Z/W Resonance Schemes @ Lepton Colliders

Ui Min

Zhuoni Qian

Sháo-Fēng Gě

葛韶锋

gesf@sjtu.edu.cn

SFG, Ui Min, Zhuoni Qian [2410.15890 [hep-ph]]

上海交通大學 SHANGHAI JIAO TONG UNIVERSITY

Jan 14, 2025 @ FCC Workshop

李政道研究所 Tsung-Dao Lee Institute

Outline

Tsung-Dao Lee Institute

1) Breit-Wigner vs Energy-Dependent Schemes

Z Lineshape Scan

Forward-Backward Asymmetries

2) Breit-Wigner vs Theoretical Schemes

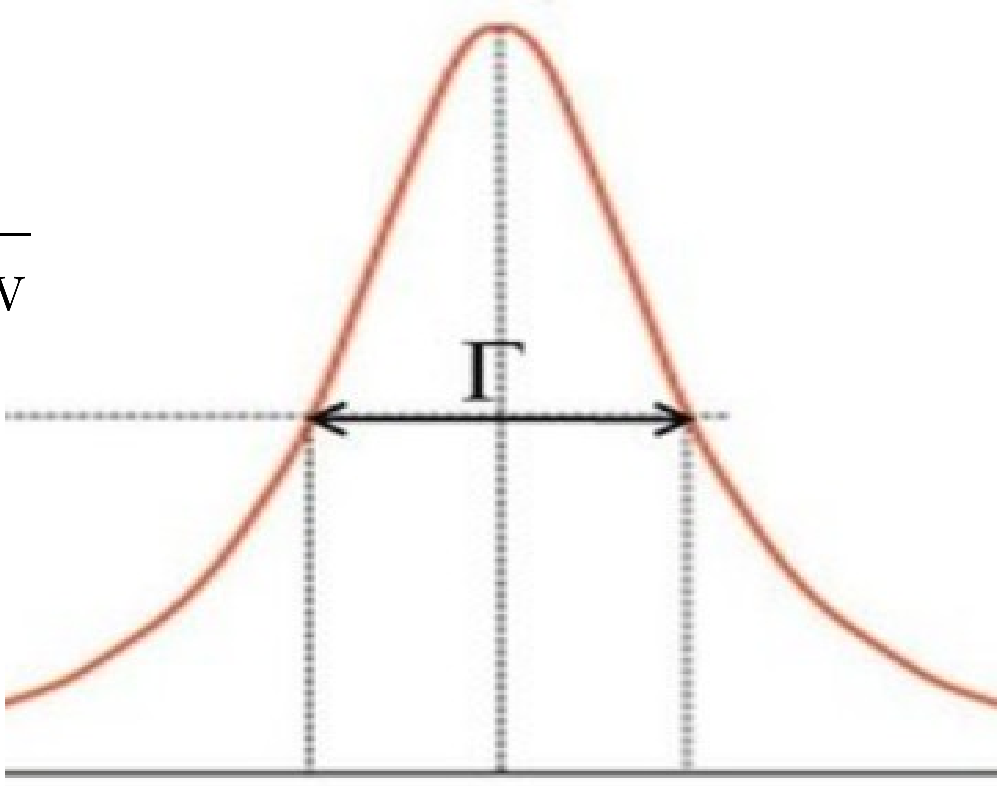
Z Lineshape Scan

WW Threshold Scan + Fermi Constant

3) Summary

Breit-Wigner Resonance

$$\mathcal{D}_{\rm BW} \sim \frac{1}{p^2 - m_{\rm BW}^2 + i m_{\rm BW} \Gamma_{\rm BW}}$$



$$|\mathcal{D}_{\rm BW}|^2 \sim \frac{1}{(p^2 - m_{\rm BW}^2)^2 + m_{\rm BW}^2 \Gamma_{\rm BW}^2}$$

