Flavor violating Higgs and Z decays at FCC-ee

8th FCC Physics Workshop, CERN

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Based on the work:

J. F. Kamenik, AK, M. Szewc, M. Tammaro and J. Zupan, Phys. Rev. D, 109.L011301

arXiv: 2306.17520



OUTLINE

- Current status of H/Z flavor-violating decays
- Analysis framework
- Results
- Conclusions

CURRENT STATUS

Decay	SM prediction	exp. bound	indir. constr.
$\mathcal{B}(h \to bs)$	$(8.9 \pm 1.5) \cdot 10^{-8}$	0.16	2×10^{-3} \bigstar
$\mathcal{B}(h \to bd)$	$(3.8 \pm 0.6) \cdot 10^{-9}$	0.16	10^{-3}
$\mathcal{B}(h \to cu)$	$(2.7 \pm 0.5) \cdot 10^{-20}$	0.16	2×10^{-2} \bigstar
$\mathcal{B}(Z \to bs)$	$(4.2 \pm 0.7) \cdot 10^{-8}$	2.9×10^{-3}	6×10^{-8} \bullet
$\mathcal{B}(Z \to bd)$	$(1.8 \pm 0.3) \cdot 10^{-9}$	2.9×10^{-3}	6×10^{-8} \bullet
$\mathcal{B}(Z \to cu)$	$(1.4 \pm 0.2) \cdot 10^{-18}$	2.9×10^{-3}	4×10^{-7} \bullet

 $\mathcal{B}(h \to bs) \equiv \mathcal{B}(h \to \overline{b}s + b\overline{s}) \qquad \land h \to u$

- $Z \rightarrow \text{had (hep-ex/0012018)}$
- ★ Meson mixing (no large cancellations)
- Global fits (flavio, smelli)

▲ $h \rightarrow$ undet (ATLAS + CMS, 2207.00043)





- Maximum likelihood estimate (MLE) for the FV branching ratios for H and Z
- - Jet tagging efficiencies $\epsilon_{\alpha}^{\beta}$
 - Detector acceptance \mathscr{A} , number of produced Z's or Higgses $N_{Z/h}$

• The likelihood is going to be a function of the FV branching ratio (**POI**) and **nuisance parameters**:

• Branching ratios of the background processes (flavor-conserving decays with mis-tagged jets)

No Monte Carlo generated data needed in this case (has been done: <u>see the ZHvvjj fit</u>, results compatible!)



1) Controlled / clean background

 $\sqrt{s} = m_Z$

1905.03764

FCC Conceptual Design Reports G. Marchiori's talk at "Higgs Performance meeting" (indico.cern.ch/event/1221257)

 $\sqrt{s} = 240 \,\text{GeV}$

Z(nunu)H(jj) @ FCC-ee

See yesterday's <u>talk by Alexis</u>

Other backgrounds ($\tau\tau$ for *Z*, DY, WW, ZZ for *h*) can be neglected

G. Marchiori's talk at "FCC Physics Workshop" (indico.cern.ch/event/1176398/)

Parameters	Nominal value	Rel. uncert. (i
$\mathcal{B}(Z \to uu + dd)$	27.01%	5.0
$\mathcal{B}(Z \to ss)$	15.84%	3.8
$\mathcal{B}(Z \to cc)$	12.03%	1.7
$\mathcal{B}(Z \to bb)$	15.12%	0.33
N_Z	5×10^{12}	10^{-3}
${\cal A}$	0.994	10^{-3}

Parameters	Nominal Value	Rel. uncert. (
$\mathcal{B}(h \to gg)$	1.4%	1.2
$\mathcal{B}(h ightarrow ss)$	0.024%	160
$\mathcal{B}(h \to cc)$	2.9%	2.8
$\mathcal{B}(h \to bb)$	56%	0.4
N_h	$6.7 imes 10^5$	0.5
${\cal A}$	0.70	0.1





