

8th FCC Physics Workshop

Measuring τ $g-2$ via $\gamma\gamma \rightarrow \tau\tau$ process at the FCC-ee

15 January 2025

D. d'Enterria, M. Pitt, M. Selvaggi
(CERN)



Outline

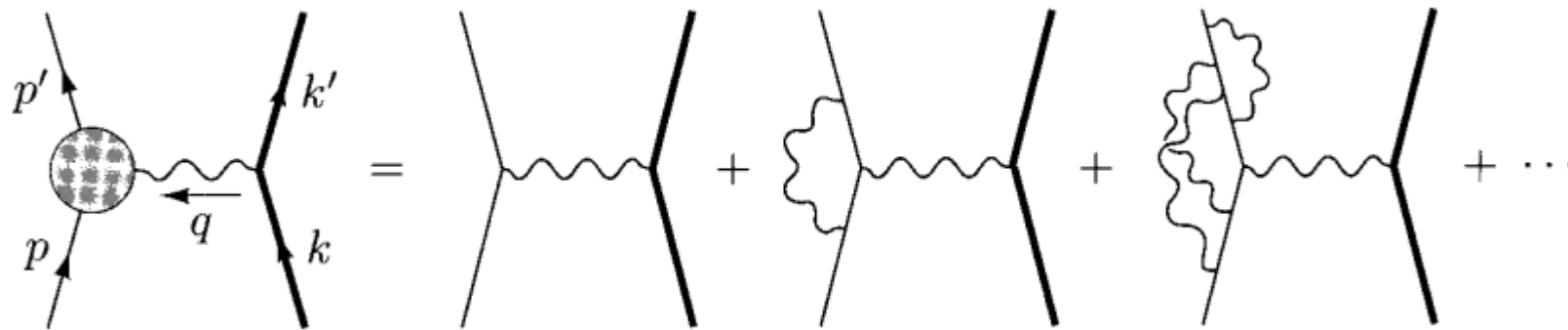
- Lepton anomalous magnetic moments ($g-2$)
- Past studies
- Feasibility at the FCC-ee

Lepton anomalous magnetic moments ($g-2$)

Photon lepton coupling

- The photon – lepton vertex (scattering of lepton from an external electromagnetic field):

$$\Gamma^\mu(p', p) = \gamma^\mu F_1(q^2) + \frac{i\sigma^{\mu\nu} q_\nu}{2m_\ell} F_2(q^2)$$



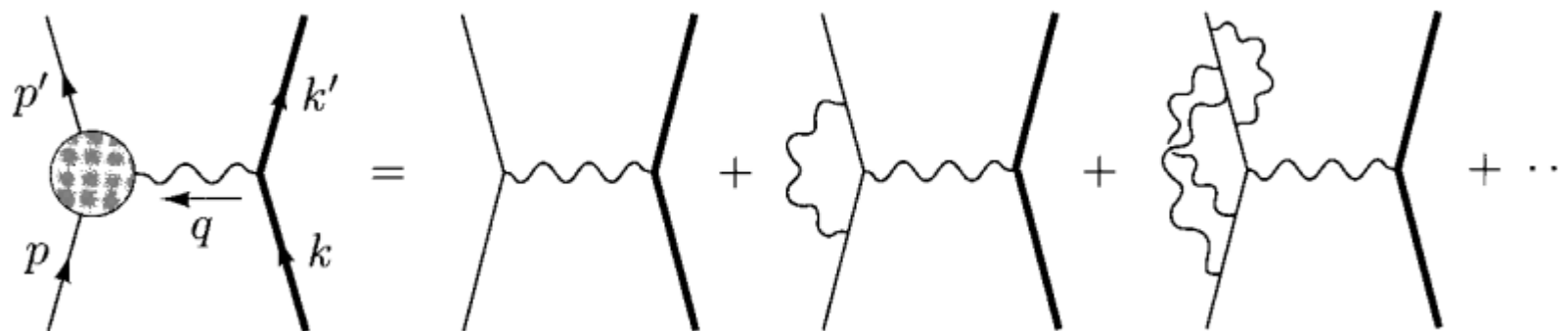
M. Peskin & D. Schroeder (Sec 6.2)