Resonance peak is at exactly the mass m_{BW}

Energy-Dependent Scheme

Tsung-Dao Lee Institute

Decay width is defined as

$$\Gamma \equiv \frac{1}{2m} \int |\mathcal{M}|^2 d\Omega \quad \text{where} \quad \int |\mathcal{M}|^2 d\Omega \propto p^2$$

$$m_{\text{parent}} \gg m_{\text{daughters}}$$

On-shell: $p^2 = m^2$ ⇒ $\Gamma \propto m$

Decay width scales with virtuality p^2 !

$$\Gamma(p^2) \equiv \frac{p^2}{m_{\text{ED}}^2} \Gamma_{\text{ED}} \quad \Rightarrow \quad \mathcal{D}_{\text{ED}} \sim \frac{1}{p^2 - m_{\text{ED}}^2 + i \frac{p^2 \Gamma_{\text{ED}}}{m_{\text{ED}}}}$$

Berends, Burgers, Hollik & van Neerven [PLB88]
Bardin, Leike, Riemann & Sachwitz [PLB88]
Bardin, Bilenky, Mitselmakher, Riemann & Sachwitz [ZPC89]

Breit-Wigner vs Energy-Dependent

$$\mathcal{D}_{\rm ED} \sim \frac{1}{p^2 - m^2_{\rm ED} + i\frac{p^2 \Gamma_{\rm ED}}{m_{\rm ED}}}$$

$$\mathcal{Z} \equiv \frac{1}{1 + \frac{\Gamma^2_{\rm ED}}{m^2_{\rm ED}}}$$

$$|\mathcal{D}_{\rm ED}|^2 \sim \frac{\mathcal{Z}}{(p^2 - \mathcal{Z} m^2_{\rm ED})^2 + \mathcal{Z}^2 m^2_{\rm ED} \Gamma^2_{\rm ED}}$$

$$|\mathcal{D}_{\rm BW}| \sim \frac{1}{(p^2 - m^2_{\rm BW})^2 + m^2_{\rm BW} \Gamma^2_{\rm BW}}$$

$$m_{\rm BW} = \sqrt{\mathcal{Z}} m_{\rm ED}$$

$$\Gamma_{\rm BW} = \sqrt{\mathcal{Z}} \Gamma_{\rm ED}$$

$$g_{\rm BW} = \mathcal{Z}^{1/4} g_{\rm ED}$$

$$1 - \sqrt{\mathcal{Z}} \sim 3.7 \times 10^{-4}$$

For both Z & W

Equivalent!

How to distinguish them?

Z-γ Interference Term

$$|\mathcal{M}_Z|^2 \sim \; |\mathcal{D}_{\rm ED}|^2 \sim \frac{\mathcal{Z}}{(p^2 - \mathcal{Z} m_{\rm ED}^2)^2 + \mathcal{Z}^2 m_{\rm ED}^2 \Gamma_{\rm ED}^2}$$

Equivalence happens for the Z mediated contribution!

$$(\mathcal{M}_Z \mathcal{M}_\gamma^*)_{\rm BW} \propto \frac{g_{Z,\rm BW}^2 e^2 (s - m_{\rm BW}^2)}{[(s - m_{\rm BW}^2)^2 + m_{\rm BW}^2 \Gamma_{\rm BW}^2]\, s}$$

$$(\mathcal{M}_Z \mathcal{M}_\gamma^*)_{\rm ED} \propto \frac{\mathcal{Z} g_{Z,\rm ED}^2 e^2 (s - m_{\rm ED}^2)}{[(s - \mathcal{Z} m_{\rm ED}^2)^2 + \mathcal{Z}^2 m_{\rm ED}^2 \Gamma_{\rm ED}^2]\, s}$$

No simultaneous equivalence for the interference term!

Z-γ Interference Term

$$\sigma_{Z\gamma} = \frac{\alpha Q_e Q_f}{6} \frac{(g_{eL} + g_{eR})(g_{fL} + g_{fR})(s - m_Z^2)}{(s - m_Z^2)^2 + \Gamma_Z^2 m_Z^2}$$

$$g_{\ell L} = -\frac{1}{2} + s_w^2$$

$$g_{\ell R} = s_w^2$$

$$\Rightarrow \quad g_{\ell L} + g_{\ell R} = -\frac{1}{2} + 2 s_w^2 \sim \mathcal{O}(1\%)$$

The interference term is unfortunately suppressed!

