# Point-to-point uncertainty on $\sqrt{s}$ from dimuon events at FCC-ee

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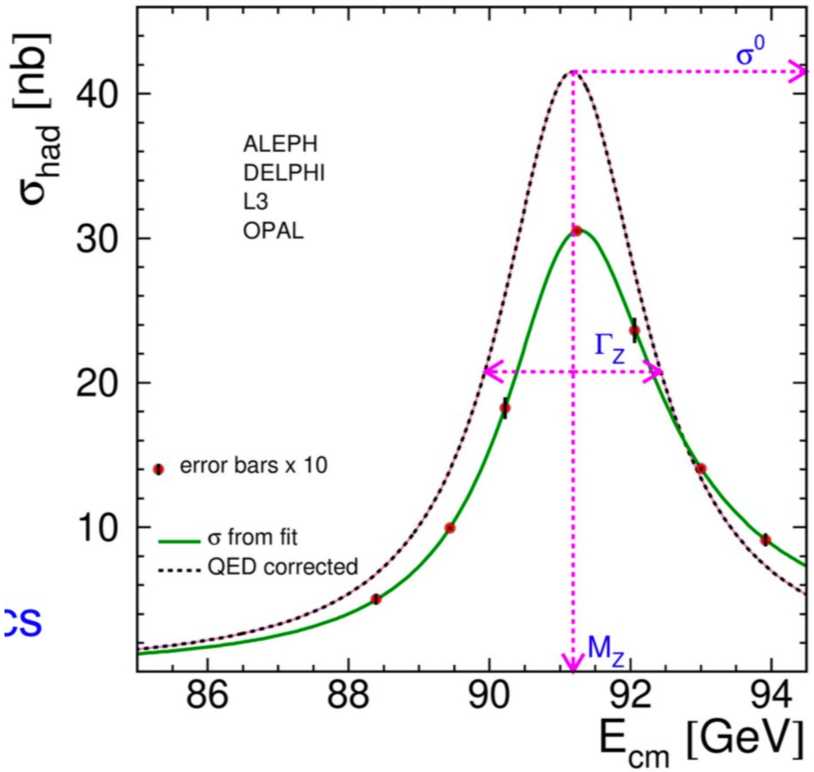
# Determination of the Z width from the line-shape

Expected statistical uncertainty: 4 keV

- Absolute calibration of √s: key for the determination of the Z mass
- But for the Z width: what matters if the relative, point-to-point uncertainty on √s, between the off-peak points used in the line-shape scan
  - Other important systematic: BES




With $\delta(\sqrt{s})_{ptp}$ ~ 10 keV, syst. uncertainty on $\Gamma_Z$ would be 5 keV, at the level of the stat. !

NB: $\delta(\sqrt{s})_{ptp}$ also important systematic for sin2thetaW from AFB(μμ). Need a few 10's of keV to reach the stat. uncertainty of 2e-6

# Point-to-point uncertainty on √s from dimuon events

Use e.g. the ”peak position” of the Mμμ distribution in dimuon events, at √s = MZ and at the off-peak points

arXiv:1909.12245

Key = exquisite momentum resolution

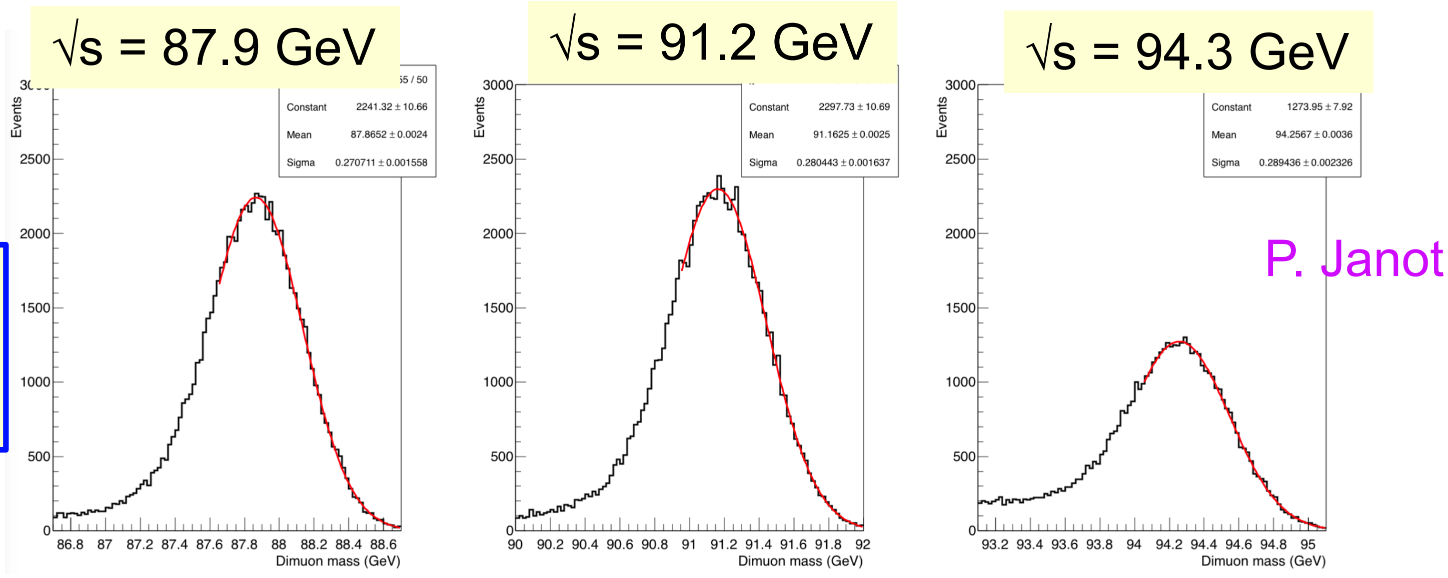



**Figure 58.** Invariant mass distribution of $10^5$ muon pairs in the CLD detector, at centre-of-mass energies of (left-to-right) 87.9, 91.2 and 94.3 GeV respectively; the width of the distribution is dominated by the muon momentum measurement uncertainty. The data correspond to 521 $pb^{-1}$, 69 $pb^{-1}$, and 257 $pb^{-1}$, which can be acquired in 4 minutes, 35 seconds and 2 minutes respectively

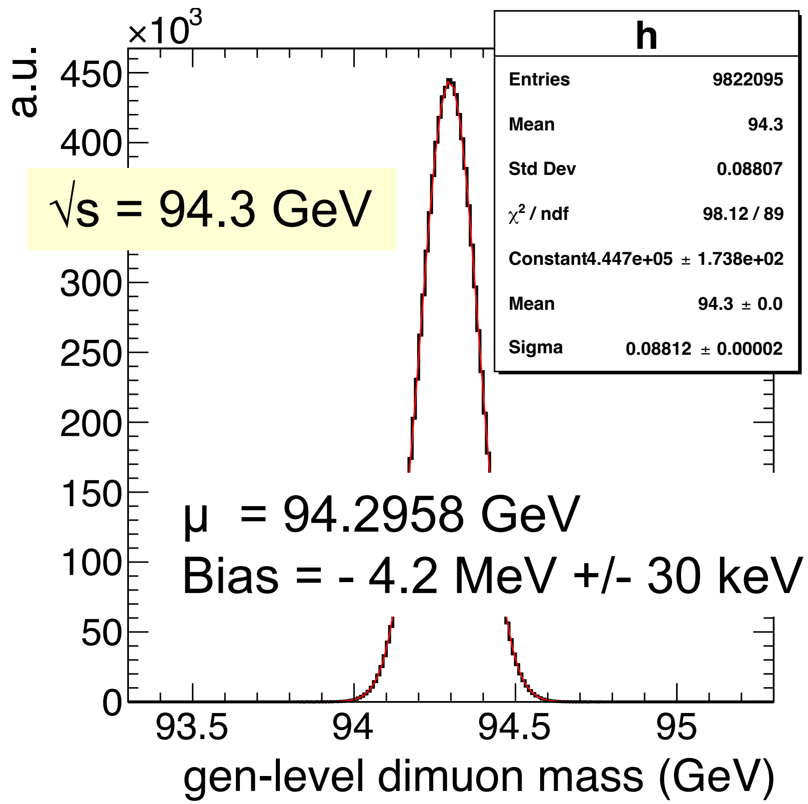
May not be good enough for an absolute calibration of √s, but could provide $\delta(\sqrt{s})_{ptp}$ to better than √2 x RDP uncertainty.