- 2) Advancements in jet flavor tagging
- Providing tagging & mistag efficiencies $\epsilon_{\beta}^{\alpha}$

The *q*-tagger rates for Higgs:

$$\epsilon^q_\beta = \{g, s, c, b\}$$

The *q*-tagger rates for *Z*:

$$\epsilon^q_\beta = \{ud, s, c, b\}$$

Applying 2 taggers, we have 8 tagger efficiencies for h/ZAssuming 1% systematics

ParticleNet: 1902.08570 Jet-Flavor tagging at FCC-ee: 2210.10322 Bedeschi, Gouskos, Selvaggi, 2202.03285





Analysis framework 3) Probabilistic model (n_b, n_s)

Tag bins $(n_b, n_s) = \{(0, 0), (0, 1), (1, 0), (2, 0), (0, 2), (1, 1)\}$

ATLAS: 2201.11428 CMS: 2004.12181 Faroughy, Kamenik, Szewc, Zupan: 2209.01222

 $\mathcal{L}(\boldsymbol{\mu}, \boldsymbol{\nu}) = \prod \operatorname{Pois}\left(N_{(n_b, n_s)}^A | \bar{N}_{(n_b, n_s)}(\boldsymbol{\mu}, \boldsymbol{\nu})\right) p(\boldsymbol{\nu})$

3) Probabilistic model

$$\mathcal{L}(\boldsymbol{\mu}, \boldsymbol{\nu}) = \prod_{(n_b, n_s)} \operatorname{Pois}\left(N_{(n_b, n_s)}^A | \bar{N}_{(n_b, n_s)}(\boldsymbol{\mu}, \boldsymbol{\nu})\right) p(\boldsymbol{\nu})$$

 $\bar{N}_{(n_b,n_s)} = \sum p(n_b,n_s|f,\nu)\bar{N}_f(\nu)$ f = bkg, bs

ATLAS: 2201.11428 CMS: 2004.12181 Faroughy, Kamenik, Szewc, Zupan: 2209.01222

Expected number of events in bin (n_b, n_s) - sum over each decay channel

(
$$u$$
) $\bar{N}_f = \mathcal{B}(Z/h \to f) N_{Z/h} \mathcal{A}$
 $\bar{N}_{bs} = \mu \mathcal{B}(Z/h \to bs)_{SM} N_{Z/h} \mathcal{A}$





3) Probabilistic model

$$\mathcal{L}(\boldsymbol{\mu}, \boldsymbol{\nu}) = \prod_{(n_b, n_s)} \operatorname{Pois}\left(N_{(n_b, n_s)}^A | \bar{N}_{(n_b, n_s)}(\boldsymbol{\mu}, \boldsymbol{\nu})\right) p(\boldsymbol{\nu})$$

$$\bar{N}_{(n_b,n_s)} = \sum_{f=bkg,bs} \left[p(n_b, n_s | f, \nu) \bar{N}_f(\nu) \qquad \bar{N}_f = \mathcal{B}(Z/h \to f) N_{Z/h} \mathcal{A} \right]$$

Given the final-state config *f*, the probability our event is found in bin (n_b, n_s)

ATLAS: 2201.11428 CMS: 2004.12181 Faroughy, Kamenik, Szewc, Zupan: 2209.01222

Expected number of events in bin (n_b, n_s) - sum over each decay channel

$$\bar{N}_{bs} = \mu \mathcal{B}(Z/h \to bs)_{\rm SM} N_{Z/s}$$





3) Probabilistic model

 $\mathcal{L}(\boldsymbol{\mu}, \boldsymbol{\nu}) = \prod$ Pois (n_b, n_s)

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$$S\left(N^{A}_{(n_b,n_s)}|\bar{N}_{(n_b,n_s)}(\boldsymbol{\mu},\boldsymbol{\nu})\right)p(\boldsymbol{\nu})$$

