Quantum tops at the FCC

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Introduction

Big interest in the theory community in the past 5 years

Measurement of entanglement in top pair production at the LHC

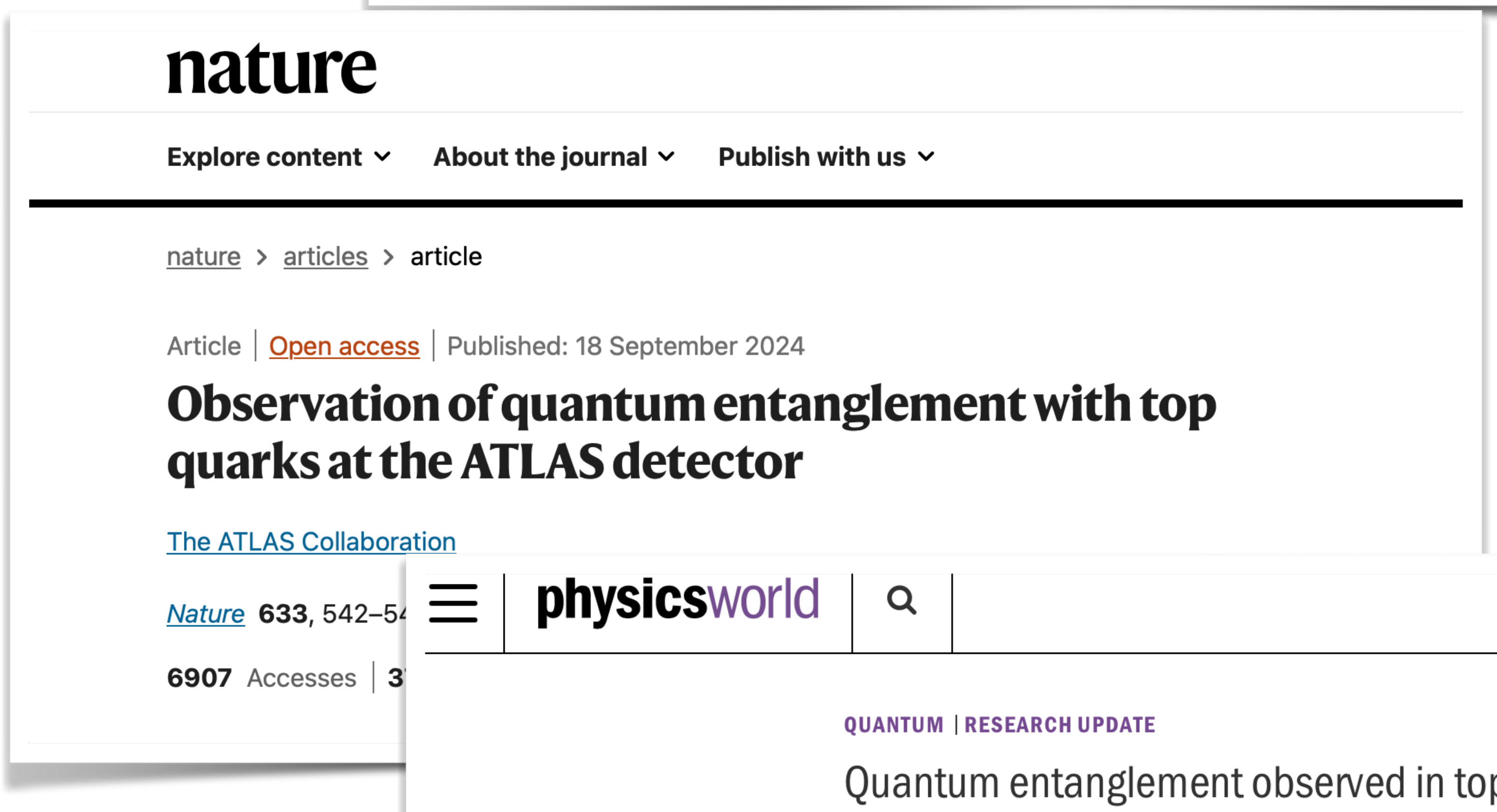
Why is this interesting?

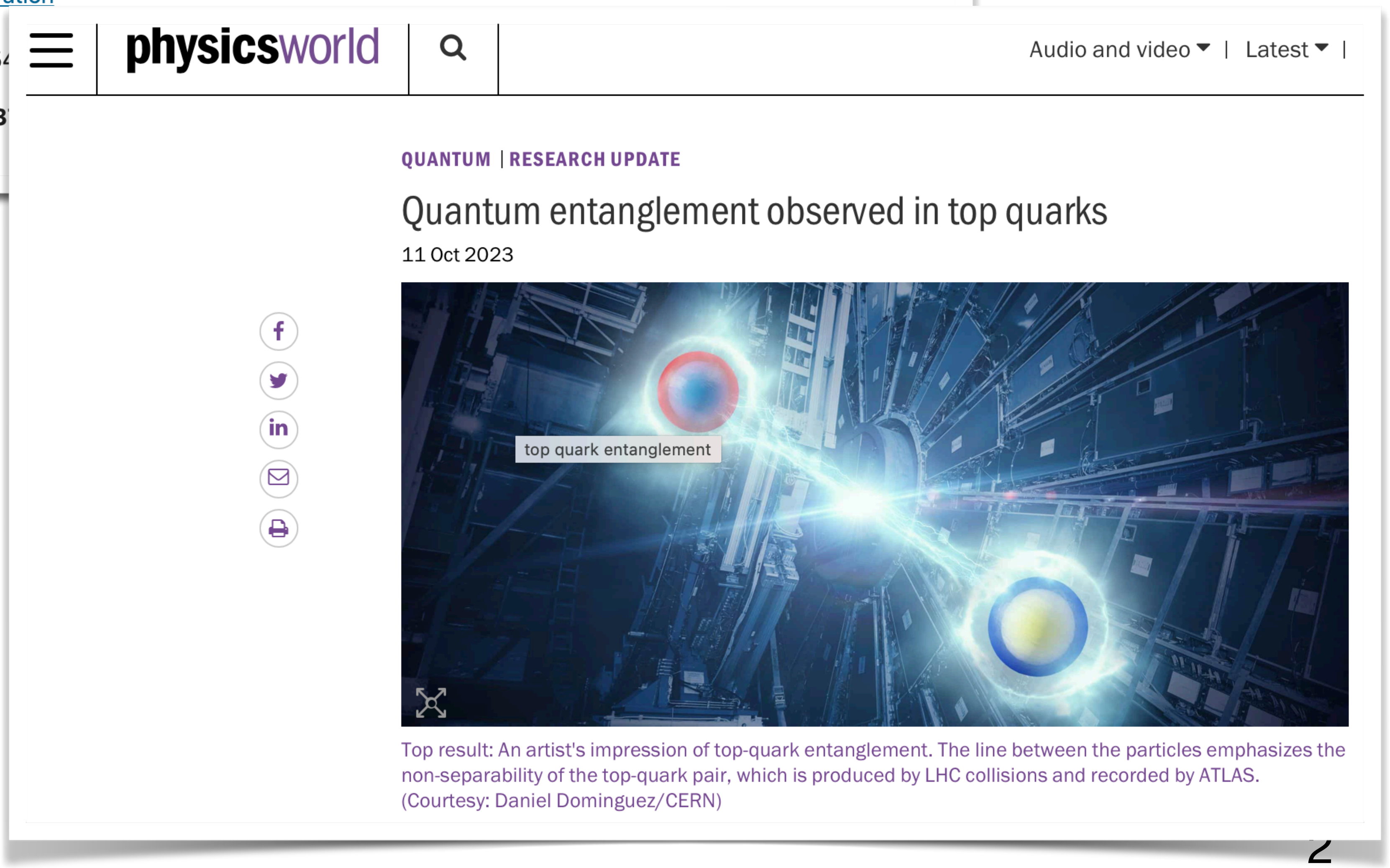
Quantum mechanics at the TeV scale!

What can we learn in particle physics using QM/QI?

New insights and information about new physics

I will talk about top quarks but other systems explored (e.g. see Luca's talk on taus tomorrow)



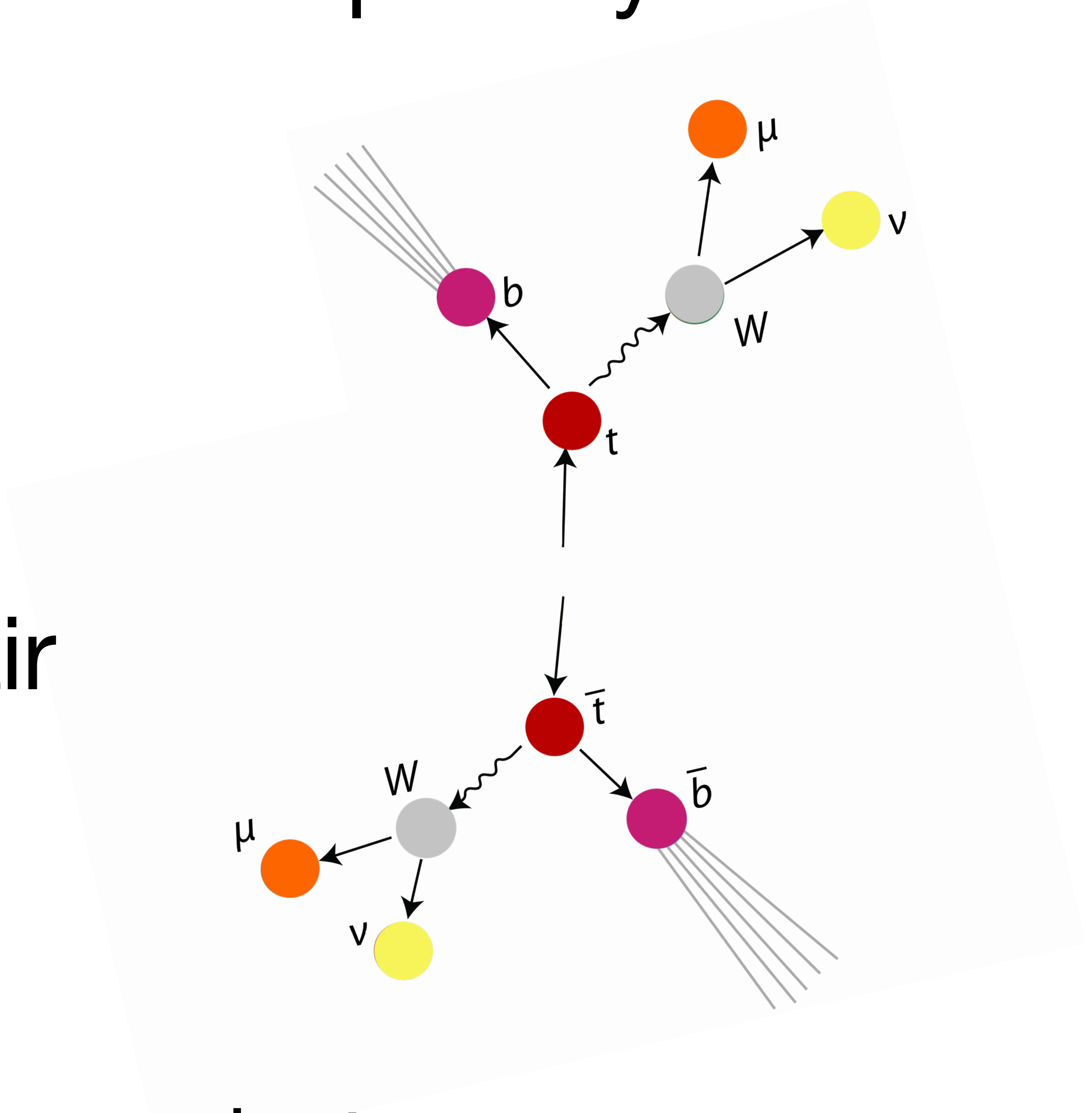


Spin density matrix

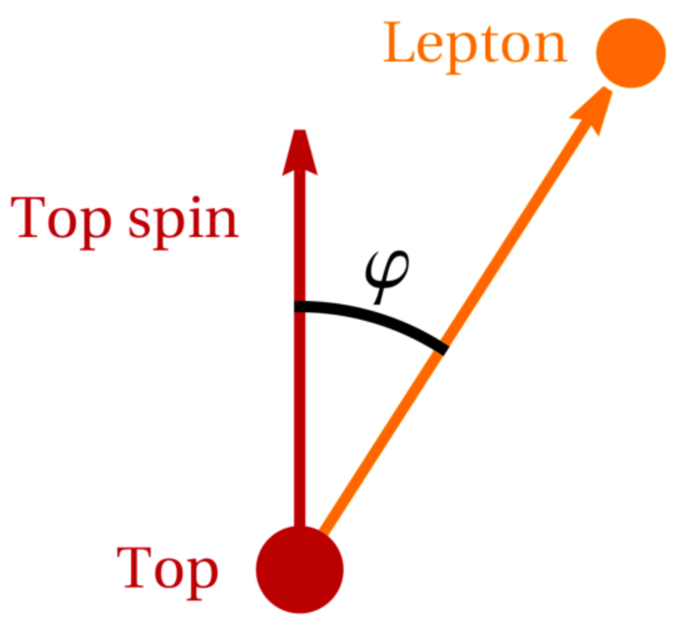
Tops produced in pairs have their spins S_i, S_j correlated: a two-qubit system

Spin density matrix:

$$\rho = \frac{1}{4}\left(\mathbb{1}\otimes\mathbb{1} + \sum_{i=1}^{3} B_i\,\sigma_i\otimes\mathbb{1} + \sum_{i=j}^{3}\bar{B}_j\,\mathbb{1}\otimes\sigma_j + \sum_{i=1}^{3}\sum_{j=1}^{3} C_{ij}\,\sigma_i\otimes\sigma_j\right)$$



15 parameters describe the quantum state of the top pair

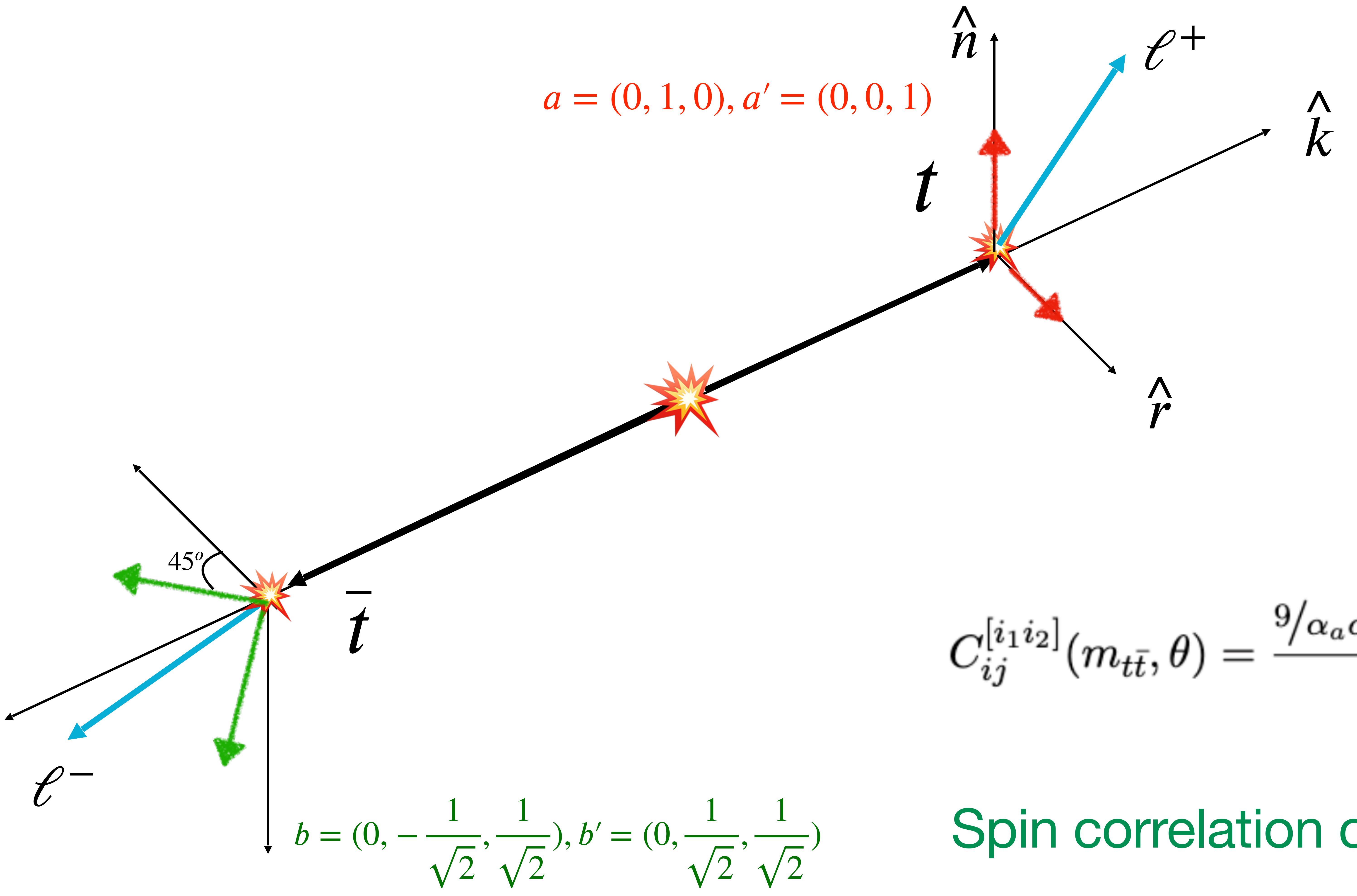


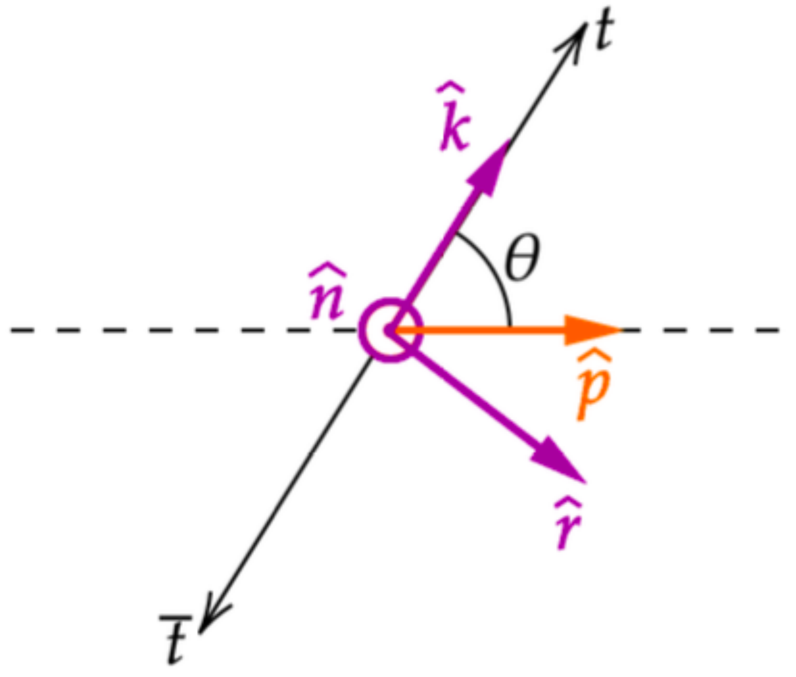
$$\langle S_i\rangle = B_i,\quad \langle \bar{S}_i\rangle = \bar{B}_j,\quad \langle S_i\bar{S}_j\rangle = C_{ij}$$

