# Tracing the bottom EW dipole operators at future lepton colliders

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based on arXiv: 2410.05398, in collaboration with Jiayin Gu, Jiayu Guo

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Bottom EW dipole operators at future lepton colliders

### **Motivation**

- "What is next beyond SM?"
- New lepton colliders are expected to be on their way
   → A telescope to high scale physics (luminosity frontier)
- Especially for circular ones  $\rightarrow$  Higgs/Z factories & EW precision test
- For new physics scale  $\Lambda \gg E$ , *v*, Standard Model Effective Field Theory (SMEFT) provides an effective and model independent tool

$$\mathcal{L}_{\text{SMEFT}} = \mathcal{L}_{\text{SM}} + \sum_{i} \frac{C_{i}^{(6)}}{\Lambda^{2}} O_{i}^{(6)} + \cdots$$

· This study will focus on EW sector about bottom dipole operators

### **Z-pole status**

#### Some Z pole observables [PDG2023]

Quantity	Exp. Value	SM prediction
M <sub>Z</sub> [GeV]	$91.1876 \pm 0.0021$	91.1882 ± 0.0020
$\Gamma_Z$ [GeV]	$2.4955 \pm 0.0023$	$2.4941 \pm 0.0009$
$\sigma_{had}$ [nb]	$41.481 \pm 0.033$	$41.482\pm0.008$
$R_e$	$20.804 \pm 0.050$	$20.736\pm0.010$
$R_b$	$0.21629 \pm 0.00066$	$0.21582 \pm 0.00002$
$R_c$	$0.1721 \pm 0.0030$	$0.17221 \pm 0.00003$
$A_{FB}^{(0,e)}$	$0.0145 \pm 0.0025$	$0.01617 \pm 0.00007$
$A_{FB}^{(0,b)}$	$0.0996 \pm 0.0016$	$0.1029 \pm 0.0002$
$A_{FB}^{(0,c)}$	$0.0707 \pm 0.0035$	$0.0735 \pm 0.0002$
$A_e$	$0.1498 \pm 0.0049$	$0.1468 \pm 0.0003$
$A_b$	$0.923 \pm 0.020$	0.9347
$A_c$	$0.670\pm0.027$	$0.6677 \pm 0.0001$

\* For Ae, only LEP 1 results shown here

- Most observables are measured precisely and consistent with theoretical prediction.
- Except  $A_{FB}$  for *b*-quark, still exist ~ 2  $\sigma$  deviation.
- At future lepton colliders, trillion Z bosons could be produced.

 $\Rightarrow$  Opportunity to reveal potential BSM NP with much improved precision.

### **SMEFT & dipole operators**

Many global fit analysises are performed



### **SMEFT & dipole operators**

#### **SMEFT dim-6 dipole operators**

(Warsaw basis, 3rd generation quarks)

$$\begin{aligned} O_{tW} &= \left(\bar{q}_{\rm L} \sigma^{\mu\nu} \tau^I t_{\rm R}\right) \tilde{H} W^{\dagger}_{\mu\nu}, \qquad O_{bW} &= \left(\bar{q}_{L} \sigma^{\mu\nu} b_{R}\right) \sigma^i H W^i_{\mu\nu}, \\ O_{tB} &= \left(\bar{q}_{\rm L} \sigma^{\mu\nu} t_{\rm R}\right) \tilde{H} B_{\mu\nu}, \qquad O_{bB} &= \left(\bar{q}_{L} \sigma^{\mu\nu} b_{R}\right) H B_{\mu\nu}. \end{aligned}$$

where,  $\sigma^{\mu\nu} = \frac{i}{2} [\gamma^{\mu}, \gamma^{\nu}], W^{i}_{\mu\nu} = \partial_{\mu}W^{i}_{\nu} - \partial_{\nu}W^{i}_{\mu} - g_{W}\varepsilon^{ijk}W^{j}_{\mu}W^{k}_{\nu}$  (similar form of  $B_{\mu\nu}$ )

