

Quantum tomography with τ leptons

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Based on:

- Quantum entanglement and Bell inequality violation at colliders, A. Barr, M. Fabbrichesi, R. Floreanini, E. Gabrielli, LM. — Prog.Part.Nucl.Phys. 139 (2024)
- Quantum tomography with τ leptons at the FCC-ee, M. Fabbrichesi, LM. Phys.Rev.D 110 (2024)
- The trace distance between density matrices, a nifty tool in new-physics searches, M. Fabbrichesi, M. Low, LM. — arXiv 2501.03311

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The τ lepton is a good candidate for these studies at collider experiments because the orientation of its spin vector in space can be reconstructed from the angular distributions of the τ decay products.

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International to the last three bins to the last three bins to the last three bins three bins to the last thre of Fig. 2.4 which comprise 99% of the events.

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- use particle physics to explore quantum information theory. performed with a series to overload

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 $\frac{1}{2}$

Focusing on $e^+e^-\to Z, \gamma\to \tau^+\tau^-$, FCC-ee would then allow us to:

- use quantum information observables and methods to test possible anomalous couplings of the τ lepton to gauge bosons.
- · study entanglement and the violation of Bell inequalities by analyzing the spin correlations of the tau lepton pairs. Let us first the analytic part of the part of the parton parto.

Theoretical Quantum Tomography 1 IUVIVIIVAI QUAI

An ensemble of bipartite systems, each formed by two qubits, is described by a 4x4 density matrix

$$
\rho = \frac{1}{4} \left[\mathbb{1} \otimes \mathbb{1} + \sum_{i} \mathbf{B}_{i}^{+} \left(\sigma_{i} \otimes \mathbb{1} \right) + \sum_{j} \mathbf{B}_{j}^{-} \left(\mathbb{1} \otimes \sigma_{j} \right) + \sum_{i,j} \mathbf{C}_{ij} \left(\sigma_{i} \otimes \sigma_{j} \right) \right]
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$$
 Pauli matrices
polarization of the τ^{\pm} lepton
spin correlations

where \overline{i} i refer to the directions used to define the orientation of the spin **Exercise is the fearm of the fearm of the fermion of the fermion of the fearm of the fearm of the mass frame we**
The fearm of the fearm of the fearm of the second we have the fearm of the second we have the second to the where i, j , refer to the directions used to define the orientation of the spin vectors in space: the {n, r, k} triad defined, in the CoM frame, by

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$$
e^{+}\underbrace{\qquad \qquad }_{\blacksquare \qquad \blacksquare \qquad \blacks
$$

with p being the direction of the incoming *e*+. The Fano coefficients B[±] and C can be computed from the amplitudes of the individual fermions of the industry of underlying production process as functions of the kinematic variables The Fano coefficients B^{\pm} and C can be computed from the amplitudes of the with p being the direction of the incoming *e*+. and C can be computed from the amplitudes of the with p built p being a file of the income *z* as functions by the Nilenial variables Id C can be computed from the amplitudes of the wing production process as functions of the kinematic variables with p being the direction of the incoming *e*+.

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\rho = \rho(\Theta, s, ...)
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B_i^+ = \text{Tr}\left[\rho(\sigma_i \otimes 1)\right]
$$
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$$
B_i^- = \text{Tr}\left[\rho(1 \otimes \sigma_i)\right]
$$
\nA. Barr. M. Fabbrichesi. R. Floreanini. F. Gabrielli. LM. Proa. Part. Nucl. Phys. 139 (2024)

A. Barr, M. Fabbrichesi, R. Floreanini, E. Gabrielli, LM, Prog.Part.Nucl.Phys. 139 (2024) $\{v_j - 11 \}$ $p(v_i \otimes v_j)$ A. Barr, M. Fabbrichesi, R. Floreanini, E. Gabrielli, LM, Prog.Part.Nucl.Phys. 139 (2024) $\qquad \qquad \qquad \qquad C_{ij} = {\rm Tr}\left[\rho (\sigma_i \otimes \sigma_j)\right]$ $\mathcal{C}_{ij} = \mathbf{1} \left[\rho(\sigma_i \otimes \sigma_j) \right]$
i. 139 (2024) The quantum tomography of the polarization density matrix is completed once the coe"cients

This aiyas us the prespects for the detection of from the scattering and the scattering the underlying process. This gives us the prospects for the detection of B*ⁱ* and C*ij* have been found. Given a Lagrangian for a theory, these quantities can be computed T the quantum tomography of the polarization density matrix is completed once the coefficients of the coefficients \mathcal{L} This gives us the prospects for the detection of... $\frac{1}{3}$ from the scattering amplitudes describing the underlying process. Experimentally, or in β This sives us the prespecte for the detection of This gives us the prospects for the detection on... $\frac{3}{100}$ This gives us the prospects for the detection of…

 is the information about the actual physical physic Entanglement ($\mathscr{C} > 0$) Equation products. In particular, $\ell_{\text{e}} > 0$

The concurrence $0 < \mathcal{C} < 1$ quantifies the amount of process *examples of entanglement* in the system.

Separated by *C interesting and the contractive contractive* by *C* and the *C interesting and the system.*

m¹² *>* 1. It is computed through the auxiliary matrix **and in the explicit**ly constructed by the explicitly constructed by $\frac{1}{2}$ using the auxiliary matrix

 $R = \rho \left(\sigma_y \otimes \sigma_y \right) \rho^* \left(\sigma_y \otimes \sigma_y \right)$ \mathbf{D} denotes the matrix with complex conjugated entries. Although non-Hermitian, the matrix \mathbf{D} $\mathbf{R} \mathbf{r} = \mu \left(\nu y \otimes \nu y \right) \mu \left(\nu y \otimes \nu y \right)$

 $\frac{1}{2}$ and the conception $\frac{2}{2}$ and $\frac{2}{3}$ for $\frac{2}{3}$ with non-negative eigenvalues $r_1 \ge r_2 \ge r_3 \le r_4$
as: with non-negative eigenvalues as: with non-negative eigenvalues $r_1^2 \ge r_2^2 \ge r_3^2 \ge r_4^2$ *.* (2.7)

$$
\mathscr{C} = \max (0, r_1 - r_2 - r_3 - r_4).
$$

Entanglement ($\mathscr{C} > 0$) $S_n = \frac{1}{2}$ \sum_{i} products. In particular, for each $\binom{w}{k}$ is the point independent

from the scattering amplitudes describing the underlying process. Experimentally, or in Monte Carlo

Let us first look at the analytic results for the parton level process *^e*+*e*[→] [↑] ^ω ⁺^ω [→]. This study already

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energies close to the *Z* boson resonance the process is instead dominated by this particle. In Fig. 3.1

yielding the maximal value of the concurrence and of the Bell inequality violation for # = ϑ*/*2. At

this region corresponds to the thin slice centered around [→]*^s* [↓] 90 GeV. At higher energies both the

The concurrence 0< \mathscr{C} <1 quantifies the amount of $\quad \cdot \quad$ We use the Horodechki condition $m_{12} > 1$, wh The concurrence 0< \mathscr{C} <1 quantifies the amount of $\begin{array}{c} 1 \\ 1 \end{array}$ We use the Horodechki condition $m_{12} > 1$, where entanglement in the system.

