
From electroweak precision observables and flavours

8th FCC Physics Workshop

Lars Röhrig^{1,2}, Stéphane Monteil²

14/01/2025

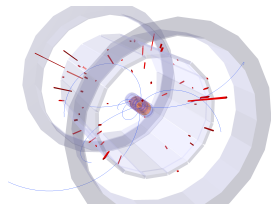
¹Department of Physics – TU Dortmund University

²Laboratoire de Physique de Clermont – Université Clermont-Auvergne

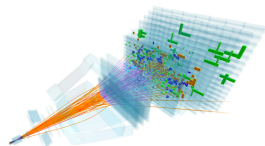
Flavours at FCC-ee

- Continuation of vibrant LHCb & Belle **flavour physics programme** with Z-pole statistics and boost

	Belle	LHCb	FCC-ee
All hadron species		✓	✓
Boost		✓	✓
High production σ		✓	
Negligible trigger losses	✓		✓
Low backgrounds	✓		✓
Initial energy constraint	✓		(✓)



Belle



LHCb

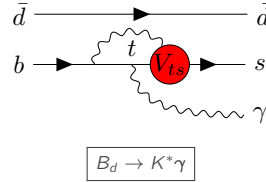
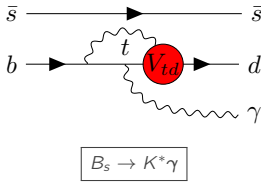
- Additionally: defines stringent **detector requirements**
→ vertexing, tracking, calorimetry, particle-ID
- E. g. vertexing requirements defined by modes with missing momentum $b \rightarrow s\tau\tau$
+ **new study in charm sector** $c \rightarrow u\nu\bar{\nu}$ [T. Hacheny tomorrow@2:40pm]

Today's outline:

- Academic exercise of rare, radiative FCNC $b \rightarrow (d, s)\gamma$ transitions to define **EM calorimetry resolution**
- Flavours in a **global context** to measure EWPOs: $\{R_b, R_c, R_s\}$ and $\{A_{FB}^b, A_{FB}^s\}$

EM calorimetry requirements from radiative decays

- $b \rightarrow (d, s)\gamma$ probe NP in loop diagrams in addition to the photon dipole operator C_7
- However: $b \rightarrow d\gamma$ signal dominated by $b \rightarrow s\gamma$ background



→ $B_s \rightarrow K^*\gamma$ **not yet observed**, estimate event yield:

$$\frac{N_{B_d}}{N_{B_s}} \approx \frac{f_{b \rightarrow B_d}}{f_{b \rightarrow B_s}} \cdot \left| \frac{V_{ts}}{V_{td}} \right|^2 \approx 92$$

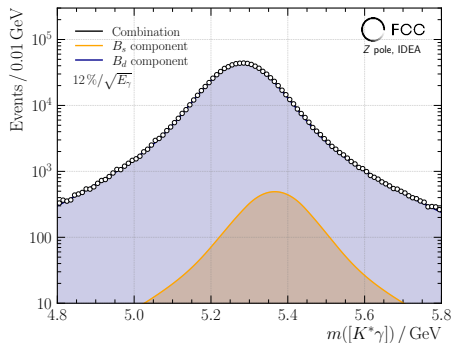
$$\rightarrow N_{B_d} \approx 30 \cdot 10^6 \quad \Rightarrow \quad N_{B_s} \approx 33 \cdot 10^4$$

- Would allow to directly measure $\frac{F_{B_d \rightarrow K^*}}{F_{B_s \rightarrow K^*}} \left| \frac{V_{ts}}{V_{td}} \right|^2$, but depends on $\Delta m = m_{B_d} - m_{B_s} = 87 \text{ MeV}$ resolution
- Limiting experimental factor: **EM calorimetry resolution**

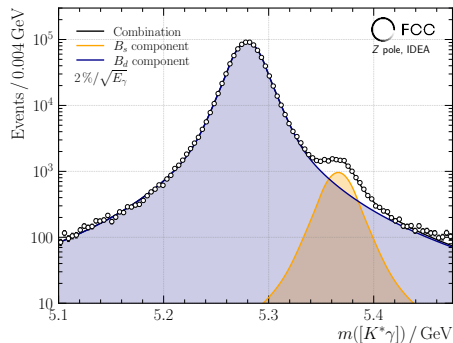
→ **Goal:** Estimate precision of $\left| \frac{V_{td}}{V_{ts}} \right|$ as function of the stochastic term of EM energy resolution

EM calorimetry requirements from radiative decays

- Analysis based on $10^6 B_d \rightarrow K^* \gamma$ events (simulated with PYTHIA8 + EvtGen + default IDEA card)
- Emulate B_s signal by scaling B_d candidates
- Use $K^* \rightarrow K \pi$ from reconstructed particles, **smear photon momentum based on MC information**



IDEA baseline with $12\%/\sqrt{E_\gamma}$.

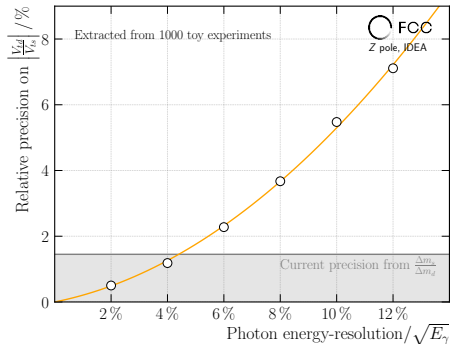
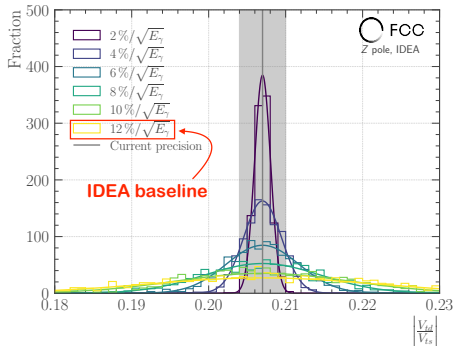


Scaled to $2\%/\sqrt{E_\gamma}$.