Extracted by measuring angular distributions of decay products

Quantum tomography is measurement of 15 parameters: 6 polarisations and 9 correlations

Kinematics





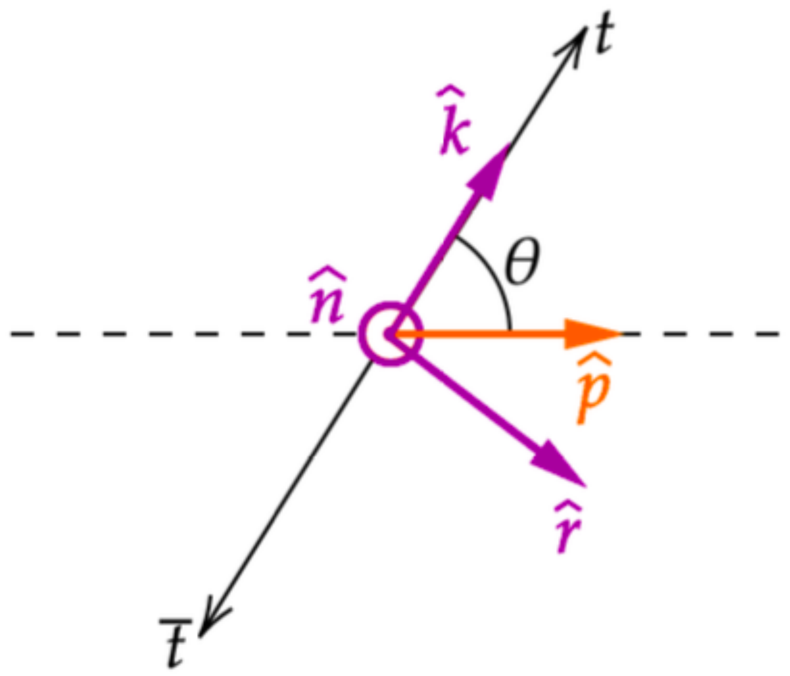
$$\hat{k} = \text{top direction}, \quad \hat{r} = \frac{\hat{p} - \hat{k}\cos\theta}{\sin\theta}, \quad \hat{n} = \frac{\hat{p} \times \hat{k}}{\sin\theta}$$

Helicity basis

$$C_{ij}^{[i_1 i_2]}(m_{t\bar{t}}, \theta) = \frac{9/\alpha_a\alpha_b \int \cos\theta_{ai}\cos\theta_{bj} \, |\mathcal{M}_{i_1 i_2 \to t\bar{t} \to a\, b\, X}|^2 \, d\pi}{\int |\mathcal{M}_{i_1 i_2 \to t\bar{t} \to a\, b\, X}|^2 \, d\pi}$$

Spin correlation coefficients are averages of angles

From spin correlations to entanglement



$$\hat{k} = \text{top direction}, \quad \hat{r} = \frac{\hat{p} - \hat{k}\cos\theta}{\sin\theta}, \quad \hat{n} = \frac{\hat{p}\times\hat{k}}{\sin\theta}$$

$$\begin{aligned} D^{(1)} &= 1/3(+C_{kk} + C_{rr} + C_{nn}), \\ D^{(k)} &= 1/3(+C_{kk} - C_{rr} - C_{nn}), \\ D^{(r)} &= 1/3(-C_{kk} + C_{rr} - C_{nn}), \\ D^{(n)} &= 1/3(-C_{kk} - C_{rr} + C_{nn}). \end{aligned}$$

$$D_{\text{min}} \equiv \min\{D^{(1)}, D^{(k)}, D^{(r)}, D^{(n)}\}$$

Entanglement markers, from the Peres-Horodecki criterion

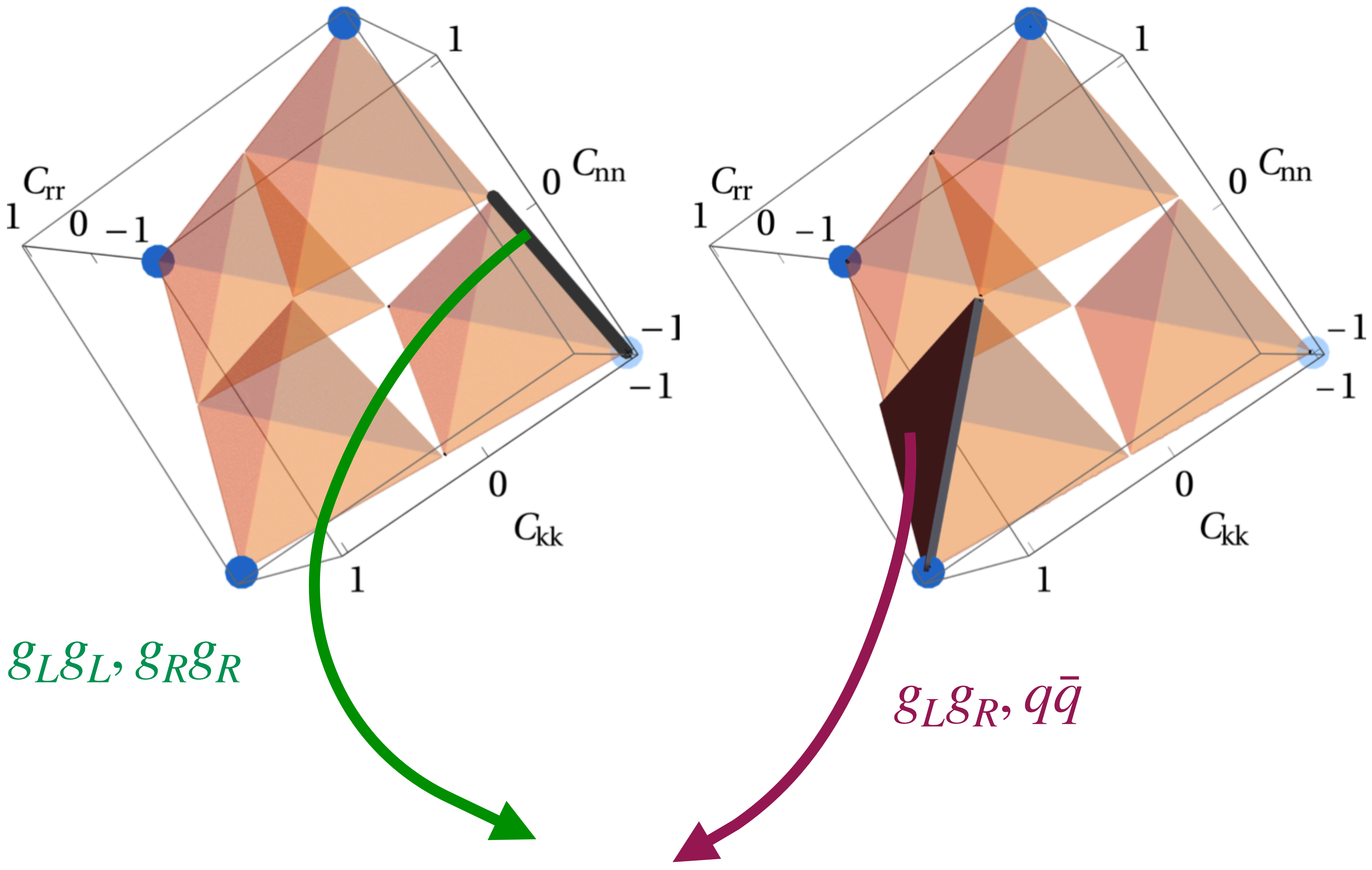
$$D_{\text{min}} < -1/3$$

for a proof see arXiv:2003.02280

Necessary and sufficient condition for entanglement

$$C = \frac{1}{2}\max\left(0, -1 - 3D_{\text{min}}\right) > 0$$

When are tops entangled?



reachable entangled states

Consider top pair production in pp collisions
Which spin states can be reached?

Threshold:

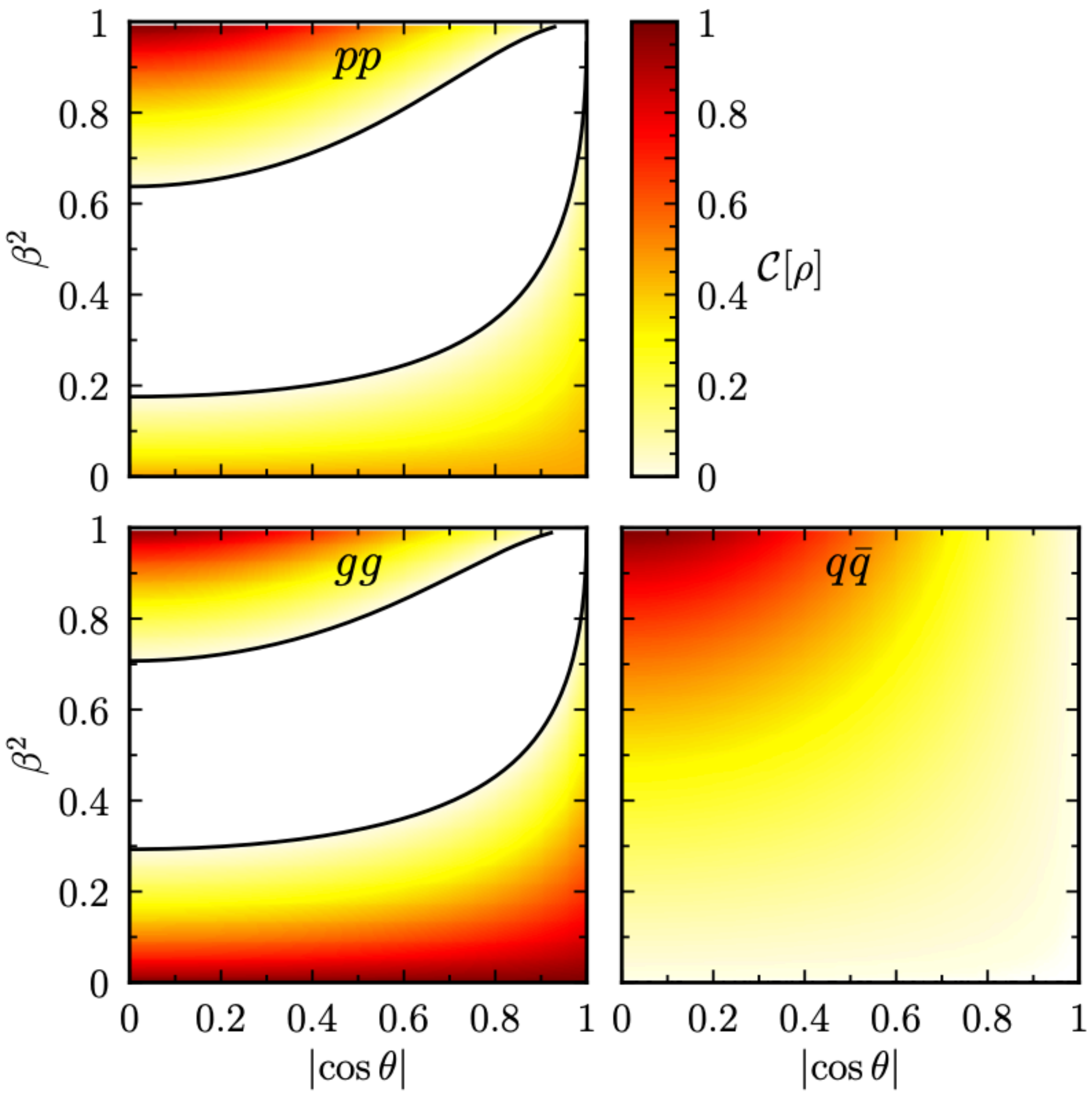
- entangled singlet state
- from same helicity gluons

Boosted:

- entangled triplet state
- for qqbar pairs and opposite helicity gluons

C. Severi, F.Maltoni, S. Tentori, EV: 2404.08049

Entanglement in the SM



Concurrence: $C = \frac{1}{2} \max(0, -1 - 3D_{\min})$

White regions: no entanglement (C<0)