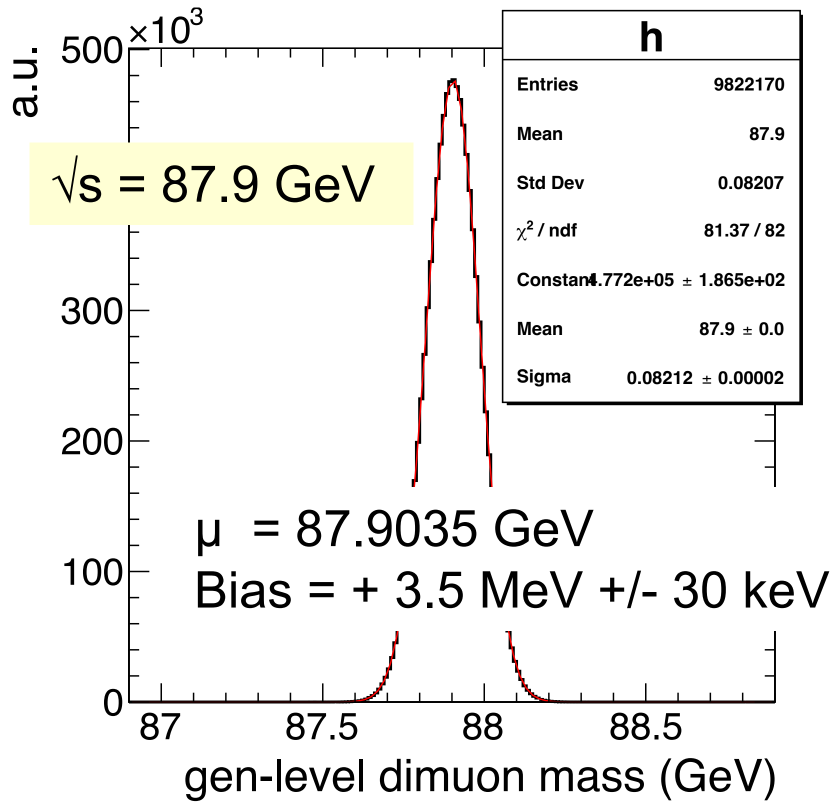
# Bias of the estimator of √s

- Any proxy to √s (e.g. the "peak position" of the Mμμ distribution, or some parameter extracted from a fit) is likely to show a bias
  - in particular due to ISR/FSR

- And this bias can depend on √s itself !

Example: no ISR, no FSR, gen-level dimuon mass. Simple gaussian fit:

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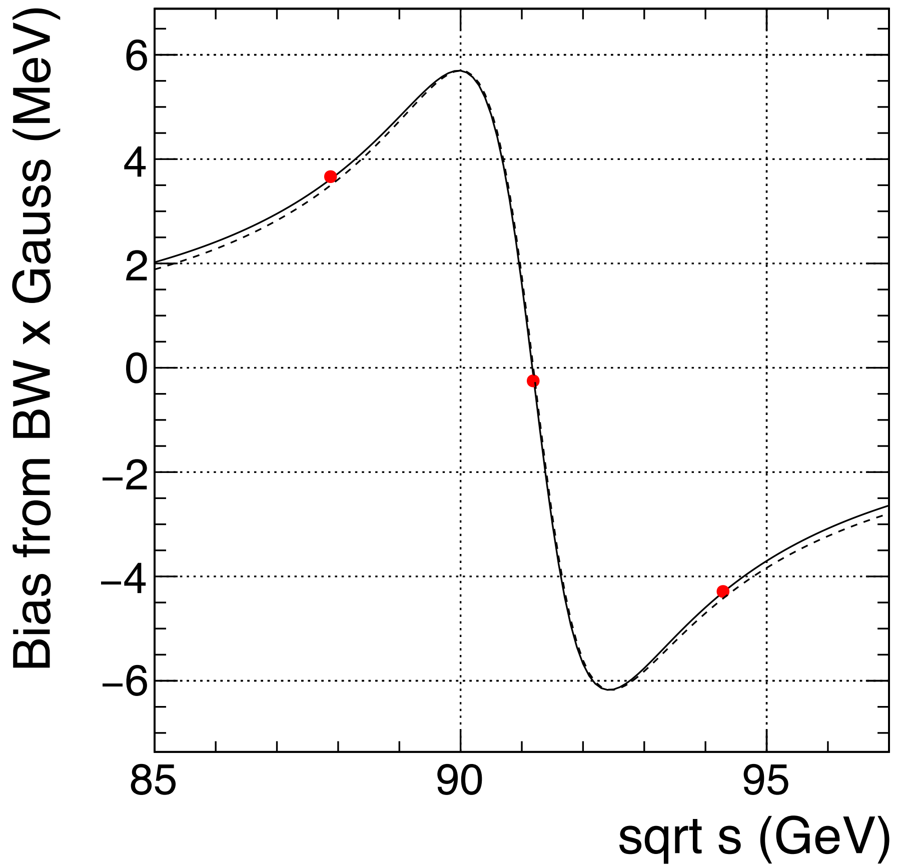
# Bias of the estimator of sqrts: simplest case

The bias in the previous plots comes from the product of the Breit-Wigner with the Gaussian that represents the beam-energy spread (BES).

- Below $M_Z$ : the BW pulls the distribution towards $M_Z$, positive bias
- Above $M_Z$ : negative bias

The value of the bias can be determined analytically by maximizing BW x Gauss(BES).

( The bias varies quadratically with the BES. )



# Samples, fit procedure

Delphes samples of ee → μμ from Whizard and KKMC
- BES, ISR and FSR
- detector: IDEA or CLD

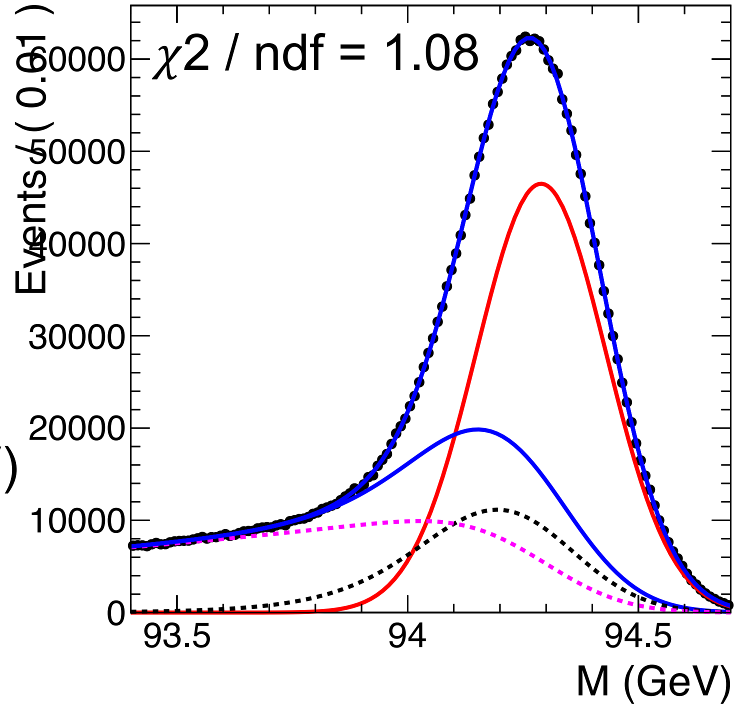
Energies: $\sqrt{s_0}$ = 91.188 GeV, $\sqrt{s_-}$ = 87.9 GeV and $\sqrt{s_+}$ = 94.3 GeV
and a few other off-peak points for checks