→ Complicated to even fit B_s signal yield with $12\%/\sqrt{E_\gamma}$ resolution + **no backgrounds** included + perfectly known signal-tail shapes

Extracting $\left| \frac{V_{td}}{V_{ts}} \right|$

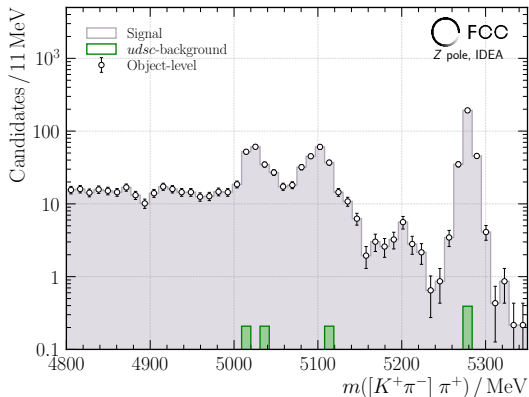
- Pseudoexperiments with fixed shape parameters, but **floating signal yields** (\rightarrow floating $\left| \frac{V_{td}}{V_{ts}} \right|$)
- Extract precision of $\left| \frac{V_{td}}{V_{ts}} \right|$ from 1σ Gaussian fit as function of EM energy resolution



- In view of the consistency check, the precision on the determination from $\Delta m_s / \Delta m_d$ is indicated
 - \rightarrow Only for an EM resolution **below** $5\% / \sqrt{E_\gamma}$ comparative result w.r.t current precision
 - \rightarrow $\mathcal{O}(5\% / \sqrt{E_\gamma})$ well **in reach with crystals** [2312.07365]

Flavours in a global context

- Precision flavour programme is **key to probe NP effects** in the SM
- Also become important in the context of **electroweak precision observables**: R_b and A_{FB}^b [Ref.]
→ probe NP in radiative and vertex corrections involving top quarks



- **Background-free** hemisphere tag for R_b and A_{FB}^b possible at FCC-ee with exclusive tagger
→ $\mathcal{O}(\sigma(R_b)/R_b) = \mathcal{O}(\sigma(A_{\text{FB}}^b)/A_{\text{FB}}^b) = 0.01\%$
- Central role to achieve $\sigma_{\text{stat.}} \approx \sigma_{\text{sys.}}$
- Concept application for $R_{C,S}$ and $A_{\text{FB}}^{C,S}$ more complicated, but ongoing

Measuring R_c with $\bar{D}^0 \rightarrow K^+\pi^-$ decays

- Double-tag equations from R_b measurement **extended** in case of R_c to benefit from **excl. b -tagger**:

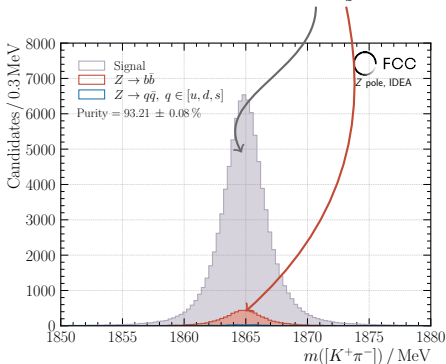
$$N_{ST}^c = 2N_{Z \rightarrow \text{had.}} (R_c \epsilon_c^c + R_b \epsilon_b^c + R_{uds} \epsilon_{uds}^c)$$

$$N_{DT}^c = N_{Z \rightarrow \text{had.}} (R_c (\epsilon_c^c)^2 C_c + R_b (\epsilon_b^c)^2 C_b + R_{uds} (\epsilon_{uds}^c)^2 C_{uds})$$

$$N_{DT}^{cb} = N_{Z \rightarrow \text{had.}} (R_c \epsilon_c^c \epsilon_c^b C_{cb} + R_b \epsilon_b^b \epsilon_b^c C_{bc} + R_{uds} \epsilon_{uds}^{uds} \epsilon_{uds}^c C_{udsc})$$

- ϵ_i^j : tag flavour j of quark-flavour i

→ **Simultaneously** measure $\{R_c, \epsilon_c^c, \epsilon_b^c\}$, remaining inputs $\{R_b, \epsilon_b^b\}$ from excl. b -tagger



Reconstruction results using winter2023 samples:

$$\rightarrow \epsilon_c^c = 6.4 \cdot 10^{-3}, \epsilon_b^c = 0.4 \cdot 10^{-3}, \epsilon_{uds}^c = 1.5 \cdot 10^{-6}$$

$$\rightarrow \sigma_{\text{stat.}}(R_c) = 3 \cdot 10^{-5}$$

- Impact of ϵ_b^c significant for $\sigma_{\text{syst.}}(R_c)$:

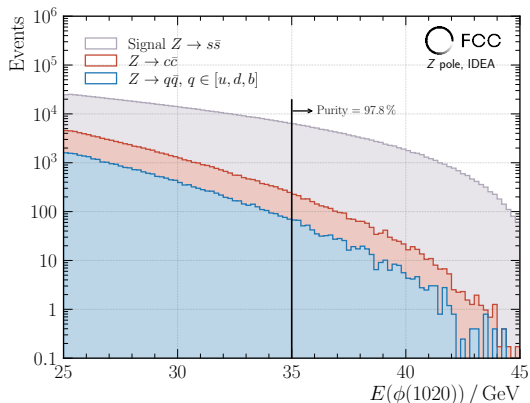
$$\rightarrow \sigma_{\text{syst.}}(R_c, \text{from } \epsilon_b^c) = 6.6 \cdot 10^{-5}$$

- Selection can be refined to remove b contamination

Commensurate $\sigma_{\text{syst.}}$ and $\sigma_{\text{stat.}}$ in reach

Measuring R_s with $\phi(1020) \rightarrow K^+K^-$ decays

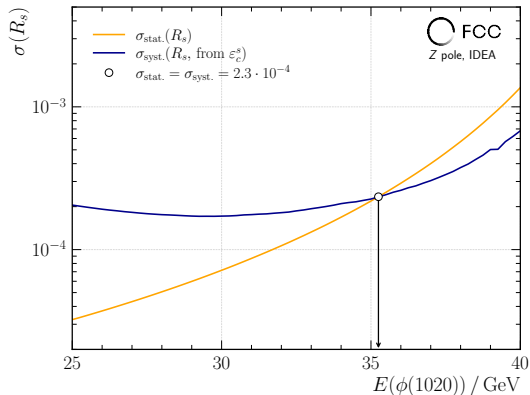
- Multivariate s -tagger not capable to **suppress background efficiently** [2202.03285]
 - Beam-like $|K^- \rangle = |\bar{u}s \rangle$ originating from interaction region suffer from u -quark **contamination**
- $\rightarrow |\phi(1020) \rangle \approx |s\bar{s} \rangle$ meson possible candidate to measure R_s (A_{FB}^s requires charge tag!)
- Validate performance from reconstructed $\phi(1020) \rightarrow K^+K^-$ mesons using winter2023 samples



