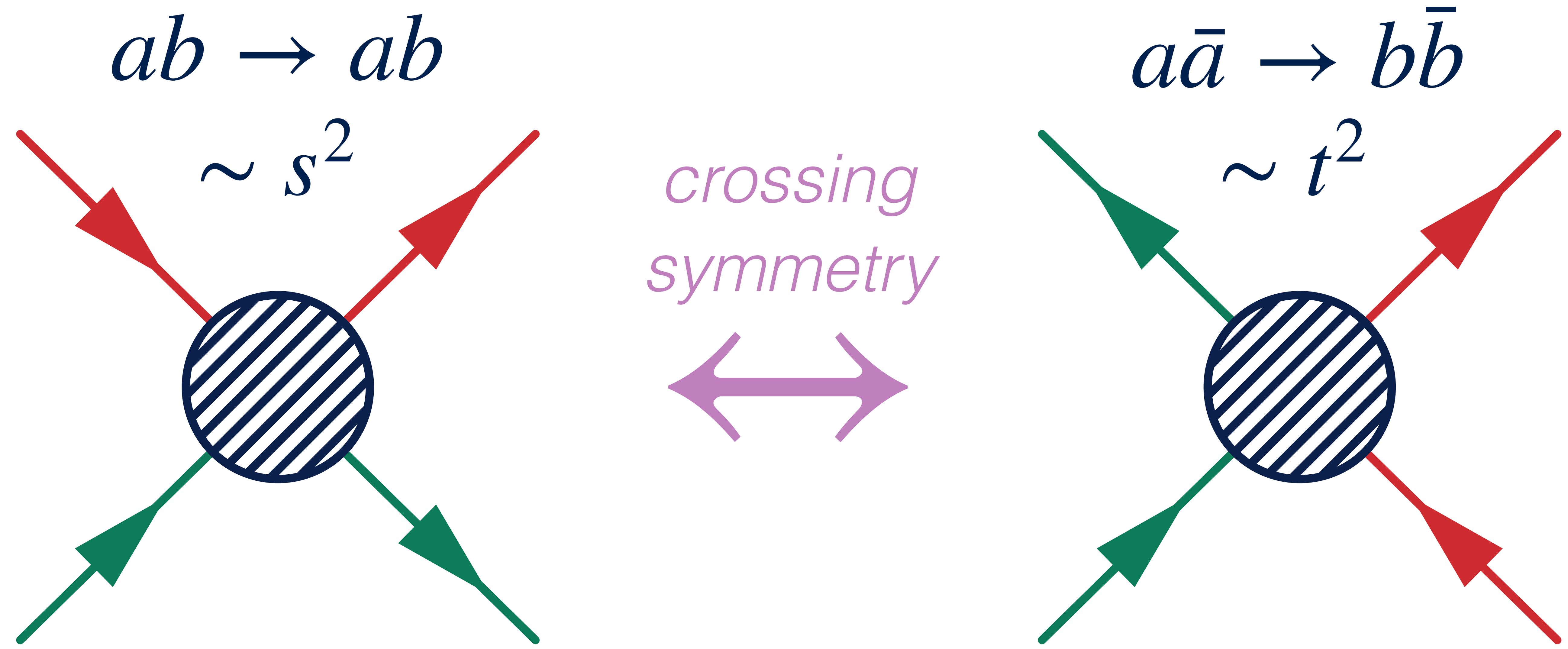
positivity bounds on elastic scattering



New angular dependence

e.g. Drell-Yan: $q\ell^+ \to q\ell^+ \leftrightarrow q\bar{q} \to \ell^+\ell^-$

Angular distributions



positivity bounds on elastic scattering

New angular dependence

e.g. Drell-Yan: $q\ell^+ \to q\ell^+ \leftrightarrow q\bar{q} \to \ell^+\ell^-$

$$\frac{d\sigma_{pp\to\ell^+\ell^-}}{dm_{\ell\ell}d\eta_{\ell\ell}d\Omega_\ell} = \frac{3}{16\pi}\frac{d\sigma_{pp\to\ell^+\ell^-}}{dm_{\ell\ell}d\eta_{\ell\ell}}\Bigg[(1+c_\theta^2) + \frac{\tilde{A}_0}{2}(1-3c_\theta^2) + \tilde{A}_1 s_{2\theta}c_\phi$$

$l \leq 2$ angular moments

$$+\frac{\tilde{A}_2}{2}s_\theta^2 c_{2\phi} + \tilde{A}_3 s_\theta c_\phi + \tilde{A}_4 c_\theta + \tilde{A}_5 s_\theta^2 s_{2\phi} + \tilde{A}_6 s_{2\theta}s_\phi + \tilde{A}_7 s_\theta s_\phi\Bigg]$$

- SM: Spin-1 photon & Z-boson $\to l \leq 2$ angular dependence
- LO is ϕ symmetric: $\tilde{A}_{1,4} \neq 0$, NLO: $\tilde{A}_{1-7} \neq 0$

[Li, KM, Yamashita, Yang, Zhang, Zhou; 2204.13121]

Angular distributions

[Alioli et al.; 2003.11615]

Higher moments: dim-8 only (SM & dim-6 contributions are 0)

[Li, KM, Yamashita, Yang, Zhang, Zhou; 2204.13121]

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$l \leq 2$

$$+\frac{\tilde{A}_2}{2}s_\theta^2 c_{2\phi}+\tilde{A}_3 s_\theta c_\phi+\tilde{A}_4 c_\theta+\tilde{A}_5 s_\theta^2 s_{2\phi}+\tilde{A}_6 s_{2\theta}s_\phi+\tilde{A}_7 s_\theta s_\phi$$

$l = 3$

$$+\frac{\tilde{B}_1^e}{2}s_\theta\left(5c_\theta^2-1\right)c_\phi+\frac{\tilde{B}_1^o}{2}s_\theta\left(5c_\theta^2-1\right)s_\phi+\frac{\tilde{B}_0}{2}\left(5c_\theta^3-3c_\theta\right)$$

$$+\tilde{B}_3^e s_\theta^3 c_{3\phi}+\tilde{B}_3^o s_\theta^3 s_{3\phi}+\tilde{B}_2^e s_\theta^2 c_\theta c_{2\phi}+\tilde{B}_2^o s_\theta^2 c_\theta s_{2\phi}$$

$l = 4$

$$+\tilde{D}_4^e s_\theta^4 c_{4\phi}+\tilde{D}_4^o s_\theta^4 s_{4\phi}+\tilde{D}_3^e s_\theta^3 c_\theta c_{3\phi}+\tilde{D}_3^o s_\theta^3 c_\theta s_{3\phi}$$

$$+\tilde{D}_2^e s_\theta^2(7c_\theta^2-1)c_{2\phi}+\tilde{D}_2^o s_\theta^2(7c_\theta^2-1)s_{2\phi}+\tilde{D}_1^e s_\theta(7c_\theta^3-3c_\theta)c_\phi$$

$$+\tilde{D}_1^o s_\theta(7c_\theta^3-3c_\theta)s_\phi+\frac{\tilde{D}_0}{2}(35c_\theta^4-30c_\theta^2+3)\Bigg]$$

[Li, KM, Yamashita, Yang, Zhang, Zhou; 2204.13121]
[Alioli et al.; 2003.11615]

Angular distributions

Higher moments: dim-8 only (SM & dim-6 contributions are 0)

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$l \leq 2$

$$+\frac{\tilde{A}_2}{2}s_\theta^2 c_{2\phi} + \tilde{A}_3 s_\theta c_\phi + \tilde{A}_4 c_\theta + \tilde{A}_5 s_\theta^2 s_{2\phi} + \tilde{A}_6 s_{2\theta}s_\phi + \tilde{A}_7 s_\theta s_\phi$$

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$$+\tilde{D}_4^e s_\theta^4 c_{4\phi} + \tilde{D}_4^o s_\theta^4 s_{4\phi} + \tilde{D}_3^e s_\theta^3 c_\theta c_{3\phi} + \tilde{D}_3^o s_\theta^3 c_\theta s_{3\phi}$$

$$+\tilde{D}_2^e s_\theta^2(7c_\theta^2-1)c_{2\phi} + \tilde{D}_2^o s_\theta^2(7c_\theta^2-1)s_{2\phi} + \tilde{D}_1^e s_\theta(7c_\theta^3-3c_\theta)c_\phi$$

$$+\tilde{D}_1^o s_\theta(7c_\theta^3-3c_\theta)s_\phi + \boxed{\frac{\tilde{D}_0}{2}(35c_\theta^4-30c_\theta^2+3)}\Big]$$