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It is computed through the auxiliary matrix $\begin{array}{ccc}\n\downarrow \\
\downarrow \\
\downarrow\n\end{array}$ using the auxiliary matrix

 $R = \rho \left(\sigma_y \otimes \sigma_y \right) \rho^* \left(\sigma_y \otimes \sigma_y \right) \qquad \qquad \mathbf{P} \qquad \qquad \mathbf{P} \qquad \qquad \mathbf{P} \qquad \qquad \mathbf{P} \qquad \mathbf{P$ \mathbf{E} denotes the matrix with complex conjugated entries. Although non-Hermitian, the matrix \mathbf{E} $R = \rho \left(\frac{\partial y}{\partial y} \otimes \frac{\partial y}{\partial y} \right)$ in terms of the eigenvalue r

 T with non-negative eigenvalues $r_1 \geq r_2 \geq r_3 \leq r_4$ if $m_1 \geq m_2 \geq m_3$ non-negative eigenvalues $r_1^2 > r_2^2 > r_3^2 > r_4^2$ for the Horodeckies that range from the r_1 **produce the set of the shown in Fig. 3.1. The plot shown in Fig. 3.1. The plot shown in Fig. 3.1.** The presence of the plot shown in Fig. 3.1. The plot shown in Fig. 3.1. The plot show $\frac{1}{2}$ and $\frac{1}{2}$ and $\frac{1}{2$ with non-negative eigenvalues $r_1^2 \geq r_2^2 \geq r_3^2 \geq r_4^2$ i $m_1 \geq m_2 \geq m_3$ as: $\mathcal{Y} = \mathcal{Y}$

$$
\mathscr{C} = \max (0, r_1 - r_2 - r_3 - r_4)
$$
 if of the matrix $M = C^T C$.

The concurrence is a quantitative estimate of the amount of the amount of entanglement in the two-qubit system.

Γ provides most $\{(\emptyset, \emptyset)$ provides most of the information about the actual physical process, that is, the one inclusive of the Bell inequality violation $(m_{12} > 1)$

 $\frac{1}{1}$ we ascure the normalism conducting $\frac{1}{1}$ and $\frac{1}{1}$

 $m_{12} \equiv m_1 + m_2$

in terms of the eigenvalues

 $m_1 > m_2 > m_3$

yielding the maximal value of the concurrence and of the Bell inequality violation for # = ϑ*/*2. At of the matrix $\ M={\bf C}^T\,{\bf C}$.

Focusing on FCC-ee working at the Z boson resonance: forming at the \angle boson resonance. FCC-ee working at the Z boson resonang

Averaging the analytical result over the \leftarrow $\boxed{m_{12}}$ angular distribution of events yields characteristic pattern in the CM energy dependence is due to the interference between the photon and **Z** Averaging the analytical result over the angular distribution of events yields the system is a system is entitled angular that the system is a weight and the concurrence is a concurrence is *C* = 0*.*4878 *.* (3.3)

$$
C = \begin{pmatrix} 0.4878 & 0 & 0 \\ 0 & -0.4878 & 0.0011 \\ 0 & 0.0011 & 1 \end{pmatrix} \qquad B^+ = B^- = \begin{pmatrix} 0 \\ 0.0001 \\ 0.2194 \end{pmatrix}
$$

 $\begin{array}{ll}\text{C} & \text{C} & \text{C} \\ \text{C} & \text{C} & \text{C} & \text{C} \end{array}$ and $\begin{array}{ll}\text{C} & \text{C} & \text{C} & \text{C} & \text{C} & \text{C} & \text{C} \end{array}$ and $\begin{array}{ll}\text{C} & \text{C} \end{array}$

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Figure 3.2: The concurrence is the Lorence of the Lorence and the Scattering and the Scattering and Scattering a might increase the signal. violation of Bell inequalities by m¹² *>* 1. As shown in Fig 3.2 the entanglement depends on the kinematic region and is maximal at " = ε*/*2. a_1 , the violation of the b_2 inequality is given by b_2 • Remark: the results hold prior to possible cuts on the scattering angle that Focusing on FCC-ee working at the Z boson resonance: forming at the \angle boson resonance. FCC-ee working at the Z boson resonang These values can be directly compared with the results of the Monte Carlo simulations. We find that

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- Remark III: I am well aware that all of this means nothing as long as I do not chow the corresponding uncertainties To gauge As for the polarization of the ω pair, we find B⁺ *ⁱ* = B[→] *ⁱ* = B*ⁱ* for *i* = *r, n, k*. Contrary to most of the dedicated Monte Carlo analysis. show the corresponding uncertainties. To gauge these we resort to a

The strategy:

• Focus on the decay mode $\tau \rightarrow \pi \nu$ (BR≃11%) for both the taus because it is clean and neutrinos are easily reconstructed.

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- Statistical errors are estimated from the variance over the 50 pseudo experiments. Systematic errors are computed from the shifts of central values due to different detector settings.

Accessing the density matrix from "data" reduction by the didney measuring measure particle angular distributions in the spin and the spin and spin and spin and powers in the spin and powers in *i*. The and the positive method is a selected the positive set of the positive set of the positive set of the s

The Fano coefficients can be experimentally reconstructed in several ways, for instance by accessing the distributions $\overline{ }$ **1** ed III severd
" *,* (3.34)

$$
\frac{1}{\sigma} \frac{d\sigma}{d\cos\theta_i^{\pm}} = \frac{1}{2} \left(1 \mp B_i^{\pm} \cos\theta_i^{\pm} \right) \qquad \frac{1}{\sigma} \frac{d\sigma}{d\cos\theta_i^{\pm} d\cos\theta_j^-} = \frac{1}{4} \left(1 + C_{ij} \cos\theta_i^{\pm} \cos\theta_j^- \right)
$$

Fano coefficients

Alternatively, quantum to model if the following distribution distributions can be reconstructed if the following distributions can be reconstructed in \mathcal{A}

ϑ here we defir τ illied $\cos v_i - u_1 v_i$, with $c_i - u$, when τ and **N** and with u being the \mathbf{r} 4 $\cos\theta^{\pm}_{i}=\vec{n}^{\pm}\cdot\hat{e}_{i}$. wit where we defined $\cos\theta_i^\pm = \vec{n}^\pm\cdot\hat{e}_i$, with $\hat{e}_i =$ **n**, **r** or **k** and with \vec{n}^\pm being the *i* are the projections of the substandary (or, et al. projection) on the rest frame of the decaying tau lepton). as computed in the rest frame of the distribution of the distributions of the *B±±* $\frac{1}{2}$ polarimetric vector for the chosen decay mode (*i.e.* the pion direction as seen in the rest frame of the decaying tau lepton).

Accessing the density matrix from "data" weighted again by the di!erential cross section. For the case of measuring the spin of *tt* $\frac{1}{2}$ particle angular distributions in the spin and the spin and spin and spin and powers in the spin and powers in *i*. The and the positive method is a selected the positive set of the positive set of the positive set of the s frequencies. The average (see Eqs. (3.34)–(3.35)–(3.34)–(3.34)–(3.34)–(3.34)–(3.35)–(3.34)–(3.35)–(3.34)–(3.34)–(3.34)–(3.34)–(3.34)–(3.34)–(3.34)–(3.34)–(3.34)–(3.34)–(3.34)–(3.34)–(3.34)–(3.34)–(3.34)–(3.34)–(3.34)–(3.3 **BING THE COENSITY MATHX TROM-CORTA** values would be perfectly symmetric. The plots are from a simulation of the process *^e*⁺*e*[→] [→] ^ω ⁺^ω [→] at [↑]*^s* = 91*.*¹⁹ GeV, by two of

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" *,* (3.34) The Fano coefficients can be experimentally reconstructed frequencies. The average (see Eqs. (3.34)–(3.35)) and standard deviation of these histograms give the mean value and uncertainty of ways, for instance by accessing the distributions

$$
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Fano coefficients

Alternatively, quantum to model if the following distribution distributions can be reconstructed if the following distributions can be reconstructed in \mathcal{A}

