SMEFT at 2 loops

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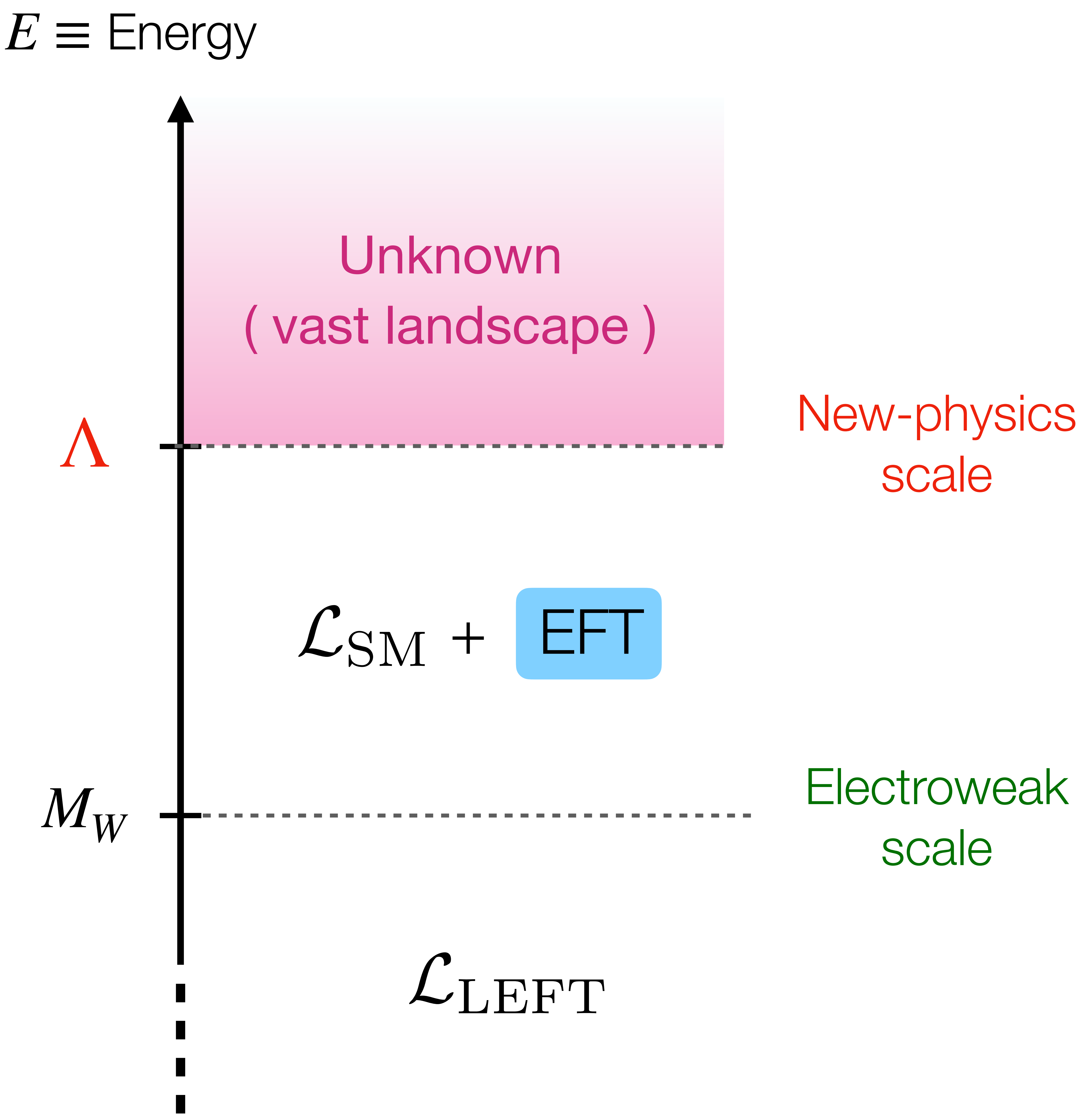
8th FCC Physics Workshop, CERN, 13-17 January 2025

The (SM)EFT approach



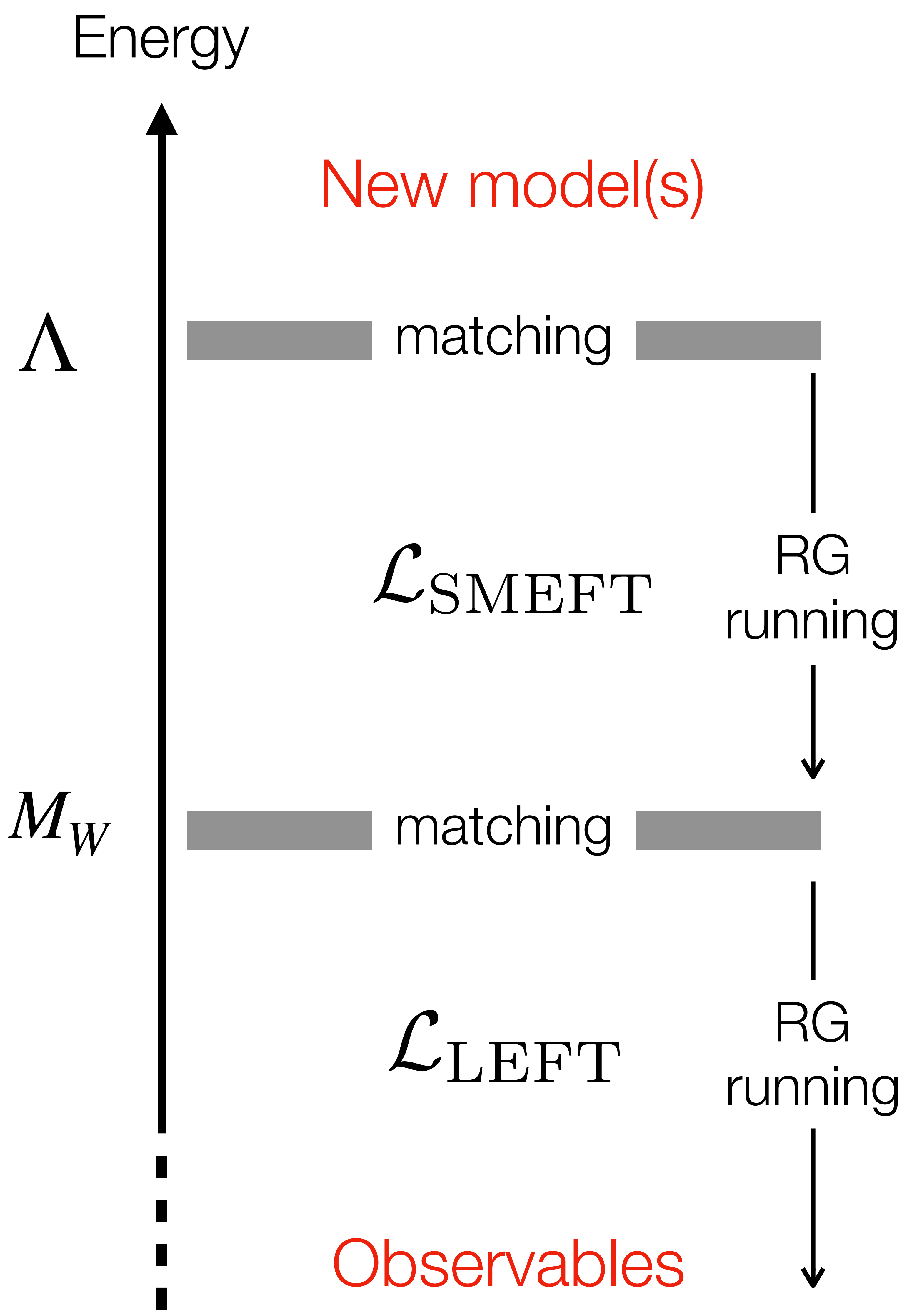
EFTs are great for parametrizing the unknown:

- Can be formulated without knowing the full theory
- Systematically improvable by adding extra terms in a double expansion in quantum corrections and E/Λ

$$\mathscr{L}_{\rm EFT}(\eta_L) = \mathscr{L}_{d=4}(\eta_L) + \sum_{\ell=0}^{\infty} \sum_{n=5}^{\infty} \sum_{k} \frac{C_{n,k}^{(\ell)}}{(16\pi^2)^{\ell}\, \Lambda^{n-4}}\, O_{n,k}(\eta_L)$$