- Generally, the leading effect from interference of dipole and SM  $\propto m_f v / \Lambda^2$
- Top is the exception, constraint of its dipole at LHC:



 $C_{tw} = [-1.2, +1.4](\Lambda / \text{TeV})^2$  and  $C_{tB} = [-1.9, +1.2](\Lambda / \text{TeV})^2$ (*ttZ*/ $\gamma$  measurement @ LHC Run2, 95% CL) [M. Schulze, Y. Soreq 1603.08911]

# $Zb\overline{b}$ dipole can be traced

• Future lepton colliders (e.g. FCC-ee/CEPC):

tremendous amount of Z events + higher precision of measurements  $\Rightarrow$  possible to trace  $Zb\bar{b}$  dipole

• Z pole observables

$$\begin{split} R_b &= \frac{\Gamma(Z \to dd)}{\sum_q \Gamma(Z \to q\bar{q})} \\ A_\ell &= \frac{\Gamma(Z \to \ell_L \bar{\ell}_L) - \Gamma(Z \to \ell_R \bar{\ell}_R)}{\Gamma(Z \to \ell\bar{\ell})} \\ A_b &= \frac{\Gamma(Z \to b_L \bar{b}_L) - \Gamma(Z \to b_R \bar{b}_R)}{\Gamma(Z \to b\bar{b})} \\ A_{FB}^b &= \frac{3}{4} A_\ell A_b \end{split}$$

• Off pole scattering

$$A_{FB}^b = \frac{\sigma_F - \sigma_B}{\sigma_F + \sigma_B}$$

• Projections of CEPC here (similar for FCC-ee)

Quantity	Projected Precision	Runs
$\Delta R_b$	$4.4 \times 10^{-5}$	Z pole
$\Delta A_e$	$1.5  imes 10^{-5}$	Z pole
$\Delta A_b$	$2.1 \times 10^{-4}$	Z pole
$\sigma(b\bar{b})$ [fb]	$275.64\pm0.12$	$\sqrt{s} = 240 \text{ GeV}$
$A_{FB}(b\bar{b})$	$0.592 \pm 0.00034$	$\sqrt{s} = 240 \text{ GeV}$
$\sigma(b\bar{b})$ [fb]	$108.33\pm0.33$	$\sqrt{s} = 360 \text{ GeV}$
$A_{FB}(b\bar{b})$	$0.602 \pm 0.0024$	$\sqrt{s} = 360 \text{ GeV}$

 $([c_{\theta}^{min}, c_{\theta}^{max}] = [-0.9, 0.9], \epsilon = 0.15)$ 

[J.D. Blas et al., 2206.08326 (Snowmass2021)]

# $Zb\overline{b}$ dipole and the Lagrangian

- SMEFTsim package was used to extract dipole coupling and calculate observables [Brivio et al. 2012.11343]
- Effective Lagrangian with effective  $\gamma/Zb\bar{b}$  couplings

$$\begin{split} \mathcal{L} &\supset -eA_{\mu}\bar{b}\gamma^{\mu}b - \frac{g}{\cos\theta_{W}}Z_{\mu}\left(g_{Lb}\bar{b}_{L}\gamma^{\mu}b_{L} + g_{Rb}\bar{b}_{R}\gamma^{\mu}b_{R}\right) \\ &+ \frac{\kappa_{bA}}{m_{b}}\left(\bar{b}\sigma^{\mu\nu}b\right)A_{\mu\nu} + \frac{\kappa_{bZ}}{m_{b}}\left(\bar{b}\sigma^{\mu\nu}b\right)Z_{\mu\nu}\,, \end{split}$$