Forward-Backward Asymmetry

李政道研究所 Tsung-Dao Lee Institute

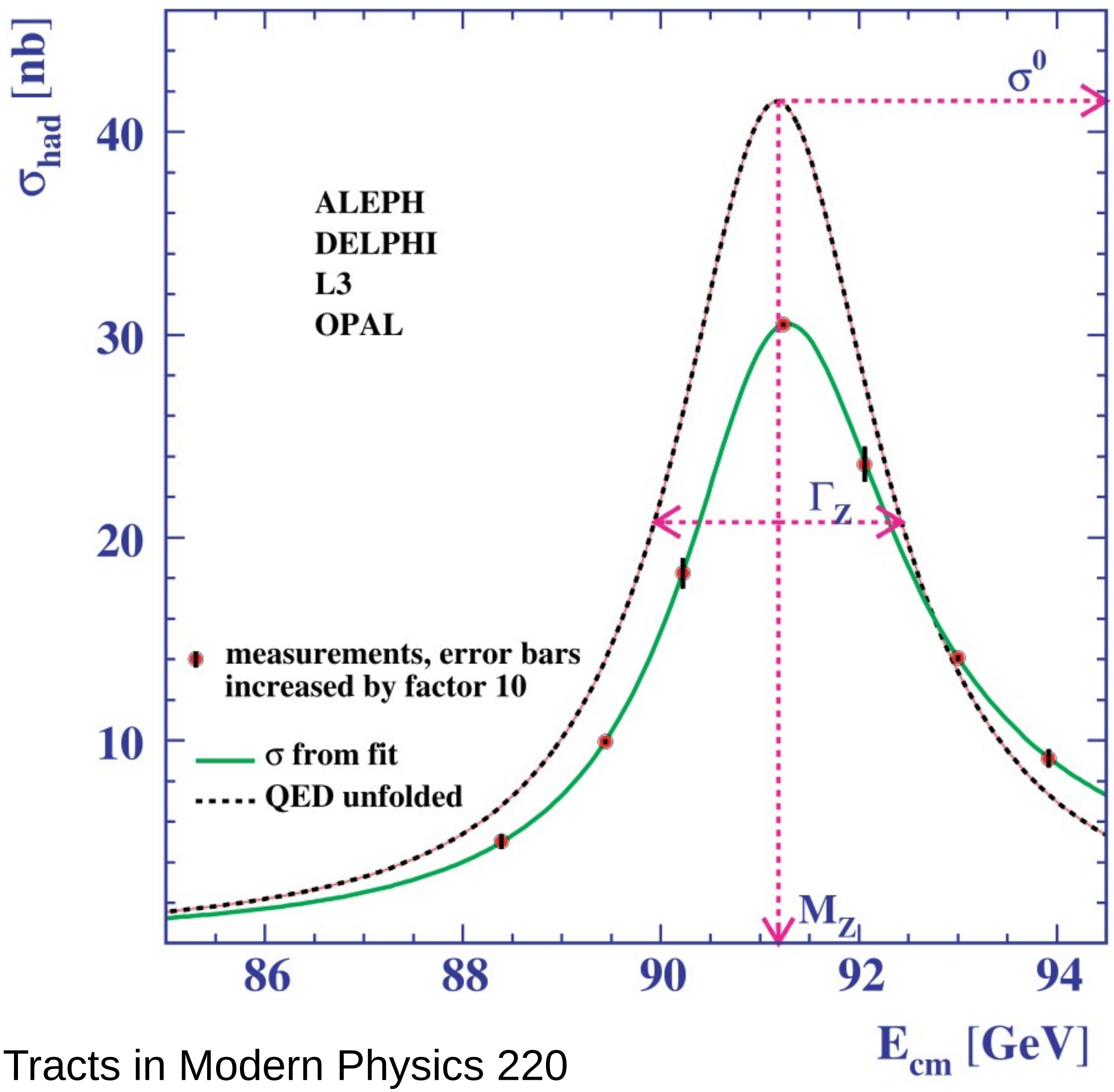
$$\sigma_{Z\gamma} = \frac{\alpha Q_e Q_f}{6} \frac{(g_{eL} + g_{eR})(g_{fL} + g_{fR})(s - m_Z^2)}{(s - m_Z^2)^2 + \Gamma_Z^2 m_Z^2}$$