UV physics

The rise of automation

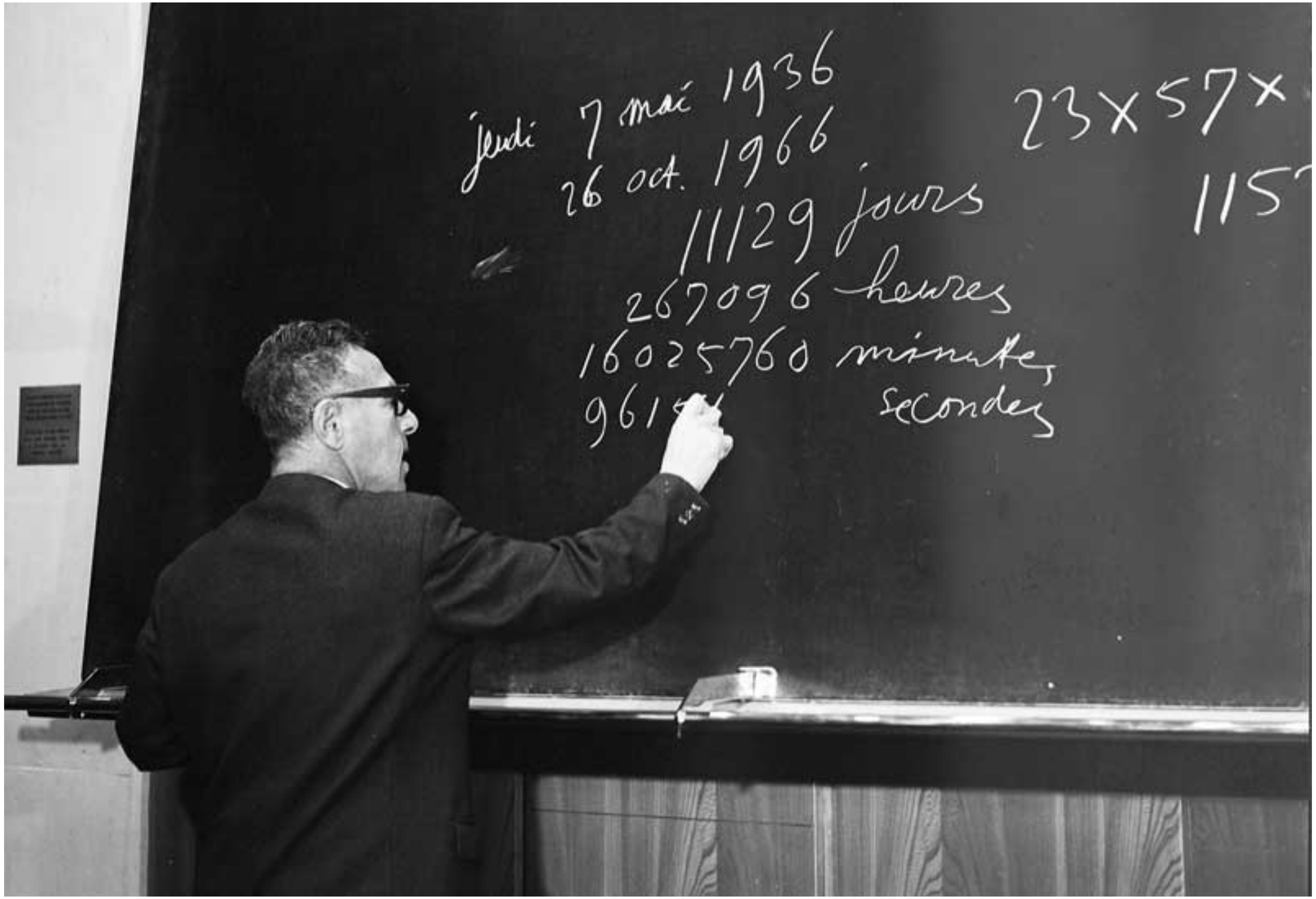


Main motivation

The vast landscape of BSM models and the repetitive nature of EFT computations call for automated solutions

The rise of automation

Wim Klein, CERN “human computer”



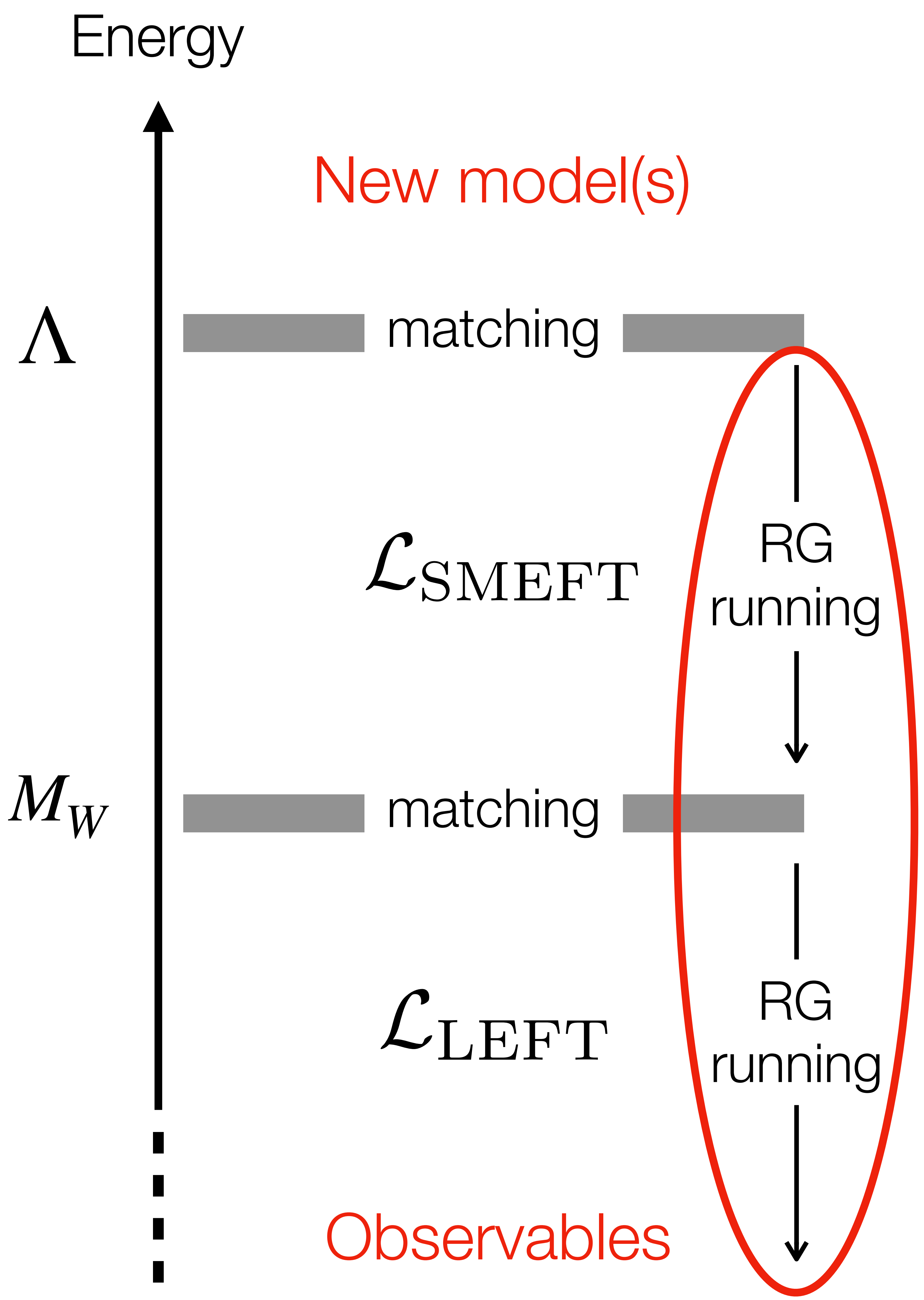
The rise of automation

CERN first electronic computer

The (SM)EFT software project:

Upgrading from “human computers” to computers

The rise of automation



JFM et al. '17 & '21

Aebischer et al. '18

Building from “human computed” one-loop results:

SMEFT running: Jenkins et al. '13, '14; Alonso et al. '14

LEFT basis: Jenkins et al. '18

SMEFT-LEFT matching: Dekens, Stoffer '19

LEFT running: Jenkins et al. '18

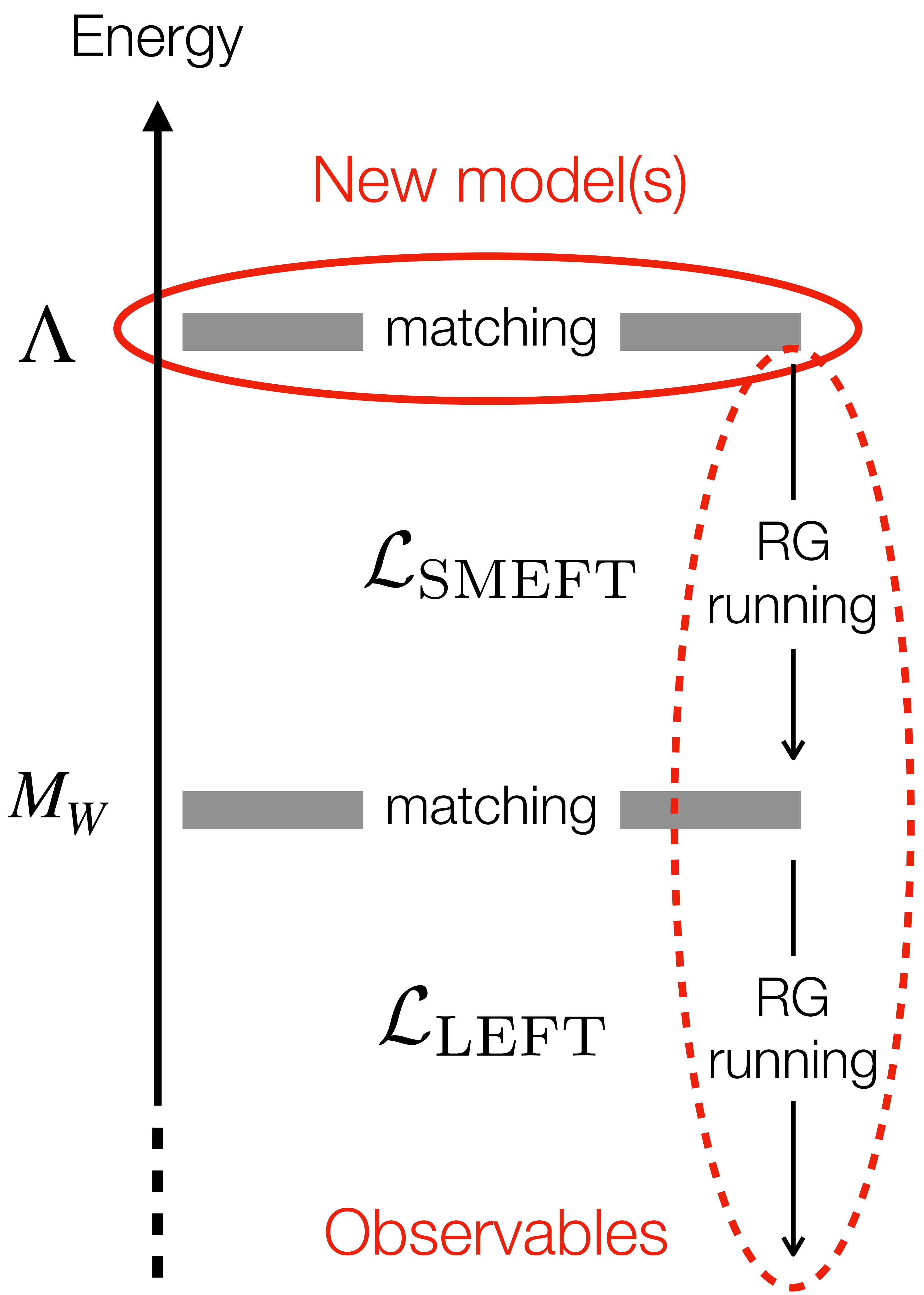
Very recent developments in the LEFT:

LEFT 2-loop RGEs:

Aebischer, Morell, Pesut, Virto, 2501.08384

Naterop, Stoffer, 2412.13251

The rise of automation



matchmakereft

Carmona et al. '22

MATCHETE

JFM et al. '23

Automated one-loop RG and matching calculations for *many* models

SOLD

Guedes et al. '23

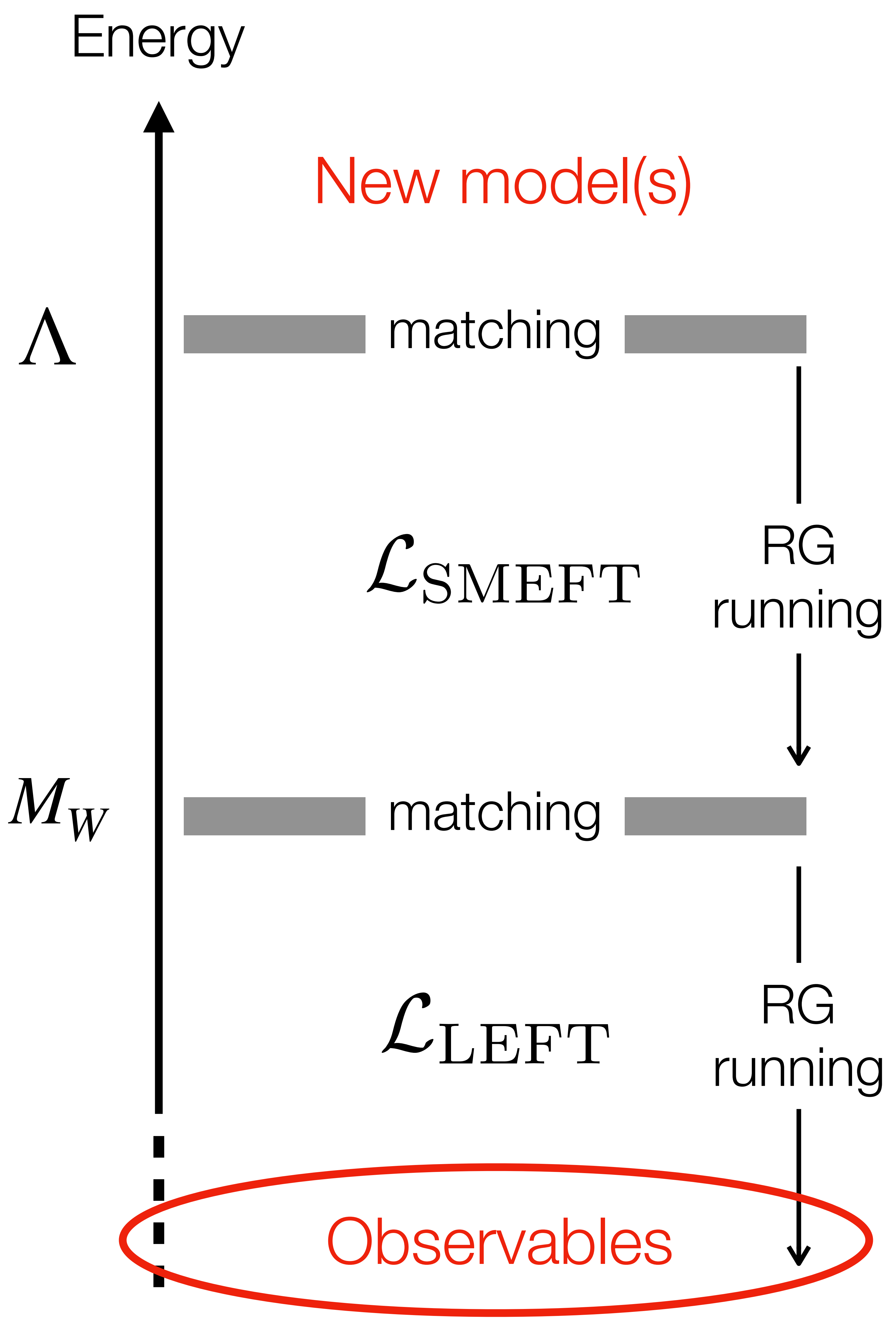
Guedes et al. '24

UV-SMEFT dictionaries

"Breaking SMEFT operators" UV-to-SMEFT mapping

Cepedello et al. '23

The rise of automation



SMEFT likelihood (smelli)