Use $(\tilde{B}_0, \tilde{D}_0)$ to constrain the space of dim-8 WCs

$$O_{8,lq\partial 3} = (\bar{\ell}\gamma_\mu \overleftrightarrow{D}_\nu \ell)(\bar{q}\gamma^\mu \overleftrightarrow{D}^\nu q)$$

$$O_{8,ed\partial 2} = (\bar{e}\gamma_\mu \overleftrightarrow{D}_\nu e)(\bar{d}\gamma^\mu \overleftrightarrow{D}^\nu d)$$

$$O_{8,lq\partial 4} = (\bar{\ell}\tau^I\gamma_\mu \overleftrightarrow{D}_\nu \ell)(\bar{q}\tau^I\gamma^\mu \overleftrightarrow{D}^\nu q)$$

$$O_{8,eu\partial 2} = (\bar{e}\gamma_\mu \overleftrightarrow{D}_\nu e)(\bar{u}\gamma^\mu \overleftrightarrow{D}^\nu u)$$

$$O_{8,ld\partial 2} = (\bar{\ell}\gamma_\mu \overleftrightarrow{D}_\nu \ell)(\bar{d}\gamma^\mu \overleftrightarrow{D}^\nu d)$$

$$O_{8,qe\partial 2} = (\bar{e}\gamma_\mu \overleftrightarrow{D}_\nu e)(\bar{q}\gamma^\mu \overleftrightarrow{D}^\nu q)$$

$$O_{8,lu\partial 2} = (\bar{\ell}\gamma_\mu \overleftrightarrow{D}_\nu \ell)(\bar{u}\gamma^\mu \overleftrightarrow{D}^\nu u)$$


Λ = 2 TeV
B_0 only (quadratic, individual)
B_0 (linear, individual)
B_0 & D_0 (quadratic, individual)
B_0 & D_0 (quadratic, profiled)
Positivity allowed
Set all others to 0
Allow all others to vary
$C_{8,lq\partial 3}$
$C_{8,lq\partial 4}$
$C_{8,ed\partial 2}$
$C_{8,eu\partial 2}$
$C_{8,ld\partial 2}$
$C_{8,lu\partial 2}$
$C_{8,qe\partial 2}$


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Set all others to 0
Allow all others to vary
$C_{8,lq\partial 4}$
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$C_{8,ld\partial 2}$
$C_{8,lu\partial 2}$
$C_{8,qe\partial 2}$
$C_{8,lq\partial 3}$

Testing positivity

Suppose we measure our WCs to be $\vec{c}_0$

[Fuks et al.; Chin. phys. C 45 (2021) 023108]

Testing positivity

Suppose we measure our WCs to be $\overrightarrow{C}_0$

Define "distance" from region allowed by elastic positivity

$$-\Delta^{-4} \equiv \min\left[\min_{\text{processes}} \frac{1}{2}\frac{d^2M(0)}{ds^2}, 0\right] = \frac{\delta(\overrightarrow{C}_0)}{\Lambda^4},$$

elastic ql scatterings

"most non-positive" direction

$$\delta(\overrightarrow{C}_0) \equiv \min\Big[-4C_{8,lq\partial 3} + 4C_{8,lq\partial 4}, -4C_{8,lq\partial 3} - 4C_{8,lq\partial 4}, -4C_{8,ed\partial 2}, -4C_{8,eu\partial 2}, -4C_{8,ld\partial 2}, -4C_{8,lu\partial 2}, -4C_{8,qe\partial 2}, 0\Big]$$

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"most non-positive" direction

Associate a scale, Δ, to positivity violation

Satisfied: $\Delta = \infty$

Violated: $\Delta = \dfrac{\Lambda}{\sqrt[4]{\delta C_{\text{min.}}}}$

[Fuks et al.; Chin. phys. C 45 (2021) 023108]

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elastic ql scatterings

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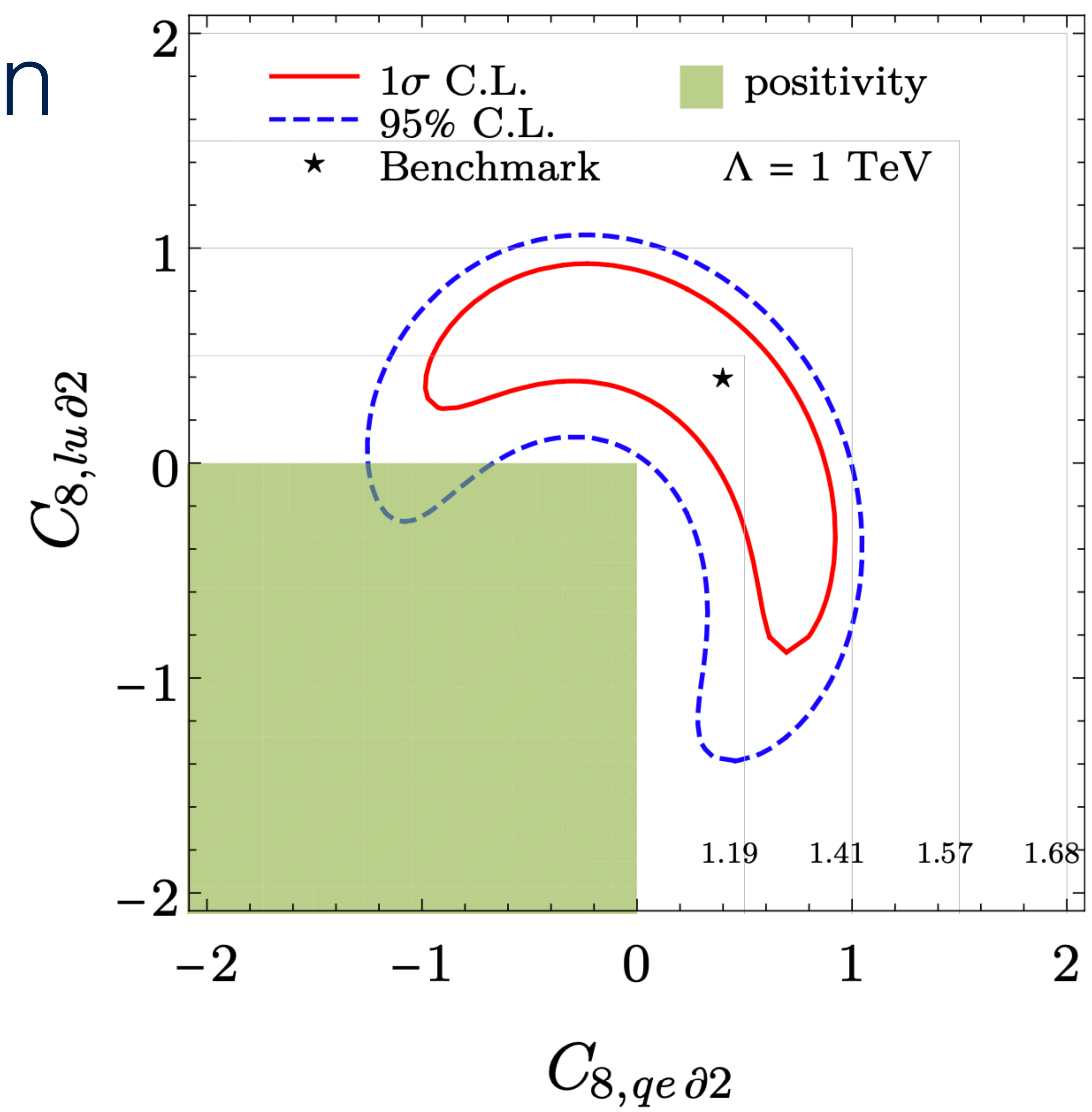
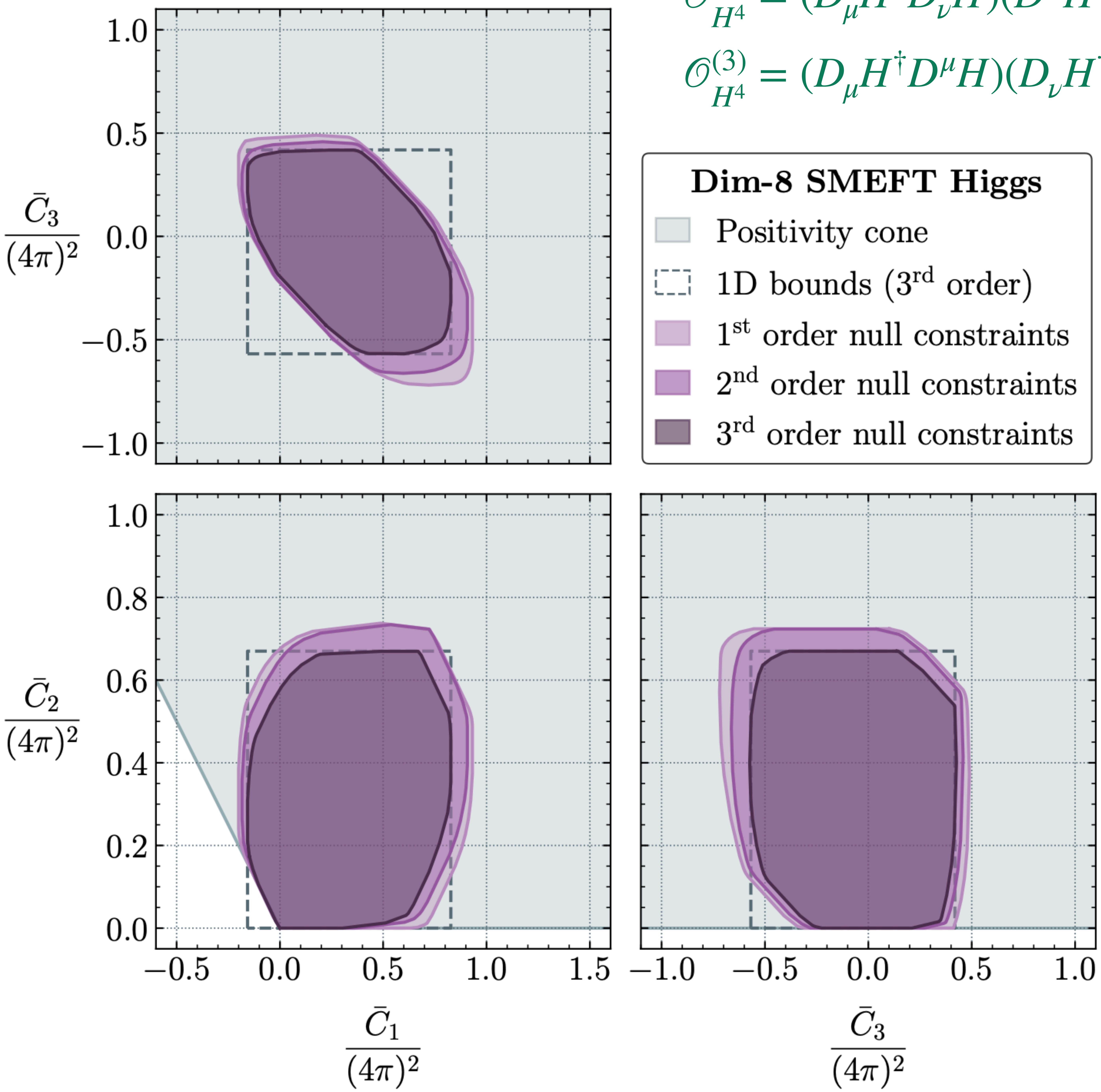
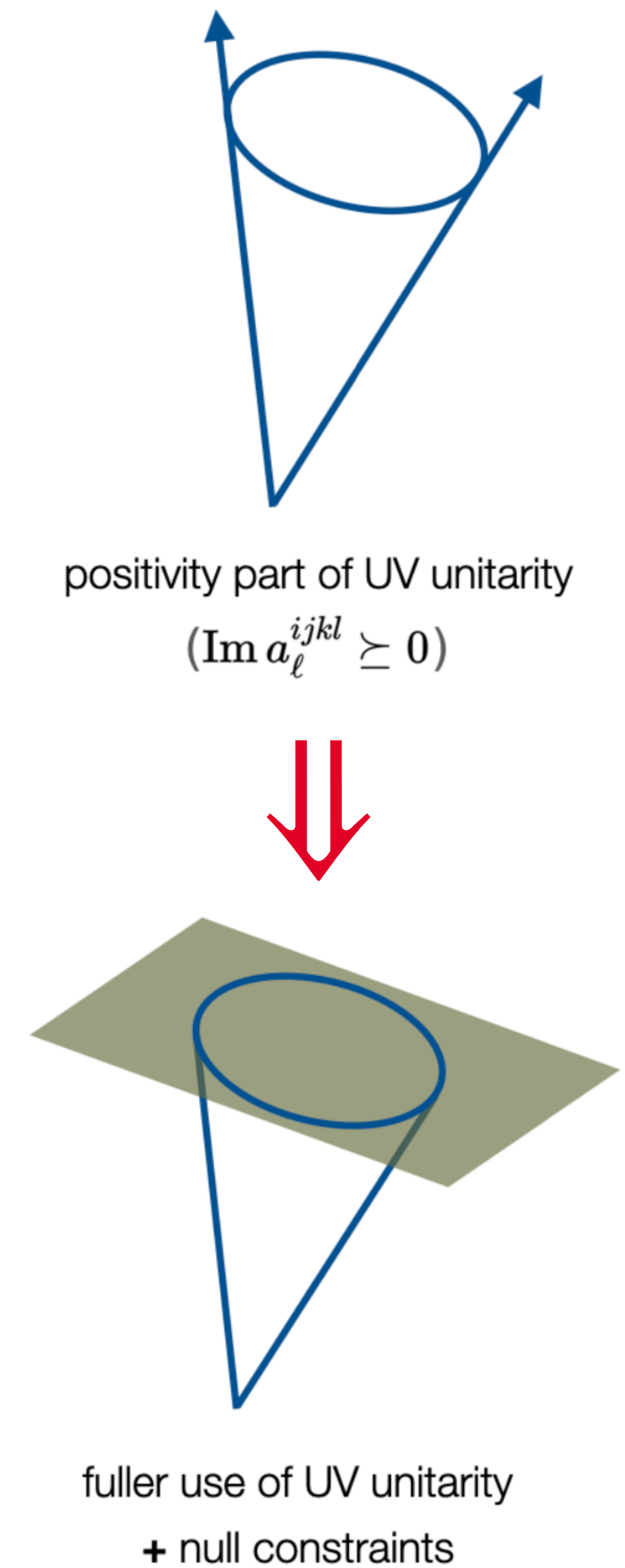


