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Quantum tomography with τ leptons

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Based on:

- Quantum entanglement and Bell inequality violation at colliders, A. Barr, M. Fabbrichesi, R. Floreanini, E. Gabrielli, LM. — Prog.Part.Nucl.Phys. 139 (2024)
- Quantum tomography with τ leptons at the FCC-ee, M. Fabbrichesi, LM. — Phys.Rev.D 110 (2024)
- The trace distance between density matrices, a nifty tool in new-physics searches, M. Fabbrichesi, M. Low, LM. — arXiv 2501.03311

8th FCC workshop, CERN, 13-16/01/2025

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Focusing on $e^+e^- \to Z, \gamma \to \tau^+\tau^-$, FCC-ee would then allow us to:

- use quantum information observables and methods to test possible anomalous couplings of the τ lepton to gauge bosons.
- study entanglement and the violation of Bell inequalities by analyzing the spin correlations of the tau lepton pairs.

Theoretical Quantum Tomography

An ensemble of bipartite systems, each formed by two qubits, is described by a 4x4 density matrix

$$\rho = \frac{1}{4}\left[\mathbb{1} \otimes \mathbb{1} + \sum_i \mathrm{B}_i^+ \; (\sigma_i \otimes \mathbb{1}) + \sum_j \mathrm{B}_j^- \; (\mathbb{1} \otimes \sigma_j) + \sum_{i,j} \mathrm{C}_{ij} \; (\sigma_i \otimes \sigma_j)\right]$$

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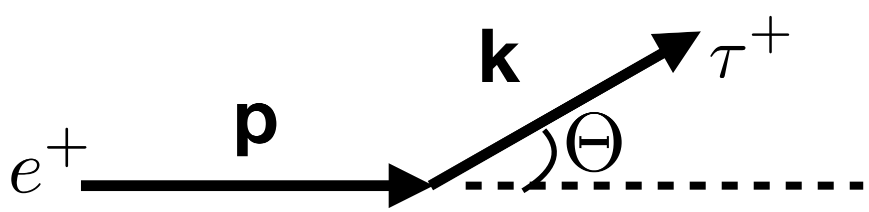
where *i*, *j*, refer to the directions used to define the orientation of the spin vectors in space: the **{n, r, k}** triad defined, in the CoM frame, by



$$\mathbf{n} = \frac{1}{\sin\Theta}(\mathbf{p}\times\mathbf{k}) \qquad \mathbf{r} = \frac{1}{\sin\Theta}(\mathbf{p} - \mathbf{k}\cos\Theta)$$

W. Bernreuther, D. Heisler, Z. Si, JHEP 12 (2015) 026

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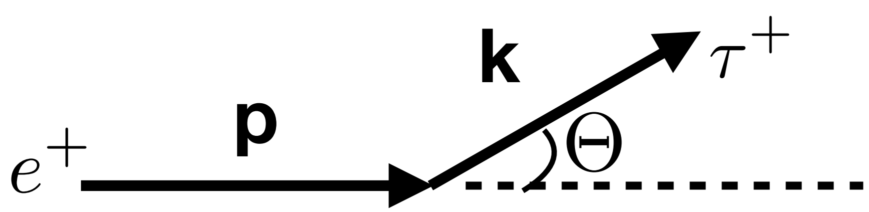

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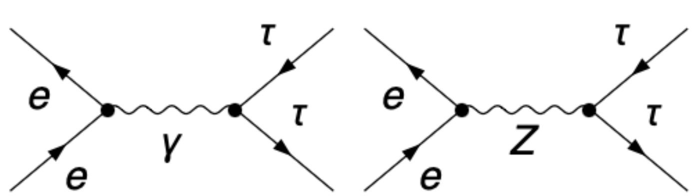
The Fano coefficients $B^\pm$ and C can be computed from the amplitudes of the underlying production process as functions of the kinematic variables



$$\rho = \rho(\Theta, s, \dots) \rightarrow \begin{cases} \mathrm{B}_i^+ = \mathrm{Tr}\left[\rho(\sigma_i\otimes\mathbb{1})\right] \\ \mathrm{B}_i^- = \mathrm{Tr}\left[\rho(\mathbb{1}\otimes\sigma_i)\right] \\ \mathrm{C}_{ij} = \mathrm{Tr}\left[\rho(\sigma_i\otimes\sigma_j)\right] \end{cases}$$

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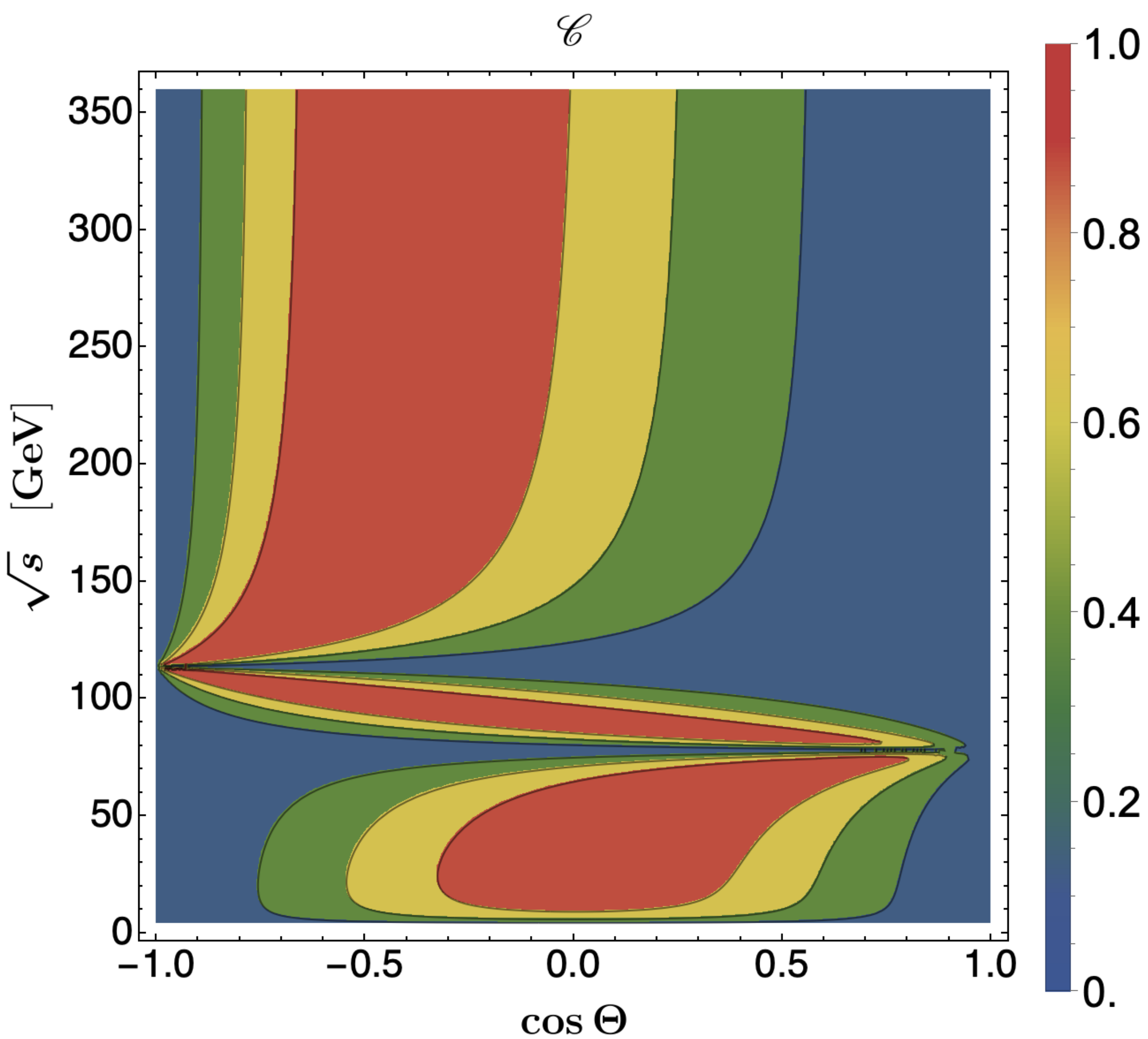
This gives us the prospects for the detection of…

Entanglement ($\mathscr{C} > 0$)



The concurrence 0< $\mathscr{C}$ <1 quantifies the amount of entanglement in the system.

It is computed through the auxiliary matrix

$$R = \rho\,(\sigma_y \otimes \sigma_y)\,\rho^*\,(\sigma_y \otimes \sigma_y)$$

with non-negative eigenvalues $r_1^2 \geq r_2^2 \geq r_3^2 \geq r_4^2$ as:

$$\mathscr{C} = \max\left(0, r_1 - r_2 - r_3 - r_4\right) .$$

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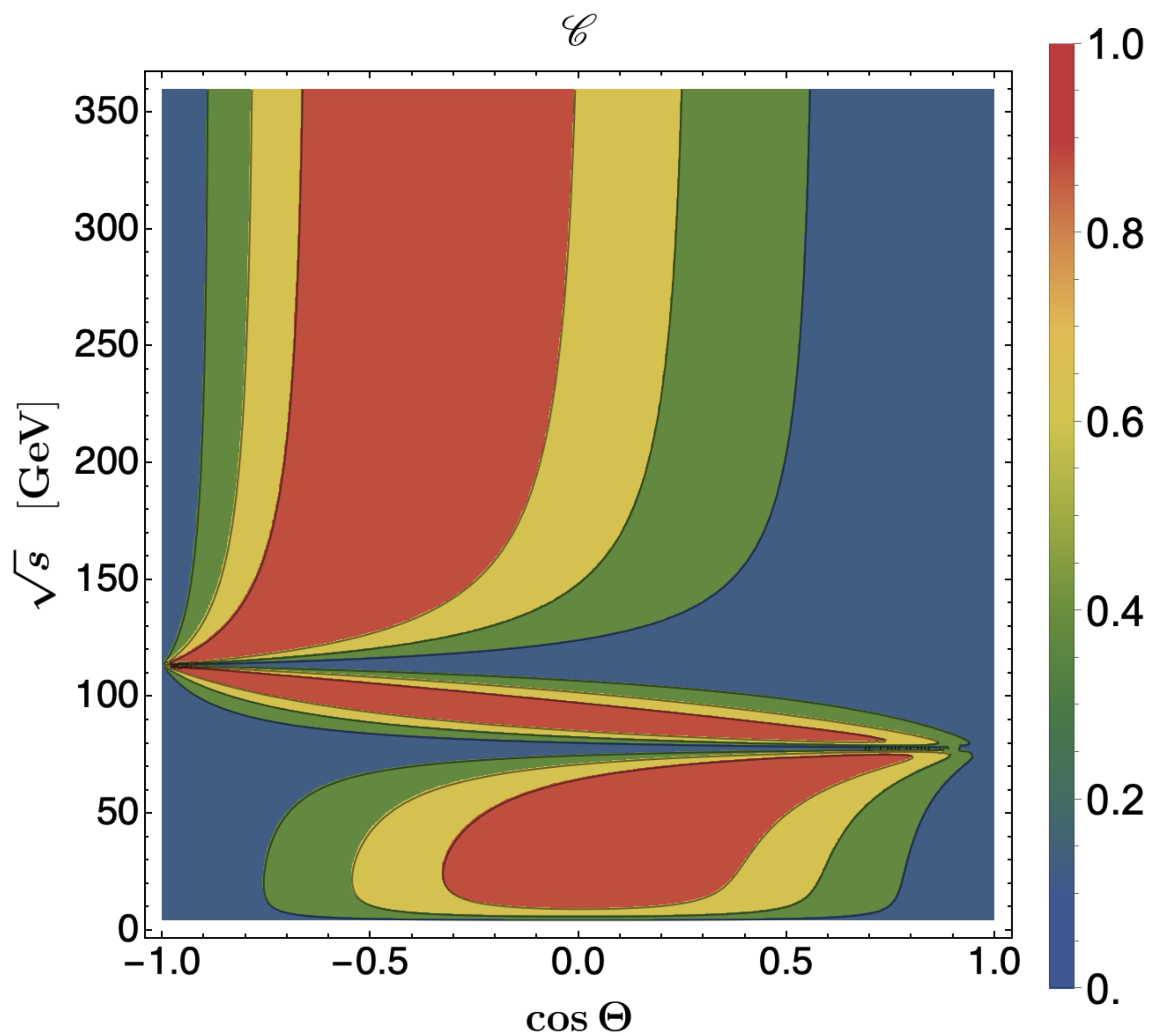