Aebischer et al. '18

flavio

Straub '16

HighPT

Allwicher et al. '22

HEPfit

De Blas et al. '19

Fitmaker

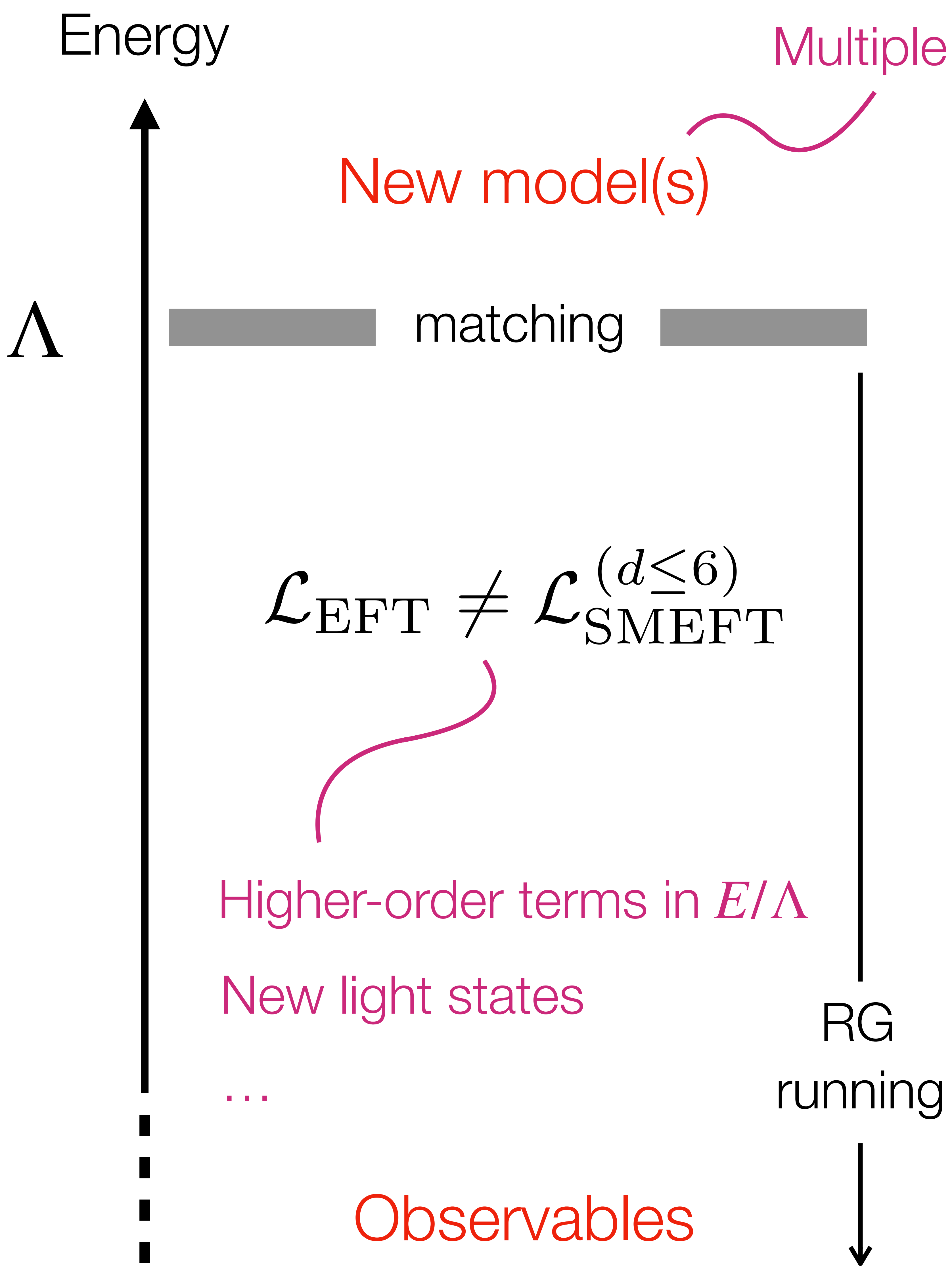
Ellis et al '20

SMEFiT

Giani et al. '23

+ others

Going beyond the state-of-the-art



Present limitations & ongoing efforts

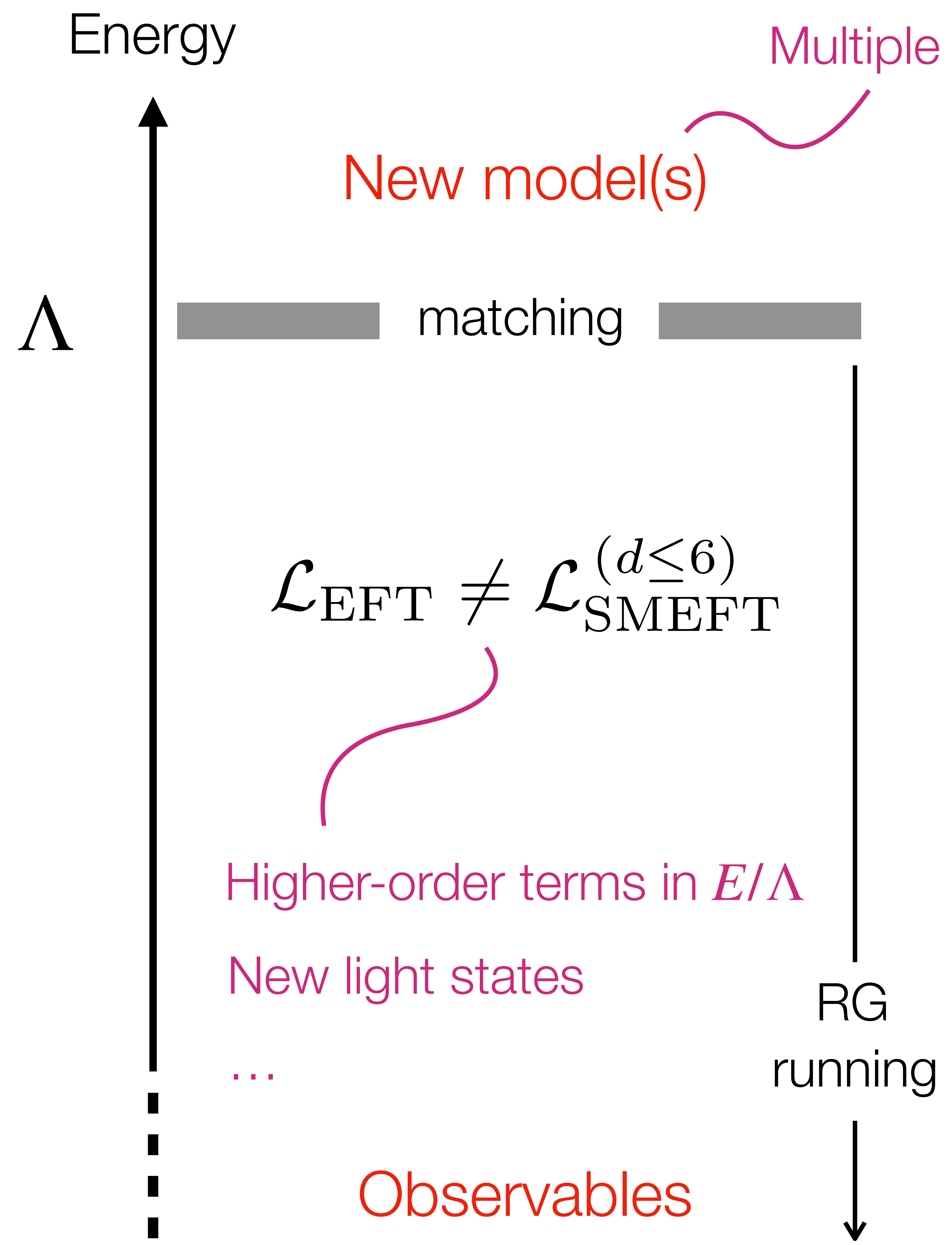
Full automation only for simpler scenarios (no heavy vectors yet!)

