

Accuracy complements energy: EW precision at Tera-Z

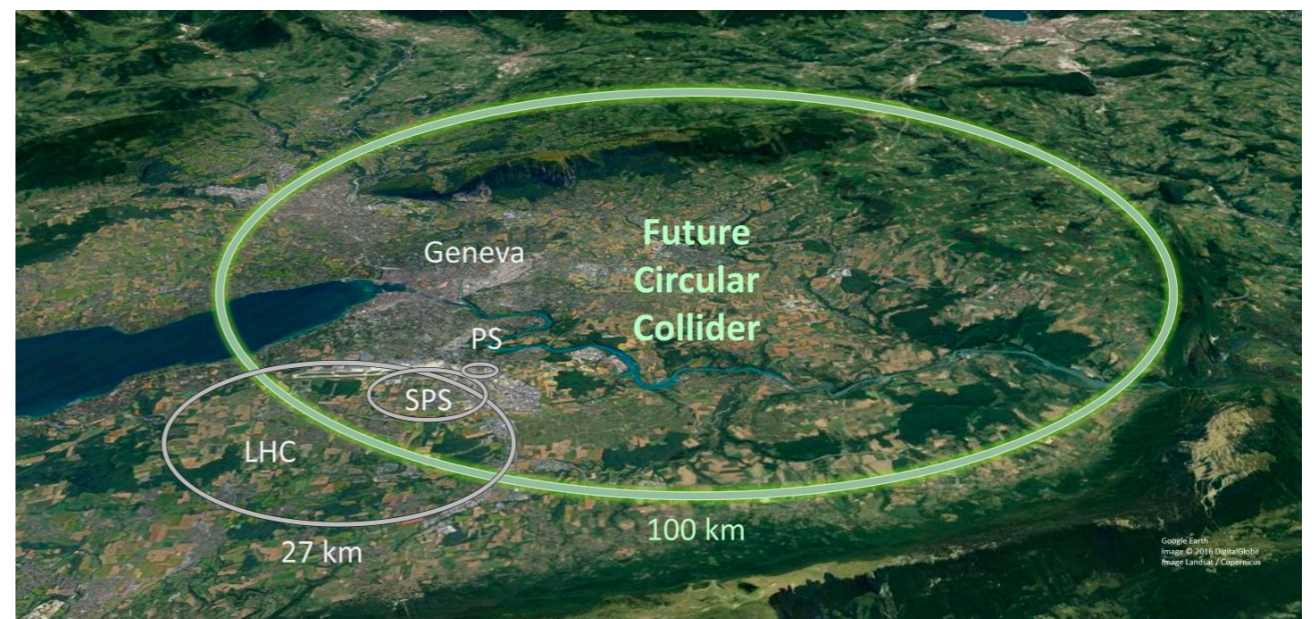
Ben A. Stefanek

GENT Fellow, IFIC Valencia

Flavor and Origin of Matter Group

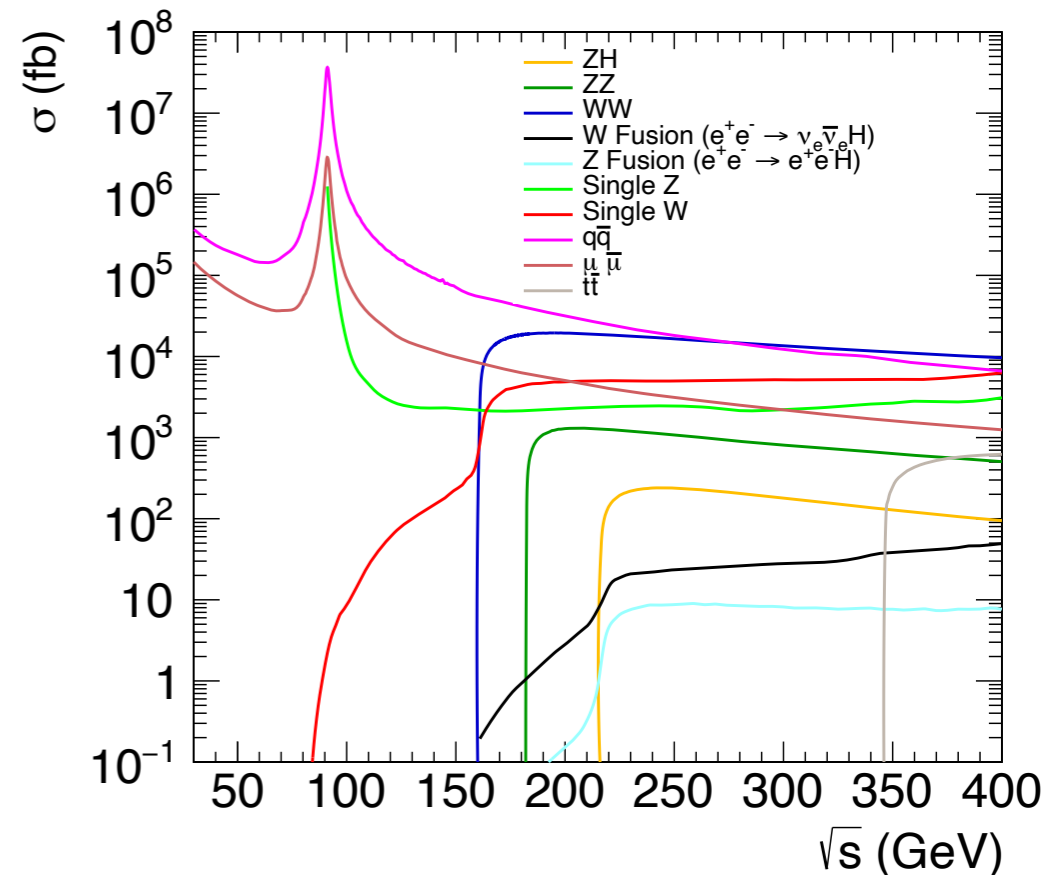
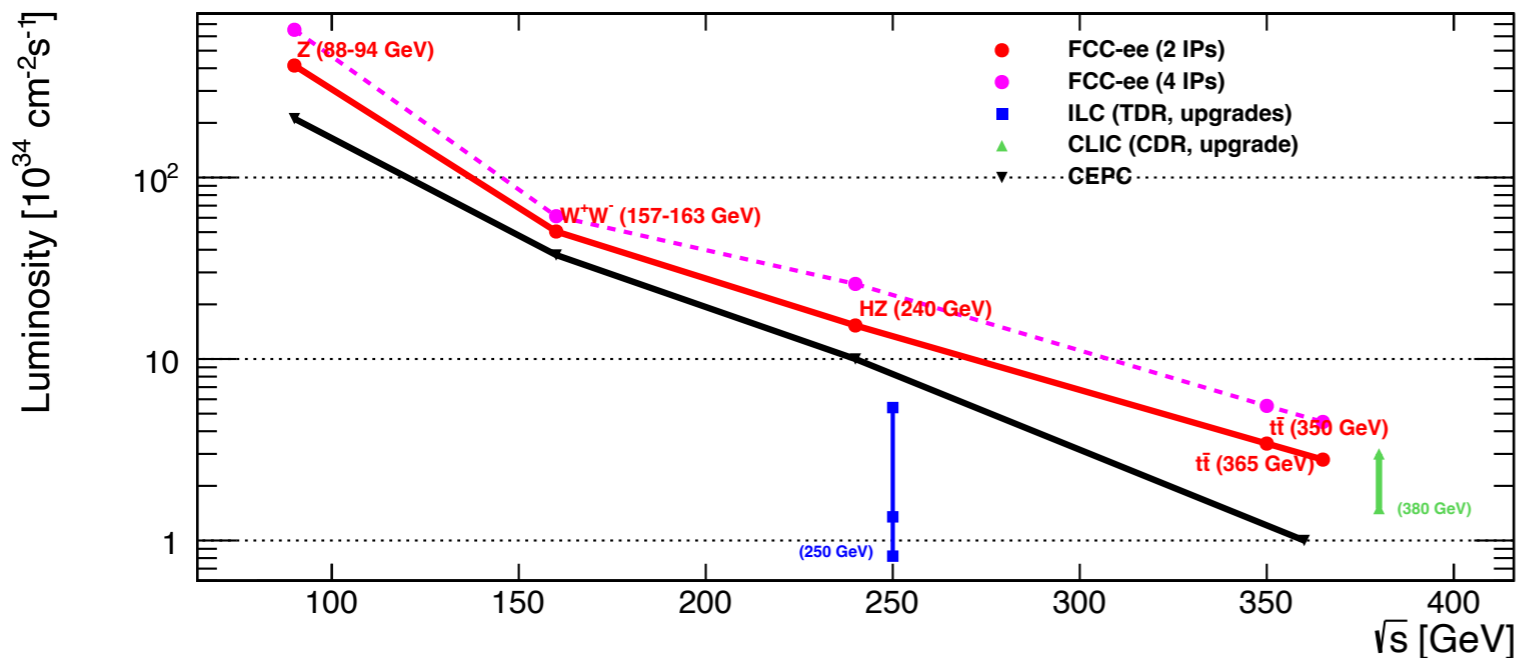
8th FCC Physics Workshop

January 14th, 2025



[Based on: [2412.14241](#), [2407.09593](#), [2311.00020](#)]

Run plan for FCC-ee



Energy \rightarrow

Working point	Z years 1-2	Z, later	WW	HZ	$t\bar{t}$	
\sqrt{s} (GeV)	88, 91, 94		157, 163	240	340–350, 365	
Lumi/IP ($10^{34} \text{ cm}^{-2} \text{ s}^{-1}$)	115	230	28	8.5	0.95	1.55
Lumi/year (ab^{-1} , 2 IP)	24	48	6	1.7	0.2	0.34
Physics goal (ab^{-1})	150		10	5	0.2	1.5
Run time (year)	2	2	2	3	1	4
Number of events	5×10^{12} Z		10^8 WW	10^6 HZ + 25k WW \rightarrow H	10^6 $t\bar{t}$ +200k HZ +50k WW \rightarrow H	

Accuracy \leftarrow

Accuracy complements energy (ACE) at FCC-ee

- As a general principle, FCC-ee will have similar sensitivity to a given EFT operator both on and off the Z-pole in two main ways:

1. The same operator enters at *leading order off the Z-pole*, as well as at *next-to-leading order on the Z-pole* (similarly for NLO vs NNLO).

$$\Delta_{Z \text{ pole}/ZH}^{\text{NLO/LO}} \sim \frac{1}{16\pi^2} \frac{\epsilon_Z}{\epsilon_{ZH}} \sqrt{\frac{N_Z}{N_{ZH}}} \sim O(1), \quad \left(\begin{array}{l} \epsilon_Z \sim 10^{-1}, \quad \epsilon_{ZH} \sim 1 \\ N_Z \sim 10^{12}, \quad N_{ZH} \sim 10^6 \end{array} \right)$$

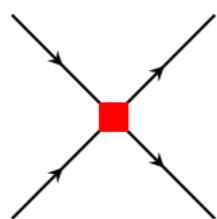
2. The same operator enters at *leading order both on and off the Z-pole*, but *receives an energy enhancement off the pole*. This wasn't the case at LEP!



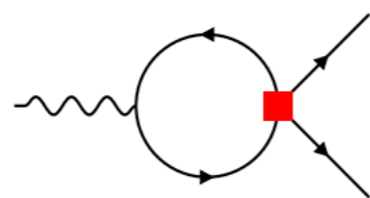
$$\Delta_{Z \text{ pole}/WW}^{\text{LO/LO}} \sim \frac{m_Z^2}{E_{WW}^2} \frac{\epsilon_Z}{\epsilon_{WW}} \sqrt{\frac{N_Z}{N_{WW}}} \sim O(1), \quad \left(\begin{array}{l} E_{WW} \sim 200 \text{ GeV} \\ N_Z \sim 10^{12}, \quad N_{WW} \sim 10^8 \end{array} \right) \quad \begin{array}{l} (N_Z/N_{WW})_{\text{LEP}} \sim 10^2 \\ (N_Z/N_{WW})_{\text{FCC}} \sim 10^4 \end{array}$$

Four fermion operators

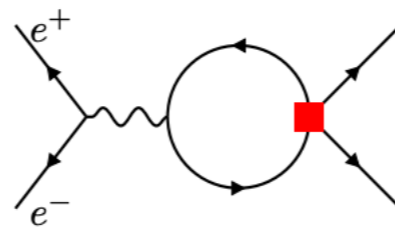
- So far, this case has received the most attention in the literature. The general summary is that **4F operators with electrons are better off pole**, while **4F operators with tops are better on pole**. Otherwise, ACE holds + there is similar sensitivity.



(a) Four-fermion operator



(b) Z-vertex correction



(c) $e^+e^- \rightarrow f\bar{f}$

- We use the recent dedicated flavor tagging study for $e^+e^- \rightarrow f\bar{f}$ at FCC-ee. Don't miss the *talk by Alessandro Valenti this evening* for the details on this very nice analysis.

