CKM determination from W decays

David Marzocca (INFN Trieste)

Based on: D.M., Manuel Szewc and Michele Tammaro *JHEP 11 (2024) 017* [2405.08880]

8th FCC Physics Workshop - CERN - 15/01/2025

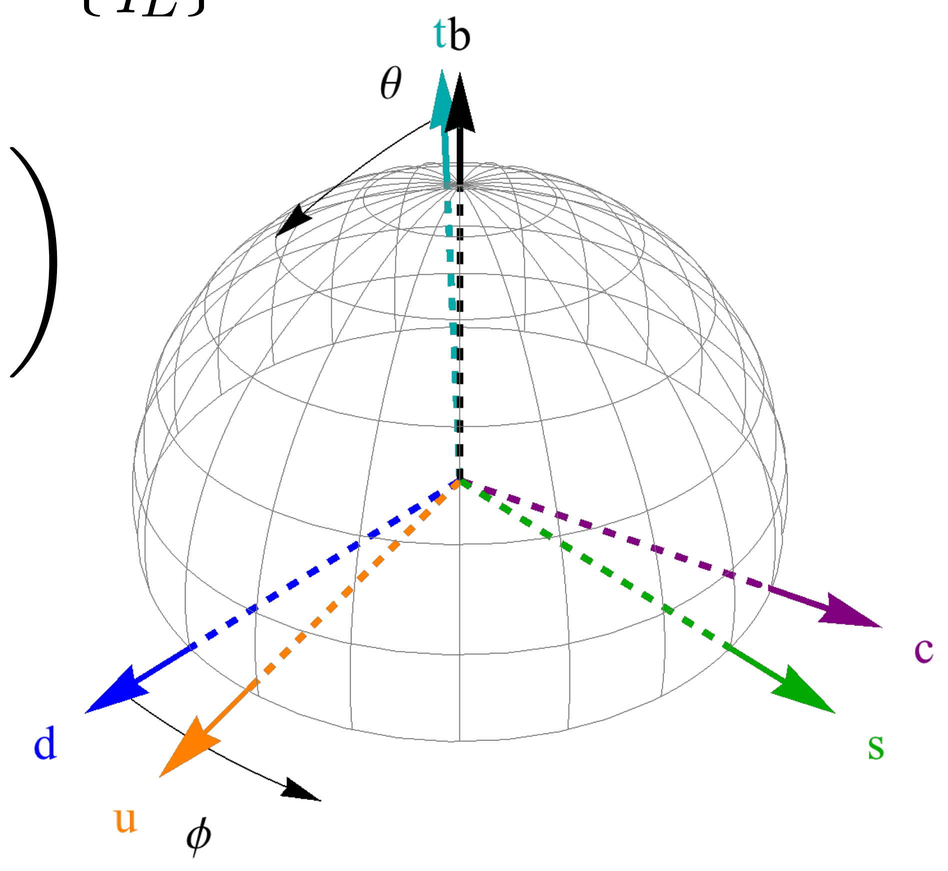
CKM matrix elements

$\mathbf{V}_{ij}$

Fundamental SM parameters:
CKM elements enter in all quark **flavour-changing transitions** and set the size of CP violation effects.

$$q_L^i = \begin{pmatrix} V_{ji}^* u_L^i \\ d_L^i \end{pmatrix}$$

$\{q_L^i\}$ space, neglecting phases



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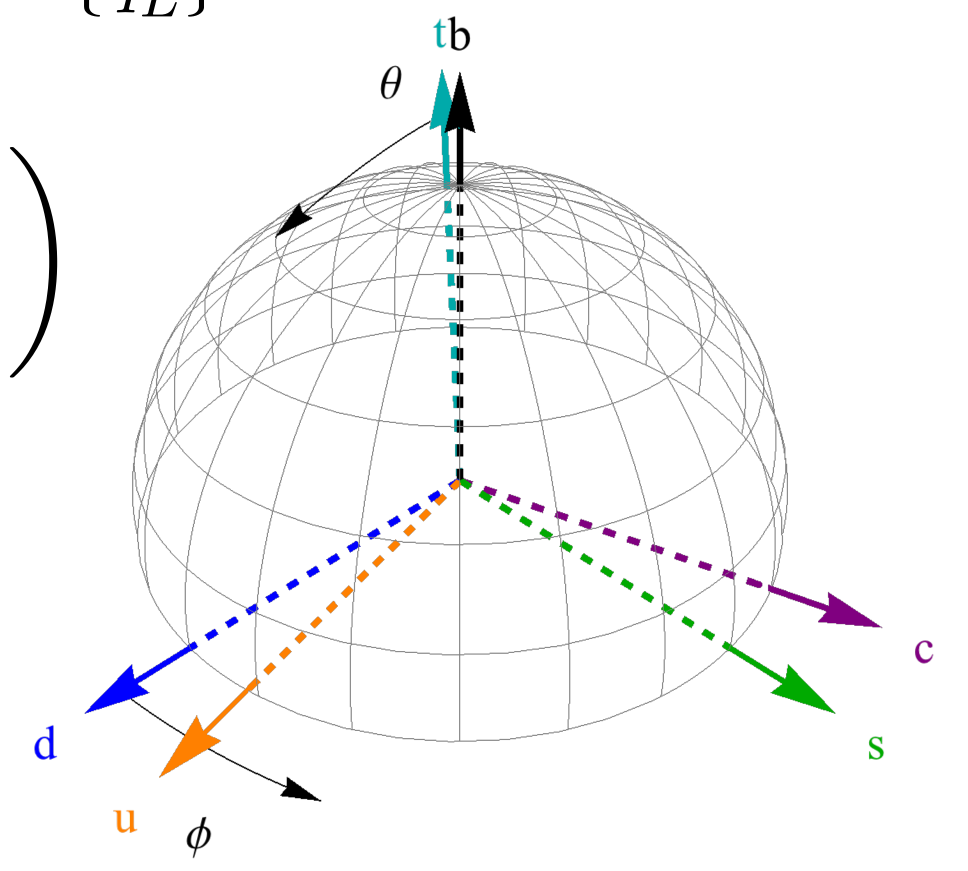

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Precise knowledge is **crucial** to derive the **strongest possible sensitivity** on **new physics from rare meson decays** and meson mixing observables.

$|V_{cb}|$ is particularly important:

$B \to K^{(*)} \nu \nu$

$B^+ \to K^+ \nu\bar{\nu}$	$(5.06 \pm 0.14 \pm 0.28) \times 10^{-6}$
$B^0 \to K_S \nu\bar{\nu}$	$(2.05 \pm 0.07 \pm 0.12) \times 10^{-6}$

Becirevic et al. 2301.06990

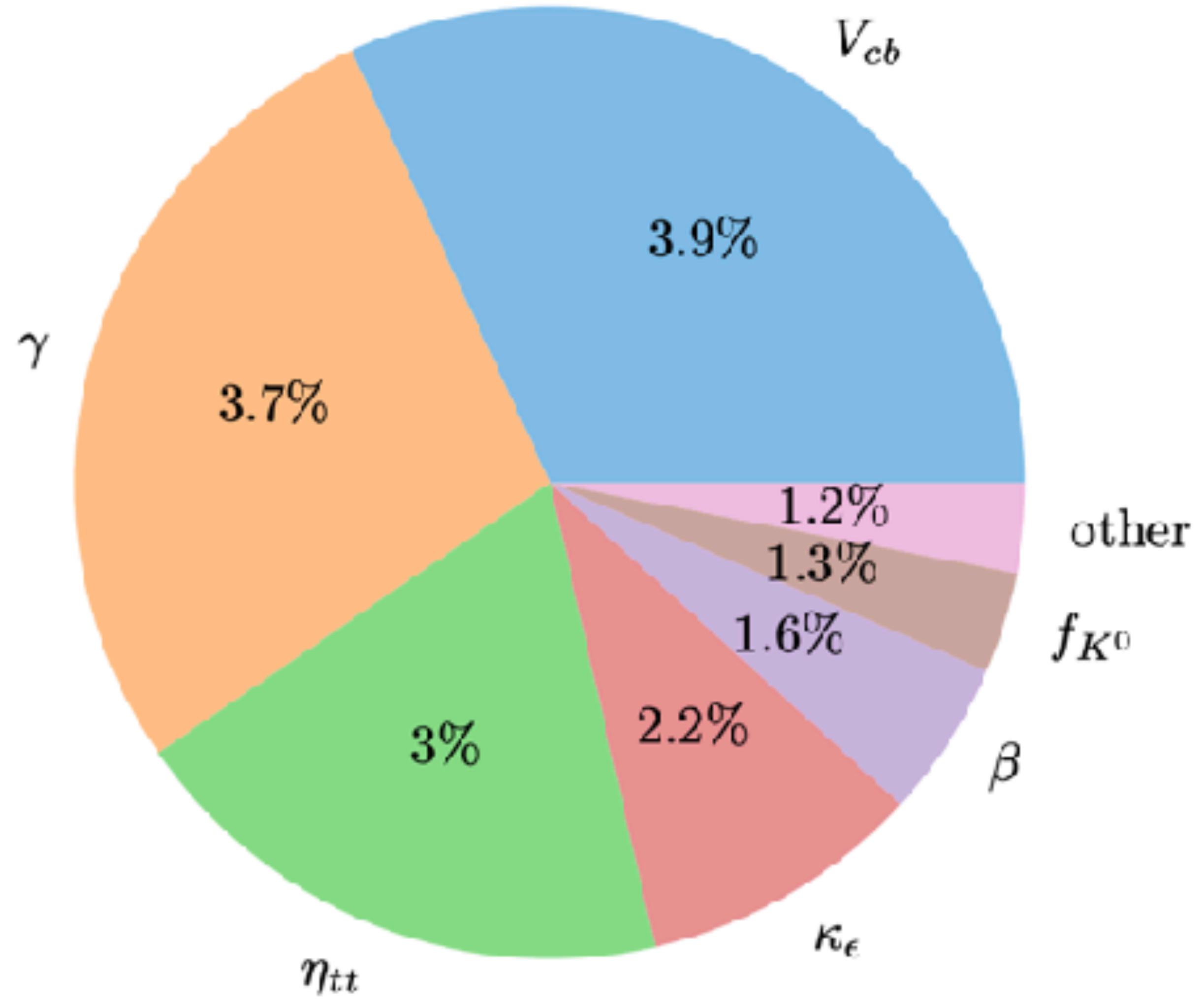
Main theory uncertainties due to:
- Hadronic form factors (Lattice QCD)
- CKM matrix elements: mostly $|V_{cb}|$

$$V_{ts} = -V_{cb}\left[1 - \frac{\lambda^2}{2}(1 - 2\bar{\rho} - 2i\bar{\eta}) + \mathcal{O}(\lambda^4)\right]$$

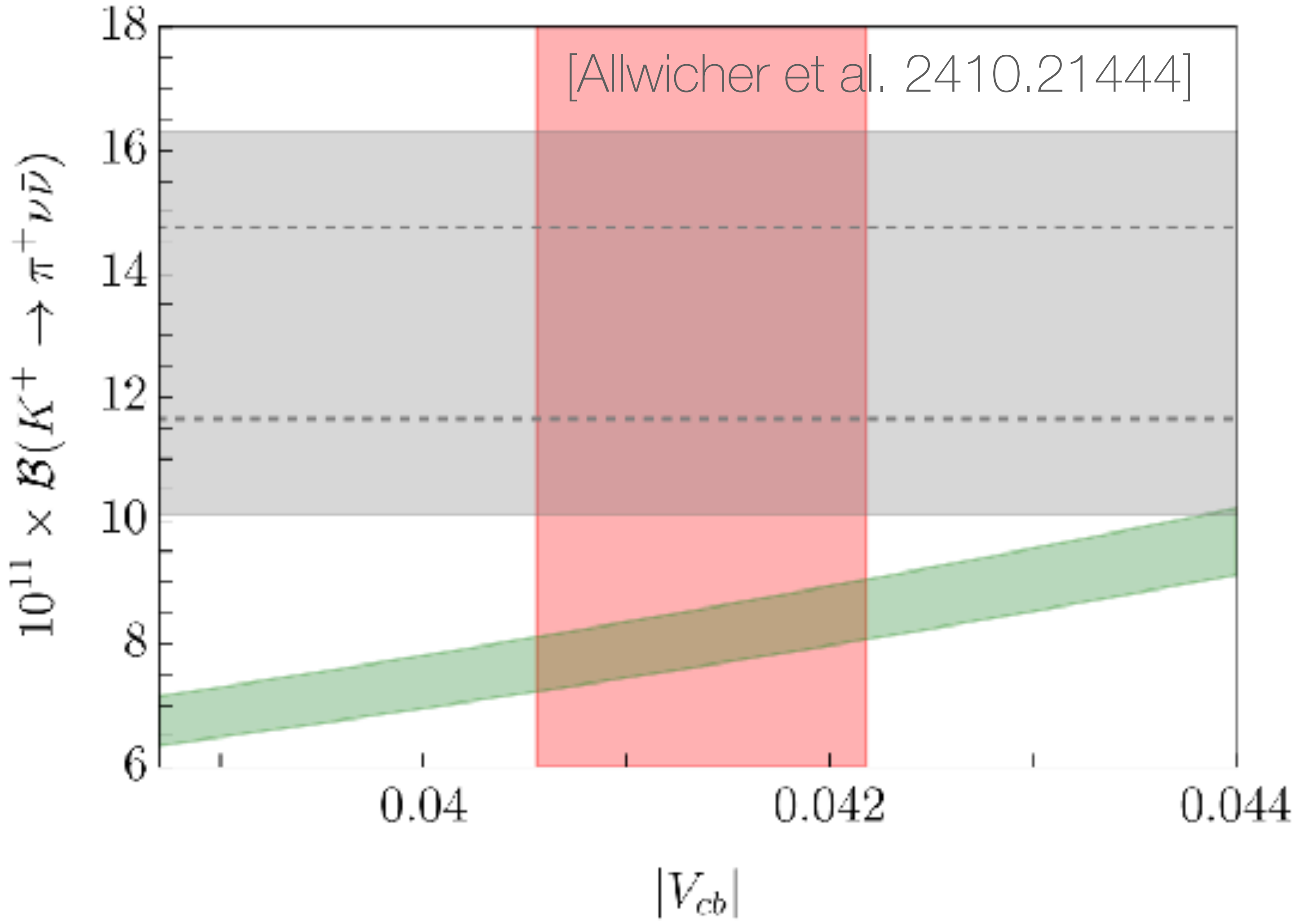
Buras et al. 1409.4557

Error budget of ε_K

[Buras, Stengl 2412.14254]