Implementation of many observables at one loop is still needed

Multiple efforts to extend this program beyond dimension 6

(Automated) inclusion of higher-loop orders in RG and matching calculations has recently started

Going beyond the state-of-the-art



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Needed to cancel the scheme-dependence of one-loop matching contributions

See [Di Noi et al, 2310.18221] for a recent example in $gg \to h$

SMEFT at higher orders: dim-8 vs 2-loop effects

As Λ increases, 2-loop effects start becoming more relevant:

Naively, at FCC:

(assuming $\Lambda \sim 10$ TeV)

$$\frac{y_t^2}{16\pi^2} \sim \frac{1}{150} \quad \text{vs.} \quad \frac{m_Z^2}{\Lambda^2} \sim \frac{2}{150^2}$$

Loop suppression

Mass suppression

If an effect is absent at tree-level (e.g. protected by symmetries), two-loop effects easily dominate over dim-8

New states: $\Theta_1 \sim \mathbf{4}_{1/2}\,, \quad \Theta_3 \sim \mathbf{4}_{3/2} \longrightarrow \boldsymbol{\Theta} \sim (\mathbf{4},\mathbf{4})$ of $SU(2)_L \times SU(2)_R$

$$\mathscr{L}_{\rm CQ} \supset -M_4^2\left(|\Theta_1|^2 + |\Theta_3|^2\right) - \lambda_4\left(H^*H^*(\varepsilon H)\Theta_1 + \frac{1}{\sqrt{3}}H^*H^*H^*\Theta_3\right) + {\rm h.c.}$$

Custodially-protected models

See Ben Stefanek's talk

[Durieux, McCollough, Salvioni, 2209.00666]

RG mixing in the SMEFT up to two loops

$$\frac{C_j(\mu)}{C_i(\mu_0)} \approx \frac{\gamma_{ji}^{(1)}}{16\pi^2} \ln\frac{\mu}{\mu_0} + \frac{\gamma_{jm}^{(1)}\gamma_{mi}^{(1)}}{2(16\pi^2)^2} \ln^2\frac{\mu}{\mu_0} + \frac{\gamma_{ji}^{(2)}}{(16\pi^2)^2} \ln\frac{\mu}{\mu_0}$$

1-loop LL effect

2-loop LL effect

2-loop NLL effect

RG mixing in the SMEFT up to two loops

Mixing $Q_i \to Q_j$

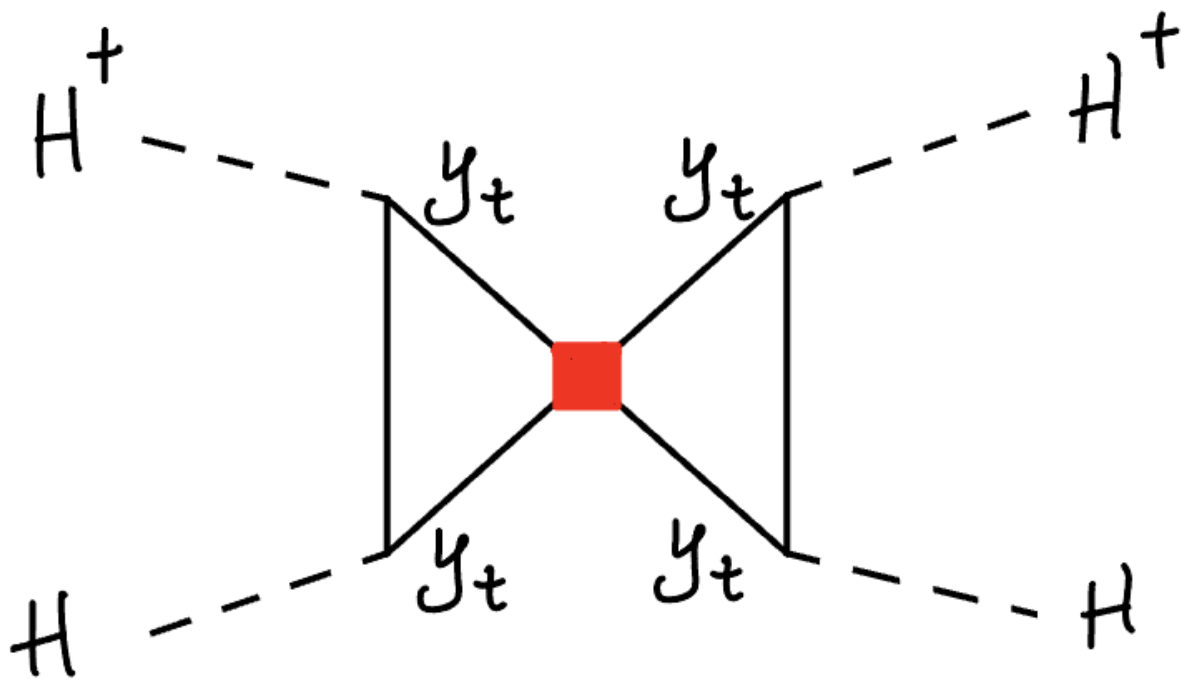
$$\frac{C_j(\mu)}{C_i(\mu_0)} \approx \frac{\gamma_{ji}^{(1)}}{16\pi^2} \ln\frac{\mu}{\mu_0} + \frac{\gamma_{jm}^{(1)}\gamma_{mi}^{(1)}}{2(16\pi^2)^2} \ln^2\frac{\mu}{\mu_0} + \frac{\gamma_{ji}^{(2)}}{(16\pi^2)^2} \ln\frac{\mu}{\mu_0}$$

Mixing $Q_i \to Q_m \to Q_j$

How relevant are 2-loop effects?

Two recent examples

1. Four-top operator mixing into C_{HD} (mostly 2-loop LL)



$$[O_{qq}^{(1)}]_{3333} \longrightarrow [O_{Hq}^{(1)}]_{33} \longrightarrow O_{HD}$$

$$T \simeq -\frac{3y_t^4}{8\pi^4\alpha}\frac{v^2}{\Lambda^2}[C_{qq}^{(1)}]_{3333}\left[\ln^2\frac{\Lambda}{M_Z} - \frac{1}{4}\ln\frac{\Lambda}{M_Z}\right]$$

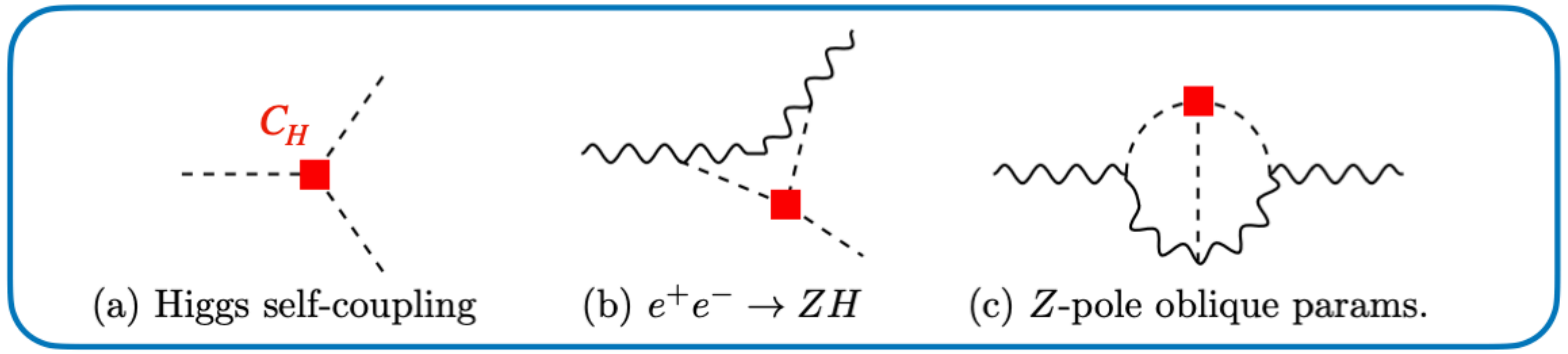
$$T \in [-0.23, 0.25] \longrightarrow \frac{[C_{qq}^{(1)}]_{3333}}{\Lambda^2} \in \frac{[-2.04, 1.87]}{\mathrm{TeV}^2}$$