Pure 3rd gen. 4F operators

$\Lambda^{[3333]}$ [TeV]	FCC-ee Z, W-pole+ τ	FCC-ee above Z-pole
$\Lambda_{\ell q}^{(1)}$	15.7	1.1
$\Lambda_{\ell q}^{(3)}$	14.0	5.1
Λ_{eu}	16.2	1.6
Λ_{ed}	1.5	1.3
Λ_{lu}	15.4	1.5
Λ_{ld}	1.5	1.3
Λ_{qe}	16.7	1.1
Λ_{ll}	1.0	1.0
Λ_{le}	2.1	1.5
Λ_{ee}	3.5	2.4
$\Lambda_{qq}^{(1)}$	13.1	2.4
$\Lambda_{qq}^{(3)}$	8.4	7.1
$\Lambda_{qu}^{(1)}$	9.4	1.4
$\Lambda_{qd}^{(1)}$	3.1	0.9
Λ_{uu}	12.1	1.9
Λ_{dd}	0.4	2.3
$\Lambda_{ud}^{(1)}$	2.8	1.9

[Greljo, Tiblom, Valenti [2411.02485](#)]

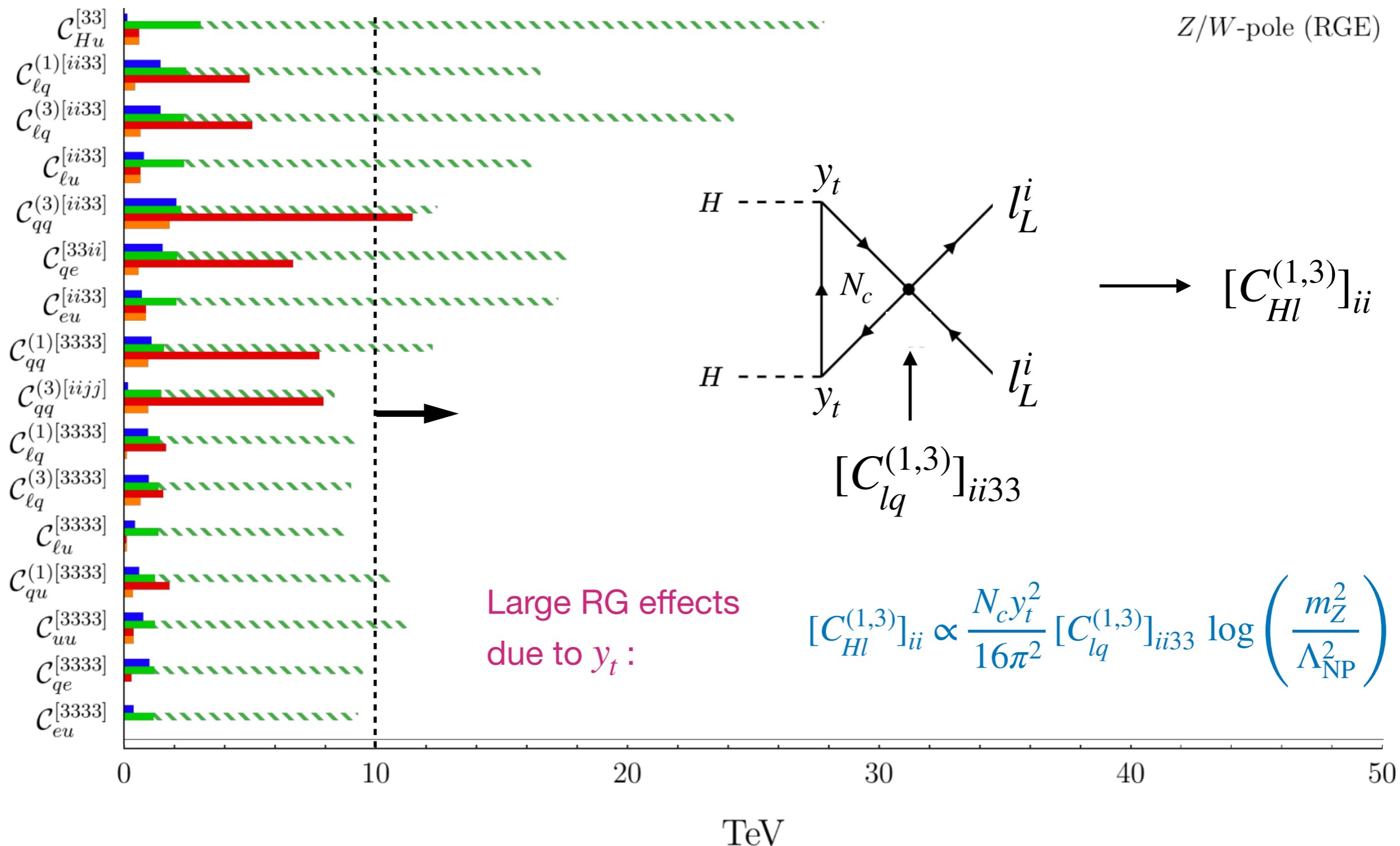
 See Alessandro's talk for more!

Four fermion operators

[Allwicher, Cornella, Isidori, BAS, [2311.00020](#)]

See also Lukas Allwicher's talk

- NLO contribution of top operators at the Z-pole (SMEFT perspective)



Higgs physics

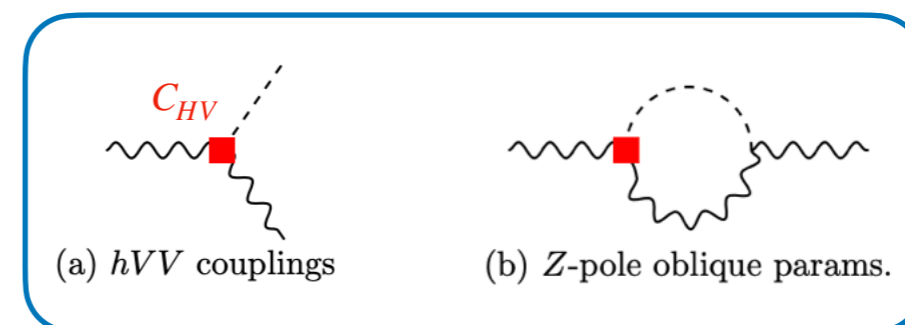
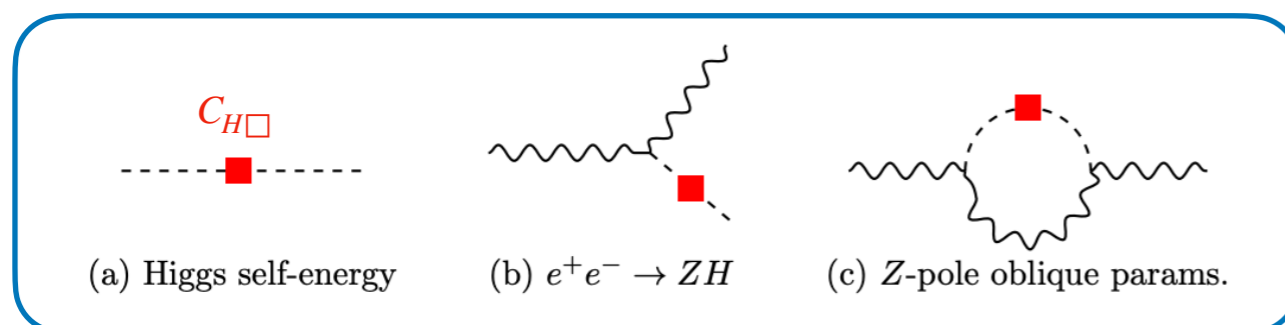
See also Sally Dawson's talk

- The cross-section $\sigma(e^+e^- \rightarrow ZH)$ is sensitive to 3 (4) dimension-6 operators at LO (NLO) that can modify Higgs couplings:

$$Q_{H\Box} = (H^\dagger H) \Box (H^\dagger H), \quad Q_H = (H^\dagger H)^3,$$

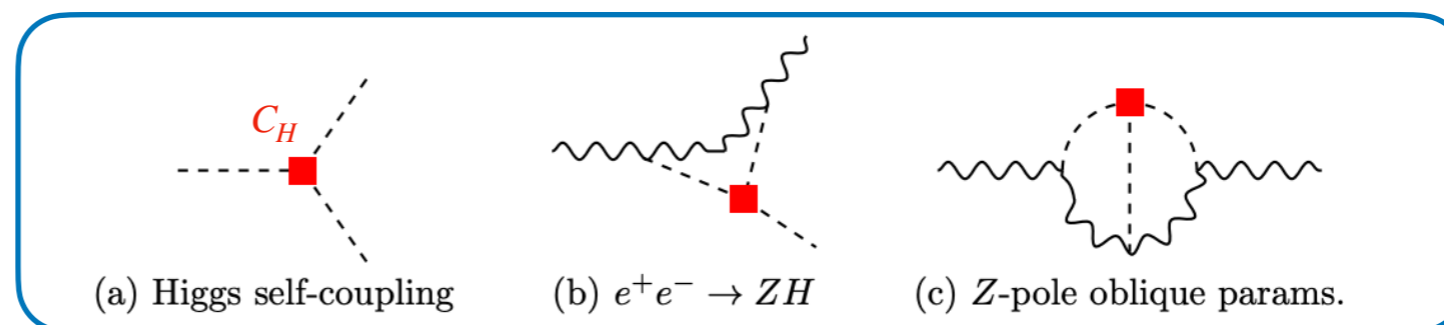
$$Q_{HW} = (H^\dagger H) W_{\mu\nu}^I W^{I\mu\nu}, \quad Q_{HB} = (H^\dagger H) B_{\mu\nu} B^{\mu\nu}.$$

- All of these operators also enter the Z-pole at one higher loop order:



Higgs self-coupling modifications starting at NLO (finite):

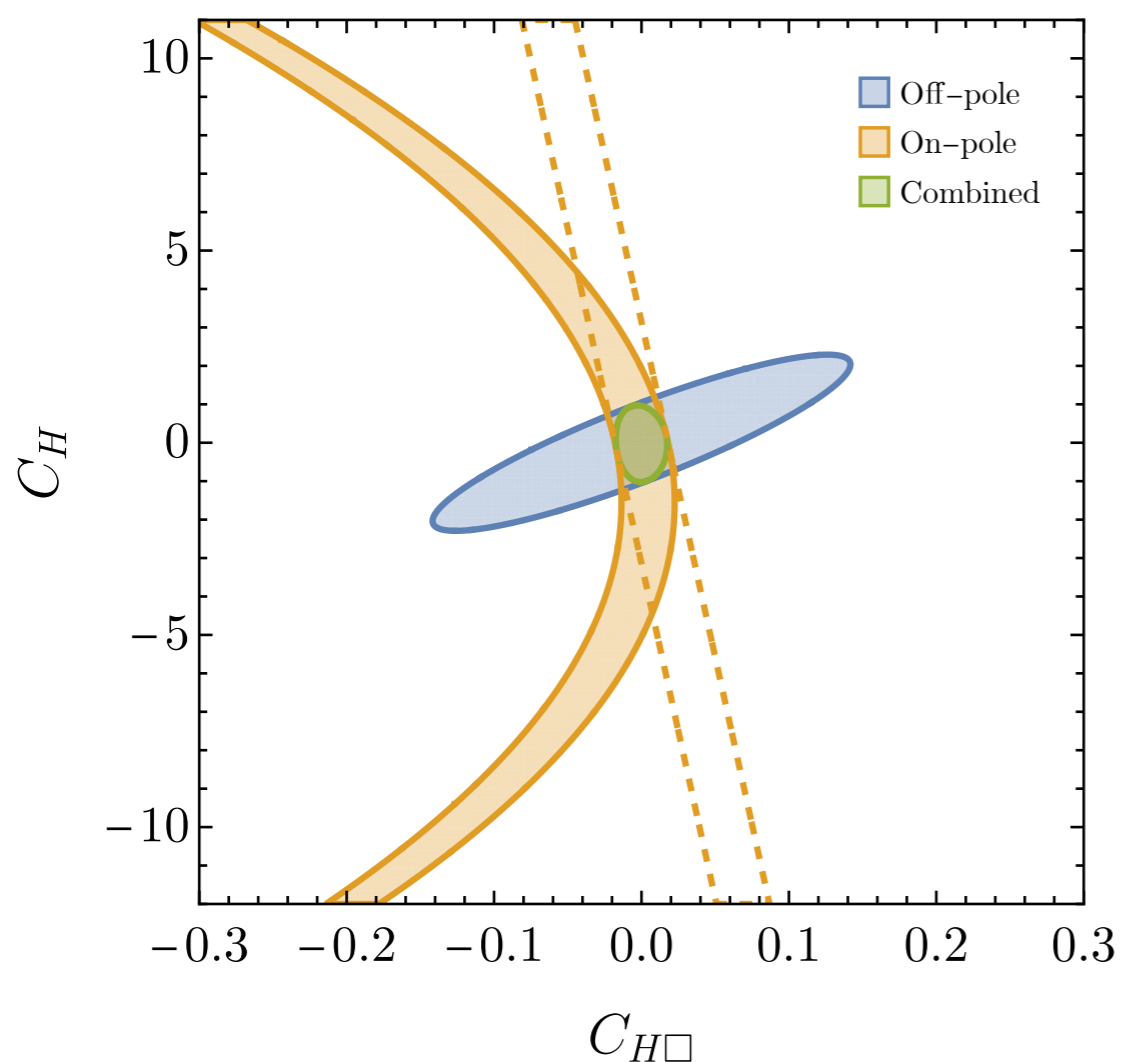
[1312.3322, 1702.07678, 1702.01737]