• Coefficients and relations { $\delta g_{Lb}$ ,  $\delta g_{Rb}$ ,  $\kappa_{bA}$ ,  $\kappa_{bZ}$ }:

$$g_{Lb} = -\frac{1}{2} + \frac{1}{3} \sin^2 \theta_W + \delta g_{Lb}, \quad g_{Rb} = \frac{1}{3} \sin^2 \theta_W + \delta g_{Rb},$$
  
$$\delta g_{Lb} = (c_{Hq}^{(1)} + c_{Hq}^{(3)}) \frac{v^2}{2\Lambda^2}, \quad \delta g_{Rb} = c_{Hb} \frac{v^2}{2\Lambda^2},$$
  
$$\kappa_{bZ} = \frac{m_b v}{\sqrt{2}\Lambda^2} \left(\cos \theta_W c_{bW} + \sin \theta_W c_{bB}\right), \quad \kappa_{bA} = \frac{m_b v}{\sqrt{2}\Lambda^2} \left(\cos \theta_W c_{bB} - \sin \theta_W c_{bW}\right).$$

- For convenience, set  $\Lambda = 1$  TeV and only keep real parts of dipole WCs
- 4 parameters included, and ratio of γ and Z diagrams varies from energy ⇒ need to include Z pole and off pole measurements.
- Central values of future measurements are assume to be SM-like.



\*global fit analysis with  $\{\delta g_{Lb}, \delta g_{Rb}, \kappa_{bA}, \kappa_{bZ}\}$  in this and following page

#### **Squared items contribution**

A generic observable has the form:

$$\sigma = \sigma_{\rm SM} + \sum_{\alpha} \sigma_{\alpha} C_a + \sum_{\alpha,\beta} \sigma_{\alpha\beta} C_{\alpha} C_{\beta}$$

where  $\sigma_{SM}$ ,  $\sigma_{\alpha}$ ,  $\sigma_{\alpha\beta}$  denote the SM, interference and squared contributions respectively. If not omit squared items (full fit)



Z pole + 240 GeV

• 2nd non-SM best fit: { $\delta g_{Lb}$ ,  $\delta g_{Rb}$ ,  $\kappa_{bA}$ ,  $\kappa_{bZ}$ } = {-3.84, -5.19, -0.105, -0.980} × 10^{-4}

### **Squared items contribution**

#### Extra off pole measurement



- · Squared terms of the dipole contributions play important role in the full fit.
- Including more off pole measurement:
  - $\Rightarrow$  may be minor contribution in the linear fit,
  - $\Rightarrow$  help to lift the 2nd non-SM best fit point.

# $\chi^2$ constrain analysis

### 1 $\sigma$ bound and correlation matrix $\rho$ (effective coupling)

	Linear fit	Correlation $\rho$			Full fit	
	$1\sigma$ bound (×10 <sup>-4</sup> )	$\delta g_{Lb}$	$\delta g_{Rb}$	$\kappa_{bZ}$	$\kappa_{bA}$	$1\sigma$ bound (×10 <sup>-4</sup> )
$\delta g_{Lb}$	±2.66	1				[-5.34, 1.87]
$\delta g_{Rb}$	±3.17	0.919	1			[-7.94, 2.32]
$\kappa_{bZ}$	±0.621	0.975	0.914	1		[-1.41, 0.398]
$\kappa_{bA}$	±0.249	0.804	0.784	0.868	1	[-0.228, 0.228]

### Z pole + 240 GeV

Z pole + 240 GeV + 360 GeV

	Linear fit	Correlation $\rho$				Full fit
	$1\sigma$ bound (×10 <sup>-4</sup> )	$\delta g_{Lb}$	$\delta g_{Rb}$	$\kappa_{bZ}$	$\kappa_{bA}$	$1\sigma$ bound (×10 <sup>-4</sup> )
$\delta g_{Lb}$	±2.64	1				[-2.58, 1.50]
$\delta g_{Rb}$	±3.15	0.918	1			[-3.30, 2.10]
$\kappa_{bZ}$	±0.616	0.975	0.913	1		[-0.624, 0.304]
$\kappa_{bA}$	±0.247	0.803	0.783	0.868	1	[-0.126, 0.168]

\*  $\chi^2 = \sum_{ij} (\hat{c}_i - \hat{c}_i^0) \sigma_{ij}^{-2} (\hat{c}_j - \hat{c}_j^0)$ , where  $\sigma_{ij}^{-2} = [\delta \hat{c}_i \rho_{ij} \delta \hat{c}_j]^{-1}$  [1411.0669]