$K^+ \to \pi^+ \nu \nu$

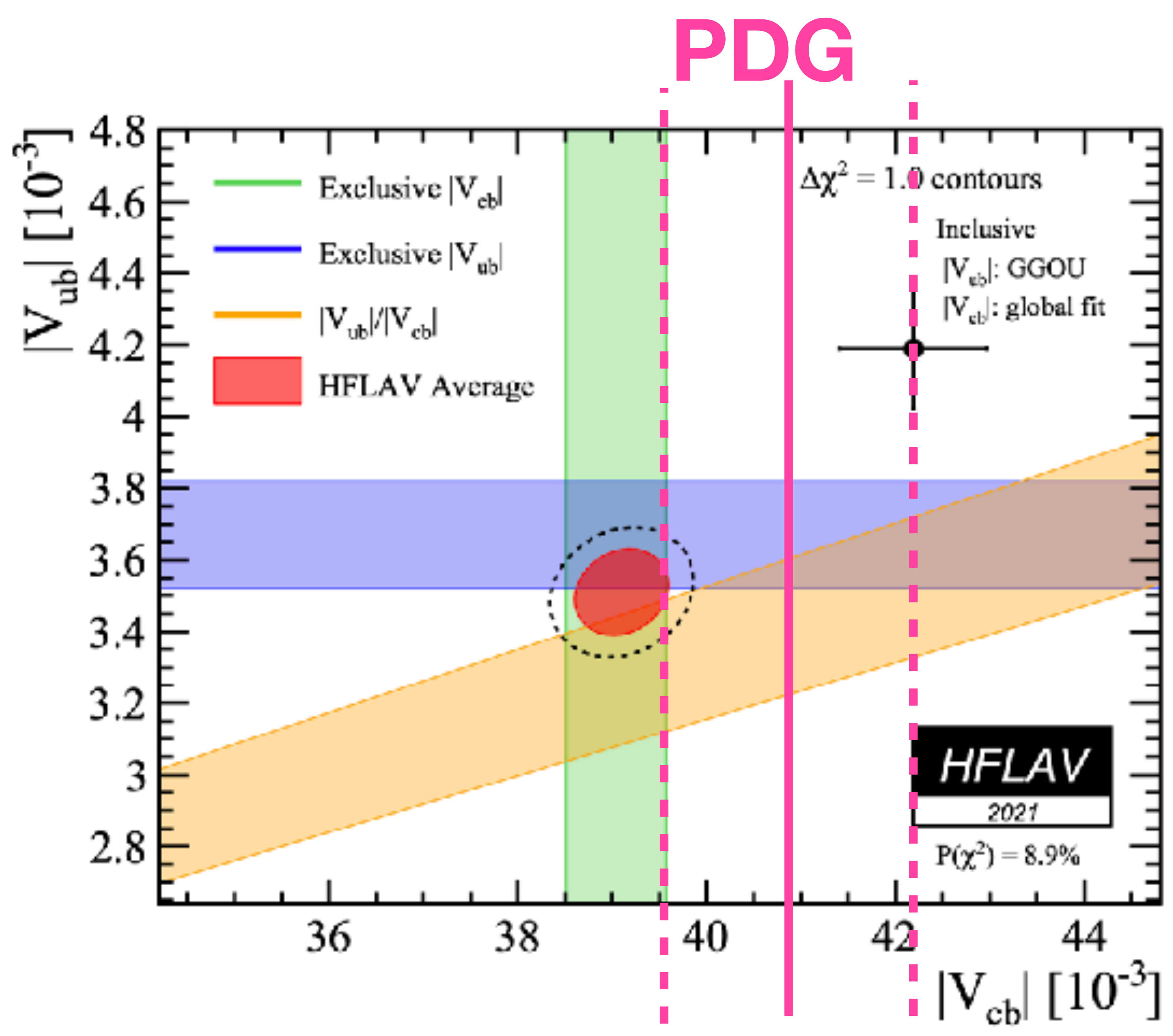


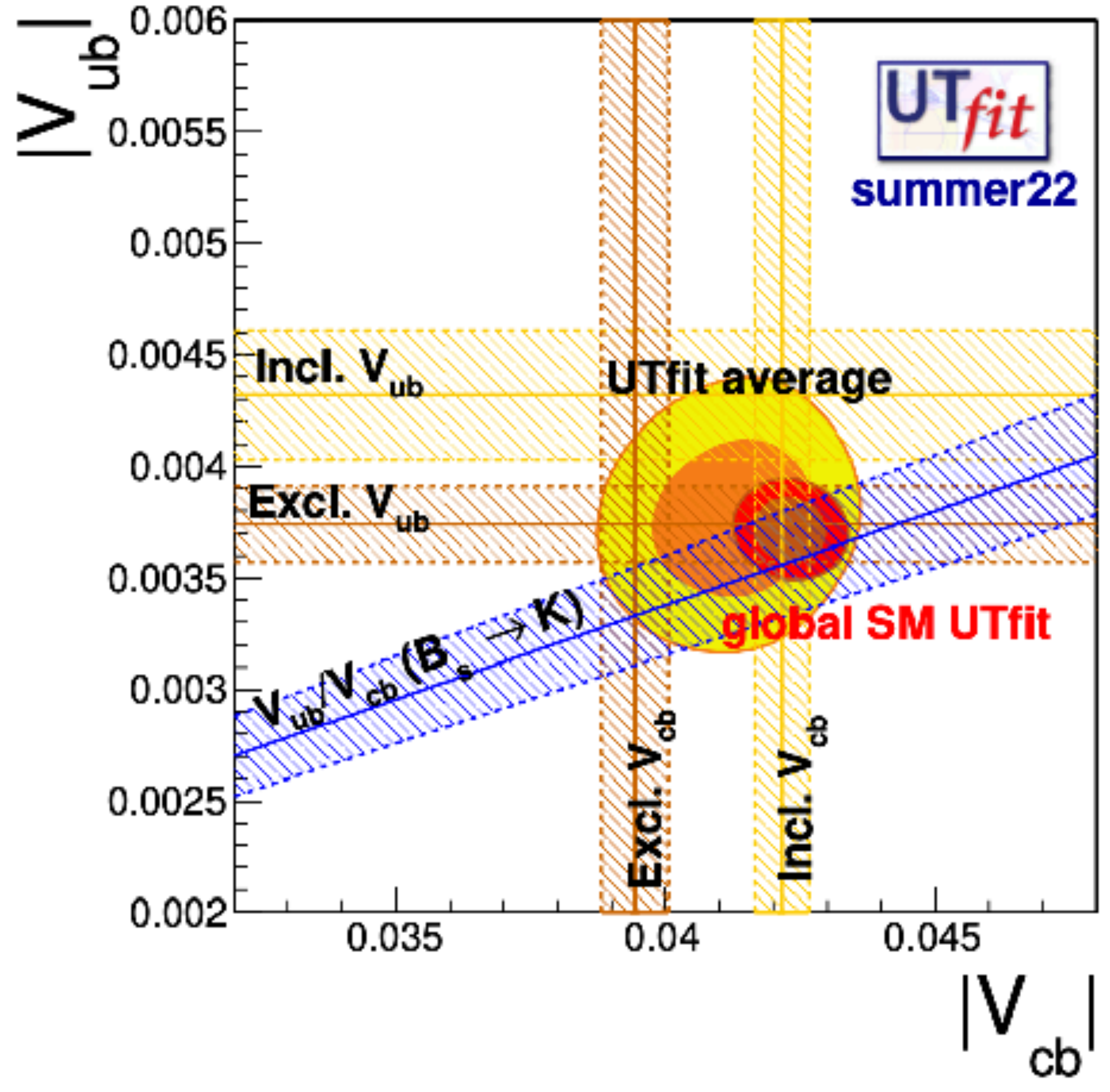
$$V_{ts}^* V_{td} = \lambda |V_{cb}|^2 \left[(\bar{\rho} - 1)\left(1 - \frac{\lambda^2}{2}\right) + i\bar{\eta}\left(1 + \frac{\lambda^2}{2}\right)\right]$$

Measuring V_{cb} from meson decays

$|V_{cb}|$ is currently measured from **tree-level semi-leptonic B decays**,
either **exclusive** $B \to D^{(*)} \ell \nu$ or **inclusive $b \to c$ transitions**, $B \to X_c \ell \nu$.

At present the $|V_{cb}|$ extraction from **inclusive vs. exclusive** decays are in **tension**.

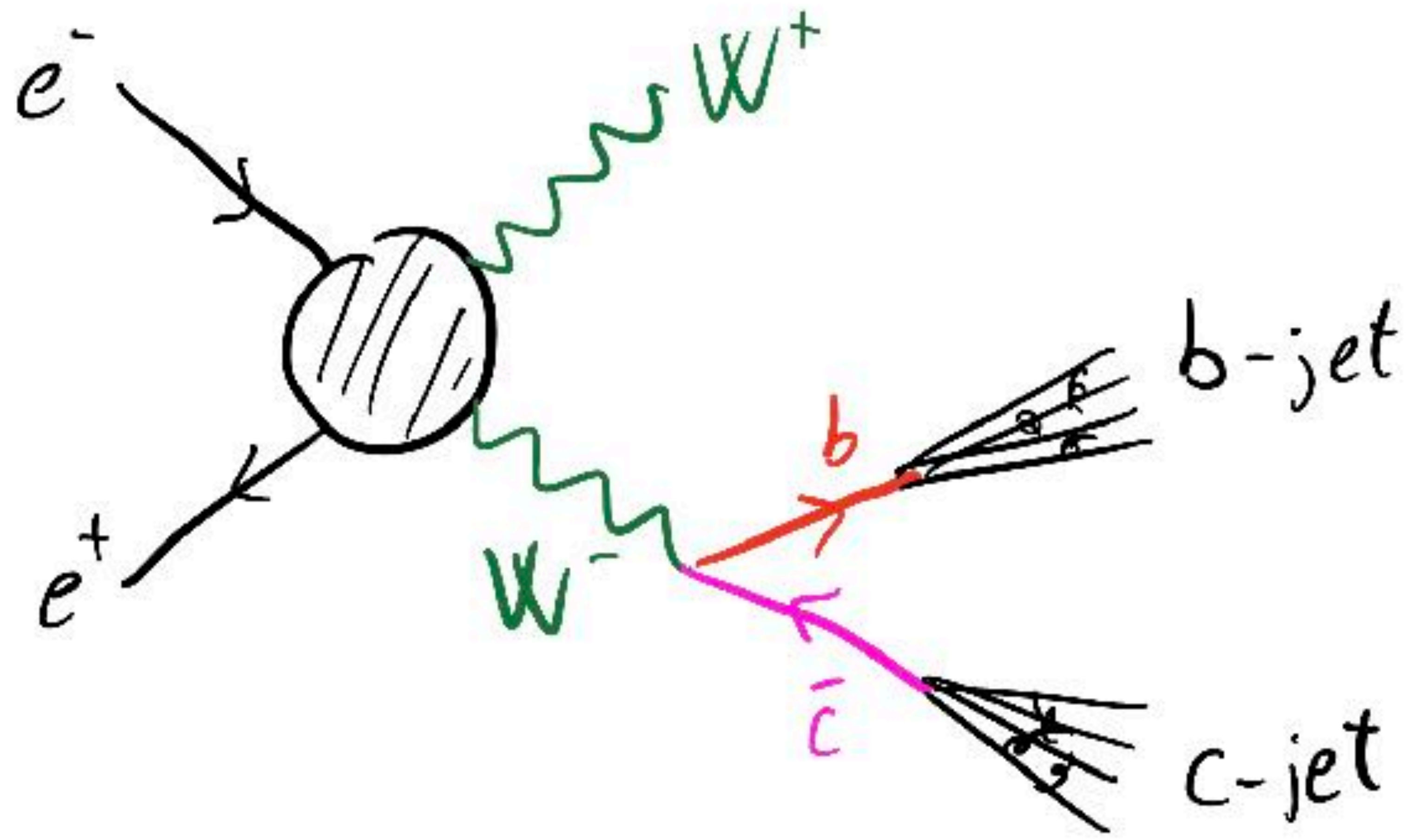




$\lvert V_{ij}\rvert$	Current (PDG)	
$\lvert V_{cs}\rvert$	0.975 ± 0.006	(0.6%)
$\lvert V_{cb}\rvert$	$(40.8 \pm 1.4) \times 10^{-3}$	(3.4%)

Independently on this tension, the extraction of **V_{cb} from semileptonic B decays**
is **already limited by systematics**: Belle-II detector performance.

A more direct measurement



CKM

$$\mathcal{L}_{gauge} \supset -\frac{g}{\sqrt{2}}\left(W^+_\mu\, \bar{u}^\alpha_L\, V_{\alpha\beta}\, \gamma^\mu\, d^\beta_L + h.c.\right)$$

→

$$\mathcal{B}(W^- \to \bar{u}_i d_j) \approx \frac{1}{2}|V_{ij}|^2 \mathcal{B}(W^- \to \text{hadrons})$$

$$\mathcal{B}(W^- \to \text{hadrons}) = (67.41 \pm 0.27)\%$$

→ $|V_{cb}|$

Extracting **CKM elements directly from on-shell W decays** could provide:

1) A completely **independent measurement** of a crucial input for flavour physics.