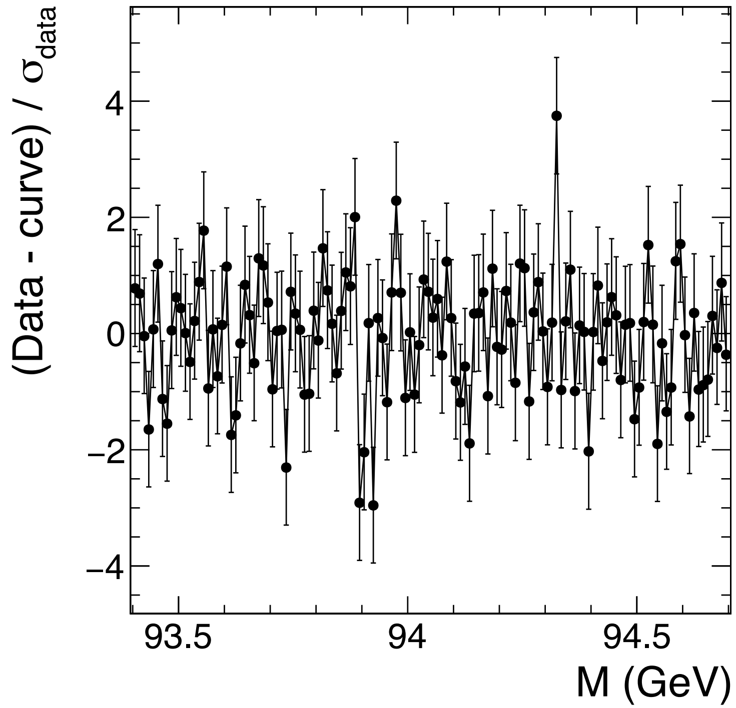
About 100 M events for each sample

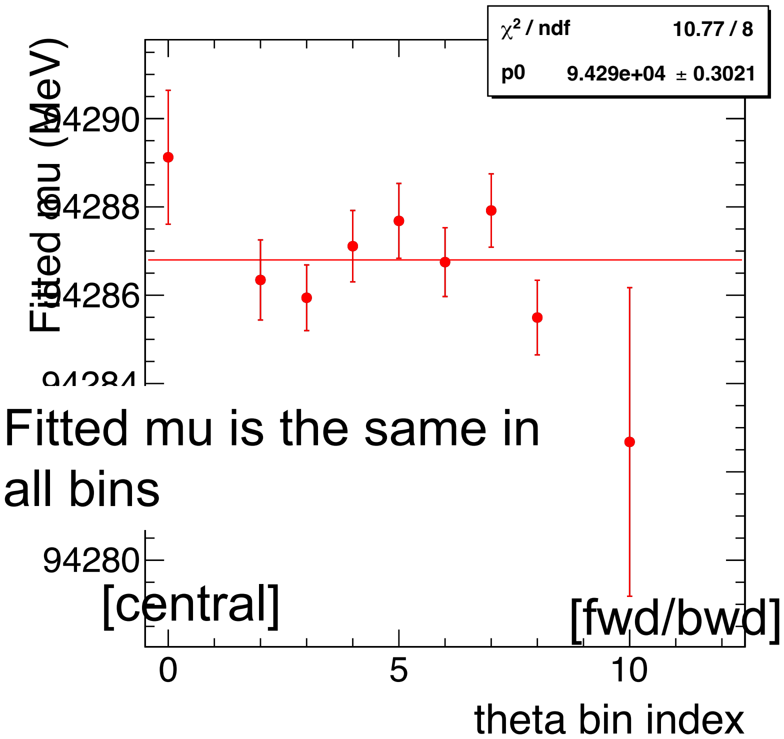
- Fit the dimuon mass distribution
  - so far, only the "raw" dimuon mass
- Fit model: Gauss ⊗ ( delta + two exponentials )
  - cf 2022 paper from G. Wilson & B. Madison, arXiv:2209.03281
  - Provides good fits – for this MC statistics
- Fits done in theta bins (angular dependence of the momentum resolution)
- To have 1D bins only: demand that the mu+ and the mu- be in the "same" theta bin (accop cut : | theta+ + theta- - Pi | < 0.1 rad )
- Keep only good fits
  - Equivalent : $\chi^2$ < Ndf + 3 x √( 2 * Ndf )
- Proxy for √s: weighted average of the mean of the Gaussian in the various theta bins

# Example fits

Reco'ed mass,
IDEA
$\sqrt{s} = \sqrt{s_+}$ (94.3 GeV)




Fitted mu is the same in all bins

Proxy for $\sqrt{s} \equiv \langle \mu \rangle$

- With 1e8 MC events : stat uncertainty on the proxy = 300 keV at 94.3 GeV (IDEA)
  - 200 keV at 87.9 and 91.2 GeV

Bias $\equiv \langle \mu \rangle - \sqrt{s}$

# In-situ determination of $\sqrt{s_+} - \sqrt{s_-}$

$$\sqrt{s_+} - \sqrt{s_-} = \langle \mu(\sqrt{s_+}) \rangle - \langle \mu(\sqrt{s_-}) \rangle - \left( \text{bias}(\sqrt{s_+}) - \text{bias}(\sqrt{s_-}) \right)$$

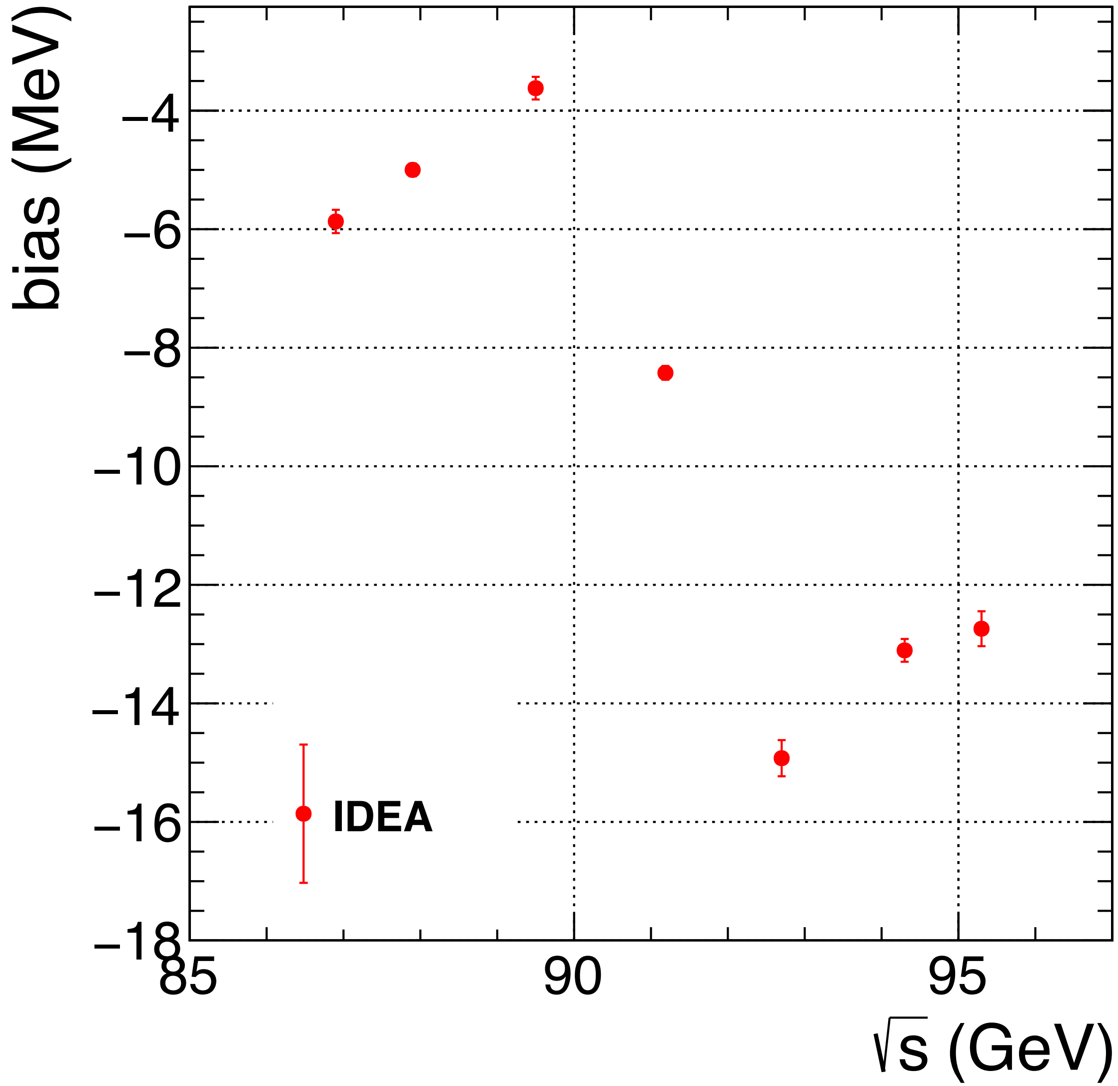
$\equiv \Delta\text{bias}$

- With 1e8 MC events, uncertainties on $\langle \mu \rangle$ = 200 - 300 keV (IDEA)
- Rescaling to the number of events expected with 40 / 125 / 40 $ab^{-1}$ at 87.9 / 91.2 / 94.3 GeV : $\langle \mu \rangle$ would be known to
  - ~ 4 keV at 91.2 GeV,
  - ~ 20 keV off-peak
- $\langle \mu(\sqrt{s_+}) \rangle - \langle \mu(\sqrt{s_-}) \rangle$ known to $20 \oplus 20 = 28$ keV (IDEA)

- $\Delta\text{bias} = \text{bias}(\sqrt{s_+}) - \text{bias}(\sqrt{s_-})$ can be predicted from MC.
- But to which precision ?
  - E.g. to which level do we need to control the modeling of ISR / FSR ?

# Dependence of the bias vs √s (IDEA)

Most of the dependence seems to come from the interplay of the Breit-Wigner with the Gaussian describing the BES (see slide 5).