This example: positivity violation measured at 1σ but not 95% C.L.

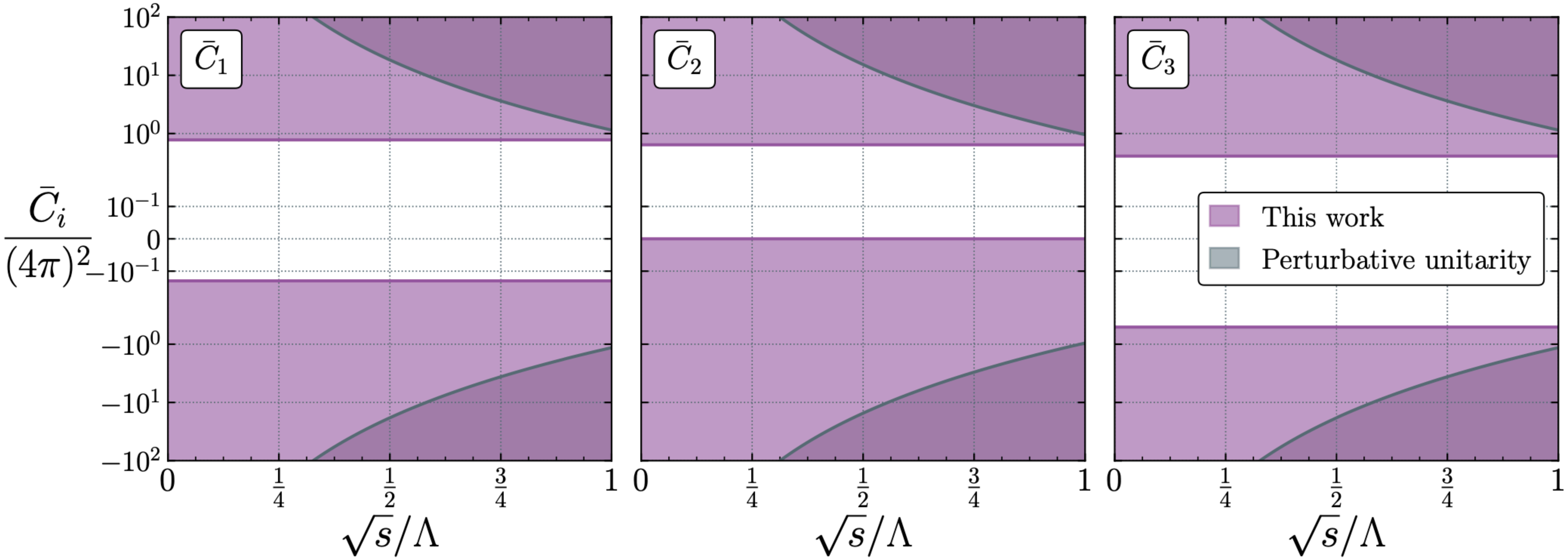
Capping the cone

$$\mathcal{O}_{H^4}^{(1)} = (D_\mu H^\dagger D_\nu H)(D^\nu H^\dagger D^\mu H)$$
$$\mathcal{O}_{H^4}^{(2)} = (D_\mu H^\dagger D_\nu H)(D^\mu H^\dagger D^\nu H)$$
$$\mathcal{O}_{H^4}^{(3)} = (D_\mu H^\dagger D^\mu H)(D_\nu H^\dagger D^\nu H)$$