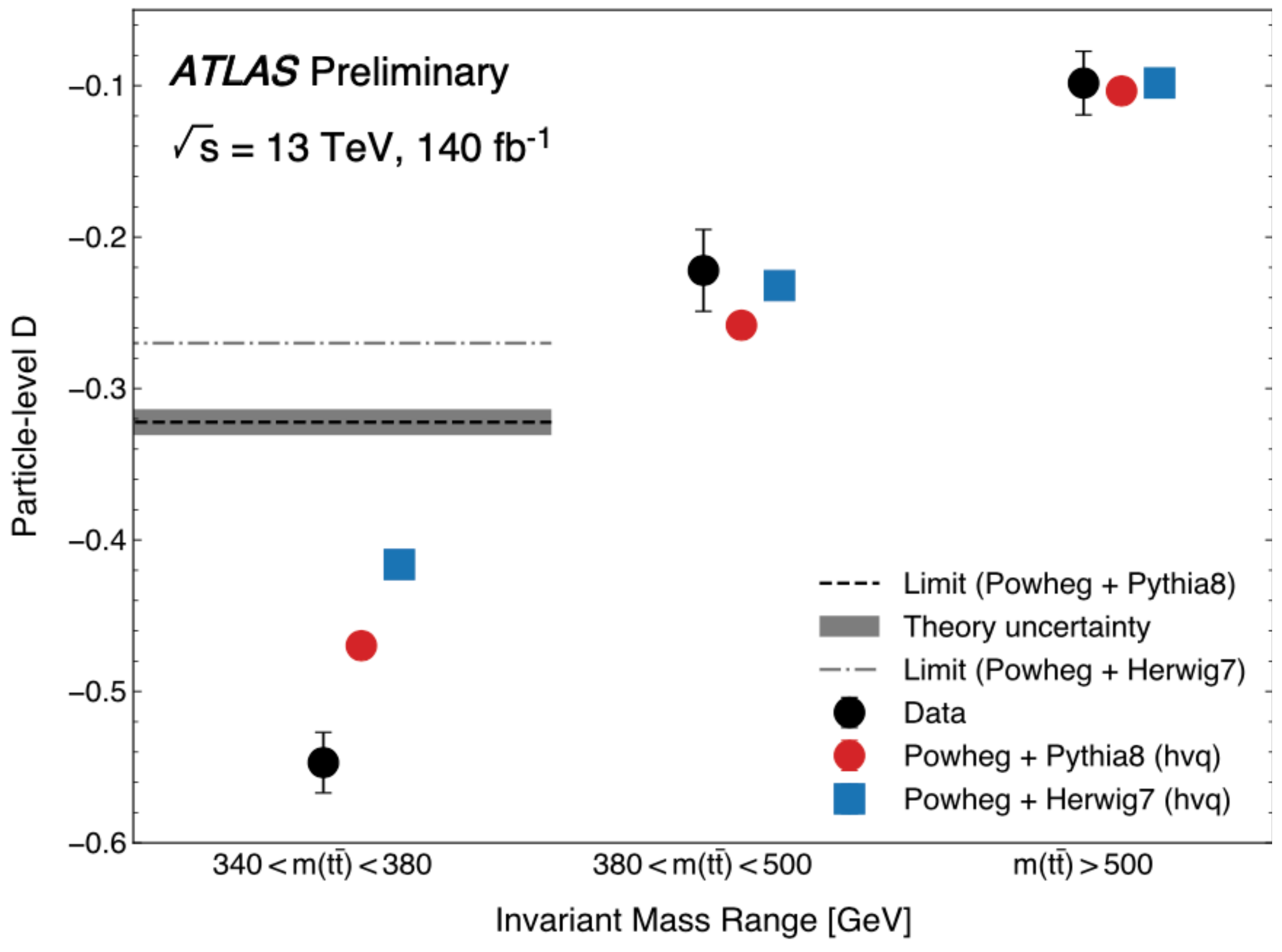
Maximal entanglement regions

At threshold: $\beta^2 = 0, \forall\theta$

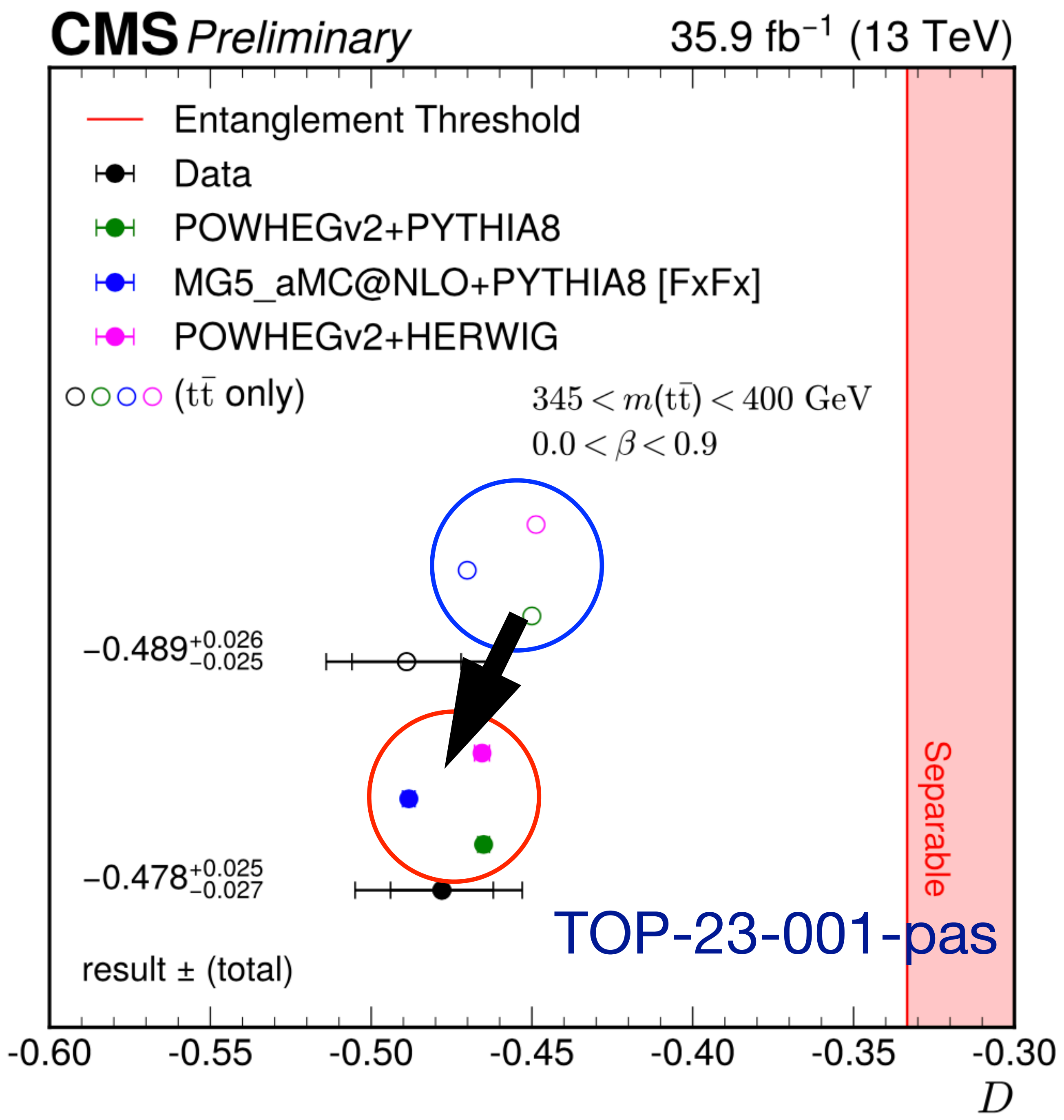
High-Energy: $\beta^2 \to 1, \cos\theta = 0$

C. Severi, C. Boschi, F. Maltoni, M. Sioli : 2110.10112

First measurements



ATLAS-CONF-2023-069



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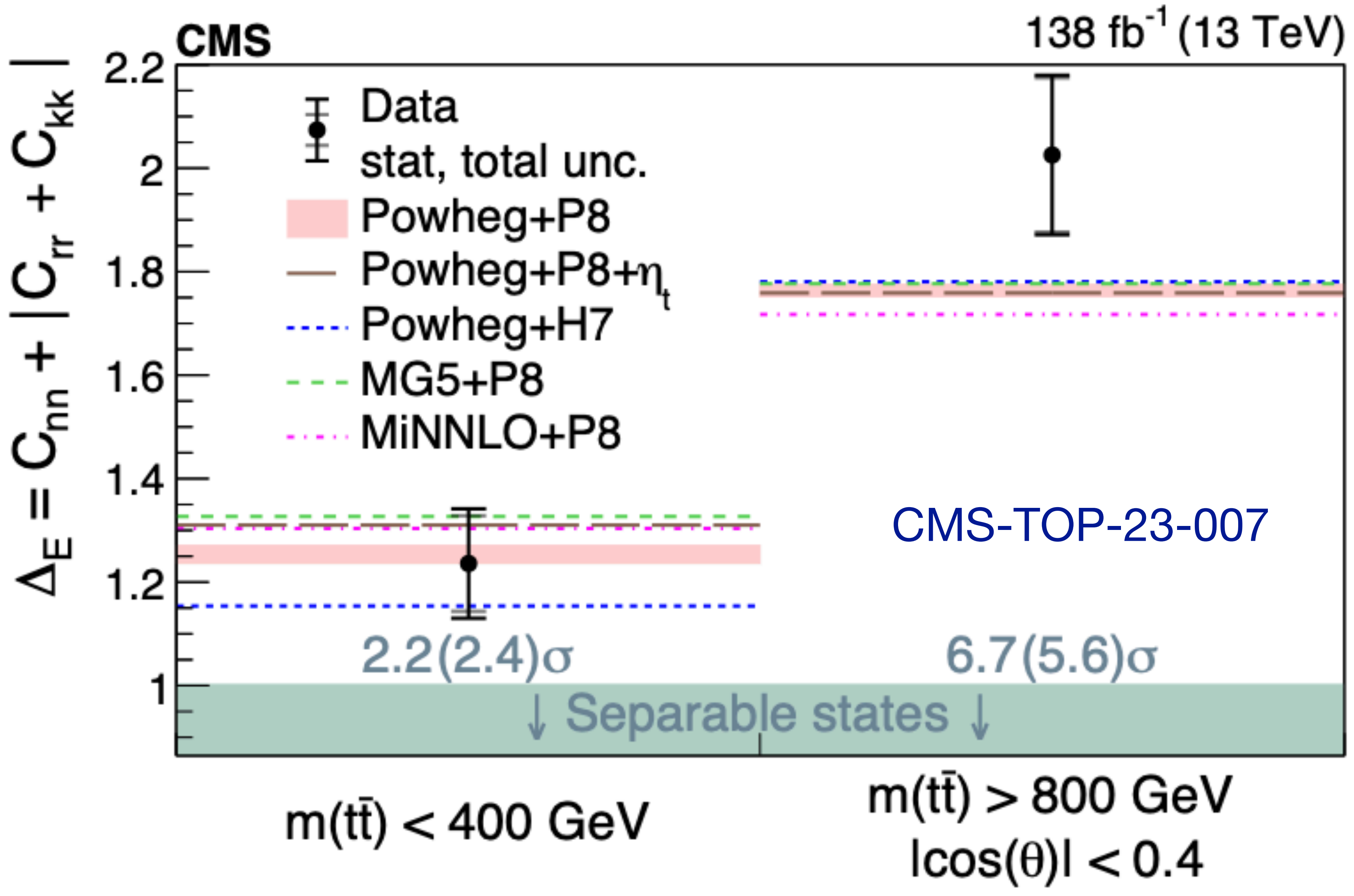


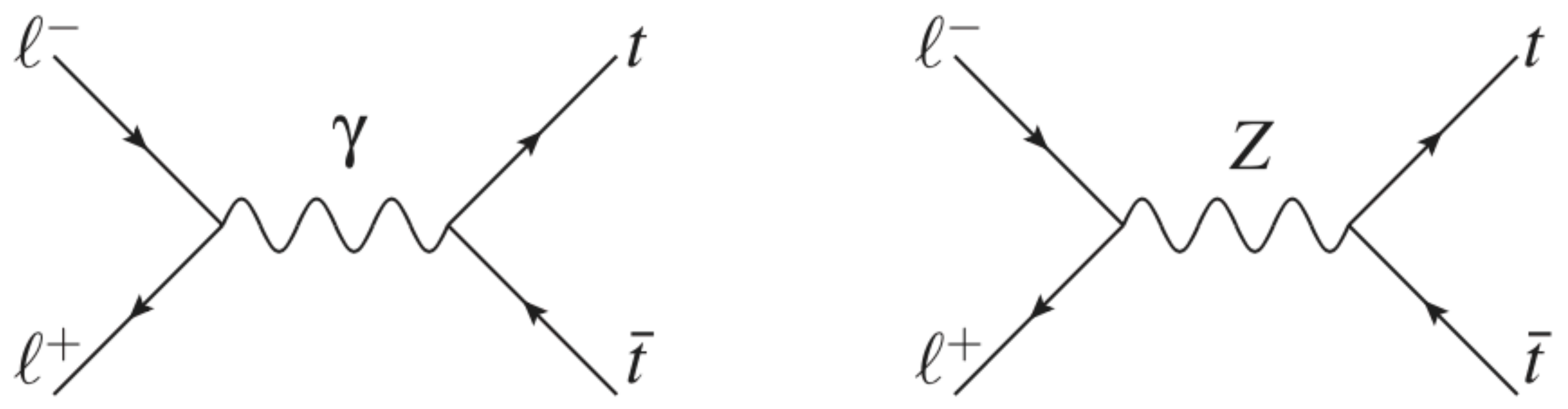
CMS-TOP-23-007

Entanglement observation at threshold by ATLAS and CMS

Entanglement observation at high energy by CMS

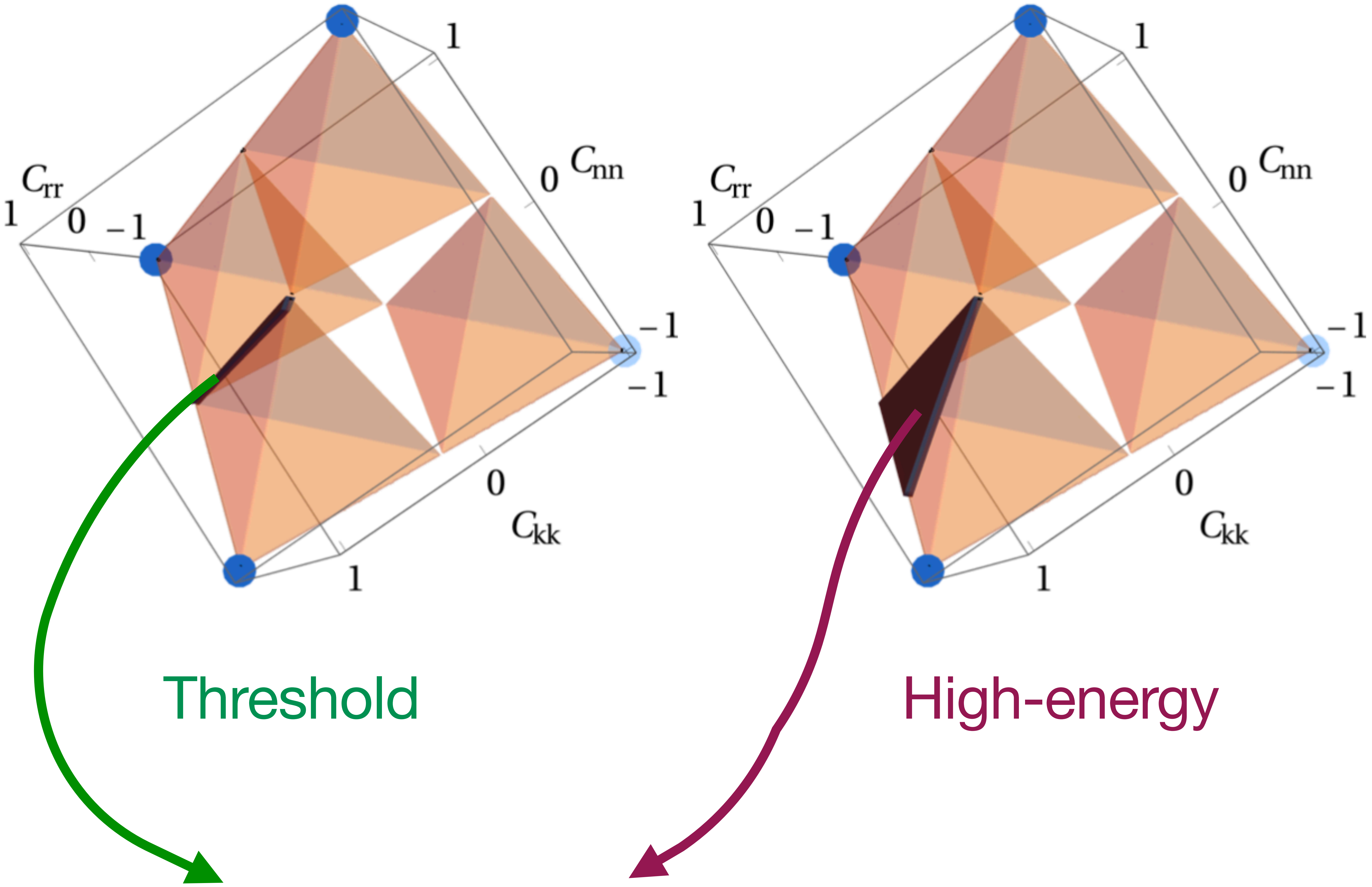
CMS-TOP-23-007

Tops in lepton colliders



$$1/3\,\mathrm{Tr}\,[\mathcal{C}] = D^{(1)} = +\frac{1}{3},$$

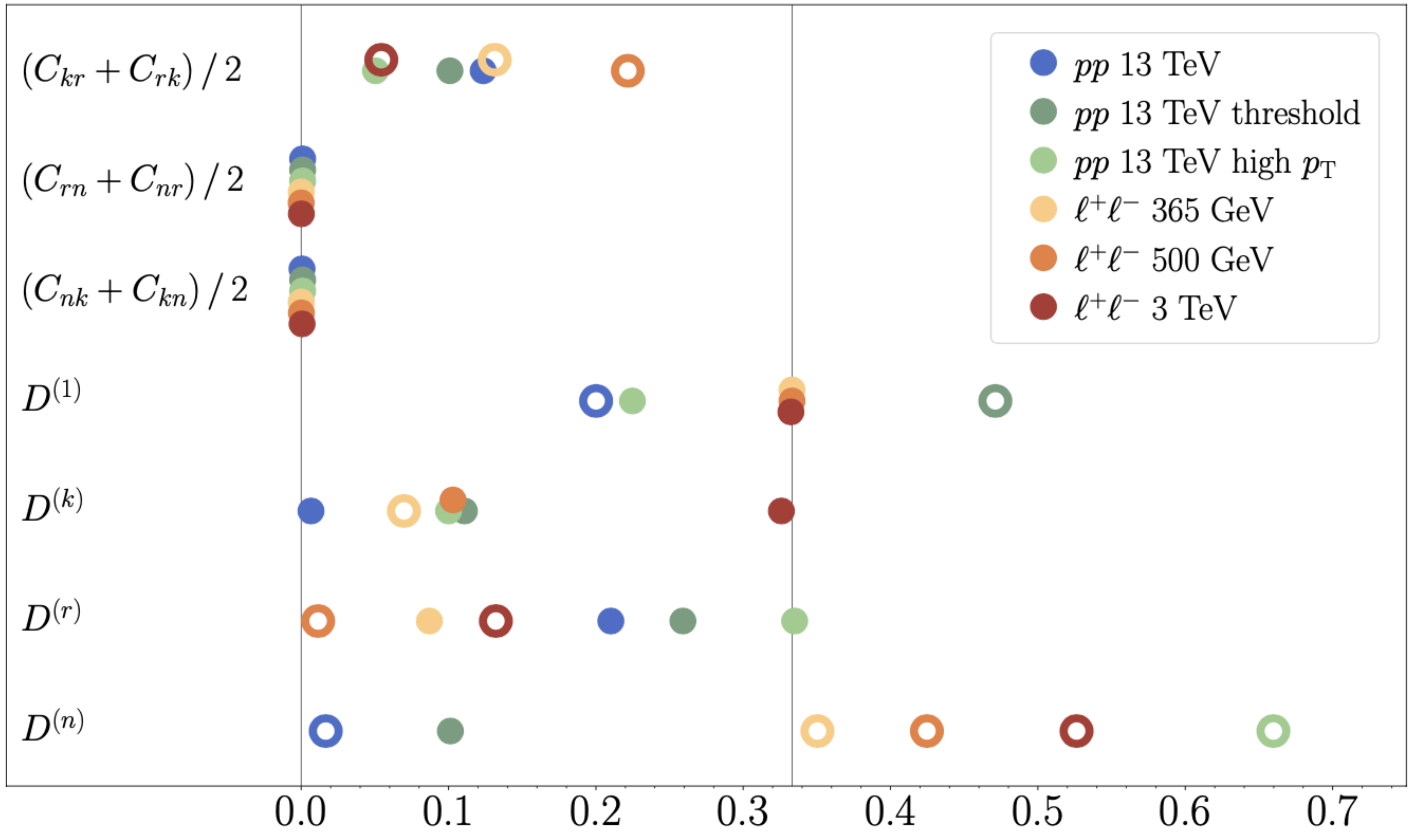
Spin-1 exchange
Spin triplet state



reachable entangled states

C. Severi, F. Maltoni, S. Tentori, EV: 2404.08049

Lepton vs pp collisions



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- Spin Triplet state $D^{(1)} = +1/3$
- Entanglement through $D^{(n)}$ for lepton colliders
- Entanglement through $D^{(1)}$ for LHC at threshold
- Entanglement through $D^{(n)}$ for LHC at high transverse momentum