2) A measurement **independent from Lattice QCD** inputs: a possible benchmark for LQCD?

3) A way to **improve the precision** **beyond** the one from **semileptonic B decays**. **Quantify?**

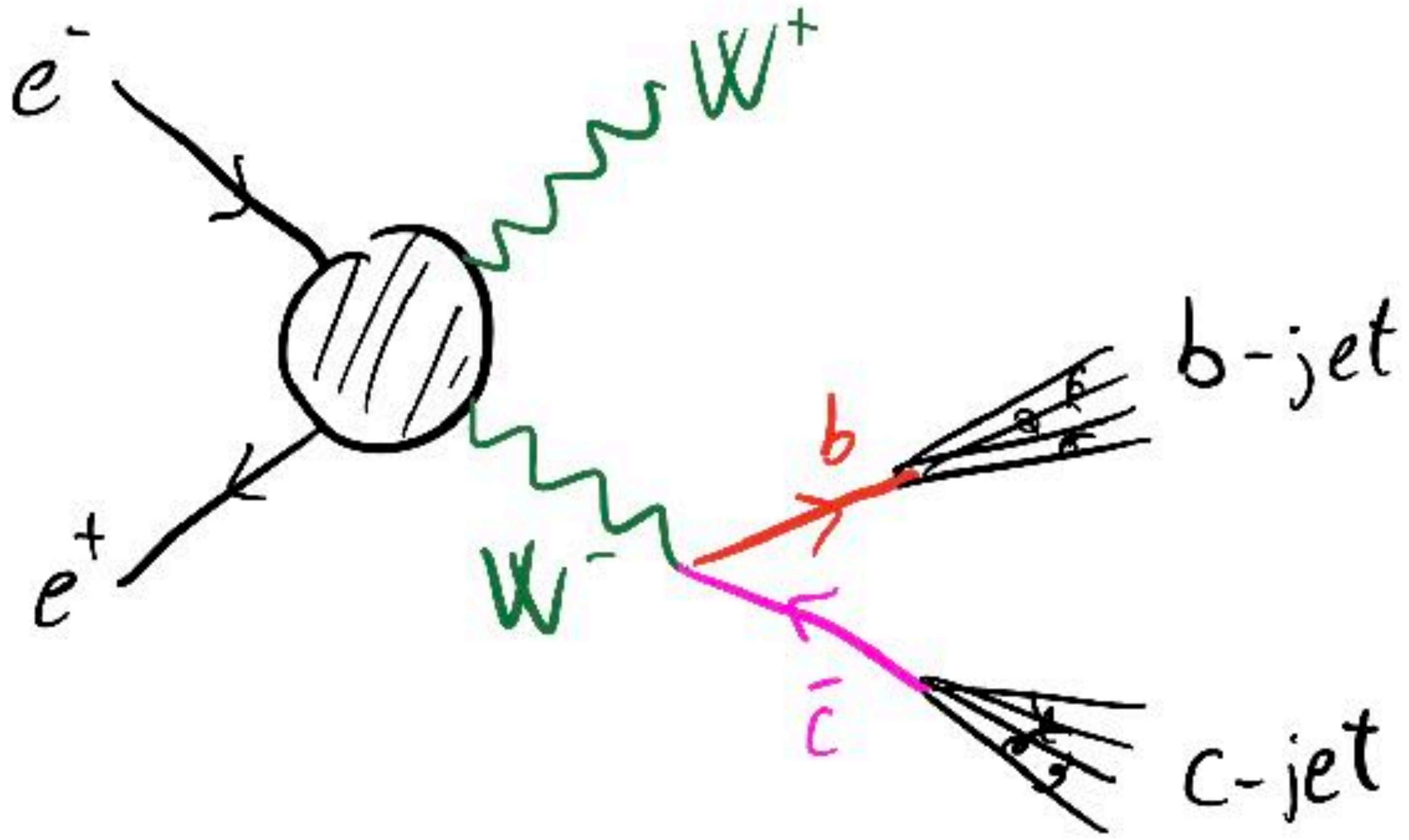
The scope

Assuming **~10⁸ W pairs** and a ***"perfect jet flavour tagger"***, the statistical precision achievable in each CKM ME would be:

$W^- \to$	$\bar{u}d$	$\bar{u}s$	$\bar{u}b$	$\bar{c}d$	$\bar{c}s$	$\bar{c}b$
BR	31.8%	1.7%	4.5×10^{-6}	1.7%	31.7%	5.9×10^{-4}
N_{ev}	64×10^6	3.4×10^6	900	3.4×10^6	63×10^6	118×10^3
$\delta^{th}_{V_{ij}}$	0.0063 %	0.027 %	1.7 %	0.027 %	0.0063 %	0.15 %

see [2401.07564]

0.0063% **0.15%**



- V_{cb} and V_{cs}: **good statistical** prospect, **good tagging**
- V_{ub}: **poor statistics**, **poor tagging** (no hope)
- Others: **good statistics**, **poor tagging**

We focus on V_{cb} and V_{cs}

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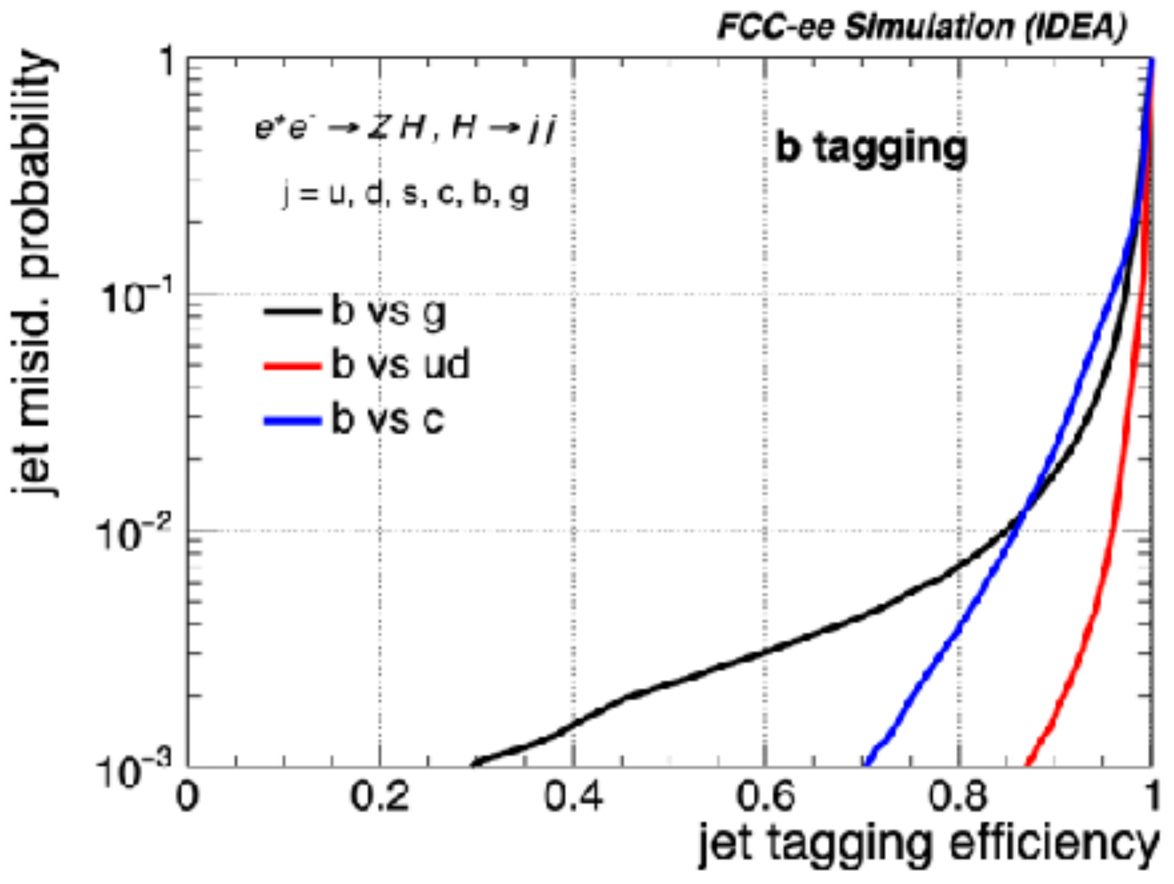
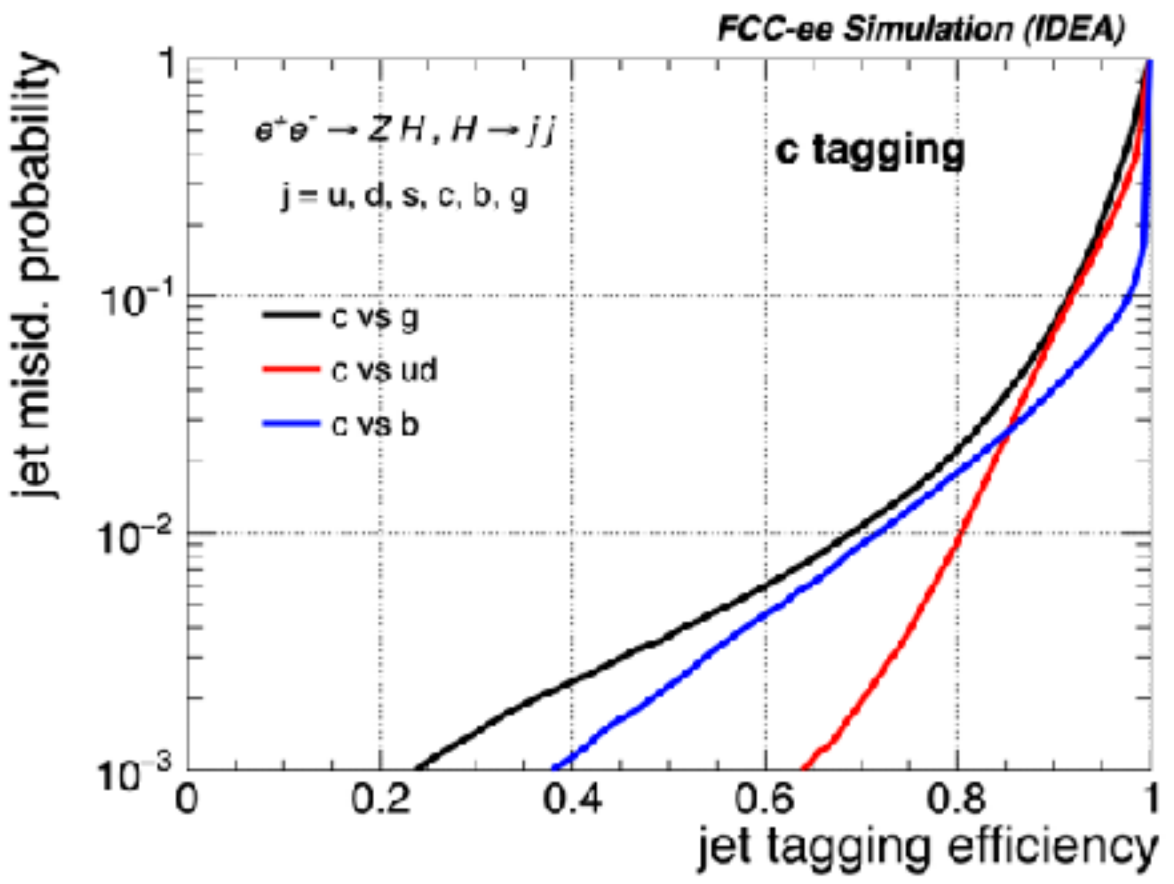
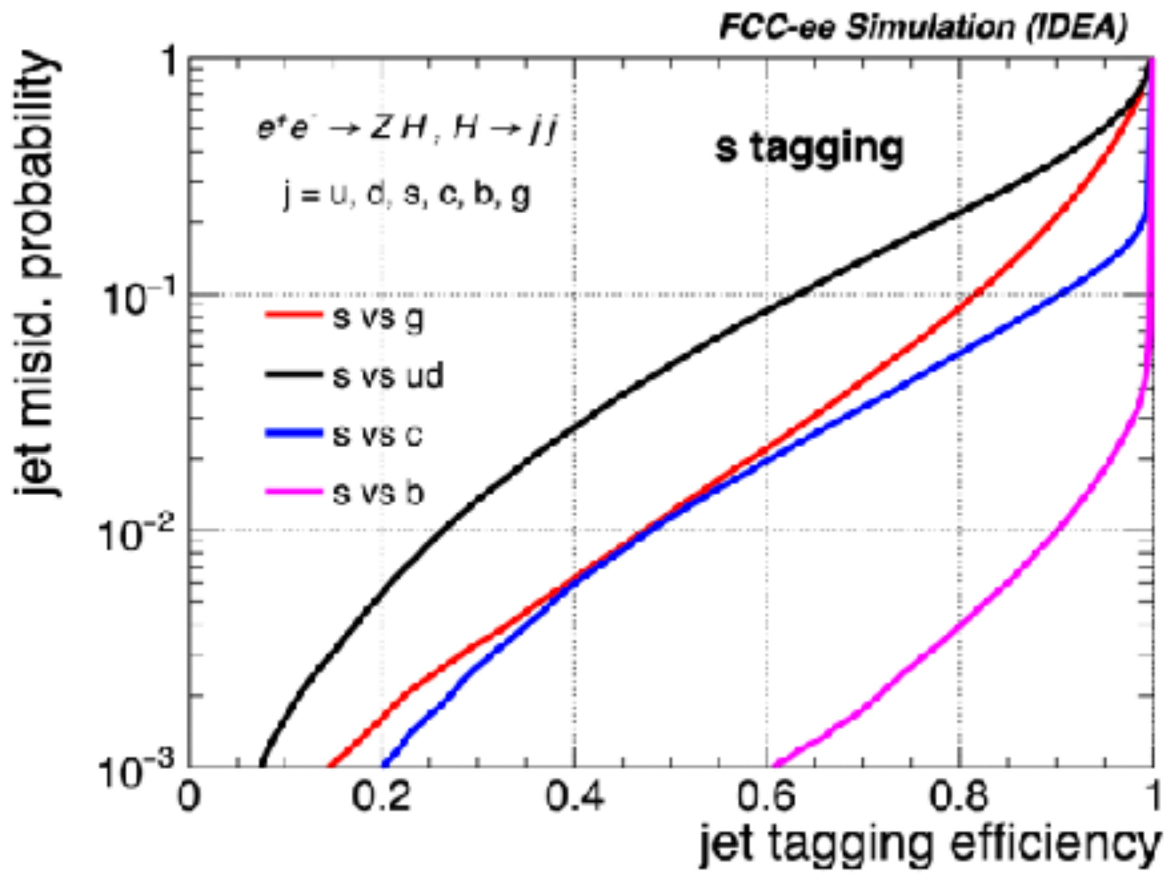
Jet Flavour Tagging

