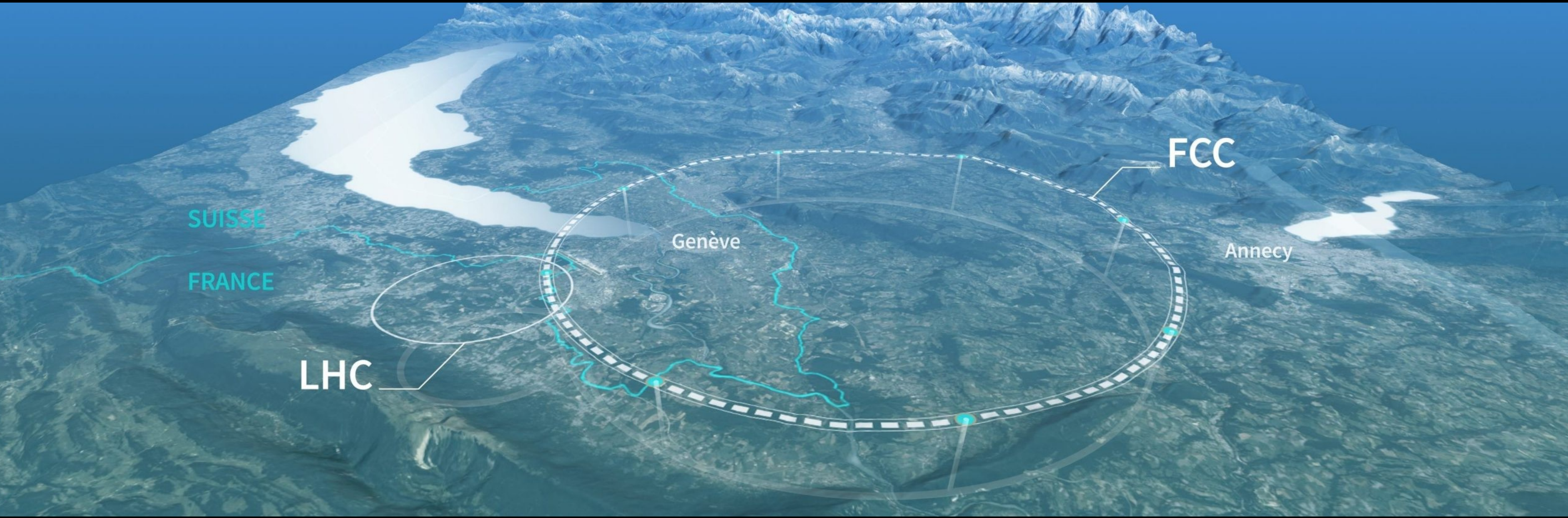
Extraction of $\alpha_{em}(m_Z^2)$ at Tera-Z



Marc Riembau
CERN

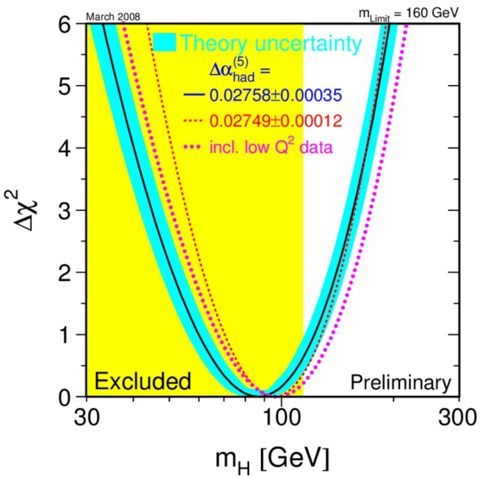
FCC workshop, 16th January 2025

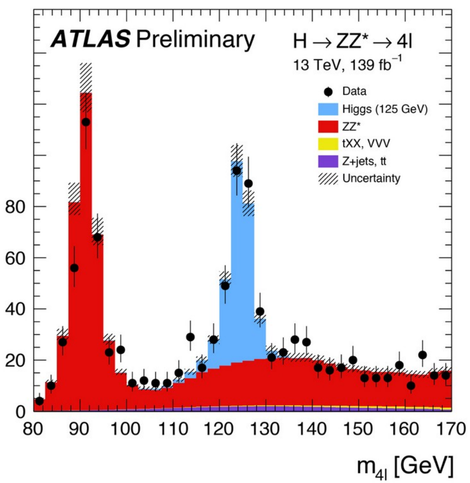
based on 2501.05508

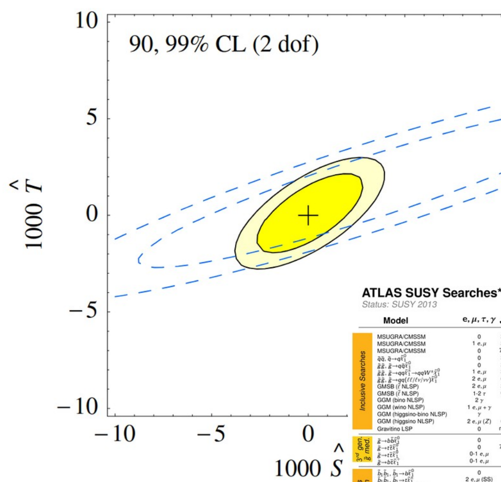
Per-mille precision at LEP already hinted at the two LHC major discoveries:

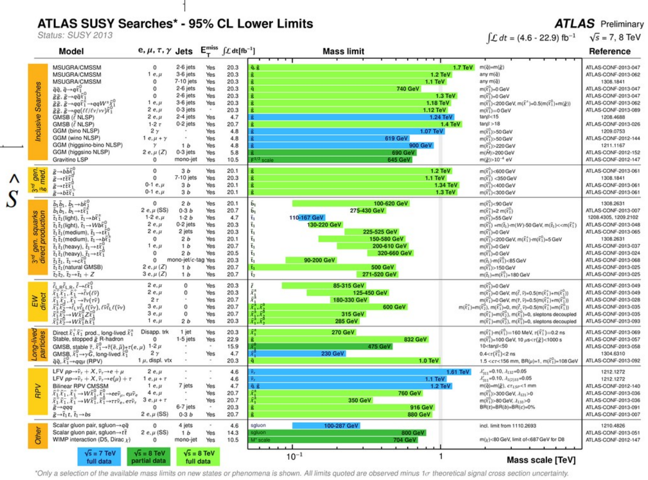
- The discovery of a scalar associated with the origin of mass

- The discovery of a mass gap above the electroweak scale



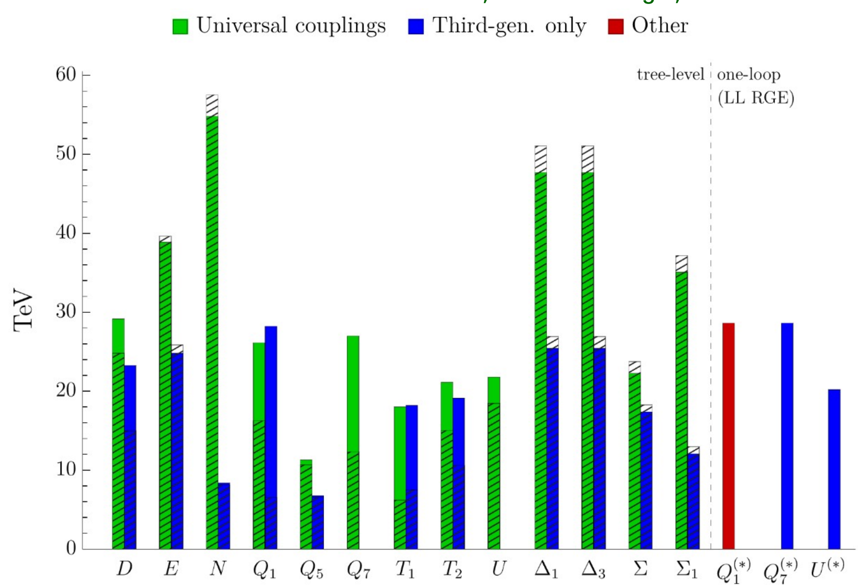






If the ~per-million precision at Tera-Z is archieved, electroweak precision observables will probe generic stuff coupling to the EW sector up to tens of TeV.