ϑ here we defir τ illied $\cos v_i - u_1 v_i$, with $c_i - u$, when τ and **N** and with u being the \mathbf{r} 4 $\cos\theta^{\pm}_{i}=\vec{n}^{\pm}\cdot\hat{e}_{i}$. wit $\mathsf{a}^\mathtt{final}\ \cos\theta^\pm = \vec{n}^\pm\cdot\hat{e}\cdot\quad$ with $\hat{e}_\pm = \mathsf{n}\ \mathsf{r}\ \mathsf{or}\ \mathsf{k}\ \mathsf{and}\ \mathsf{with}\ \vec{n}^\pm\ \mathsf{h}\mathsf{a}\ \mathsf{in}\ \mathsf{a}\ \mathsf{the}\ \mathsf{a}\ \mathsf{b}\ \mathsf{a}$ *i* are the projections of the substandary (or, et al. projection) on the rest frame of the decaying tau lepton). as computed in the rest frame of the distribution of the distributions of the *B±±* $\frac{1}{2}$ vector for the chosen decay mode (i.e. the pion direction as seen where we defined $\cos\theta^{\pm}_{i}=\vec{n}^{\pm}\cdot\hat{e}_{i}$, with $\hat{e}_{i}=\mathsf{n},$ **r** or The process of the process of the process of the polarization in the polarization $\frac{1}{2}$ polanticute vector for the criosen decay mode *(i.e. the piol*)
3. In the rest frame of the decaying tau lenton) in the rest frame of the decaying tau lepton). igiven by projecting ones of Eq. (3.31) which components of the averages of t where we defined $\cos\theta^{\pm}_i=\vec{n}^{\pm}\cdot\hat{e}_i$, with $\hat{e}_i=$ **n**, **r** or **k** and with \vec{n}^{\pm} being the in the rest frame of the decaying tau lepton). polarimetric vector for the chosen decay mode (*i.e.* the pion direction as seen

Alternatively, the Fano coefficients can be computed as the averages The sain boson of the seit boson parado as the declayer when we consider the decay in the decay $\frac{1}{2}$ Alternatively, the Fano coefficients can be computed as the averages \mathbf{b} taking the average the set of \mathbf{b}

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B_i^{\pm} = \frac{3}{\kappa_{\pm}} \frac{1}{\sigma} \int d\Omega^{\pm} \frac{d\sigma}{d\Omega^{\pm}} (\vec{n}^{\pm} \cdot \hat{e}_i), \qquad C_{ij} = \frac{9}{\kappa_{+} \kappa_{-}} \frac{1}{\sigma} \int d\Omega^{\pm} d\Omega^{-} \frac{d\sigma}{d\Omega^{\pm} d\Omega^{-}} (\vec{n}^{\pm} \cdot \hat{e}_i)(\vec{n}^{\pm} \cdot \hat{e}_j)
$$

spin analyzing power:

$$
k_{\pm} = \pm 1
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Accessing the density matrix from "data" reduction by the didney measuring measure particle angular distributions in the spin and the spin and the spin analysing powers in the spin and the spin *i*. The and the positive method is a selected the positive set of the positive set of the positive set of the s frequencies. The average (see Eqs. (3.34)–(3.35)–(3.34)–(3.34)–(3.34)–(3.34)–(3.35)–(3.34)–(3.35)–(3.34)–(3.34)–(3.34)–(3.34)–(3.34)–(3.34)–(3.34)–(3.34)–(3.34)–(3.34)–(3.34)–(3.34)–(3.34)–(3.34)–(3.34)–(3.34)–(3.34)–(3.3 **BING THE COENSITY MATHX TROM-CORTA** values would be perfectly symmetric. The plots are from a simulation of the process *^e*⁺*e*[→] [→] ^ω ⁺^ω [→] at [↑]*^s* = 91*.*¹⁹ GeV, by two of

The Fano coefficients can be experimentally reconstructed in several ways, for instance by accessing the distributions $\overline{ }$ **1** ed III severd
" *,* (3.34) The Fano coefficients can be experimentally reconstructed frequencies. The average (see Eqs. (3.34)–(3.35)) and standard deviation of these histograms give the mean value and uncertainty of ways, for instance by accessing the distributions

$$
\frac{1}{\sigma} \frac{d\sigma}{d\cos\theta_i^{\pm}} = \frac{1}{2} \left(1 \mp B_i^{\pm} \cos\theta_i^{\pm} \right) \qquad \frac{1}{\sigma} \frac{d\sigma}{d\cos\theta_i^{\pm} d\cos\theta_j^{-}} = \frac{1}{4} \left(1 + C_{ij} \cos\theta_i^{\pm} \cos\theta_j^{-} \right)
$$

Fano coefficients

Alternatively, quantum to model if the following distribution distributions can be reconstructed if the following distributions can be reconstructed in \mathcal{A}

ϑ here we defir τ illieu $\cos v_i - u \cdot v_i$, with $v_i - u$, when τ is the air with u being the state τ \mathbf{r} 4 $\cos\theta^{\pm}_{i}=\vec{n}^{\pm}\cdot\hat{e}_{i}$. wit $\mathsf{a}^\mathtt{final}\ \cos\theta^\pm = \vec{n}^\pm\cdot\hat{e}\cdot\quad$ with $\hat{e}_\pm = \mathsf{n}\ \mathsf{r}\ \mathsf{or}\ \mathsf{k}\ \mathsf{and}\ \mathsf{with}\ \vec{n}^\pm\ \mathsf{h}\mathsf{a}\ \mathsf{in}\ \mathsf{a}\ \mathsf{the}\ \mathsf{a}\ \mathsf{b}\ \mathsf{a}$ *i* are the projections of the substandary (or, et al. projection) on the rest frame of the decaying tau lepton). as computed in the rest frame of the distribution of the distributions of the *B±±* $\frac{1}{2}$ vector for the chosen decay mode (i.e. the pion direction as seen where we defined $\cos\theta^{\pm}_{i}=\vec{n}^{\pm}\cdot\hat{e}_{i}$, with $\hat{e}_{i}=\mathsf{n},$ **r** or The process of the process of the process of the polarization in the polarization $\frac{1}{2}$ polanticute vector for the criosen decay mode *(i.e. the piol*)
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B_i^{\pm} = \frac{3}{\kappa_{\pm}} \frac{1}{\sigma} \int d\Omega^{\pm} \frac{d\sigma}{d\Omega^{\pm}} (\vec{n}^{\pm} \cdot \hat{e}_i)
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$$
C_{ij} = \frac{9}{\kappa_{+} \kappa_{-}} \frac{1}{\sigma} \int d\Omega^{\pm} d\Omega^{-} \frac{d\sigma}{d\Omega^{\pm} d\Omega^{-}} (\vec{n}^{\pm} \cdot \hat{e}_i) (\vec{n}^{\pm} \cdot \hat{e}_j)
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T+ rest frame and researd $cos \theta^+$ begat to the T-rest frame and researd V^+ rest frame and record $\cos \theta_i$, poost to the thest frame and record $\alpha^- = \pi$. $\cos\theta_i$. The result is a series of histograms which give us the Fano coefficients. These momenta are the only information that we need to extract from the numerical simulation that w **For every simulated event** $\texttt{\texttt{t}}^{\texttt{+}}$ rest frame and record $\cos\theta_i^+$, boost to the $\texttt{\texttt{t}}^{\texttt{-}}$ rest frame and record $\boldsymbol{\mu}$ recently between $\boldsymbol{\mu}$, which we mostly follow in the remainder of the remainder of this section. $\cos \theta_i^-$. The result is a series of histograms which give us the Fano ΄ ever∣ $\dot{\mathbf{m}}$ For every simulated event, we boost to the CoM frame (ISR), boost to the

Y axes: relative frequencies; x axes: values of the products.