Exquisite jet flavour tagging performance at FCCee

[Bedeschi, Gouskos, Selvaggi 2202.03285 + updates 1, 2]







ε^q_β : probability of tagging a β-jet as a q-jet

FCC (IDEA) working point

	b	s	c	u	d	g
ϵ^b_β	0.8	0.0001	0.003	0.0005	0.0005	0.007
ϵ^c_β	0.02	0.008	0.8	0.01	0.01	0.01
ϵ^s_β	0.01	0.9	0.1	0.3	0.3	0.2

Similar performance at CEPC [2205.08553, 2310.03440]

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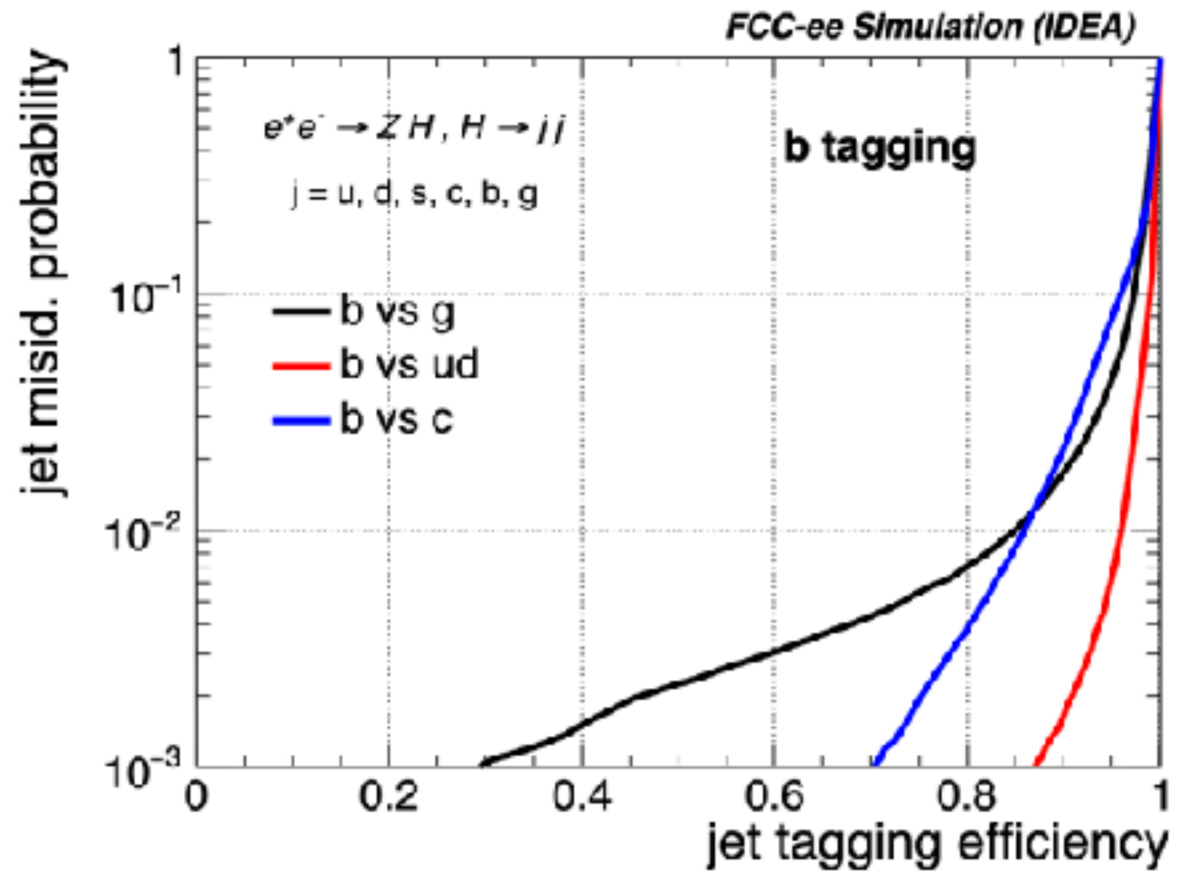

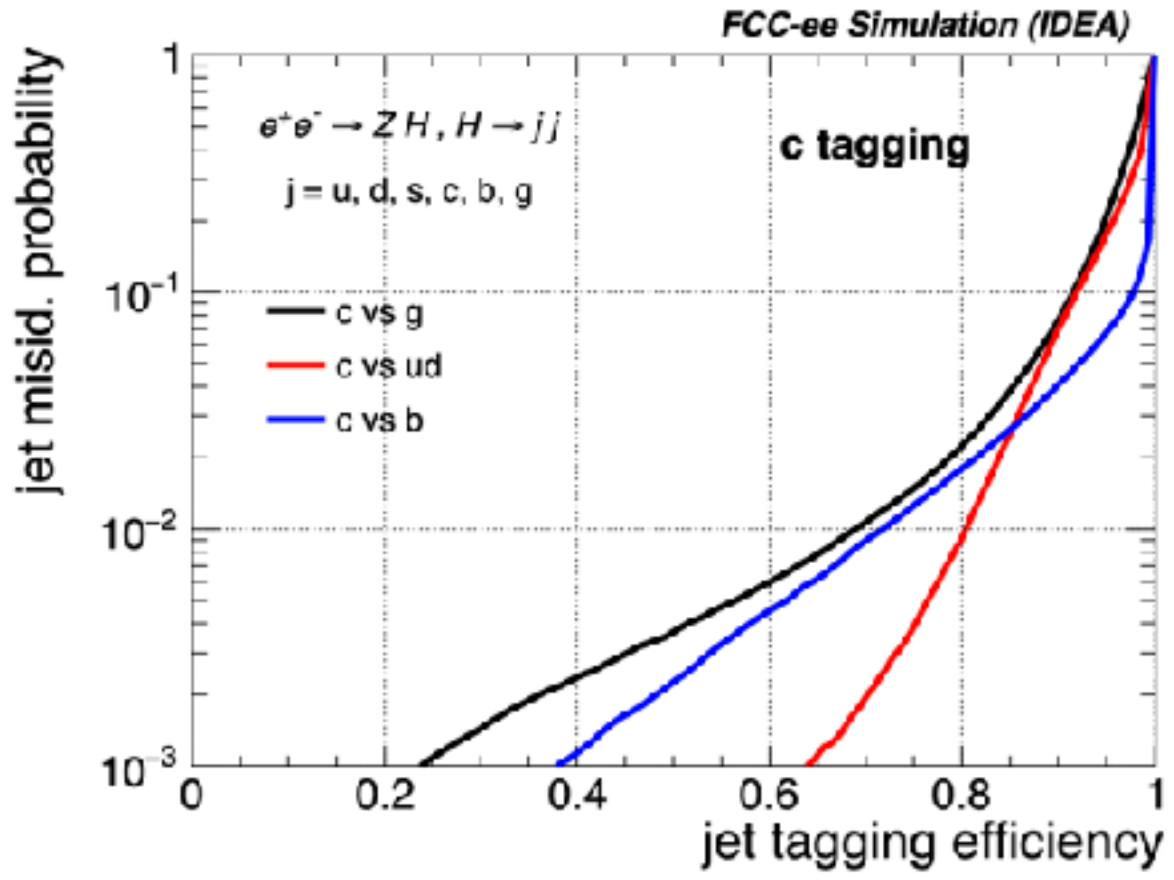

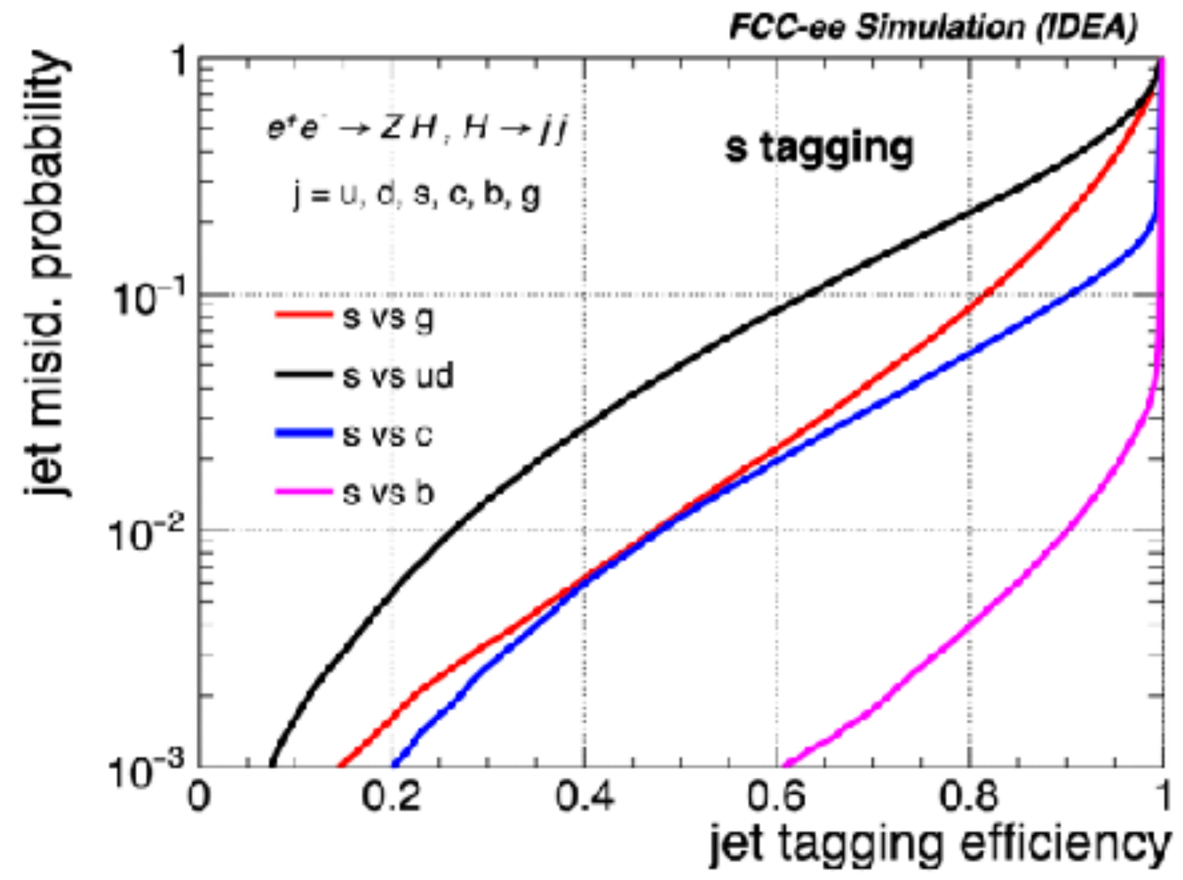

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Precise calibration thanks to ~**10^{12} Z decays** (R_b method [hep-ex/9810002]) allows **O(0.1%) precision** in the jet tagging efficiencies.

The impact of the **extrapolation of these jet tagging from the Z-pole run up to the WW threshold** should be carefully assessed.

First estimates

Using BDT-based **ILD jet-tagging performances** as a reference

[Charles et al 2006.04824]

	b	c	uds
Eff b-jet tagger	25%		
Eff c-jet tagger	10%	50%	2%

$$\delta V_{cb} \simeq 0.4\%$$

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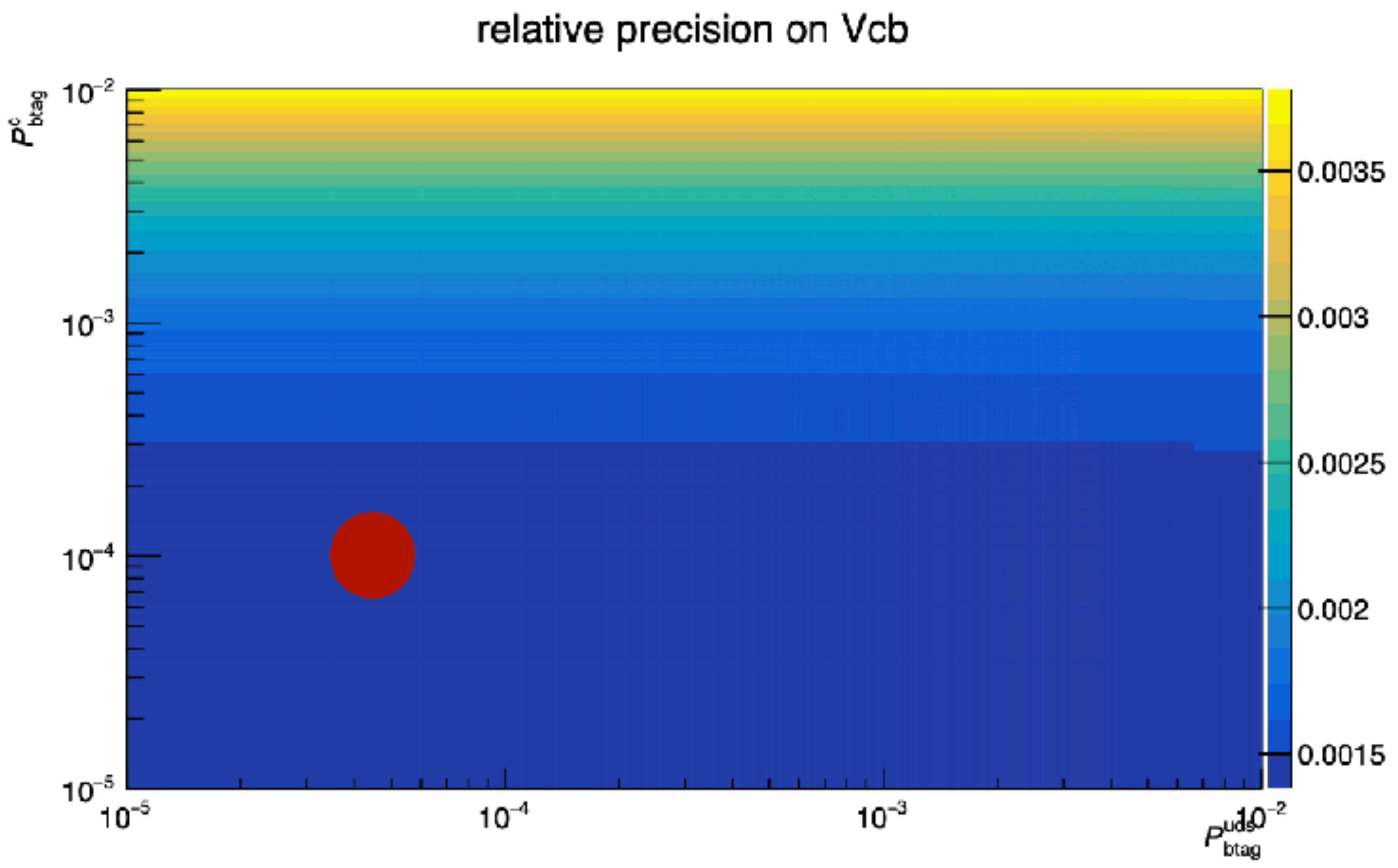
Revised using with optimized GNN-based IDEA performance data