# $\chi^2$ constrain analysis

### 1 $\sigma$ bound and correlation matrix $\rho$ (in Warsaw basis)

	Linear fit	Correlation $\rho$			Full fit	
	$1\sigma$ bound (×10 <sup>-2</sup> )	$(c_{Hq}^{(1)} + c_{Hq}^{(3)})$	$c_{Hb}$	$C_{bB}$	$c_{bW}$	$1\sigma$ bound (×10 <sup>-2</sup> )
$(c_{Hq}^{(1)} + c_{Hq}^{(3)})$	±0.877	1				[-1.76, 0.617]
$c_{Hb}$	±1.05	0.919	1			[-2.62, 0.765]
$C_{bB}$	±6.82	0.933	0.887	1		[-9.90, 5.10]
$c_{bW}$	±6.17	0.978	0.908	0.938	1	[-17.0, 3.75]

### Z pole + 240 GeV

Z pole + 240 GeV + 360 GeV

	Linear fit	Correlation $\rho$			Full fit	
	$1\sigma$ bound (×10 <sup>-2</sup> )	$(c_{Hq}^{(1)} + c_{Hq}^{(3)})$	$c_{Hb}$	$C_{dB}$	$C_{dW}$	$1\sigma$ bound (×10 <sup>-2</sup> )
$(c_{Hq}^{(1)} + c_{Hq}^{(3)})$	±0.871	1				[-0.851, 0.495]
$c_{Hb}$	±1.04	0.918	1			[-1.09, 0.693]
$c_{bB}$	±6.76	0.932	0.887	1		[-4.80, 3.75]
$c_{bW}$	±6.13	0.978	0.907	0.939	1	[-7.20, 2.80]

\* 
$$\chi^2 = \sum_{ij} (\hat{c}_i - \hat{c}_i^0) \sigma_{ij}^{-2} (\hat{c}_j - \hat{c}_j^0)$$
, where  $\sigma_{ij}^{-2} = [\delta \hat{c}_i \rho_{ij} \delta \hat{c}_j]^{-1}$  [1411.0669]

# **Recall the LEP** $A_{FB}^{0,b}$ **discrepancy**



\*Constraints from LEP in  $\{\kappa_{bZ}, \delta g_{Lb}\}$  and  $\{\kappa_{bZ}, \delta g_{Rb}\}$  plane individually here.

\*Existing problem:  $|\kappa_{bZ}| \sim 0.001$  to generate such  $A_{FB}^{0,b}$ . Assuming  $c_{bW}, c_{bB} \sim 1/16\pi^2 \Rightarrow \Lambda \lesssim 10^2$  GeV...

### Summary

- SMEFT & dipole
  - Dipoles are usually overlooked in the analysises (Except the heavy top).
  - dim-6 dipole operators might contribute to the  $A_{FB}$  inconsistency.
- $\bullet$  Future lepton collider offers opportunity to trace  $Zb\bar{b}$  dipole
  - Z pole measurements  $\Rightarrow$  only flat constraint.
  - Off Z pole measurement  $\Rightarrow$  interference of  $\gamma$  and Z diagram  $\Rightarrow$  closed constraint.
  - Quadratic items also give contribution (full fit).
  - · All runs are essential: extra off pole measurements lift the non-SM best-fit.
  - Our estimation (Z pole + 240 + 360 GeV ):  $k_{bZ} = [-0.624, 0.304] \times 10^{-4}, k_{bA} = [-0.126, 0.168] \times 10^{-4}.$
- More efforts are needed and in progress
  - Further combined analysis, distributions ...
  - More off pole analysises at other future lepton colliders ...
  - Appropriate models to contribute such  $A_{FB}^{0,b}$  discrepancy...

Thank you!