[Maura, BAS, You, 2412.14241] 6

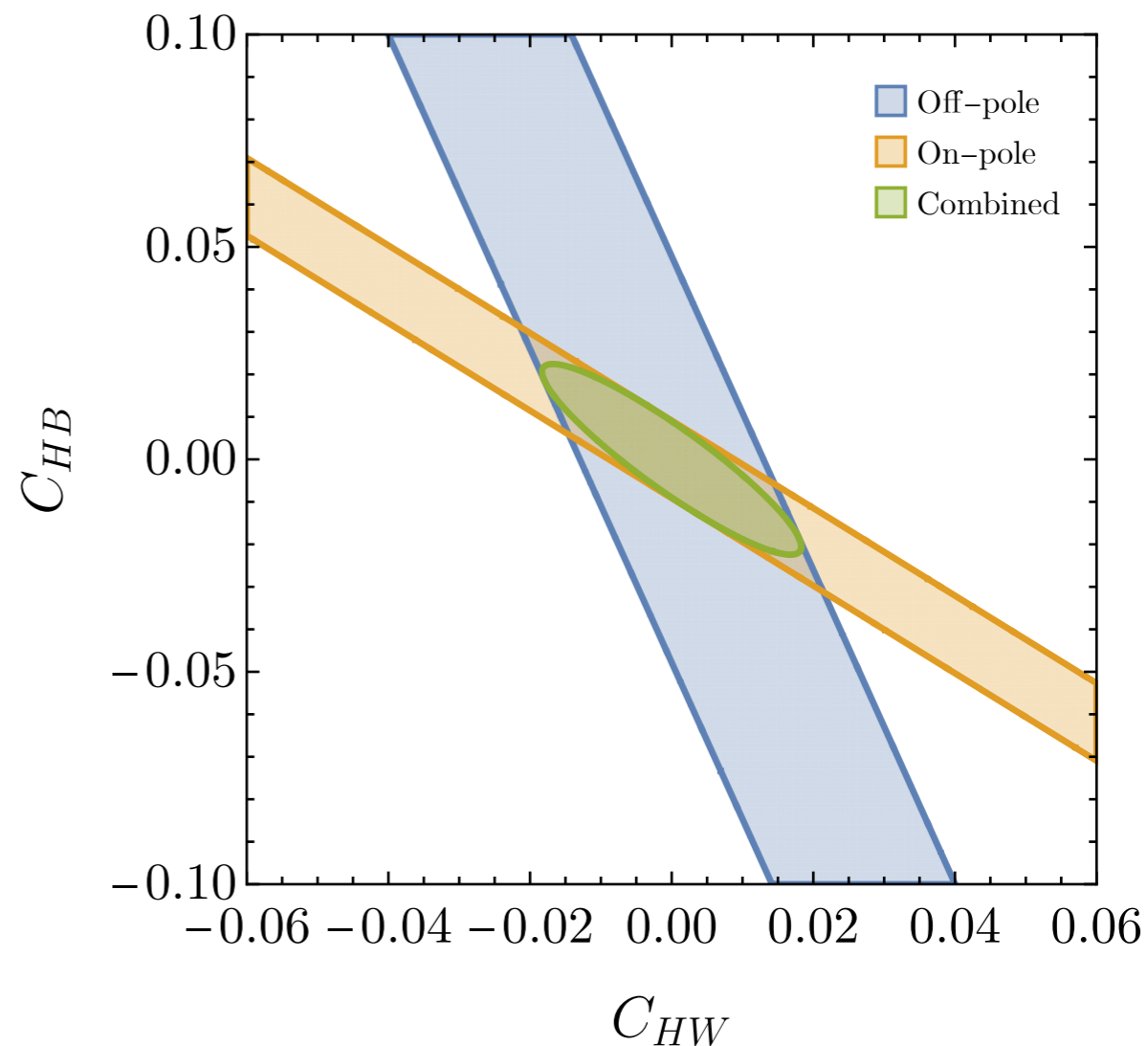
Accuracy complements energy for Higgs physics

- Since the Z-pole contributions have a relative one-loop suppression, we expect similar sensitivity via the first principle of accuracy complements energy:



NNLO Oblique $10^6 \hat{S} = -0.71 C_H - 0.25 C_H^2$,
Parameters: $10^6 \hat{T} = 1.2 C_H + 0.36 C_H^2$.

[1702.07678, 1702.01737]



*All Wilson coefficients are in units of TeV^{-2} and are renormalized at 1 TeV

[Maura, BAS, You, 2412.14241] 7

Gauge sector (2- and 3-point functions)

- We look at modifications of the EW gauge boson propagators (all runs) as well as modifications of gauge 3-point functions (aTGC).



Recall: $\Pi_{VV}(p^2) = \Pi_{VV}(0) + p^2\Pi'_{VV}(0) + p^4\Pi''_{VV}(0) + \dots$

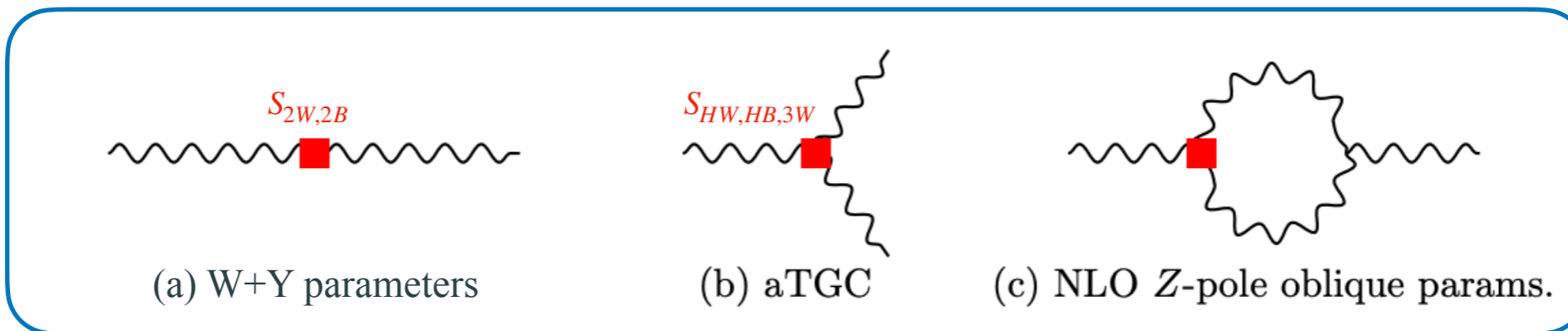


aTGC

$$\begin{aligned} \mathcal{O}_{HW} &= i(D^\mu H)^\dagger \tau^I (D^\nu H) W_{\mu\nu}^I, \\ \mathcal{O}_{HB} &= i(D^\mu H)^\dagger (D^\nu H) B_{\mu\nu}, \\ \mathcal{O}_{3W} &= \varepsilon_{IJK} W_{\mu}^{I\nu} W_{\nu}^{J\rho} W_{\rho}^{K\mu}. \end{aligned}$$

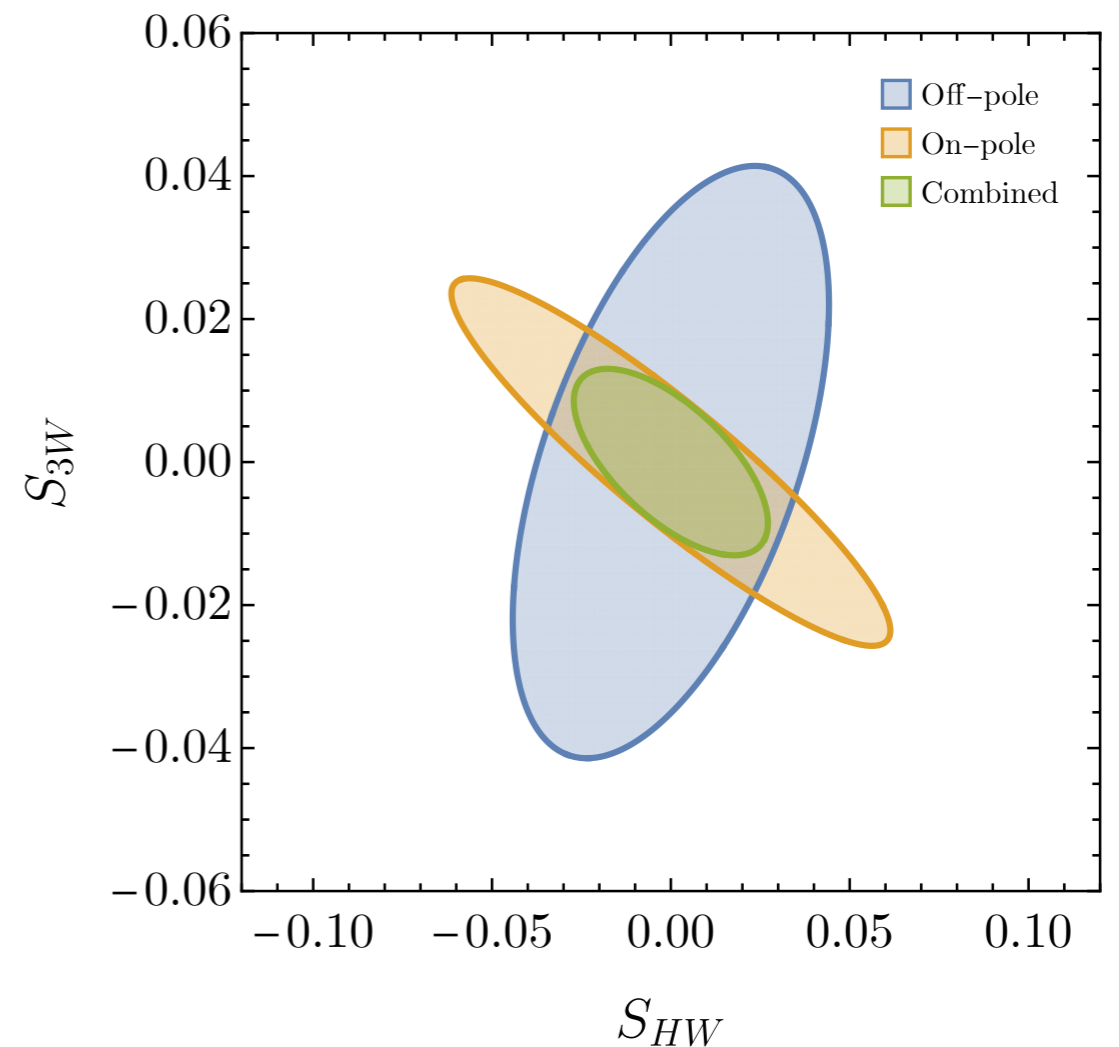
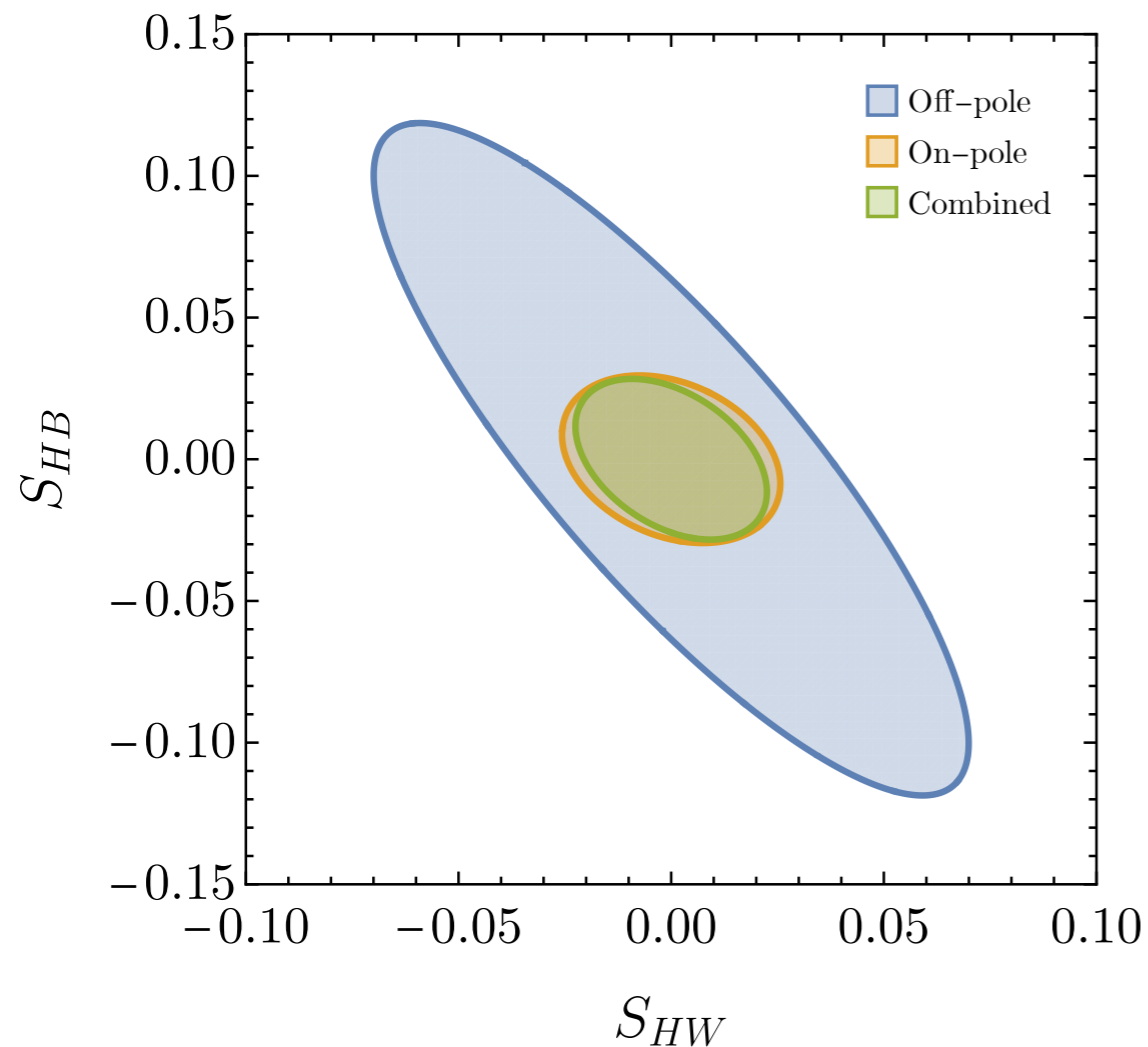
W+Y parameters

$$\begin{aligned} \mathcal{O}_{2B} &= -\frac{1}{2}(\partial^\mu B_{\mu\nu})(\partial_\rho B^{\rho\nu}), \\ \mathcal{O}_{2W} &= -\frac{1}{2}(D^\mu W_{\mu\nu}^I)(D_\rho W^{I\rho\nu}), \end{aligned}$$



Gauge sector: Anomalous triple gauge couplings

- Again, Z-pole contributions have a relative one-loop suppression. The Z-pole gives a better constraint on \mathcal{O}_{HB} , otherwise the sensitivity is similar.