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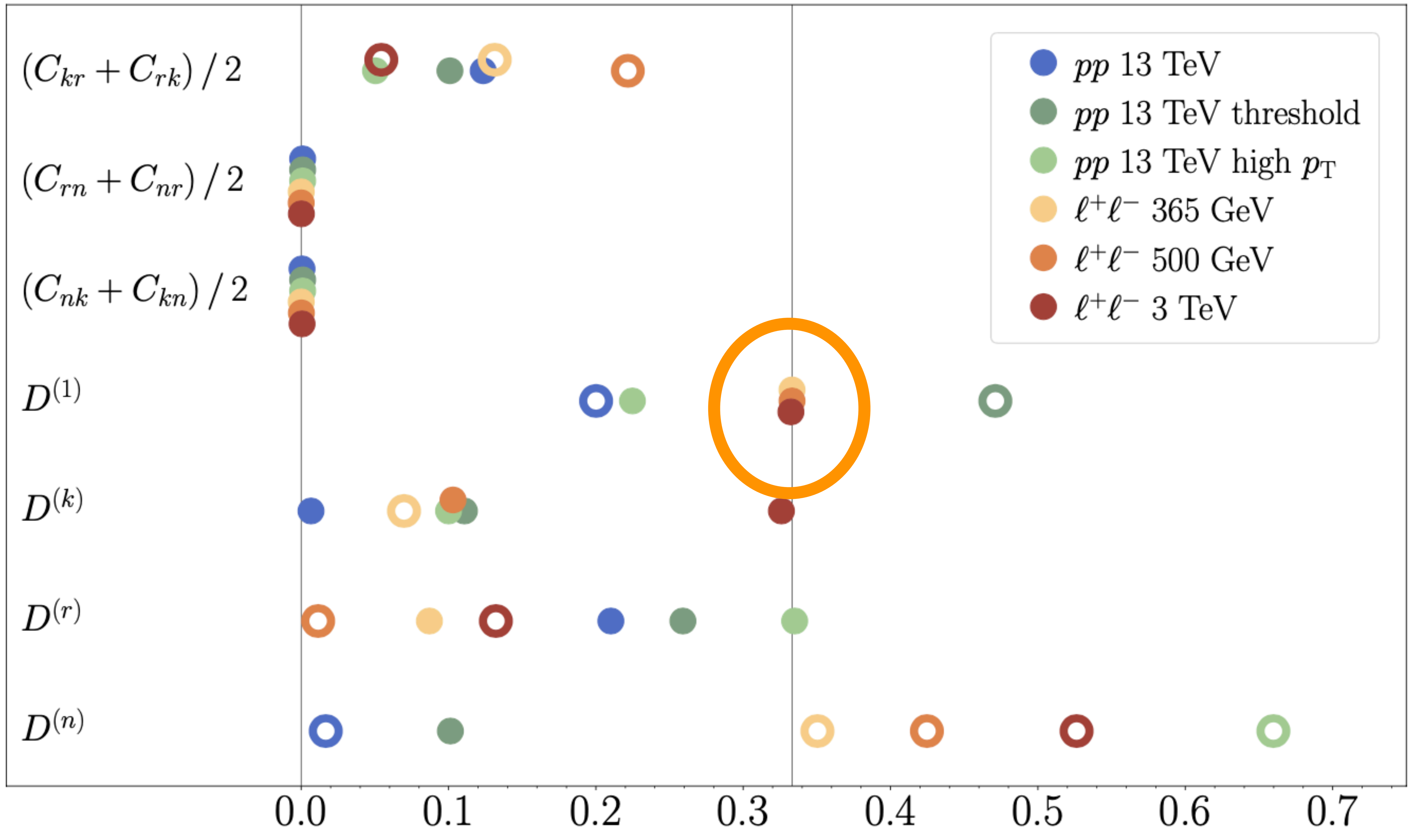

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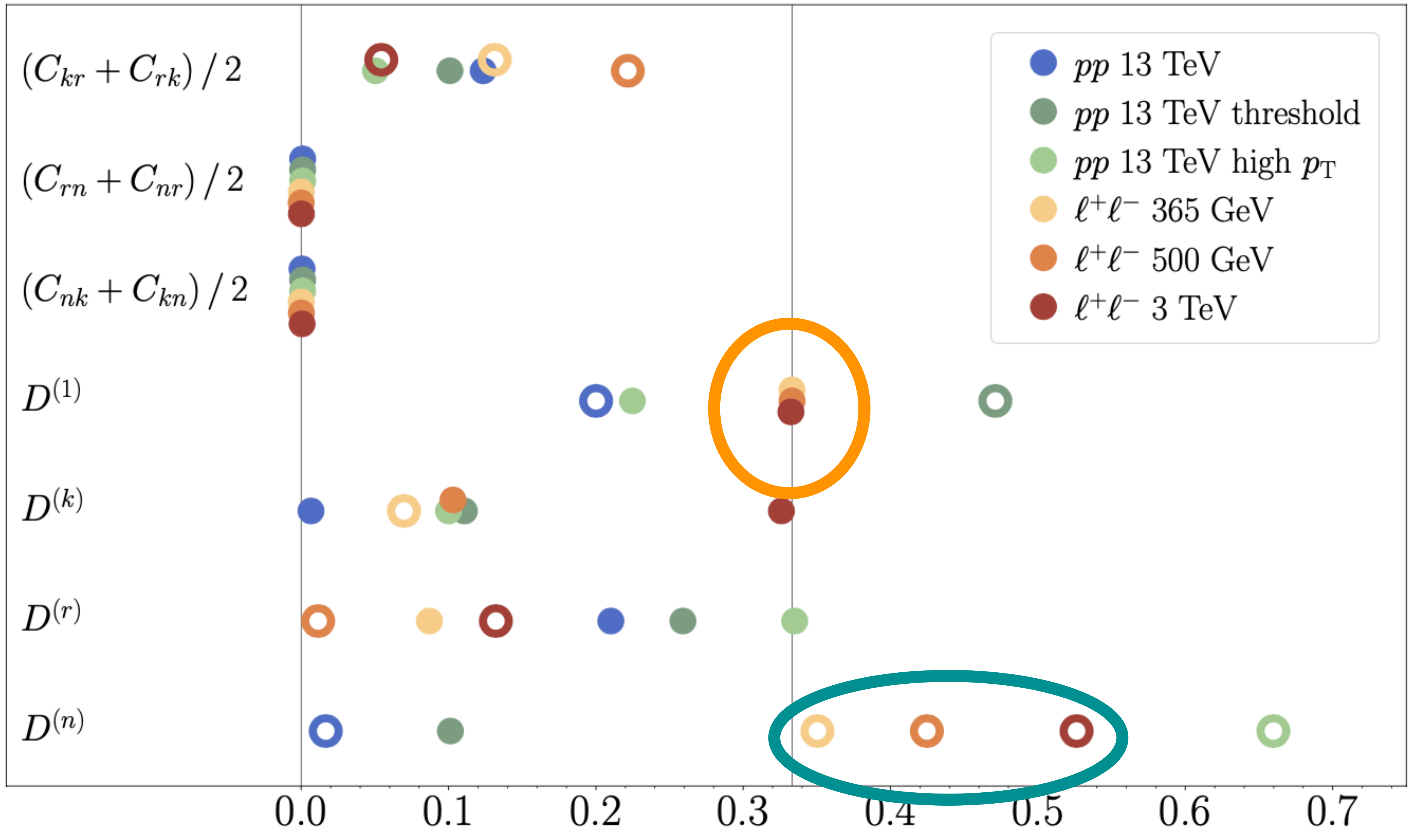

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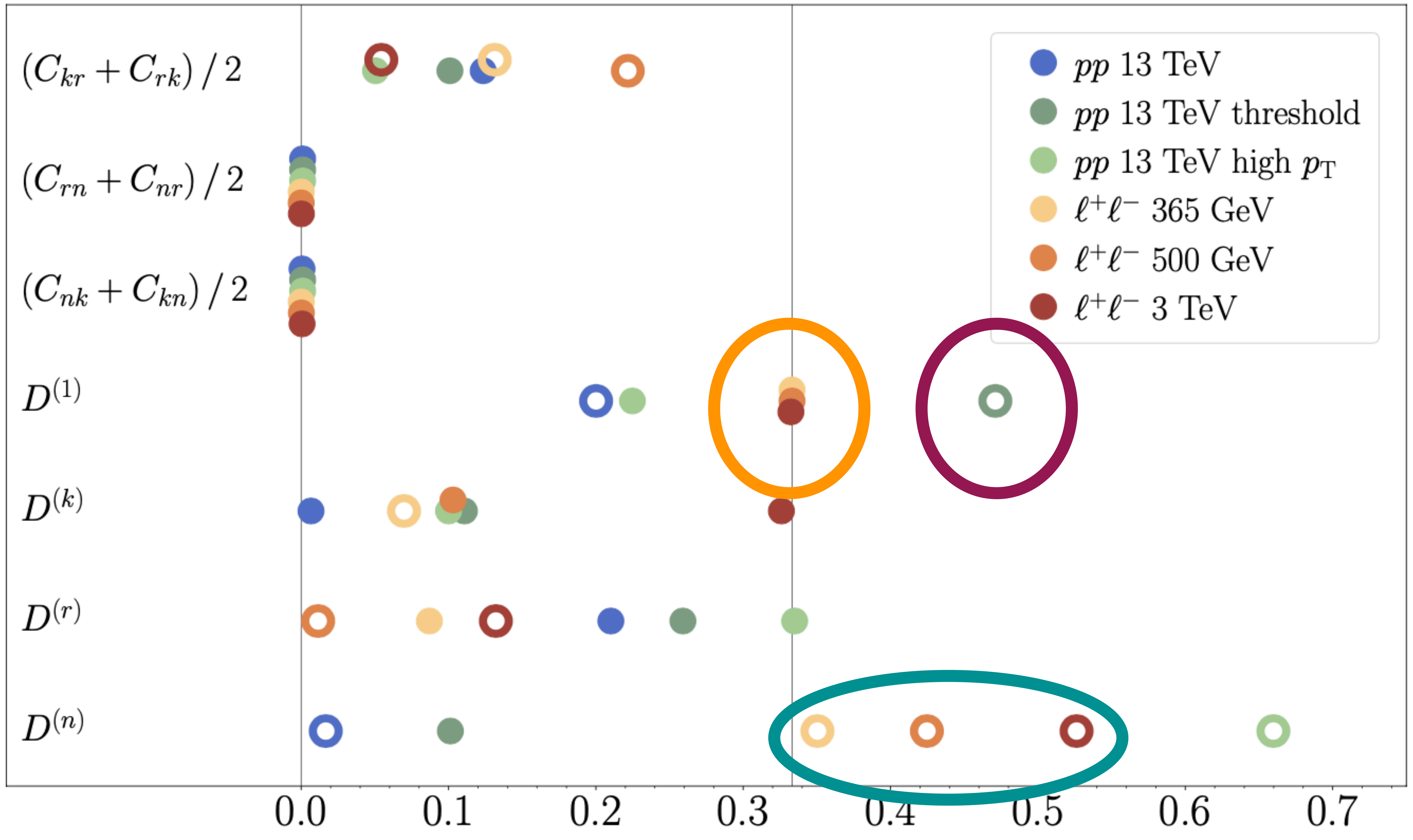

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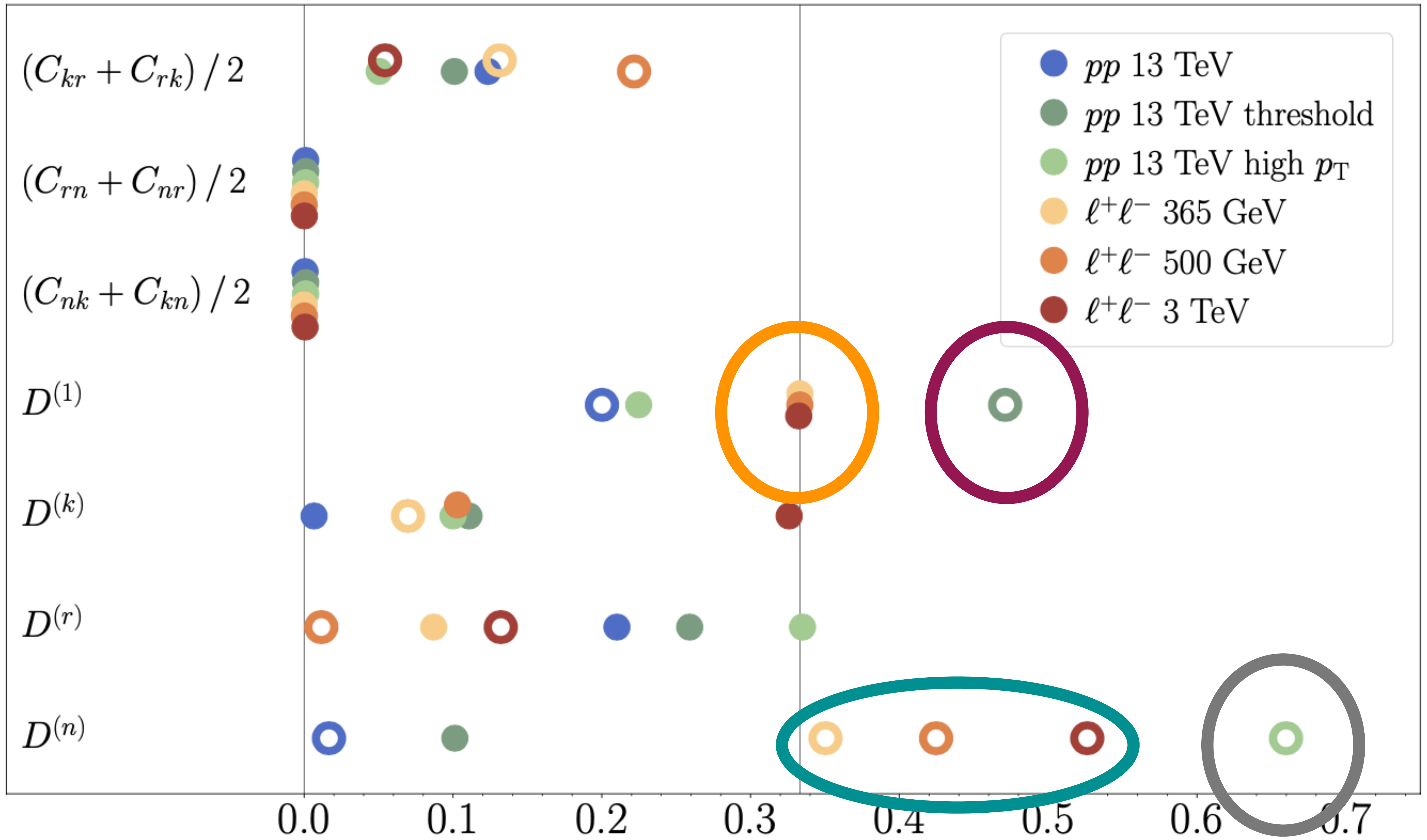