Comparisons

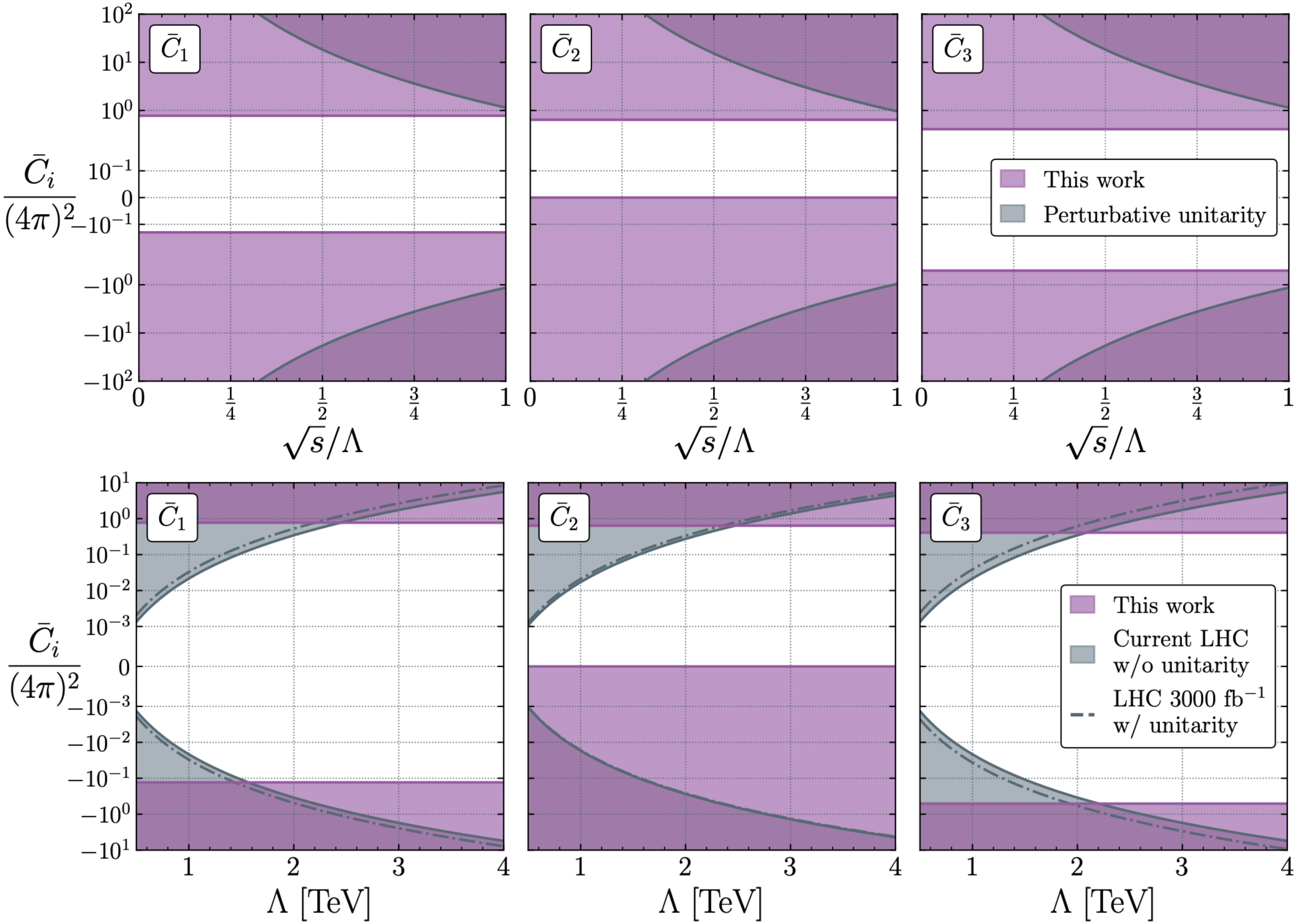


Perturbative unitarity in the EFT

[Almeida, Eboli & Gonzelez-Garcia; PRD 101 (2020) 11, 113003]

[Chen, KM, Wu, Zhang & Zhou; JHEP 03 (2024) 180]

Comparisons



Perturbative unitarity in the EFT

[Almeida, Eboli & Gonzelez-Garcia; PRD 101 (2020) 11, 113003]

HL-LHC projections from VBS

[Capati et al.; JHEP 09 (2022) 038]

(See R. Covarelli's talk)

Conclusions & open questions

Positivity means that **dimension-8 is special**

- Heavy new physics must *unambiguously* show up there *"Inverse problem"*
- Important to control theory uncertainties in dim-6 EFT analyses

How best to use the information from positivity?

- Theory prior for statistical analyses $\Rightarrow$ improved sensitivity
- Test the fundamental axioms of QFT

Devise positivity-sensitive experimental observables

- Angular distributions in $a\bar{a} \to b\bar{b}$

Future collider potential is largely unexplored

- Important part of the EFT programme beyond dim-6

CERN TH institute/COMETA COST action topical workshop

Positivity, Amplitudes, and Phenomenology

7–11 Apr 2025
CERN
Europe/Zurich timezone

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TH workshop secretariat

thworkshops.secretariat...

This CERN TH Institute, jointly hosted by the COMETA COST action, aims to connect the formal, phenomenological and experimental communities to discuss recent developments in the realm of first-principle theoretical constraints on scattering amplitudes relevant for the effective field theory (EFT) interpretation of collider data.

An overarching goal of the meeting will be to investigate concrete ways in which positivity and related constraints can connect collider and other data to fundamental properties of physics in the deep ultraviolet. Examples include the possibility of using the constraints as a prior in statistical interpretations, designing phenomenological studies to test positivity at present and future colliders, and exploring theoretical connections between positivity and outstanding problems in BSM physics.

The workshop will last 5 days (from Monday afternoon until Friday morning) and be all plenary with sessions dedicated to different sub-topics, including one day dedicated to experimental-theory exchange. The programme will be kept relatively light, with plenty of discussion time.

7-11th of April 2025
stay tuned!

Gauthier Durieux
Ilaria Brivio
Joe Davighi
Ken Mimasu
Tevong You
Tim Cohen

https://indico.cern.ch/event/1488316/
https://th-dep.web.cern.ch/events/positivity-amplitudes-and-phenomenology

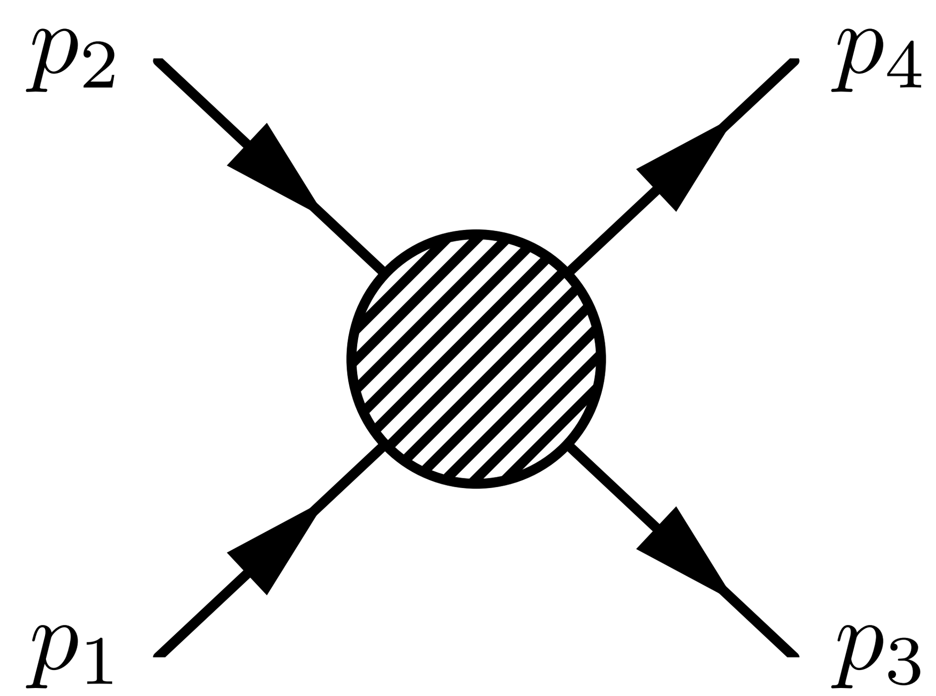
Backup

FC run parameters

[Fuks et al.; 2009.02212]

Scenario	Beam polarization $P(e^-, e^+)$	Runs (luminosity @ energy), [ab^{-1}] @ [GeV] 1	2	3	4
CEPC	None	2.6@161	5.6@240		
FCC-ee	None	10@161	5@240	0.2@350	1.5@365
ILC-500	(−80%, 30%)	0.9@250	0.135@350	1.6@500	
	(80%, −30%)	0.9@250	0.045@350	1.6@500	
ILC-1000	(−80%, 30%)	0.9@250	0.135@350	1.6@500	1.25@1000
	(80%, −30%)	0.9@250	0.045@350	1.6@500	1.25@1000
CLIC	(−80%, 0%)	0.5@380	2@1500	4@3000	
	(80%, 0%)	0.5@380	0.5@1500	1@3000	

New angular dependence



$\rightarrow \mathscr{A}(s,t)$

$$s = (p_1 + p_2)^2 \qquad t = (p_1 - p_3)^2 \qquad \cos\theta \sim 1 - \frac{2t}{s}$$

$\mathscr{A}_{SM}$: spin-1 $\rightarrow \propto \cos\theta \sim \dfrac{t}{s}$

- Differential cross section $|\mathscr{A}|^2 \sim t, t^2$: $Y_{l\leq 2,m}$
- QCD corrections factorise, $l \leq 2$ unchanged
- Leading higher l contribution: NLL EW Sudakov $\sim \dfrac{\alpha^2}{16\pi^2}\log\dfrac{t}{m_W^2}$

$\mathscr{A}_{BSM}$: new Lorentz structures

- Higher spin states or contact interactions (4F operators)

Dim 6 (E^2)

$$\mathscr{A} \sim s, t \Rightarrow |\mathscr{A}|^2 : \; l \leq 2$$

Dim 8 (E^4)

$$\mathscr{A} \sim s^2, t^2 \Rightarrow \mathscr{A}_{SM}\mathscr{A}_{EFT} : \; l \leq 3$$