- Purity $\approx 98\%$ for $E(\phi(1020)) > 35 \text{ GeV}$
- $\rightarrow \epsilon_s^s = 10^{-3}, \epsilon_c^s = 2 \cdot 10^{-5}, \epsilon_{udb}^s = 2 \cdot 10^{-6}$

Measuring R_s with $\phi(1020) \rightarrow K^+K^-$ decays

- Multivariate s -tagger not capable to **suppress background efficiently** [2202.03285]
 - Beam-like $|K^- \rangle = |\bar{u}s \rangle$ originating from interaction region suffer from u -quark **contamination**
- $|\phi(1020) \rangle \approx |s\bar{s} \rangle$ meson possible candidate to measure R_s (A_{FB}^s requires charge tag!)
- Validate performance from reconstructed $\phi(1020) \rightarrow K^+K^-$ mesons using winter2023 samples

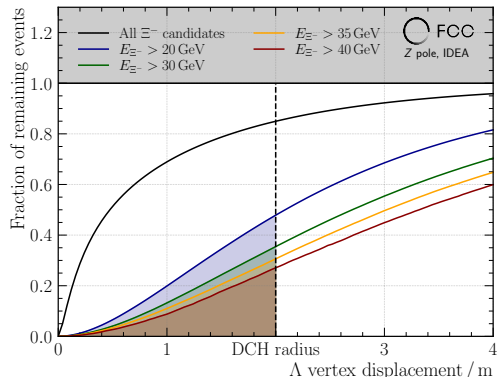
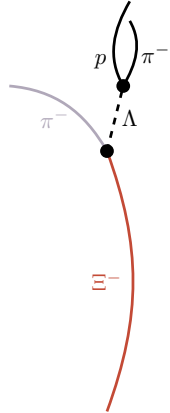


- Purity $\approx 98\%$ for $E(\phi(1020)) > 35$ GeV
- $\epsilon_s^s = 10^{-3}$, $\epsilon_c^s = 2 \cdot 10^{-5}$, $\epsilon_{udb}^s = 2 \cdot 10^{-6}$
- $Z \rightarrow c\bar{c}$ contribution significant for $\sigma_{\text{syst.}}(R_s)$

$$\mathcal{O}(\sigma(R_s)) = 3 \cdot 10^{-4} \text{ in reach}$$

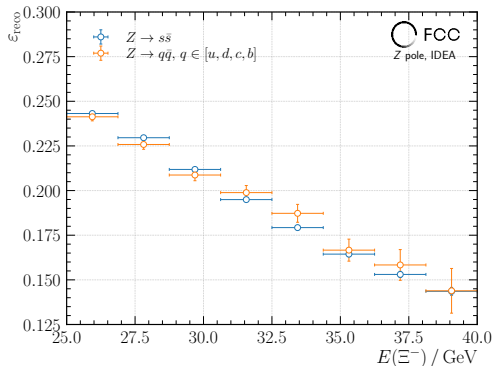
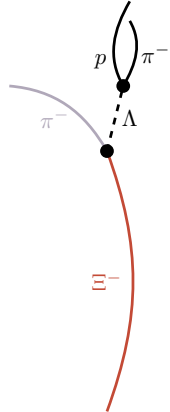
s-quark charge measurement

- A_{FB}^s relies on the charge tag of the s quark
- Unambiguous, pure charge tagger would vanish the systematics
- Use **beam-like** ($E(\Xi^-) \gtrsim 35$ GeV) $|\Xi^- \rangle = |ds\bar{s}\rangle$ in $\Xi^- \rightarrow \Lambda\pi^-$ decays
 - Complication: $\tau(\Xi^-) = 1.6 \cdot 10^{-10}$ s $\Rightarrow \langle L(\Xi^-) \rangle = 1.2$ m
 - Significant fraction of produced Ξ^- final-state particles **outside of tracking volume**



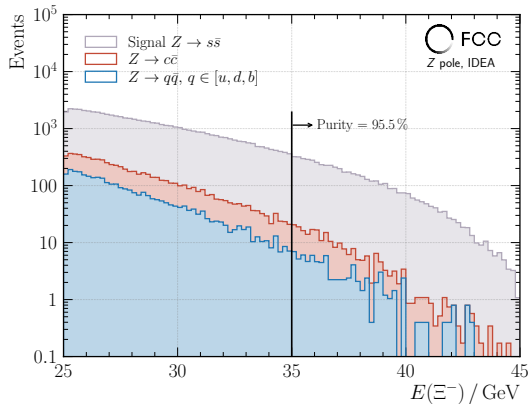
s-quark charge measurement

- A_{FB}^s relies on the charge tag of the s quark
- Unambiguous, pure charge tagger would vanish the systematics
- Use **beam-like** ($E(\Xi^-) \gtrsim 35$ GeV) $|\Xi^- \rangle = |ds\bar{s}\rangle$ in $\Xi^- \rightarrow \Lambda\pi^-$ decays
 - Complication: $\tau(\Xi^-) = 1.6 \cdot 10^{-10}$ s $\Rightarrow \langle L(\Xi^-) \rangle = 1.2$ m
 - Significant fraction of produced Ξ^- final-state particles **outside of tracking volume**
- For now: vertex Λ candidates with additional π^- track with $\mathcal{O}(\epsilon_{\text{reco}}) = 15\%$