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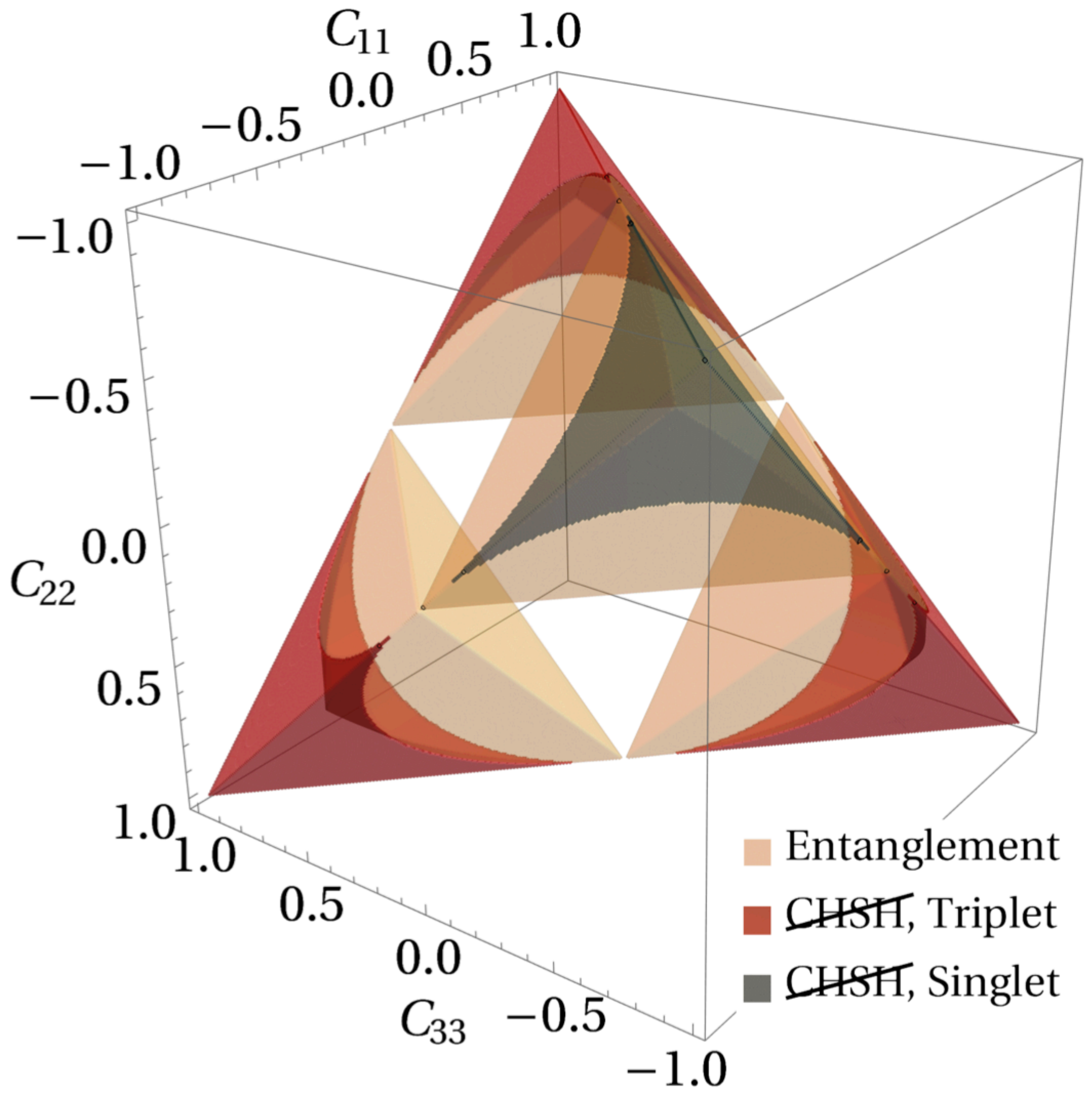
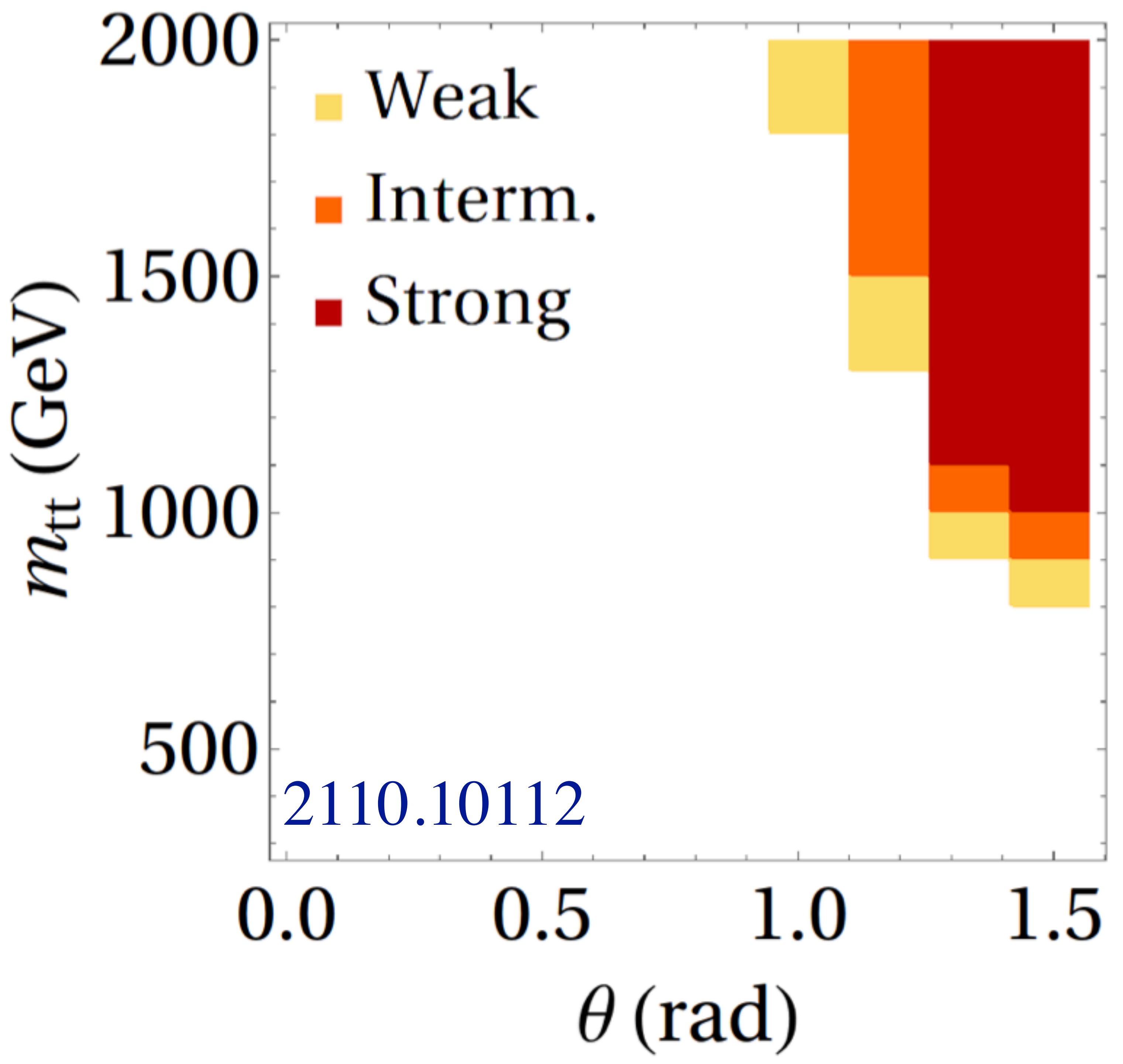
How about Bell inequalities?





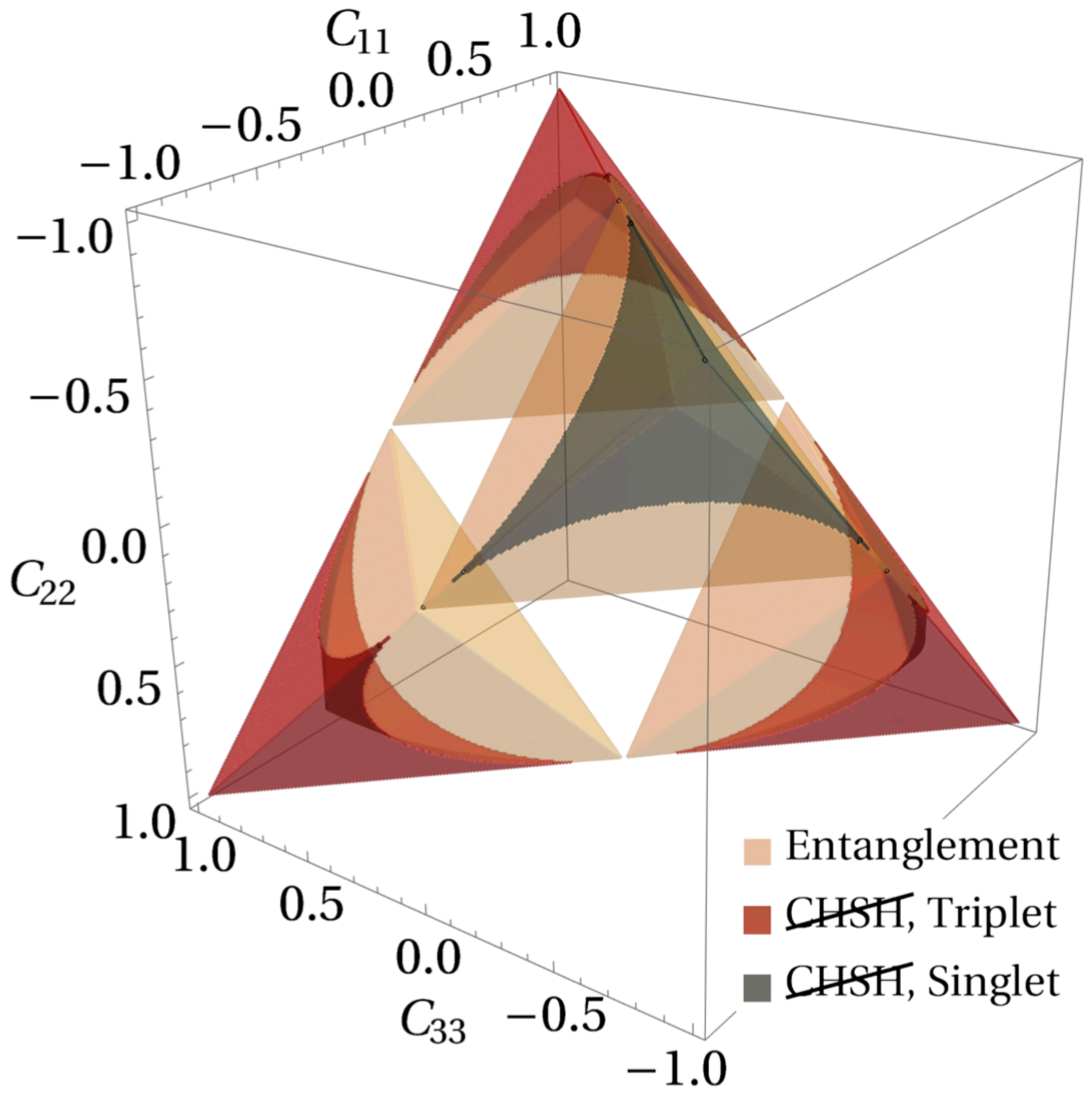
CHSH $$\left|\sum_{ij} C_{ij}\left(a_i b_j - a_i b'_j + a'_i b_j + a'_i b'_j\right)\right| \leq 2$$

$$\max_{a\,a'\,b\,b'}\left|\sum_{ij} C_{ij}\left(a_i b_j - a_i b'_j + a'_i b_j + a'_i b'_j\right)\right| = 2\sqrt{\lambda + \lambda'},$$

two largest eigenvalues of $C^T C$

Much harder to see Bell inequalities violation at the LHC

How about Bell inequalities?



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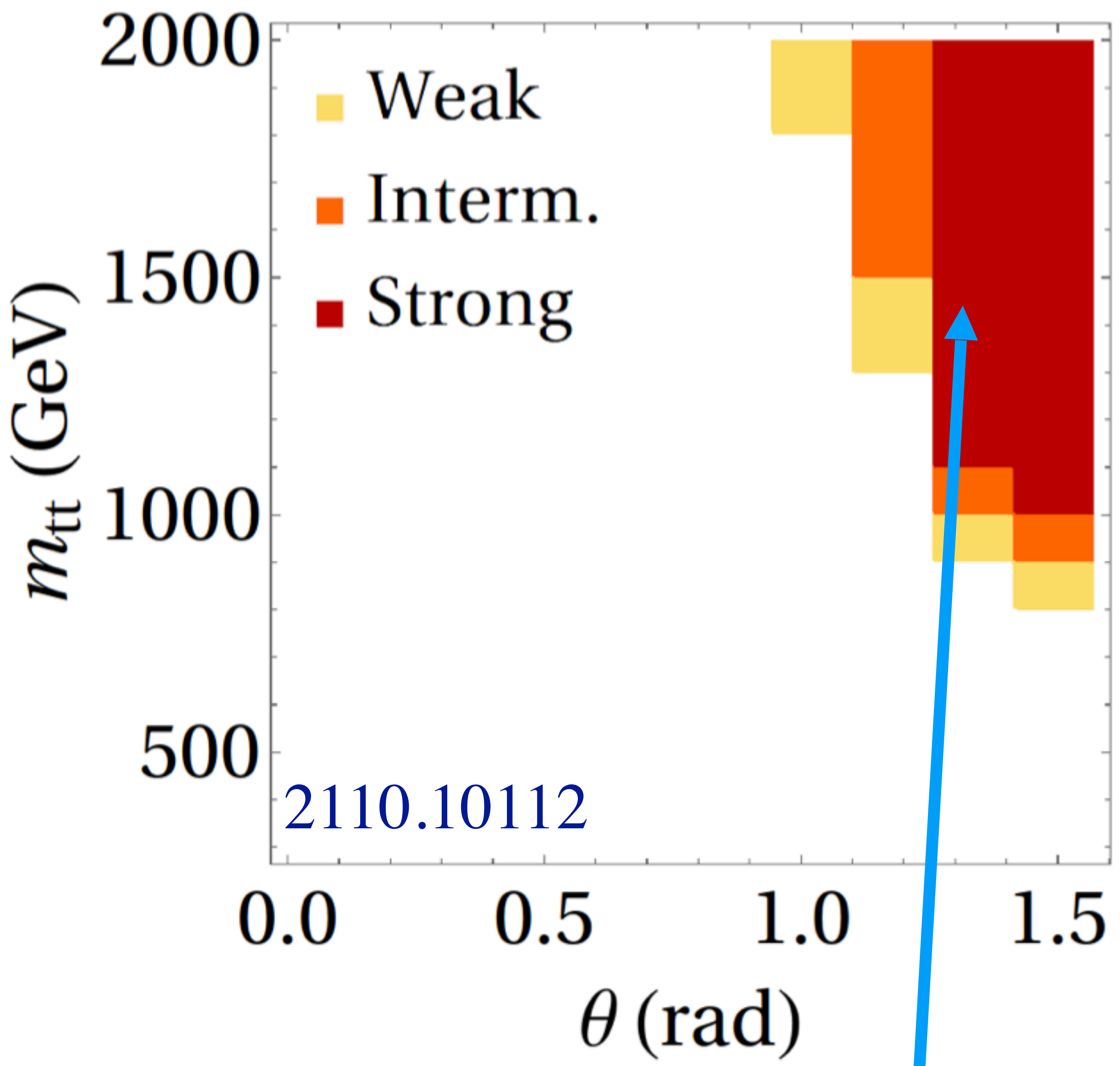
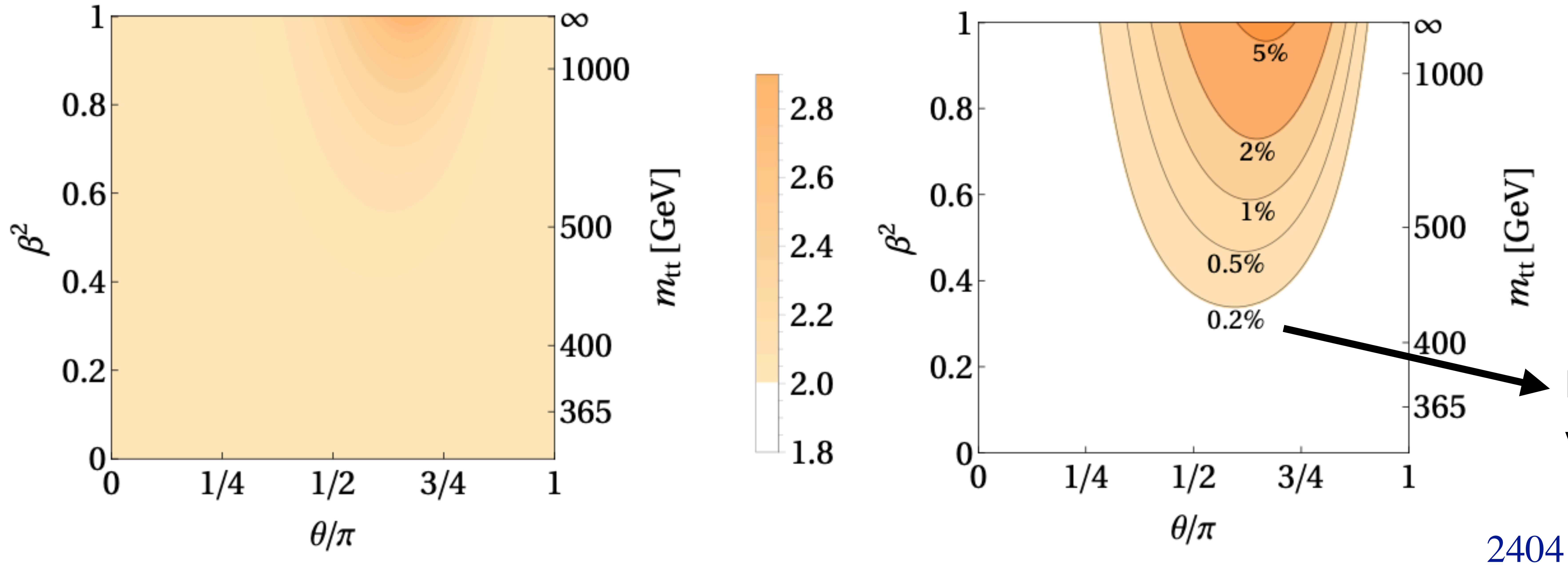
two largest eigenvalues of $C^T C$



$\sqrt{2}\left|-C_{rr} + C_{nn}\right| > 2$

Much harder to see Bell inequalities violation at the LHC

Bell inequalities at lepton colliders



Experimental accuracy needed to establish Bell violation

2404.08049

$$\langle a\,b + a\,b' + a'\,b - a'\,b' \rangle \equiv \langle\, \mathcal{B}(a, a', b, b')\,\rangle > 2, \quad \Longrightarrow \quad \text{Bell violation.}$$

Bell violation everywhere, but B~2

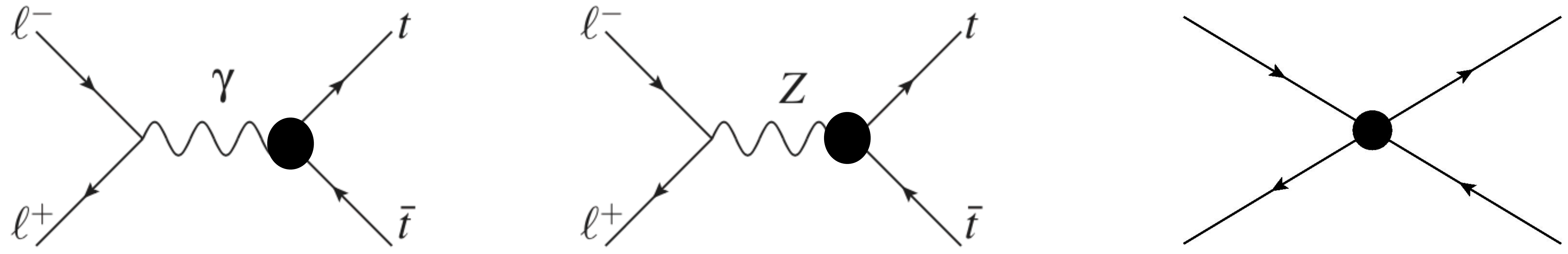
Better prospects of Bell violation at higher energy lepton colliders (extremely hard at 365 GeV)

Using QI for new physics

Can they tell us anything interesting/new?