Angular dependence

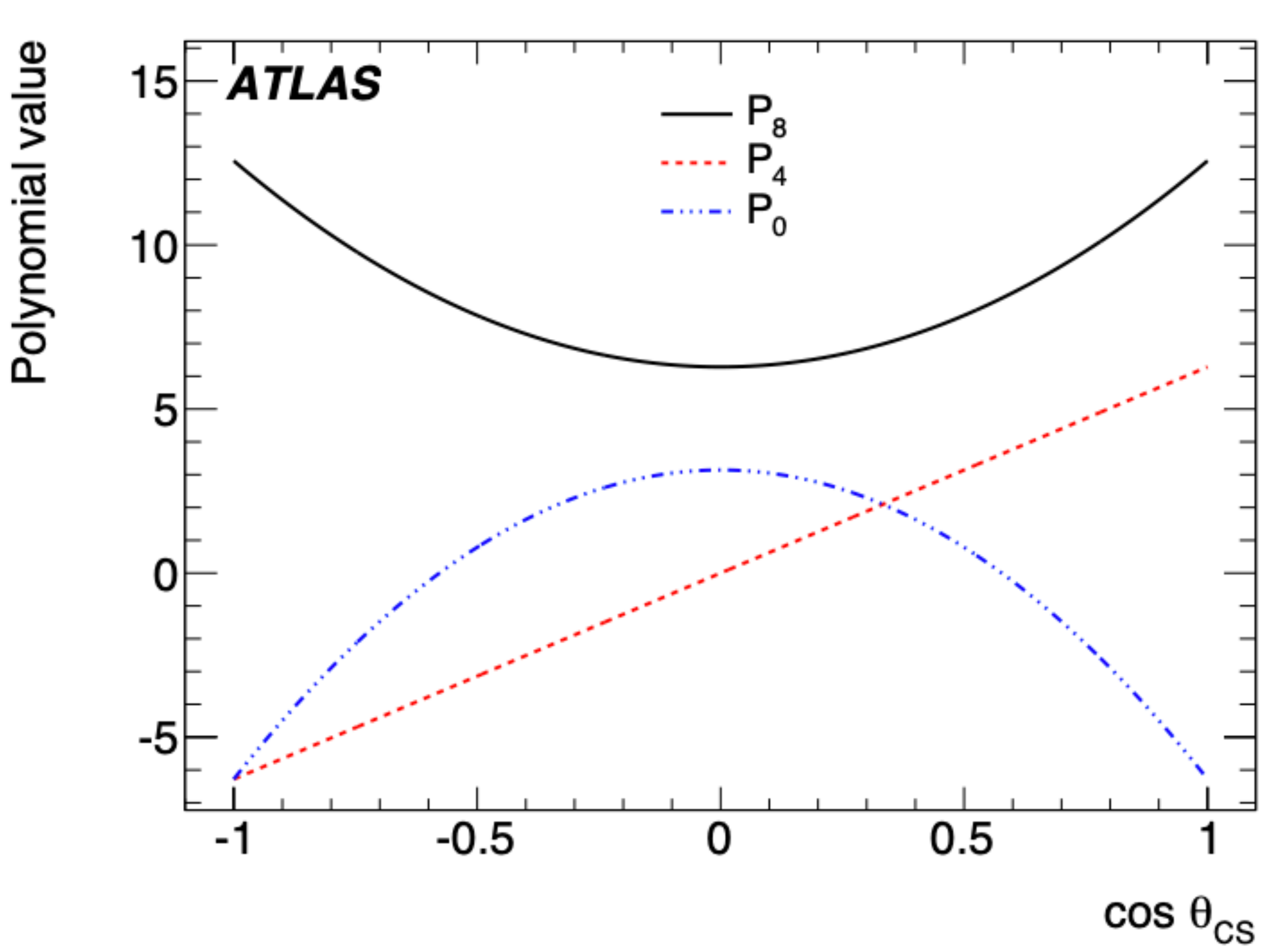
Extracting the $\tilde{A}_i$: moments of spherical harmonics *

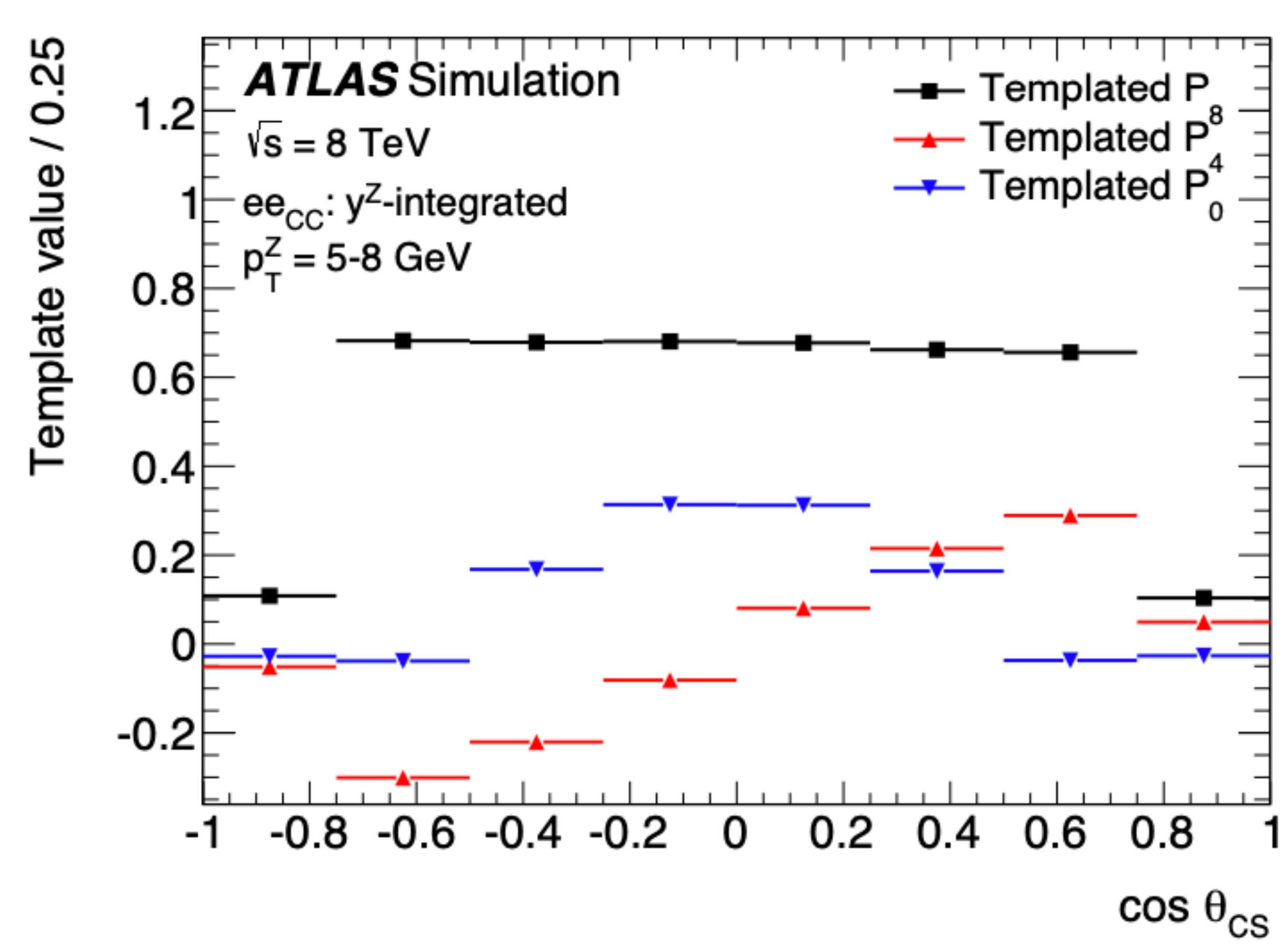
$$\langle f(\theta,\phi)\rangle \equiv \left(\frac{d\sigma}{dm\,d\eta\,d\Omega}\right)^{-1}\int d\Omega_\ell \,\frac{d\sigma}{dm\,d\eta\,d\Omega}\, f(\theta,\phi) \qquad f(\theta,\phi) \propto \left\{Y_{0,0}, Y_{1,0}, Y_{1,\pm1}, Y_{2,0}, Y_{2,\pm1}, Y_{2,\pm2}\right\}$$

- A.K.A. weighted sum of the basis functions over event sample
- $\tilde{A}_i$'s are linear functions of the $\langle Y_{l,m}\rangle$