$$\sigma_F - \sigma_B \propto \frac{3s}{64\pi}(g_{eL}^2 - g_{eR}^2)(g_{fL}^2 - g_{fR}^2)$$

$$+\frac{3 Q_e Q_f \alpha (s - M_Z^2)}{8} (g_{eL} - g_{eR}) (g_{fL} - g_{fR})$$

The interference term is enhanced!

Z Lineshape Scan



Roth, Springer Tracts in Modern Physics 220

Z Lineshape Scan + A_{FB}

Tsung-Dao Lee Institute

$$\sigma_{\rm had} \equiv \sum_q \sigma_F^q + \sigma_B^q \qquad A_{FB}^f \equiv \frac{\sigma_F^f - \sigma_B^f}{\sigma_F^f + \sigma_B^f}$$

Required precision: $\Delta m_Z \lesssim 34\,\mathrm{MeV}$ $\quad \Delta\Gamma_Z \lesssim 0.9\,\mathrm{MeV}$

Need both of them simultaneously!

$\Delta\chi^2_{\rm min}$	ALEPH	DELPHI	L3	OPAL	Combined
$\sigma_{\rm had}$	0.106	0.0083	0.0127	0.0529	0.183
$\sigma_{\rm had} + A_{FB}^{\mu}$	8.34	10.1	5.70	14.1	37.9
$\sigma_{\rm had} + A_{FB}^{b,c}$	1.19	0.878	0.010*	0.734	2.97
$\sigma_{\rm had} + A_{FB}^{\mu,b,c}$	8.42	9.67	5.58*	13.3	37.0

CEPC: **490** ($\sigma_{\rm had} + A^{\mu}$), **$5.1\times10^5$** ($\sigma_{\rm had} + A^{\mu} + A^{q}$)

Outline

Tsung-Dao Lee Institute

1) Breit-Wigner vs Energy-Dependent Schemes

Z Lineshape Scan

Forward-Backward Asymmetries

2) Breit-Wigner vs Theoretical Schemes

Z Lineshape Scan

WW Threshold Scan + Fermi Constant

3) Summary

Quantum Description

Tsung-Dao Lee Institute

A fundamental particle is described by wave function

Schodinger Equation

$$\psi \longrightarrow e^{-iEt}\psi$$

This form is Intrinsically for stable particles!

How about unstable particles?

$$E \longrightarrow E - \frac{i}{2}\Gamma$$

Then the wave function decays with time

$$\psi(t) \propto e^{-\frac{1}{2}\Gamma t}$$

Willenbrock & Valencia [PLB91]
Willenbrock [2203.11056]

Physical Pole for Unstable Particle

李政道研究所 Tsung-Dao Lee Institute

It is much more convenient to define mass & decay width in the rest frame.

$$m - \frac{i}{2}\Gamma$$

A physical pole can be generally parametrized as

$$\frac{1}{p^2 - \mu^2} \xrightarrow{\mu \equiv m - \frac{i}{2}\Gamma} \frac{1}{p^2 - m^2 + \frac{1}{4}\Gamma^2 + i m \Gamma}$$

Resonance peak is at $m^2 - \frac{1}{4}\Gamma^2$

Willenbrock & Valencia [PLB91]
Willenbrock [2203.11056]