Allwicher, McCullough, Renner '24



All the statistics are useless unless there is an herculean effort to bring experimental and theoretical systematic uncertainties below this level.

- It requires reaching ~10^{-6} precision in every aspect:
  - beam quality,
  - detector performance,
  - theoretical predictions,
  - MonteCarlo simulations with >10^{13} events,
  - ...

  20 years is perhaps not that much if we want everything ready...

- In this talk I will discuss a much simpler aspect:

  Do we know the SM well enough to even talk about 10^{-6} level predictions?

In the SM, the electroweak sector has 3 input parameters:

- Fermi constant G_F , given by $1.1663787(6) \cdot 10^{-5}$ GeV^{-2}

  (~ 10^{-7} relative precision) ✓

- Z mass m_Z , given by 91.1876(21) GeV

  but expected to be measured at 10^{-6} precision at FCC-ee

- Electromagnetic coupling $\alpha_{\rm em}$, given by 1/137.0359991496(330)

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  But it needs to run to the Z pole!

  (~ 10^{-4} relative precision)

The electromagnetic coupling seems a bottleneck for the precision electroweak program at Tera-Z

It is a bottleneck for interpreting the measurements:

- In the SM (at tree level), the effective mixing angle is fixed to $\sin^2\theta_W^{eff}\cos^2\theta_W^{eff} = \frac{\sqrt{2}G_F m_Z^2}{\pi\alpha_{em}}$

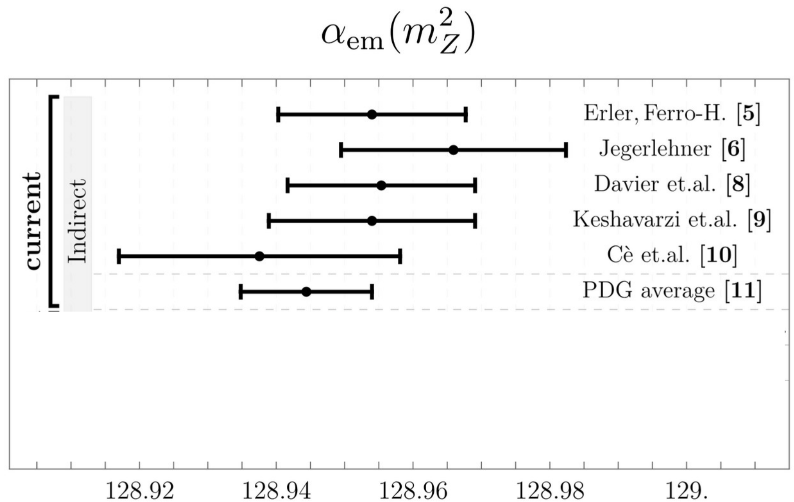
Deviation interpreted in terms of new physics, encoded in $\widehat{S}$

$$\delta(\sin^2\theta_W^{eff})/\sin^2\theta_W^{eff} \times 10^5 = \frac{\widehat{S}}{5\cdot 10^{-6}} - \frac{\delta(\alpha_{em}^{-1})}{10^{-3}}$$

- The electromagnetic coupling α_{em}^{-1} must be known at 10^{-3},

equivalent to 10^{-5} relative precision

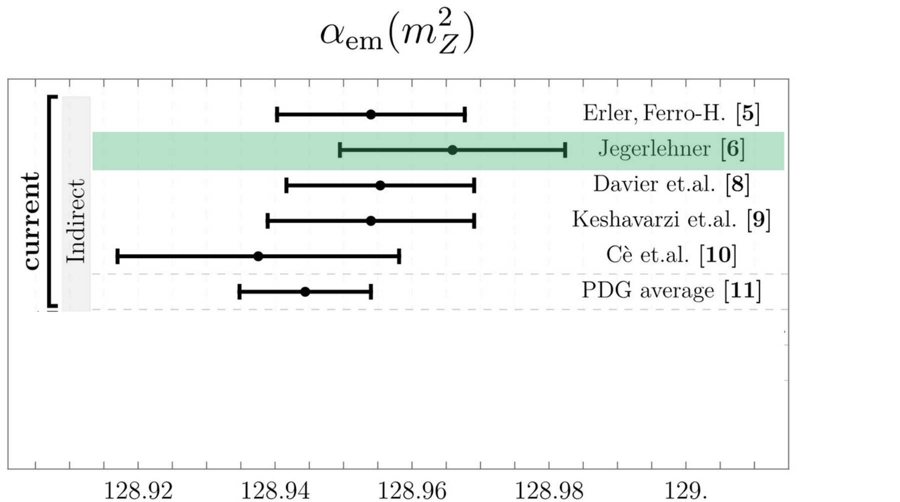
Summary of electromagnetic coupling determinations:

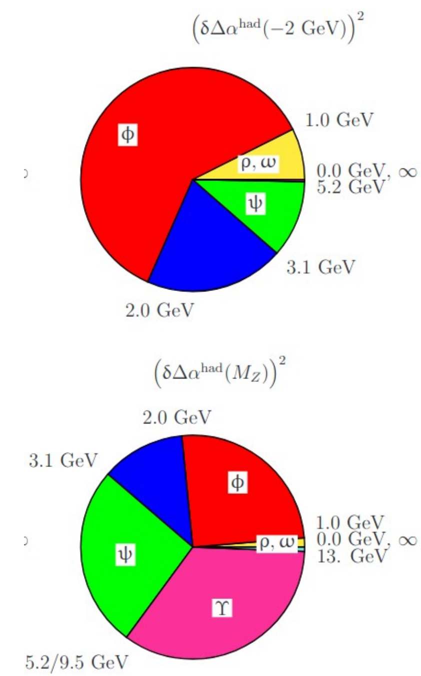


Different approaches lead to consistent and similarly precise values,

all around the 10^{-4} relative uncertainty on $\alpha_{em}(m_Z)$

Summary of electromagnetic coupling determinations:





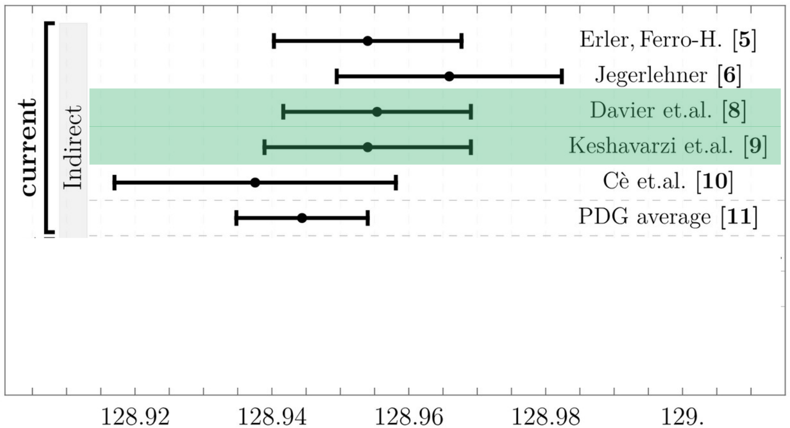
$$\alpha\left(M_Z^2\right) = \alpha^{\rm data}\left(-M_0^2\right) + \left[\alpha\left(-M_Z^2\right) - \alpha\left(-M_0^2\right)\right]^{\rm pQCD} + \left[\alpha\left(M_Z^2\right) - \alpha\left(-M_Z^2\right)\right]^{\rm pQCD}$$

$$\Delta = \Delta\alpha_{\rm had}^{(5)}(M_Z^2) - \Delta\alpha_{\rm had}^{(5)}(-M_Z^2) = 0.000038 \pm 0.000005$$