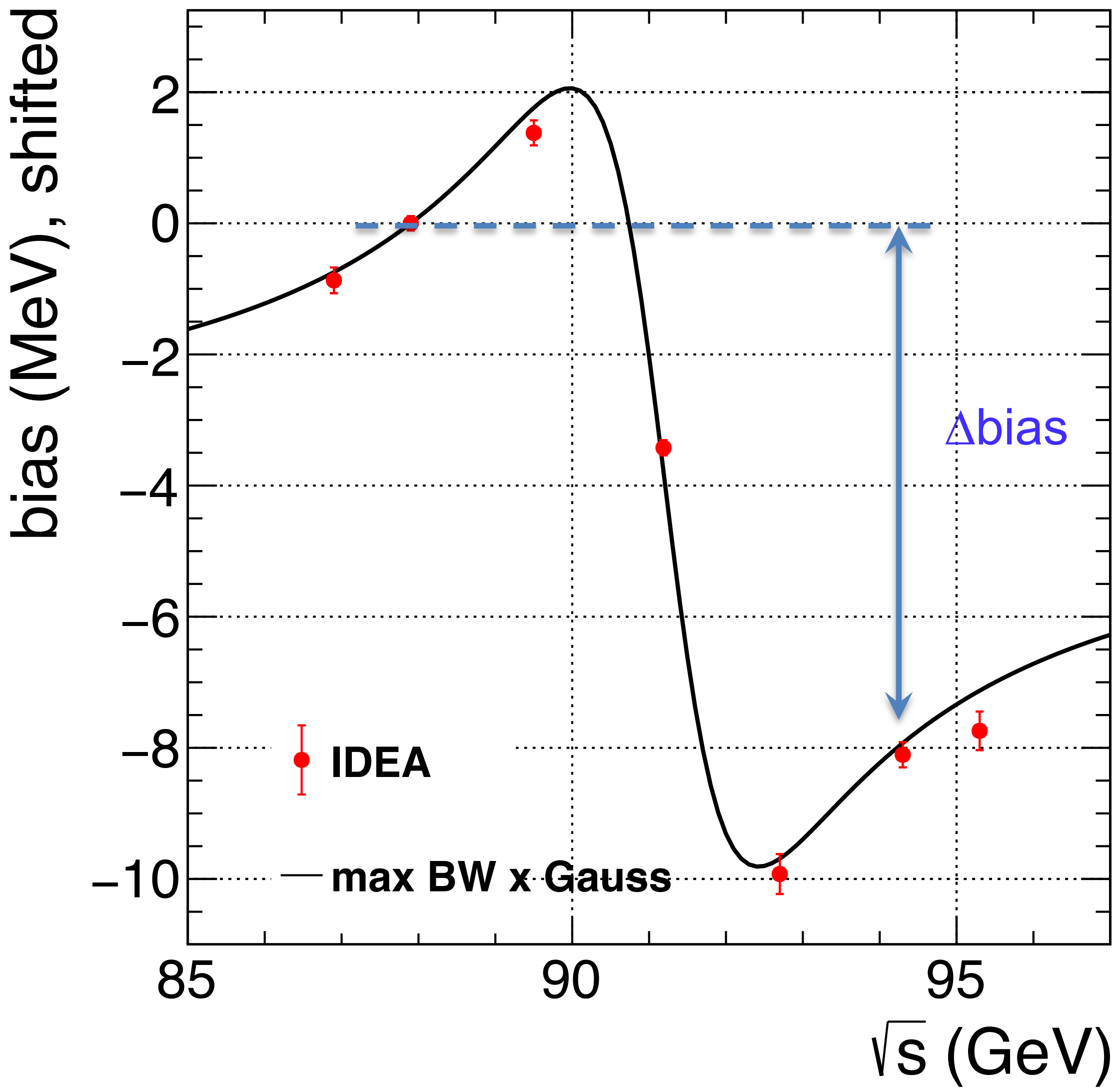


# Dependence of the bias vs √s (IDEA)

Most of the dependence seems to come from the interplay of the Breit-Wigner with the Gaussian describing the BES (see slide 5).

Same shape, modulo a constant shift.

Shift defined such that the bias at 87.9 GeV is zero.

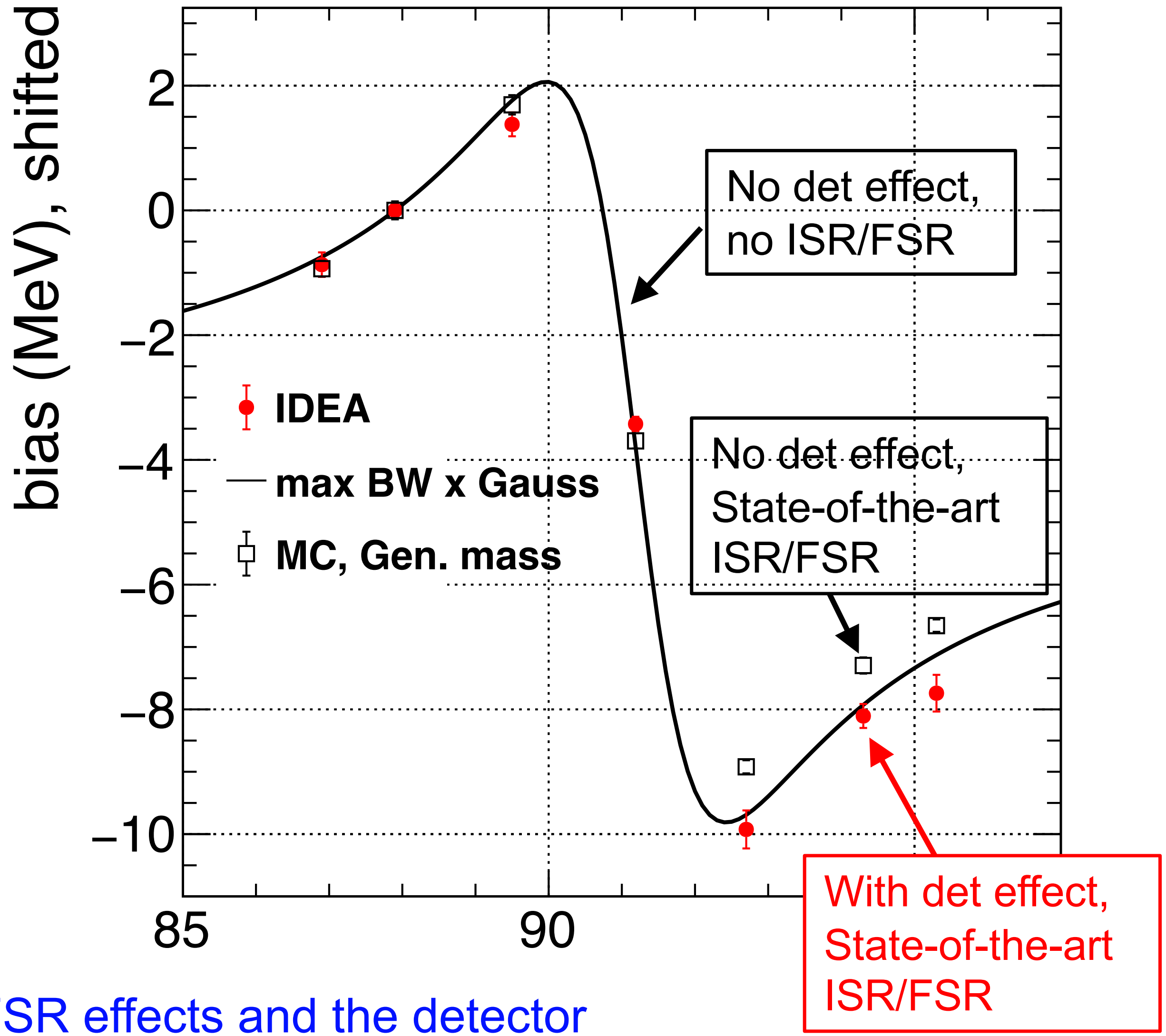


# Dependence of the bias vs √s (IDEA)

To which precision do we know the point at $\sqrt{s_+}$ = 94.3 GeV ?

- Black symbols vs curve: difference between radiations and no radiation at all
- Red vs black symbols: difference between detector-level and gen-level

Full difference of ~ 500 keV.

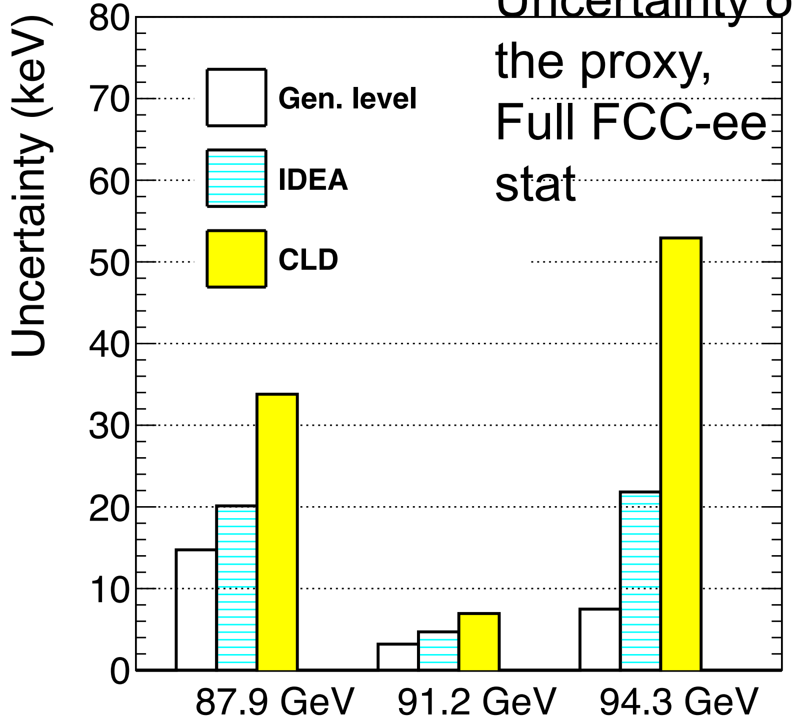



Would need to know the ISR/FSR effects and the detector response to 1% to ensure a systematic uncertainty on Δbias below 5 keV. Probably within reach.