[Haisch, Schnell, 2410.13304; Stefanek, 2407.09593]

2. C_H at the Z-pole (finite 2-loop)

Higgs self-coupling modifications starting at NLO (finite):

[1312.3322, 1702.07678, 1702.01737]



[Maura, BAS, You, 2412.14241]

See Ben Stefanek's talk

2-loop RG and matching calculations

The functional approach in a nutshell

Functional matching

- **Lagrangian:** $\mathscr{L}_{\rm UV}$ with fields $\eta = (\eta_H\ \eta_L)^\intercal$ and hierarchy $m_H \gg m_L$

- **Background field method:** shift *all* fields $\eta \to \hat{\eta} + \eta$

  $\hat{\eta}$: background fields (satisfy the quantum EOM) [Tree lines in Feynman graphs]

  $\hat{\eta}$: quantum fluctuations [Loop lines in Feynman graphs]

- **Quantum effective action:**

$$e^{i\,\Gamma_{\rm UV}[\hat{\eta}]} = \int \mathscr{D}\eta\ \exp\left(i\int {\rm d}^d x\ \mathscr{L}_{\rm UV}(\eta + \hat{\eta})\right)$$

Goal: Evaluate the path integral
("integrate out" the quantum fluctuations)
and isolate the EFT contribution

Functional matching

- **Expanding the Lagrangian in η:**

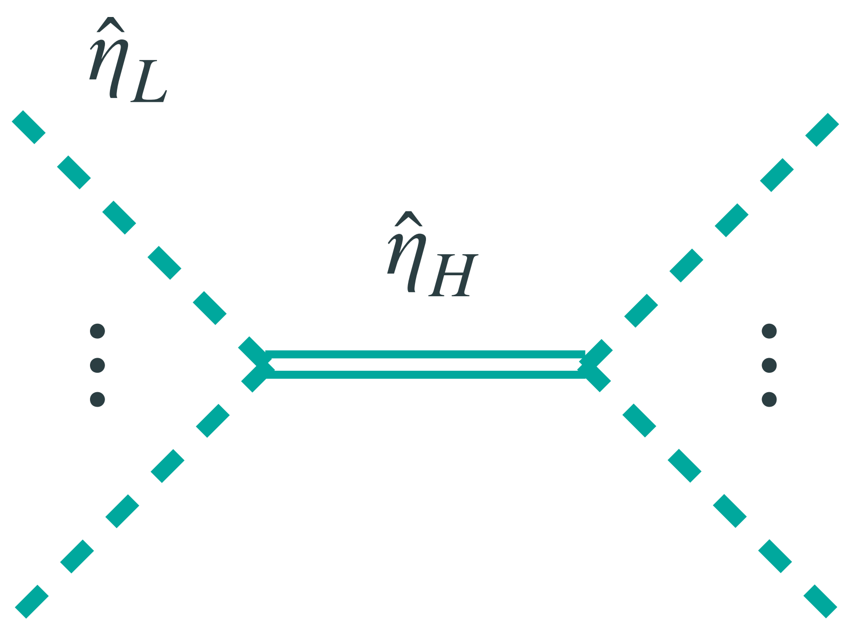
$$\mathscr{L}_{\rm UV}(\hat\eta+\eta) = \mathscr{L}_{\rm UV}(\hat\eta) + \frac{\delta\mathscr{L}_{\rm UV}}{\delta\eta_a}\bigg|_{\eta=\hat\eta}\eta_a + \frac{1}{2}\eta_a(x)\frac{\delta^2\mathscr{L}_{\rm UV}}{\delta\eta_a(x)\,\delta\eta_b(x')}\bigg|_{\eta=\hat\eta}\eta_b(x') + \mathcal{O}(\eta^3)$$

- **Tree-level:** $\mathscr{L}^{(0)}_{\rm EFT} = \mathscr{L}_{\rm UV}\big(\hat\eta_L, \hat\eta_H(\hat\eta_L)\big)$

— Substitute $\hat\eta_H$ by its EOM expanded in m_H^{-1}

[Simpler than computing Feynman graphs]

$$\frac{\delta\mathscr{L}_{\rm UV}}{\delta\eta_H}\bigg|_{\eta=\hat\eta} = 0$$



Functional matching

- **Expanding the Lagrangian in η:**

$$\mathscr{L}_{\rm UV}(\hat\eta+\eta) = \mathscr{L}_{\rm UV}(\hat\eta) + \underbrace{\frac{\delta \mathscr{L}_{\rm UV}}{\delta \eta_a}\bigg|_{\eta=\hat\eta}}_{0}\eta_a + \frac{1}{2}\,\eta_a(x)\,\underbrace{\frac{\delta^2 \mathscr{L}_{\rm UV}}{\delta\eta_a(x)\,\delta\eta_b(x')}\bigg|_{\eta=\hat\eta}}_{\int_{x'} \mathcal{Q}_{ab}(x,x')}\,\eta_b(x') + \mathcal{O}(\eta^3)$$

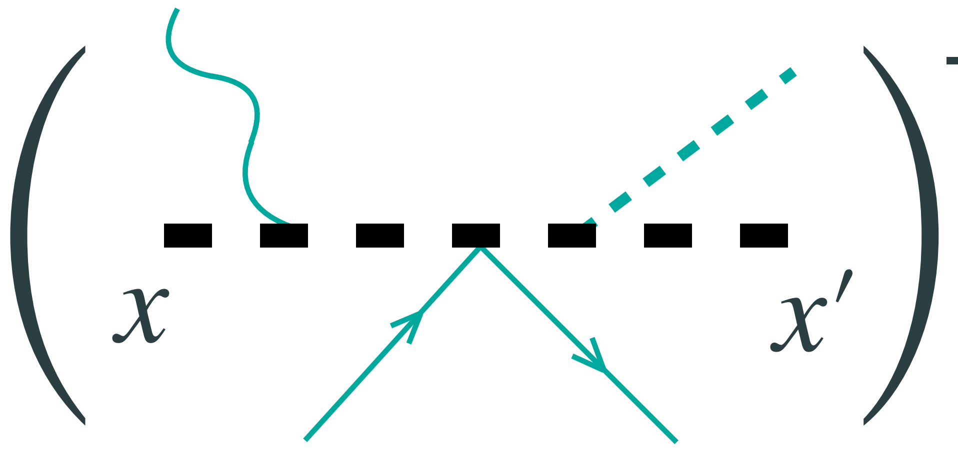
- **Inverse quantum-field propagator:**

$$\mathcal{Q}_{ab}(x,x') = Q_{ac}\left(\hat\eta(x), \hat D_x^{\mu_1}\cdots\hat D_x^{\mu_n}\hat\eta(x), \hat D_x^{\mu}\right) \boldsymbol{U_{cb}(x,x')}\,\delta(x-x')$$

Wilson line

[parallel transport $x \leftrightarrow x'$]

$$\left(\; \right)^{-1}$$



$$\hat D_x^{\mu_1}\cdots\hat D_x^{\mu_n}\,U(x,x')\Big|_{x=x'} = p_n\left(G^{\mu\nu}, D^\mu G^{\nu\rho}, \ldots\right)$$

[Kuzenko, McArthur, ’03]

[JFM, Moreno-Sánchez, Palavrić, Thomsen, 2412.12270]