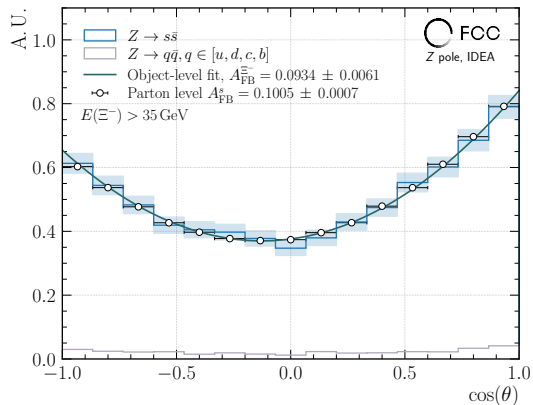
A_{FB}^s : some numbers and outlook

- Purity above $> 95\%$ in reach for Ξ^- baryons



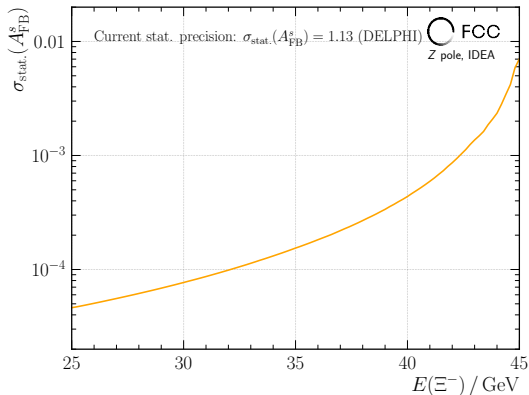
A_{FB}^s : some numbers and outlook

- Purity above $> 95\%$ in reach for Ξ^- baryons
 - Requires an **adequate correction** for detector acceptance effects
- However: for $E(\Xi^-) > 35 \text{ GeV}$ **accurate approximation** of s-quark direction



A_{FB}^s : some numbers and outlook

- Purity above $> 95\%$ in reach for Ξ^- baryons
 - Requires an **adequate correction** for detector acceptance effects
- However: for $E(\Xi^-) > 35$ GeV **accurate approximation** of s-quark direction

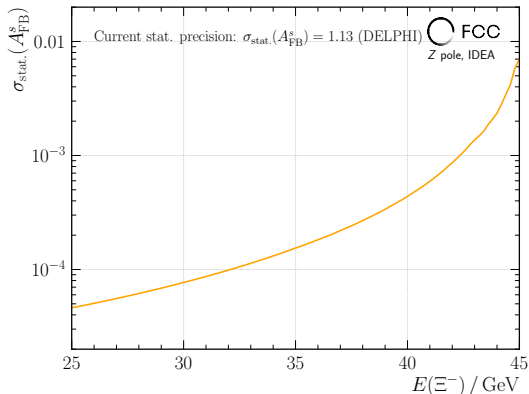


- Full analysis **lacks the proper analysis tools**
- All numbers presented rely on PYTHIA8's hadronisation fraction for $s \rightarrow \{\phi(1020), \Xi^-\}$

$$\mathcal{O}(\sigma(A_{\text{FB}}^s)) = 1.5 \cdot 10^{-4} \text{ in reach}$$

A_{FB}^s : some numbers and outlook

- Purity above $> 95\%$ in reach for Ξ^- baryons
 - Requires an **adequate correction** for detector acceptance effects
- However: for $E(\Xi^-) > 35$ GeV **accurate approximation** of s-quark direction



- Full analysis **lacks the proper analysis tools**
- All numbers presented rely on PYTHIA8's hadronisation fraction for $s \rightarrow \{\phi(1020), \Xi^-\}$

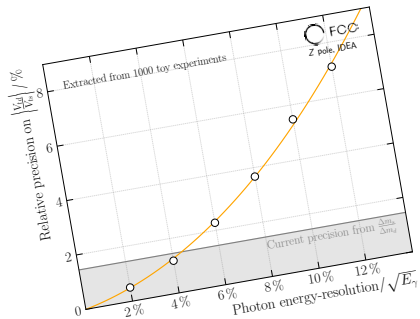
$$\mathcal{O}(\sigma(A_{\text{FB}}^s)) = 1.5 \cdot 10^{-4} \text{ in reach}$$

- Discussion of systematic uncertainties like QCD corrections to be done
- Expected to be subdominant in **presence of energy cuts**

Conclusions & Outlook

- Flavour physics programme at FCC-ee opens up a multitude to probe NP effects

→ Defines stringent detector requirements, e. g. EM calorimetry resolution



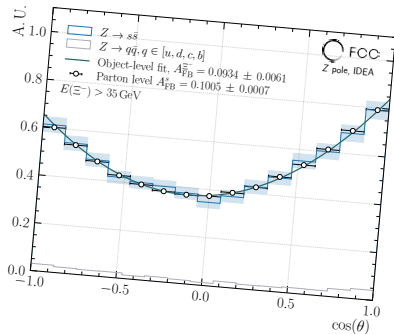
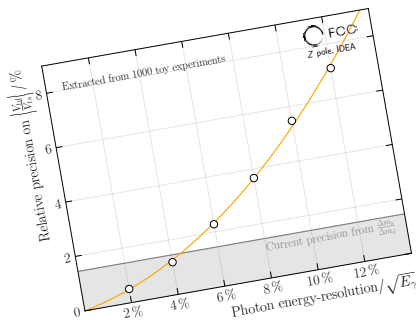
Conclusions & Outlook

- Flavour physics programme at FCC-ee opens up a multitude to probe NP effects

→ Defines stringent detector requirements, e. g. EM calorimetry resolution

- Also allows to study EWPOs at a **new level of precision**

- Complement EWPOs in the charm sector with A_{FB}^C (but no showstoppers identified so far)

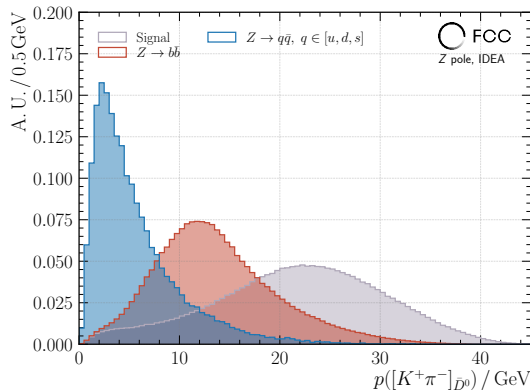
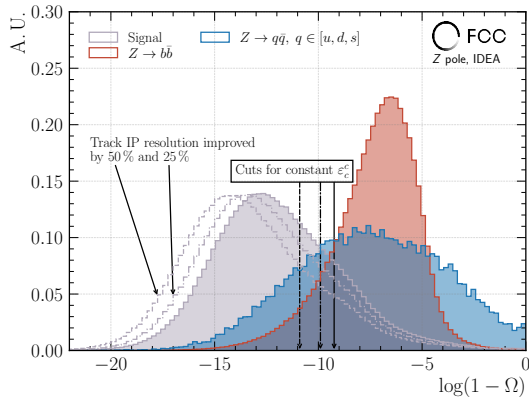