# Summary of uncertainties with the full FCC-ee statistics

Hence, in : $\sqrt{s_+} - \sqrt{s_-} = <\mu(\sqrt{s_+})> - <\mu(\sqrt{s_-})> - (\text{bias}(\sqrt{s_+}) - \text{bias}(\sqrt{s_-}))$
the uncertainty of the second term is subleading.

125 / 40 / 40 ab$^{-1}$ at
$\sqrt{s}$ = 91.2, 87.9 and 94.3GeV




Uncertainty on the proxy, Full FCC-ee stat

- Potential to control the point-to-point systematic uncertainty on $\sqrt{s}$ to ~ 28 keV (20 $\oplus$ 20 keV) with the resolution of the IDEA tracker
  - O(2x) worse with CLD samples

- Requires that the momentum scale ( B ) is stable to 20 keV / 100 GeV = 2e-7 !
  - NMR probes ? …
  - or in-situ, using low mass resonances
    - demands excellent momentum resolution for soft(er) tracks

# Stability of the momentum scale: using J / ψ ?

J / ψ mostly produced in Z → bb events. N ( J / ψ → μμ ) ≈ N ( Z → μμ ) / 150
But much better mass resolution for J / ψ → μμ

Reconstruction:
- Use the thrust axis to separate the events in two hemispheres
- Build candidates by fitting to a common vertex pairs of opposite-charge secondary tracks that belong to the same hemisphere

Mass resolution for J / ψ → μμ ~ 2 MeV

Simple scaling from Z → μμ evts w/o ISR/FSR : from 40 $ab^{-1}$ at 87.9 GeV, the position of the mass peak of J / ψ → μμ is determined to :

$$30\,\mathrm{keV} \times \sqrt{10^7/8.9\,.\,10^9} \times \sqrt{150} \times \frac{2\,\mathrm{MeV}}{90\,\mathrm{MeV}} = 0.28\,\mathrm{keV}$$

cf slide 4, from $10^7$ μμ evts w/o rad

Scale to the statistics of J / ψ → μμ at 87.9 GeV

Ratio of resolutions

Relative precision: 0.28 keV / 3 GeV = 9 $10^{-8}$

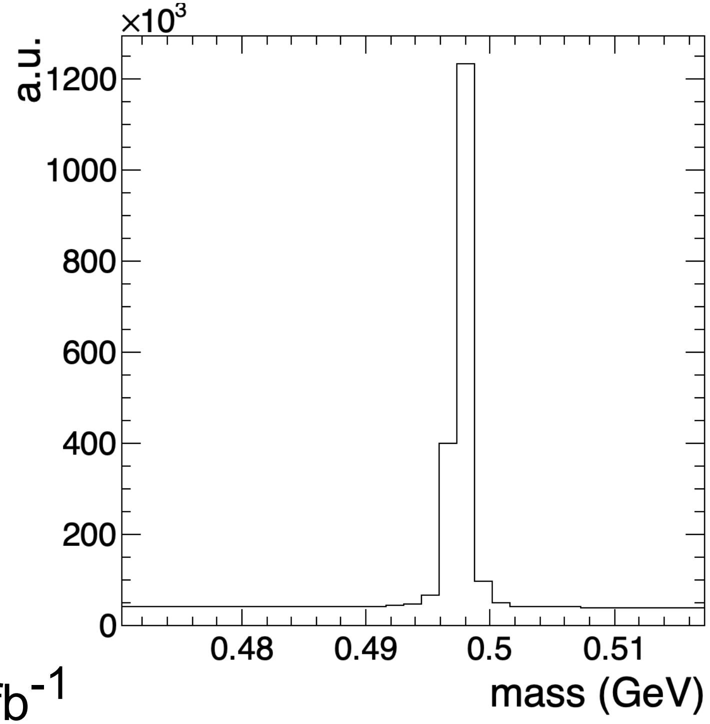
Split the 40 $ab^{-1}$ in e.g. 100 subsamples: monitoring of the scale stability to 9 $10^{-7}$
4.5x larger than the target

# Stability of the momentum scale using $K_S \to \pi^+\pi^-$

Roughly one $K_S \to \pi^+\pi^-$ decay in every second Z → had. event !

- Ks candidates: fit to a common vertex pairs of opposite-charge secondary tracks that belong to the same hemisphere
  - $\chi^2 < 10$
  - window on the vertex mass

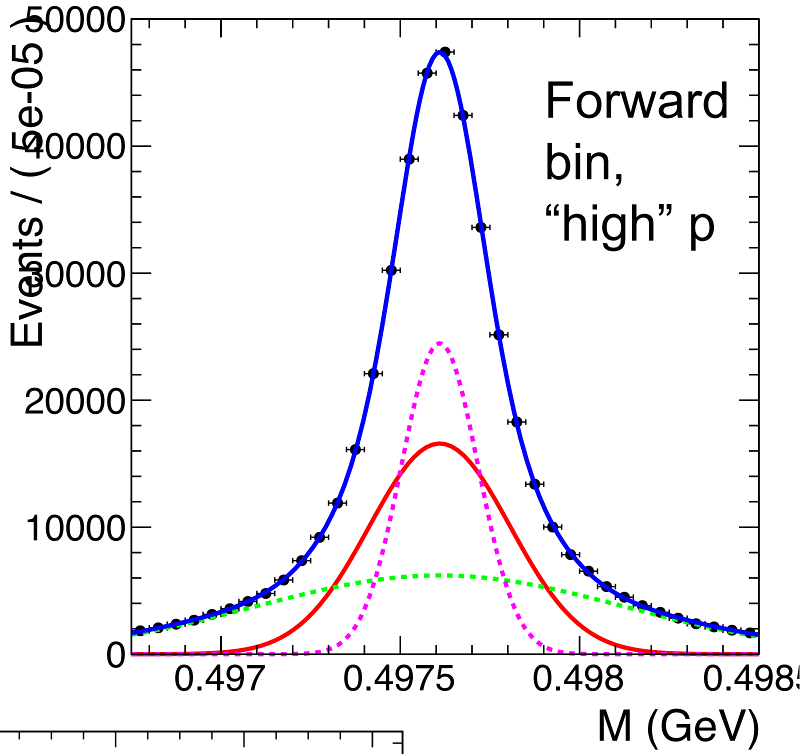
Background is low (large range shown here on purpose) :