Asimov dataset $\mu = 0$ (upper limits) $u_i =
u_{i,0}$

3) Probabilistic model

$\mathcal{L}(\boldsymbol{\mu}, \boldsymbol{\nu}) = \prod$ Pois (n_b, n_s)

ATLAS: 2201.11428 CMS: 2004.12181 Faroughy, Kamenik, Szewc, Zupan: 2209.01222

$$S\left(N^A_{(n_b,n_s)}|\bar{N}_{(n_b,n_s)}(\boldsymbol{\mu},\boldsymbol{\nu})\right)p(\boldsymbol{\nu})$$

Insert now the tagging efficiencies and other nuisance parameters and derive bounds on the branching ratios!





For the Higgs, statistics dominated

2D scan. Common TPR $\epsilon_b^b = \epsilon_s^s$ and FPR $\epsilon_{gcb}^s = \epsilon_{gsc}^b$

Obtain identical results w.r.t. full analysis if we take on the plot the point TPR = 0.8, FPR = max($\epsilon_{h}^{s}, \epsilon_{s}^{b}$) (Medium WP):

 $\epsilon^{b}_{\beta;\text{Med}} = \{0.007, 0.0001, 0.003, 0.80\}$ $\epsilon^s_{\beta;\text{Med}} = \{0.09, 0.80, 0.06, 0.004\}$ $\epsilon^q_\beta = \{g, s, c, b\}$







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 $\epsilon^{b}_{\beta;\text{Med}} = \{0.007, 0.0001, 0.003, 0.80\}$ $\epsilon_{\beta;\text{Med}}^s = \{0.09, \, 0.80, \, 0.06, \, \boxed{0.004}\}$ $\epsilon^q_\beta = \{g, s, c, b\}$

 $|\mathcal{B}(h \to bs) \lesssim 9.6 \times 10^{-4}|$ @ 95 % CL









Can set relevant phenomenological bounds for the Higgs flavor-violating couplings w.r.t. low-energy measurements



Can set relevant phenomenological bounds for the Higgs flavor-violating couplings w.r.t. low-energy measurements

$$\mathcal{L} \supset y_{db}(\bar{d}_L b_R)h + y_{bd}(\bar{b}_L d_R)h +$$

Matching to WET, wilson for running, flavio/smelli

Dashed line - Using only the *b* tagger



Can set relevant phenomenological bounds for the Higgs flavor-violating couplings w.r.t. low-energy measurements

$$\mathcal{L} \supset y_{sb}(\bar{s}_L b_R)h + y_{bs}(\bar{b}_L s_R)h + 1$$

Matching to WET, wilson for running, flavio/smelli

Solid line - Using both *b* and *s* taggers Dashed line - Using only the *b* tagger



Can set relevant phenomenological bounds for the Higgs flavor-violating couplings w.r.t. low-energy measurements

$$\mathcal{L} \supset y_{uc}(\bar{u}_L c_R)h + y_{cu}(\bar{c}_L u_R)h + 1$$

Matching to WET, wilson for running, flavio/smelli

Different Working Points (solid / dashed line)

No *u*-tagger used (however, performance would be improved!)





For the Z, cannot put relevant bounds. Systematics (rel. error on tagging efficiencies $\epsilon_{\alpha}^{\beta}$, 1%) dominated

ay	SM prediction	exp. bound	indir. co
$\rightarrow bs$)	$(4.2 \pm 0.7) \cdot 10^{-8}$	2.9×10^{-3}	6×10^{-8}
			_
	(TPR, FPR, $\Delta \epsilon^{\alpha}_{\beta} / \epsilon^{\alpha}_{\beta}$)	$\mathcal{B}(Z \to bs) \ (95\% \ \mathrm{CL})$	
	$(0.4, 10^{-4}, 1\%)$	1.8×10^{-6}	
	$(0.4, 10^{-4}, 0.1\%)$	1.8×10^{-7}	
	$(0.2, \ 10^{-5}, \ 1\%)$	4.2×10^{-7}	
	$(0.2, 10^{-5}, 0.1\%)$	4.2×10^{-8}	