Eff. \	*b*	*c*	*uds*
b-tag	0.87		
c-tag	1	0.65	0.0001

S. Monteil [7th FCCee Physics Workshop 2024 slides]



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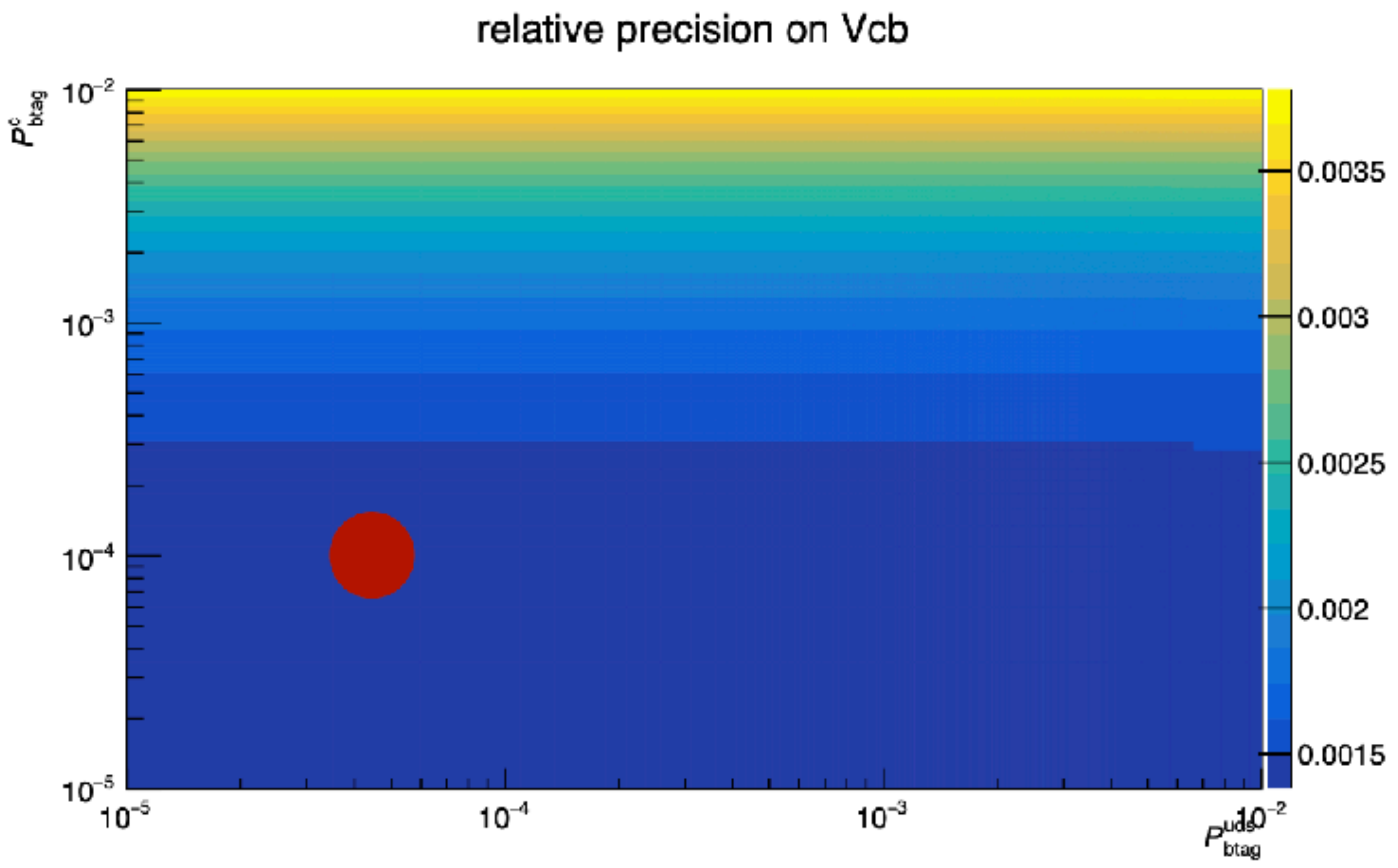

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Questions: What are the **prospects for V_{cs}?**

How do they depend on **systematic uncertainties?**

[DM, Szewc, Tammaro 2405.08880]

Strategy

Each $W \to u_i d_j$ depends on CKM elements as (normalised with hadronic Br to reduce QCD uncertainties)

$$\frac{\mathcal{B}_{ij}}{\mathcal{B}_{\text{had}}} \approx \frac{|V_{ij}|^2}{\sum_{l=u,c;\, m=d,s,b} |V_{lm}|^2}$$

Including acceptance, the expected fraction of events per channel is:

$$F_{ij} = \mathcal{A}_W \times \mathcal{B}_{ij}$$

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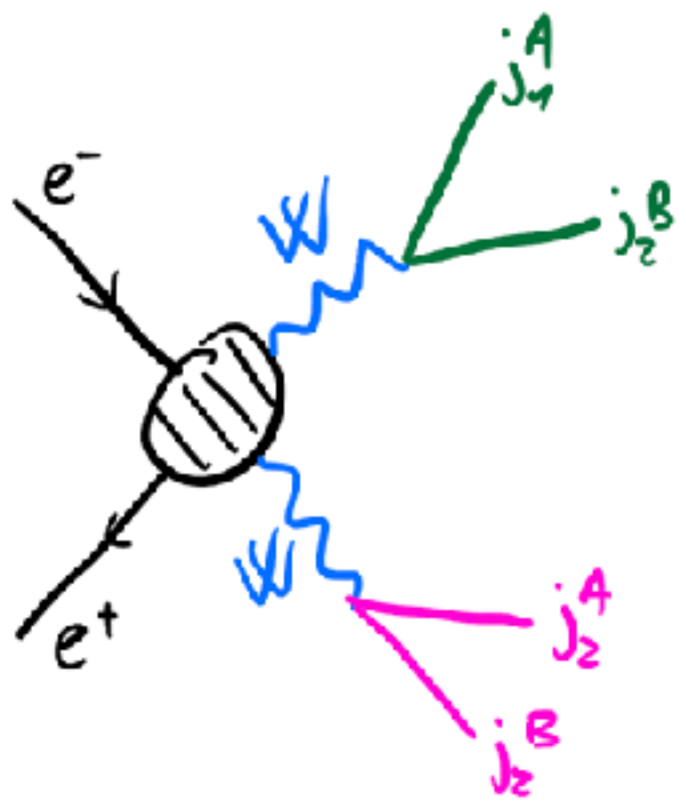
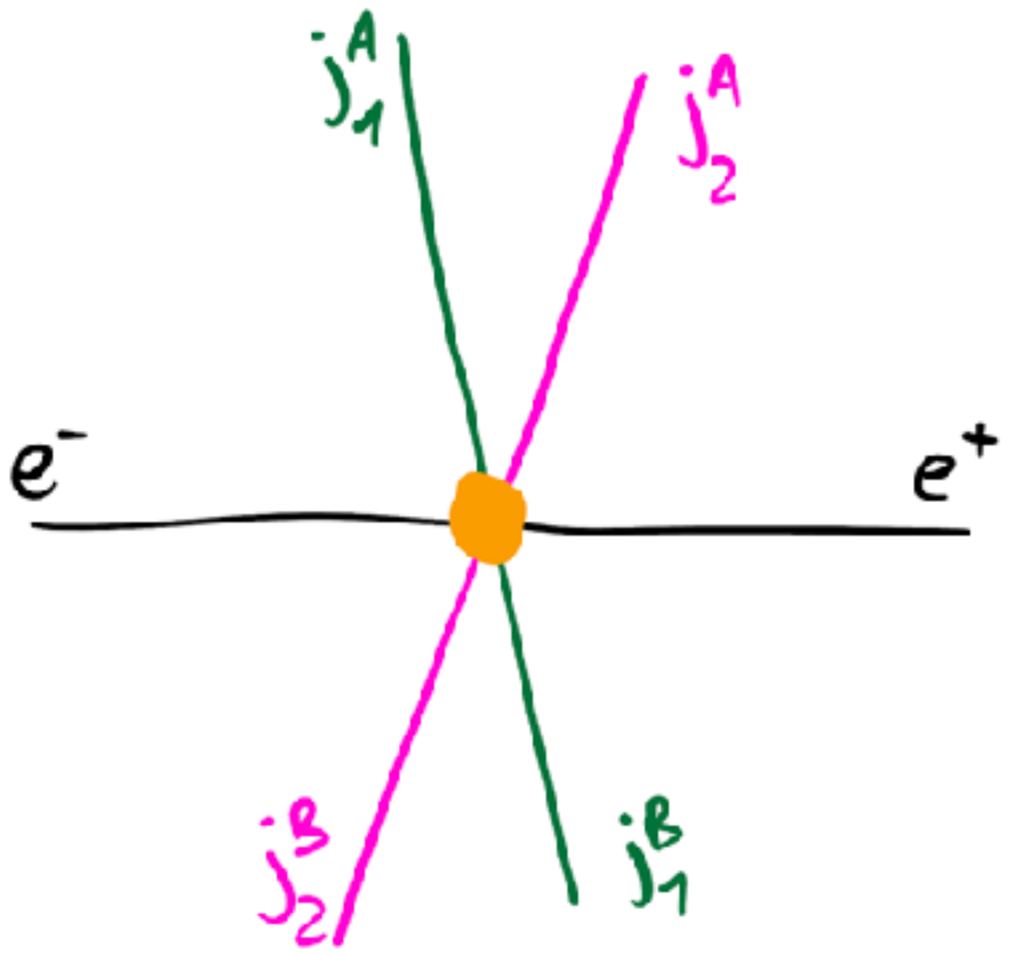
$$e^+e^- \to W^+W^- \to (u_i\bar{d}_j)\,(d_k\bar{u}_z)$$





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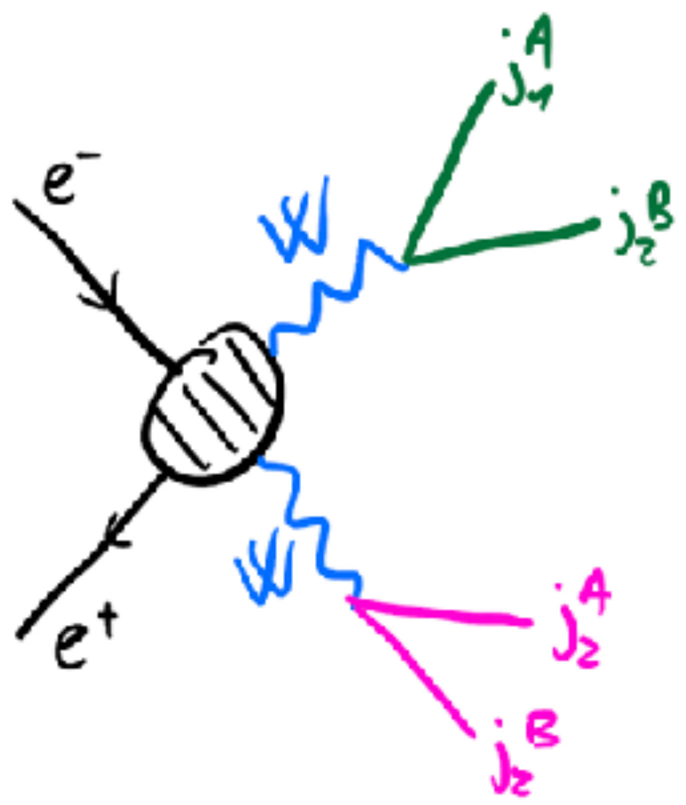