Backup

R_C : kinematic cuts in the phase space

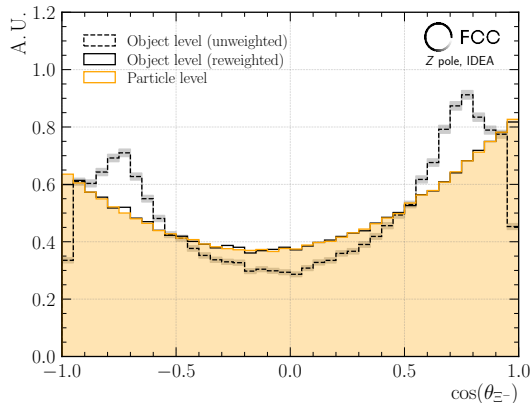
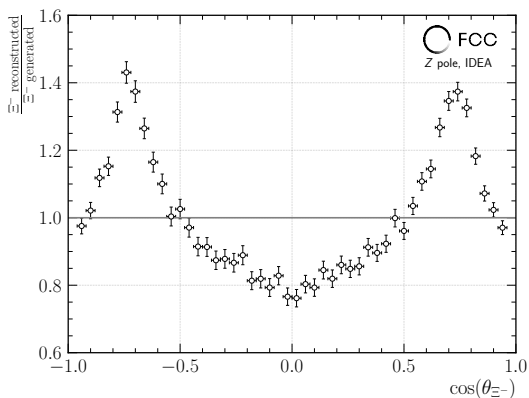
- Simple cuts to suppress backgrounds from the most obvious processes

$$\rightarrow \text{FD}(\bar{D}^0) < 3 \text{ mm}, d_0(K) < 1 \text{ mm}, N_{\text{SL, hem.}} = 0, p(\bar{D}^0) > 16 \text{ GeV}, \Omega = \frac{\vec{r} \cdot \vec{p}(\bar{D}^0)}{|\vec{r}| \cdot |\vec{p}(\bar{D}^0)|} < -9$$



A_{FB}^S : correction for detector acceptance effects

- Angular acceptance given by volume in front of final-state decay particles



Exclusive reconstruction: included B^+ decay-modes

Mode	$\text{Br}(B^+ \rightarrow XY) / \%$	$\text{Br}(X \rightarrow \text{final state}) / \%$		$\sum \text{Br} / \%$
$J/\psi K^+$	0.102 ± 0.002	$J/\psi \rightarrow e^+e^-$ $J/\psi \rightarrow \mu^+\mu^-$	5.971 ± 0.032 5.961 ± 0.033	0.012
$\bar{D}^0 \rho^+$	1.340 ± 0.180	$\bar{D}^0 \rightarrow K^+\pi^-\pi^0$	14.400 ± 0.500	0.545
$\bar{D}^0 \pi^+\pi^-\pi^+$	0.560 ± 0.210	$\bar{D}^0 \rightarrow K^+\pi^-2\pi^0$	8.860 ± 0.230	0.723
$\bar{D}^0 \pi^+$	0.468 ± 0.013	$\bar{D}^0 \rightarrow K^+2\pi^-\pi^+$	8.220 ± 0.140	0.909
$[\bar{D}^0\pi^+]_{D^*(2010)^+} \pi^-\pi^-\pi^0$	10.160 ± 4.740	$\bar{D}^0 \rightarrow K^+2\pi^-\pi^+\pi^0$ $\bar{D}^0 \rightarrow K^+\pi^-$	4.300 ± 0.400 3.947 ± 0.030	0.950
$D^- \pi^+\pi^-$	0.107 ± 0.005	$D^+ \rightarrow K^-2\pi^+$ $D^+ \rightarrow K^-2\pi^+\pi^0$	9.380 ± 0.160 6.250 ± 0.180	0.966
$D_s^+ \bar{D}^0$	0.900 ± 0.090	$D_s^+ \rightarrow [\pi^+\pi^-\pi^0]_\eta \pi^+\pi^0$ $D_s^+ \rightarrow [\pi^+\pi^-\pi^0]_\eta [\pi^+\pi^0]_{\rho^+}$ $D_s^+ \rightarrow K^+K^-\pi^+\pi^0$ $D_s^+ \rightarrow K^+K^-\pi^+$ $D_s^+ \rightarrow 2\pi^+\pi^-$ $D_s^+ \rightarrow K^+K^-2\pi^+\pi^-$ $D_s^+ \rightarrow 3\pi^+2\pi^-$	9.500 ± 0.500 8.900 ± 0.800 5.500 ± 0.240 5.380 ± 0.100 1.080 ± 0.040 0.860 ± 0.150 0.790 ± 0.080	1.081

Exclusive reconstruction: included B_d^0 decay-modes

Mode	Br($B^0 \rightarrow$ final state) / %	Σ Br / %
$J/\psi K^+ \pi^-$	0.014	0.014
$D^*(2010)^- \pi^+ \pi^+ \pi^- \pi^0$	0.473	0.487
$D^*(2010)^- \pi^+ \pi^0$	0.403	0.891
$D^*(2010)^- \pi^+ \pi^+ \pi^-$	0.194	1.084
$D^- \pi^+ \pi^+ \pi^-$	0.094	1.178
$D^*(2010)^- \pi^+$	0.074	1.252
$D^*(2010)^- D_s^+$	0.069	1.321
$D^- \pi^+$	0.039	1.360
$D^- D_s^+$	0.036	1.396
$D^*(2010)^- D^0 K^+$	0.026	1.422
$D^- D^0 K^+$	0.007	1.429

Exclusive reconstruction: included B_s^0 decay-modes

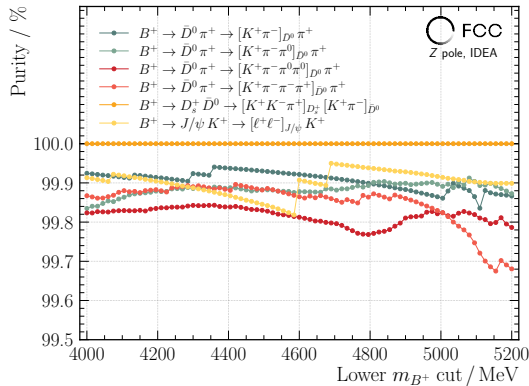
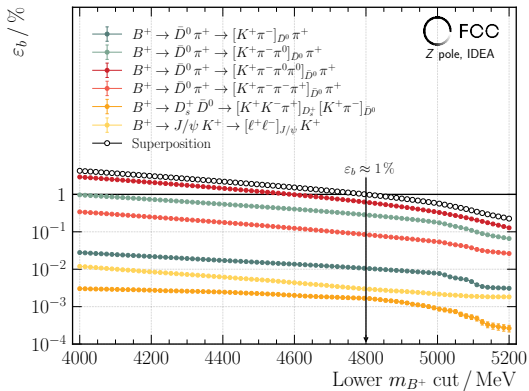
Mode	Br($B_s^0 \rightarrow$ final state) / %	Σ Br / %
$D_s^- [\pi^+ \pi^0]_{\rho^+}$	0.218	0.218
$D_s^- \pi^+ \pi^+ \pi^-$	0.195	0.413
$D_s^*(2010)^- \pi^+ \pi^+ \pi^-$	0.194	0.607
$D_s^- \pi^+$	0.095	0.702
$D_s^+ D_s^-$	0.045	0.747
$D^0 K^- \pi^+$	0.041	0.789