 Y X axes: relative frequencies; x axes: \mathbb{R} values of the products. parton is a set of the process *experiment is a signal of the process* \mathcal{L} **violation of Bellin interest in the Bellin Street in the Bellin Street in the Bellin Street in the Bellin Street in**

To have a better sense of the actual experimental ex and detector effects we obtain: tion including for the charged pool to the computation of the closest and of the closest approach vector discussed provided by a contract of the closest approach vector discussed by a contract of the closest approach vecto are achieved for a diserent value of the scattering angle due to the scattering angle due to the diserent interpret inter \Box including \Box R, momenta reconstruction the physics at Belle II. Here we study the region at the *Z*-boson resonance. particles **extend the formulation** is the process of the process \mathbb{R}^n and \mathbb{R}^n are process in the process of \mathbb{R}^n and the process of \mathbb{R}^n and the process of \mathbb{R}^n and the process of \mathbb{R}^n

$$
\mathbf{C} = \begin{pmatrix}\n0.4819 \pm 0.0079 & -0.0073 \pm 0.0082 & -0.0016 \pm 0.0089 \\
-0.0066 \pm 0.0082 & -0.4784 \pm 0.0084 & 0.0016 \pm 0.0070 \\
-0.0002 \pm 0.0080 & -0.0004 \pm 0.0087 & 1.000 \pm 0.0074\n\end{pmatrix}
$$
\n
$$
\mathbf{B}^{+} = \begin{pmatrix}\n-0.0028 \pm 0.0042 \\
-0.0001 \pm 0.0049 \\
-0.0001 \pm 0.0049 \\
0.2198 \pm 0.0044\n\end{pmatrix} \quad \mathbf{B}^{-} = \begin{pmatrix}\n-0.0039 \pm 0.0048 \\
0.0017 \pm 0.0049 \\
0.2207 \pm 0.0044\n\end{pmatrix}
$$

well in agreement with the theoretical **and vectimates seen before:** m¹² = 1*.*239 *±* 0*.*017 *.* (3.12) characteristic pattern in the CM energy dependence is due to the interference between the photon and **Z** also well in agreement with the theoretical The coe#cients C*ij* and B*ⁱ* averaged over the angular distribution of the ω pair are found to be at the leading order even though the initial beam is made of unpolarized electrons and positrons. This feature is due to the parity violating electroweak interactions and makes the study of the ω leptons at this energy particularly interesting. The two plots in Fig. 3.3 show the behavior of the coe#cients B*±*

$$
C = \begin{pmatrix} 0.4878 & 0 & 0 \\ 0 & -0.4878 & 0.0011 \\ 0 & 0.0011 & 1 \end{pmatrix}
$$

$$
B^{+} = B^{-} = \begin{pmatrix} 0 \\ 0.0001 \\ 0.2194 \end{pmatrix}
$$

Quantum information with taus @ FCC-ee and in the scattering and the sc 0 ↑0*.*4878 0*.*0011 Quantum information with taus @ FCC-ee These contributions are, again, subdominant with respect to the statistical errors in Eq. (3.11) and Eq. (3.12). They become important as we rescale our statistical uncertainties toward the FCC target

luminosity and will eventually come to dominate.

As to the prospects for detecting entanglement and the violation of the Bell inequality at FCC-ee with tau leptons, we find C = $\frac{1}{2}$ 0 ↑0*.*4878 0*.*0011 $\mathbb{R} \to \mathbb{R}$ ects for detecting entanglement and the violation of the Bell Eq. (3.12). They become important as we rescale our statistical uncertainties toward the FCC target As to the prospects for detecting entang α nequality at FCC-ee with tau leptons, we find As to the prospects for detecting entanglement and the violat *C* = 0*.*4805 *±* 0*.*0063*|*stat *±* 0*.*0012*|*syst *,* (3.17)

e.
Se $\mathscr{C}= 0.4805\pm 0.0063|_{\text{stat}}\pm 0.0012|_{\text{syst}}$ \parallel and the Horodeckies condition gives the Horodeckies condition gives $\frac{1}{2}$

These values can be directly compared with the results of the Monte Carlo simulations. We find that $\mathfrak{m}_{12}=1.239\pm0.017|_{\rm stat}\pm0.008|_{\rm syst}$ and the Horodecki's condition gives the Horodecki's condition gives the Horodecki's condition gives the Horode
The Horodecki's condition gives the Horodecki's condition gives the Horodecki's condition gives the Horodecki
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in line with the given theoretical predictions: $\mathscr{C}=0.4878$, $\mathfrak{m}_{12}=1.238$. m¹² = 1*.*239 *±* 0*.*017*|*stat *±* 0*.*008*|*syst *,* (3.18) in line with the given theoretical predict Equations Eqs. (3.17)–(3.18) are the main result of the present work. The overall significance of

As shown in Fig 3.2 the entanglement depends on the kinematic region and is maximal at " = ε*/*2.

Quantum information with taus @ FCC-ee and in the scattering and the sc 0 ↑0*.*4878 0*.*0011 Quantum information with taus @ FCC-ee These contributions are, again, subdominant with respect to the statistical errors in Eq. (3.11) and Eq. (3.12). They become important as we rescale our statistical uncertainties toward the FCC target

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Similarly, the violation of the Bell inequality is given by quoted statistical uncertainties are bound to shrink by a factor of about 70 if • the above results use our benchmark luminosity of 17.6 fb-1, hence the the full 150 ab⁻¹ luminosity is utilized. $\frac{1}{2}$ the shows requite use our benchmark luminosity of 17.6 fb-1, bence the deviations on the errors once the errors are added in the expected significance with 150 absolute significance
The expected significance with 150 absolute with 150 absolute significance with 150 ab quoted statistical dricertainties are bound to shinnk by a factor of about 70 in deviations on ϵ is the errors are added in ϵ and ϵ in ϵ as in ϵ of ϵ ϵ ϵ ϵ of datasets with ϵ can be estimated by retaining only the systematic error in Eq. (3.18) and it is about 30 standard in Eq. (3.18
30 standard in Eq. (3.18) and it is about 30 standard in Eq. (3.18) and it is about 30 standard in Eq. (3.18)

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Se $\mathscr{C}= 0.4805\pm 0.0063|_{\text{stat}}\pm 0.0012|_{\text{syst}}$ \parallel and the Horodeckies condition gives the Horodeckies condition gives $\frac{1}{2}$

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The expected significance with 150 absolute with 150 absolute significance with 150 ab quoted statistical dricertainties are bound to shinnk by a factor of about 70 in deviations on ϵ is the errors are added in ϵ and ϵ in ϵ as in ϵ of ϵ ϵ ϵ ϵ of datasets with ϵ can be estimated by retaining only the systematic error in Eq. (3.18) and it is about 30 standard in Eq. (3.18
30 standard in Eq. (3.18) and it is about 30 standard in Eq. (3.18) and it is about 30 standard in Eq. (3.18)
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1 • the quoted systematic uncertainties are computed by evaluating the shift in the values of the observables obtained with and without ISR+detector effects. To this we add a further shift obtained for a different tuning of the detector parameters. *iii are director parameters iii* **are director of the polarizations of the polarizations of the polarizations of the state of the state of the polarizations of the state of the state of the state** the quoted systematic uncertainties are computed by evaluating the shift in
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3.3 Polarizations of the series of the s that the these coe coe coe coe coefficients are equal, B+ and B+ and

As shown in Fig 3.2 the entanglement depends on the kinematic region and is maximal at " = ε*/*2.