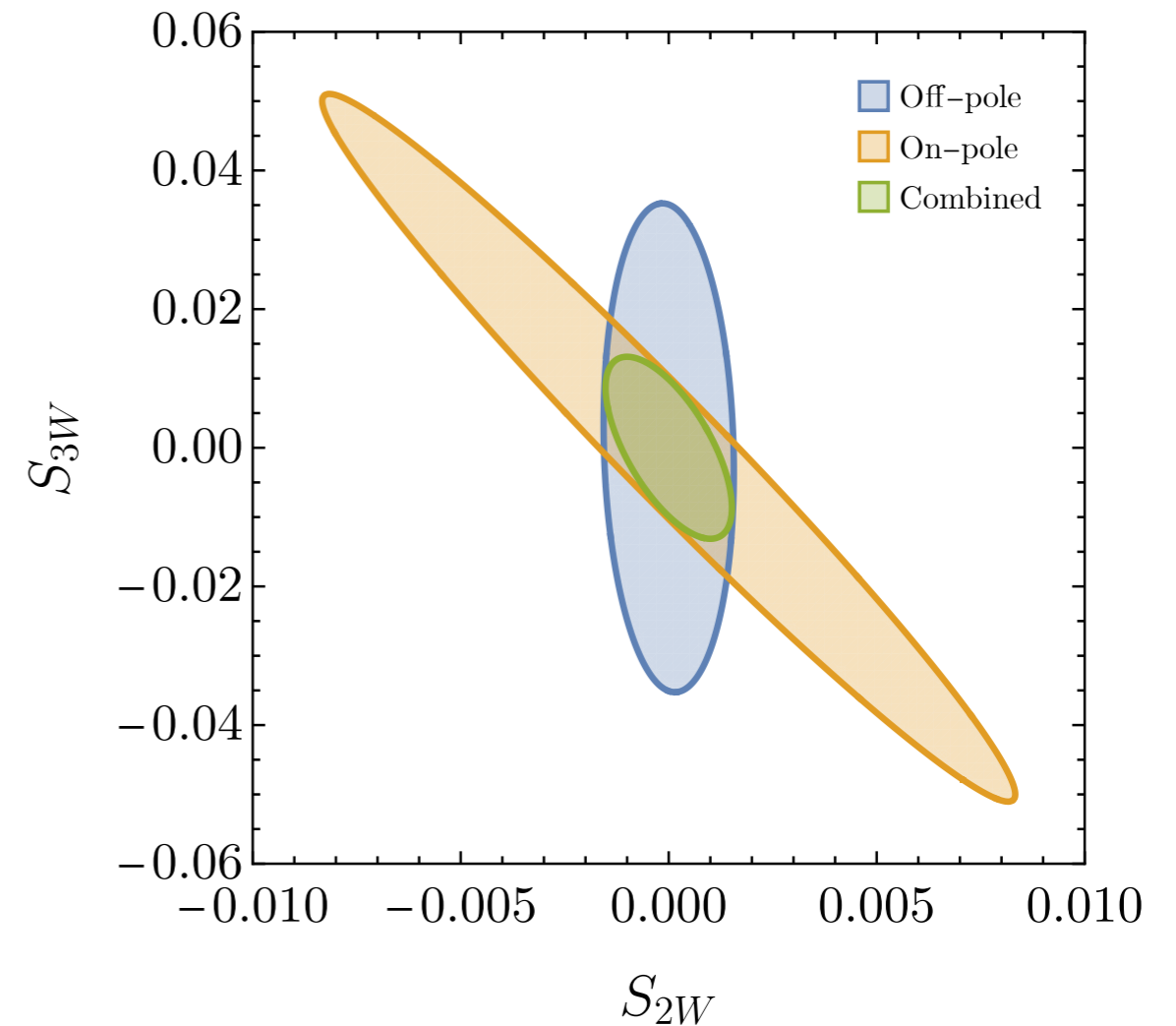
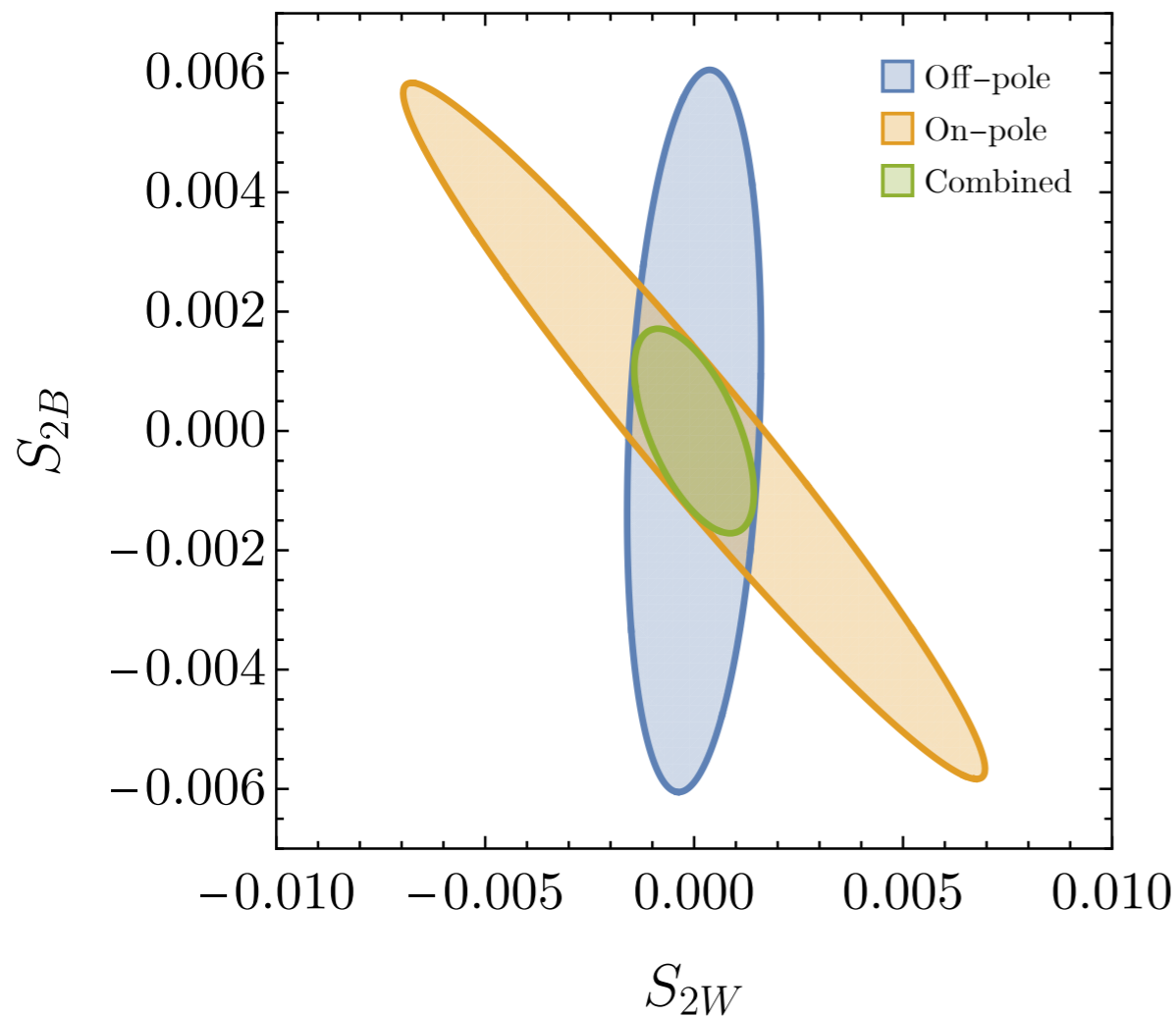
$$\mathcal{O}_{HB} = \mathcal{O}_B - \frac{1}{2}y_h g_1 Q_{HB} - \frac{1}{4}g_2 Q_{HWB},$$

$$\mathcal{O}_{HW} = \mathcal{O}_W - \frac{1}{4}g_2 Q_{HW} - \frac{1}{2}y_h g_1 Q_{HWB},$$

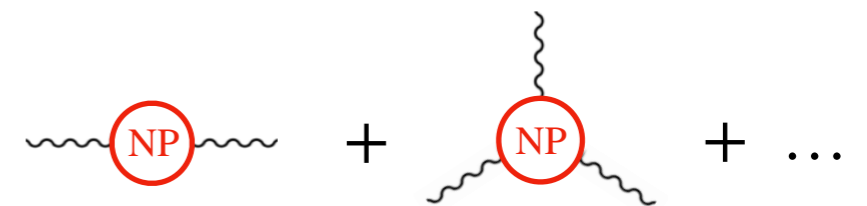
*Otherwise flat direction for off-pole data is broken by $\sigma(ZH)$ sensitivity to $\mathcal{O}_{HW,HB}$.

Gauge sector: W+Y and correlation with aTGC

- The W+Y parameters contribute at LO both on and off the pole, but the off-pole energy enhancement is compensated by Z-pole statistics.



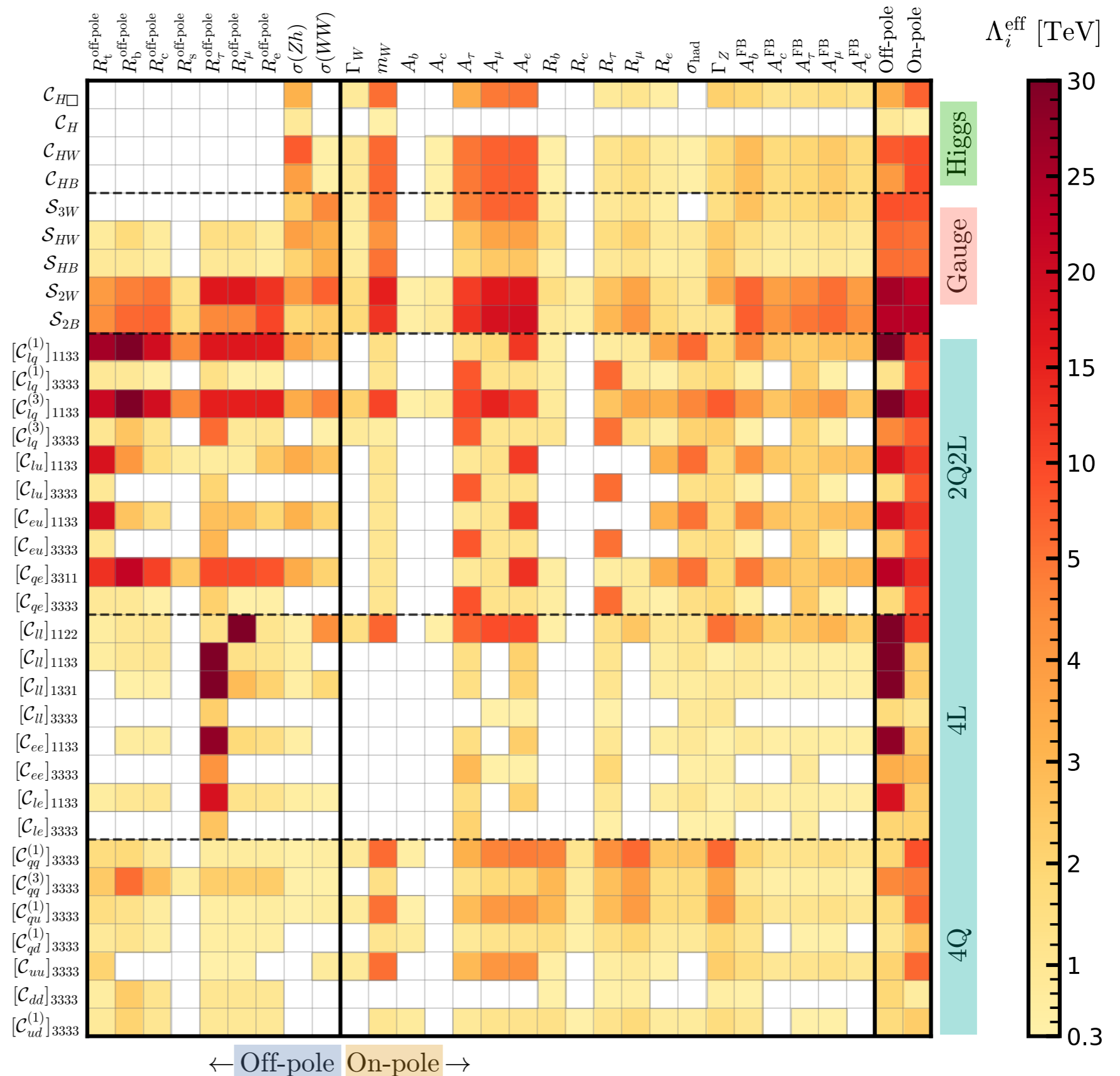
*In agreement with 2411.02485, both W+Y can be constrained at the 10^{-5} level, a factor of 10 better than current leading bounds from LHC.



Accuracy complements energy: EFT summary plot

Some comments

- All Z-pole contributions are NLO except W+Y.
- Still, the typical sensitivity is in the 10 TeV ballpark.
- Most important Z-pole observables: m_W, A_I
- Good complementarity on and off the pole for the Higgs and gauge sectors.
- Z-pole always wins or competes for 4F operators with tops.
- Off pole wins for operators with electrons, otherwise the two are complementary.