Exclusive reconstruction: included Λ_b^0 decay-modes

Mode	$\text{Br}(\Lambda_b^0 \rightarrow XY) / \%$	$\text{Br}(X \rightarrow \text{final state}) / \%$		$\sum \text{Br} / \%$
$\Lambda_b^0 \rightarrow \Lambda_c^+ \pi^+ \pi^- \pi^-$	0.760 ± 0.110	$\Lambda_c^+ \rightarrow p K^- \pi^+$	6.280 ± 0.320	0.082
		$\Lambda_c^+ \rightarrow p K^- \pi^+ \pi^0$	4.460 ± 0.300	

Increasing the tagging efficiency

- For R_b , also partially reconstructed candidates are b -taggers
- Releasing the B^+ -mass constraint significantly increases ε_b



Quantitative summary

B^+ decay-mode	$\epsilon_{\text{reco}} / \%$	Purity / %	B^+ signal width / MeV
$\bar{D}^0 \pi^+ \rightarrow [K^+ \pi^-]_{\bar{D}^0} \pi^+$	77.17 ± 2.99	99.93 ± 0.11	7.0
$\bar{D}^0 \pi^+ \rightarrow [K^+ \pi^- \pi^0]_{\bar{D}^0} \pi^+$	64.89 ± 1.41	99.89 ± 0.09	32.8
$\bar{D}^0 \pi^+ \rightarrow [K^+ \pi^- \pi^0 \pi^0]_{\bar{D}^0} \pi^+$	49.95 ± 2.68	99.81 ± 0.07	35.1
$\bar{D}^0 \pi^+ \rightarrow [K^+ \pi^- \pi^- \pi^+]_{\bar{D}^0} \pi^+$	72.63 ± 6.90	99.73 ± 0.27	9.7
$D_s^+ \bar{D}^0 \rightarrow [K^+ K^- \pi^+]_{D_s^+} [K^+ \pi^-]_{\bar{D}^0}$	78.57 ± 22.39	100.00	5.6
$J/\psi K^+ \rightarrow [\ell^+ \ell^-]_{J/\psi} K^+$	85.87 ± 4.13	99.90 ± 0.24	7.4

b -quark partial-decay width ratio

R_b : systematic uncertainties from ALEPH

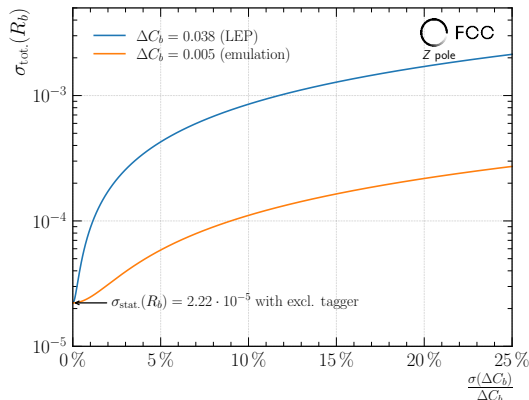
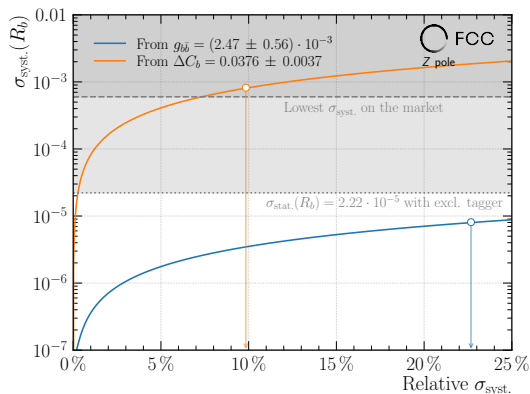
- Systematic uncertainties enter where quantities have been estimated from MC simulations: ϵ_{udsc} , ΔC_q
- For ϵ_{udsc} , predictions depend on assumed impact parameter resolution and efficiency for vertex-detector hits to be associated to a track
- Physical parameters that enter the calculation of ϵ_{udsc}

$$\begin{aligned} \Delta R_b = & \pm 0.00047 \quad \text{Monte Carlo statistics} \\ & \pm 0.00017 \quad \text{Event selection} \\ & \pm 0.00084 \quad \text{Physics uncertainty} \\ & \pm 0.00046 \quad \text{Tracking uncertainty} \\ & \pm 0.00027 \quad \text{Hemisphere correlations uncertainty} \end{aligned}$$

R_b : systematic uncertainties comparison

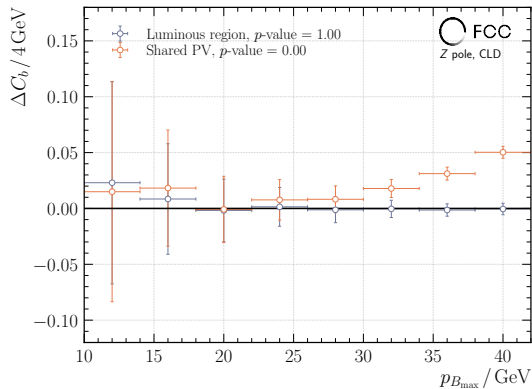
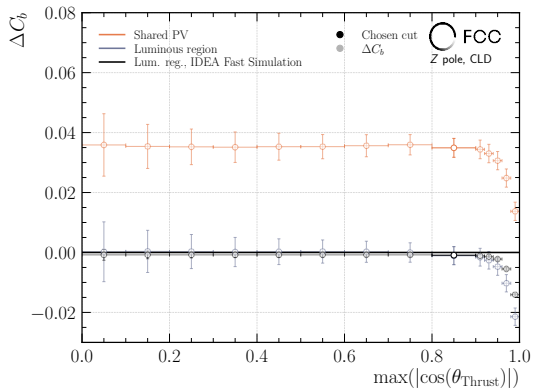
- Gluon splitting rate $g_{b\bar{b}} = 0.00247(56)$ as source of $\sigma_{\text{sys.}}(R_b)$ negligible compared to ΔC_b