Quantum information observables for HEP α) α anti α int The use \mathcal{L}_{max} and the top \mathcal{L}_{max} and \mathcal{L}_{max} and \mathcal{L}_{max} The ware was the model the heaviest among the leptons, as the top quark among the top quark among the quarks—could be the top im intormation observables for $\mathsf{H}\mathsf{H}\mathsf{P}$ electroweak Lorentz-invariant vertex !*^µ* between the *Z*-boson and the ω lepton up to dimension five

Can entanglement tell us something about new physics? Lets introduce some anomalous couplings for the τ lepton **electronic Invariant vertex invariant vertex invariant vertex invariant vertex invariant vertex invariant vert**
Lepton final final final final fields and the up to dimension fields in the up to dimension fields in the USD operators can be written as 2
2 ... *for the τ lept*

$$
i\,\frac{g}{2\,\cos\theta_W}\,\bar\tau\,\Gamma^\mu(q^2)\,\tau\,Z_\mu(q) =\,\,i\,\,\frac{g}{2\,\cos\theta_W}\,\bar\tau\,\bigg[\gamma^\mu F_1^V(q^2) + \gamma^\mu\gamma_5 F_1^A(q^2) + \frac{i\sigma^{\mu\nu}q_\nu}{2m_\tau}F_2(q^2) + \frac{\sigma^{\mu\nu}\gamma_5q_\nu}{2m_\tau}F_3(q^2)\bigg]\,\tau\,Z_\mu(q)
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i \frac{g}{2 \cos \theta_W} \bar{\tau} \Gamma^{\mu}(q^2) \tau Z_{\mu}(q) = i \frac{g}{2 \cos \theta_W} \bar{\tau} \left[\gamma^{\mu} F_1^V(q^2) + \gamma^{\mu} \gamma_5 F_1^A(q^2) + \frac{i \sigma^{\mu \nu} q_{\nu}}{2 m_{\tau}} F_2(q^2) + \frac{\sigma^{\mu \nu} \gamma_5 q_{\nu}}{2 m_{\tau}} F_3(q^2) \right] \tau Z_{\mu}(q)
$$

$$
F_1^{V, A}(q^2) = F_1^{V, A}(0) + \frac{q^2}{m_Z^2} C_1^{V, A} \blacktriangleleft
$$

$$
F_1^A(0) = g_V = -1/2 + 2 \sin^2 \theta_W
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F_1^A(0) = -g_A = 1/2
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 F^V ₂ as well as $F_{2,3}$ (*m*² *Z* (m_Z^2) , via a x^2 test where we and give limits on the coe"cients *CV,A* C_1 , as well as $T_{2,3}(m_Z)$, vid a χ ² test writere we vary tries for χ one at a time. Then, we constrain $C_1^{A,V}$ as well as F_2 $_3(m_Z^2)$, via a x^2 test where we of the way $m_Z^2)$, via a χ^2 test where we vary the Then, we constrain $C^{A,V}_{1}$ as well as $\,F_{2,3}(m^{2}_{Z})$, via a χ² test where we vary the parameters one at a time.

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i \frac{g}{2 \cos \theta_W} \bar{\tau} \Gamma^{\mu}(q^2) \tau Z_{\mu}(q) = i \frac{g}{2 \cos \theta_W} \bar{\tau} \left[\gamma^{\mu} F_1^V(q^2) + \gamma^{\mu} \gamma_5 F_1^A(q^2) + \frac{i \sigma^{\mu \nu} q_{\nu}}{2 m_{\tau}} F_2(q^2) + \frac{\sigma^{\mu \nu} \gamma_5 q_{\nu}}{2 m_{\tau}} F_3(q^2) \right] \tau Z_{\mu}(q)
$$

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used to constrain the parameters in Eq. (3.24): one observable is the concurrence *C* defined in Eq. (2.7)

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Can entanglement tell us something about new physics? Lets introduce some anomalous couplings for the τ lepton **electronic Invariant vertex invariant vertex invariant vertex invariant vertex invariant vertex invariant vert**
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$$

$$
F_1^{V, A}(q^2) = F_1^{V, A}(0) + \frac{q^2}{m_Z^2} C_1^{V, A} \blacktriangleleft
$$

$$
F_1^A(0) = g_V = -1/2 + 2 \sin^2 \theta_W
$$

$$
F_1^A(0) = -g_A = 1/2
$$

 F^V ₂ as well as $F_{2,3}$ (*m*² *Z* (m_Z^2) , via a x^2 test where we and give limits on the coe"cients *CV,A* C_1 , as well as $T_{2,3}(m_Z)$, vid a χ ² test writere we vary tries for χ one at a time. Then, we constrain $C_1^{A,V}$ as well as F_2 $_3(m_Z^2)$, via a x^2 test where we of the way $m_Z^2)$, via a χ^2 test where we vary the Then, we constrain $C^{A,V}_{1}$ as well as $\,F_{2,3}(m^{2}_{Z})$, via a χ² test where we vary the parameters one at a time.

used to constrain the parameters in Eq. (3.24): one observable is the concurrence *C* defined in Eq. (2.7)

By the way, the concurrence is more sensitive than the cross section if the relative uncertainty is the same:

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Is that the best that this quantum stuff can do?

Nope! Rather than using 'quantum information observables' like entanglement,
magic, discord, we can use the density matrix itself. In quantum information *Meen two density matrices is often quantified with the* **of 60.6000+1000 examples** in the corresponding to the corresponding the corresponding to 11.4 These limits are comparable since they are obtained with a similar number of events and utilize similar number of α magic, discord, we can use the density matrix itself. In quantum information theory, the distance between two density matrices is often quantified with the trace distance: theory the distance hetween two Toory, the distance between two denoty matrices is siten quant

$$
\mathscr{D}^T(\rho,\varsigma) = \frac{1}{2} \operatorname{Tr} \sqrt{(\rho-\varsigma)^{\dagger}(\rho-\varsigma)} \ge 0
$$

By the way, the concurrence is more sensitive than the cross section if the relative uncertainty is the same: χ^2 trace distance and fidelity utilize the full density matrices of the two target quantum systems and By the way, the concurrence is more sensitive than the cross section if the Incurrence is more sensitive than the eress section if the strate the internal structure of the top description of α In the following introducement integration in the following terms in the section of the section of the section strate the internal structure of the top of the top the top the top of the top quark in QCD by means of recent
of recent Links in the concentration By the way, the concurrence is more sensitive than the cross section if the r elative uncertainty is the same: r

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 P \overline{X} **Example 2** = \overline{X} = \over theory the distance hetween two Toory, the distance between two denoty matrices is siten quant The trace distance between two density matrices ε and ϑ is defined as [26] The trace distance between two density matrices ε and ϑ is defined as [26] *^D^T* (ε*,*ϑ) = ¹ $\frac{1}{2}$ generalizing the Kolmogorov distance used for probabilities distributions. The trace distributions. The trace d
The trace distance is trace distance is trace distance is the trace distance is the trace distance is the trac a metric on the space of density operators and remains in variant units invariant unitary transformations: \mathbf{a} a metric on the space of density operators and remains invariant unitary transformations: $\mathbf{1}$ \mathbf{i} Tr ! (ε → ϑ)*†*(ε → ϑ) ↑ 0 *,* (2.1) theory, the distance between two density matrices is often quantified with the

$$
\mathscr{D}^{T}(\rho,\varsigma) = \frac{1}{2} \operatorname{Tr} \sqrt{(\rho-\varsigma)^{\dagger}(\rho-\varsigma)} \ge 0
$$

 $T_{\rm max}$ shows the bounds obtained with pseudo-experiments containing each $T_{\rm max}$ paring two qubit $\rho = \frac{1}{2} |1 + r \cdot \sigma|$, $\varsigma = \frac{1}{2} |1 + \vec{s} \cdot \vec{\sigma}|$ gives: $\frac{1}{10}$ the Kolmogorov distance used for probabilities distributions. The trace is not probabilities distributions. The trace is not probabilities distributions. The trace is not probabilities distributions. The trace As an example, comparing two qubit $\rho = \frac{1}{2} |1 + \vec{r} \cdot \vec{\sigma}|$, $\varsigma = \frac{1}{2} |1 + \vec{s} \cdot \vec{\sigma}|$ gives: *D^T* (ε*,*ϑ) = *D^T* (*U* ε*U†, U* ϑ *U†*), with *U* a unitary matrix. To see the e!ect of the trace distance 1 2 $\left[\mathbb{1} + \vec{r} \cdot \vec{\sigma}\right]$, $\varsigma = \frac{1}{2}$ 2 **1** $\rho = \frac{1}{2} \left[\mathbb{1} + \vec{r} \cdot \vec{\sigma} \right]$, $\varsigma = \frac{1}{2} \left[\mathbb{1} + \vec{s} \cdot \vec{\sigma} \right]$ gives: $=\frac{1}{2}\Big[\mathbb{1}+\vec{r}\cdot\vec{\sigma}\Big]$, $\varsigma=$ 1 2 $\left[\mathbb{1} + \vec{s} \cdot \vec{\sigma}\right]$ gives: $\frac{1}{2}$ As all example, companity two qubit P^2

$$
\mathscr{D}^T(\rho,\varsigma) = \frac{\|\vec{r} - \vec{s}\|}{2}
$$

So, re-doing the analysis using only trace distance and cross section gives:

68% and 95% joint confidence intervals (2 parameters); assuming negligible systematics affecting quantum tomography

Outlook

- The FCC-ee offers unprecedented possibilities for analyzing the spin correlations of tau lepton pairs via quantum tomography.
- The method gives access to entanglement and to the violation of Bell inequalities with significances well above the 5σ level:

Outlook ω*C |*syst = *|*0*.*4807 → 0*.*4805*|* = 0*.*0002 *,* (3.15) ωm12*|*syst = *|*1*.*232 → 1*.*230*|* = 0*.*002 *.* (3.16)

- The FCC-ee offers unprecedented possibilities for analyzing the spin correlations of tau lepton pairs via quantum tomography.
	- The method gives access to entanglement and to the violation of Bell inequalities with significances well above the 5σ level: luminosity and will eventually come to dominate. Bell inequalities with significances well above the 50 level:

 $\textcircled{\textbf{\textit{c}}} = 0.4805 \pm 0.0063|_{\text{stat}} \pm 0.0012|_{\text{syst}} \text{ } ,$

These contributions are, again, subdominant with respect to the statistical errors in Eq. (3.11) and

 \bullet $\mathfrak{m}_{12} = 1.239 \pm 0.017|_{\text{stat}} \pm 0.008|_{\text{syst}}$

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- Quantum information observables and methods can be ported to high-energy physics and employed in new physics searches where the systematic errors where added linearly.
- Rather than entangiement, magic and other esotenc quant use trace distance, fidelity and other tools designed to compare deviations on concerning the expected significance with 150 absolute significance with 150 ab \sim 1 of data above with 150 ab \sim 1 of data above with 150 ab \sim 1 of data above with 150 absolute 3.1 of data above with 150 ab the Bell inequality violation (obtained with a benchmark luminosity of 17.6 fb→1) is about 13 standard with 13 • Hather than entangiement, magic and other esoteric quantit • Rather than entanglement, magic and other esoteric quantities I'd

Outlook ω*C |*syst = *|*0*.*4807 → 0*.*4805*|* = 0*.*0002 *,* (3.15) ωm12*|*syst = *|*1*.*232 → 1*.*230*|* = 0*.*002 *.* (3.16)

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- catchy) slogan, could you really find anything cooler to do while running at the *Z* resonance? These significances can be increased by means of the increased by means of the select events with scattering with scattering α angles close to " = ε*/*2, for which the values of *C* and m¹² are larger (see Fig. 3.2). • Even if "it from bit" were to turn out to be merely an empty (albeit

Backup

The $m₁₂$ bias $\frac{1}{1}$ in the amount of violation with a specific choice of violation with $\frac{1}{1}$

Values of m_{12} and related standard error as a function of the size of the sample used in the Monte Carlo analysis:

The $m₁₂$ bias $\frac{1}{1}$ in the amount of violation with a specific choice of violation with $\frac{1}{1}$

Values of m_{12} and related standard error as a function of the size of the sample used in the Monte Carlo analysis:

No need to worry about the bias as we use samples of size N>10⁵, resulting in a value of m₁₂ well compatible with the expected theoretical estimate (the dashed green line). Than 200 events because more many containers with a set of the less than 2.00 events of the

Modeling the ISR a Gaussian smearing of the Monte Carlo truth pion momenta and closest approach vector taking the Λ that the uncertainties are small if typical pion momenta of the order of the order of 10

To model the effect of ISR we pollute our dataset with events characterized by lower CoM energy down to 89 GeV, using the relative weights indicated by the plot below obtained with Pythia 8.

Modeling the detector effects and systematic errors *pT* gev 10
ε το

To simulate the detector we apply a gaussian smearing to the pion momenta and tracks using two settings: $\frac{1}{2}$ measure of the experimental systematic uncertainty. We have chosen the following two sets *P*

response and limitations which the results of the results of the results of the two sets in the two sets is an measure of the experimental systematic uncertainty of the following two sets is the following two sets is the σ_{p_T} *pT* $= 3 \times 10^{-5} \oplus 0.3 \times 10^{-3} \frac{p_T}{\text{GeV}}$ **c** $\sigma_{\theta, \phi} = 0.1 \times 10^{-3} \text{rad}$ **c** $\sigma_b = 3 \mu \text{m}$ ω↑ *pT* $\frac{p_7}{p_7}$ $= 3 \times 10^{-5} \oplus 0.6 \times 10^{-3} \frac{p_T}{g_T}$ (a) $\sigma_0 A = 0.1 \times 10^{-3}$ rad (2.16) $\sigma'_1 = 5 \mu$ m $f(t)$ measure of the experimental systematic uncertainty. We have chosen the following two sets $\frac{\sigma_{p_T}}{p_T} = 3 \times 10^{-5} \oplus 0.3 \times 10^{-3} \frac{1}{\text{G}}$ $\frac{1}{\rm GeV}$ and $\sigma_{\theta,\phi} = 0.1 \times 10^{-3} \, \mathrm{rad}$ and $\sigma_b = 3 \, \mu \mathrm{m} \oplus \sigma_c$ $\frac{\sigma'_{p_T}}{g_{p_T}} = 3 \times 10^{-5} \oplus 0.6 \times 10^{-3} \frac{\mu}{C}$ $\sigma_{\rm d}^3 \frac{p_T}{\rm GeV}$ **i** $\sigma_{\theta,\phi} = 0.1 \times 10^{-3}$ rad **i** $\sigma'_b = 5 \,\mu{\rm m} \oplus \frac{15 \,\mu{\rm m}}{\sin^2/3} \frac{\rm GeV}{\rm cm}$ for the tracks proper and for the tracks proper and the tracks proper and the tracks p $\sigma_b = 3 \, \mu\text{m} \oplus$ $15 \,\mu{\rm m}$ $\sin^{2/3} \Theta$ GeV *pT* for the impact parameters and for the reconstruction of the vector of closest approach. These sets $\sigma_{\theta,\phi} = 0.1 \times 10^{-3}$ rad $\sigma'_b = 5 \,\mu\text{m} \oplus \frac{19 \,\mu\text{m}}{\sin^{2/3} \Theta} \frac{\text{GeV}}{p_T}$ in the detector or anomalies in signal collection, where momentum is momentum is momentum in signal with the m momenta tracks impact parameter = 3 [→] ¹⁰→⁵ [↑] ⁰*.*³ [→] ¹⁰→³ *^p^T* GeV and ^ωω*,*^ε = 0*.*¹ [→] ¹⁰→³ rad (2.15) or ω↑ *pT pT* $\sigma_{\theta,\phi} = 0.1 \times 10^{-3} \,\text{rad}$ $\frac{1}{\text{I}}$ $\sigma_b = 3 \,\mu\text{m} \oplus \frac{15 \,\mu\text{m}}{\text{sin}^{2/3} \Theta} \frac{\text{GeV}}{p_T}$ for the tracks proper and 1×10 $\frac{1}{2}$ $\frac{1}{2}$ $\overline{3}$ *pT* $\sigma'_b = 5 \,\mu\mathrm{m} \oplus$ $15 \,\mu{\rm m}$ $\sin^{2/3} \Theta$ GeV *pT* for the impact parameters and for the reconstruction of the vector of closest approach. These sets response and limitations while the di!erence in the results obtained by means of the two sets is a measure of the experimental systematic uncertainty. We have chosen the following two sets $\frac{r_{\perp}}{p_{\perp}}$ $\frac{1}{2}$ $\frac{F_T}{p_T} = 3 \times 10^{-9} \oplus 0.3 \times 10^{-9} \frac{1}{\text{GeV}}$ **c** $\sigma_{\theta, \phi} = 0.1 \times 10^{-9} \text{rad}$ **c** $\sigma_b = 3 \mu \text{m} \oplus \frac{1}{\text{GeV}}$ $\sigma_{p_T}^\prime$ *pT* $\frac{\sigma'_{p_T}}{p_T} = 3 \times 10^{-5} \oplus 0.6 \times 10^{-3} \frac{p_T}{\text{GeV}}$ **c** $\sigma_{\theta, \phi} = 0.1 \times 10^{-3} \text{rad}$ **c** $\sigma'_{b} = 5 \,\mu\text{m}$ for the tracks proper and measure of the experimental systematic uncertainty. We have chosen the following two sets two sets of two sets $\sigma_{\theta} = \frac{1}{\sqrt{3}} \frac{\rho_1}{\sqrt{3}}$ and $\sigma_b = 0.1 \times 10^{-3}$ rad $\sigma_b = 5 \mu \text{m} \oplus \frac{1}{\text{s}}$