Only then SM precision reachable (with very ambitious tagger performance) $\epsilon_b^s \gtrsim 10^{-3}$, limited by vertexing at FCC-ee Barchetta, Collins, Riedler: 2112.13019





The assessment of the FCC-ee potential Model-dependent analyses - type-III 2HDM

$$\mathcal{L}_{2\text{HDM}} \supset -\frac{\sqrt{2}m_i}{v} \delta_{ij} \bar{q}_L^i H_1 d_R^j - \sqrt{2} Y_{ij}^d \bar{q}_L^i H_2 d_R^j - \frac{\sqrt{2}m_i}{v} \delta_{ij} \bar{q}_L'^i \tilde{H}_1 u_R^j - \sqrt{2} Y_{ij}^u \bar{q}_L'^i \tilde{H}_2 u_R^j$$

$$\begin{pmatrix} G^+ \\ (v+h_1+iG^0) \end{pmatrix}, \quad H_2 = \begin{pmatrix} H^+ \\ \frac{1}{\sqrt{2}}(h_2+iA) \end{pmatrix}$$

$$\mathcal{L}_{2\text{HDM}} \supset -\frac{\sqrt{2}m_i}{v} \delta_{ij} \bar{q}_L^i H_1 d_R^j - \sqrt{2} Y_{ij}^d \bar{q}_L^i H_2 d_R^j - \frac{\sqrt{2}m_i}{v} \delta_{ij} \bar{q}_L'^i \tilde{H}_1 u_R^j - \sqrt{2} Y_{ij}^u \bar{q}_L'^i \tilde{H}_2 u_R^j$$

$$H_1 = \begin{pmatrix} G^+ \\ \frac{1}{\sqrt{2}} (v+h_1+iG^0) \end{pmatrix}, \quad H_2 = \begin{pmatrix} H^+ \\ \frac{1}{\sqrt{2}} (h_2+iA) \end{pmatrix} \qquad \qquad \begin{pmatrix} h_1 \\ h_2 \end{pmatrix} = \begin{pmatrix} c_\alpha & s_\alpha \\ -s_\alpha & c_\alpha \end{pmatrix} \begin{pmatrix} h \\ H \end{pmatrix}$$

$$\mathcal{L}_{\text{WET}} \supset C_2(\bar{s}_R b_L)^2 + C_2'(\bar{s}_L b_R)^2 + C_4(\bar{s}_L b_R)(\bar{s}_R b_L)$$

$$C_{2} = -\frac{\left(Y_{bs}^{d*}\right)^{2}}{2} \left(\frac{s_{\alpha}^{2}}{m_{h}^{2}} + \frac{c_{\alpha}^{2}}{m_{H}^{2}} - \frac{1}{m_{A}^{2}}\right),$$

$$C_{2}' = -\frac{\left(Y_{sb}^{d}\right)^{2}}{2} \left(\frac{s_{\alpha}^{2}}{m_{h}^{2}} + \frac{c_{\alpha}^{2}}{m_{H}^{2}} - \frac{1}{m_{A}^{2}}\right),$$

$$C_{4} = -\left(Y_{bs}^{d*}Y_{sb}^{d}\right) \left(\frac{s_{\alpha}^{2}}{m_{h}^{2}} + \frac{c_{\alpha}^{2}}{m_{H}^{2}} + \frac{1}{m_{A}^{2}}\right)$$

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Match

The assessment of the FCC-ee potential Model-dependent analyses - type-III 2HDM



 $\mathcal{L}_{\text{WET}} \supset C_2(\bar{s}_R b_L)^2 + C_2'(\bar{s}_L b_R)^2 + C_4(\bar{s}_L b_R)(\bar{s}_R b_L)$