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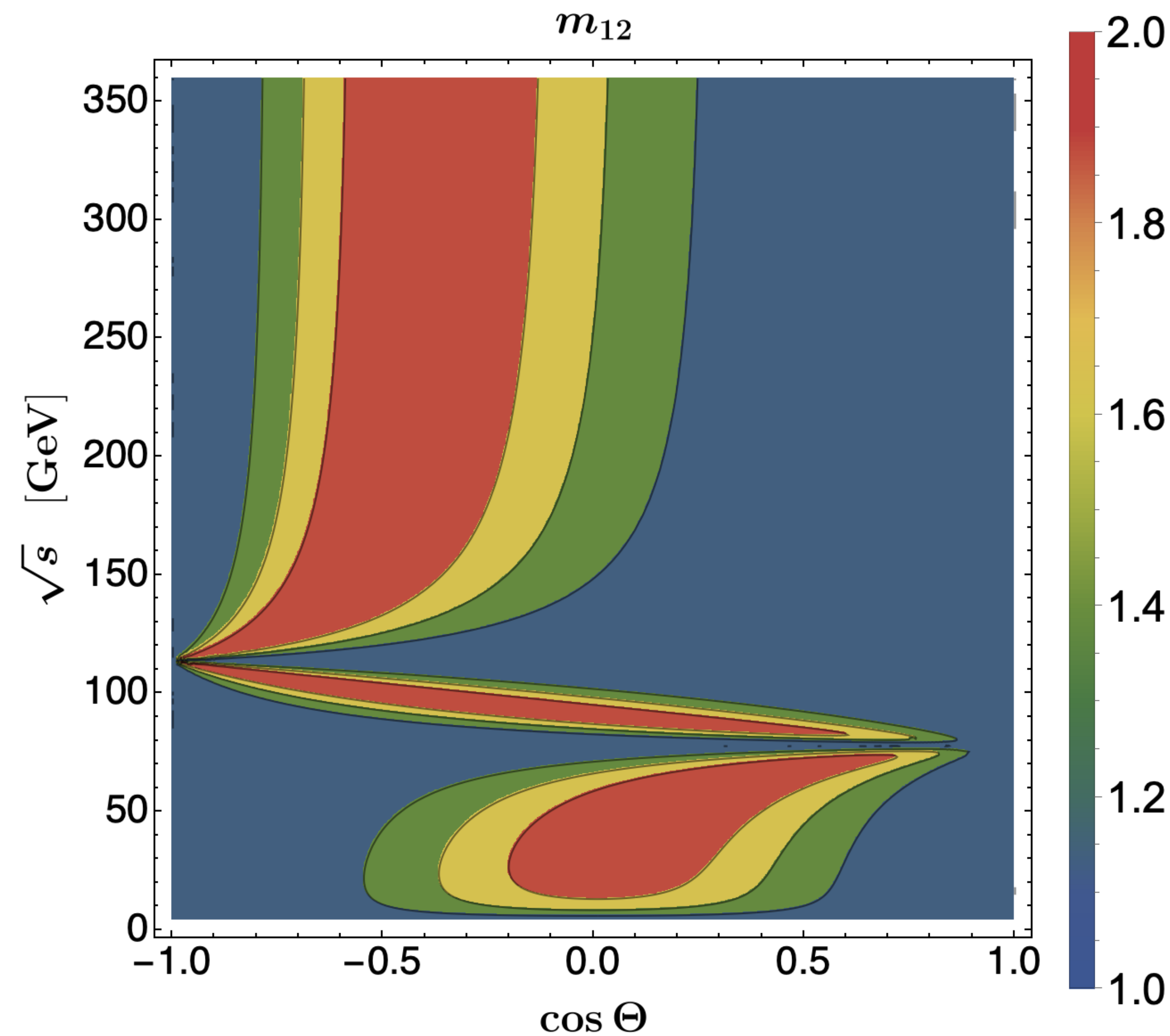
Bell inequality violation ($m_{12} > 1$)



We use the Horodechki condition $m_{12} > 1$, where the parameters is expressed as

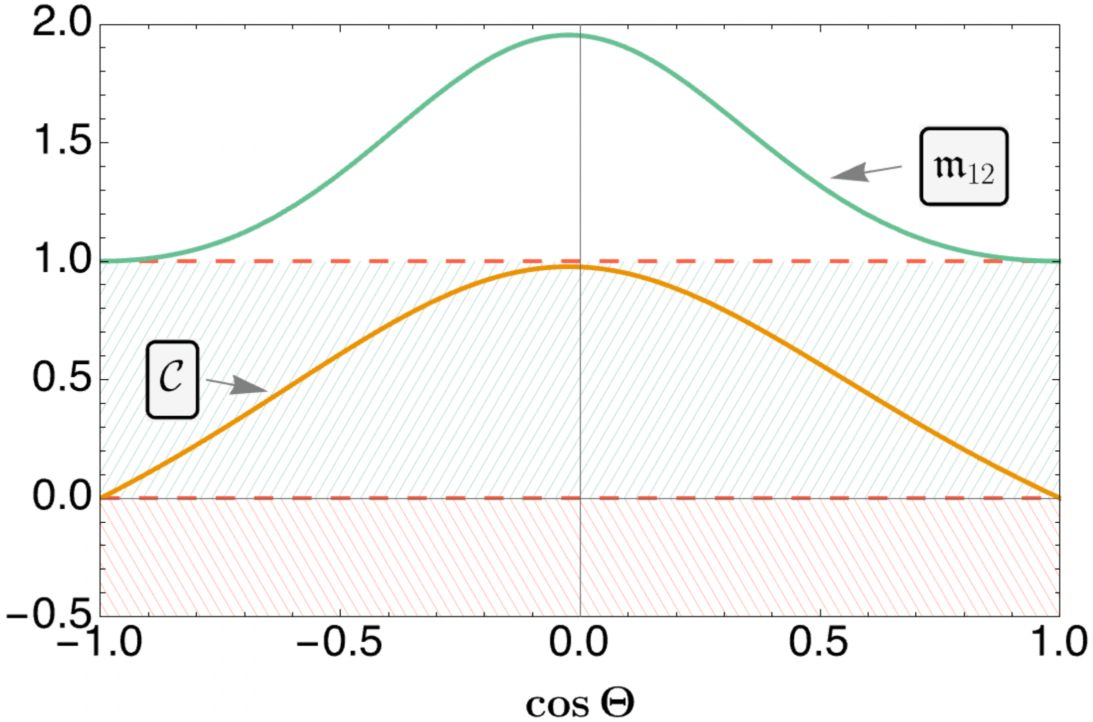
$$\mathfrak{m}_{12} \equiv m_1 + m_2$$

in terms of the eigenvalues

$$m_1 \geq m_2 \geq m_3$$

of the matrix $M = \mathrm{C}^T\,\mathrm{C}$.

Focusing on FCC-ee working at the Z boson resonance:



Averaging the analytical result over the angular distribution of events yields

$$\mathrm{C} = \begin{pmatrix} 0.4878 & 0 & 0 \\ 0 & -0.4878 & 0.0011 \\ 0 & 0.0011 & 1 \end{pmatrix} \qquad \mathrm{B}^+ = \mathrm{B}^- = \begin{pmatrix} 0 \\ 0.0001 \\ 0.2194 \end{pmatrix}$$

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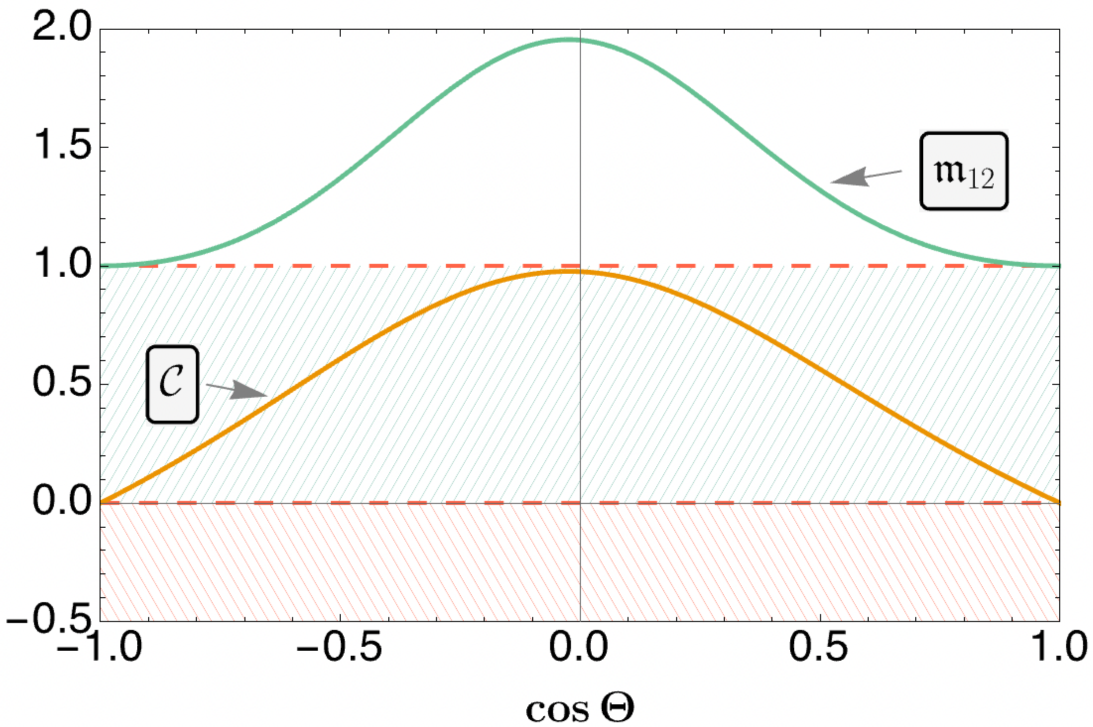