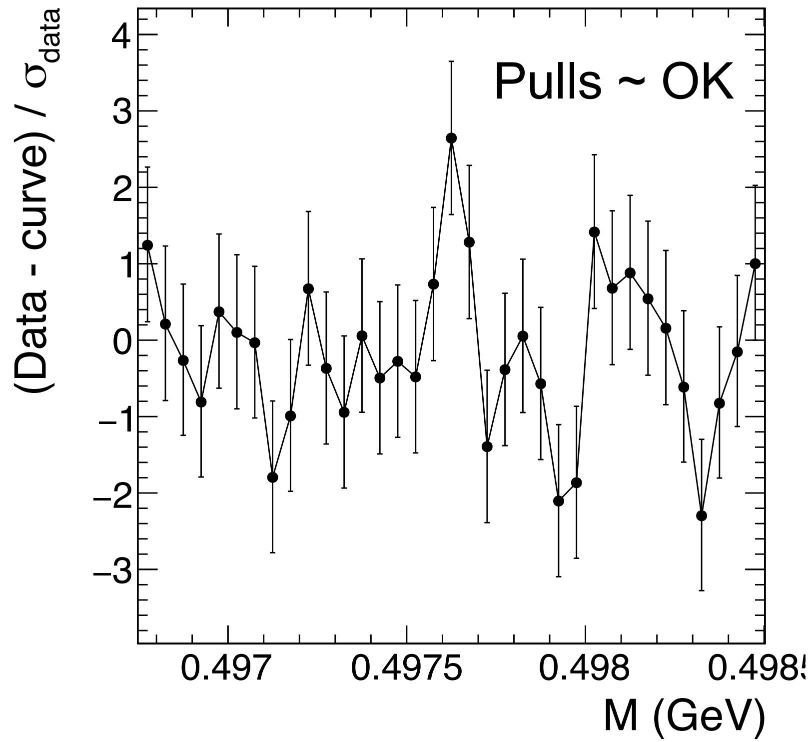


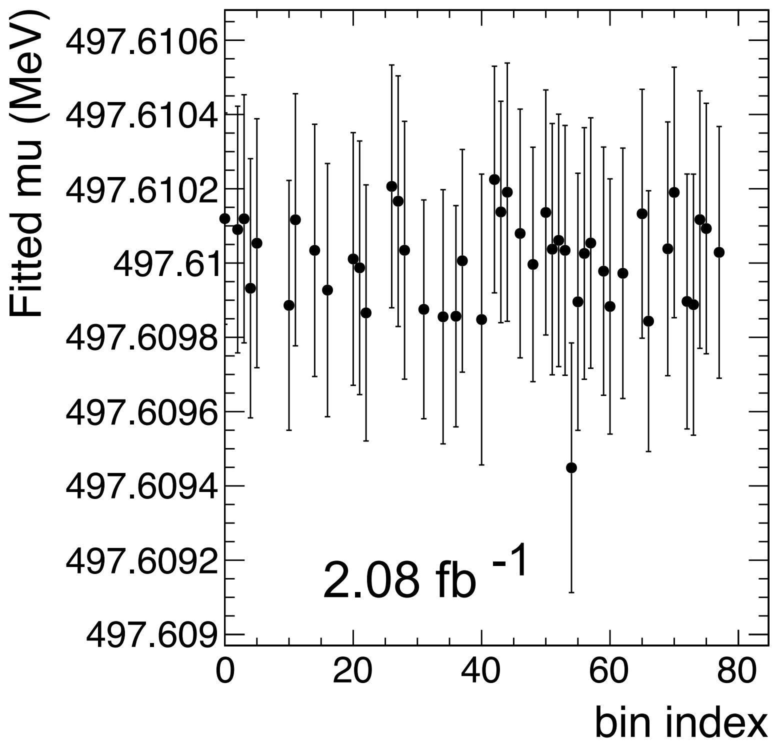

- Z → had MC : 91.2 GeV, luminosity used = 2.08 $fb^{-1}$

- Fits of the mass distribution:
  - Model = sum of 3 gaussians with same mean μ + constant
  - fits made in bins of ( θ, $p^+$, $p^-$ )
    - p bins : 0.5 < p < 2 GeV ; 2 < p < 5 GeV ; p > 5 GeV

# Fit results




Fitted μ consistent across the bins.
Uncertainty on the peak position (on the mean of the Gaussians) from that of the weighted average:
0.05 keV with 2 fb$^{-1}$ at 91.2 GeV

With 40 ab$^{-1}$ at 87.9 GeV :
Relative uncertainty = 3 10 $^{-9}$

Stability with 100 subsamples: to 3 10 $^{-8}$

Can be monitored in O(50) angular bins within the target of 2 10 $^{-7}$

# Conclusions

- Potential to control the point-to-point systematic uncertainty on $\sqrt{s}$ to ~ 28 keV with the resolution of the IDEA tracker
  - O(2x) worse with CLD samples

- Requires that the momentum scale is stable to 20 keV / 100 GeV = 2e-7 !
  - Using $K_S \rightarrow \pi^+\pi^-$ decays, potential to monitor the stability of the scale to that level with the IDEA tracker – thanks to excellent momentum resolution for soft tracks

- This 28 keV uncertainty translates into a 11 keV uncertainty on the Z width.

| Uncertainty | $\Gamma_Z$ [keV] |
|---|---|
| Absolute | 2.5 |
| Point-to-point | 11 |
| Sample size | 1 |
| Energy spread | 5 |
| Total $\sqrt{s}$ related | 12 |
| FCC-ee statistical | 4 |

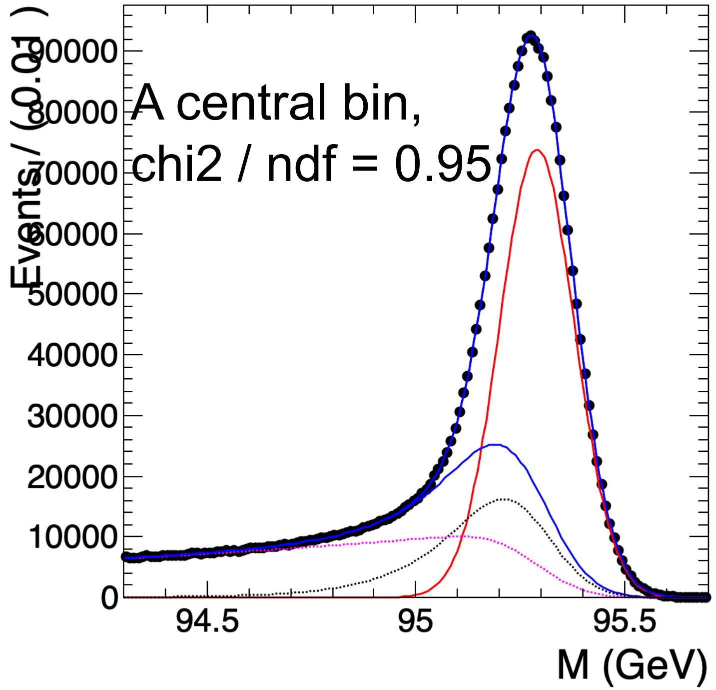
cf Guy Wilkinson on Monday: We are approaching regime where $\Gamma_Z$ may not be $E_{CM}$-systematics limited

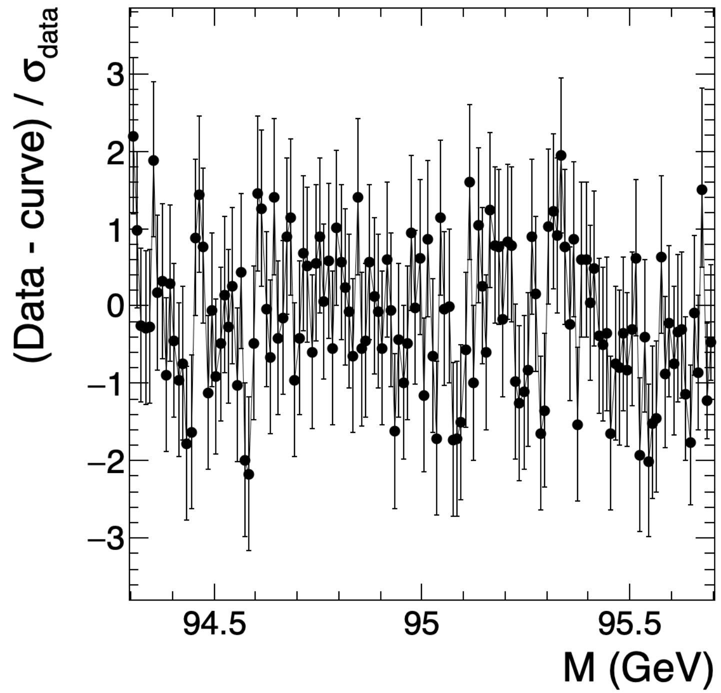
Need to check other uncertainty components (e.g. relative normalisation)