$$N_b = 2N_Z \left(R_b \varepsilon_{b_{1,2}}^{Z \rightarrow b\bar{b}} \varepsilon_{E_B}^{Z \rightarrow b\bar{b}} + (1 - R_b) g_{b\bar{b}} \varepsilon_{udsc_{1,2}}^{g \rightarrow b\bar{b}} \varepsilon_{E_B}^{g \rightarrow b\bar{b}} \right)$$



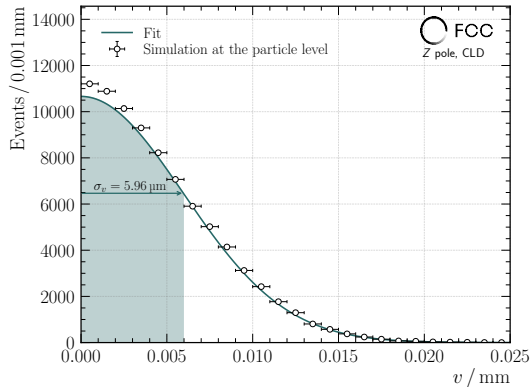
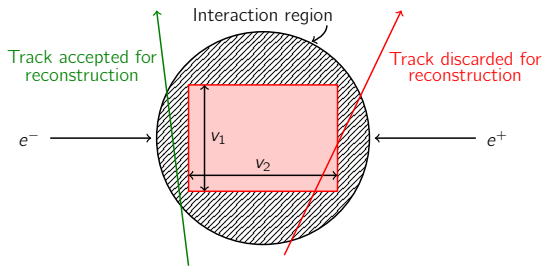
R_b : systematic uncertainties

- PV reconstruction main source of $\Delta C_b \neq 0$
- However: detector acceptance effects + gluon splitting studied as well



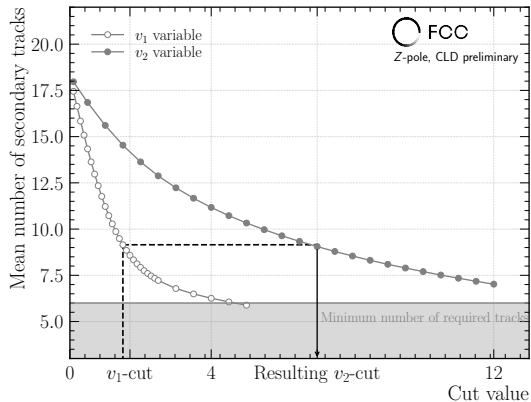
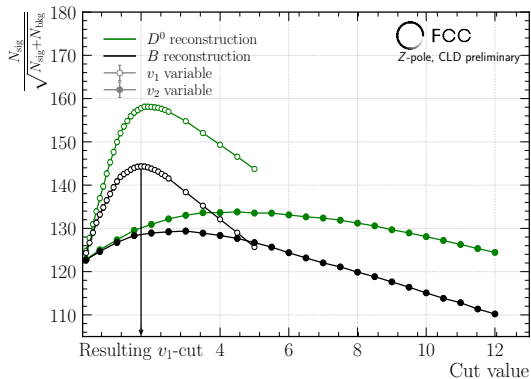
Luminous region selection

- Selection of tracks outside of luminous region to overcome PV limitations
- Maximise $v_1 = d_0 / \sqrt{\sigma_{d_0}^2 + \sigma_v^2}$ and $v_2 = z_0 / \sqrt{\sigma_{z_0}^2 + \sigma_v^2}$ w.r.t. specific FOM



Luminous region selection: FOMs

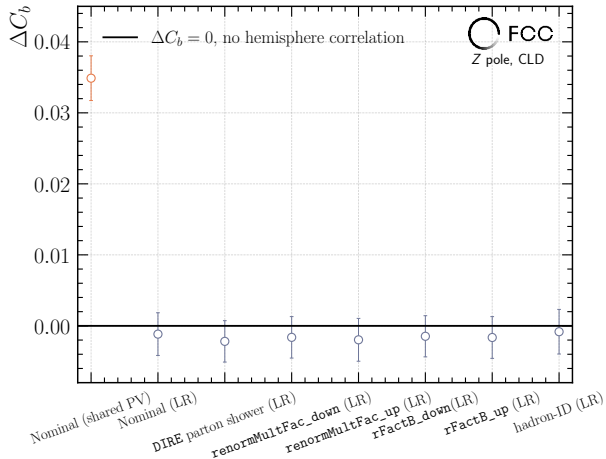
- Evaluate \bar{D}^0 and B^+ significance + mean number of secondary tracks



$$\rightarrow v_1 \leq 3 \ \& \ v_2 \leq 8$$

ΔC_b : systematic uncertainties

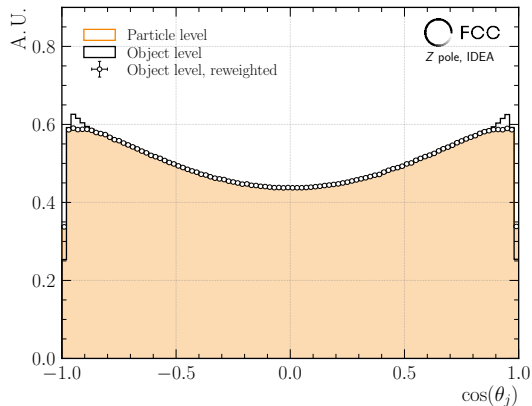
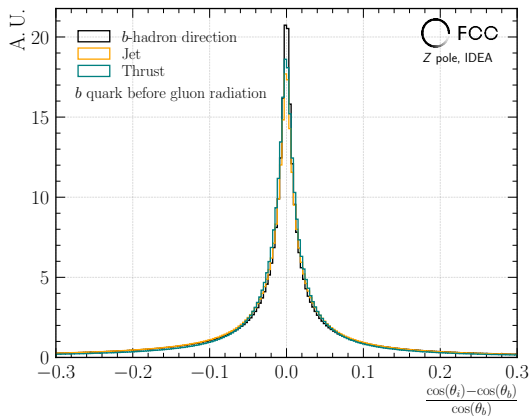
- First investigation of systematic uncertainties for ΔC_b
 - Varying inputs: DIRE parton shower, renormalisation scale, b fragmentation, track ID
- No significant impact on ΔC_b



b -quark forward-backward asymmetry

Different b -quark direction estimators

- Usual choices at LEP: thrust-axis
- Existing study @FCC-ee: b -quark direction from b -tagged jets
- Taking into account jet-reconstruction efficiency effects