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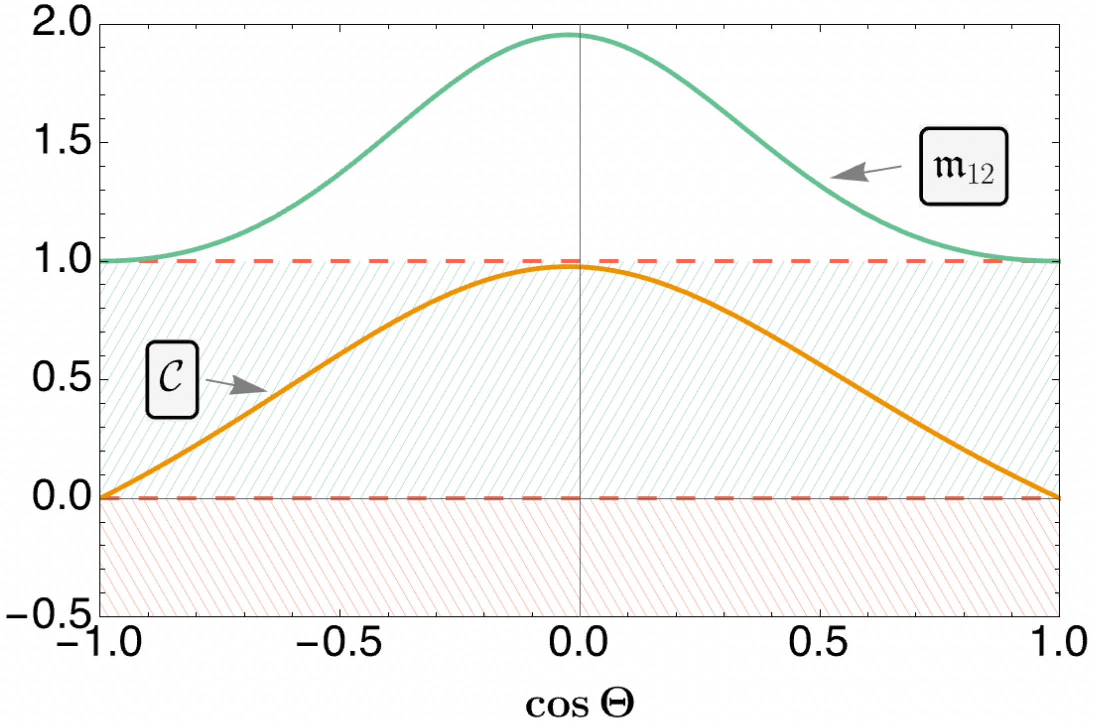

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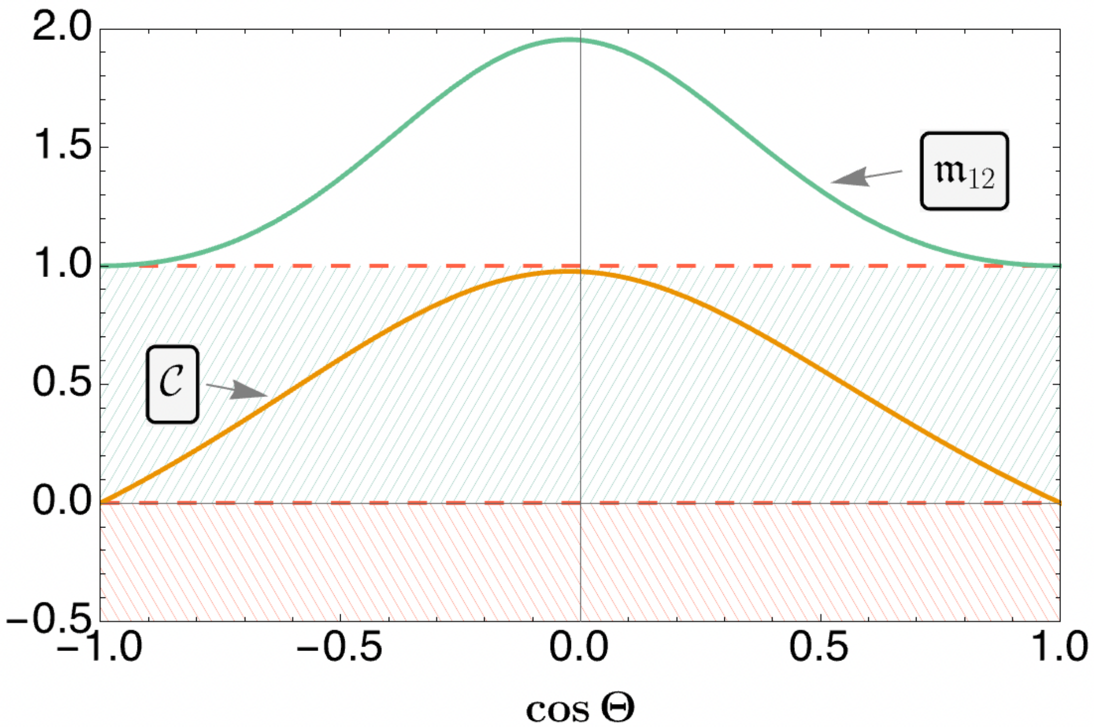

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- Remark III: I am well aware that all of this means nothing as long as I do not show the corresponding uncertainties. To gauge these we resort to a dedicated Monte Carlo analysis.

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- For each pseudo experiment we reconstruct the Fano coefficients, including in the analysis
    - Neutrino and tau momenta reconstruction
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- Statistical errors are estimated from the variance over the 50 pseudo experiments. Systematic errors are computed from the shifts of central values due to different detector settings.

Accessing the density matrix from "data"

The Fano coefficients can be experimentally reconstructed in several ways, for instance by accessing the distributions

$$\frac{1}{\sigma}\frac{\mathrm{d}\sigma}{\mathrm{d}\cos\theta_i^{\pm}} = \frac{1}{2}\left(1 \mp B_i^{\pm}\cos\theta_i^{\pm}\right) \qquad \frac{1}{\sigma}\frac{\mathrm{d}\sigma}{\mathrm{d}\cos\theta_i^{+}\,\mathrm{d}\cos\theta_j^{-}} = \frac{1}{4}\left(1 + C_{ij}\,\cos\theta_i^{+}\,\cos\theta_j^{-}\right)$$

Fano coefficients

where we defined $\cos\theta_i^{\pm} = \vec{n}^{\pm}\cdot\hat{e}_i$, with $\hat{e}_i =$ **n**, **r** or **k** and with $\vec{n}^{\pm}$ being the polarimetric vector for the chosen decay mode (*i.e.* the pion direction as seen in the rest frame of the decaying tau lepton).

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Alternatively, the Fano coefficients can be computed as the averages

$$B_i^{\pm} = \frac{3}{\kappa_{\pm}}\frac{1}{\sigma}\int \mathrm{d}\Omega^{\pm}\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega^{\pm}}(\vec{n}^{\pm}\cdot\hat{e}_i), \qquad C_{ij} = \frac{9}{\kappa_{+}\kappa_{-}}\frac{1}{\sigma}\int \mathrm{d}\Omega^{+}\mathrm{d}\Omega^{-}\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega^{+}\mathrm{d}\Omega^{-}}(\vec{n}^{+}\cdot\hat{e}_i)(\vec{n}^{-}\cdot\hat{e}_j)$$