[Maura, BAS, You, [2412.14241](#)]

Accuracy complements energy in specific UV models

Real singlet scalar model (with Z_2 symmetry)

- *Why do we care about it?* Simplest extension of the SM that allows for a first order EW phase transition and hardest “loryon” to probe experimentally.

$$\mathcal{L}_\phi = \frac{1}{2}(\partial_\mu\phi)^2 - \frac{1}{2}m_\phi^2\phi^2 - \frac{1}{2}\kappa|H|^2\phi^2 - \frac{1}{4!}\lambda_\phi\phi^4$$

- Integrating out ϕ at 1 loop generates finite contributions to only two operators, namely $Q_{H\Box}$ and Q_H . The matching conditions at the scale m_ϕ are

$$C_{H\Box} = -\frac{1}{16\pi^2} \frac{\kappa^2}{24m_\phi^2}, \quad C_H = -\frac{1}{16\pi^2} \frac{\kappa^3}{12m_\phi^2}.$$

Off-pole:

LO (ZH)

NLO (ZH)

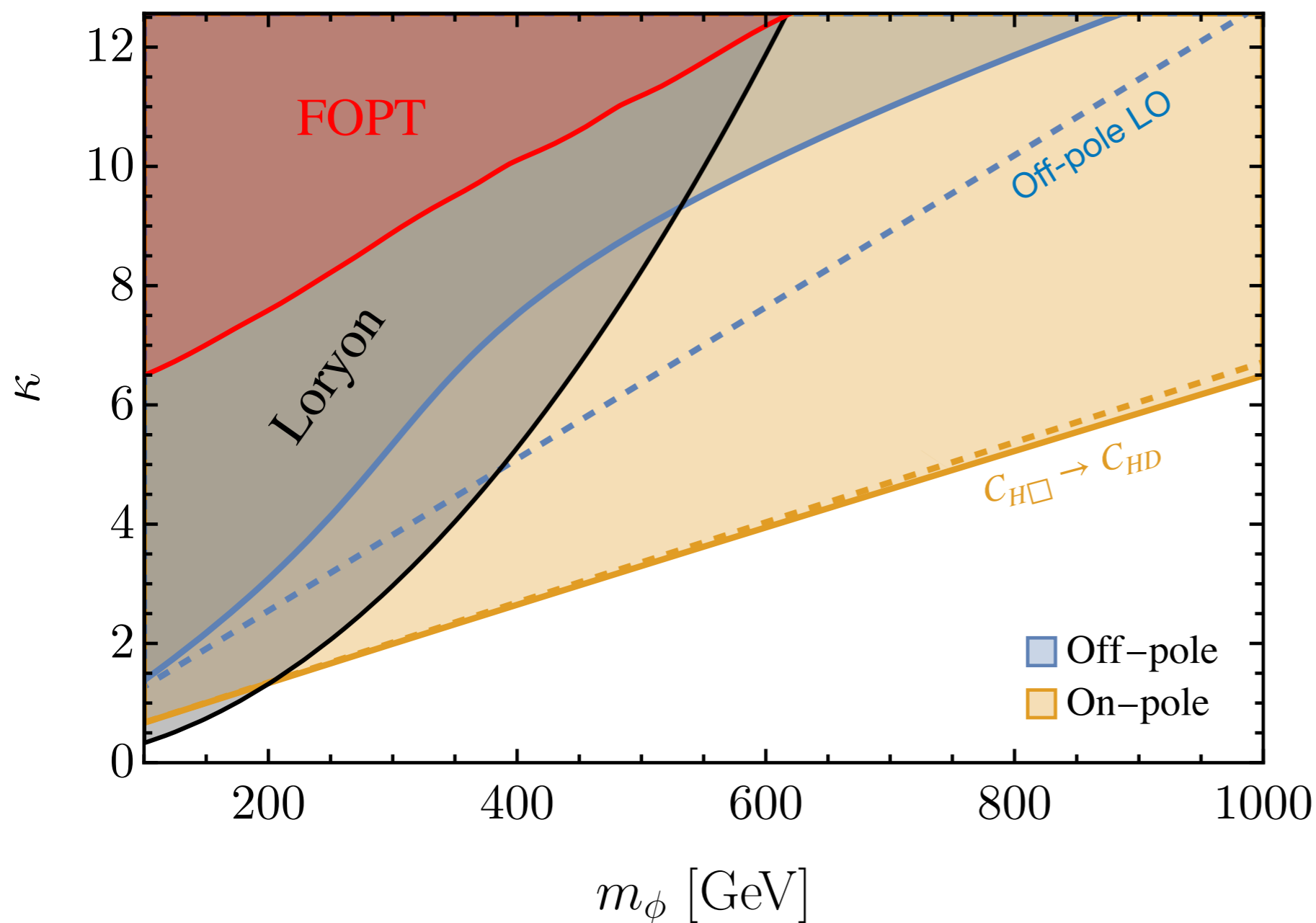
On-pole:

NLO

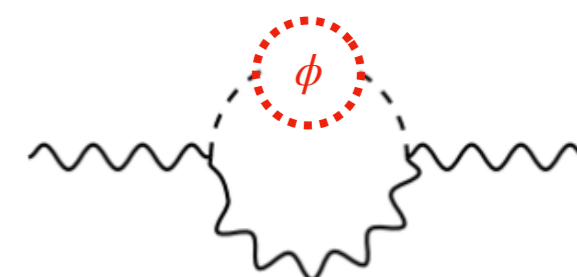
NNLO

Real singlet scalar model (with Z_2 symmetry)

- Full NLO result gives a weaker off-pole constraint due to a partial cancellation between the $Q_{H\Box}$ and Q_H contributions to $\sigma(ZH)$. Better constraint from Z-pole!



- Both Z-pole and ZH can exclude the region where a first order EWPT can occur.
- EFT breaks down for $m_\phi \lesssim v_{EW}$, to know the correct result for low mass, need to compute S+T at 2-loops in the RSS model:



[Maura, BAS, You, WIP]

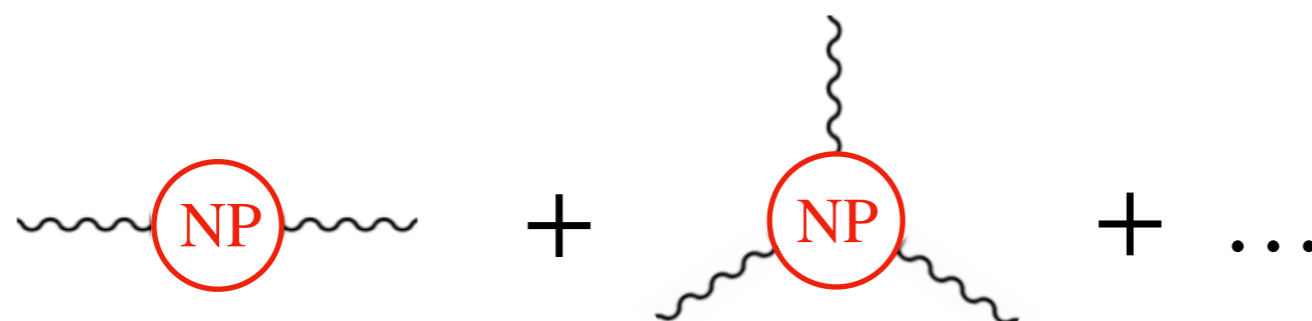
Weakly interacting massive particles

- *Why do we care about them?* Completely generic possibility that BSM states could carry EW charges, one of the simplest models for dark matter.
- Assuming an n -tuple of $SU(2)_L$ with hypercharge Y that interacts with the SM only via EW gauge interactions, the full d6 EFT Lagrangian reads

$$\mathcal{L}_{\text{EFT}}^{d=6} = -\frac{S_{2B}}{2}(\partial^\mu B_{\mu\nu})(\partial_\rho B^{\rho\nu}) - \frac{S_{2W}}{2}(D^\mu W_{\mu\nu}^I)(D_\rho W^{I\rho\nu}) + S_{3W} \epsilon_{IJK} W_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$$

with the following matching conditions at the scale M_{WIMP} :

$$S_{2B} = \frac{g_1^2}{16\pi^2} \frac{nY^2}{30M_{\text{WIMP}}^2} N_2, \quad S_{2W} = \frac{g_2^2}{16\pi^2} \frac{n(n^2-1)}{360M_{\text{WIMP}}^2} N_2, \quad S_{3W} = \frac{g_2^3}{16\pi^2} \frac{n(n^2-1)}{2160M_{\text{WIMP}}^2} N_3,$$



RS, CS, MF, DF

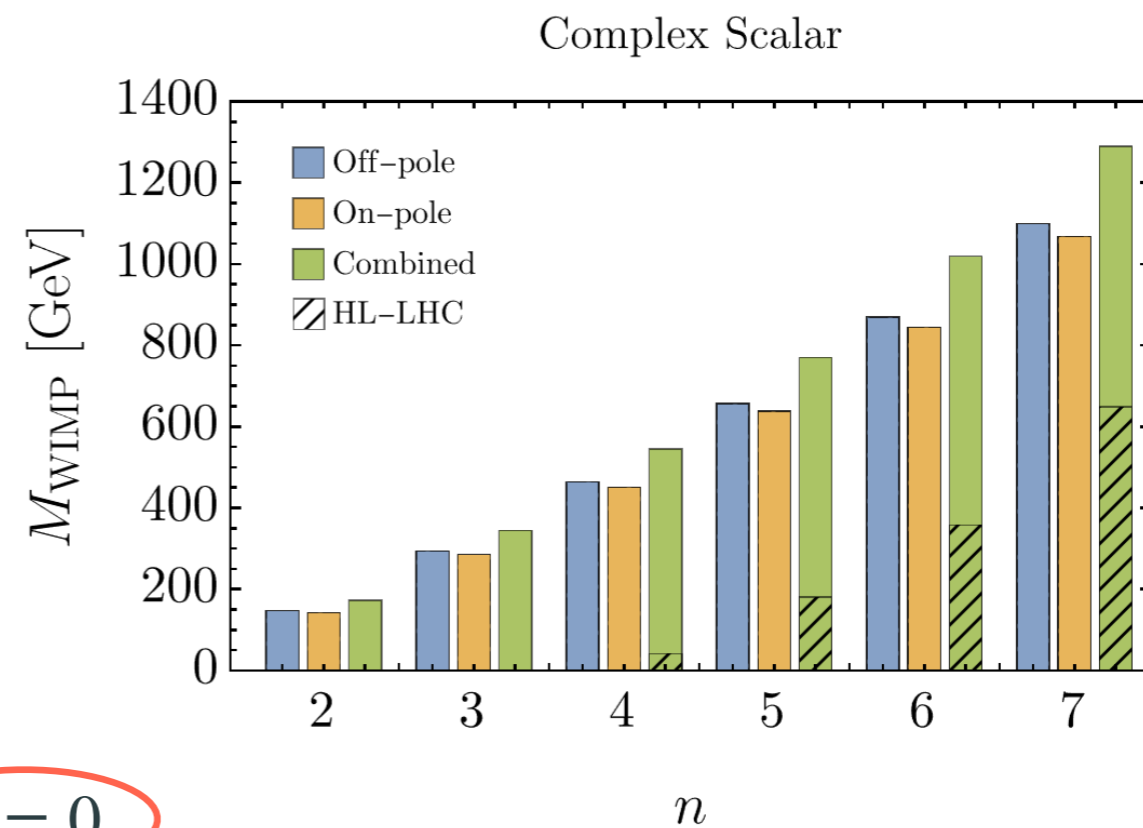
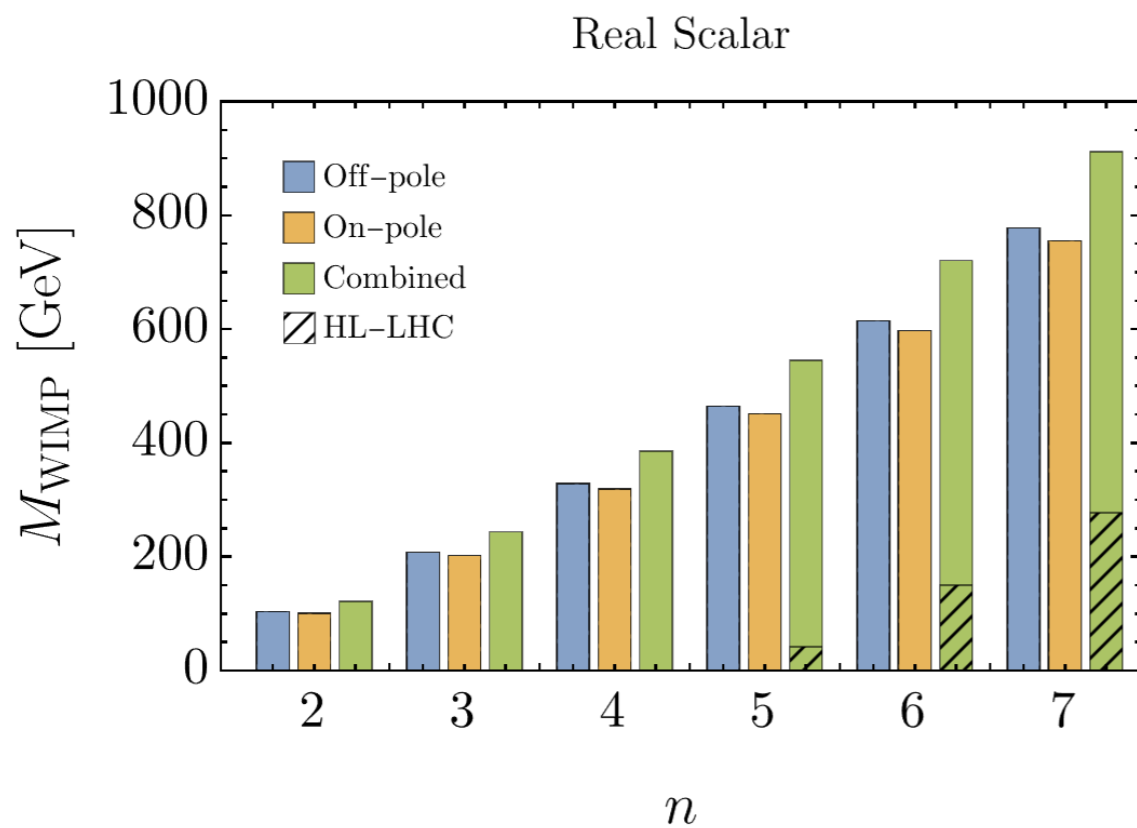
$$N_2 = 1/2, 1, 4, 8$$