Breit-Wigner vs Theoretical

Tsung-Dao Lee Institute

$$\mathcal{D}_{\rm BW} \sim \frac{1}{p^2 - m_{\rm BW}^2 + i m_{\rm BW} \Gamma_{\rm BW}}$$

$$\frac{m_{\rm th}}{m_{\rm BW}} = \frac{\Gamma_{\rm BW}}{\Gamma_{\rm th}} \simeq 1 + \frac{\Gamma_Z^2}{8 m_Z^2} \equiv R_Z$$

Equivalent!

$$\mathcal{D}_{\rm th} \sim \frac{1}{p^2 - m_{\rm th}^2 + \frac{1}{4}\Gamma_{\rm th}^2 + i m_{\rm th} \Gamma_{\rm th}}$$

The equivalence happens at the amplitude level already!

Reciprocal Scaling Behaviors

李政道研究所 Tsung-Dao Lee Institute

$$\frac{1}{p^2 - m_{\mathrm{BW}}^2 + i m_{\mathrm{BW}} \Gamma_{\mathrm{BW}}} \quad \text{vs} \quad \frac{1}{p^2 - m_{\mathrm{th}}^2 + \frac{1}{4}\Gamma_{\mathrm{th}}^2 + i m_{\mathrm{th}} \Gamma_{\mathrm{th}}}$$

Being extracted from data, the imaginary parts should be the same!

$$\mathrm{Data} = m_{\mathrm{BW}} \Gamma_{\mathrm{BW}} = m_{\mathrm{th}} \Gamma_{\mathrm{th}} \quad \Rightarrow \quad \Gamma \propto \frac{1}{m}$$

Different!

Theoretical predictions should be

$$\Gamma_Z \propto m_Z$$

$$\Gamma_Z = \frac{m_Z}{24\pi} \beta_f \left[g_L^2 + g_R^2 - (g_L^2 - 6 g_L g_R + g_R^2) \frac{m_f^2}{m_Z^2} \right]$$

$$\beta_f \equiv \sqrt{1 - 4 m_f^2 / m_Z^2}$$

Constrained Fit

李政道研究所 Tsung-Dao Lee Institute

$$\chi^2_Z \equiv \left(\frac{m_{\rm th}/R_Z - m_Z}{\Delta m_Z} \right)^2 + \left(\frac{\Gamma_{\rm th} R_Z - \Gamma_Z}{\Delta \Gamma_Z} \right)^2$$

Required precision: $\Delta m_Z \lesssim 8.4\,\mathrm{MeV}$ $\Delta \Gamma_Z \lesssim 0.23\,\mathrm{MeV}$

Need both of them simultaneously!

	Δm_Z (MeV)	$\Delta\Gamma_Z$ (MeV)
LEP	2.1	2.3
CEPC	0.1	0.025
FCC-ee	0.1	0.025
ILC	0.2	0.12

$\chi^2_{Z,\min} = 326$ (CEPC & FCC-ee) 14.3 (ILC)