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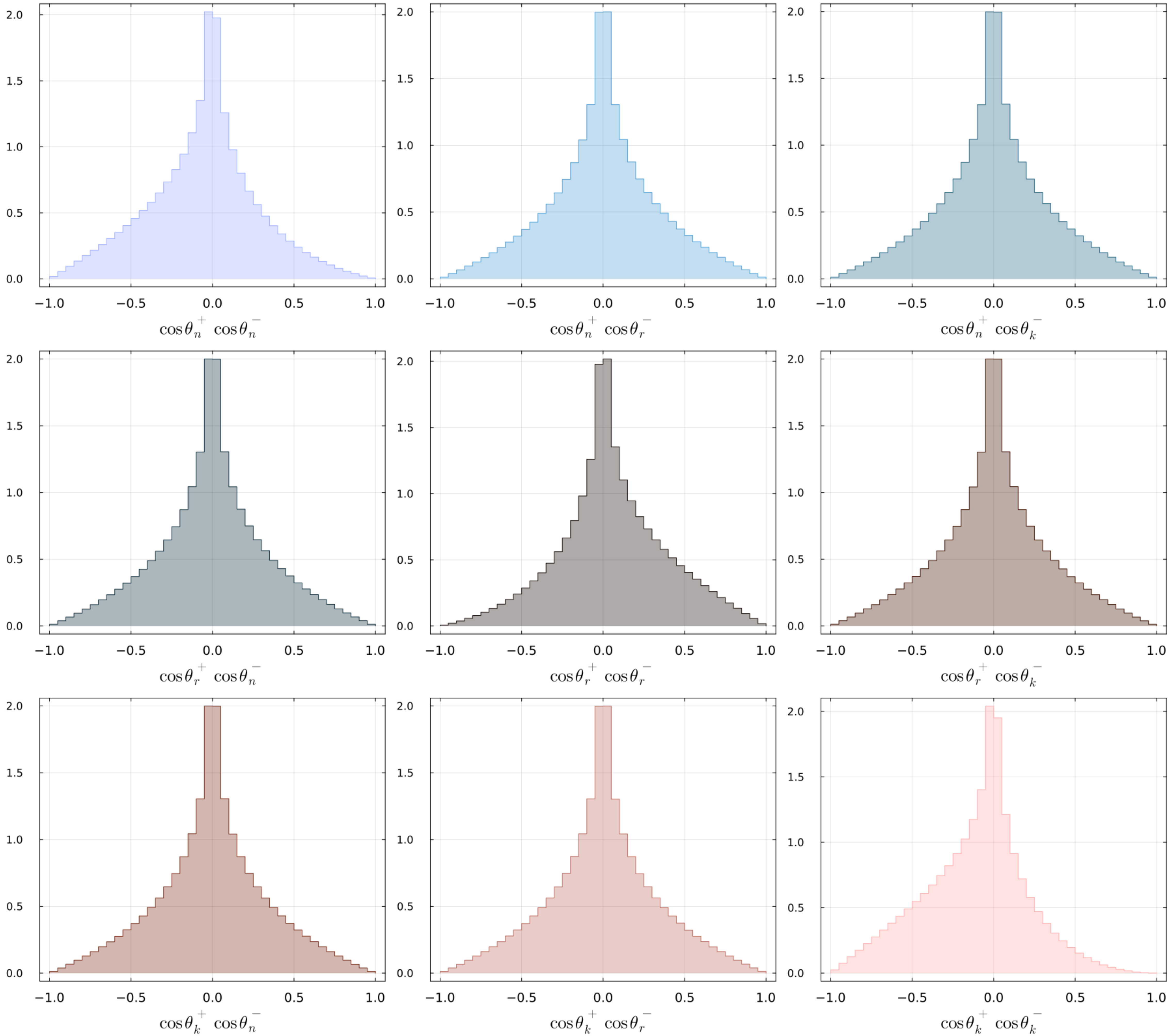
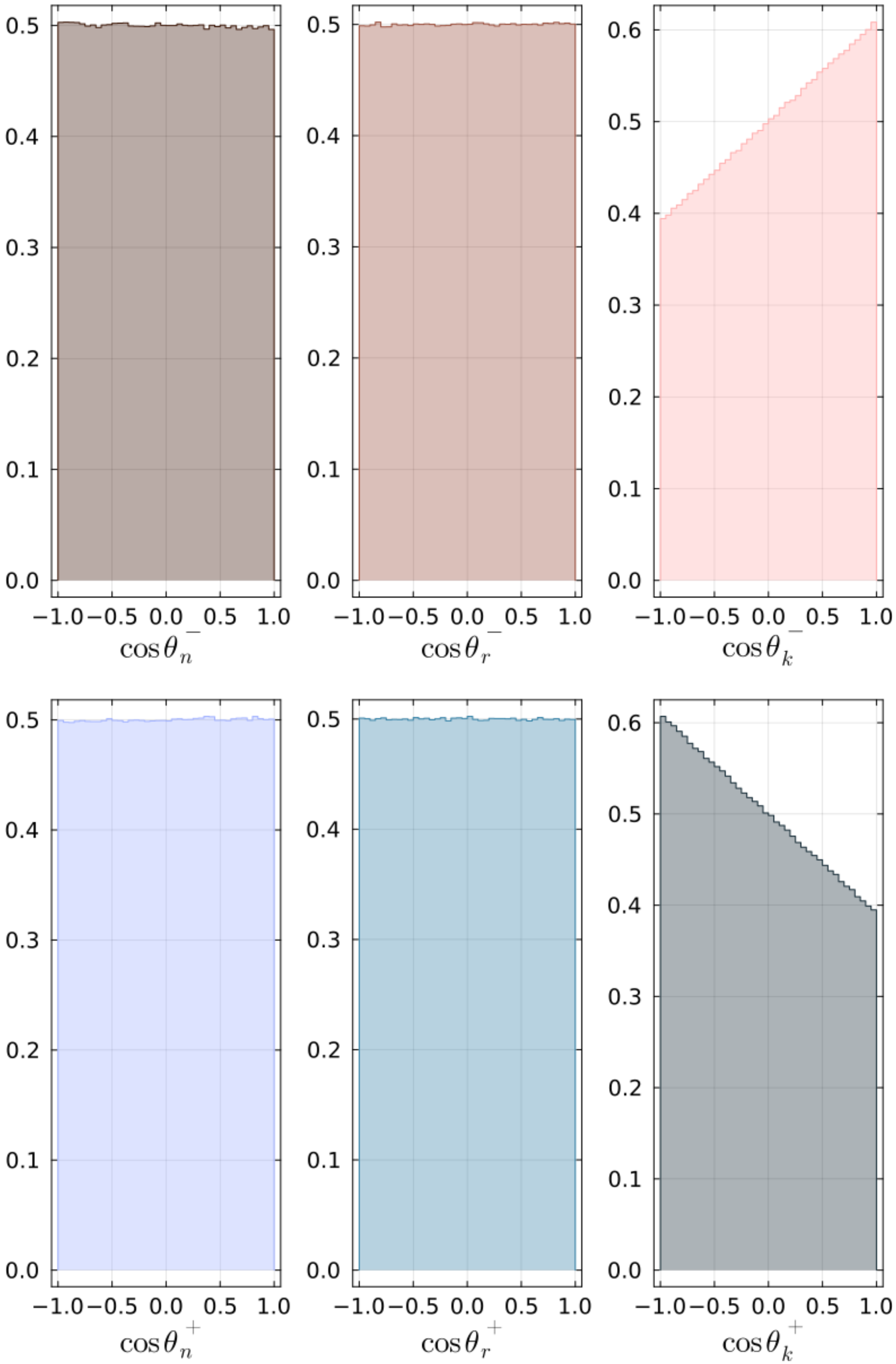
For every simulated event, we boost to the CoM frame (ISR), boost to the τ^+ rest frame and record $\cos\theta_i^{+}$, boost to the τ^- rest frame and record $\cos\theta_i^{-}$. The result is a series of histograms which give us the Fano coefficients.



Y axes: relative frequencies; x axes: values of the products



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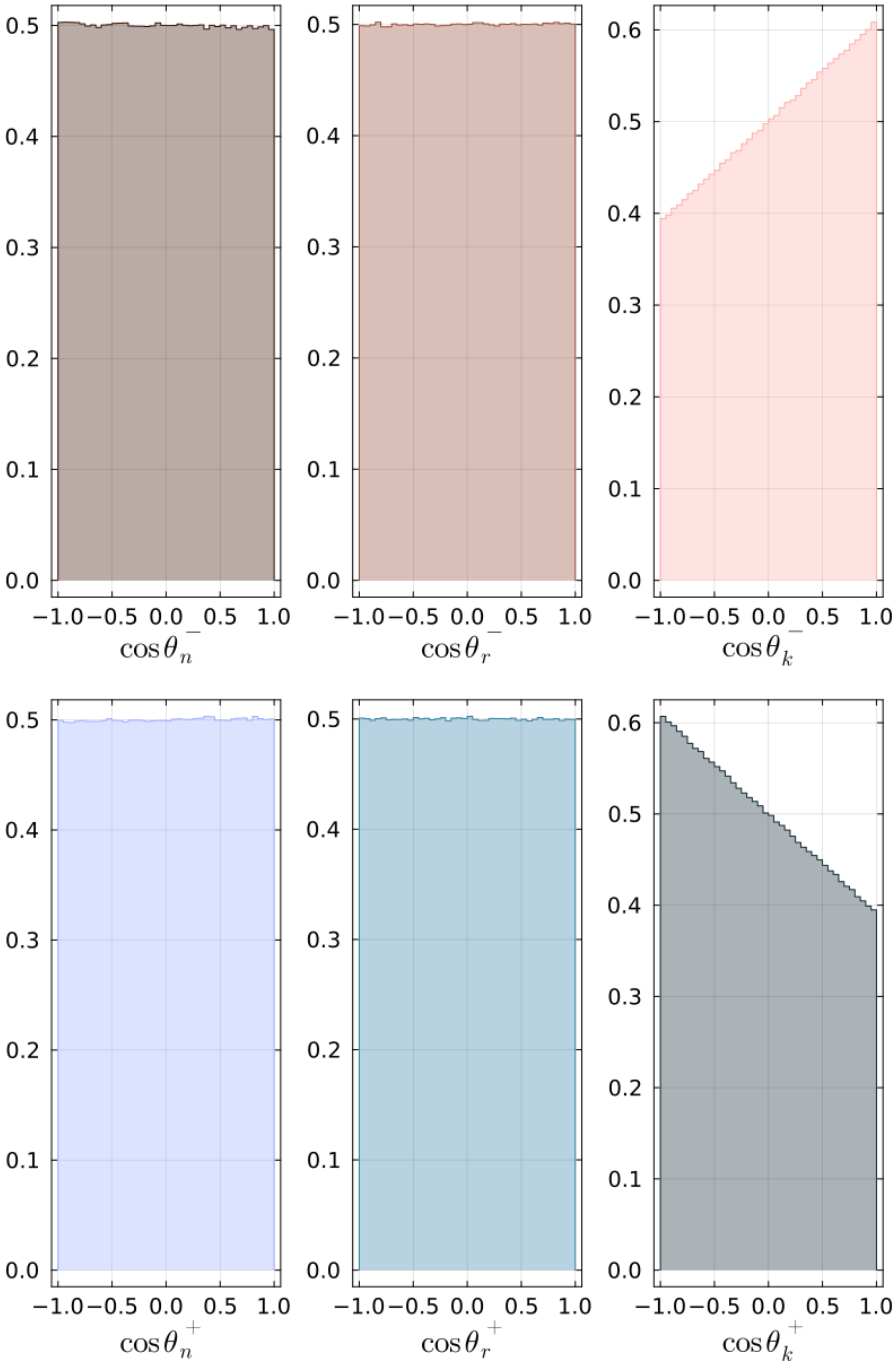

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Including ISR, momenta reconstruction and detector effects we obtain:

$$\mathrm{C} = \begin{pmatrix} 0.4819 \pm 0.0079 & -0.0073 \pm 0.0082 & -0.0016 \pm 0.0089 \\ -0.0066 \pm 0.0082 & -0.4784 \pm 0.0084 & 0.0016 \pm 0.0070 \\ -0.0002 \pm 0.0080 & -0.0004 \pm 0.0087 & 1.000 \pm 0.0074 \end{pmatrix}$$

$$\mathrm{B}^+ = \begin{pmatrix} -0.0028 \pm 0.0042 \\ -0.0001 \pm 0.0049 \\ 0.2198 \pm 0.0044 \end{pmatrix} \qquad \mathrm{B}^- = \begin{pmatrix} -0.0039 \pm 0.0048 \\ 0.0017 \pm 0.0049 \\ 0.2207 \pm 0.0044 \end{pmatrix}$$

well in agreement with the theoretical estimates seen before:

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Quantum information with taus @ FCC-ee

As to the prospects for detecting entanglement and the violation of the Bell inequality at FCC-ee with tau leptons, we find

$$\mathscr{C} = 0.4805 \pm 0.0063|_{\text{stat}} \pm 0.0012|_{\text{syst}}$$

$$\mathfrak{m}_{12} = 1.239 \pm 0.017|_{\text{stat}} \pm 0.008|_{\text{syst}}$$

in line with the given theoretical predictions: $\mathscr{C} = 0.4878$, $\mathfrak{m}_{12} = 1.238$.

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- the quoted systematic uncertainties are computed by evaluating the shift in the values of the observables obtained with and without ISR+detector effects. To this we add a further shift obtained for a different tuning of the detector parameters.