Functional matching

- **Expanding the Lagrangian in η:**

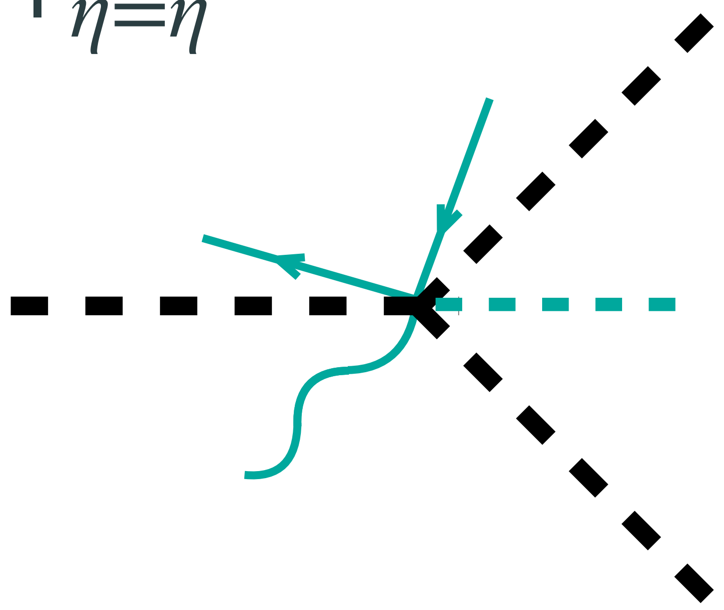
$$\int_{x'} \mathcal{Q}_{ab}(x,x') \equiv$$

$$\mathcal{L}_{\mathrm{UV}}(\hat{\eta}+\eta) = \mathcal{L}_{\mathrm{UV}}(\hat{\eta}) + \underbrace{\left.\frac{\delta \mathcal{L}_{\mathrm{UV}}}{\delta \eta_a}\right|_{\eta=\hat{\eta}}}_{0} \eta_a + \frac{1}{2}\eta_a(x) \left.\frac{\delta^2 \mathcal{L}_{\mathrm{UV}}}{\delta\eta_a(x)\,\delta\eta_b(x')}\right|_{\eta=\hat{\eta}} \eta_b(x') + \mathcal{O}(\eta^3)$$

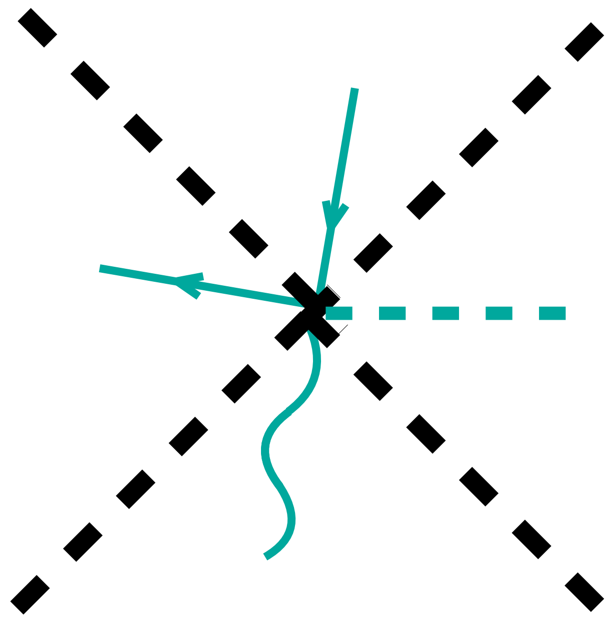
Higher-loop orders

- **Inverse quantum-field propagator:**

$$\mathcal{C}_{IJK} \equiv \left.\frac{\delta^3 \mathcal{L}_{\mathrm{UV}}}{\delta\eta_I\,\delta\eta_J\,\delta\eta_K}\right|_{\eta=\hat{\eta}}$$



$$\mathcal{D}_{IJKL} \equiv \left.\frac{\delta^4 \mathcal{L}_{\mathrm{UV}}}{\delta\eta_I\,\delta\eta_J\,\delta\eta_K\,\delta\eta_L}\right|_{\eta=\hat{\eta}}$$



N.B.: $\eta_I \equiv \eta_a(x)$

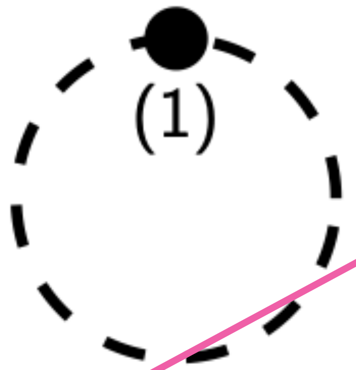
Going beyond one loop

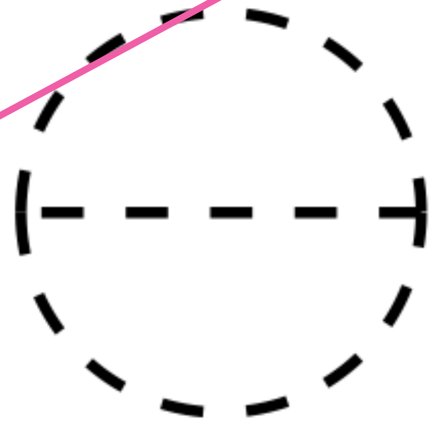
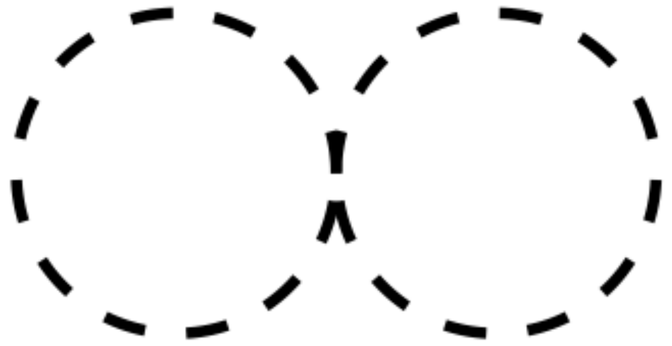
[JFM, Palavrić, Thomsen, 2311.13630]

[JFM, Moreno-Sánchez, Palavrić, Thomsen, 2412.12270]

$$\Gamma_{\text{UV}}[\hat{\eta}] = S_{\text{UV}}[\hat{\eta}] - i \ln \int \mathscr{D}\eta \exp\left[i \left(\frac{1}{2} \bar{\eta}_I \mathcal{Q}_{IJ} \eta_J + \frac{1}{3!} \eta_K \eta_J \eta_I \mathscr{C}_{KJI} + \frac{1}{4!} \eta_L \eta_K \eta_J \eta_I \mathscr{D}_{IJKL} + \ldots \right) \right]$$

$$= S_{\text{UV}}[\hat{\eta}] + \frac{i\hbar}{2} \text{STr} \ln \mathcal{Q} + \frac{i\hbar^2}{2} \mathcal{Q}^{-1}_{IJ} \mathcal{Q}^{(1)}_{JI} + \frac{\hbar^2}{12} \mathscr{C}_{IJK} \mathcal{Q}^{-1}_{IL} \mathcal{Q}^{-1}_{JM} \mathcal{Q}^{-1}_{KN} \mathscr{C}_{LMN} - \frac{\hbar^2}{8} \mathcal{Q}^{-1}_{IJ} \mathscr{D}_{IJKL} \mathcal{Q}^{-1}_{KL} + \mathcal{O}(\hbar^3)$$

$= S_{\text{UV}}[\hat{\eta}] + \frac{i}{2} \log$  $+ \frac{i}{2}$ 

$+ \frac{1}{12}$  $- \frac{1}{8}$  $+ \mathcal{O}(\hbar^3)$

Tree level — **One loop** — **Two loops**

Every two-loop contribution is included here

(at all orders in the EFT expansion!)