Hadronic contribution to $\alpha_{em}(-(2\mathrm{GeV})^2)$ very different than $\alpha_{em}(m_Z)$

Summary of electromagnetic coupling determinations:

$\alpha_{\text{em}}(m_Z^2)$

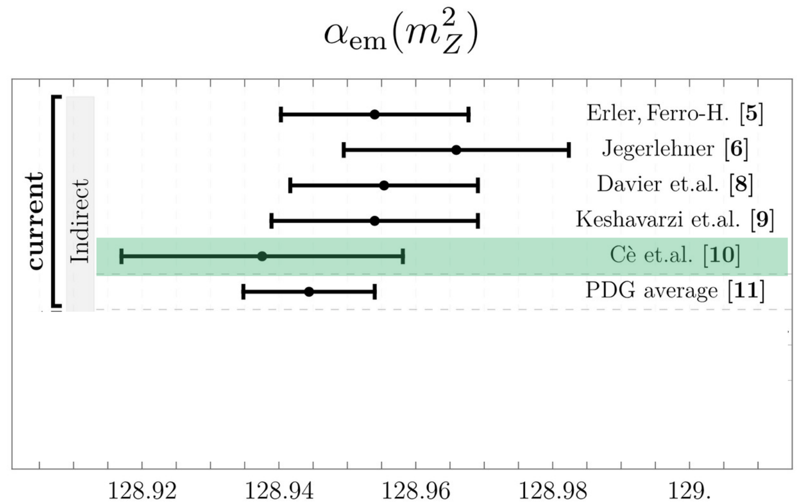


Direct integration of $\Delta\alpha_{\text{had}}(m_Z^2) = \frac{\alpha}{3\pi}\int_{4m_\pi^2}^{\infty}\frac{ds}{s}R(s)\frac{m_Z^2}{m_Z^2 - s}$ using e+e- data.

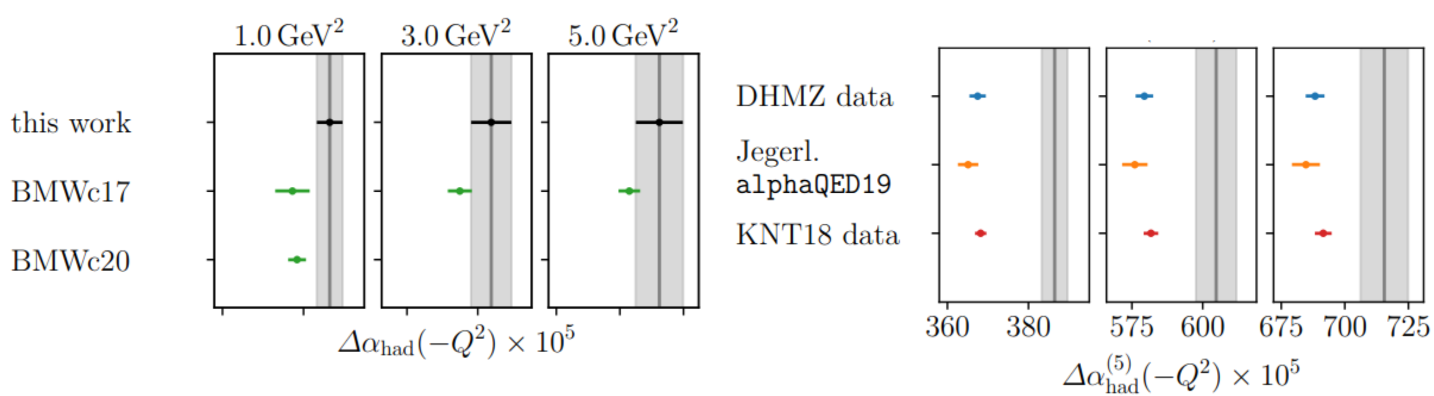
Analysis	$\Delta\alpha_{\text{had}}^{(5)}(M_Z^2) \times 10^4$	$\alpha^{-1}(M_Z^2)$
DHMZ10 [51]	275.59 ± 1.04	128.952 ± 0.014
HLMNT11 [40]	276.26 ± 1.38	128.944 ± 0.019
FJ17 [47]	277.38 ± 1.19	128.919 ± 0.022
DHMZ17 [54]	276.00 ± 0.94	128.947 ± 0.012
KNT18	276.11 ± 1.11	128.946 ± 0.015
DHMZ19 [37]	276.10 ± 1.00	128.946 ± 0.013
KNT19 [This work]	276.09 ± 1.12	128.946 ± 0.015

Table from Keshavarzi et al, [1911.00367]

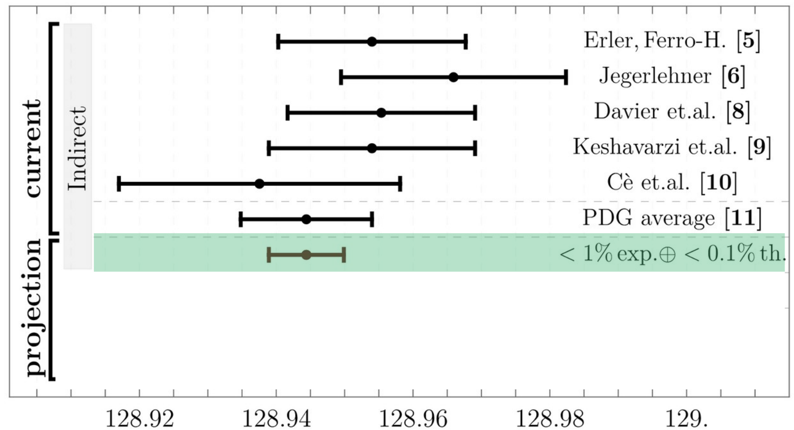
Summary of electromagnetic coupling determinations:



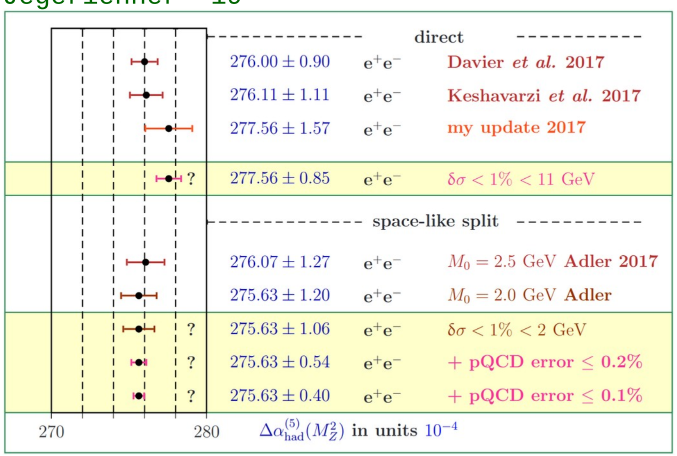
Lattice computation of $\alpha_{em}(-Q^2)$