Quantum information observables for HEP

Can entanglement tell us something about new physics? Lets introduce some anomalous couplings for the τ lepton

$$i\,\frac{g}{2\,\cos\theta_W}\,\bar{\tau}\,\Gamma^{\mu}(q^2)\,\tau\,Z_{\mu}(q) = \; i\,\frac{g}{2\,\cos\theta_W}\,\bar{\tau}\left[\gamma^{\mu}F_1^V(q^2) + \gamma^{\mu}\gamma_5 F_1^A(q^2) + \frac{i\sigma^{\mu\nu}q_{\nu}}{2m_{\tau}}F_2(q^2) + \frac{\sigma^{\mu\nu}\gamma_5 q_{\nu}}{2m_{\tau}}F_3(q^2)\right]\tau\,Z_{\mu}(q)$$

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$$F_1^{V,A}(q^2) = F_1^{V,A}(0)+\frac{q^2}{m_Z^2}C_1^{V,A}$$

$$\begin{cases} F_1^V(0) = g_V = -1/2+2\,\sin^2\theta_W \\ F_1^A(0) = -g_A = 1/2 \end{cases}$$

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$$i\,\frac{g}{2\,\cos\theta_W}\,\bar{\tau}\,\Gamma^\mu(q^2)\,\tau\,Z_\mu(q) = i\,\frac{g}{2\,\cos\theta_W}\,\bar{\tau}\left[\gamma^\mu F_1^V(q^2) + \gamma^\mu\gamma_5 F_1^A(q^2) + \frac{i\sigma^{\mu\nu}q_\nu}{2m_\tau}F_2(q^2) + \frac{\sigma^{\mu\nu}\gamma_5 q_\nu}{2m_\tau}F_3(q^2)\right]\tau\,Z_\mu(q)$$

$$F_1^{V,A}(q^2) = F_1^{V,A}(0) + \frac{q^2}{m_Z^2}C_1^{V,A}$$

$$\begin{cases} F_1^V(0) = g_V = -1/2 + 2\,\sin^2\theta_W \\ F_1^A(0) = -g_A = 1/2 \end{cases}$$

Then, we constrain $C_1^{A,V}$, as well as $F_{2,3}(m_Z^2)$, via a χ² test where we vary the parameters one at a time.

$\mathscr{O}_a$	σ_a^I	limits I (L = 17.6 fb^{-1})	σ_a^{II}	limits II (L = 150 ab^{-1})
$\mathscr{C}$	0.006	$-0.002 \le F_2(m_Z^2) \le 0.003$	0.001	$-0.001 \le F_2(m_Z^2) \le 0.001$
$\mathscr{C}_{odd}$	0.009	$-0.001 \le F_3(m_Z^2) \le 0.001$	0.006	$-0.0004 \le F_3(m_Z^2) \le 0.0005$
σ_T	0.05 pb	$-0.009 \le C_1^V \le 0.010$	0.02 pb	$-0.004 \le C_1^V \le 0.004$
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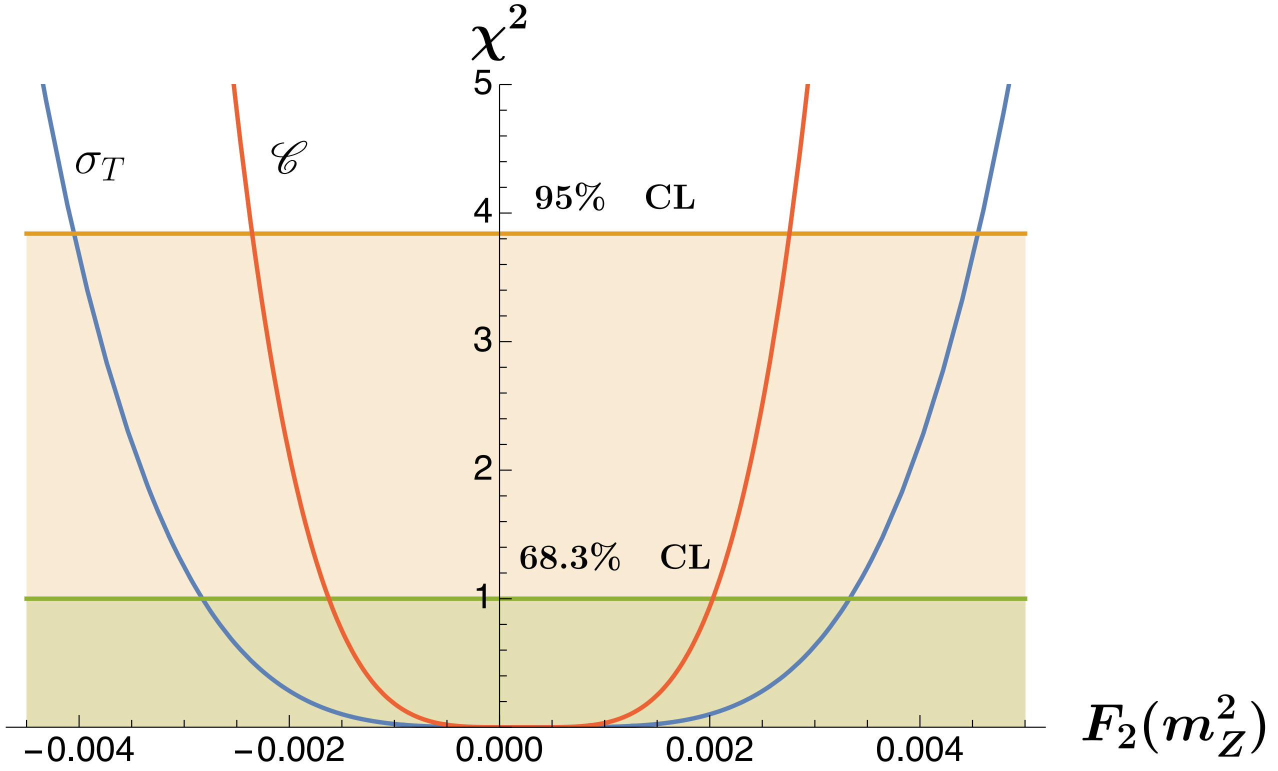
our benchmark

FCC-ee

$$\mathscr{C}_{odd} = \frac{1}{2}\sum_{i<j}|C_{ij} - C_{ji}|$$

By the way, the concurrence is more sensitive than the cross section if the relative uncertainty is the same:



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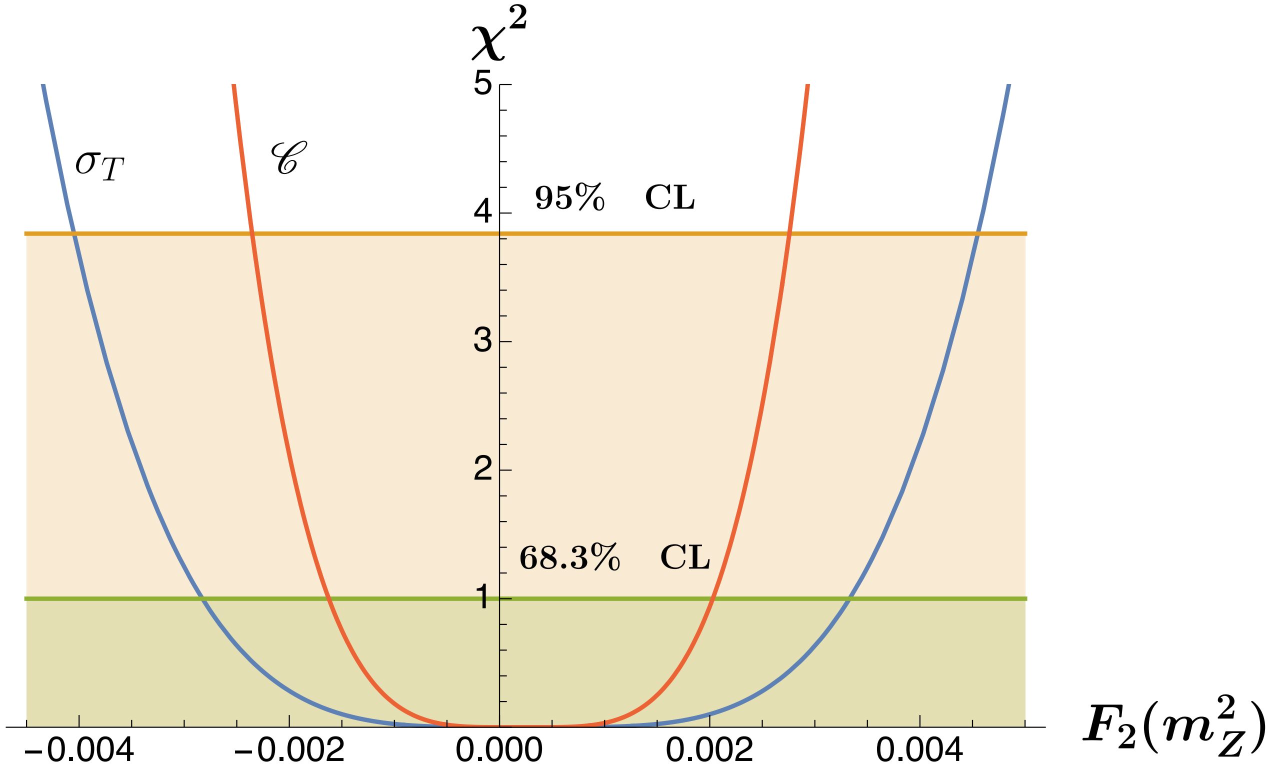

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Nope! Rather than using 'quantum information observables' like entanglement, magic, discord, we can use the density matrix itself. In quantum information theory, the distance between two density matrices is often quantified with the trace distance:

$$\mathscr{D}^T(\rho, \varsigma) = \frac{1}{2}\,\mathrm{Tr}\,\sqrt{(\rho - \varsigma)^\dagger(\rho - \varsigma)} \geq 0$$