Towards SMEFT RGEs at 2 loops

No phenomenology this time, apologies

Two-loop RGEs in the bosonic SMEFT (bSMEFT)

We have implemented functional methods at 2-loops into a custom version (still private) of MATCHETE

[fermions not yet implemented, ongoing progress]

[Born, JFM, Kvedaraitė, Thomsen, 2410.07320]

First application: 2-loop RGEs in the bosonic SMEFT (bSMEFT) :

$$\mathscr{L}_{bSMEFT} = -\frac{1}{4}B^{\mu\nu}B_{\mu\nu} - \frac{1}{4}W^{I\,\mu\nu}W^I_{\mu\nu} - \frac{1}{4}G^{A\,\mu\nu}G^A_{\mu\nu} + D_\mu H^\dagger D^\mu H + \mu^2 H^\dagger H - \frac{\lambda}{2}(H^\dagger H)^2 + \mathscr{L}_{\text{gf.}} + \mathscr{L}_{\text{gh.}} + \mathscr{L}_\theta$$

+

	X^2H^2
C_{HB}	$H^\dagger H\, B^{\mu\nu}\, B_{\mu\nu}$
C_{HW}	$H^\dagger H\, W^{I\,\mu\nu}\, W^I_{\mu\nu}$
C_{HG}	$H^\dagger H\, G^{A\,\mu\nu}\, G^A_{\mu\nu}$
$C_{H\widetilde{B}}$	$H^\dagger H\, B^{\mu\nu}\, \widetilde{B}_{\mu\nu}$
$C_{H\widetilde{W}}$	$H^\dagger H\, W^{I\,\mu\nu}\, \widetilde{W}^I_{\mu\nu}$
$C_{H\widetilde{G}}$	$H^\dagger H\, G^{A\,\mu\nu}\, \widetilde{G}^A_{\mu\nu}$
C_{HWB}	$(H^\dagger \sigma^I H)\, B^{\mu\nu}\, W^I_{\mu\nu}$
$C_{H\widetilde{W}B}$	$(H^\dagger \sigma^I H)\, B^{\mu\nu}\, \widetilde{W}^I_{\mu\nu}$

	X^3
C_W	$f^{IJK}\, W^{I\,\nu}_\mu\, W^{J\,\rho}_\nu\, W^{K\,\mu}_\rho$
C_G	$f^{ABC}\, G^{A\,\nu}_\mu\, G^{B\,\rho}_\nu\, G^{C\,\mu}_\rho$
$C_{\widetilde{W}}$	$f^{IJK}\, \widetilde{W}^{I\,\nu}_\mu\, W^{J\,\rho}_\nu\, W^{K\,\mu}_\rho$
$C_{\widetilde{G}}$	$f^{ABC}\, \widetilde{G}^{A\,\nu}_\mu\, G^{B\,\rho}_\nu\, G^{C\,\mu}_\rho$
	H^4D^2 and H^6
C_H	$(H^\dagger H)^3$
$C_{H\Box}$	$(H^\dagger H)\Box(H^\dagger H)$
C_{HD}	$(H^\dagger D_\mu H)^*\,(H^\dagger D^\mu H)$

Some results in the bSMEFT at NLL

$$\frac{dC_i}{d\ln\mu} = \frac{\beta_i^{(1)}}{(16\pi^2)} + \frac{\beta_i^{(2)}}{(16\pi^2)^2} + \ldots$$

$$\beta^{(1)}_{C_{HD}} = \left(\frac{9}{2}g_L^2 + 6\lambda - \frac{5}{6}g_Y^2\right)C_{HD} + \frac{20}{3}g_Y^2 C_{H\Box}$$

$$\beta^{(1)}_{C_{H\Box}} = \left(12\lambda - \frac{4}{3}g_Y^2 - 4g_L^2\right)C_{H\Box} + \frac{5}{3}g_Y^2 C_{HD}$$

$$\begin{aligned}\beta^{(2)}_{C_{HD}} = &\left[\lambda\left(\frac{5}{2}g_Y^2 - \frac{45}{2}g_L^2\right) + \frac{299}{216}g_Y^4 + \frac{41}{2}g_Y^2 g_L^2 - \frac{1}{8}g_L^4 - 36\lambda^2\right]C_{HD}\\ &+\left(\frac{70}{27}g_Y^4 - \frac{227}{9}g_Y^2 g_L^2 - \frac{136}{3}\lambda g_Y^2\right)C_{H\Box}\\ &+\left(32 g_Y^3 g_L - 68 g_Y g_L^3 - 96\lambda g_Y g_L\right)C_{HWB}\\ &+\left(\frac{32}{3}g_Y^4 + 12 g_Y^2 g_L^2 - 48\lambda g_Y^2\right)C_{HB}\\ &+28 g_Y^2 g_L^2 C_{HW} + 26 g_Y^2 g_L^3 C_W\end{aligned}$$

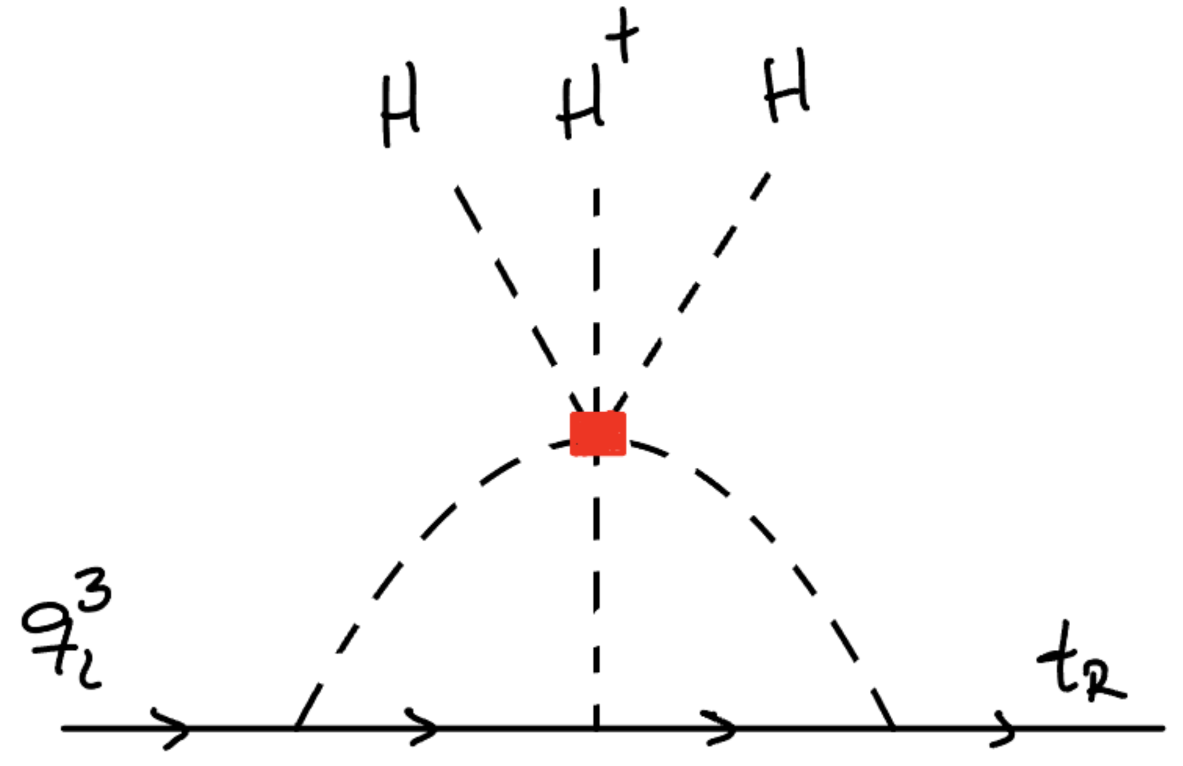
Enlarged mixing: Most operators mix at NLL

Some results in the bSMEFT at NLL

$$\frac{dC_i}{d\ln\mu} = \frac{\beta_i^{(1)}}{(16\pi^2)} + \frac{\beta_i^{(2)}}{(16\pi^2)^2} + \dots$$

C_H does not mix into any other bSMEFT operator, even at two loops

→ This is is no longer the case in the complete SMEFT



[Top-Yukawa correction]

This result was already anticipated using amplitude methods

[Bern, Parra-Martinez, Sawyer, 2005.12917]

Summary and conclusions

- The EFT program has entered an **era of automation**, considerably simplifying BSM analyses

- **Functional methods** play a crucial role in RG and matching calculations, especially when combined with computer tools
  - — Compact, systematic, and manifestly gauge invariant at all steps

- First results towards the **2-loop SMEFT RGE** already available
  - — 2-loop LL (from the one-loop anomalous dimension) often capture dominant 2-loop effects, but NLL effects give a richer operator-mixing structure
  - — FCC-ee will reach sufficient precision to probe some of these 2-loop effects. However, a systematic phenomenological exploration is still missing

Thank you