$$N_3 = 1/2, 1, -1, -2$$

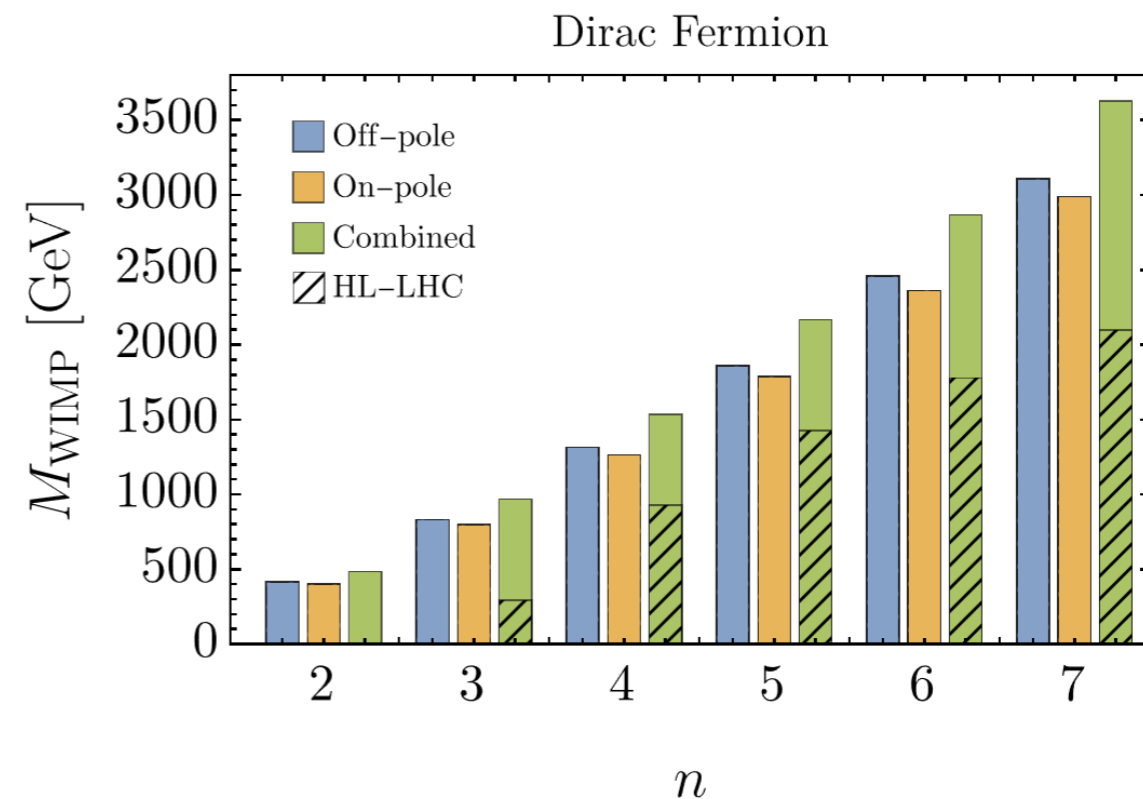
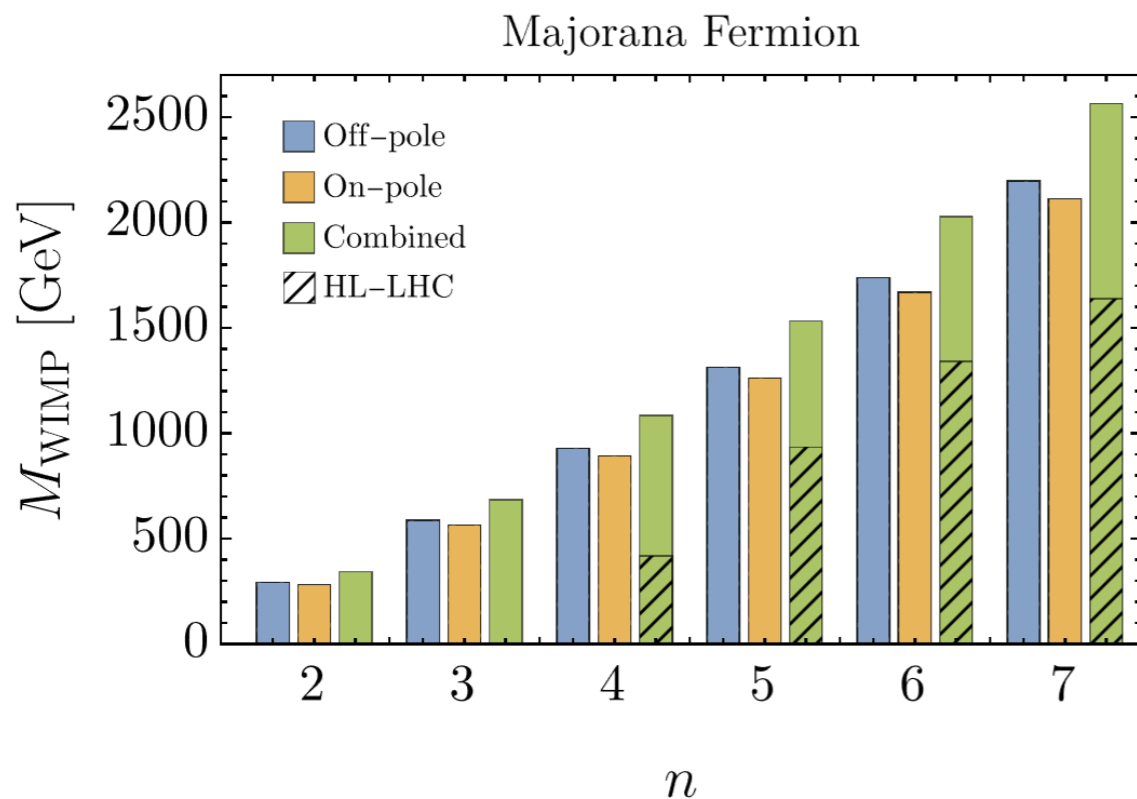
Weakly interacting massive particles

[Maura, BAS, You, [2412.14241](#)]

[Di Luzio, Gröber, Panico, [1810.10993](#)]

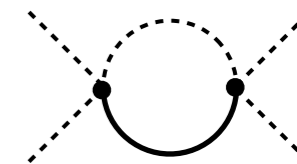


$Y = 0$



Custodial quadruplet model

4 Higgs operators at 1-loop:



- *Why do we care about it?* At tree level, the model generates only the $|H|^6$ operator. Interesting example for Higgs factories as it allows for sizable Higgs self-coupling deviations, with effects in other operators relegated to the 1-loop level.

New states: $\Theta_1 \sim \mathbf{4}_{1/2}, \quad \Theta_3 \sim \mathbf{4}_{3/2} \longrightarrow \Theta \sim (\mathbf{4}, \mathbf{4})$ of $SU(2)_L \times SU(2)_R$

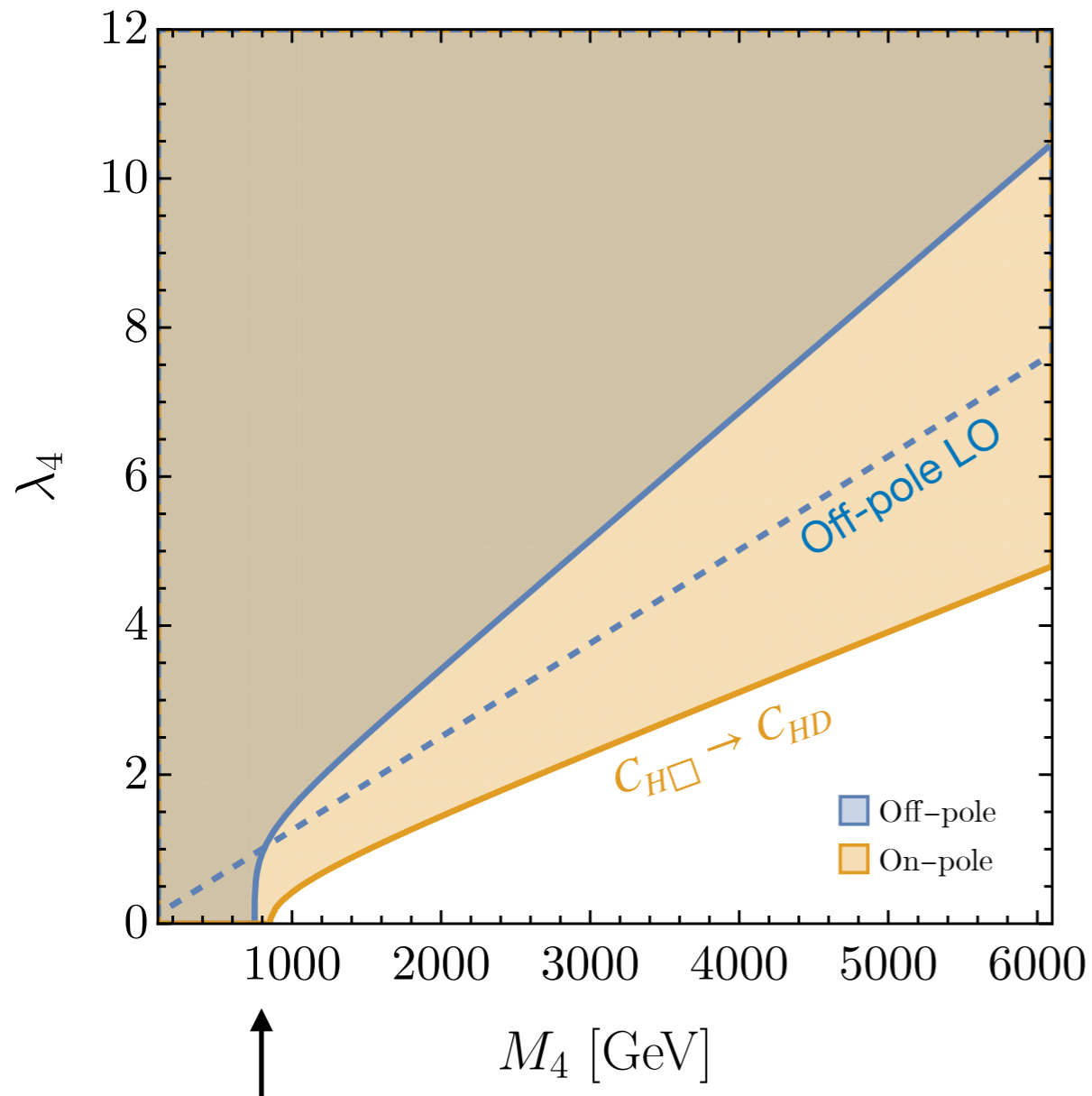
$$\mathcal{L}_{\text{CQ}} \supset -M_4^2 \left(|\Theta_1|^2 + |\Theta_3|^2 \right) - \lambda_4 \left(H^* H^* (\varepsilon H) \Theta_1 + \frac{1}{\sqrt{3}} H^* H^* H^* \Theta_3 \right) + \text{h.c.}$$

-The full 1-loop matching at dimension-6 can be written in two lines:

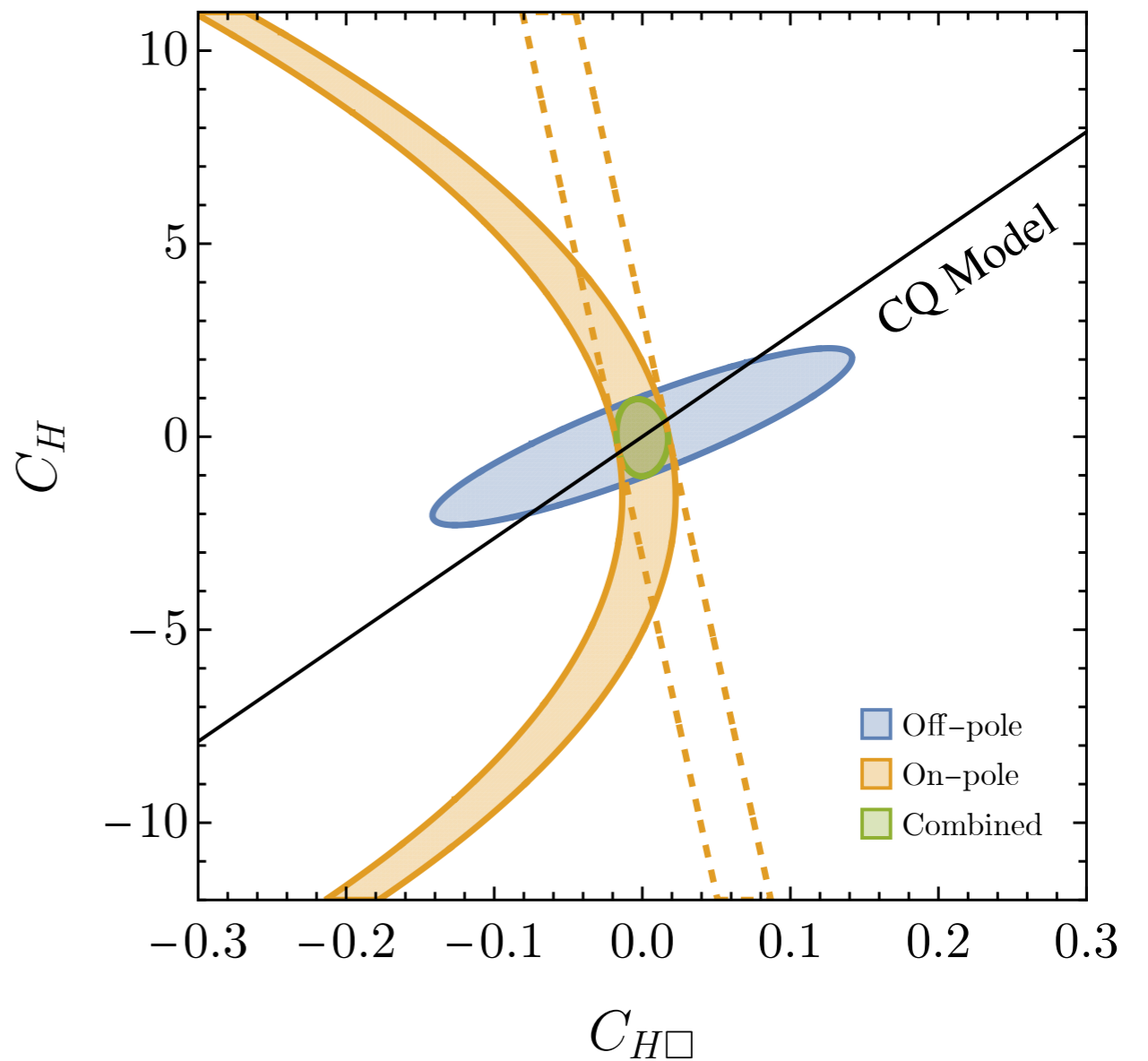
$$\begin{aligned} \mathcal{L}_{\text{CQ}}^{\text{d=6}} = & \frac{2}{3} \frac{\lambda_4^2}{M_4^2} \left(1 + \frac{21\lambda_{\text{SM}}}{16\pi^2} \right) |H|^6 + \frac{\lambda_4^2}{4\pi^2 M_4^2} |H|^2 \square |H|^2 - \frac{\lambda_4^2}{3\pi^2 M_4^2} |H|^2 (H^\dagger D^2 H + \text{h.c.}) \\ & + \frac{1}{48\pi^2 M_4^2} \left[\frac{g_2^3}{3!} \epsilon_{IJK} W_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu} - \frac{g_1^2}{2} (\partial^\mu B_{\mu\nu})(\partial_\rho B^{\rho\nu}) - \frac{g_2^2}{2} (D^\mu W_{\mu\nu}^I)(D_\rho W^{I\rho\nu}) \right] \end{aligned}$$

Custodial quadruplet model

- While $C_{H\Box}$ is 1-loop and C_H is tree, the reverse is true in how they affect $\sigma(ZH)$, so they contribute similarly, but with the opposite sign. Again, partial cancellation!