$\alpha_{\text{em}}(m_Z^2)$



Jegerlehner '19



$$0.4 \cdot 10^{-4} \quad \text{in} \quad \Delta_{\text{had}}(m_Z)$$

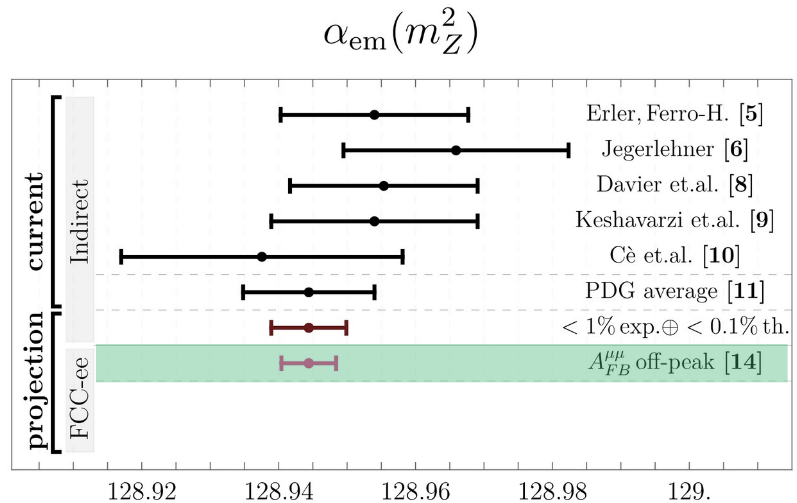
⇕

$$5 \cdot 10^{-3} \quad \text{absolute unc. in} \quad \alpha_{\text{em}}^{-1}$$

⇕

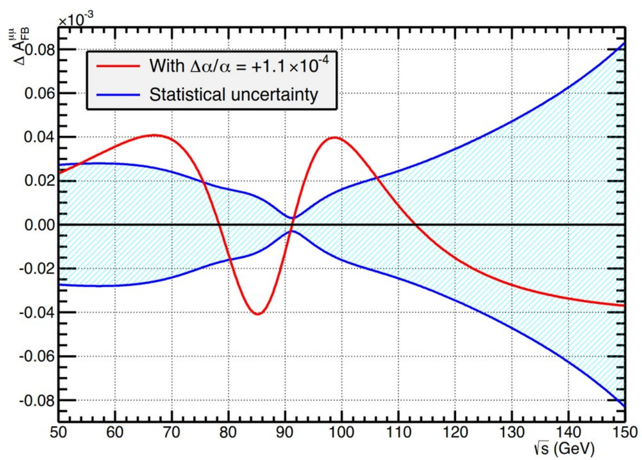
$$4 \cdot 10^{-5} \quad \text{relative unc. in} \quad \alpha_{\text{em}}$$

- Projections of indirect determinations well above 10^{-5}
- Potential data/lattice tensions...
- Alternative, direct, independent determination highly desirable

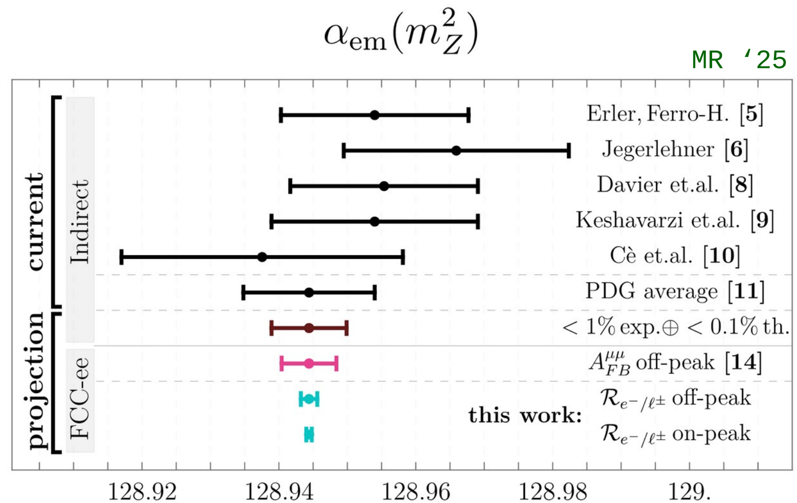


Proposal for a direct determination of $\alpha_{em}(m_Z)$ at Tera-Z in Janot '16

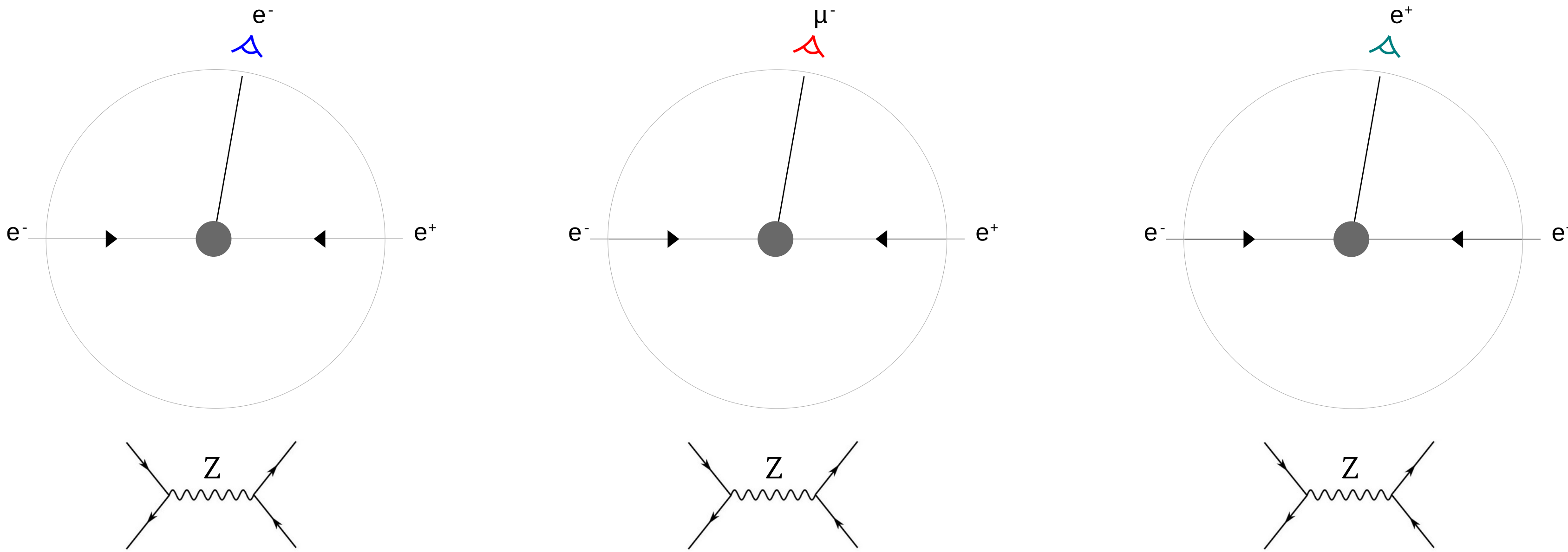
Based on measuring the muon forward-backward asymmetry on-peak and off-peak.



- Reaches a $3 \cdot 10^{-5}$ relative sensitivity, statistically limited.
- Completely independent of hadronic data.