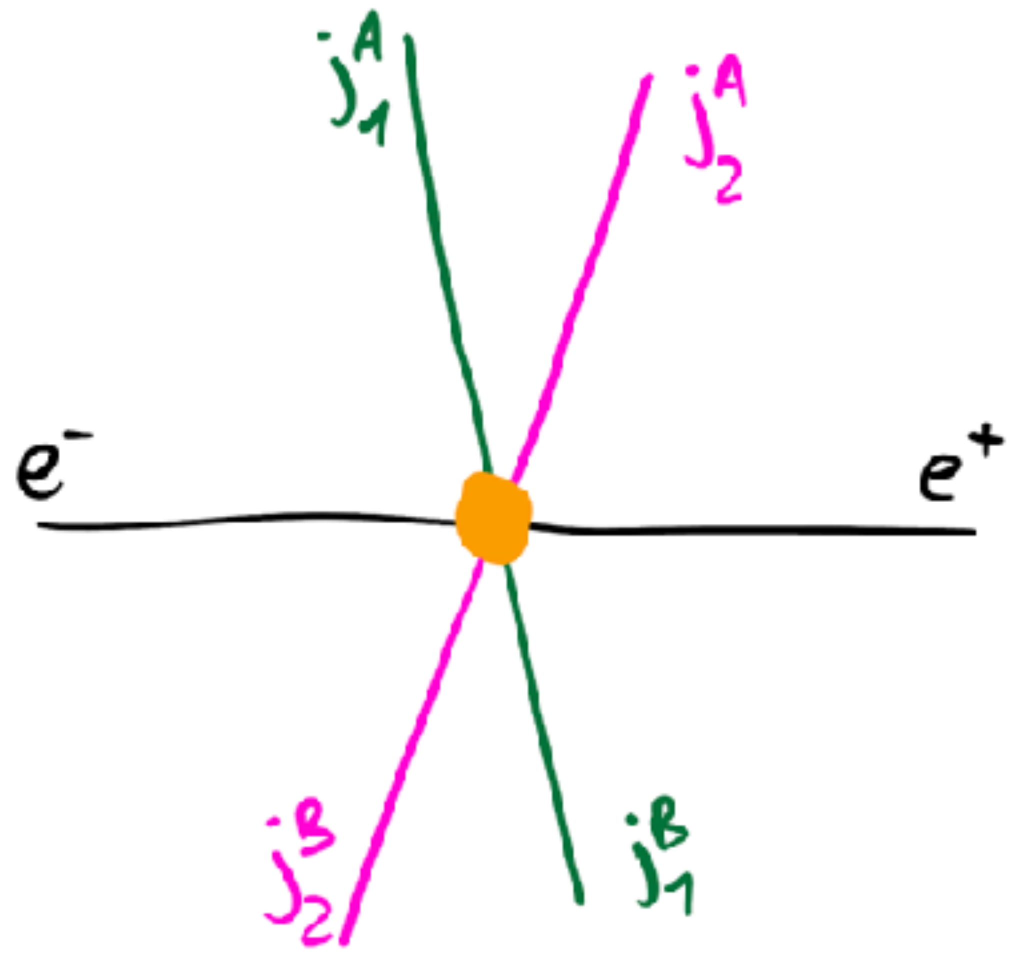

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Example for V_{cb}:

For each **pair of jets W → j^A j^B** in an event we **count the number of jets tagged as b and c:**

$$(n_b, n_c) = \{(0,0), (1,0), (0,1), (2,0), (0,2), (1,1)\}$$

Each of this "bin" has a **probability $P_W(n_b, n_c)$**. it **depends on F_{ij} and tagging efficiencies**.

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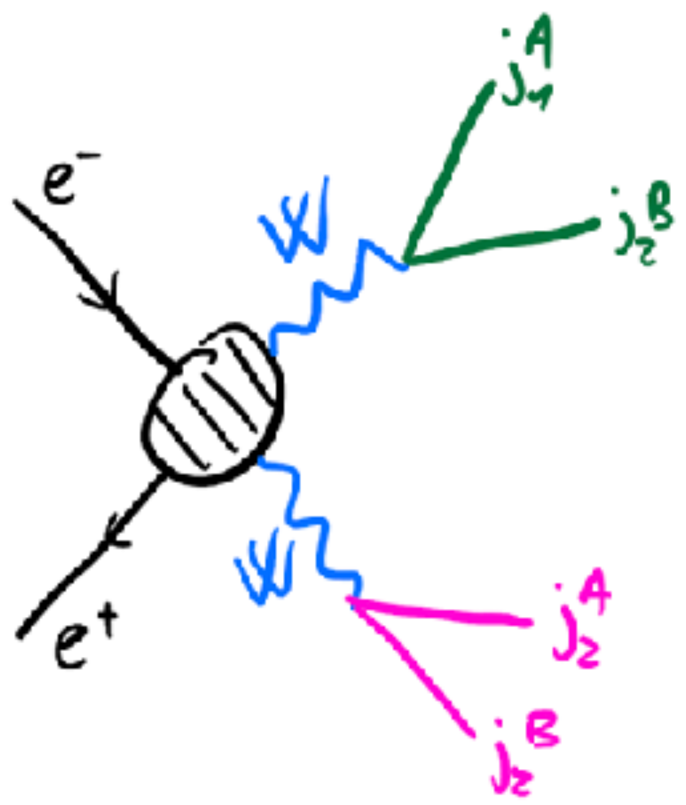

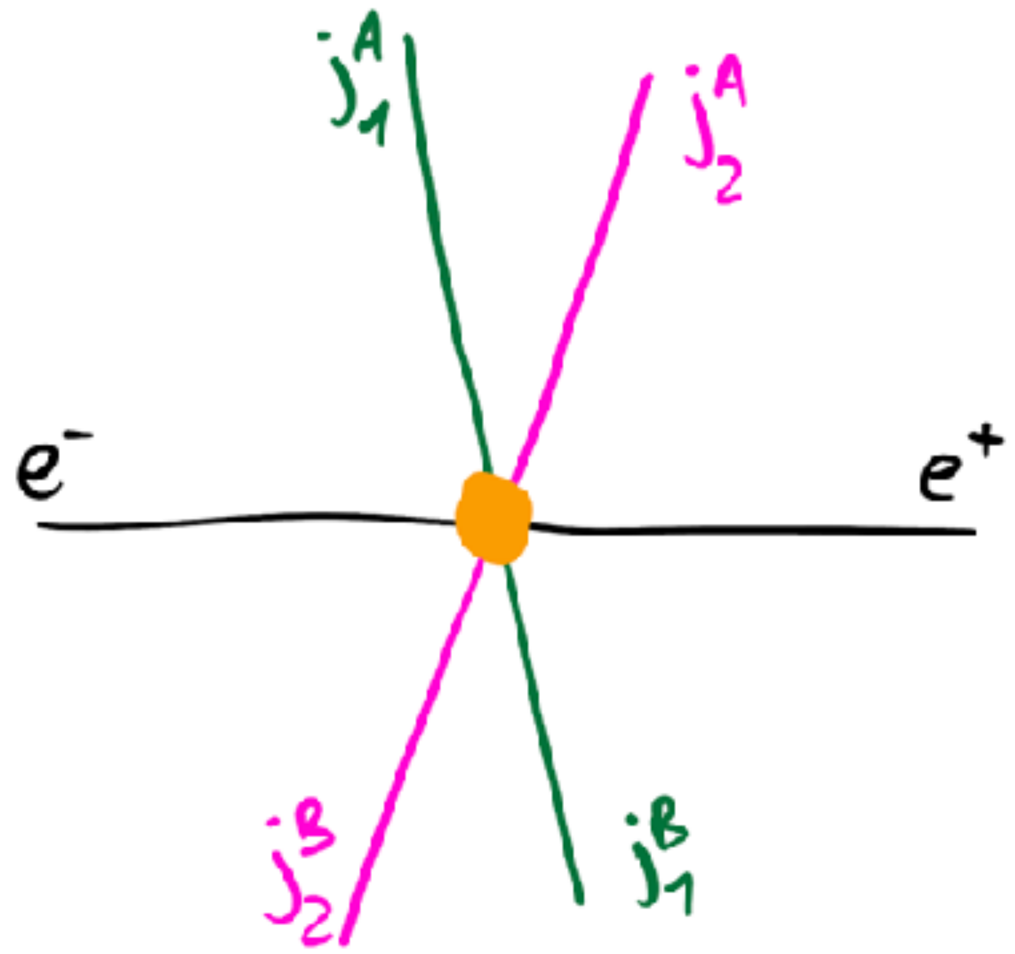

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Each event has 2 pairs, so we have two sets of bins:

$$B_{bc;1} \equiv (n_{b;1}, n_{c;1}) \qquad B_{bc;2} \equiv (n_{b;2}, n_{c;2})$$

The final **number of expected events** in each counting bin is:

$$N_{B_{bc;1},B_{bc;2}} = N_{WW}\, P_W(B_{bc;1})\, P_W(B_{bc;2})$$

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$$P_W(1,1) = \sum_{u_i=u,c}\sum_{d_j=d,s,b} (\epsilon^c_{u_i}\epsilon^b_{d_j} + \epsilon^b_{u_i}\epsilon^c_{d_j})\mathcal{B}_{ij}\mathcal{A}_W \ ,$$

$$P_W(0,2) = \sum_{u_i=u,c}\sum_{d_j=d,s,b} \epsilon^c_{u_i}\epsilon^c_{d_j}\mathcal{B}_{ij}\mathcal{A}_W \ ,$$

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Tagging efficiencies

$\varepsilon^q{}_\beta$

To each we assign a **relative uncertainty** (equal for all)

$$\delta_\epsilon \equiv \delta\epsilon/\epsilon$$

[DM, Szewc, Tammaro 2405.08880]

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The flavour decomposition of the two pairs of jets is the following (2q2g category):

W_1	$(\bar{d}d)$	$(\bar{s}s)$	$(\bar{b}b)$	$(\bar{u}u)$	$(\bar{c}c)$	$(\bar{d}g)$	$(\bar{s}g)$	$(\bar{b}g)$	$(\bar{u}g)$	$(\bar{c}g)$
W_2	(gg)	(gg)	(gg)	(gg)	(gg)	(dg)	(sg)	(bg)	(ug)	(cg)
σ [fb]	14	14	6.9	20	22	15	15	8.7	19	22
N_{ev} [10^6]	1.7	1.7	0.83	2.4	2.7	1.8	1.8	1.1	2.3	1.6

Very small mistag rates of a gluon-jet into a b- or c-jet: 0.007 and 0.01

We checked that the impact on CKM extraction is negligible.

SM corrections

$$\frac{\mathcal{B}_{ij}}{\mathcal{B}_{\rm had}} \approx \frac{|V_{ij}|^2}{\sum_{l=u,c;\, m=d,s,b} |V_{lm}|^2}$$

QCD and EW corrections mostly cancel in this ratio:
small kinematical effects, could be easily taken into account.

$$\Gamma^{(0)} = \frac{\sqrt{2} G_F N_c}{24\pi} \sum_{\rm quarks\, i,j} \frac{\kappa\left(m_W^2, m_{q,i}^2, m_{q',j}^2\right)}{m_W}\left(2m_W - m_{q,i}^2 - m_{q',j}^2 - \frac{(m_{q,i}^2 - m_{q',j}^2)^2}{m_W^2}\right)|V_{ij}|^2$$

$$\Gamma_{\rm had}^{W} = \Gamma^{(0)}\left[1 + \sum_{i=1}^{4} c_{\rm QCD}^{(i)} \cdot \left(\frac{\alpha_s}{\pi}\right)^i + \delta_{\rm ewk}(\alpha) + \delta_{\rm mixed}(\alpha\alpha_s)\right]$$

Hadronic width can be computed at N^3LO, used to extract precise value of $\alpha_s(m_W)$.
[e.g. d'Entrerria, Srebre 1603.06501 + d'Entrerria, Jacobsen 2005.04545]

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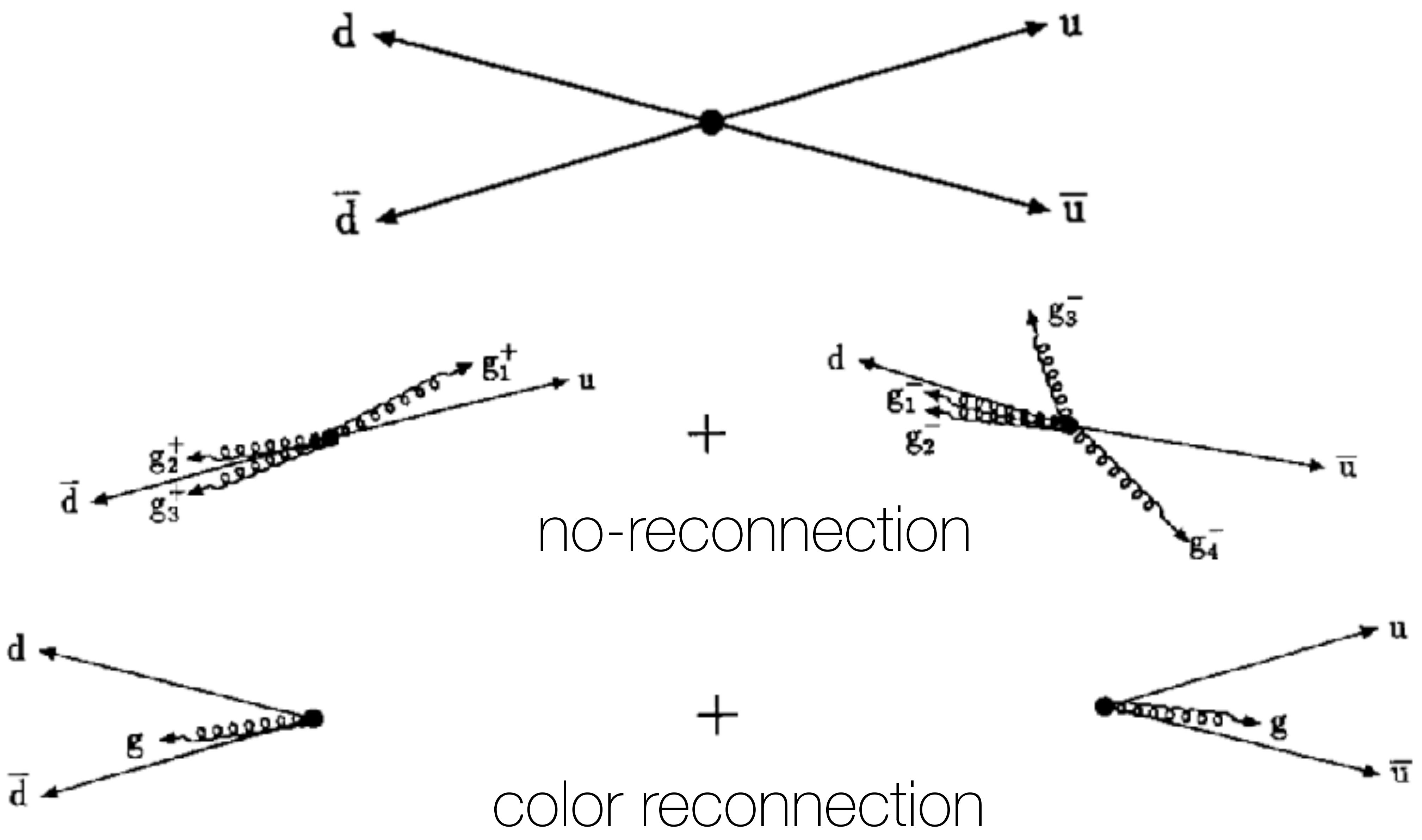
Color reconnection can affect the hadron distribution in the WW → 4j process.
Its understanding is crucial for a precise m_W measurement: modelled in showering algorithms.