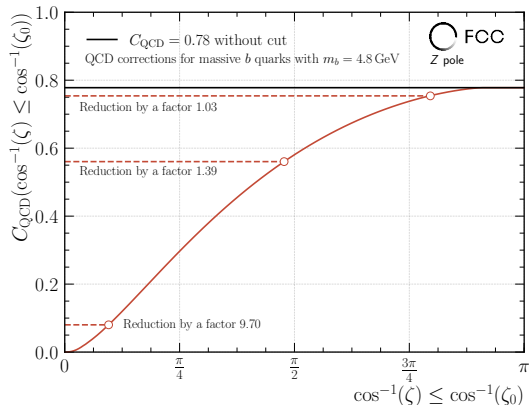
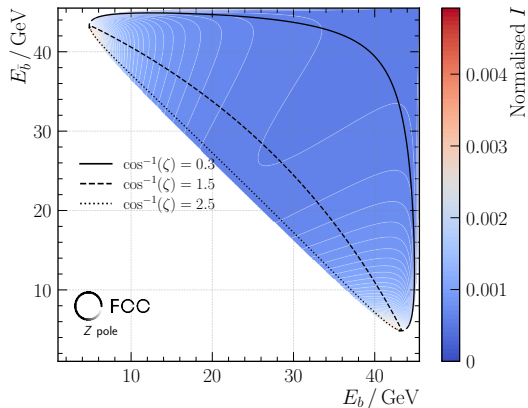


Calculations of C_{QCD} for jet-jet acollinearity

- C_{QCD} as function of acollinearity $\cos(\zeta(x, \bar{x})) = \frac{x\bar{x} + \mu^2 + 2(1-x-\bar{x})}{\sqrt{x^2 - \mu^2}\sqrt{\bar{x}^2 - \mu^2}}$ with $x = 2E_b/\sqrt{s}$ and $\mu = 2m_b/\sqrt{s}$

$$C_{\text{QCD}} \approx \int_{x_{\min}}^{x_{\max}} \int_{\bar{x}_{\min}(x)}^{\bar{x}_{\max}(x)} \frac{2\bar{x}^2(1 - \cos(\zeta(x, \bar{x})))}{3(1-x)(1-\bar{x})} d\bar{x} dx,$$

$$I(x, \bar{x}) = \frac{(x^2 + \bar{x}^2) \cdot (1 - \cos(\zeta(x, \bar{x})))}{3(1-x)(1-\bar{x})}$$



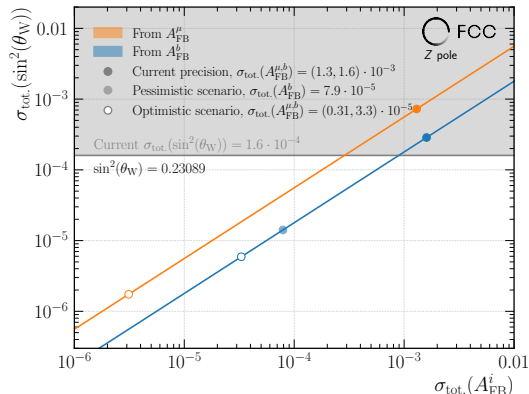
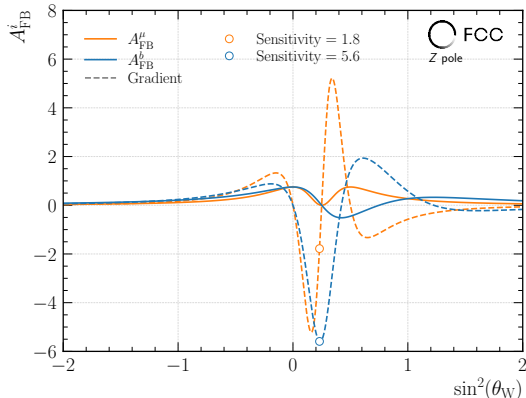
Impact on SM parameters: weak mixing angle

- Precision of A_{FB}^b impacts precision of $\sin^2(\theta_W)$

$$A_{\text{FB}} = \frac{3}{4} A_e A_b, \quad \text{with} \quad A_f = \frac{2v_f a_f}{v_f^2 + a_f^2},$$

$$a_f = T_f, \quad \text{and} \quad v_f = T_f - 2Q_f \sin^2(\theta_W)$$

→ 3× more sensitivity from A_{FB}^b to $\sin^2(\theta_W)$ than from A_{FB}^μ (fractional charge)



Impact on SM parameters: top-quark mass

- Precision of m_t from top-quark loops in Z propagator and $\sin^2(\theta_W^{\text{eff.}}) = \xi \sin^2(\theta_W)$ and $\xi = 1 + \Delta\rho \cotan^2(\theta_W)$

$$\Delta\rho = 3x_t + 3x_t^2(19 - \pi^2), \quad \text{and} \quad x_t = \frac{G_F m_t^2}{8\sqrt{2}\pi^2}$$

- In addition: account for vertex corrections with top quarks

$$\Delta\tau = -2x_t - \frac{G_F m_t^2}{6\sqrt{2}\pi^2} \cdot (1 - \cos(\theta_W)) \ln\left(\frac{m_t}{m_W}\right) - 2x_t^2 \cdot \left(2 - \frac{\pi^2}{3}\right),$$

$$\Rightarrow v_f \rightarrow \bar{v}_f = \sqrt{\rho_f} \left(T_f - \frac{2Q_f \sin^2(\theta_W^{\text{eff.}})}{1 + \Delta\tau} \right),$$

$$\Rightarrow a_f \rightarrow \bar{a}_f = \sqrt{\rho_f} T_f, \quad \text{with} \quad \rho_f = \frac{(1 + \Delta\tau)^2}{1 - \Delta\rho}$$