*Lower λ_4 -independent bound on the mass comes from W+Y parameters.



*Off-pole direction not fully flat because we include $\sigma(ZH)$ at 240+365 GeV.

Conclusions

- The Tera-Z run has access at NLO (or even NNLO) to many Wilson coefficients that are typically thought to be better constrained at LO off the pole. It is a simple counting argument to see that, in general, a similar sensitivity to these Wilson coefficients is expected at Tera-Z.
- The same is true for operators that enter both on and off pole at LO, but are energy enhanced off the pole. The prototypical example here is the electroweak $W+Y$ parameters, which can be constrained at the 10^{-5} level in both cases.
- A Tera-Z program will thus anticipate much of the BSM physics at higher energy runs. Accuracy will complement energy since on- and off-pole data can be combined to break flat directions and increase the overall FCC-ee sensitivity to new physics.
- The dominant Tera-Z probes are higher loop contributions to the oblique $S+T$ parameters (seen as shifts in m_W and A_I). Because many operators contribute to these beyond LO, making our analysis fully rigorous via a global SMEFT fit seems very difficult (especially at 2 loops).
- However, one can disentangle the various contributions and make concrete statements in the context of specific UV models. To illustrate this point, we gave several well-motivated examples where the model sensitivity does indeed benefit from combining on- and off-pole data.

Backup

Non-universality of composite Higgs models

- The composite sector will unavoidably generate other large top+H operators at the high scale m_*

These operators are usually ignored via the following arguments:

- Some operators are **phenomenologically irrelevant** at LO.
- Model building tricks exist to kill the LO contribution of **the most dangerous operators**, e.g. $Zbb \propto C_{Hq}^{(1)} + C_{Hq}^{(3)}$.
- The rest are subdominant to **universal constraints**.

Flavor non-universal operators	
EW vertex corrections	
$\mathcal{O}_{Hq}^{(1)} = (H^\dagger i \overleftrightarrow{D}_\mu H)(\bar{q}_L^3 \gamma^\mu q_L^3)$	$\mathcal{O}_{Hq}^{(3)} = (H^\dagger i \overleftrightarrow{D}_\mu^I H)(\bar{q}_L^3 \gamma^\mu \tau^I q_L^3)$
$\mathcal{O}_{Ht} = (H^\dagger i \overleftrightarrow{D}_\mu H)(\bar{t}_R \gamma^\mu t_R)$	$\mathcal{O}_{tD} = g_1(\bar{t}_R \gamma^\mu t_R) \partial^\nu B_{\mu\nu}$
$\mathcal{O}_{qD}^{(1)} = g_1(\bar{q}_L^3 \gamma^\mu q_L^3) \partial^\nu B_{\mu\nu}$	$\mathcal{O}_{qD}^{(3)} = g_2(\bar{q}_L^3 \gamma^\mu \tau^I q_L^3) D^\nu W_{\mu\nu}^I$
4-fermion operators	
$\mathcal{O}_{qq}^{(1)} = (\bar{q}_L^3 \gamma^\mu q_L^3)(\bar{q}_L^3 \gamma_\mu q_L^3)$	$\mathcal{O}_{qq}^{(3)} = (\bar{q}_L^3 \gamma^\mu \tau^I q_L^3)(\bar{q}_L^3 \gamma_\mu \tau^I q_L^3)$
$\mathcal{O}_{qt}^{(1)} = (\bar{q}_L^3 \gamma^\mu q_L^3)(\bar{t}_R \gamma_\mu t_R)$	$\mathcal{O}_{qt}^{(8)} = (\bar{q}_L^3 \gamma^\mu T^A q_L^3)(\bar{t}_R \gamma_\mu T^A t_R)$
$\mathcal{O}_{tt} = (\bar{t}_R \gamma^\mu t_R)(\bar{t}_R \gamma_\mu t_R)$	
Dipoles and Yukawas	
$\mathcal{O}_{tB} = g_1(\bar{q}_L^3 \sigma^{\mu\nu} t_R) \tilde{H} B_{\mu\nu}$	$\mathcal{O}_{tW} = g_2(\bar{q}_L^3 \sigma^{\mu\nu} \tau^I t_R) \tilde{H} W_{\mu\nu}^I$
$\mathcal{O}_{tG} = g_3(\bar{q}_L^3 \sigma^{\mu\nu} T^A t_R) \tilde{H} G_{\mu\nu}^A$	$\mathcal{O}_{tH} = (H^\dagger H)(\bar{q}_L^3 \tilde{H} t_R)$

Universal operators in composite Higgs models

- Now let's have a look at the operators we can write only involving the Higgs (and gauge fields of course). We work here in the SILH basis:

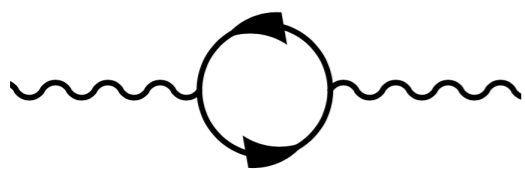
Flavor universal bosonic operators	
$\mathcal{O}_H = \frac{1}{2} \partial_\mu (H^\dagger H) \partial^\mu (H^\dagger H)$	$\mathcal{O}_T = \frac{1}{2} (H^\dagger \overleftrightarrow{D}_\mu H) (H^\dagger \overleftrightarrow{D}^\mu H)$
$\mathcal{O}_W = i \frac{g_2^2}{2} (H^\dagger \overleftrightarrow{D}_\mu^I H) D_\nu W^{I \mu \nu}$	$\mathcal{O}_B = i \frac{g_1^2}{2} (H^\dagger \overleftrightarrow{D}_\mu H) \partial_\nu B^{\mu \nu}$
$\mathcal{O}_{2W} = -\frac{g_2^2}{2} (D^\mu W_{\mu\nu}^I) (D_\rho W^{I \rho \nu})$	$\mathcal{O}_{2B} = -\frac{g_1^2}{2} (\partial^\mu B_{\mu\nu}) (\partial_\rho B^{\rho\nu})$

\mathcal{O}_H : Higgs coupling modifications

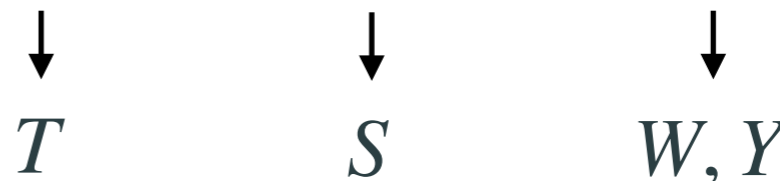
\mathcal{O}_T : Peskin-Takeuchi T parameter

\mathcal{O}_{W+B} : Peskin-Takeuchi S parameter

$\mathcal{O}_{2W,2B}$: $W + Y$ parameters



Recall: $\Pi_{VV}(p^2) = \Pi_{VV}(0) + p^2 \Pi'_{VV}(0) + p^4 \Pi''_{VV}(0) + \dots$

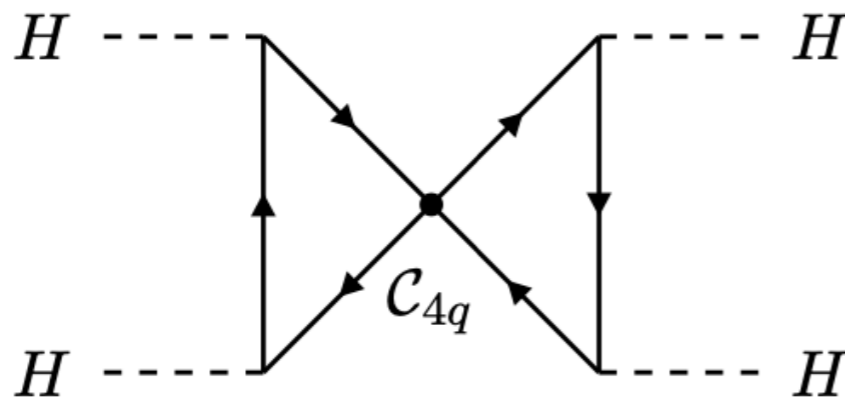


The full 2-loop contribution to the T parameter

- While the double-log contribution is expected to dominate, in general the full 2-loop contribution of 4-top operators to the T parameter takes the form of a second-order logarithmic polynomial. E.g. for C_{tt} , we have:

$$[\mathcal{C}_{HD}]_{2\text{-loop}} = \frac{N_c(N_c + 1)}{4\pi^2} \alpha_t^2 \left[\underbrace{\log^2(\mu^2/m_*^2)}_{1\text{-loop RGE}} + \underbrace{c_1 \log(\mu^2/m_*^2)}_{2\text{-loop RGE}} + \underbrace{c_2}_{\text{finite}} \right] C_{tt}.$$

- The $O(1)$ constants c_1+c_2 cannot be obtained from the 1-loop RG equations. In particular, c_1 corresponds to the 2-loop anomalous dimension. To get all contributions, we need to do a 2-loop computation:



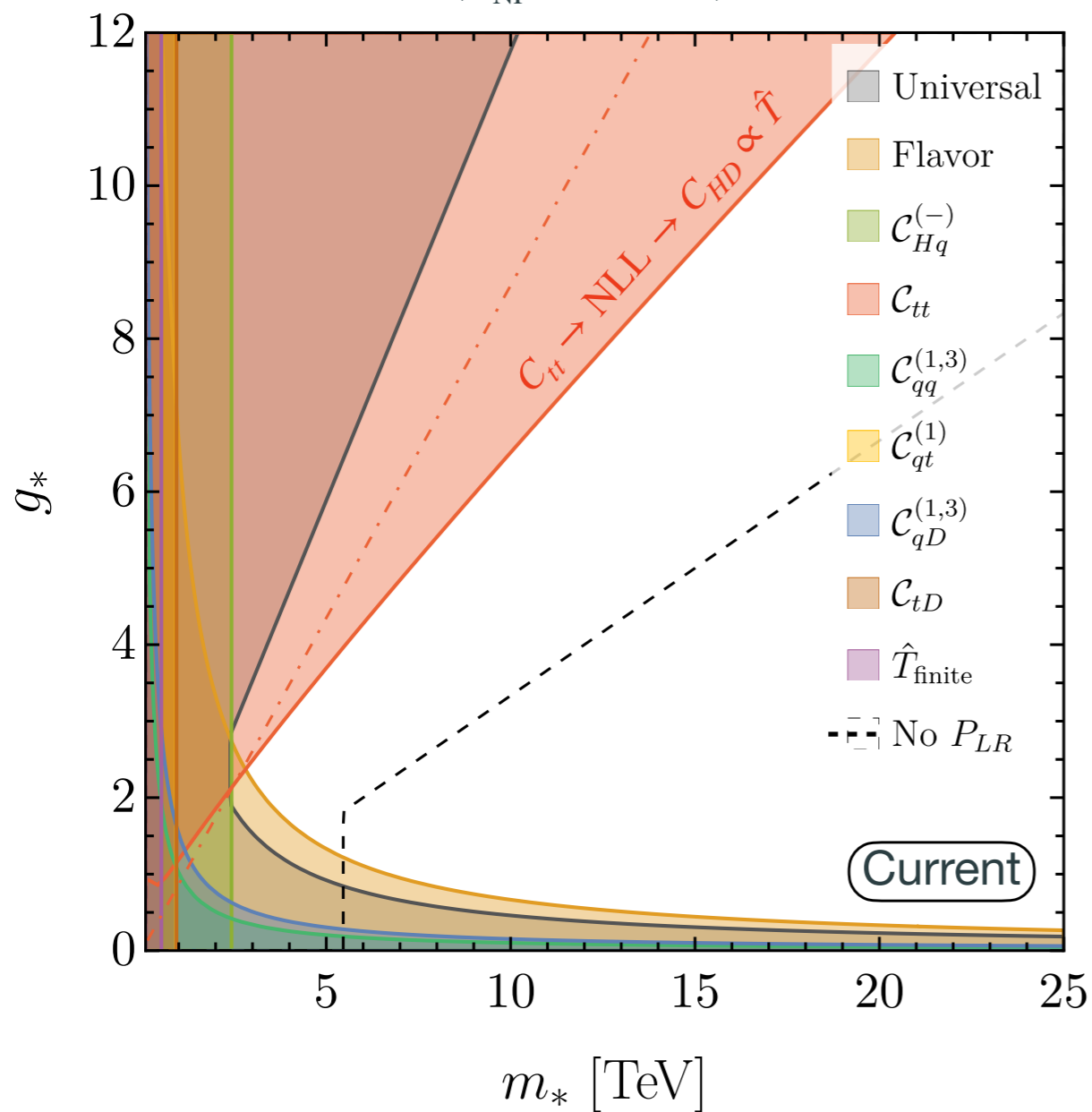
U. Haisch and L. Schnell, Precision tests of third-generation four-quark operators: matching SMEFT to LEFT, to appear soon