*P*CC Collaboration, A. Abada et al., PCC-ee: The Lepton Collider: Future Circular Collider Conceptual Design
Report Volume 2, Eur. Phys. J. ST 228 (2019). **boration, A. Abada et** *pT* FCC Collaboration, A. Abada et al., FCC-ee: The Lepton Collider: Future Circ a noise that comes from the imperfection of the readouts. In the simulation we retain the leading it., I CO-ee. The Lepton Comuer. I uture Circular Comuer Conceptual Design
[228 (2019). FCC Collaboration, A. Abada et al., FCC-ee: The Lepton Collider: Future Circular Collider Conceptual Design *b* and it, A. Abada e
 b and it, Phys. J. Report Volume 2, Eur. Phys. J. ST 228 (2019). $\overline{\text{cm}}$ $\overline{\text{cm}}$ $\frac{1}{2}$ FCC Colla
Report Vo FCC Collaboration, A. Abada et al., FCC-ee: The Lepton Collider: Future Circular Collider Conceptual Design
Report Volume 2, Eur. Phys. J. ST 228 (2019).

F. Azzi and E. Perez, Exploring requirements and detector solutions for FCC- $\left(2021\right).$ *p*
 p Azzi and F. Perez. Exploring requirements and detector solutions for ECC-ee. Fur. Phys. J. Plus 136 $f(\Delta \theta \Delta \theta)$ P. Azzi and E. Perez, Exploring requirements and detector solutions for FCC-ee, Eur. Phys. J. Plus 136 $\left(2021\right)$. for the impact parameters and for the vector of the vector of the vector of the vector of closest approach. These sets approach \sim (2021).

provides a useful benchmark to guide our analysys. A constant smearing originates in imperfections in the use the difference in the results obtained with the two s a noise that comes from the imperfection of the simulation of the readouts. In the simulation we retain the lea c in the quoted uncertainties and account for the relations by performing for the relations by performing α $\frac{10}{6}$ use the difference in the requite enterined with the two este provides and difference in the reduce optunious with the two octons. systematic error. obtained with the two sets to estimate the \mathbf{C} contractions in the quoted uncertainties and account for the relations \mathbf{C} the relationship of the relation and G internet the results obtained with the two sets to estimate the α We use the difference in the results obtained with the two **s** in the detector or anomalies in signal collection, whereas the term scaling with the momentum is 15 *µ*m sin2*/*³ " $\overline{}$ \overline{a} ^T *in the results* 15 *µ*m sin2*/*³ " $\overline{}$ *pT* We use the difference in the results obtained with the two sets to estimate the for the impact parameters and for the reconstruction of the vector of closest approach. These sets

Momenta reconstruction

Momenta construction equations: four from the sum of the ω -lepton momenta, which is constrained to satisfy \mathbf{M} . The eight unknown components of the neutrino momenta can be reconstructed by means of eight \mathbf{M} equations: four from the sum of the sum of the which is constrained to satisfy the which is constrained to satisfy α amonto roomatruation As opposed to other processes, like top quark and *W*-boson decays, the reconstruction of the Λ *pµ* ^ω⁺ ⁺ *^p^µ* ^ω[→] ⁼ *^p^µ ^e*+*e*[→] *,* (2.9)

The 8 components of neutrino momenta are reconstructed via the following constraints $\overline{}$. The eight unknown components of the neutrino momenta can be reconstructed by means of eight equations: four from the sum of the which is constrained to satisfy the unit of the which is constrained to satisfy the satisfy of the which is constrained to satisfy the satisfy of the which is constrained to satisfy the \sim and four from the mass-shell conditions \sim 8 components of neutrino momenta are reconstructed via the following and four from the mass-shell conditions of the mass-shell conditions of the mass-Consequently, the vector of closest approach, identified from the continuation of the trajectories of the pions emitted in the decay, can be measured and used and used as to resolve the two-fold degeneracy arising the twoand o comp

neutrino momenta and, therefore, of the warehouse the momenta is almost perfect perfect. The warehouse the warehouse

$$
p_{\tau^+}^{\mu} + p_{\tau^-}^{\mu} = p_{e^+e^-}^{\mu} \qquad (p_{\tau^+} - p_{\pi^+})^2 = m_{\nu}^2 = 0 \qquad (p_{\tau^-} - p_{\pi^-})^2 = m_{\nu}^2 = 0
$$

$$
p_{\tau^+}^2 = m_{\tau}^2 \qquad p_{\tau^-}^2 = m_{\tau}^2
$$

yielding two possible solutions. We break the degeneracy by computing the vector of closest approach for both the solutions As the posemble central the mean the acgent ac_gon and *y* companing the reconstruction of the reconstruction o neutrino modelle approach for bour the dollations vector of closest approach for both the solutions neutrino momenta and, therefore, of the ω momenta is almost perfect. The reason is that the ω ing two peopible polutions. We break the degeneracy by computing the yielding two possible solutions. We break the degeneracy by Consequently, the vector of closest approach, identified from the continuation of the trajectories of the

The system of equations is seen order and a two-fold degeneracy arises (see the Appendix in α $\Delta t = 0$ direction of the π $\Delta t = 0$ direction of the π $\frac{1}{2}$ direction of the π direction of the π- ρ direction of the π

$$
\mathbf{d}_{min} = \mathbf{d} + \frac{\left[(\mathbf{d} \cdot \mathbf{n}_{+})(\mathbf{n}_{-} \cdot \mathbf{n}_{+}) - \mathbf{d} \cdot \mathbf{n}_{-} \right] \mathbf{n}_{-} + \left[(\mathbf{d} \cdot \mathbf{n}_{-})(\mathbf{n}_{-} \cdot \mathbf{n}_{+}) - \mathbf{d} \cdot \mathbf{n}_{+} \right] \mathbf{n}_{+}}{1 - (\mathbf{n}_{-} \cdot \mathbf{n}_{+})^{2}}
$$

 π -decay, vertex ay ver
. X *[|]*p→*[|]* and ⁿ⁺ ⁼ ry v in which p*±* are the two decay vertices v \pm of the weight \pm leptons is \pm leptons in \pm leptons in \pm leptons is

direction of the π^+ π- decay vertex direction of the π⁺

$$
\mathbf{d}=\mathbf{v}_+-\mathbf{v}_-
$$

in which p*[±]* are their momenta. The distance between the two decay vertices v*[±]* of the ω *[±]* leptons is The performance of what will be the actual detectors of Γ actual detectors of Γ can only be presumed from the presumed from th π+ decay vertex

in and by comparing them with the "measured" one The vertical connection of continues the backward continuations of the two continuations of the two charged pio by comparing them with the "measured" one. The pairs of the pairs of the charged pions and the final state pions appearing the final state of the final state of the final state pions appearing in the final state of the final state of the final state of the final st and by comparing them with the "measured" one.