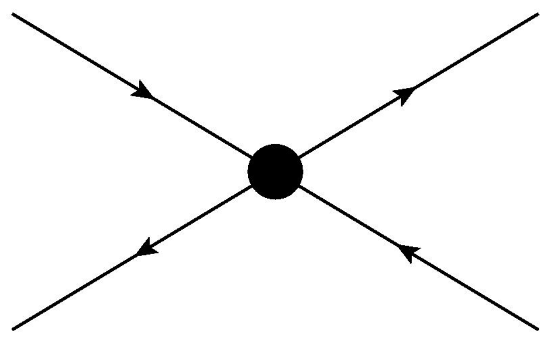
- SMEFT → New Interactions of SM particles

$$\mathcal{L}_{EFT} = \mathcal{L}_{SM} + \sum_i \frac{C_i^{(6)} O_i^{(6)}}{\Lambda^2} + \mathcal{O}(\Lambda^{-4})$$



SMEFT in lepton colliders

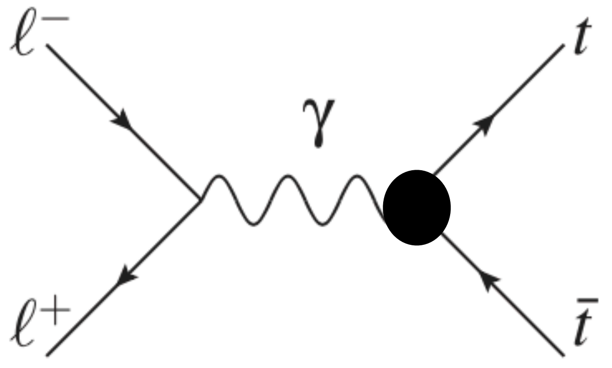
$$\mathcal{O}_{Q\ell}^{(1)} = (\bar{Q}_L\gamma^\mu Q_L)(\bar{\ell}_L\gamma_\mu \ell_L),$$
$$\mathcal{O}_{Q\ell}^{(3)} = (\bar{Q}_L\gamma^\mu \sigma_I Q_L)(\bar{\ell}_L\gamma_\mu \sigma^I \ell_L),$$
$$\mathcal{O}_{Qe} = (\bar{Q}_L\gamma^\mu Q_L)(\bar{\ell}_R\gamma_\mu \ell_R),$$
$$\mathcal{O}_{t\ell} = (\bar{t}_R\gamma^\mu t_R)(\bar{\ell}_L\gamma_\mu \ell_L),$$
$$\mathcal{O}_{te} = (\bar{t}_R\gamma^\mu t_R)(\bar{\ell}_R\gamma_\mu \ell_R).$$

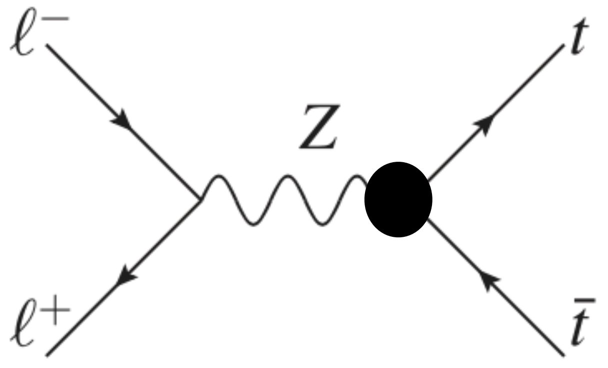


4-fermion operators

$$\mathcal{O}_{\phi Q}^{(1)} = i(\phi^\dagger \overleftrightarrow{D}_\mu \phi)(\bar{Q}_L\gamma^\mu Q_L),$$
$$\mathcal{O}_{\phi Q}^{(3)} = i(\phi^\dagger \overleftrightarrow{D}_{\mu I} \phi)(\bar{Q}_L\gamma^\mu \sigma^I Q_L),$$
$$\mathcal{O}_{\phi t} = i(\phi^\dagger \overleftrightarrow{D}_\mu \phi)(\bar{t}_R\gamma^\mu t_R),$$

current operators





$$\mathcal{O}_{tW} = (\bar{Q}_L\gamma^{\mu\nu}\sigma_I t_R)\,\tilde{\phi}\,W^I_{\mu\nu},$$
$$\mathcal{O}_{tB} = (\bar{Q}_L\gamma^{\mu\nu} t_R)\,\tilde{\phi}\,B_{\mu\nu}.$$

dipole operators

Degrees of freedom

$$c_{Q\ell}^{(3)} + c_{Q\ell}^{(1)},$$
$$c_{\mathrm{VV}} = \frac{1}{4}\big(c_{Q\ell}^{(1)} - c_{Q\ell}^{(3)} + c_{te} + c_{t\ell} + c_{Qe}\big),$$
$$c_{\mathrm{AV}} = \frac{1}{4}\big(-c_{Q\ell}^{(1)} + c_{Q\ell}^{(3)} + c_{te} + c_{t\ell} - c_{Qe}\big),$$
$$c_{\mathrm{VA}} = \frac{1}{4}\big(-c_{Q\ell}^{(1)} + c_{Q\ell}^{(3)} + c_{te} - c_{t\ell} + c_{Qe}\big),$$
$$c_{\mathrm{AA}} = \frac{1}{4}\big(c_{Q\ell}^{(1)} - c_{Q\ell}^{(3)} + c_{te} - c_{t\ell} - c_{Qe}\big).$$

$$c_{\phi Q}^{(3)} + c_{\phi Q}^{(1)},$$
$$c_{\phi \mathrm{V}} = \frac{1}{2}\big(c_{\phi t} + c_{\phi Q}^{(1)} - c_{\phi Q}^{(3)}\big),$$
$$c_{\phi \mathrm{A}} = \frac{1}{2}\big(c_{\phi t} - c_{\phi Q}^{(1)} + c_{\phi Q}^{(3)}\big).$$

$$c_{tZ} = c_{\mathrm{W}}\,c_{tW} - s_{\mathrm{W}}\,c_{tB},$$
$$c_{t\gamma} = s_{\mathrm{W}} c_{tW} + c_{\mathrm{W}} c_{tB},$$

Structure of spin correlations within SMEFT

Degeneracy between possible structures arising from SM and EFT

SM:

$$A^{[0]} = F^{[0]}\left(\beta^2 c_\theta^2 - \beta^2 + 2\right)$$
$$A^{[1]} = 2\,F^{[1]}\,c_\theta$$
$$A^{[2]} = F^{[2]}\,\left(1 + c_\theta^2\right)$$

BSM:

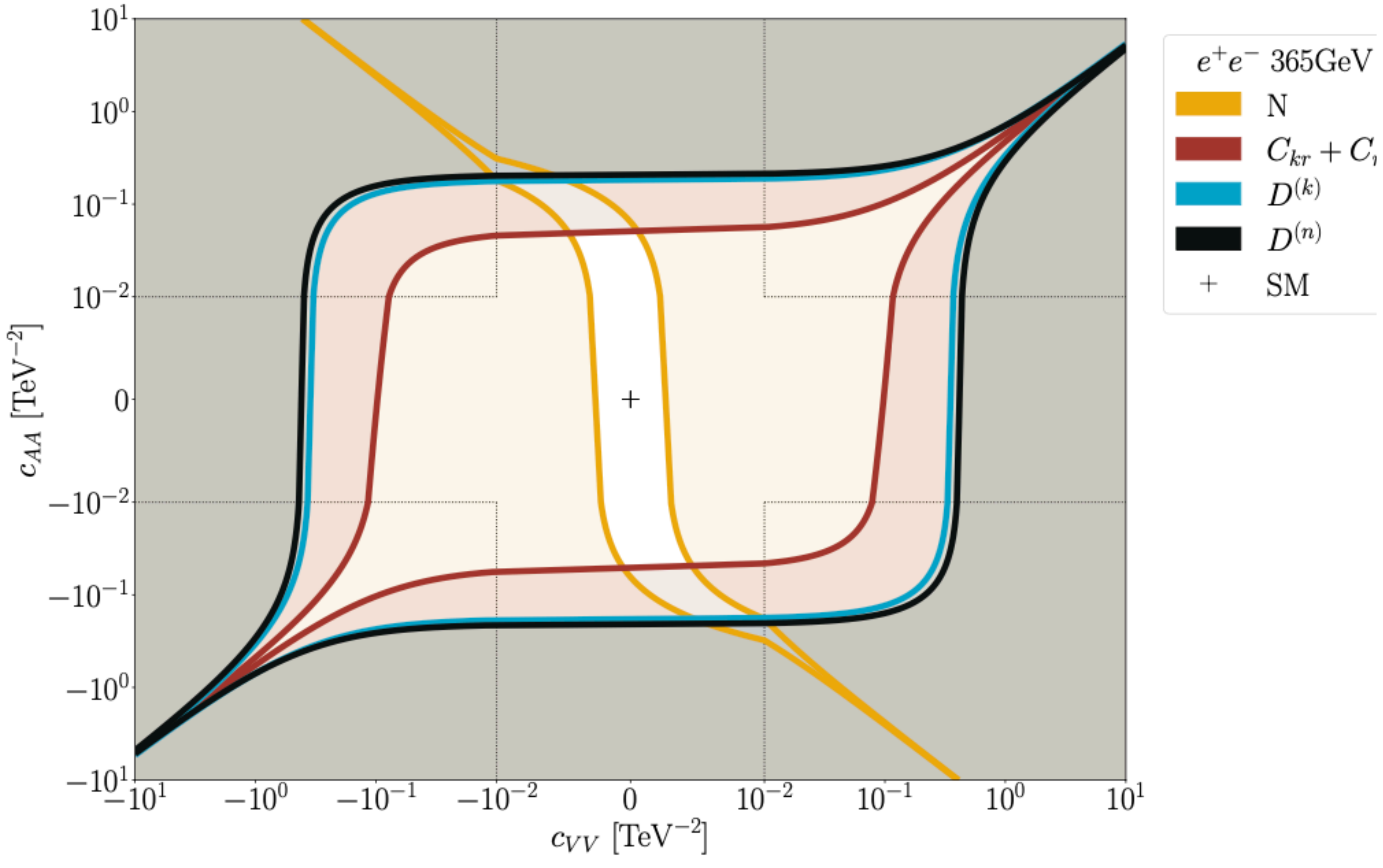
$$A^{[6,0,\mathrm{D}]} = F^{[6,0,\mathrm{D}]}$$
$$A^{[6,1,\mathrm{D}]} = F^{[6,1,\mathrm{D}]}\,c_\theta$$
$$A^{[8,\mathrm{DD}]} = F^{[8,\mathrm{DD}]}\left(-\beta^2 c_\theta^2 - \beta^2 + 2\right)$$

		$\mathcal{M}_1$: Q_t, g_{Vt}, c_{VV}, c_{VA}, $c_{\phi\mathrm{V}}$	$\mathcal{M}_1$: g_{At}, c_{AV}, c_{AA}, $c_{\phi\mathrm{A}}$	$\mathcal{M}_1$: $c_{\mathrm{t}Z}$, $c_{\mathrm{t}\gamma}$
$\mathcal{M}_2$	Q_t, g_{Vt}, c_{VV}, c_{VA}, $c_{\phi\mathrm{V}}$	$A^{[0]}$	$A^{[1]}$	$A^{[6,0,D]}$
	g_{At}, c_{AV}, c_{AA}, $c_{\phi\mathrm{A}}$	$A^{[1]}$	$A^{[2]}$	$A^{[6,1,D]}$
	$c_{\mathrm{t}Z}$, $c_{\mathrm{t}\gamma}$	$A^{[6,0,D]}$	$A^{[6,1,D]}$	$A^{[8,\mathrm{DD}]}$

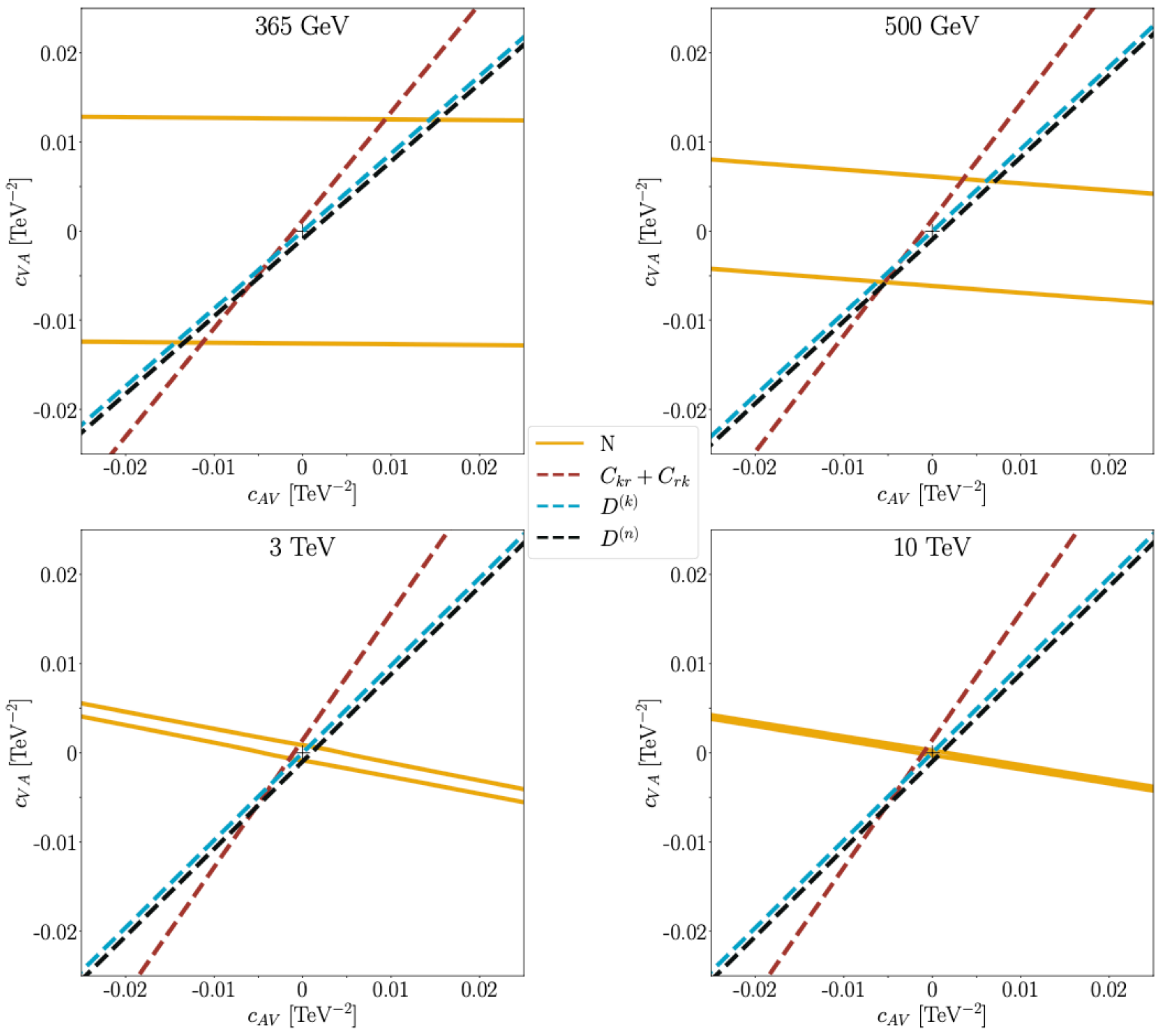
New structures related to dipole operators, the rest gives linear combinations of pre-existing structures

C. Severi, F. Maltoni, S. Tentori, EV: 2404.08049

Breaking degeneracies with Quantum Obs



C. Severi, F. Maltoni, S. Tentori, EV: 2404.08049[hep-ph]

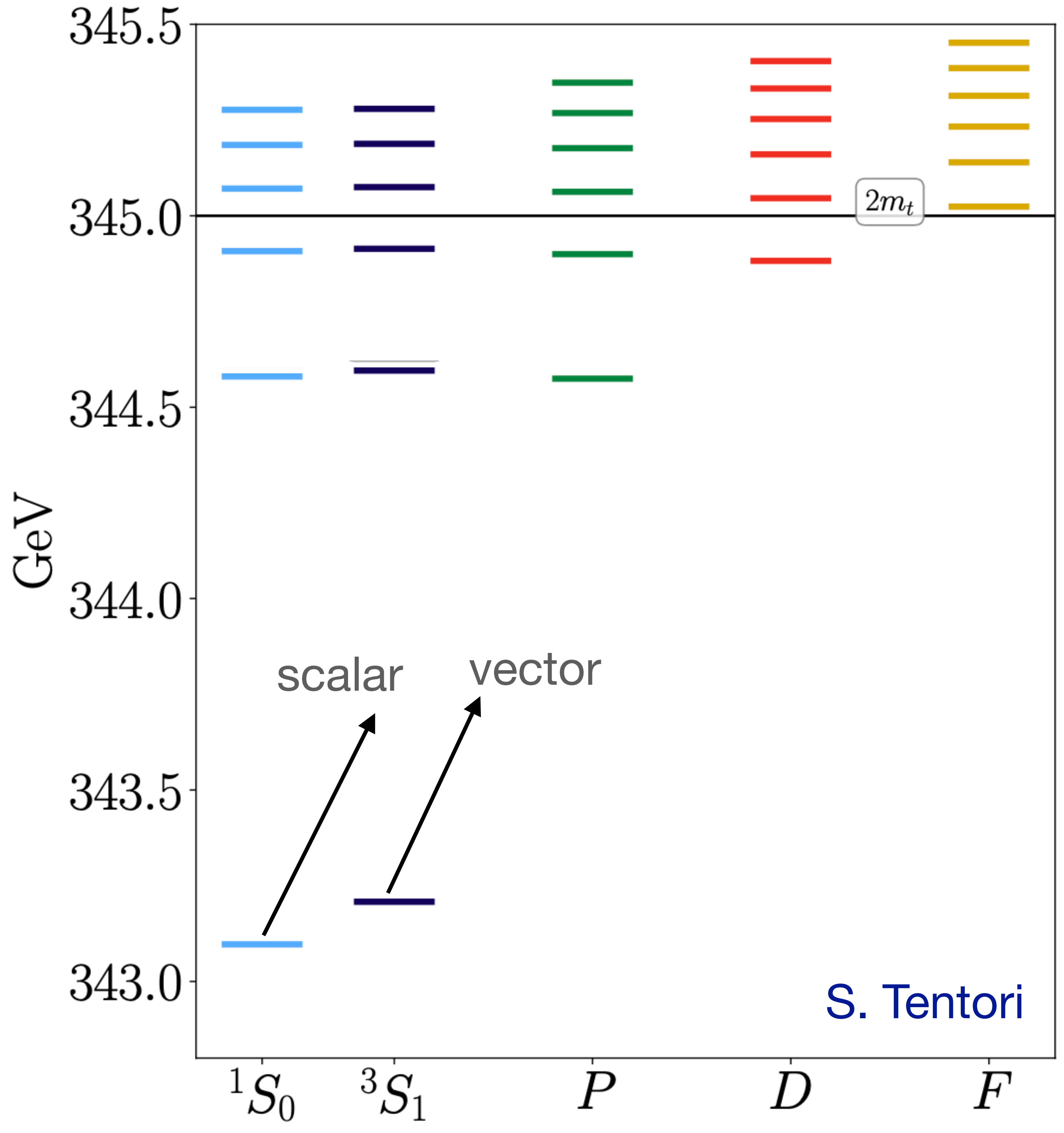


Spin correlation observables probe different linear combinations of Wilson coefficients