* In practice, finite experimental acceptance

- Spoils the orthonormality of spherical harmonics



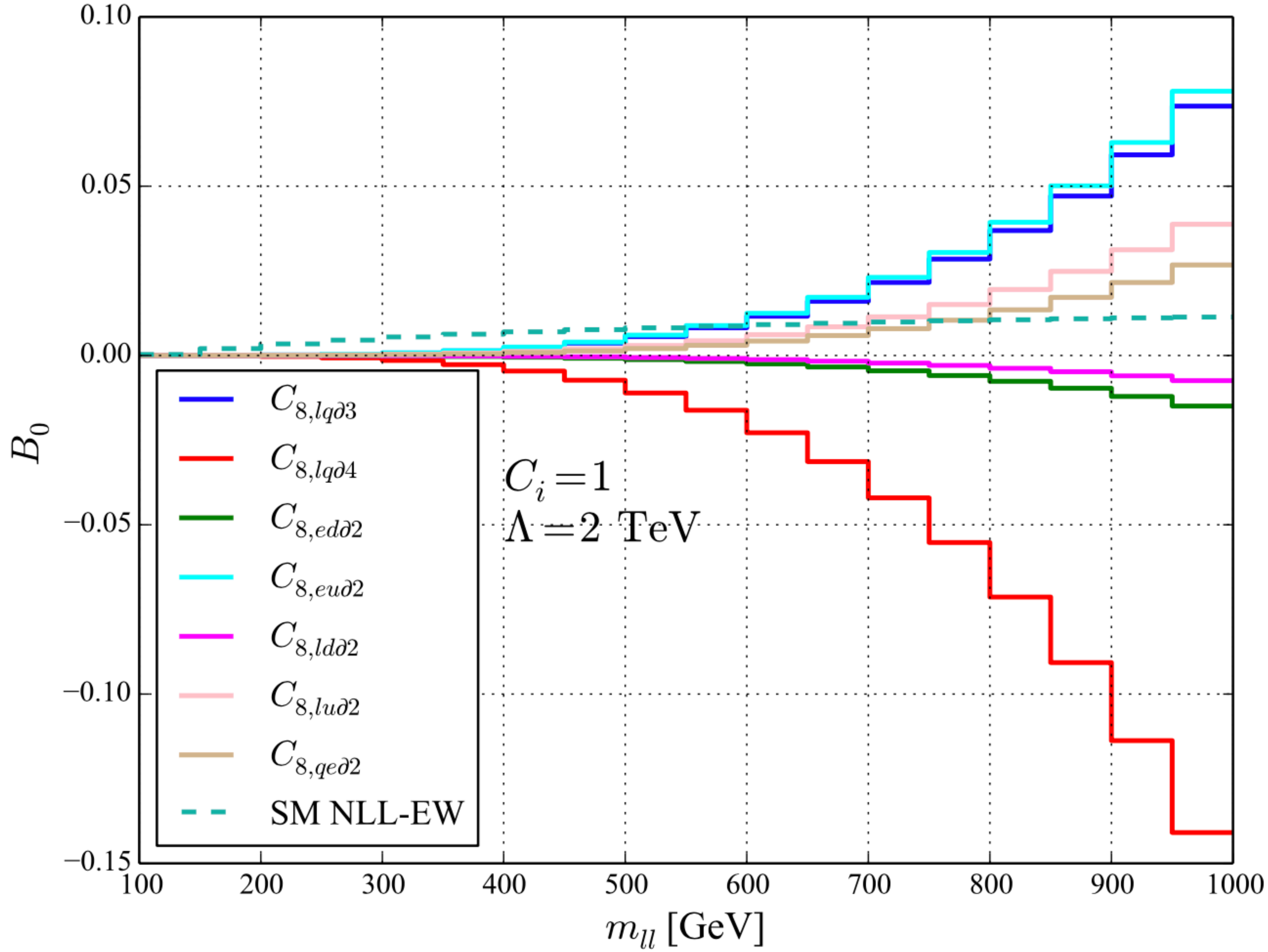


Extracted by fit to signal templates

[CMS; PLB 750 (2015) 154-175]
[ATLAS; JHEP 08 (2016) 159]

$\tilde{B}_0(m_{\ell^+\ell^-})$

[Alioli et al.; PLB 809 (2020) 135703]

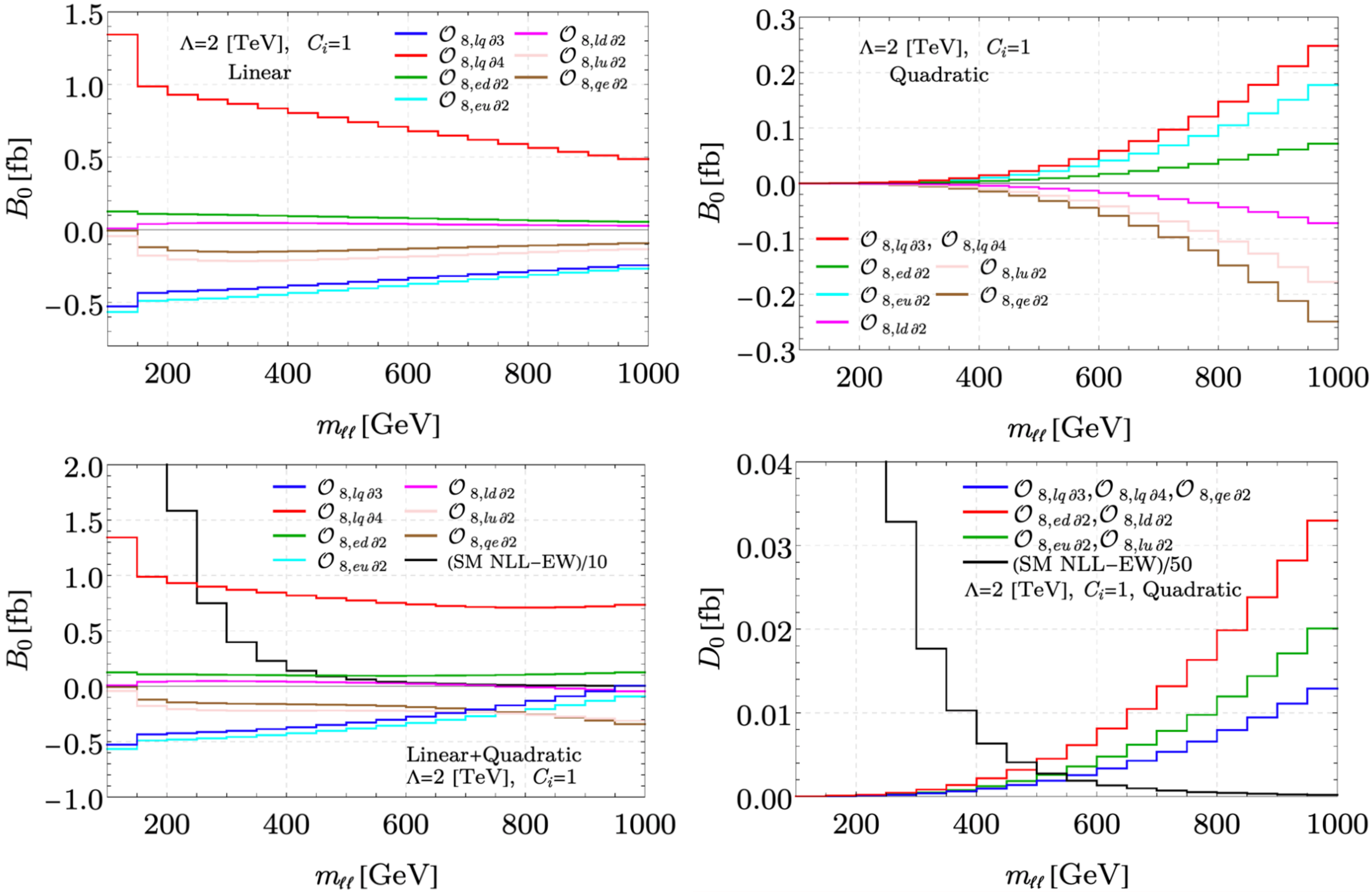


[Li, KM, Yamashita, Yang, Zhang, Zhou; JHEP 10 (2022) 107]

PDF: `NNPDF_nlo_as_0118_luxqed`

LHC predictions

$$\sqrt{s} = 14\,\mathrm{TeV}$$



LHC sensitivity

1 TeV cut to mitigate impact of quadratics

Consider 10 × 10 square $\{m_{\ell\ell}, \eta_{\ell\ell}\}$ binning:

$$m_{\ell\ell}\colon \{100,\ 190,\ 280,\ 370,\ 460,\ 550,\ 640,\ 730,\ 820,\ 910,\ 1000\}\,\text{GeV},$$
$$\eta_{\ell\ell}\colon \{-5,\ -4,\ -3,\ -2,\ -1,\ 0,\ 1,\ 2,\ 3,\ 4,\ 5\},$$

Binned $\Delta\chi^2$, combining (B_0, D_0), for $L_{\text{int.}} = 3000\,\text{fb}^{-1}$

$$\chi^2(C_i) \equiv \Delta\chi^2(C_i) = \sum_i \left(B_0^i(\vec{C}), D_0^i(\vec{C})\right) \cdot \mathbf{V}^{-1} \cdot \left(B_0^i(\vec{C}), D_0^i(\vec{C})\right) \leq 3.84,$$