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in and by comparing them with the "measured" one The vertical connection of continues the backward continuations of the two continuations of the two charged pio easured" one. \mathbf{b} The pairs of the pairs of the charged pions and the final state pions appearing the final state of the final state of the final state pions appearing in the final state of the final state of the final state of the final st and by comparing them with the "measured" one.

Entanglement is the "*spooky action at a distance*" that *keeps binding two quantum systems* that share a common history, despite their spatial separation.

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Mathematically, *it follows from the postulates of quantum mechanics and from the superposition principle.* Take a bipartite system formed by A and B

• iv postulate:
$$
\mathcal{H}_{A\cup B} = \mathcal{H}_A \otimes \mathcal{H}_B \implies |n_i\rangle = |a_i\rangle \otimes |b_i\rangle
$$
 can describe $(A \cup B)$
 $|a_i\rangle \in \mathcal{H}_A, |b_i\rangle \in \mathcal{H}_B$

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• superposition: $\ket{\psi} = \sum_i$ *i* $c_i \ket{n_i}$ can also describe ($A \cup B$)

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• superposition: $\ket{\psi} = \sum_i$ *i* $c_i \ket{n_i}$ can also describe ($A \cup B$)

The subsystems A and B are *entangled if the (pure) state*|*ψ*⟩ *of the system:*

 $|\psi\rangle \neq |\psi_A\rangle \otimes |\psi_B\rangle \quad \forall |\psi_A\rangle \in \mathcal{H}_A, |\psi_B\rangle \in \mathcal{H}_B$

Entanglement and Bell inequalities **P** e!ectively describes *S* as an open system, in this case as a system interacting with the apparatus used to measure **the observed in general in general in general decoherence can be modelled as an amic of the modelled assessment** the system interaction with the form interaction with the form (2.6), the form (2.6), the form (2.6), the only one that form (2.6), the only one that form (2.6), the only one that α

Entanglement is the "*spooky action at a distance*" that *keeps binding two* quantum systems that share a common history, despite their spatial separation. The characteristic properties of \mathcal{S} and \mathcal{S} is the possibility of \mathcal{S} and \mathcal{S} are latents as \mathcal{S} and \mathcal{S} and \mathcal{S} are latents as \mathcal{S} and \mathcal{S} are latents and \mathcal{S} α and is, correlations among the initial physics. In that cannot be accounted for by classical physics. In initial

Mathematically, it follows from the postulates of quantum mechanics and ϵ from the superpection principle. Take a binartite system formed by Λ and R the internation of protocols and the real intervalization of various appearation of \mathcal{B} *from the superposition principle.* Take a bipartite system formed by A and B

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 $|a_i\rangle \in \mathcal{H}_A, |b_i\rangle \in \mathcal{H}_B$

• superposition: $\ket{\psi} = \sum_i$ *i* $c_i \ket{n_i}$ can also describe ($A \cup B$) the constituents of a promising of a promising elementary particles as seen as seen as seen at colliders seems a promising $\sum_{i=1}^n a_i = a_i$ \bullet superposition: $\ket{\psi} = \sum c_i \ket{n}$ i

The subsystems A and B are entangled if the (pure) state $|\psi\rangle$ of the system: $|\psi\rangle\neq|\psi_A\rangle\otimes|\psi_B\rangle\quad\forall|\psi_A\rangle\in\mathcal{H}_A,\,|\psi_B\rangle\in\mathcal{H}_B$ *O* sensor positive *f* 1 can the 2 can tend in a specific in and (points) cance | *Y* / Crane cyclemn

A state (density matrix) ω *of S is called separable if and only if it can be written as a linear convex* For a *mixed state,* described by a *density matrix ρ*, this generalizes to

$$
\rho \neq \sum_{ij} p_{ij} \,\rho_i^{(A)} \otimes \rho_j^{(B)} , \quad \text{with} \quad p_{ij} > 0 \quad \text{and} \quad \sum_{ij} p_{ij} = 1
$$

Entanglement and Bell inequalities **P** e!ectively describes *S* as an open system, in this case as a system interacting with the apparatus used to measure **the observed in general in general in general decoherence can be modelled as an amic of the modelled assessment** the system interaction with the form interaction with the form (2.6), the form (2.6), the form (2.6), the only one that form (2.6), the only one that form (2.6), the only one that α

Entanglement is the "*spooky action at a distance*" that *keeps binding two* quantum systems that share a common history, despite their spatial separation. The characteristic properties of \mathcal{S} and \mathcal{S} is the possibility of \mathcal{S} and \mathcal{S} are latents as \mathcal{S} and \mathcal{S} and \mathcal{S} are latents as \mathcal{S} and \mathcal{S} are latents and \mathcal{S} α and is, correlations among the initial physics. In that cannot be accounted for by classical physics. In initial

Mathematically, it follows from the postulates of quantum mechanics and ϵ from the superpection principle. Take a binartite system formed by Λ and R the internation of protocols and the real intervalization of various appearation of \mathcal{B} *from the superposition principle.* Take a bipartite system formed by A and B

• iv postulate:
$$
\mathcal{H}_{A\cup B} = \mathcal{H}_A \otimes \mathcal{H}_B \implies |n_i\rangle = |a_i\rangle \otimes |b_i\rangle
$$
 can describe $(A \cup B)$
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Physically, entanglement is the hallmark of quantum mechanics as classical configurations are described by product states. *the form of (2.11) are called entangled or non-separable, and exhibit quantum correlations.*

Scientist and Two Colleagues Find It Is Not 'Complete' Even Though 'Correct.'

SEE FULLER ONE POSSIBLE

Believe a Whole Description of 'the Physical Reality' Can Be Provided Eventually.

Einstein saw entanglement as a bug of quantum mechanics (*spooky* was not meant as a compliment!). The problem is the *non-local nature* of the correlations sourced by entanglement.

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So, is *quantum mechanics incomplete*?

This was the question until 1964, when J. Bell identified an objective way to distinguish between the two frameworks.

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 P(*A, BP*^{*a*}^{*it, a*</sub> for independent measo, we mante it has to}
- *Locality*: for independent measurements it has to hold $P(A \cap B) = P(A)P(B)$ – no action at a *distance! n*₁ *p*₂ *a*₂ *n*₂ *n*₂ *e*₂ *a*₂ *p*₂ *m*₂ *p*₂ *n*₂ *h*₂ $\frac{1}{2}$ is called $\frac{1}{2}$ if $\frac{2}{3}$, nonelocal otherwise. Checking the validity of the hypothesis (2.36) is usually done by performing a Bell test, that is, by putting under experimental scruting u

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Two independent observers (A, B) have, each, two observables at their disposal (\hat{A}_1, \hat{A}_2 and \hat{B}_1, \hat{B}_2) all with possible outcomes 0 or 1. They test a *bipartite system* and look at the combination of expectation values (i.e. combination of average probabilities) given by (CHSH version)

$$
\mathcal{I}_2 = \langle \hat{A}_1 \hat{B}_1 \rangle + \langle \hat{A}_1 \hat{B}_2 \rangle + \langle \hat{A}_2 \hat{B}_1 \rangle - \langle \hat{A}_2 \hat{B}_2 \rangle
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Theorem (Bell): if locality and realism hold, then $\mathcal{I}_2 \leq 2$.

• When we compute the same quantity with the rules of quantum mechanics α answers compare the edition data big 11.1.1. The rates of data bell integration α we obtain $\mathcal{I}_2 \leq 2\sqrt{2}$, hence *measuring* $2 < \mathcal{I}_2 \leq 2\sqrt{2}$ would strongly favor *I*² ↔ 2 *.* (2.38) *quantum mechanics over hidden-variable theories.* $\frac{1}{\sqrt{2}}$ 2, hence *measuring* $2 < \mathcal{I}_2 \leq 2$ $\frac{1}{\sqrt{2}}$ 2