→ Breaking degeneracies

Old New Physics: Threshold effects

- Quasi-Bound State of top and antitop
- Energy states obtained by solving Schrödinger equation with QCD potential
- Described by NRQCD
- Ground state n=1 S-wave
- Spin-singlet vs spin-triplet depending on production mode
  - spin singlet for pp and spin triplet for e^+e^-
- Most results obtained for e^+e^- threshold



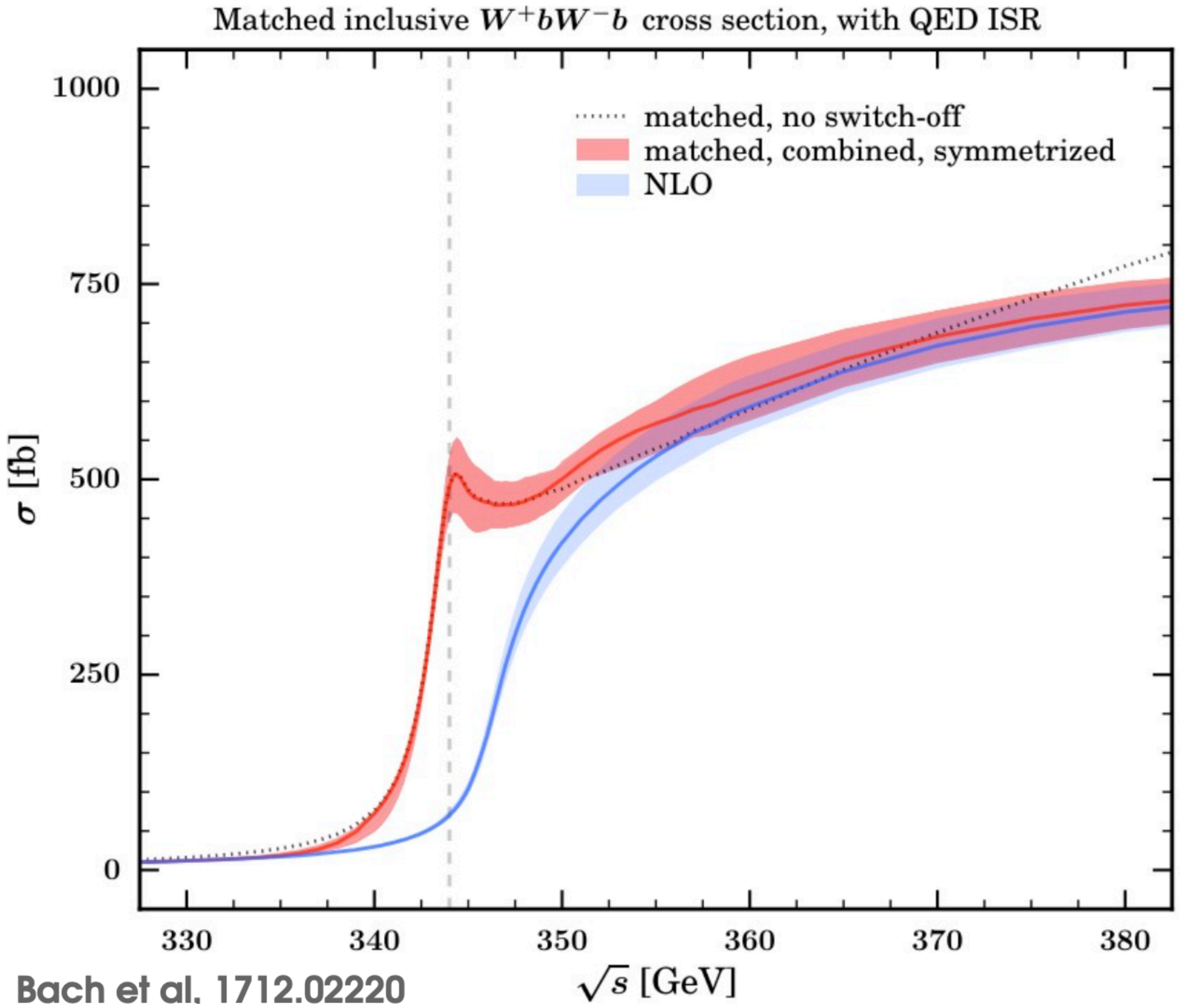
$$\left[(E + i\Gamma_t) - \left(\frac{\boldsymbol{\nabla}^2}{m_t} + V(\boldsymbol{r})\right)\right] G(\boldsymbol{r}, E + i\Gamma_t) = \delta^{(3)}(\boldsymbol{r})$$

$$V_{\rm QCD}(r, \mu_B) = C^{[\rm col]}\,\frac{\alpha_s(\mu_B)}{r}\left[1 + \frac{\alpha_s}{4\pi}\left(2\,\beta_0 \log(e^{\gamma}\mu_B\, r) + \frac{31}{9}C_A - \frac{10}{9}n_f\right) + \mathcal{O}(\alpha_s^2)\right]$$

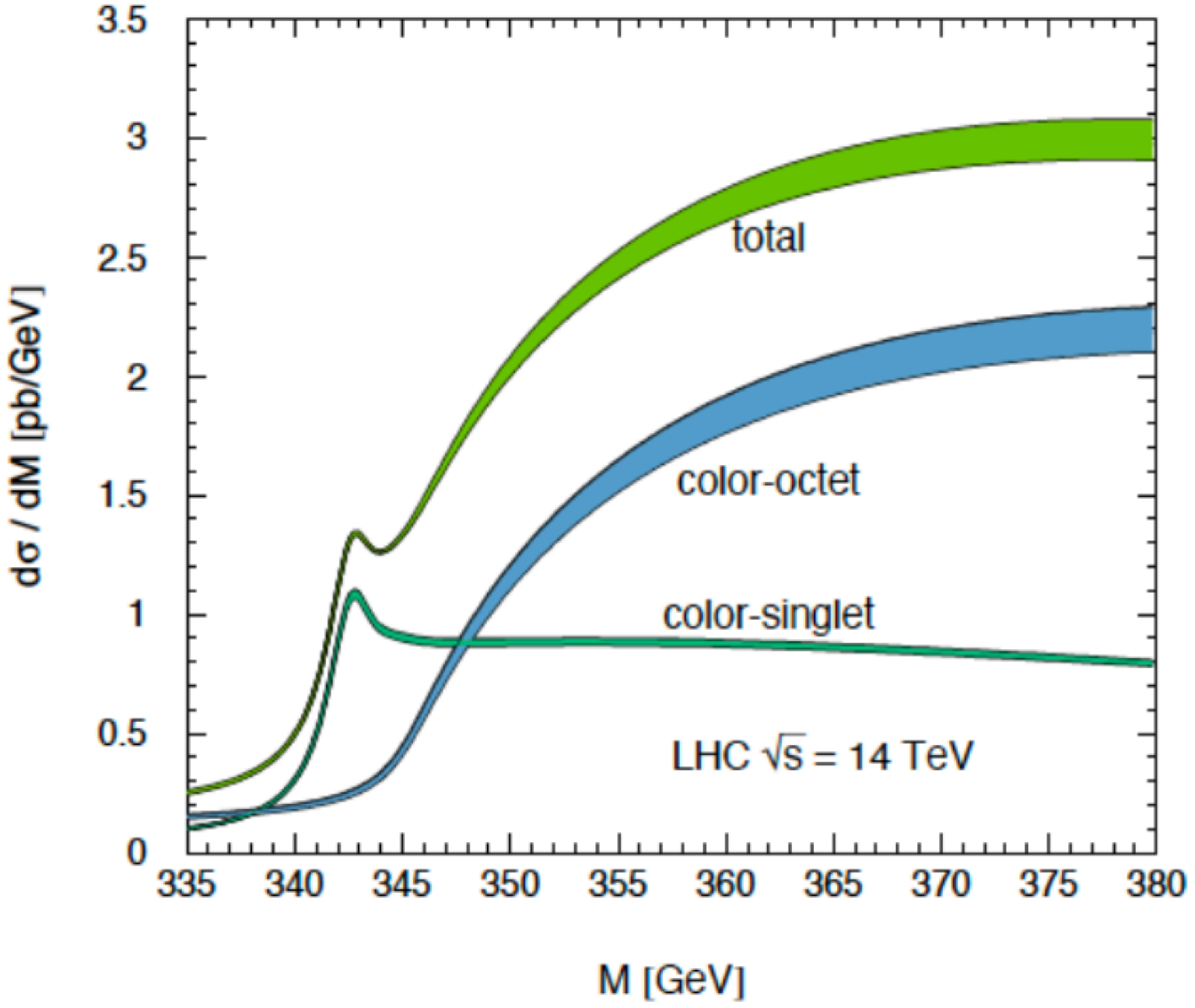
What do we know about toponium?

e^+e^-

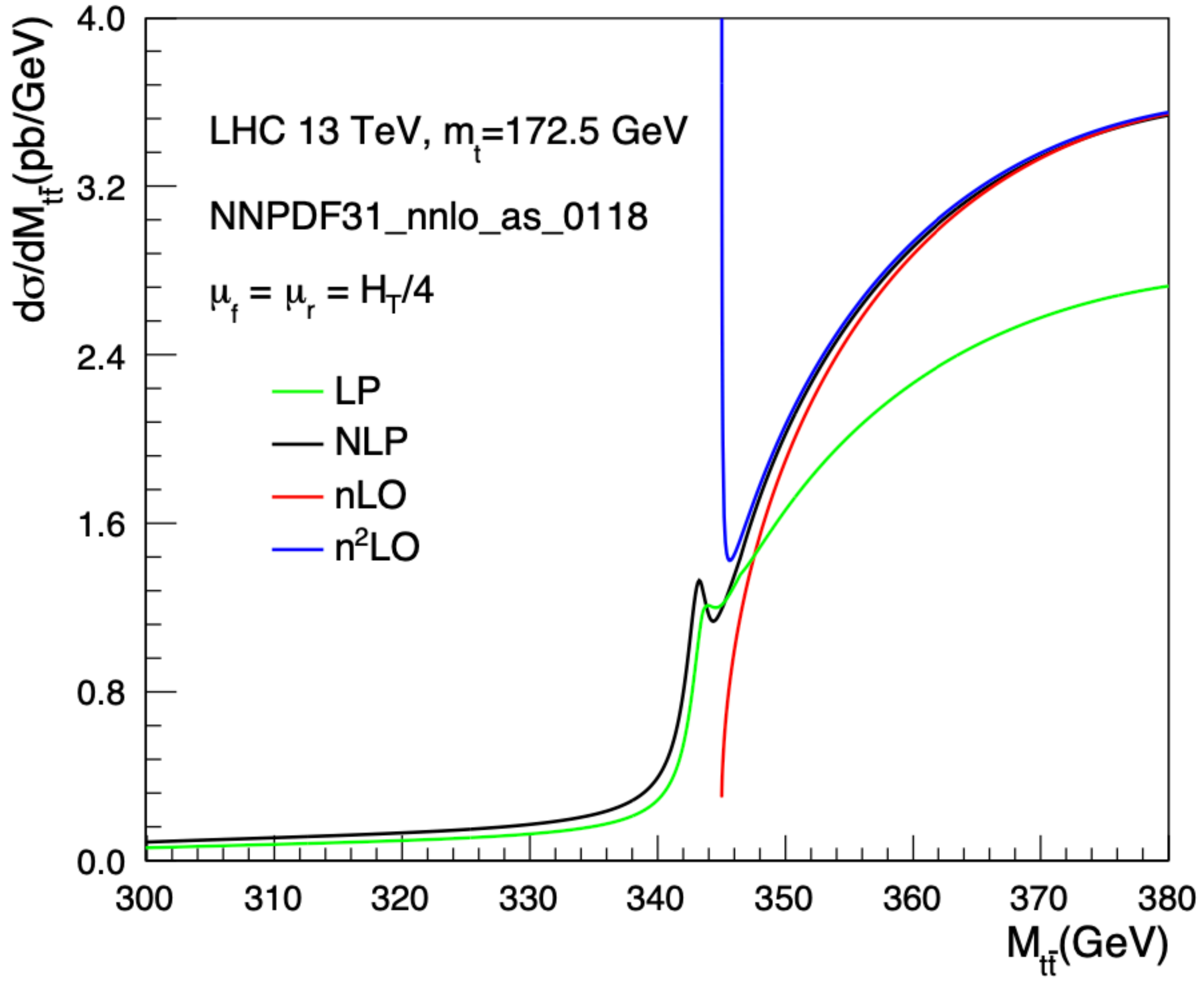
LHC results



Bach et al. 1712.02220



Kiyo, Kühn, Steinhauser, Moch, Uwer arXiv: 0812.0919



Ju, Wang, Wang, Xu, Xu and Li Lin Yang arXiv:2004.03088

Fully differential NLO+LL, Coulomb Resummation

Coulomb Resummation

Any computation needs matching between below threshold, toponium region, continuum

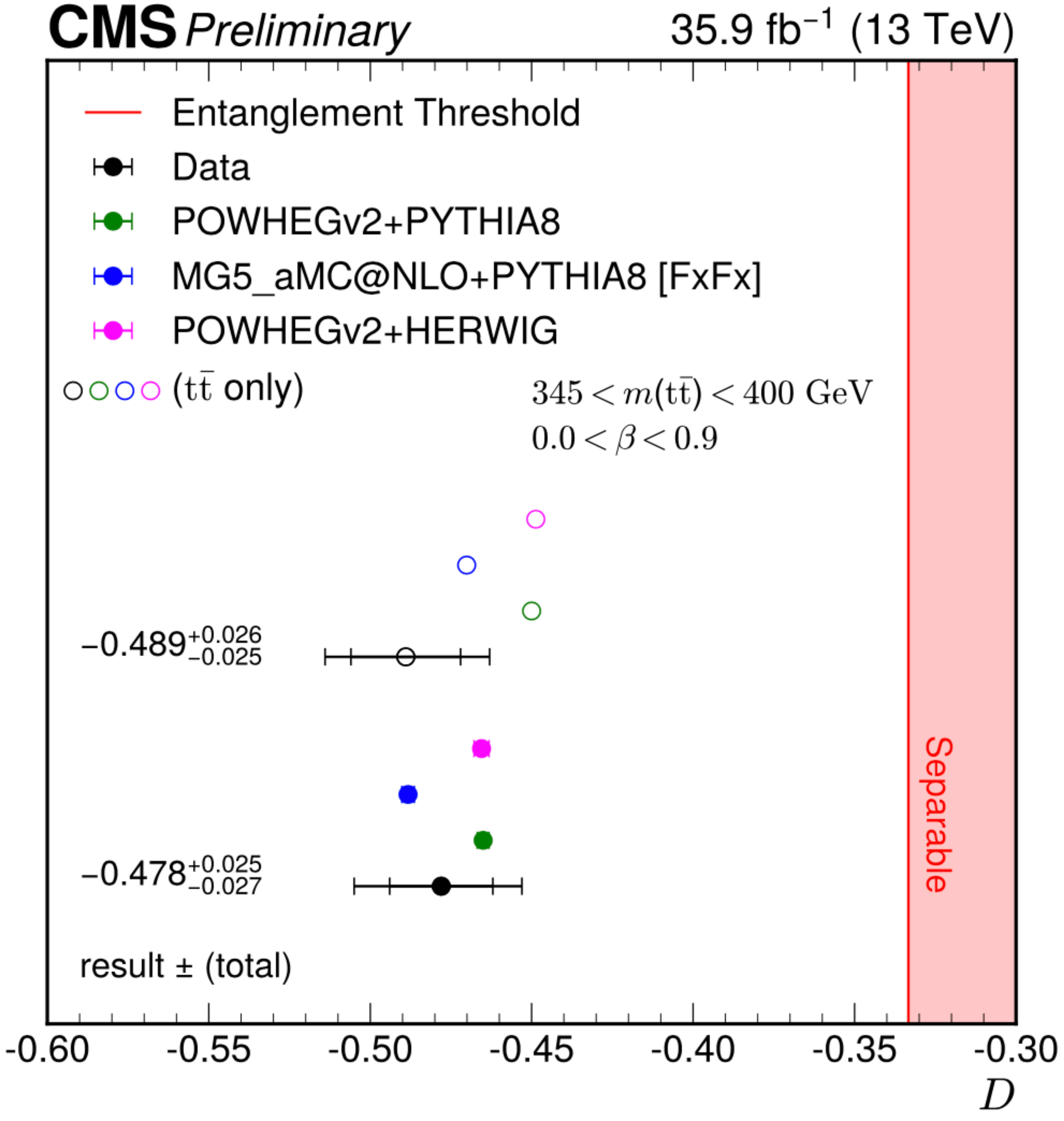
Toponium modelling

Best theory computations for bound states are not available in Monte Carlo generators
We can approximate their impact in the Monte Carlo by introducing a toy model with a resonance