$$c_1 = -1/2 \text{ and } c_2 = 0^*$$

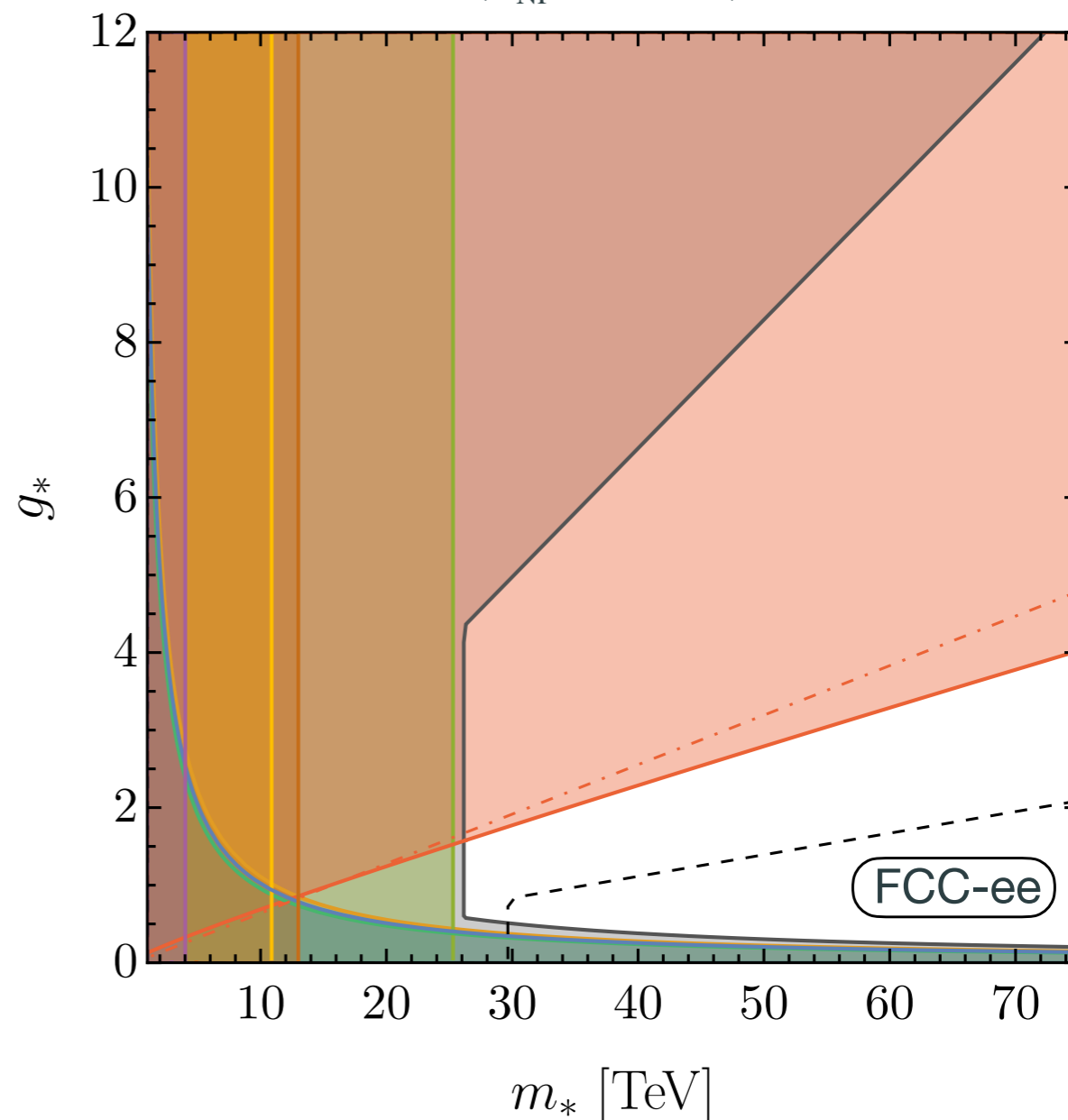
Results: Right compositeness

- Right compositeness has $\epsilon_L = y_t/g_*$, $\epsilon_R = 1$. Flavor constraints: $C_{B_s} \propto \frac{g_*^2}{m_*^2} \epsilon_L^4$

($\Lambda_{\text{NP}} = 2.5 \text{ TeV}$)

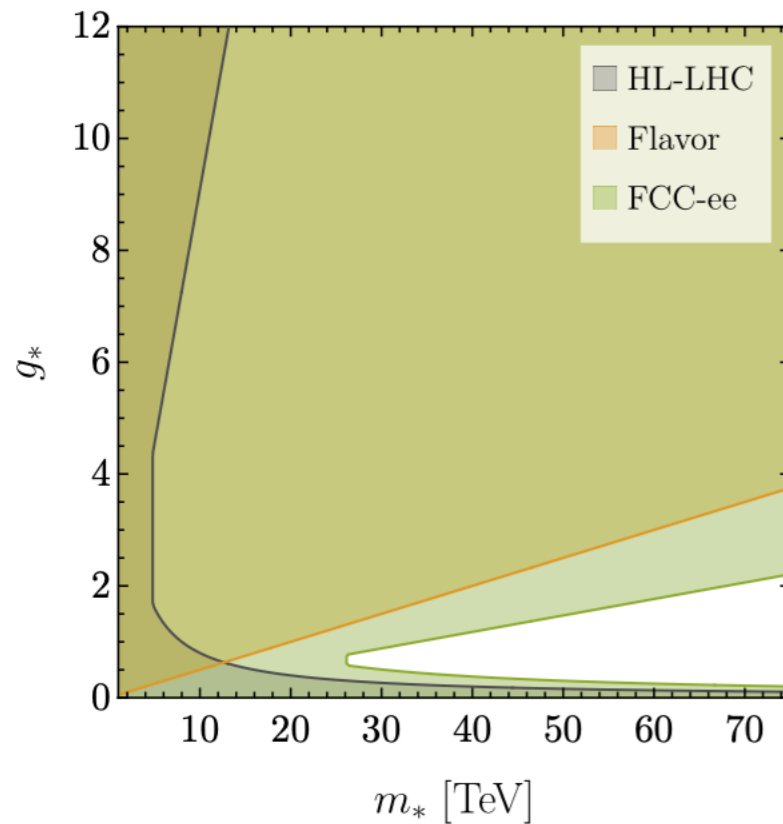


($\Lambda_{\text{NP}} = 25 \text{ TeV}$)

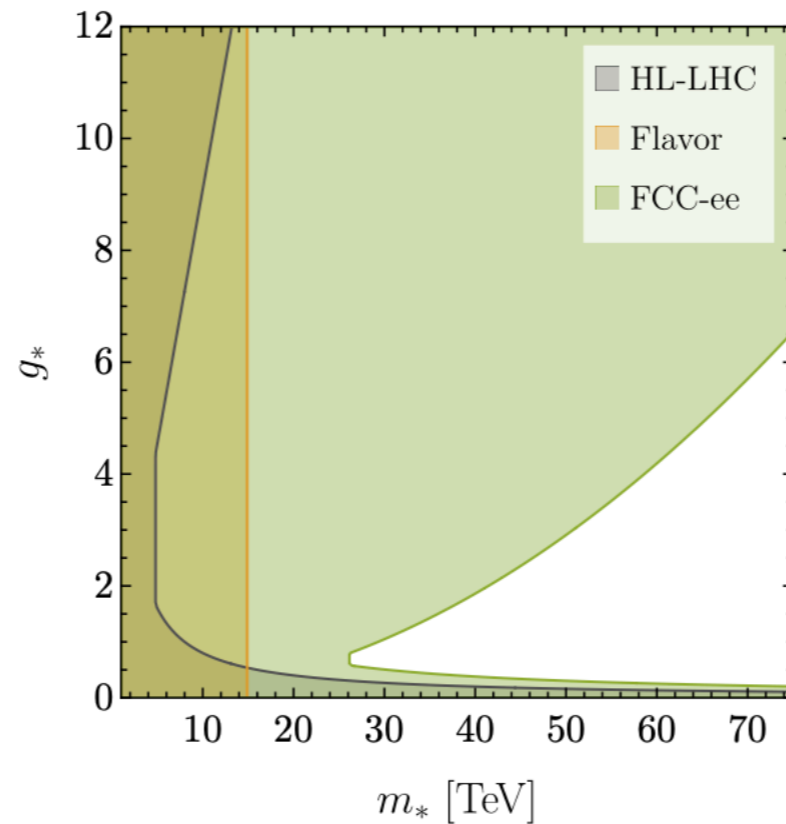


Future summary plots

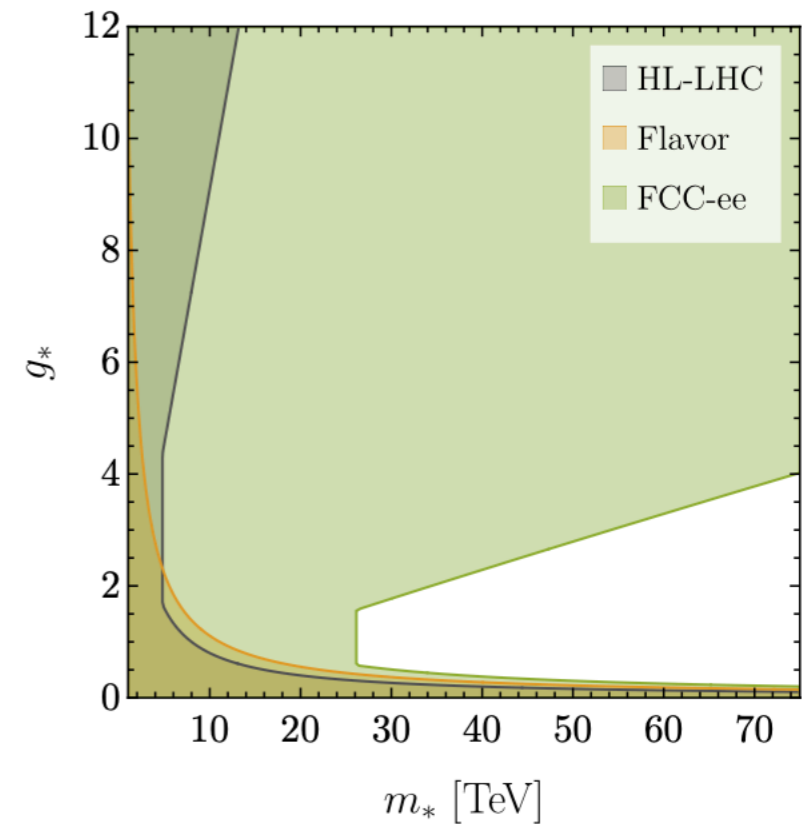
- Flavor non-universal RG effects give the best bound for $g_* \gtrsim 1$, while universal effects are only better for $g_* < 1$. Interestingly: $\langle H \rangle \sim f = m_*/g_*$



(a) Left compositeness



(b) Mixed compositeness

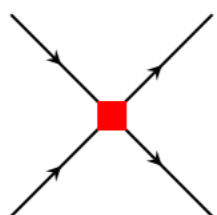


(c) Right compositeness

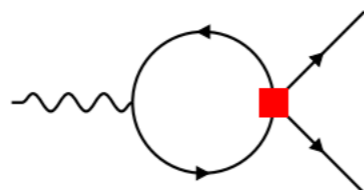
- In all cases, FCC-ee dominates over other sectors, setting a mixing-independent bound of $m_* \gtrsim 25$ TeV. Adds the most new info in the mixed + right comp. cases.

Suspects for complementary probes via AcE

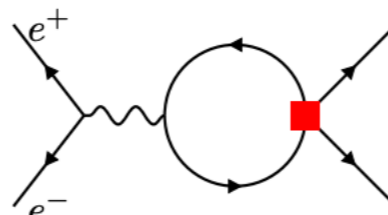
1. *Four fermion operators* (receive energy enhancement off-pole)



(a) Four-fermion operator



(b) Z-vertex correction

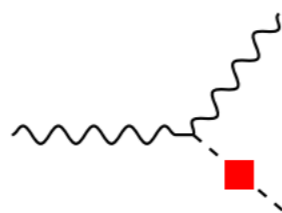


(c) $e^+e^- \rightarrow \bar{f}f$

2. *Higgs physics* (enter Z-pole obs. at one higher loop order)



(a) Higgs self-energy



(b) $e^+e^- \rightarrow ZH$

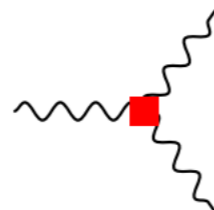


(c) Z-pole oblique params.

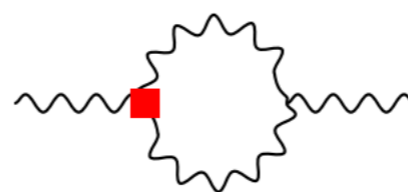
3. *Gauge two- and three-point functions* (both effects at play)



(a) LO Z-pole oblique params.



(b) aTGC



(c) NLO Z-pole oblique params.