**Backups** 

# **Backups**



### **Quadratic items contribution - backup**

**Preferred region** (Full fit, in Warsaw basis) Z pole + 240 GeV



## Backups

	${\cal L}_6^{(6)}-\psi^2 X H$		$\mathcal{L}_{c}^{(7)} - \psi^2 H^2 D$
$Q_{eW}$	$(\bar{l}_p\sigma^{\mu\nu}e_r)\sigma^iHW^i_{\mu\nu}$	$O^{(1)}$	$(H^{\dagger}i\overleftrightarrow{D}H)(\overline{I}\gamma^{\mu}I)$
$Q_{eB}$	$(\bar{l}_p \sigma^{\mu\nu} e_r) H B_{\mu\nu}$	$Q_{Hl}$	$(H^{\dagger}iD_{\mu}H)(\bar{i}_{p}\gamma^{\prime}i_{r})$
$Q_{uG}$	$(\bar{q}_p \sigma^{\mu\nu} T^a u_r) \widetilde{H} G^a_{\mu\nu}$	$Q_{Hi}$	$(H^{\dagger}i D^{*}_{\mu}H)(l_{p}\sigma^{*}\gamma^{\mu}l_{r})$
$Q_{uW}$	$(\bar{q}_p \sigma^{\mu u} u_r) \sigma^i \widetilde{H} W^i_{\mu u}$	$Q_{He}$	$(H^{\dagger}i D_{\mu}H)(\bar{e}_p\gamma^{\mu}e_r)$
$Q_{uB}$	$(\bar{q}_p \sigma^{\mu\nu} u_r) \widetilde{H} B_{\mu\nu}$	$Q_{Hq}^{\left( 1 ight) }$	$(H^{\dagger}i\overleftrightarrow{D}_{\mu}H)(\bar{q}_{p}\gamma^{\mu}q_{r})$
$Q_{dG}$	$(\bar{q}_p \sigma^{\mu\nu} T^a d_r) H G^a_{\mu\nu}$	$Q_{Hq}^{\left( 3 ight) }$	$(H^{\dagger}i\overleftrightarrow{D}{}^{i}_{\mu}H)(ar{q}_{p}\sigma^{i}\gamma^{\mu}q_{r})$
$Q_{dW}$	$(\bar{q}_p \sigma^{\mu\nu} d_r) \sigma^i H W^i_{\mu\nu}$	$Q_{Hu}$	$(H^{\dagger}i\overleftrightarrow{D}_{\mu}H)(ar{u}_{p}\gamma^{\mu}u_{r})$
$Q_{dB}$	$(\bar{q}_p \sigma^{\mu\nu} d_r) H B_{\mu\nu}$	$Q_{Hd}$	$(H^{\dagger}i\overleftrightarrow{D}_{\mu}H)(\bar{d}_{p}\gamma^{\mu}d_{r})$
		$Q_{Hud} + {\rm h.c.}$	$i(\widetilde{H}^{\dagger}D_{\mu}H)(ar{u}_{p}\gamma^{\mu}d_{r})$

[arXiv:2012.11343]

Dipole operators of dim-6 SMEFT in the Warsaw basis.

### **Global SMEFT fit**



[arXiv:2206.08326]