The assessment of the FCC-ee potential **Model-dependent** analyses - Insertion of vectorlike quarks

$$(D_L, D_R) \sim (\mathbf{1}, -1/3)$$

$$(Q_L, Q_R) \sim (\mathbf{2}, -1/3)$$

$$(Q_L, Q_R) \sim ($$



Conclusion

- The role of FCC-ee for flavor physics is irrefutable
- The FCC-ee is probing very interesting regions of parameter spaces for H and **Higgs-related models**
 - Excluding flat directions

• FCC-ee not competitive with low-energy measurements for Z regarding FV BRs





Conclusion

Decay	SM prediction	exp. bound	indir. constr.	FCC-ee bound
$\mathcal{B}(h \to bs)$	$(8.9 \pm 1.5) \cdot 10^{-8}$	0.16	2×10^{-3} \bigstar	9.6×10^{-4}
$\mathcal{B}(h \to bd)$	$(3.8 \pm 0.6) \cdot 10^{-9}$	0.16	10^{-3}	5×10^{-3}
$\mathcal{B}(h \to cu)$	$(2.7 \pm 0.5) \cdot 10^{-20}$	0.16	2×10^{-2} \bigstar	$2.5 imes 10^{-3}$
$\mathcal{B}(Z \to bs)$	$(4.2 \pm 0.7) \cdot 10^{-8}$	2.9×10^{-3}	6×10^{-8} –	$O(10^{-6})$
$\mathcal{B}(Z \to bd)$	$(1.8 \pm 0.3) \cdot 10^{-9}$	2.9×10^{-3}	6×10^{-8} \bullet	$O(10^{-6})$
$\mathcal{B}(Z \to cu)$	$(1.4 \pm 0.2) \cdot 10^{-18}$	2.9×10^{-3}	4×10^{-7} \bullet	2.3×10^{-3}

.

Thank you!

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Backup

Z FCNC fits

$$C_{VL} = \frac{(g_{sb}^L)^2}{2m_Z^2}, \qquad C_{VR} = \frac{(g_{sb}^R)^2}{2m_Z^2}, \qquad C_{VLR} = \frac{g_{sb}^L g_{sb}^R}{m_Z^2}$$

Global minimum not reachable in this 2D parameter space





Z FCNC fits

$$C_{VL} = \frac{(g_{sb}^L)^2}{2m_Z^2}, \qquad C_{VR} = \frac{(g_{sb}^R)^2}{2m_Z^2}, \qquad C_{VLR} = \frac{g_{sb}^L g_{sb}^R}{m_Z^2}$$





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 $\mathcal{L} \supset g_{sb}^L(\bar{s}_L\gamma_\mu b_L)Z^\mu + g_{sb}^R(\bar{s}_R\gamma_\mu b_R)Z^\mu$ $+y_{sb}(\bar{s}_L b_R)h + y_{bs}(\bar{b}_L s_R)h + h.c.$



For the Z, that is not the case

VLQs - Lagrangians

$$-\mathcal{L}_{\text{int}} \supset y_d^{ij} \bar{q}_L^i H d_R^j + y_u^{ij}$$
$$-\mathcal{L}_Q = y_d^{ij} \bar{q}_L^i H d_R^j + y_u^{ij} \bar{q}_L^i \tilde{H} u_R^j$$

$$(D_L, D_R) \sim (\mathbf{1}, -1/3)$$

$\bar{q}_L^i \tilde{H} u_R^j + y_D^i \bar{q}_L^i H D_R + M_D \bar{D}_L D_R + \text{h.c.}$ $+ y_D^i \bar{Q}_L H d_R^i + y_U^i \bar{Q}_L \tilde{H} u_R^i + M_Q \bar{Q}_L Q_R + \text{h.c.}$

$$\mathcal{L}_{\mathrm{VLQ}}^Q \supset \frac{g}{2c_W} X_{ij}^Q \big(\bar{d}^i \gamma^\mu P_R d^j \big) Z_\mu + X_{ij}^Q \frac{m_j}{v} \big(\bar{d}^i P_R d^j \big) h + \text{h.c.}$$