Outline

Tsung-Dao Lee Institute

1) Breit-Wigner vs Energy-Dependent Schemes

Z Lineshape Scan

Forward-Backward Asymmetries

2) Breit-Wigner vs Theoretical Schemes

Z Lineshape Scan

WW Threshold Scan + Fermi Constant

3) Summary

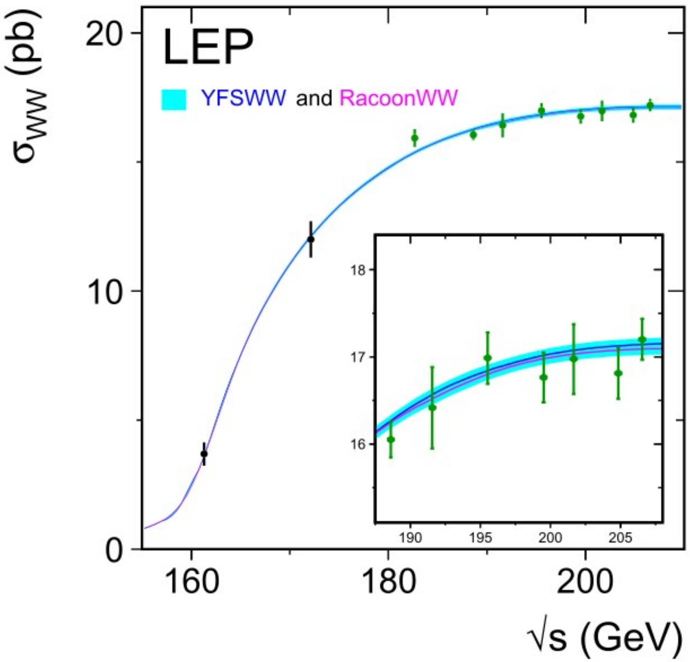
WW Threshold Scan

$$\chi^2_W \equiv \left(\frac{m_{W,\text{th}}/R_W - m_W}{\Delta m_W}\right)^2 + \left(\frac{\Gamma_{W,\text{th}} R_W - \Gamma_W}{\Delta \Gamma_W}\right)^2$$

$$\Gamma_W = \frac{m_W}{24\pi} g^2 \beta_{12} \left[1 - \frac{m_1^2 + m_2^2}{2 m_W^2} - \frac{(m_1^2 - m_2^2)^2}{2 m_W^4}\right]$$

Unfortunately, the mass & decay width precisions are not enough!

	Δm_W (MeV)	$\Delta\Gamma_W$ (MeV)
CEPC	0.5	2.0
FCC-ee	0.4	1.2
ILC	2.4	2.0



1302.3415

Fermi Constant G_F

$$G_{F,\mathrm{BW}} \approx \frac{g^2}{4\sqrt{2} m^2_{W,\mathrm{BW}}} \left(1 - \frac{\overline{\Gamma}^2_W}{2 m^2_W}\right)$$

Momentum transfer in muon decay $p^2 < m^2_\mu$

$$\overline{\Gamma}_W = \frac{p^2}{m^2_W} \frac{2}{5} \Gamma_W(m^2_W) \sim 10^{-7} \Gamma_W(m^2_W)$$

Fermi constant is free of decay width @ tree level

$$G_{F,\mathrm{BW}} \approx \frac{g^2}{4\sqrt{2} m^2_{W,\mathrm{BW}}} \approx \frac{g^2}{4\sqrt{2} m^2_{W,\mathrm{th}}}$$

Fermi constant fixes m_W to the same value!

WW Threshold + Fermi Constant

李政道研究所 Tsung-Dao Lee Institute

$$\chi^2_W \equiv \left(\frac{m_{W,\mathrm{th}}/R_W - m_W}{\Delta m_W}\right)^2 + \left(\frac{\Gamma_{W,\mathrm{th}} R_W - \Gamma_W}{\Delta \Gamma_W}\right)^2 + \left(\frac{G_{F,\mathrm{th}} - G_F}{\Delta G_F}\right)^2$$