- B_0 & D_0 are correlated: statistical covariance matrix $\mathbf{V}$

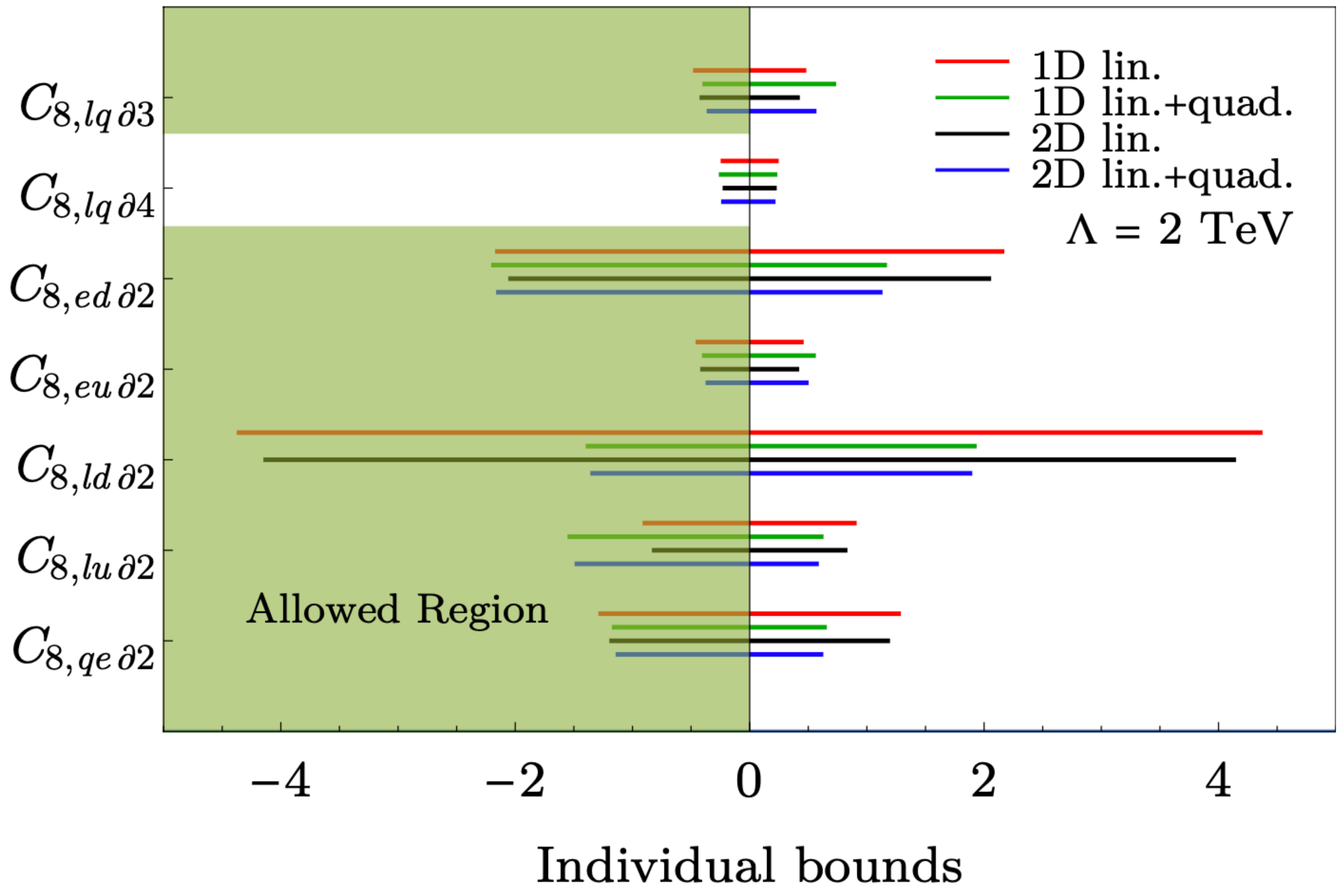
$$V_{ij} = \frac{1}{L}\int_{m_{\text{min.}}}^{m_{\text{max.}}} dm_{\ell\ell} \int_{\eta_{\text{min.}}}^{\eta_{\text{max.}}} d\eta_{\ell\ell} \int_{-1}^{1} dc_\theta \frac{d\sigma_{pp\to\ell^-\ell^+}}{d\eta_{\ell\ell}\, dm_{\ell\ell}\, dc_\theta} \cdot F_{ij}(c_\theta),$$

(co)variance of weighted average(s)

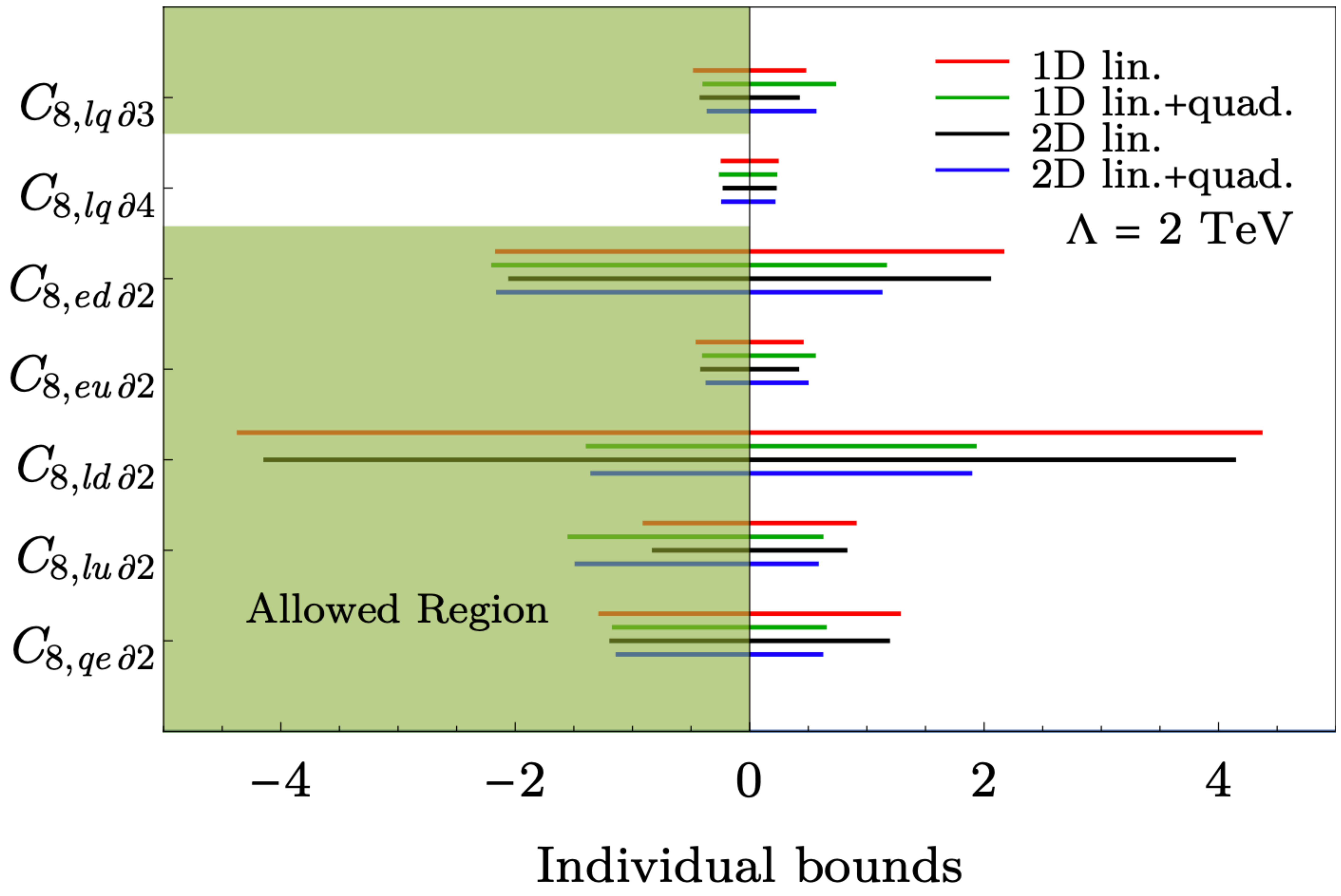
$$F_{11} = \frac{448\pi}{9}\left(Y_3^0(c_\theta)\right)^2; \quad F_{22} = \frac{36\pi^3}{49}\left(Y_4^0(c_\theta)\right)^2; \quad F_{12} = F_{21} = \sqrt{\frac{16}{7}}4\pi^2 Y_3^0(c_\theta) Y_4^0(c_\theta)$$