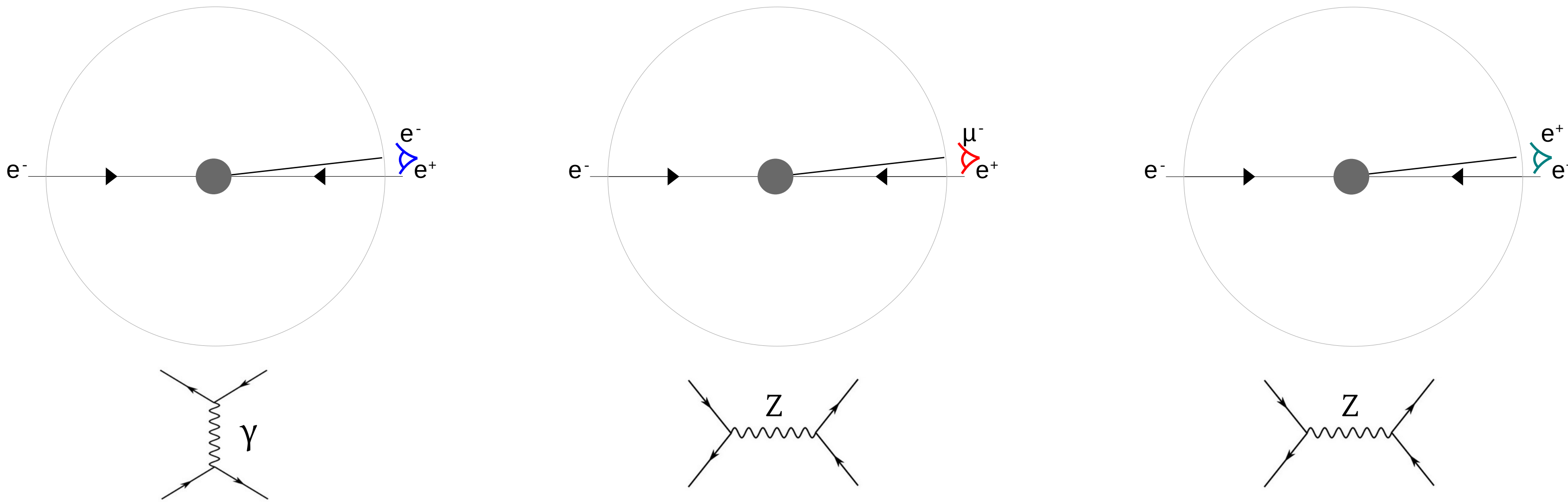


In the rest of the talk I present another proposal for a direct extraction at Tera-Z, which reaches a 10^{-5} level of statistical sensitivity



- In the central region, Z s-channel exchange dominates the rate.

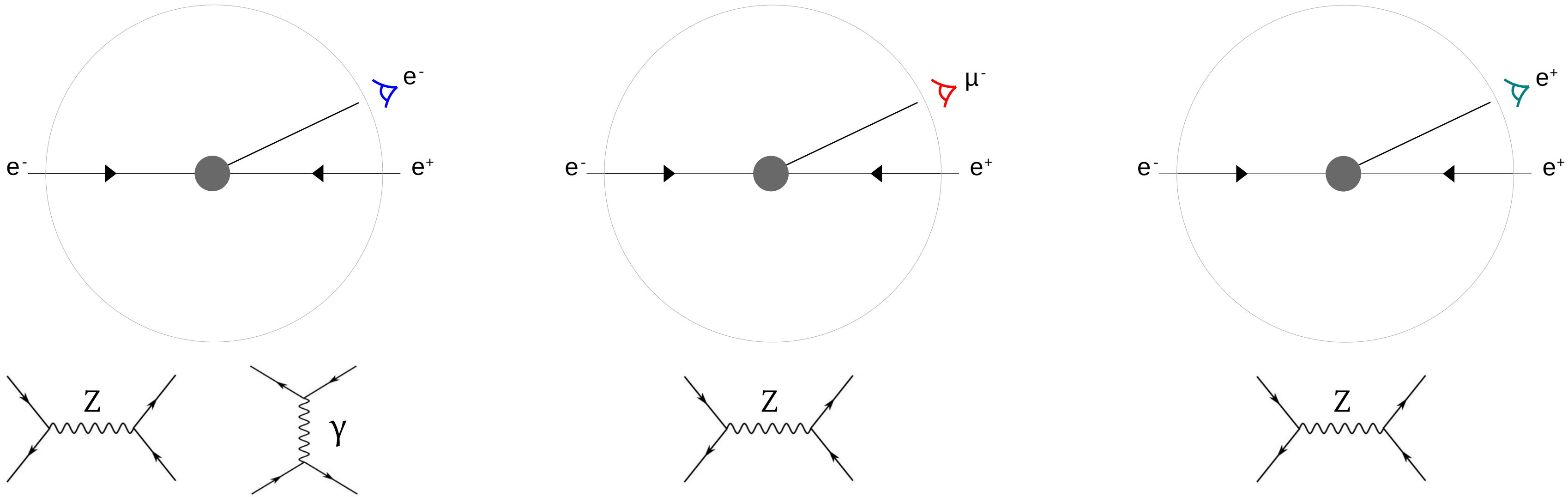
  The rate is controlled by $G_F \cdot m_Z^2$, so no sensitivity to α_{em} (considering mixing angle independent).



- In the forward region, muons and electrons are still dominated by Z, but electrons are dominated by forward photon pole.

- Measurement of electron production for angles between 62 and 88 mrad allow determination of luminosity at 10^{-4} level. Extraction of α_{em} correlated with luminosity measurement.

  Comparably low rate of muons and positrons in the luminosity region.



- For the electron channel, both processes comparable at some angle, given by

$$z = \frac{1-\cos\theta}{2} \quad \rightarrow \quad \frac{1}{2}z^2 \simeq \frac{\Gamma_Z^2}{m_Z^2}\frac{1}{\mathcal{Z}^2} \quad \text{with} \quad \mathcal{Z} = \frac{\sqrt{2}G_F m_Z^2}{\pi\alpha}(g_V^2 + g_A^2)$$

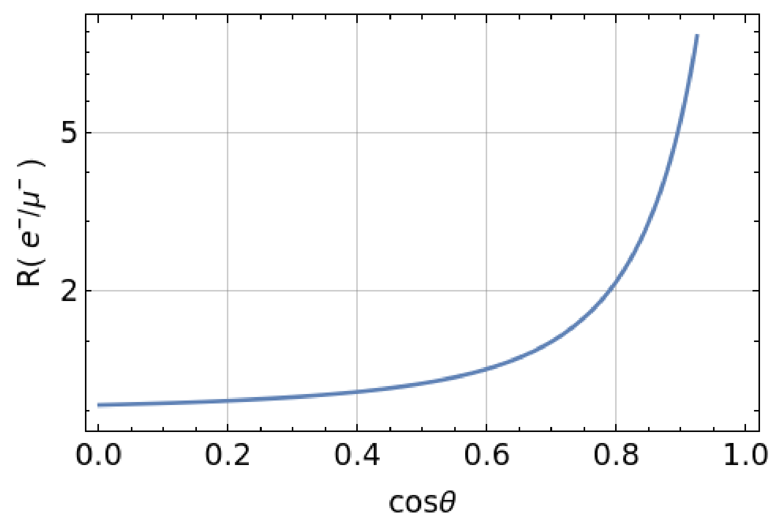
  Numerically, $\cos\theta \simeq 0.8$ or $\theta \simeq 35^\circ$

- This is well within the detector coverage, and in a region with large statistics for electrons, but also for muons and positrons.