### **EWPOs FCC-ee**

Quantity	current	ILC250	ILC-GigaZ	FCC-ee	CEPC	CLIC380
$\Delta \alpha(m_Z)^{-1} (\times 10^3)$	17.8*	17.8*		3.8 (1.2)	17.8*	
$\Delta m_W$ (MeV)	12*	0.5(2.4)		0.25(0.3)	0.35(0.3)	
$\Delta m_Z$ (MeV)	$2.1^{*}$	0.7(0.2)	0.2	0.004 (0.1)	0.005(0.1)	2.1*
$\Delta m_H$ (MeV)	170*	14		2.5 (2)	5.9	78
$\Delta \Gamma_W$ (MeV)	42*	2		1.2(0.3)	1.8(0.9)	
$\Delta \Gamma_Z$ (MeV)	$2.3^{*}$	1.5(0.2)	0.12	$0.004 \ (0.025)$	0.005(0.025)	2.3*
$\Delta A_e ~(\times 10^5)$	190*	14(4.5)	1.5(8)	0.7 (2)	1.5	64
$\Delta A_{\mu} (\times 10^5)$	$1500^{*}$	82(4.5)	3 (8)	2.3(2.2)	3.0(1.8)	400
$\Delta A_{\tau} (\times 10^5)$	400*	86(4.5)	3 (8)	0.5 (20)	1.2(6.9)	570
$\Delta A_b (\times 10^5)$	2000*	53 (35)	9 (50)	2.4 (21)	3 (21)	380
$\Delta A_c (\times 10^5)$	2700*	140(25)	20 (37)	20 (15)	6 (30)	200
$\Delta \sigma_{\rm had}^0$ (pb)	37*			0.035 (4)	0.05(2)	37*
$\delta R_e (\times 10^3)$	$2.4^{*}$	0.5(1.0)	0.2(0.5)	0.004 (0.3)	0.003(0.2)	2.7
$\delta R_{\mu} (\times 10^3)$	$1.6^{*}$	0.5(1.0)	0.2(0.2)	0.003(0.05)	0.003(0.1)	2.7
$\delta R_{\tau} (\times 10^3)$	$2.2^{*}$	0.6(1.0)	0.2(0.4)	0.003(0.1)	0.003(0.1)	6
$\delta R_b \ ( imes 10^3)$	$3.0^{*}$	0.4(1.0)	0.04(0.7)	0.0014 (< 0.3)	0.005(0.2)	1.8
$\delta R_c( imes 10^3)$	17*	0.6(5.0)	0.2(3.0)	0.015(1.5)	0.02(1)	5.6

Table 3: EWPOs at future  $e^+e^-$ : statistical error (experimental systematic error).  $\Delta$ 

FCC-ee $\sqrt{s}$ [GeV]	Final state	$\mathcal{L}\left[fb^{-1}\right]$	$\sigma$ [fb]	$A_{FB}$	$[c_{\theta}^{\min},c_{\theta}^{\max}]$	$\epsilon$	
	$e^-e^+$		$77330.4 \pm 3.87$	$0.96{\pm}0.00001388$	[-0.9, 0.9]	0.98	
	$\mu^{-}\mu^{+}$		$1870.84{\pm}0.612$	$0.521 {\pm} 0.000279$	[-0.95, 0.95]	0.98	
240	$\tau^{-}\tau^{+}$	5000	$1589.15{\pm}0.564$	$0.506 {\pm} 0.000306$	[-0.9, 0.9]	0.9	
	cc		$93.38 {\pm} 0.1367$	$0.62 {\pm} 0.00115$	[-0.9, 0.9]	0.03	
	$b\overline{b}$		$275.64{\pm}0.235$	$0.592{\pm}0.000687$	[-0.9, 0.9]	0.15	
	$e^-e^+$		$34221.5 {\pm} 4.72$	$0.957{\pm}0.0000399$	[-0.9, 0.9]	0.98	
	$\mu^{-}\mu^{+}$		$787.74{\pm}0.725$	$0.488 {\pm} 0.000803$	[-0.95, 0.95]	0.98	Įa
365	$\tau^{-}\tau^{+}$	1500	$669.11 {\pm} 0.668$	$0.473 {\pm} 0.00088$	[-0.9, 0.9]	0.9	
	$c\overline{c}$		$38.11 {\pm} 0.1594$	$0.595 {\pm} 0.00336$	[-0.9, 0.9]	0.03	
	$b\overline{b}$		$105.12{\pm}0.2647$	$0.603 {\pm} 0.00201$	[-0.9, 0.9]	0.15	

[arXiv:2206.08326]