Gustafson, Pettersson, Zerwas '88, Sjostrand and Khoze '93,
Christiansen and Sjöstrand [1506.09085]

$$W^+W^- \to u\,\bar{d} + d\,\bar{u}$$

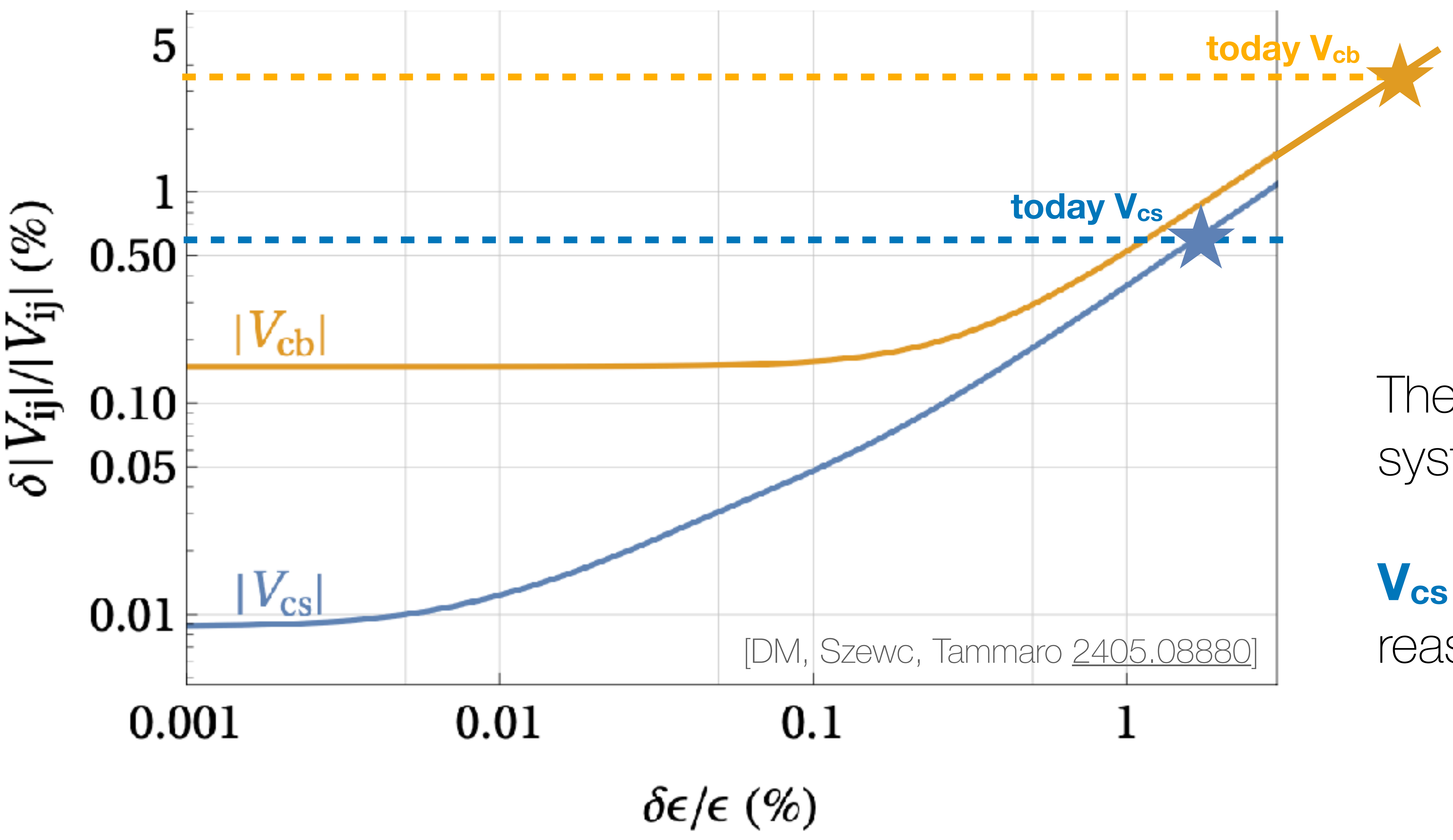


We assume any related systematic uncertainties can be **"described" by the systematic uncertainties** associated to the jet tagging efficiencies.

<u>Open question</u>: what is the impact of this in CKM extraction?

Results

Fixing the **efficiencies working point** at the **FCC (IDEA)** one.



Parameter	Value
N_{WW}	3×10^8
$\mathrm{Br}(W \to \mathrm{had})$	0.6741
$\mathrm{Br}(W \to \ell\nu)$	0.3278
$\mathcal{A}_W$	0.9

The precision on V_{cb} saturates at per-mille level of systematic uncertainties, due to limited statistics.

V_{cs} instead is never statistically limited for any reasonable value of systematic uncertainties.

Considerable improvement in V_{cb} and V_{cs} extraction compared to present (and future) measurements are expected, **for any systematic uncertainty below the 1% level**.

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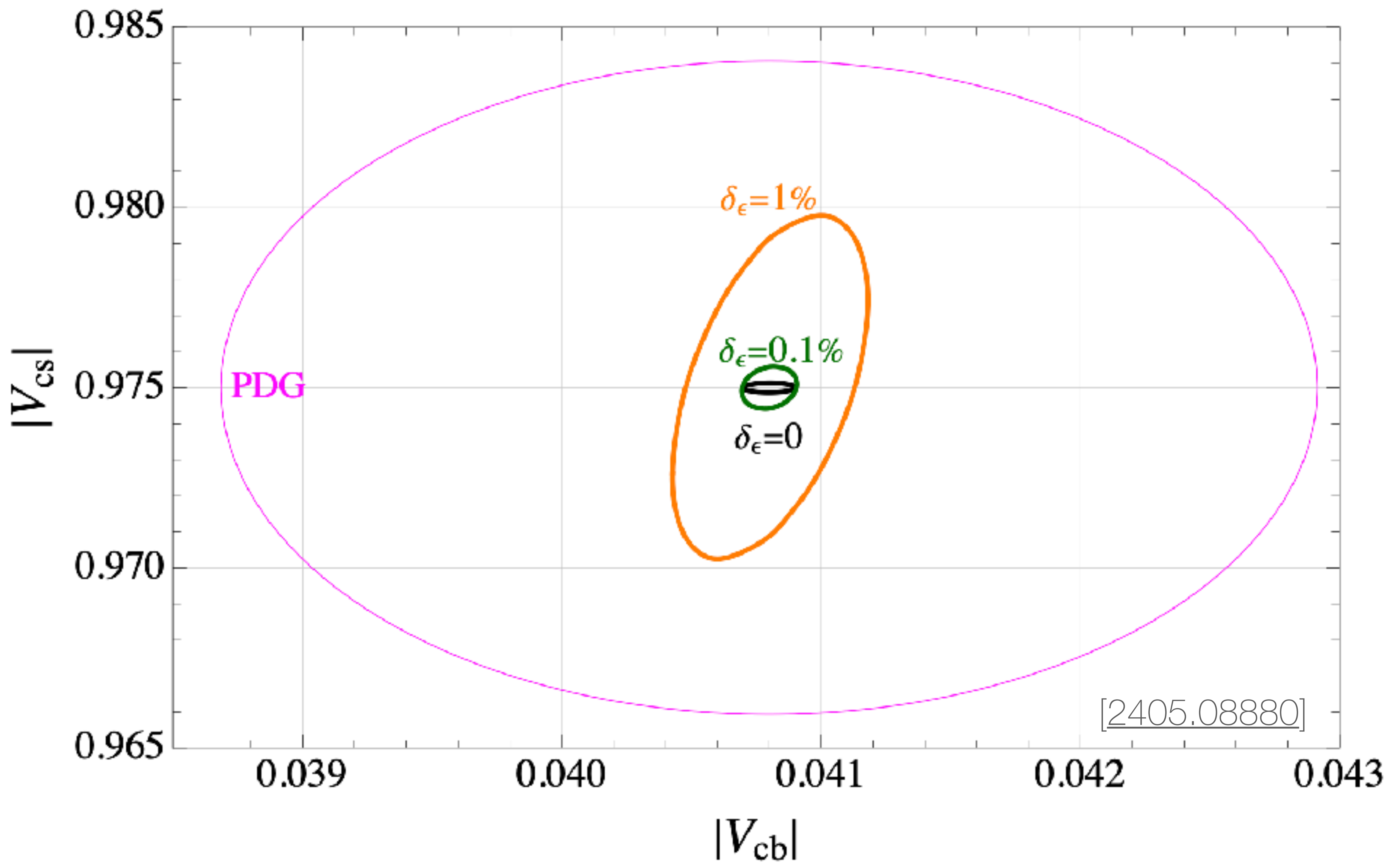

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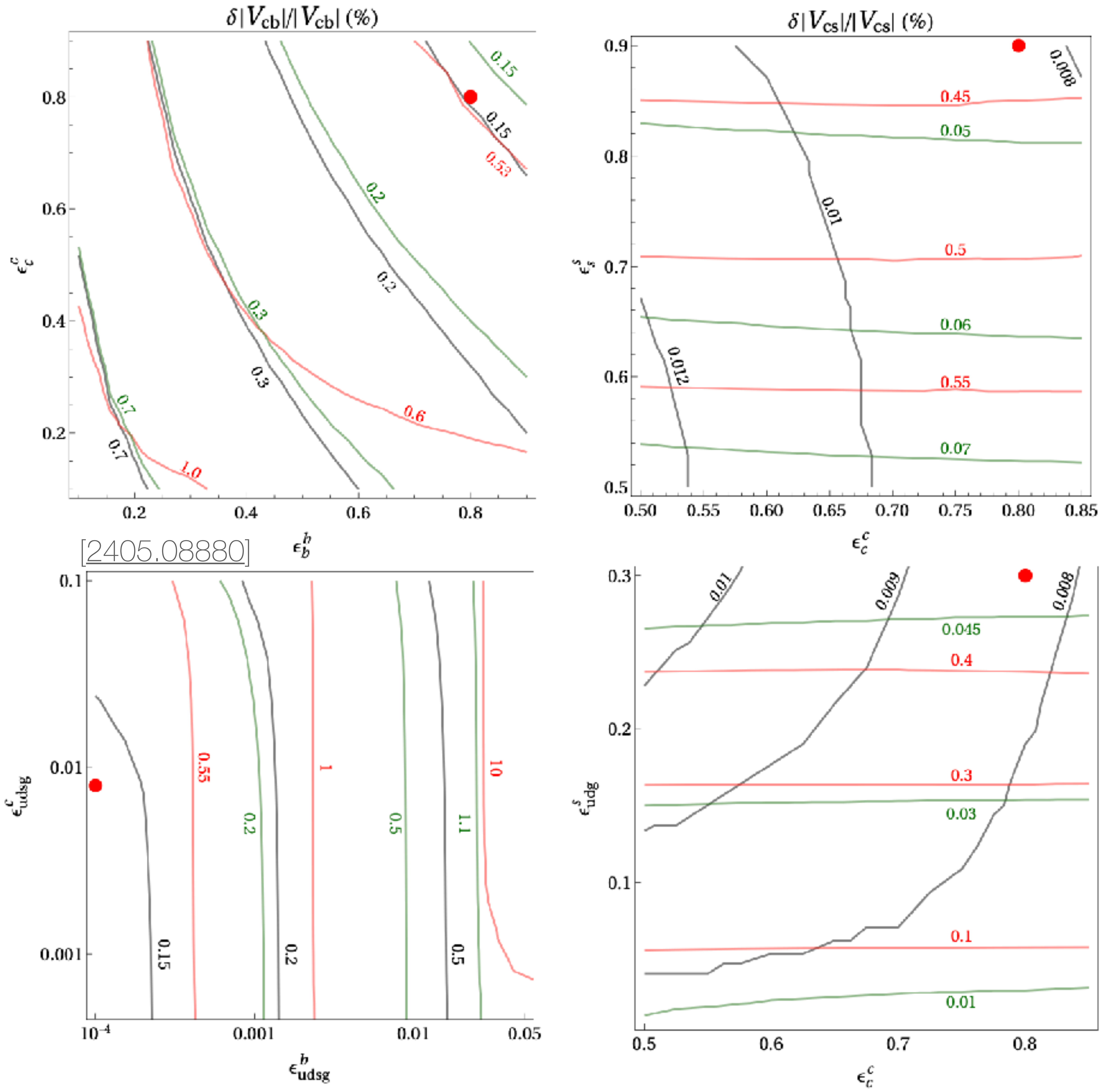
$\lvert V_{ij}\rvert$	Current (PDG)		FCC-ee ($\delta_\epsilon = 1\%$)	FCC-ee ($\delta_\epsilon = 0.1\%$)
$\lvert V_{cs}\rvert$	0.975 ± 0.006	(0.6%)	0.36%	0.05%
$\lvert V_{cb}\rvert$	$(40.8 \pm 1.4) \times 10^{-3}$	(3.4%)	0.52%	0.16%

Considerable improvement in V_{cb} and V_{cs} extraction compared to present (and future) measurements are expected, **for any systematic uncertainty below the 1% level**.