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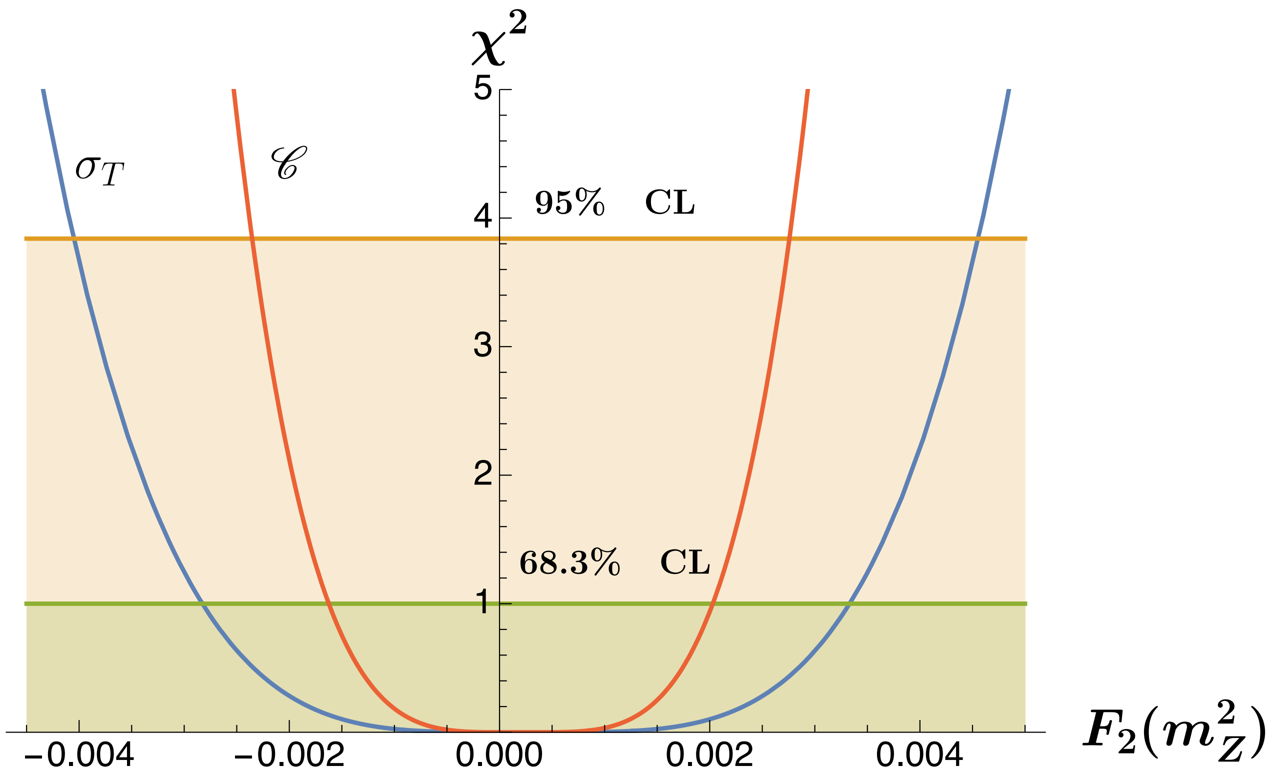

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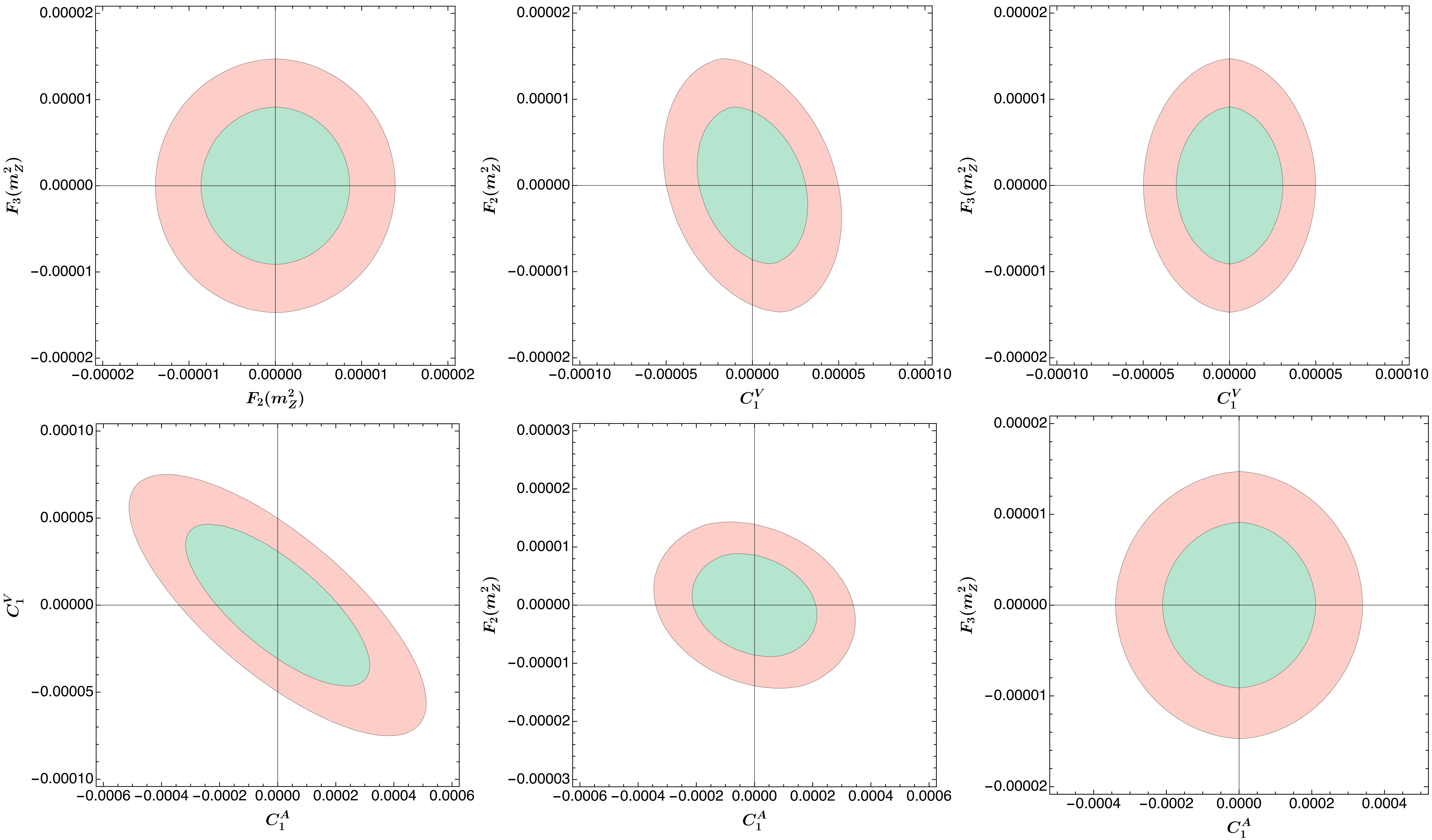
As an example, comparing two qubit $\rho = \frac{1}{2}\left[\mathbb{1} + \vec{r} \cdot \vec{\sigma}\right]$, $\varsigma = \frac{1}{2}\left[\mathbb{1} + \vec{s} \cdot \vec{\sigma}\right]$ gives:

$$\mathscr{D}^T(\rho, \varsigma) = \frac{\|\vec{r} - \vec{s}\|}{2}$$

So, re-doing the analysis using only trace distance and cross section gives:



68% and 95% joint confidence intervals (2 parameters); assuming negligible systematics affecting quantum tomography

Outlook

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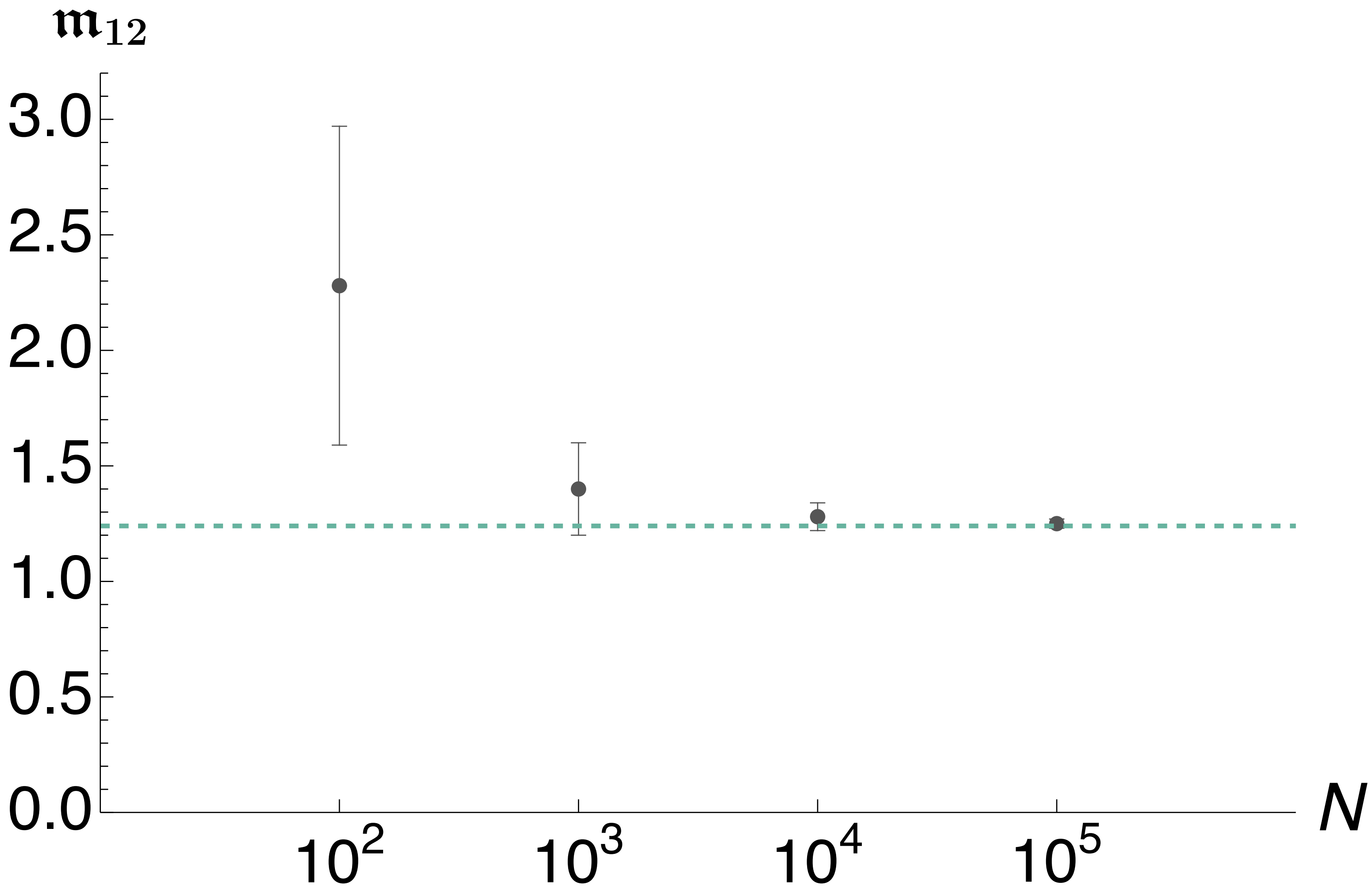
  - Rather than entanglement, magic and other esoteric quantities I'd use trace distance, fidelity and other tools designed to compare quantum states

- Even if "it from bit" were to turn out to be merely an empty (albeit catchy) slogan, could you really find anything cooler to do while running at the *Z* resonance?