- Variances dominated by SM, computed @ NLO QCD with `mg5`

Individual bounds on C_i



Individual bounds on C_i



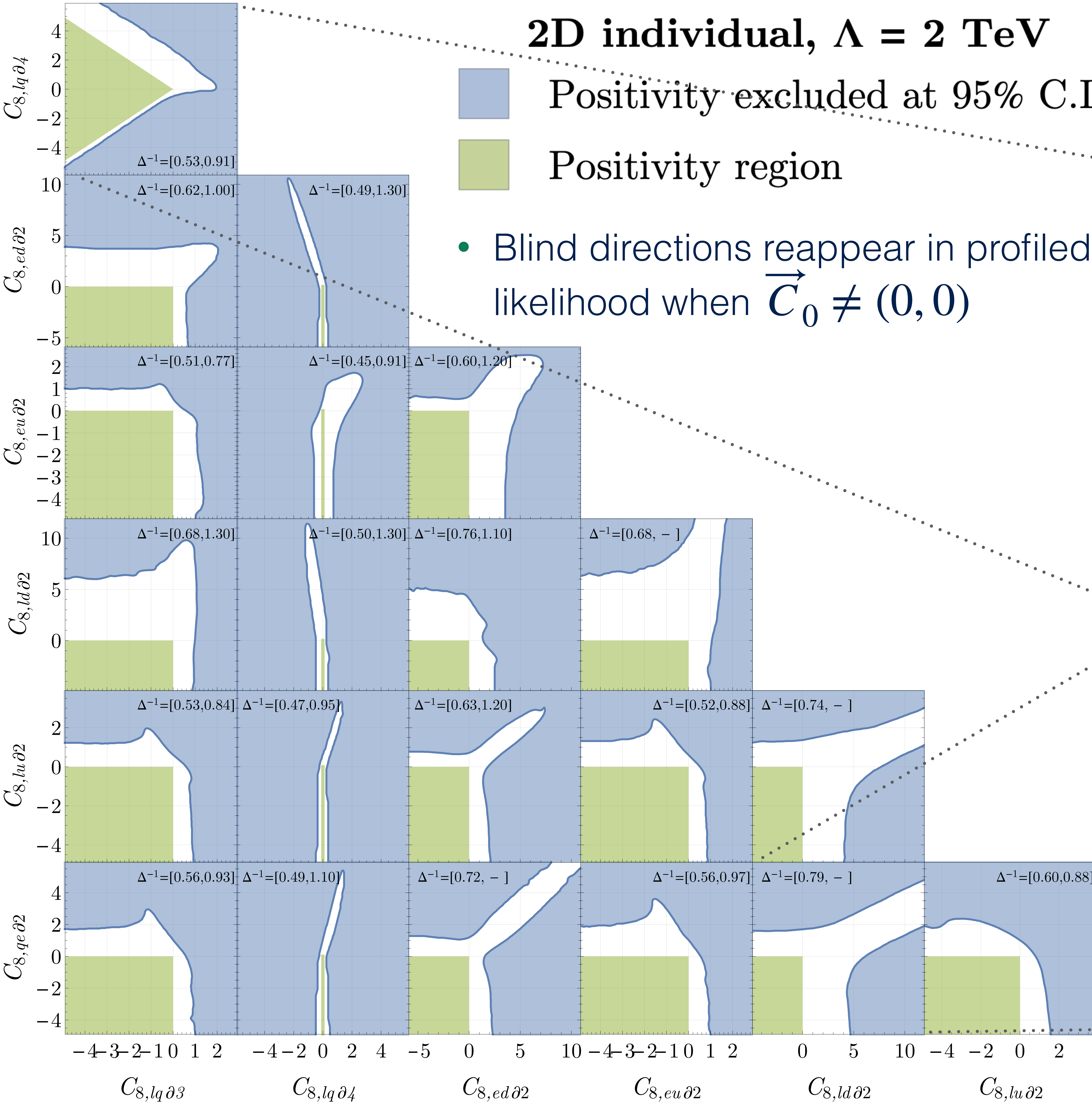
A priori restricted parameter space to consider

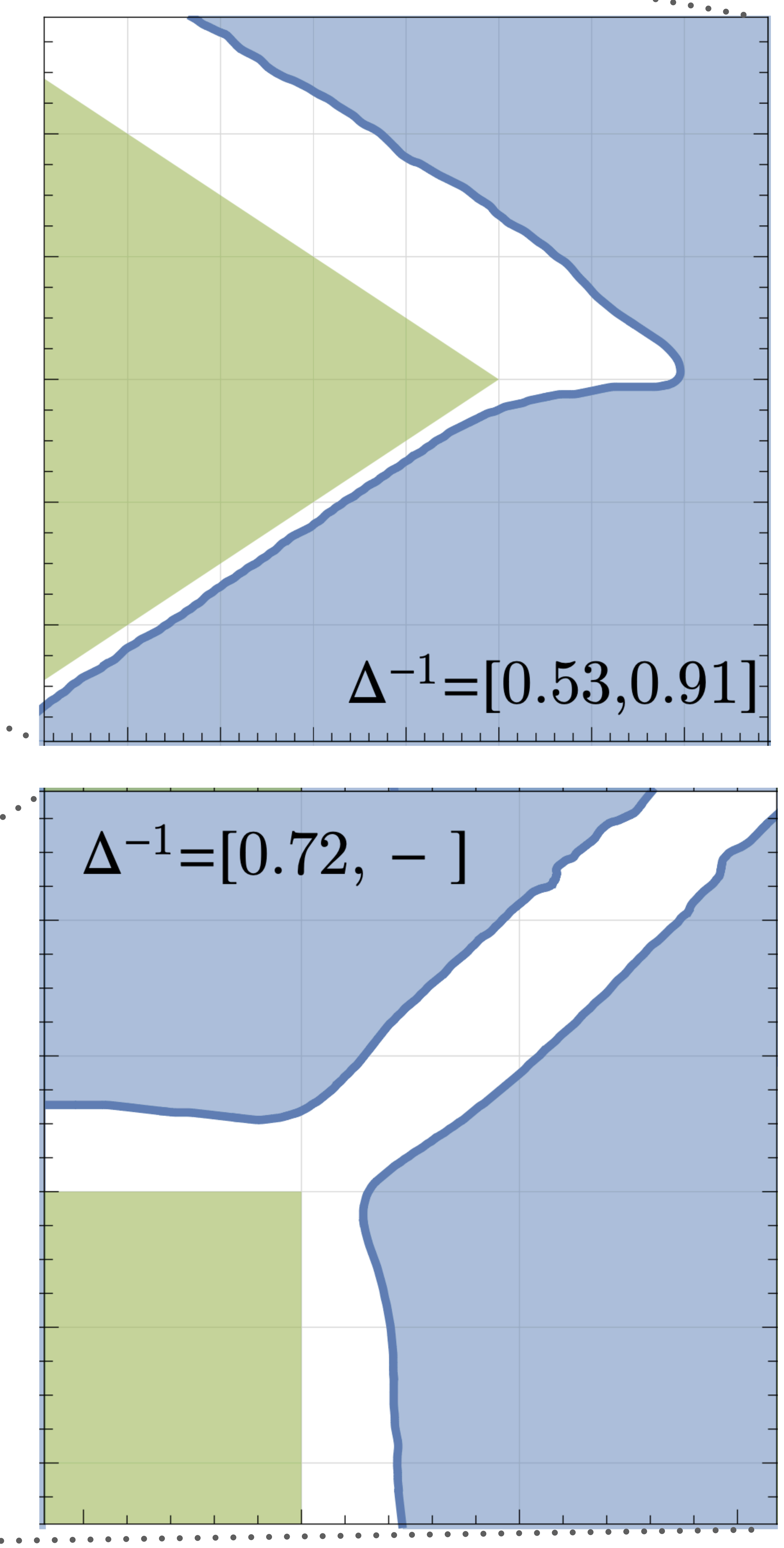
Can also be used to search for violations of positivity

Connection to the “inverse problem”

2D individual, Λ = 2 TeV

- Blind directions reappear in profiled likelihood when $\overrightarrow{C}_0 \neq (0,0)$





More information?

Positivity cone uses "half" of UV amplitude information

$$\frac{1}{2}\frac{d^2 M_{ijkl}(0)}{ds^2} = \sum_X \int d\Pi_X \int_{\Lambda^2}^{\infty} \frac{d\mu}{2\pi\mu^3}\left(m_{ij}m^*_{kl} + m_{i\tilde{l}}m^*_{k\tilde{j}}\right) \qquad m_{ij} \equiv M_{ij\to X}(\mu)$$

- Partial wave coefficients, $a_{ijkl}(\mu)$, are also bounded from above $\quad 0 < \rho_\ell^{iiii} \leq 2$
- In addition to $s \leftrightarrow u$ crossing symmetry, we have $s \leftrightarrow t$ $\quad \rho_\ell^{ijkl} \equiv \mathrm{Im}[a_\ell^{ijkl}]$

$$\rho_\ell^{ijkl} = (-1)^\ell \rho_\ell^{jikl} = (-1)^\ell \rho^{ijlk}$$

[Caron-Huot & Van Duong; JHEP 05 (2021) 280]
[Du, Zhang & Zhou; JHEP 12 (2021) 115]

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$s \leftrightarrow t$ crossing leads to a series of null constraints

$$0 = \sum_\ell 16(2\ell+1)\int_{\Lambda^2}^{\infty}\frac{d\mu}{\mu^{r+4}}\Big[C_{r,i_r}(\ell)\rho_\ell^{ijkl}(\mu) + D_{r,i_r}(\ell)\rho_\ell^{ijlk}(\mu) + E_{r,i_r}(\ell)\rho_\ell^{ikjl}(\mu) + F_{r,i_r}(\ell)\rho_\ell^{iklj}(\mu) + G_{r,i_r}(\ell)\rho_\ell^{iljk}(\mu) + H_{r,i_r}(\ell)\rho_\ell^{ilkj}(\mu)\Big]$$

Testing positivity

7D case: does the allowed region intersect positivity region?

- $\Delta^{-1} = [\Delta^{-1}_{\rm low}, \Delta^{-1}_{\rm high}]$, $\Delta_{\rm low}$ gives conservative estimate (highest scale)
- Uniformly sample a ball of radius 2, with $\Lambda = 1\,{\rm TeV}$