Dependence on the working point



— Statistics only

— 1% Systematics • FWP

— 0.1% Systematics

V_{cb}:

- ~ **linear dependence on ε^b_b and ε^c_c** for small syst., less for 1% syst.
- Important a **good rejection of light jets from b-tagger**
- Looser rejection for the c-tagger acceptable

(since if the b is tagged correctly, the only alternative decay is W→ ub, which is very rare)

V_{cs}:

- With systematics, the **sensitivity is driven by the s-tagger (ε^s_s and ε^s_{udg})**.
- c-tagging performance less critical.

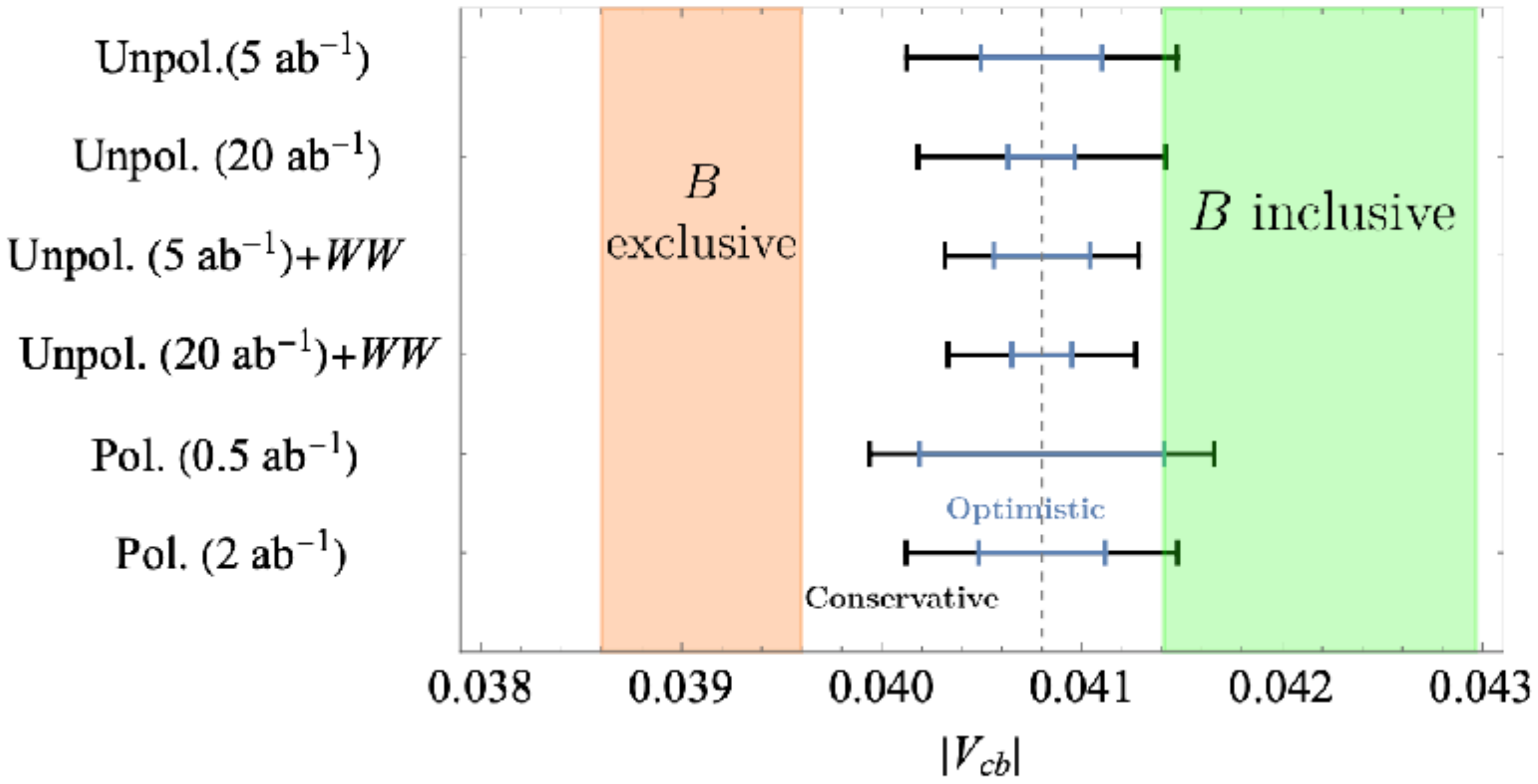
Semileptonic channel

The semileptonic channel pays the price of **smaller leptonic W branching ratio**.
On the other hand, it is **free from Drell-Yan background and color reconnection effects**.

A recent paper performed a **full detector simulation** to obtain the extraction of **$|V_{cb}|$** from **semileptonic events.**

[Liang, Feng-Li, Zhu, Schen, Ruan 2406.01675]

$\varepsilon^b{}_b \sim 91\%, \quad \varepsilon^b{}_c \sim 4\%, \quad \varepsilon^b{}_{sdu} \sim 0.5\%$



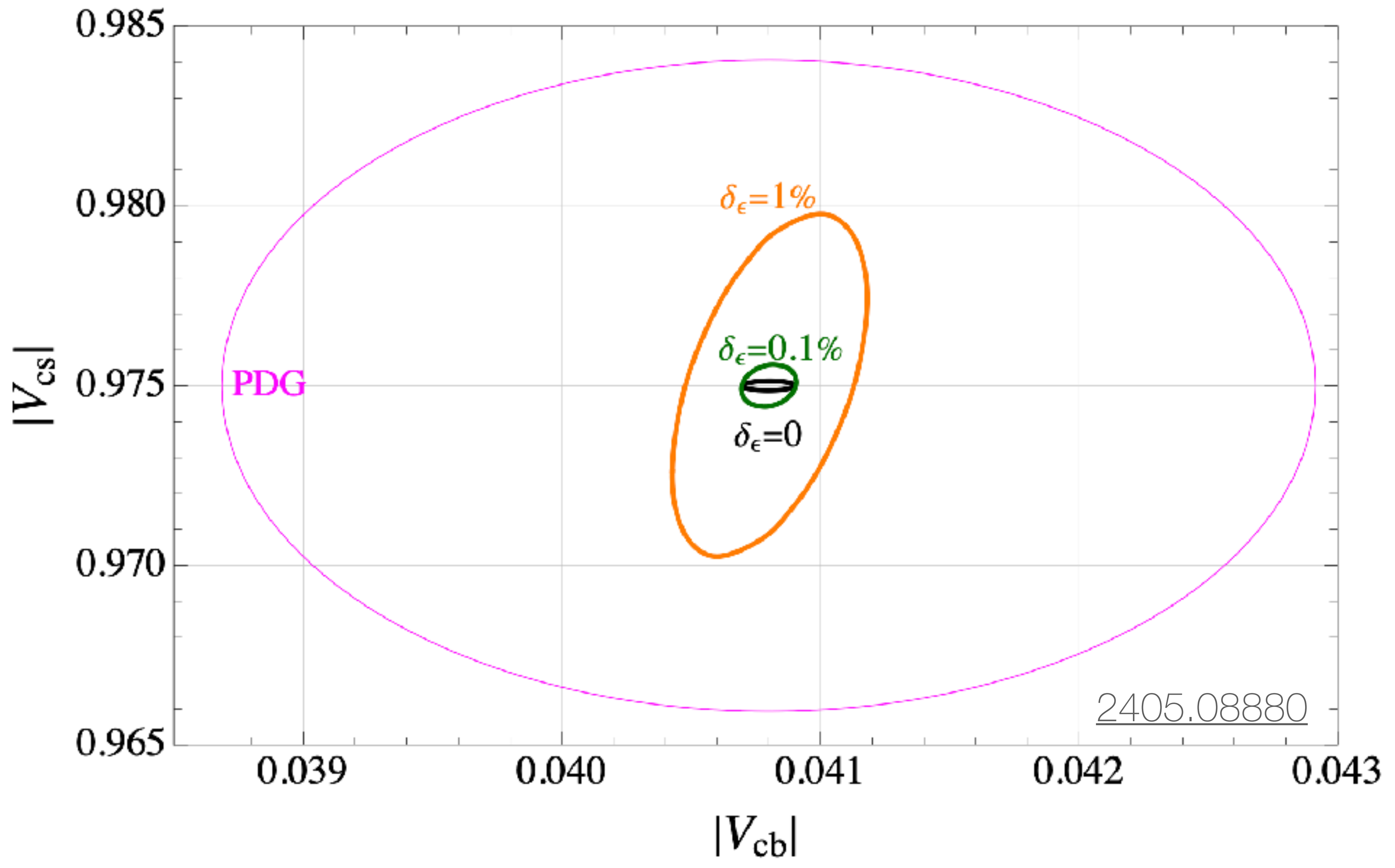
$$\delta V_{cb} \simeq \pm 0.34\%(\text{stat.}) \pm 0.2 - 1.5\ \%(\text{syst})$$

The largest source of uncertainty is determined as the one associated to flavour tagging (and mistagging rates) of b/c jets.

Conclusions and Outlook

Conclusions:

- **V_{cb}** and **V_{cs}** measurement from W decays should clearly be an important goal in flavour physics at future e^+e^- colliders
- The prospects are **well beyond anything possible via meson decays** (ultimately limited by QCD uncertainties): one order of magnitude improvement.
- This will permit the full exploitation of New Physics sensitivity from rare flavour-changing processes (both from Higgs/EW factories and from legacy flavour factories).



Open questions:

- Review of the **state-of-the-art Flavour Tagging** (FT) algorithms (**detector requirements?**)
- FT **calibration** methods and related **systematics**.
- **Full simulation study** for the WW → 4j channel.

Extra

Measuring V_{cs} from meson decays

$|V_{cs}|$ is currently measured from **tree-level leptonic D_s or semi-leptonic D decays**.

Experiments: Belle, CLEO-c, BABAR, BESIII

$D_s \to \mu\nu, \tau\nu$

$\oplus$

f_{D_s} : Lattice QCD

$$f_{D_s} = (249.9 \pm 0.5)\,\mathrm{MeV}$$

$|V_{cs}| = 0{,}984 \pm 0{,}012$

mostly experimental uncertainty

$D \to K \ell \nu$

$f_+^{DK}(0)$: Lattice QCD

$$f_+^{DK}(0) = 0.7385 \pm 0.0044$$

$|V_{cs}| = 0{,}972 \pm 0{,}007$

mostly form factor uncertainty

PDG average:

$$|V_{cs}| = 0.975 \pm 0.006$$

Statistical procedure

[DM, Szewc, Tammaro 2405.08880]

$$N_{B_{bc;1},B_{bc;2}}(\mu,\nu) = N_{WW} \prod_{i=1,2} \sum_f p(n_{b;i}, n_{c;i}|f,\nu) F_f(\mu,\nu)$$

$f = \{ud, us, ub, cd, cs, cb\}$

probability that the final state f goes in the bin (n_b, n_c)

Let us define the probability that each hadronic W decay goes in the bin (n_b, n_c):

$$P_W(n_b, n_c) \equiv \sum_f p(n_b, n_c|f,\nu) F_f(\mu,\nu)$$

$$N_{B_{bc;1},B_{bc;2}} = N_{WW}\, P_W(B_{bc;1})\, P_W(B_{bc;2})$$

$$P_W(1,1) = \sum_{u_i=u,c}\sum_{d_j=d,s,b} (\epsilon^c_{u_i}\epsilon^b_{d_j} + \epsilon^b_{u_i}\epsilon^c_{d_j})\mathcal{B}_{ij}\mathcal{A}_W\ ,$$

$$P_W(0,2) = \sum_{u_i=u,c}\sum_{d_j=d,s,b} \epsilon^c_{u_i}\epsilon^c_{d_j}\mathcal{B}_{ij}\mathcal{A}_W\ ,$$

$$P_W(2,0) = \sum_{u_i=u,c}\sum_{d_j=d,s,b} \epsilon^b_{u_i}\epsilon^b_{d_j}\mathcal{B}_{ij}\mathcal{A}_W\ ,$$

$$P_W(0,1) = \sum_{u_i=u,c}\sum_{d_j=d,s,b} (\epsilon^c_{u_i}(1-\epsilon^c_{d_j}-\epsilon^b_{d_j}) + \epsilon^c_{d_j}(1-\epsilon^c_{u_i}-\epsilon^b_{u_i}))\mathcal{B}_{ij}\mathcal{A}_W\ ,$$

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$$P_W(0,0) = \sum_{u_i=u,c}\sum_{d_j=d,s,b} (1-\epsilon^c_{u_i}-\epsilon^b_{u_i})(1-\epsilon^c_{d_j}-\epsilon^b_{d_j})\mathcal{B}_{ij}\mathcal{A}_W\ ,$$

The likelihood is then given by (Gaussian approx):

$$-2\ln\mathcal{L} = \sum_{B_{bc;1},B_{bc;2}} \frac{\left(N_{B_{bc;1},B_{bc;2}} - N^A_{B_{bc;1},B_{bc;2}}\right)^2}{N_{B_{bc;1},B_{bc;2}}} + \chi^2_{\rm tag}$$

$$\chi^2_{\rm tag} = \sum_{q=b,c}\sum_{\beta=b,s,c,u,d,g}\left(\frac{\epsilon^q_\beta - \hat\epsilon^q_\beta}{\delta_\epsilon \hat\epsilon^q_\beta}\right)^2$$