- This allows to define two observables, sensitive to α_{em} and $\sin\theta_W^{eff}$ simultaneously, insensitive to luminosity normalization, and with the large on-peak Tera-Z statistics.

$$\mathcal{R}_{e^-/\mu^-}(\theta) = \frac{\sigma(e^-e^+ \to e^-(\theta) + X)}{\sigma(e^-e^+ \to \mu^-(\theta) + X)} \qquad \mathcal{R}_{e^-/e^+}(\theta) = \frac{\sigma(e^-e^+ \to e^-(\theta) + X)}{\sigma(e^-e^+ \to e^+(\theta) + X)}$$

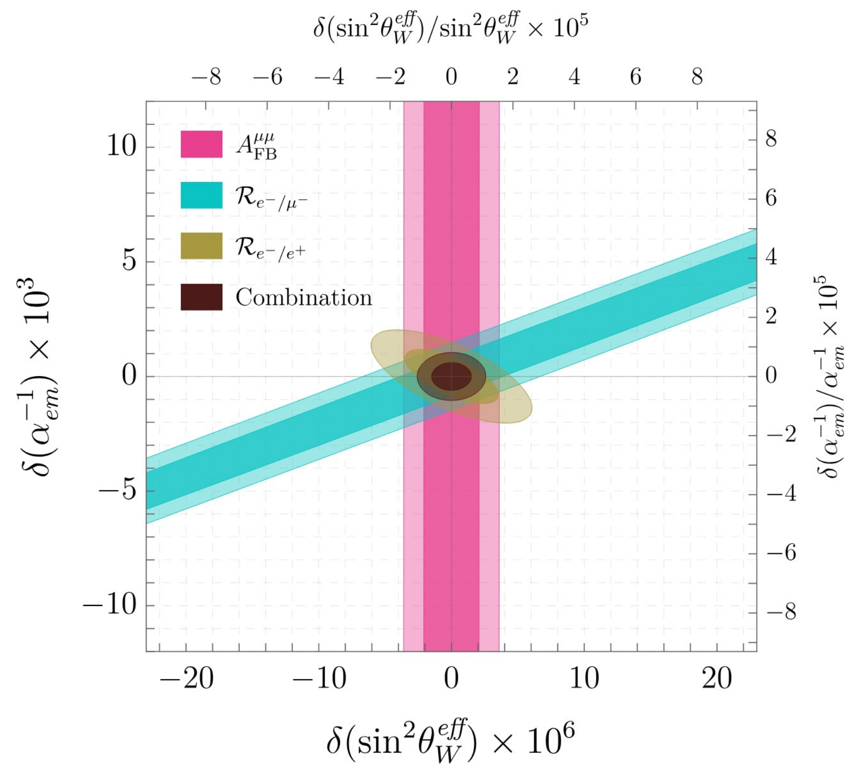
- They compare the number (density) of electrons with the one of muons and positrons



Statistical sensitivity:

- It provides a target for the rest of uncertainties
- It represents the ultimate reach given a finite set of data

Statistical sensitivity:



Statistical sensitivity on the electromagnetic coupling below the 10^{-5} level

The sensitivity relies on the region $\cos\theta > 0.8$:

Detector coverage up to

- $\cos\theta = 0.99$ ($\theta \sim 8^{o}$) $\rightarrow$ $\delta\alpha_{em}/\alpha_{em} \sim 0.5 \cdot 10^{-5}$
- $\cos\theta = 0.98$ ($\theta \sim 11^{o}$) $\rightarrow$ $\delta\alpha_{em}/\alpha_{em} \sim 0.5 \cdot 10^{-5}$
- $\cos\theta = 0.95$ ($\theta \sim 18^{o}$) $\rightarrow$ $\delta\alpha_{em}/\alpha_{em} \sim 0.7 \cdot 10^{-5}$
- $\cos\theta = 0.85$ ($\theta \sim 32^{o}$) $\rightarrow$ $\delta\alpha_{em}/\alpha_{em} \sim 1.5 \cdot 10^{-5}$

- Coverage only up to $\cos\theta = 0.8$ and below very rapidly degrades any sensitivity to α_{em}

  Region between $\sim 30^{o}$ and $\sim 10^{o}$ crucial.

Considerations for the systematic uncertainties

- Particle miss-id between electrons and muons below the 10^{-5} level at LEP.
  To contribute, it requires a double miss-id: likely negligible.

- Charge miss-identification at 0.5% level at LEP.

  As long as FCC-ee detectors have charge-id better than 0.2% in $\theta<20^{\circ}$, under control.

  Ok for muons, study required for electrons.

  Charge miss-id will be measured with great precision due to 10^8 $Z\to\mu^{\pm}\mu^{\pm}$ and $Z\to e^{\pm}e^{\pm}$

- Id-efficiency may have nontrivial θ dependence.

  If dependence is independent of the sign of the charge, it drops out in the e^-/e^+ ratio,

  but may render the precise measurement of the e^-/μ^- ratio unfeasible.

- Beam energy spread has a sizable impact and must be taken into account.

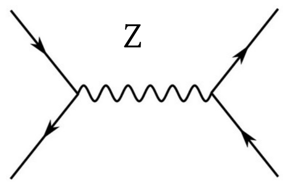
  It can be measured at the *per-mille* level every four minutes using 10^6 $Z\to\mu^-\mu^+$ events, [1909.12245]

  leading to a negligible impact on the uncertainty.

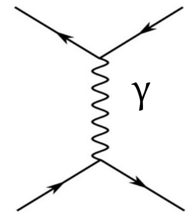
- Studies are needed to understand how forward we can go...

Parametric uncertainties

- Loop corrections bring new parametric uncertainties.



$\sim\alpha_{em}(m_Z)$



$\sim\alpha_{em}(t)$, $t=-m_Z^2/2(1-\cos\theta)$

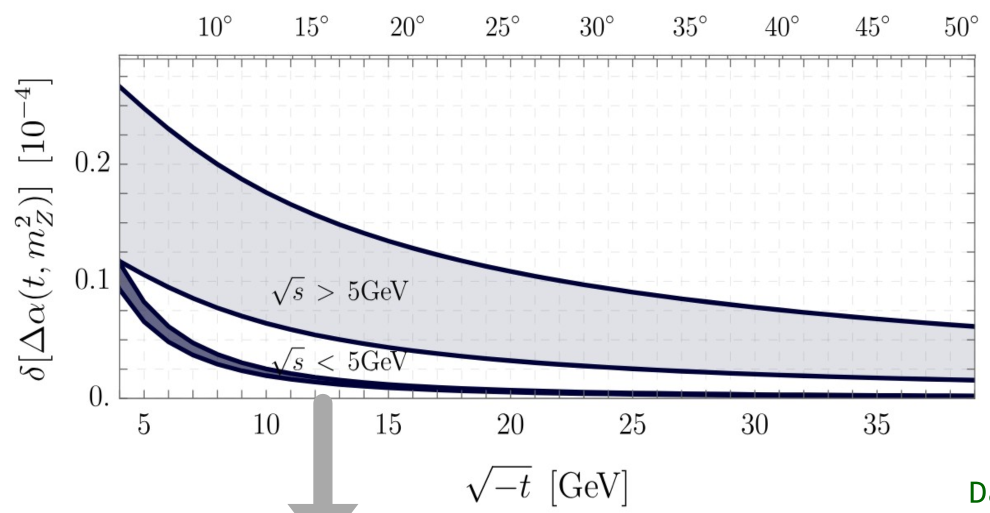
- Forward photon diagram depends on the running coupling at a given momentum exchange:

$$\alpha(m_Z^2) \simeq \alpha(t) - \alpha \times \left(\Delta\alpha(t) - \Delta\alpha(m_Z^2)\right)$$