### **EWPOs CEPC**

Quantity	current	ILC250	ILC-GigaZ	FCC-ee	CEPC	CLIC380
$\Delta \alpha(m_Z)^{-1} (\times 10^3)$	17.8*	17.8*		3.8 (1.2)	17.8*	
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$\Delta m_Z$ (MeV)	2.1*	0.7(0.2)	0.2	0.004 (0.1)	0.005(0.1)	$2.1^{*}$
$\Delta m_H$ (MeV)	170*	14		2.5 (2)	5.9	78
$\Delta \Gamma_W$ (MeV)	42*	2		1.2(0.3)	1.8(0.9)	
$\Delta \Gamma_Z$ (MeV)	$2.3^{*}$	1.5(0.2)	0.12	$0.004 \ (0.025)$	0.005(0.025)	$2.3^{*}$
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$\Delta A_c (\times 10^5)$	2700*	140(25)	20 (37)	20 (15)	6 (30)	200
$\Delta \sigma_{\rm had}^0$ (pb)	37*			0.035 (4)	0.05(2)	37*
$\delta R_e (\times 10^3)$	$2.4^{*}$	0.5(1.0)	0.2(0.5)	0.004 (0.3)	0.003(0.2)	2.7
$\delta R_{\mu} (\times 10^3)$	$1.6^{*}$	0.5(1.0)	0.2(0.2)	0.003(0.05)	0.003(0.1)	2.7
$\delta R_{\tau} (\times 10^3)$	$2.2^{*}$	0.6(1.0)	0.2(0.4)	0.003(0.1)	0.003(0.1)	6
$\delta R_b \ ( imes 10^3)$	$3.0^{*}$	0.4(1.0)	0.04(0.7)	0.0014 (< 0.3)	0.005(0.2)	1.8
$\delta R_c( imes 10^3)$	17*	0.6(5.0)	0.2(3.0)	0.015(1.5)	0.02(1)	5.6

Table 3: EWPOs at future  $e^+e^-$ : statistical error (experimental systematic error).  $\Delta$ 

CEPC $\sqrt{s}$ [GeV]	Final state	$\mathcal{L} \left[ \mathrm{fb}^{-1} \right]$	$\sigma$ [fb]	$A_{FB}$	$[c_{\theta}^{\min},c_{\theta}^{\max}]$	e	
	$e^-e^+$		$77330.4 {\pm} 1.937$	$0.96{\pm}0.00000694$	[-0.9, 0.9]	0.98	
	$\mu^{-}\mu^{+}$		$1870.84{\pm}0.306$	$0.521{\pm}0.0001395$	[-0.95, 0.95]	0.98	
240	$\tau^-\tau^+$	5000	$1589.15 {\pm} 0.282$	$0.506 {\pm} 0.000153$	[-0.9, 0.9]	0.9	
	cc		$93.38 {\pm} 0.0683$	$0.62 {\pm} 0.000574$	[-0.9, 0.9]	0.03	
	$b\overline{b}$		$275.64{\pm}0.1174$	$0.592{\pm}0.0003434$	[-0.9, 0.9]	0.15	
	$e^-e^+$		$35147.9 \pm 5.85$	$0.957 {\pm} 0.0000482$	[-0.9, 0.9]	0.98	
360	$\mu^{-}\mu^{+}$		$810.18 \pm 0.9$	$0.4885 {\pm} 0.00097$	[-0.95, 0.95]	0.98	[arXiv:2206.08326]
	$\tau^{-}\tau^{+}$	1500	$688.17 {\pm} 0.83$	$0.474 {\pm} 0.001061$	[-0.9, 0.9]	0.9	
	$c\overline{c}$		$39.22 {\pm} 0.198$	$0.596 {\pm} 0.004056$	[-0.9, 0.9]	0.03	
	$b\overline{b}$		$108.33 \pm 0.329$	$0.602 \pm 0.002425$	[-0.9, 0.9]	0.15	

## $\chi^2$ constrain comparison - backup

#### Present Z pole data (PDG) vs Z pole estimation in the future

\*(analyze individually for  $\delta g_{Lb} - \delta g_{Rb}$  and  $C_{bB} - C_{bW}$  only in this page)



Flat constraint only  $\Rightarrow$  We need off Z pole run for combined analysis.