 $(Q_L, Q_R) \sim (\mathbf{2}, 1/6)$



3) Probabilistic model

$$\mathcal{L}(\boldsymbol{\mu}, \boldsymbol{\nu}) = \prod_{(n_b, n_s)} \operatorname{Pois}\left(N_{(n_b, n_s)}^A | \bar{N}_{(n_b, n_s)}(\boldsymbol{\mu}, \boldsymbol{\nu})\right) p(\boldsymbol{\nu})$$

 $(n_b, n_s) = \{(0, 0), (0, 1), (1, 0), (2, 0), (0, 2), (1, 1)\}$

ATLAS: 2201.11428 CMS: 2004.12181 Faroughy, Kamenik, Szewc, Zupan: 2209.01222

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$$\bar{N}_{(n_b,n_s)} = \sum_{f=bkg,bs} p(n_b, n_s | f, \nu) \bar{N}_f(\nu)$$

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Analysis framework 3) Probabilistic model

$$\bar{N}_{(n_b,n_s)} = \sum_{f=bkg,bs} p(n_b,n_s|f,\nu)\bar{N}_f(\nu)$$

$$p(n_b, n_s | f, \nu) = \sum_{\substack{n_{b;1} = 0}}^{\min(n_b, 1) \min(n_s, 1 - n)} \sum_{\substack{n_{s;1} = 0}}^{\min(n_s, 1 - n)$$

Possible configurations for the bin $(n_b, n_s) = (1, 1)$: $(n_{b;1}, n_{b;2}, n_{s;1}, n_{s;2}) = (1, 0, 0, 1)$ $(n_{b;1}, n_{b;2}, n_{s;1}, n_{s;2}) = (0, 1, 1, 0)$

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 $(b_{b;1})$

 $p(n_{b;1}|j_1)p(n_{s;1}|j_1, n_{b;1})p(n_{b;2}|j_2)p(n_{s;2}|j_2, n_{b;2})$

Analysis framework 3) Probabilistic model

$$\bar{N}_{(n_b,n_s)} = \sum_{\substack{f=b \text{kg},bs}} p(n_b, n_s | f, \nu) \bar{N}_f(\nu)$$
$$= \sum_{\substack{n_{b;1}=0}}^{\min(n_b,1)\min(n_s,1-n_{b;1})} p(n_{b;1} | j_1) p(n_{s;1} | j_1, n_{b;1}) p(n_{b;2} | j_2) p(n_{s;2} | j_2, n_{b;2})$$

$$\bar{N}_{(n_b,n_s)} = \sum_{f=b \text{kg},bs} p(n_b, n_s | f, \nu) \bar{N}_f(\nu)$$

$$p(n_b, n_s | f, \nu) = \sum_{n_{b;1}=0}^{\min(n_b,1)\min(n_s,1-n_{b;1})} \sum_{n_{s;1}=0}^{\min(n_b,1)\min(n_s,1-n_{b;1})} p(n_{b;1} | j_1, n_{b;1}) p(n_{b;2} | j_2) p(n_{s;2} | j_2, n_{b;2})$$

(anti)-b-tagging probability if $n_{b;1} = 1(0)$:

$$p(n_{b;1}|j_1) = \mathbf{E}$$

(anti)-s-tagging probability if $n_{s;1} = 1(0)$, conditioned over the $n_{b;1}$ flavor tagging:

 $p(n_{s;1}|j_1, n_{b;1})$

ATLAS: 2201.11428 CMS: 2004.12181 Faroughy, Kamenik, Szewc, Zupan: 2209.01222

Binom $(k, n, p) \equiv \binom{n}{k} p^k (1-p)^{n-k}$ $\operatorname{Binom}(n_{b;1}, 1, \epsilon_1^b)$

$$= \operatorname{Binom}\left(n_{s;1}, 1 - n_{b;1}, \frac{\epsilon_1^s}{1 - \epsilon_1^b}\right)$$