- vector resonance for lepton collisions
- psedoscalar resonance for proton collisions

$$m_\psi = m_\eta \simeq 2m_t - 2\,\text{GeV}, \quad \text{and} \quad \Gamma_\psi = \Gamma_\eta \simeq 2\,\Gamma_t$$

Peak of resonance fitted to match the results obtained by the resummed computation
CMS toponym simulation based on: Fuks et al. 2102.11281, 2411.18962



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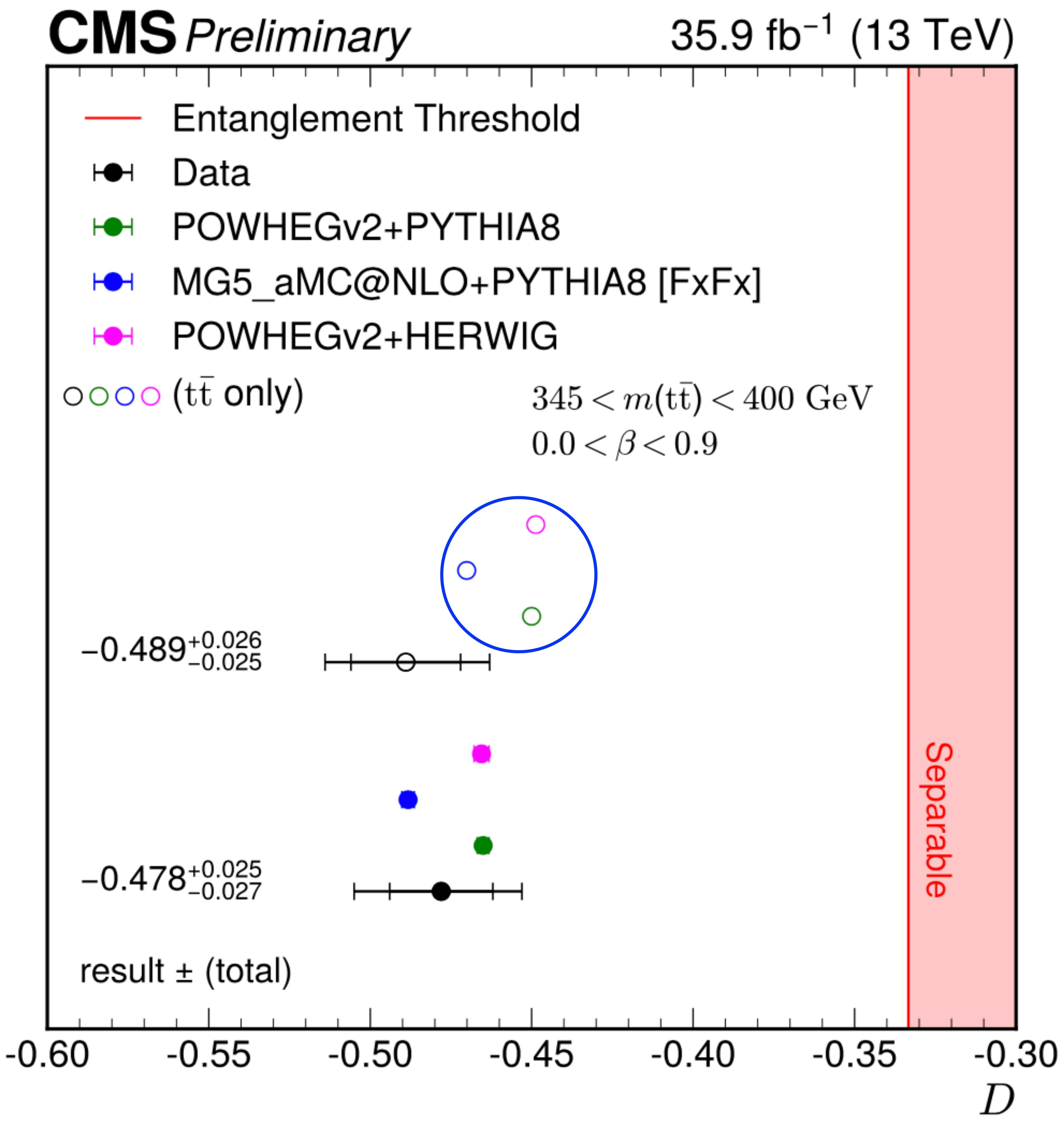


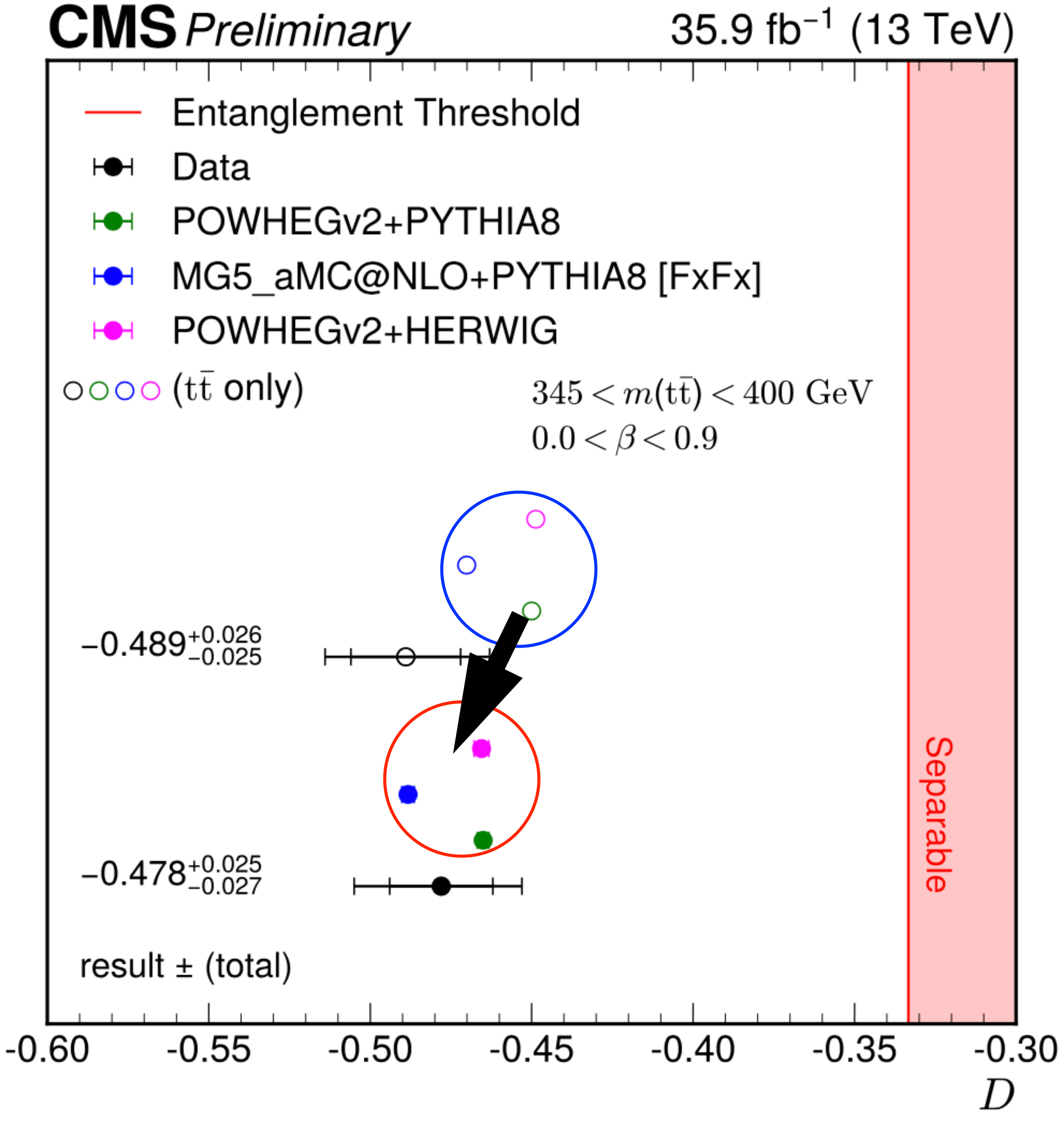
Toponium modelling

Best theory computations for bound states are not available in Monte Carlo generators
We can approximate their impact in the Monte Carlo by introducing a toy model with a resonance

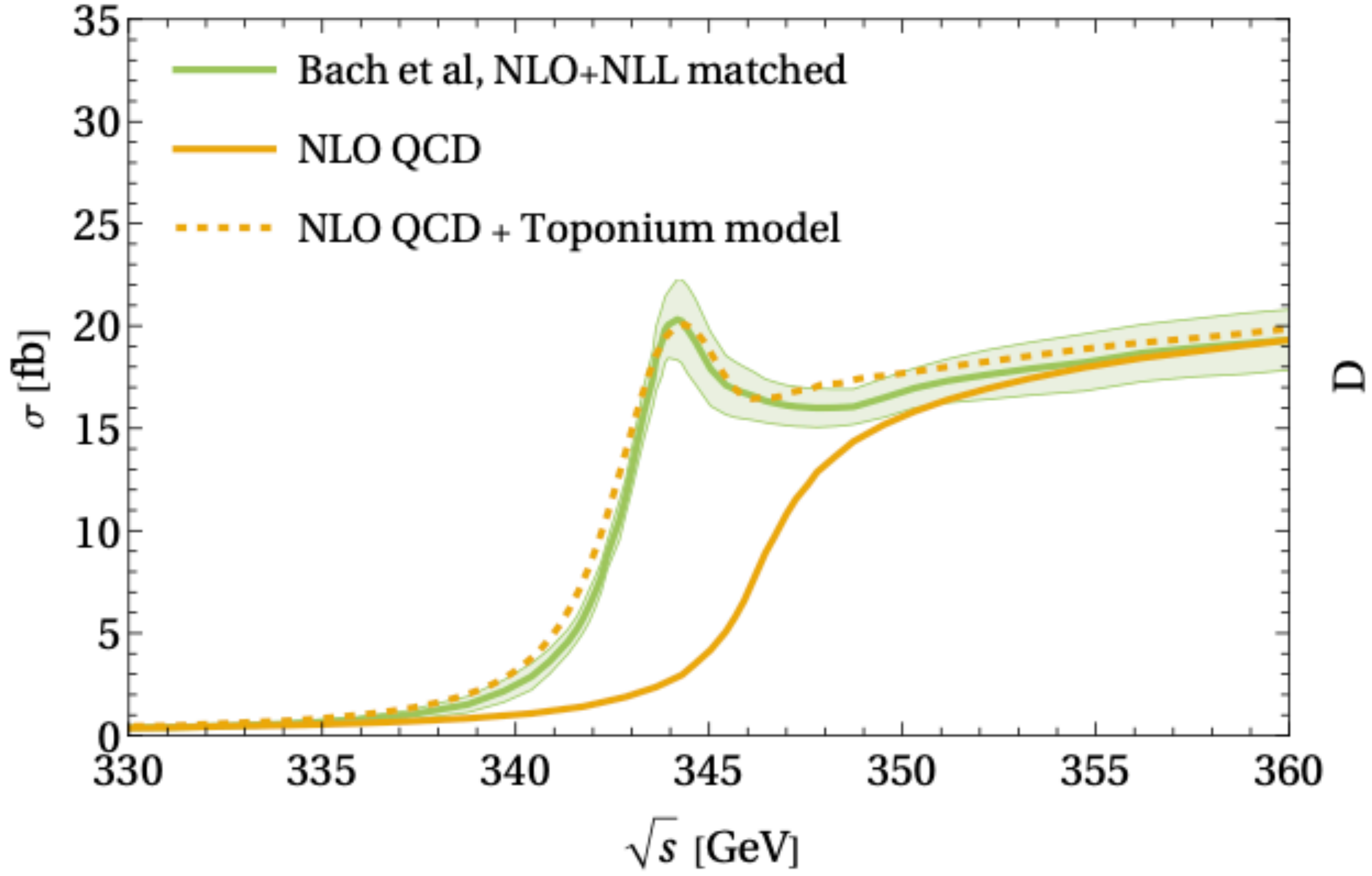
- vector resonance for lepton collisions
- psedoscalar resonance for proton collisions

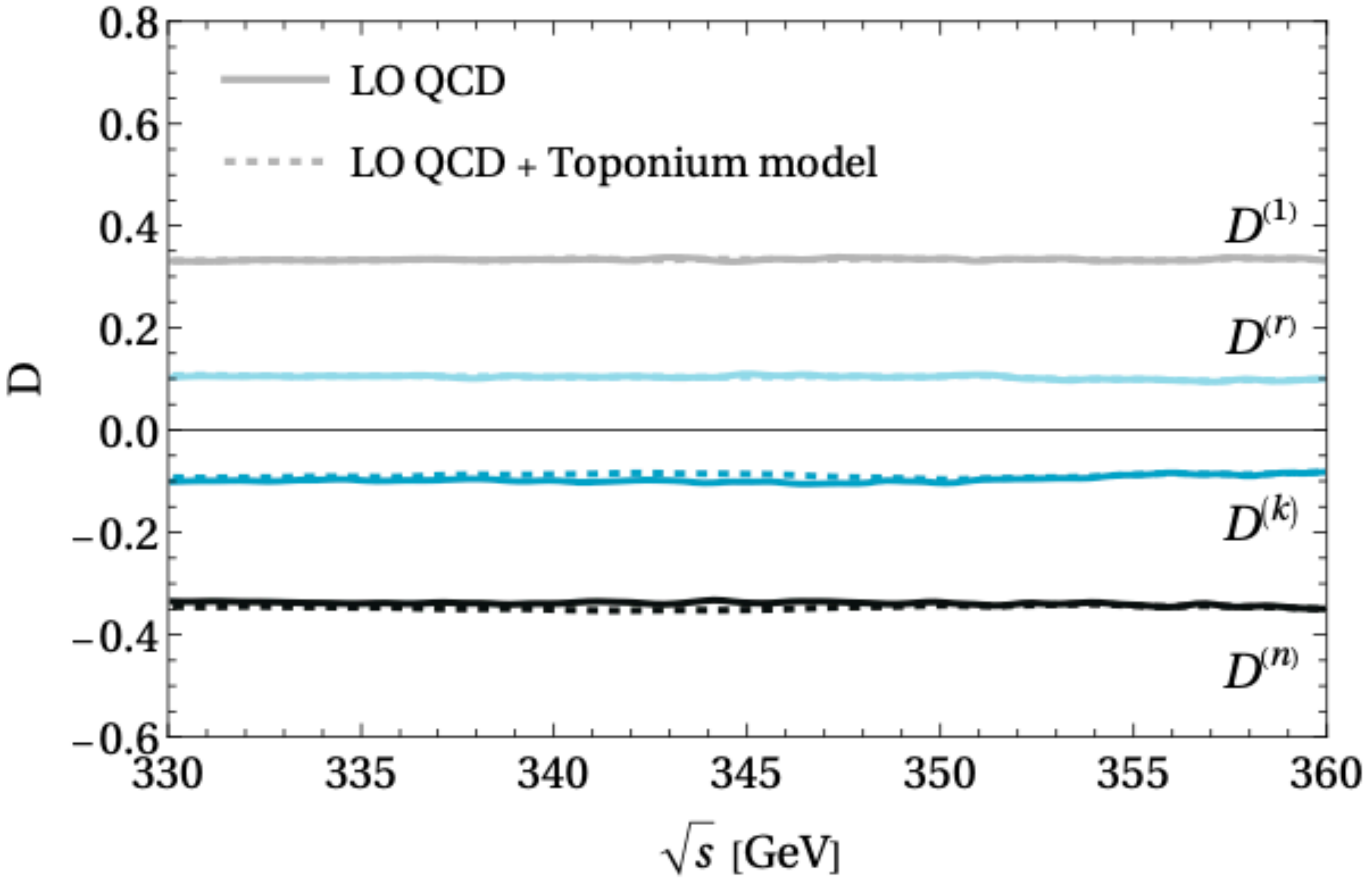
$$m_\psi = m_\eta \simeq 2m_t - 2\,\mathrm{GeV}, \quad \text{and} \quad \Gamma_\psi = \Gamma_\eta \simeq 2\,\Gamma_t$$

Peak of resonance fitted to match the results obtained by the resummed computation
CMS toponym simulation based on: Fuks et al. 2102.11281, 2411.18962



Toponium in e^+e^-





C. Severi, F. Maltoni, S. Tentori, EV: 2404.08049

Bound state effects have an impact on the lineshape (increase of cross-section)

No impact on entanglement markers (unlike the LHC)

Vector resonance leads to the same spin correlations as the EW Standard Model

Conclusions

- A new era of quantum observables at colliders is here
- Ideas and methods of QM adjusted to high energy physics
- First measurements, and lots of studies already here
- Top pairs an ideal testing ground, different degrees of correlations can be observed
- QI observables are not only fun but can also help to probe new physics
- SMEFT introduce new structures, thus probing new linear combinations between coefficients
- QI observables can break degeneracies between operators when combined with standard observables

Thank you for your attention