	Δm_W (MeV)	$\Delta \Gamma_W$ (MeV)
CEPC	0.5	2.0
FCC-ee	0.4	1.2
ILC	2.4	2.0

→

	$\Delta\chi^2_{min}$
CEPC	**169**
FCC-ee	**263**
ILC	**7.3**

$$G_F = 1.166\,378\,8\,(6) \times 10^{-5}\,\mathrm{GeV}^{-2}$$

Outline

Tsung-Dao Lee Institute

1) Breit-Wigner vs Energy-Dependent Schemes

Z Lineshape Scan

Forward-Backward Asymmetries

2) Breit-Wigner vs Theoretical Schemes

Z Lineshape Scan

WW Threshold Scan + Fermi Constant

3) Summary

Summary

李政道研究所
Tsung-Dao Lee Institute

Breit-Wigner

$$\frac{1}{p^2 - m_{\rm BW}^2 + i m_{\rm BW} \Gamma_{\rm BW}}$$

m, Γ, g changes

Z lineshape + A_{FB}

m & Γ changes

Z: reciprocal scaling

W: + Fermi constant G_F

Energy-Dependent

$$\frac{1}{p^2 - m_{\rm ED}^2 + i \frac{p^2 \Gamma_{\rm ED}}{m_{\rm ED}}}$$

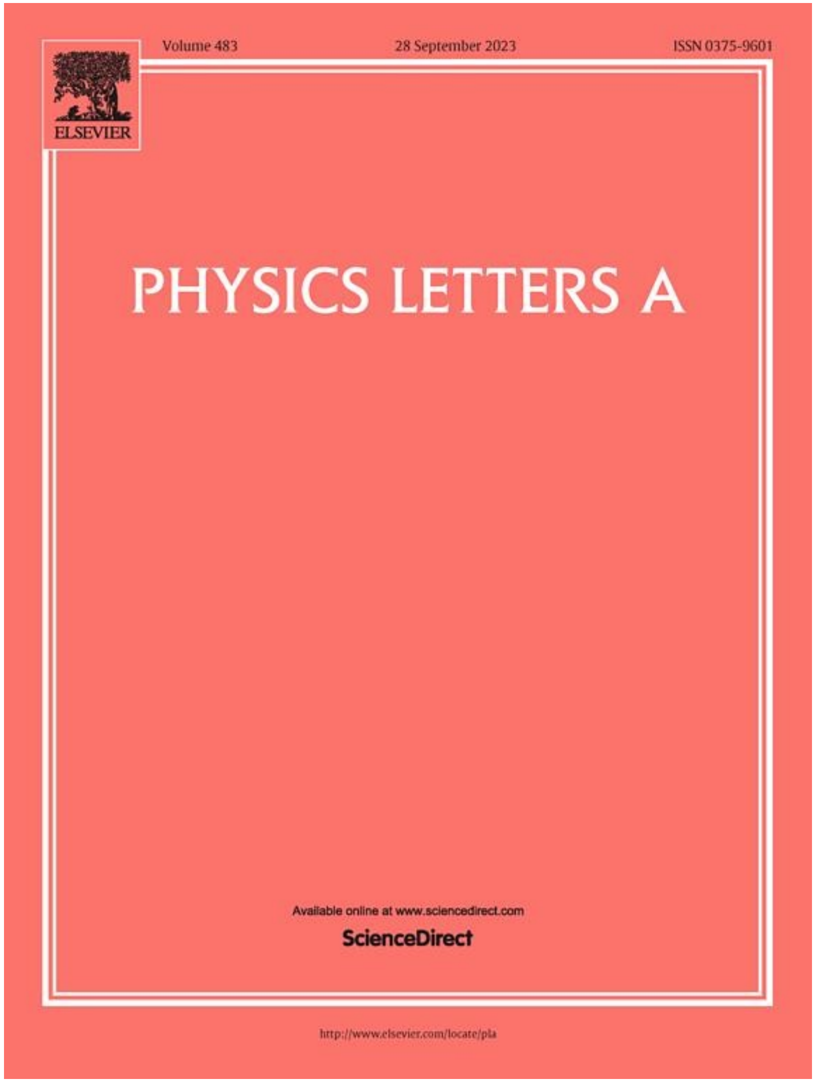
Theoretical

$$\frac{1}{p^2 - m_{\rm th}^2 + \frac{1}{4}\Gamma_{\rm th}^2 + i m_{\rm th} \Gamma_{\rm th}}$$

1. **Lepton collider can test one important aspect of quantum & field theories.**
2. **Resonance scheme can affect precision measurements.**

《 Physics Letters A 》

李政道研究所
Tsung-Dao Lee Institute



Aims & Scope

- Nonlinear science,
- Statistical physics,
- Mathematical and computational physics,
- AMO and physics of complex systems,
- Plasma and fluid physics,
- Optical physics,
- General and cross-disciplinary physics,
- Biological physics and nanoscience,
- Astrophysics, Particle physics and Cosmology.

PLA now also accepts submissions on Particle, Astrophysics, and Cosmology! You are welcome to submit.