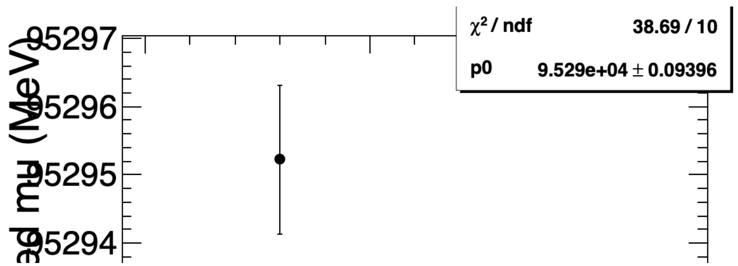
# Backup and old slides

# Example fits

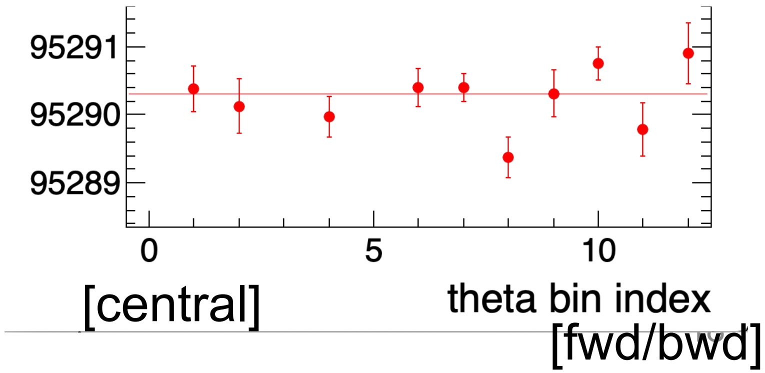
Gen-level mass
√s = 95.3 GeV

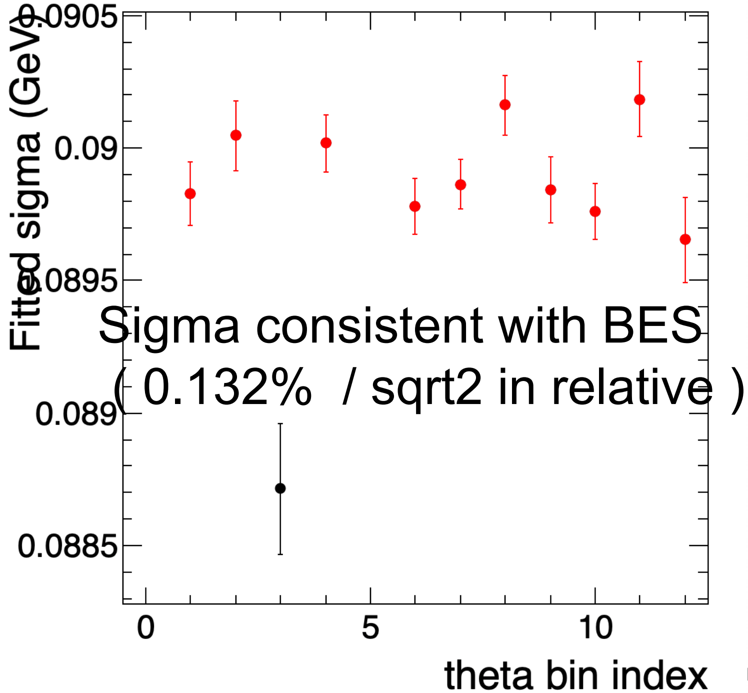


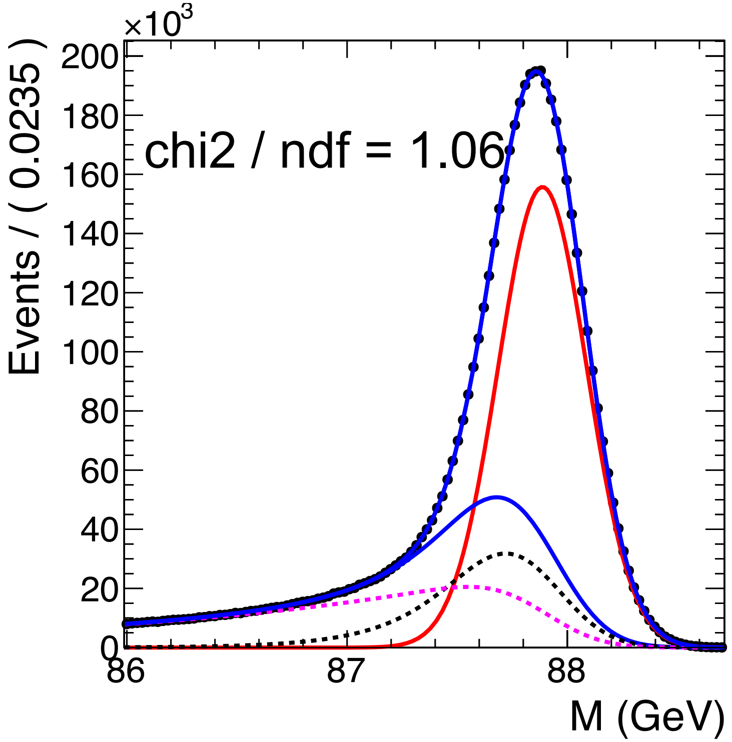

Fitted mu is the same in all bins
( outliars are removed )

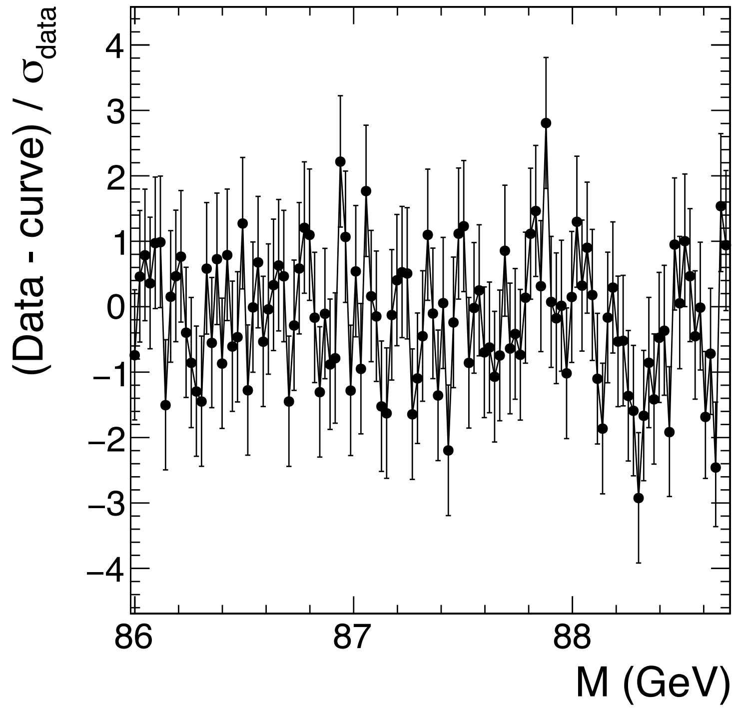


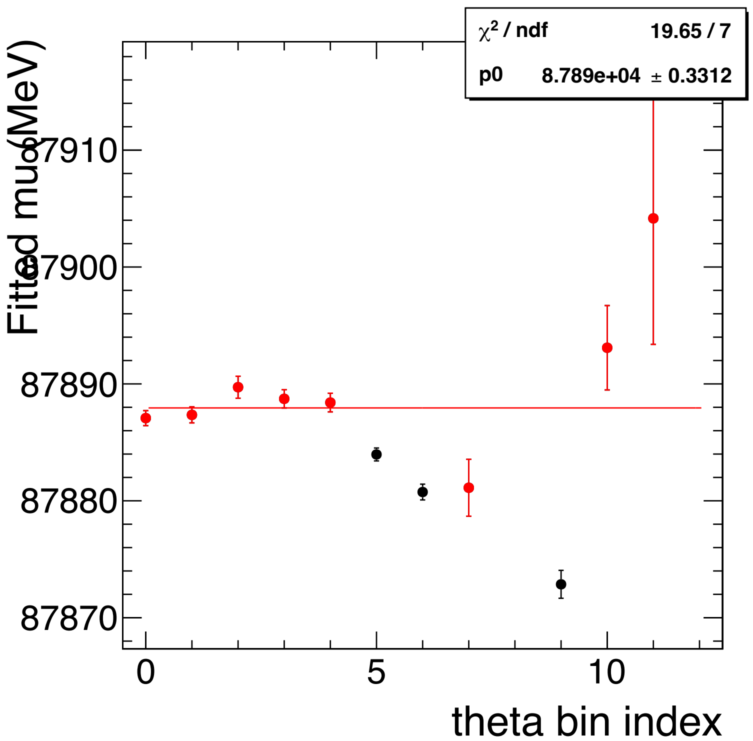



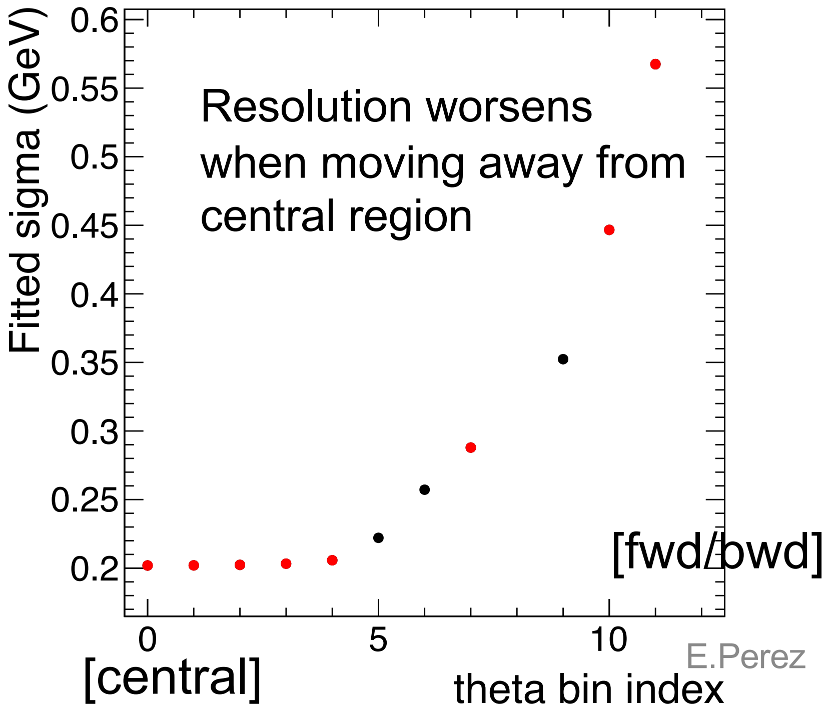
# Example fits (2)