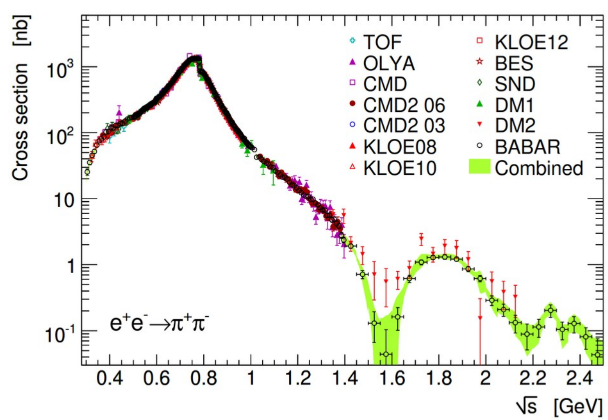
$$\Delta\alpha(t, m_Z^2) \equiv \Delta\alpha_{\rm had}(t) - \Delta\alpha_{\rm had}(m_Z^2),$$

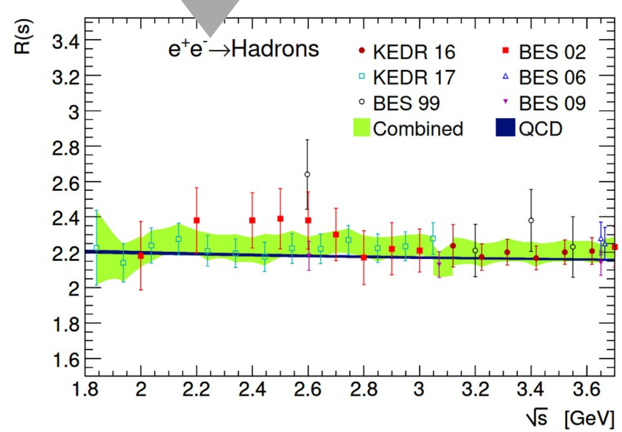
$$= \frac{\alpha}{3\pi}\int_{2m_\pi^2}^{\infty} \frac{ds}{s} R(s) \left(\frac{-t}{s-t} + \frac{m_Z^2}{s-m_Z^2}\right)$$

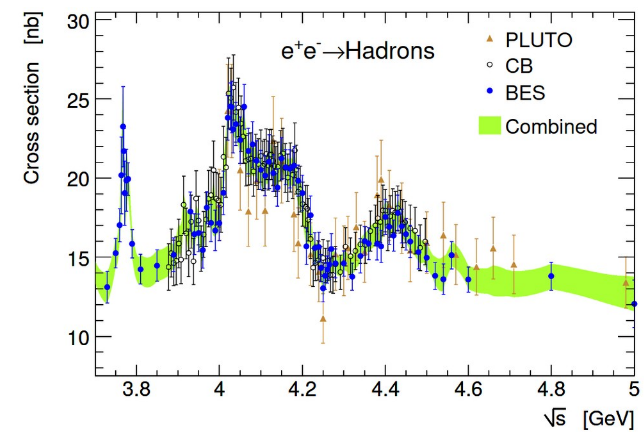
Is this under control at the 10^{-5} level?



Davier et.al. ‘17, ‘19

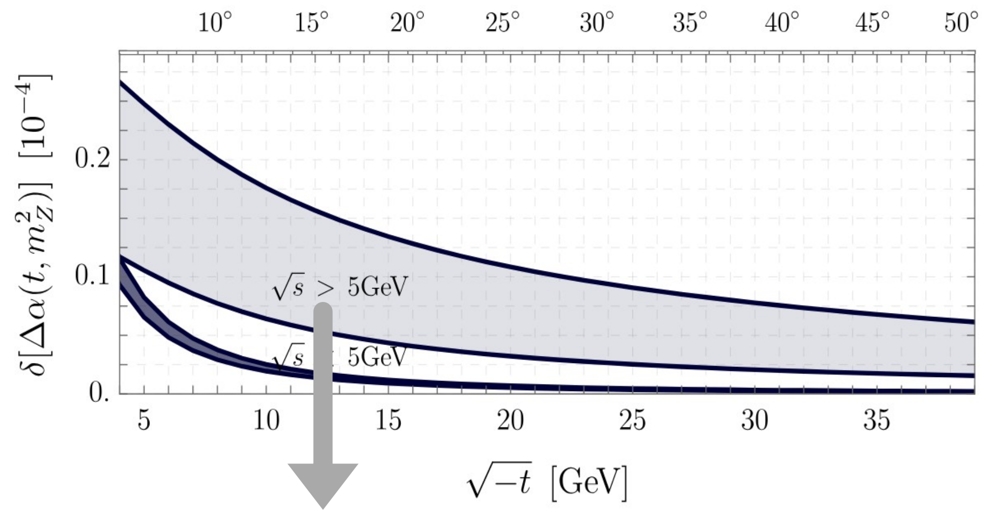






The kernel suppresses contributions for $s \ll t, m_Z$.
Current precision on data gives sub-10^{-5} unc. on $\alpha(t)/\alpha(m_Z)$

Is this under control at the 10^{-5} level?



pQCD contribution computed using `rhad` [Harlander, Steinhauser '02]

Band represents two types of param. uncertainties:

Upper boundary:

$m_c = 1.27 \pm 0.02$ GeV
$m_b = 4.18 \pm 0.03$ GeV
$\alpha_S = 0.118 \pm 0.0016$
$\mu = \sqrt{s} \cdot 2^{\pm 1}$

Lower boundary: $\mu = \sqrt{s} \cdot 2^{\pm 1}$

Lower boundary might be realistic for FCC-ee, since unc. dominated by α_S.
Uncertainty at the 10^{-5} level. No significant obstruction for interpreting measurements in terms of $\alpha_{em}(m_Z)$

Parametric uncertainties

- Top contribution to the electroweak vacuum polarization.
- Z boson coupling to matter shifted by the T parameter:

$$4\sqrt{2}G_F m_Z^2 \;\to\; 4\sqrt{2}G_F m_Z^2 \frac{1}{1-\Delta\rho} \qquad \text{with} \qquad \Delta\rho = \frac{N_c\sqrt{2}G_F m_t^2}{16\pi^2}$$

- This implies $\delta\mathcal{Z}/\mathcal{Z} = 10^{-5} \times \frac{\delta m_t}{90\,\mathrm{MeV}}$

  Unless we know the top mass below the ~100 MeV level, it may dominate the uncertainty.

- The tt-run at FCC will measure m_t at the 17MeV level. It is absolutely needed.
  If avaliable, the parametric uncertainty due to the top quark becomes negligible.

- We might ask however the chronologically relevant question: what about Tera-Z without the tt run?