Backup

The m_{12} bias

Values of m_{12} and related standard error as a function of the size of the sample used in the Monte Carlo analysis:



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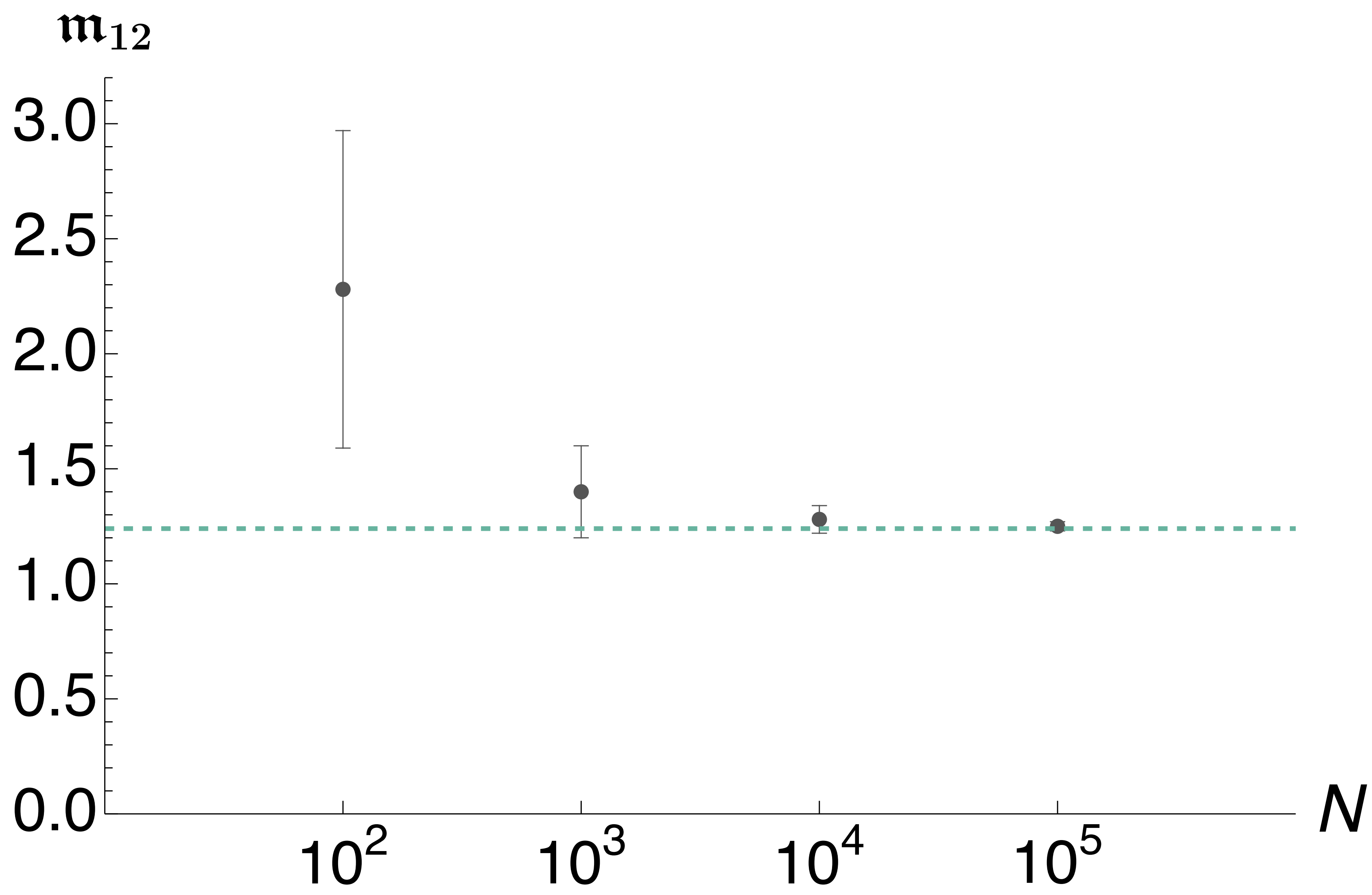
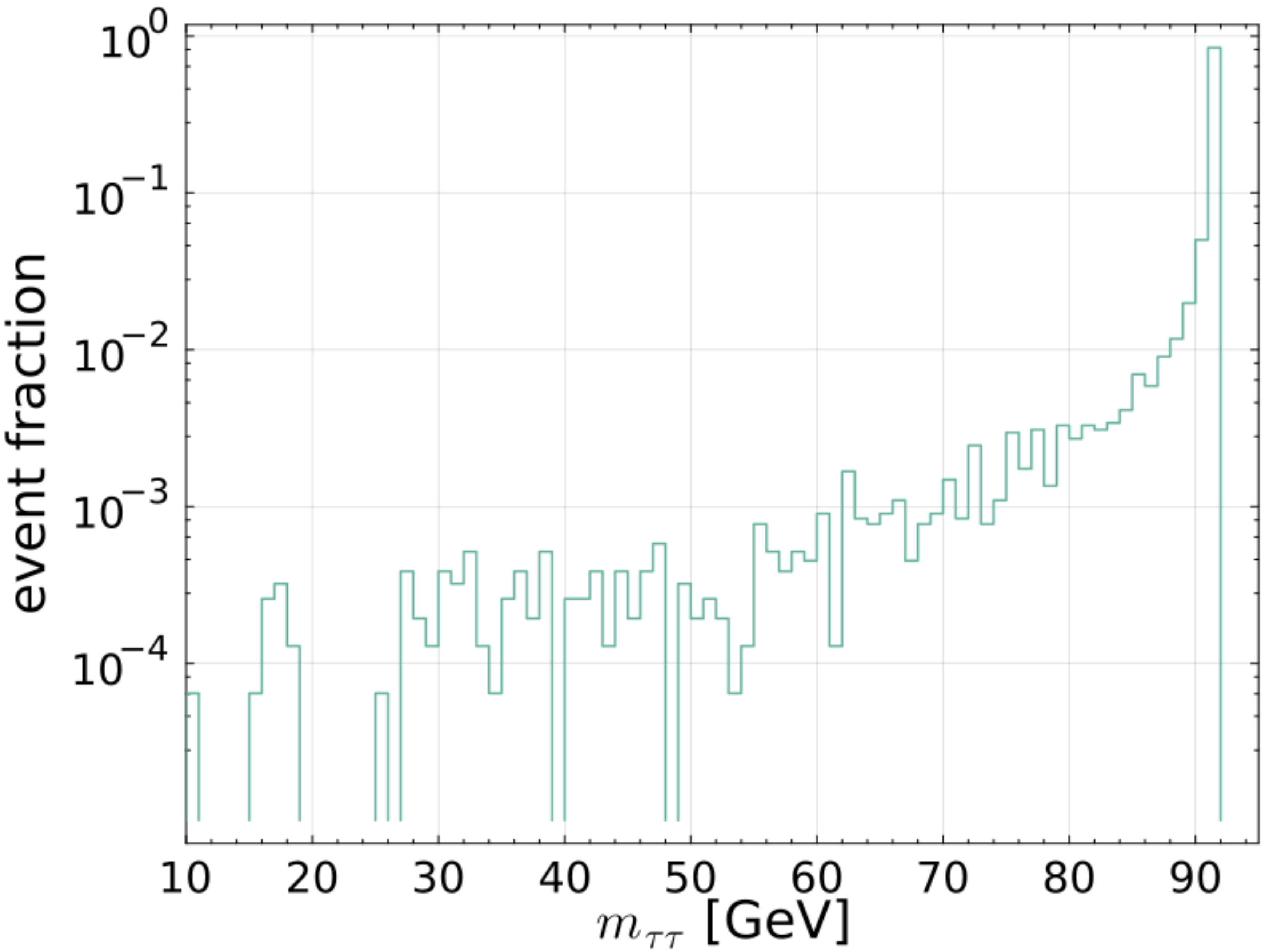


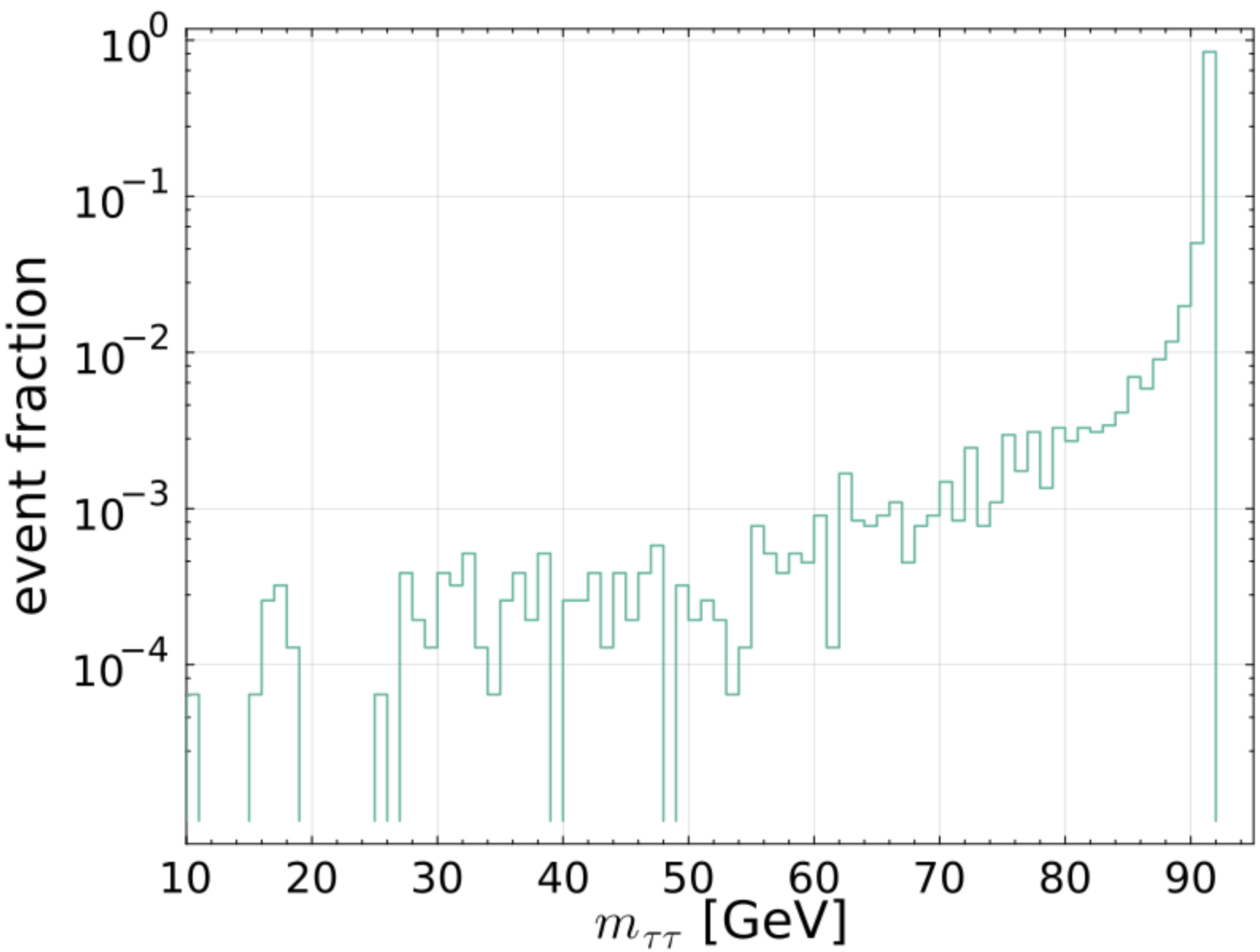
No need to worry about the bias as we use samples of size $N > 10^5$, resulting in a value of m_{12} well compatible with the expected theoretical estimate (the dashed green line).