Reco'ed mass,
CLD
√s = 87.9 GeV



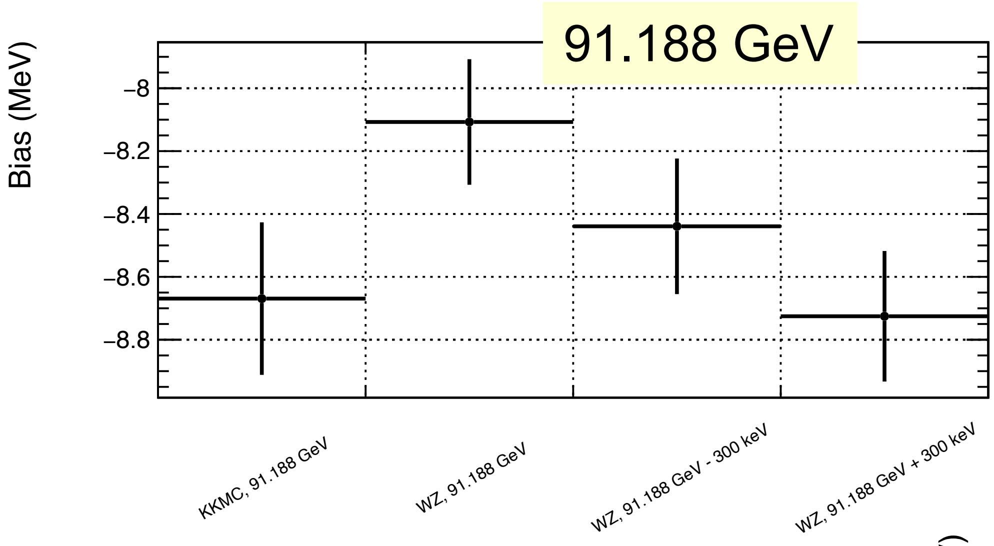



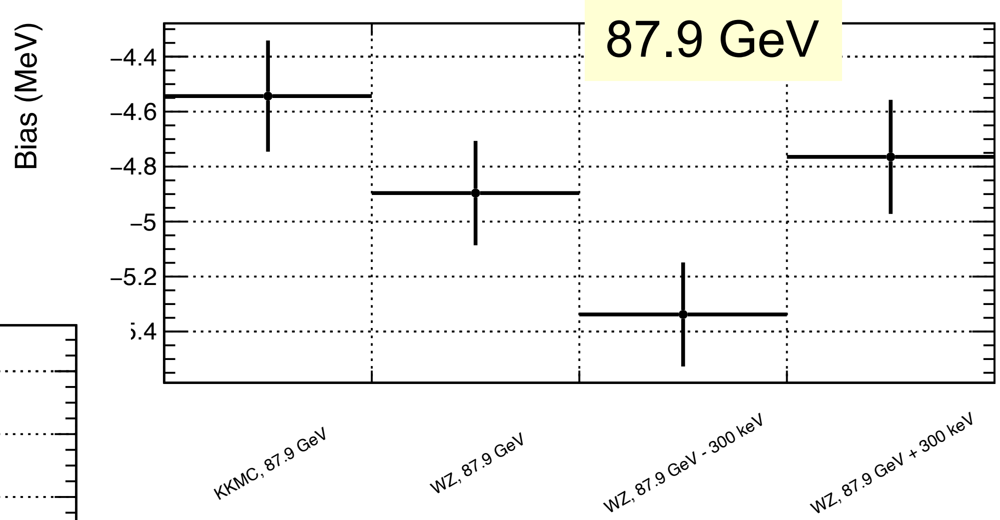
# Bias, numerical values (IDEA)

Reco'ed mass, IDEA

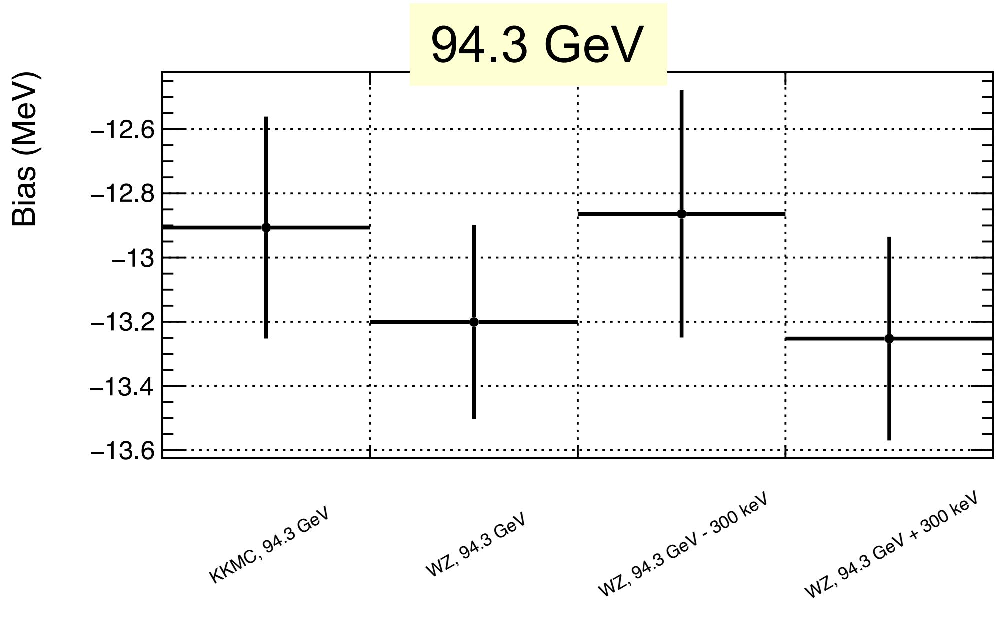



Within uncertainties (~ 200 keV with current MC statistics) :

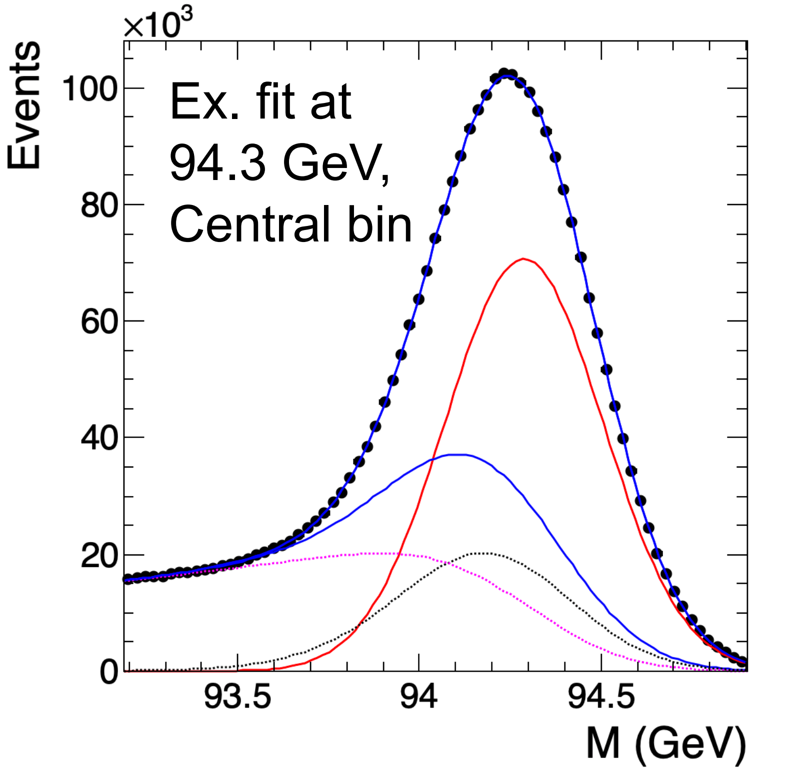
- KKMC ~ Whizard
- Bias is "locally constant" when √s varies by +/- 300 keV (√s will be known from RDP to within 100 keV or better)

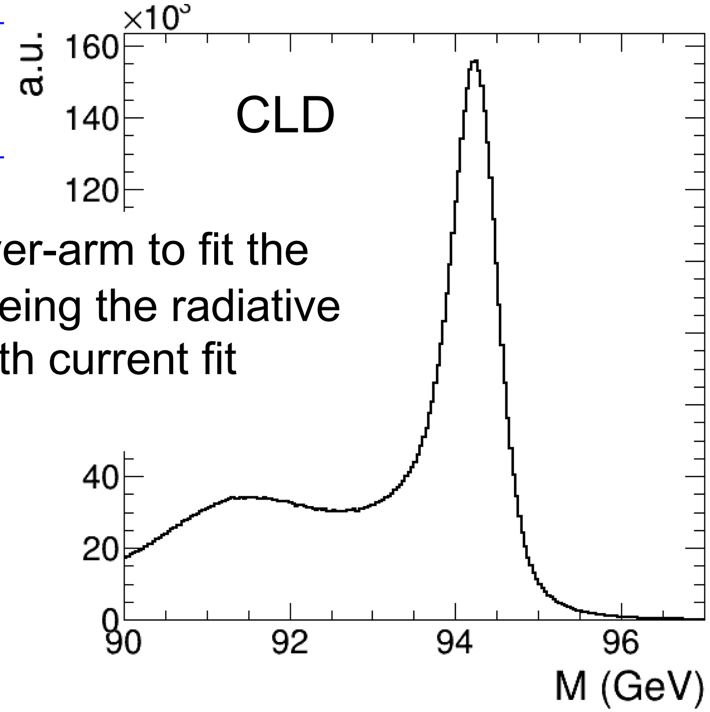



Bias = proxy ( average mu) - √s



# CLD samples (Delphes)




NB: Limited lever-arm to fit the exp. tail w/o seeing the radiative return bump with current fit model.

Shifted by about -13 MeV compared to the fits to the IDEA samples. Dependence w.r.t. √s similar.