The shift due to the top induces a shift as well on the Z boson width,

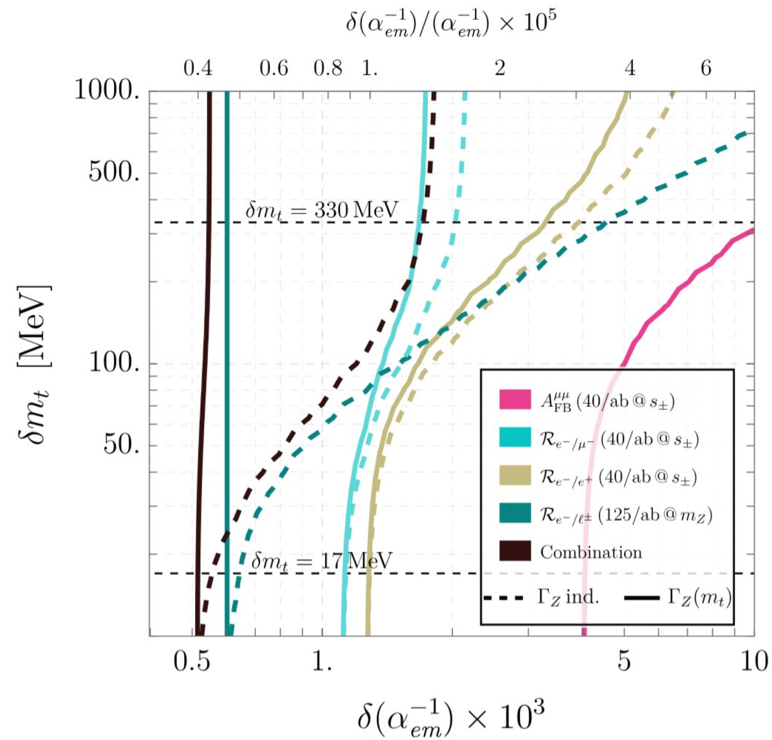
$$\Gamma_Z \quad \to \quad \Gamma_Z \frac{1}{1 - \Delta\rho}$$

- This implies that, on-peak, the Z-boson s-channel exchange, enhanced by

$$\mathcal{Z}/\Gamma_Z$$

is independent of $\Delta\rho$. Off-peak measurements are sensitive to m_t, though.

- Current theoretical unc. on the width is 400MeV, much larger than the FCC-ee 11MeV-level measurement, corresponding to $0.5 \cdot 10^{-5}$ relative precision.



- As noted, 100MeV unc. on mt (much below interpretability uncertainty at LHC) required in some cases.

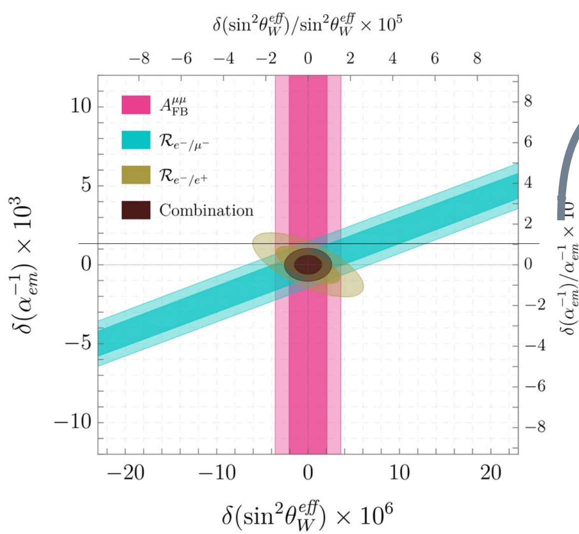
- If on-peak precision can be reached, it is unsensitive to m_t. Off-peak are much more robust, and a 10^{-5} level measurement seems safe.

- Note of caution: it is unclear whether a global fit might provide extra handle on m_t with only Tera-Z data!

So it seems that $\alpha_{em}(m_Z)$ might be extracted at the 10^{-5} level at Tera-Z.

- What about our initial motivation?

$$\delta(\sin^2\theta_W^{eff})/\sin^2\theta_W^{eff} \times 10^5 = \frac{\widehat{S}}{5\cdot 10^{-6}} - \frac{\delta(\alpha_{em}^{-1})}{10^{-3}}$$



Just right! Not by accident:
Both observables have same 10^{-5} stat precision.

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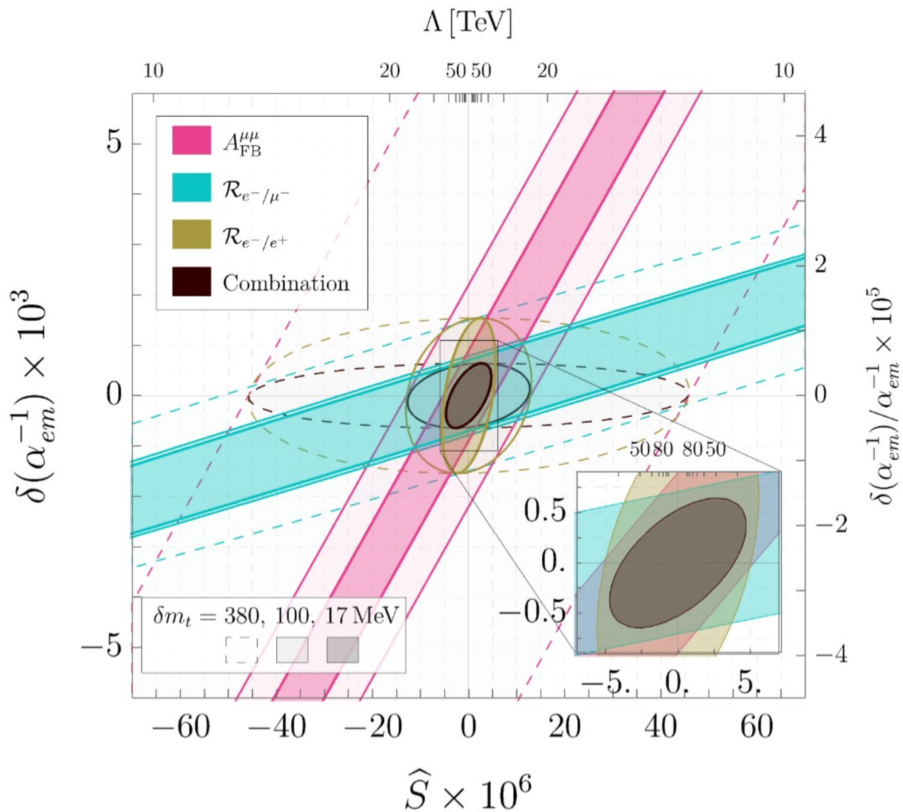
- What about our initial motivation?

$$\delta(\sin^2\theta_W^{eff})/\sin^2\theta_W^{eff} \times 10^5 = \frac{\widehat{S}}{5\cdot 10^{-6}} - \frac{\delta(\alpha_{em}^{-1})}{10^{-3}} - \frac{\delta m_t}{65\,\mathrm{MeV}}$$

- The electromagnetic coupling is no longer a bottleneck for electroweak precision.

  The top mass is. Needs to be measured at ttbar FCC-ee run.



- The ~40TeV scale reached once A_{FB} is combined with e^-/μ^- and e^-/e^+ ratios for α_{em} and ttbar run for m_t.
- Precision on M_t is strongly correlated with NP reach.
- As before, unclear whether a global fit might provide extra handle for m_t. Likely model dependent.

Conclusions

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- Current measurements and future projections of an indirect determination of the em coupling are insufficient for the ambitious electroweak program of the Tera-Z phase of FCC-ee.

- The insane statistics of Tera-Z provides itself a solution in the proposed e^-/μ^- and e^-/e^+ ratios, in combination with A_{FB}.

- Many (relevant) stuff to do: computation at higher order in PT, experimental systematics, detector requirements, refinement of HVP treatment, embedding in global fit...

- Precision of Tera-Z is a revolution. Likely plenty of unforeseen challenges and opportunities.

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Thank you!