Modeling the ISR



Modeling the ISR

To model the effect of ISR we pollute our dataset with events characterized by lower CoM energy down to 89 GeV, using the relative weights indicated by the plot below obtained with Pythia 8.



Modeling the detector effects and systematic errors

To simulate the detector we apply a gaussian smearing to the pion momenta and tracks using two settings:

momenta	tracks	impact parameter
$\frac{\sigma_{p_T}}{p_T} = 3 \times 10^{-5} \oplus 0.3 \times 10^{-3} \frac{p_T}{\text{GeV}}$	$\sigma_{\theta,\phi} = 0.1 \times 10^{-3}\,\text{rad}$	$\sigma_b = 3\,\mu\text{m} \oplus \frac{15\,\mu\text{m}}{\sin^{2/3}\Theta}\frac{\text{GeV}}{p_T}$
$\frac{\sigma'_{p_T}}{p_T} = 3 \times 10^{-5} \oplus 0.6 \times 10^{-3} \frac{p_T}{\text{GeV}}$	$\sigma_{\theta,\phi} = 0.1 \times 10^{-3}\,\text{rad}$	$\sigma'_b = 5\,\mu\text{m} \oplus \frac{15\,\mu\text{m}}{\sin^{2/3}\Theta}\frac{\text{GeV}}{p_T}$

FCC Collaboration, A. Abada et al., FCC-ee: The Lepton Collider: Future Circular Collider Conceptual Design Report Volume 2, Eur. Phys. J. ST 228 (2019).

P. Azzi and E. Perez, Exploring requirements and detector solutions for FCC-ee, Eur. Phys. J. Plus 136 (2021).

We use the difference in the results obtained with the two sets to estimate the systematic error.

Momenta reconstruction

The 8 components of neutrino momenta are reconstructed via the following constraints

$$p^{\mu}_{\tau^+} + p^{\mu}_{\tau^-} = p^{\mu}_{e^+e^-} \qquad (p_{\tau^+} - p_{\pi^+})^2 = m_{\nu}^2 = 0 \qquad (p_{\tau^-} - p_{\pi^-})^2 = m_{\nu}^2 = 0$$

$$p^2_{\tau^+} = m^2_{\tau} \qquad p^2_{\tau^-} = m^2_{\tau}$$

yielding two possible solutions. We break the degeneracy by computing the vector of closest approach for both the solutions

direction of the π⁻

$$\mathbf{d}_{min} = \mathbf{d} + \frac{[(\mathbf{d}\cdot\mathbf{n}_+)(\mathbf{n}_-\cdot\mathbf{n}_+) - \mathbf{d}\cdot\mathbf{n}_-]\,\mathbf{n}_- + [(\mathbf{d}\cdot\mathbf{n}_-)(\mathbf{n}_-\cdot\mathbf{n}_+) - \mathbf{d}\cdot\mathbf{n}_+]\,\mathbf{n}_+}{1-(\mathbf{n}_-\cdot\mathbf{n}_+)^2}$$

π⁻ decay vertex

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$$\mathbf{d} = \mathbf{v}_+ - \mathbf{v}_-$$

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and by comparing them with the “measured” one.

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direction of the π^+

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Mathematically, *it follows from the postulates of quantum mechanics and from the superposition principle.* Take a bipartite system formed by A and B

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Physically, *entanglement is the hallmark of quantum mechanics* as classical configurations are described by product states.

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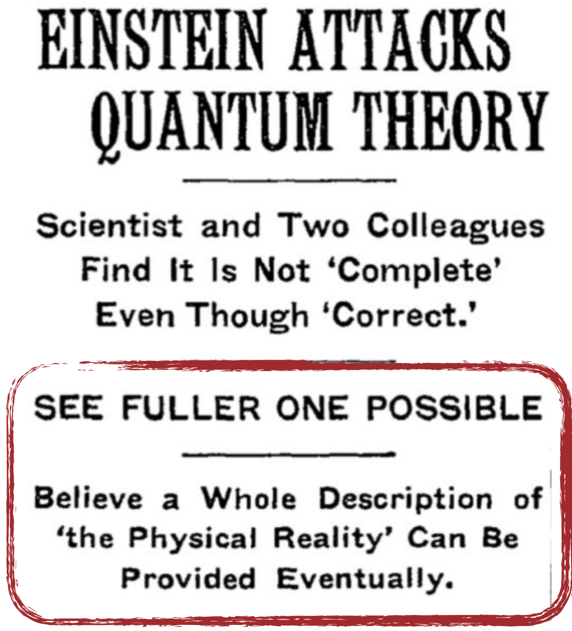

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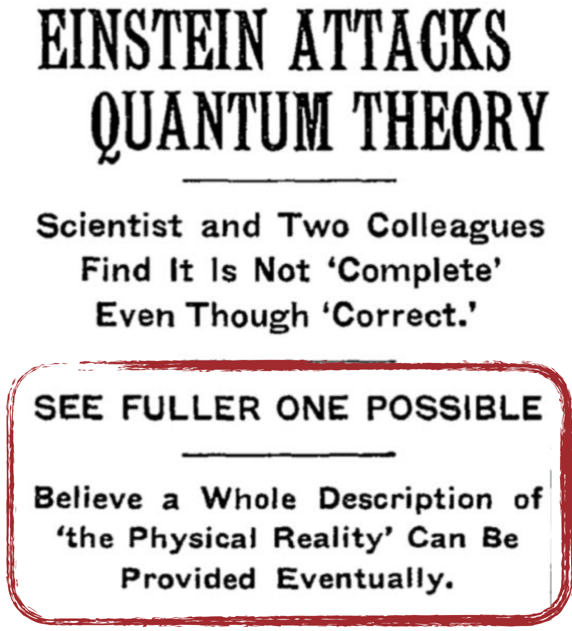
- *Realism*: The Born rule arises from unknown hidden variable λ; *everything is deterministic—no collapse!*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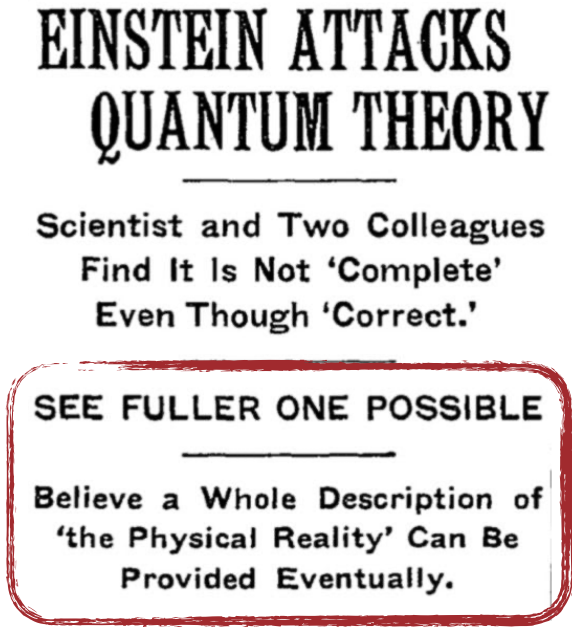

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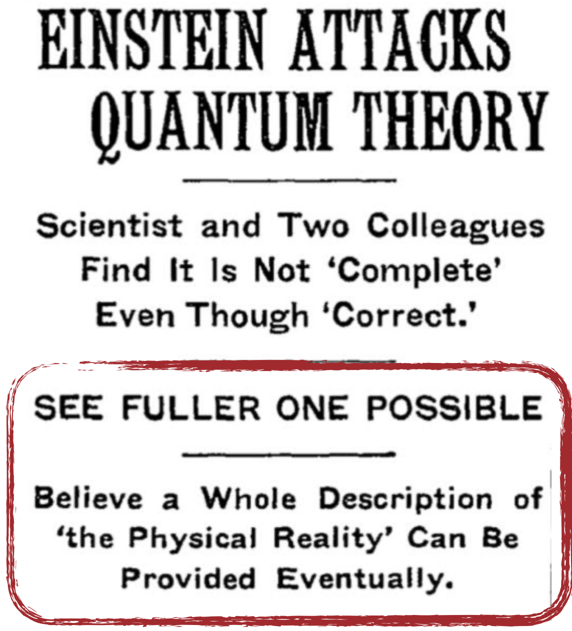
Two *independent observers (A, B)* have, each, *two observables* at their disposal ($\hat{A}_1, \hat{A}_2$ and $\hat{B}_1, \hat{B}_2$) all with possible outcomes 0 or 1. They test a *bipartite system* and look at the combination of expectation values (i.e. combination of average probabilities) given by (CHSH version)

$$\mathcal{I}_2 = \langle \hat{A}_1 \hat{B}_1 \rangle + \langle \hat{A}_1 \hat{B}_2 \rangle + \langle \hat{A}_2 \hat{B}_1 \rangle - \langle \hat{A}_2 \hat{B}_2 \rangle$$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So, is *quantum mechanics incomplete*?

This was the question until 1964, when J. Bell identified an objective way to distinguish between the two frameworks.

Two *independent observers (A, B)* have, each, *two observables* at their disposal ($\hat{A}_1, \hat{A}_2$ and $\hat{B}_1, \hat{B}_2$) all with possible outcomes 0 or 1. They test a *bipartite system* and look at the combination of expectation values (i.e. combination of average probabilities) given by (CHSH version)

$$\mathcal{I}_2 = \langle \hat{A}_1 \hat{B}_1 \rangle + \langle \hat{A}_1 \hat{B}_2 \rangle + \langle \hat{A}_2 \hat{B}_1 \rangle - \langle \hat{A}_2 \hat{B}_2 \rangle$$

Theorem (*Bell*): *if locality and realism hold, then* $\mathcal{I}_2 \leq 2$.

- When we compute the same quantity with the rules of *quantum mechanics we obtain* $\mathcal{I}_2 \leq 2\sqrt{2}$, hence *measuring* $2 < \mathcal{I}_2 \leq 2\sqrt{2}$ *would strongly favor quantum mechanics over hidden-variable theories.*