Photon lepton coupling

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$F_2(0) = 1$



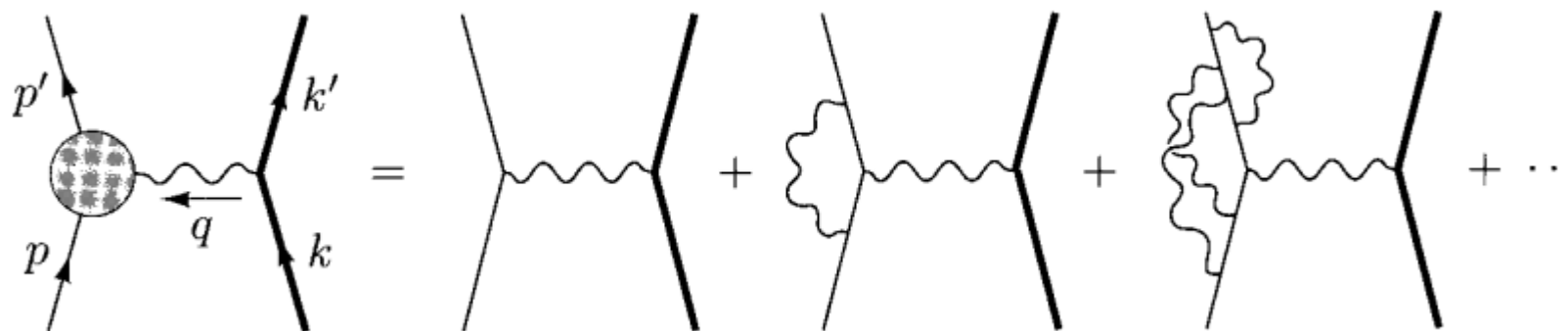
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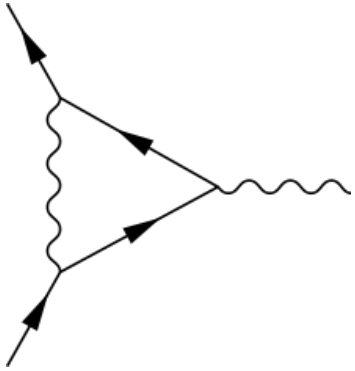
$F_2(0) = 1$ $F_2(0) = \frac{g-2}{2} \equiv a_\ell$



M. Peskin & D. Schroeder (Sec 6.2)

Photon lepton coupling

- Corrections to photon-lepton coupling



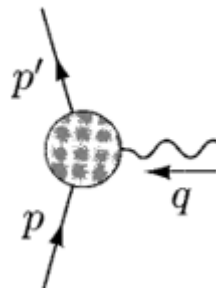
$$a_{\ell}^{LO} = \frac{\alpha}{2\pi} \approx 0.0011613758$$

J. Schwinger Phys. Rev. 73 (1948) 416



Photon lepton coupling

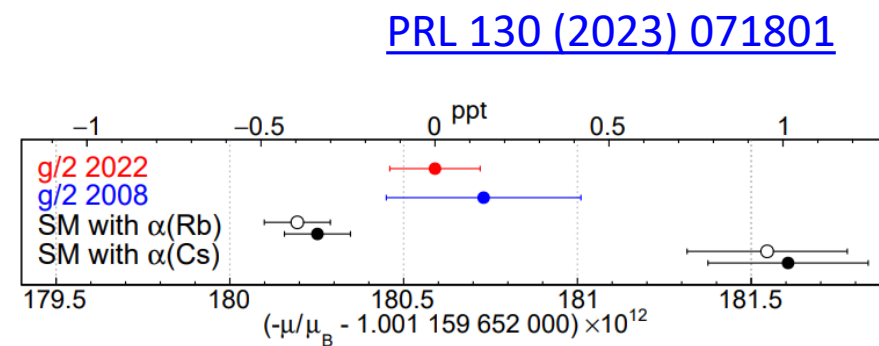
- Corrections to photon-lepton coupling



$$a_e = a_e^{NnLO}(\text{QED}) + \overbrace{a_e(\text{Weak})}^{1.7 \times 10^{-12}} + \overbrace{a_e(\text{Hadron})}^{0.03 \times 10^{-12}} = \mathbf{1159652181.6(7)} \times 10^{-12}$$

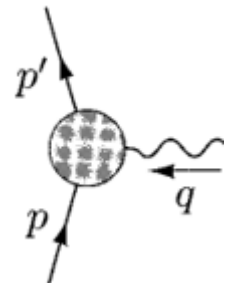
$$a_e^{NnLO}(\text{QED}) = \frac{1}{2} \left(\frac{\alpha}{\pi}\right) + C_2 \left(\frac{\alpha}{\pi}\right)^2 + C_3 \left(\frac{\alpha}{\pi}\right)^3 + C_4 \left(\frac{\alpha}{\pi}\right)^4 + C_5 \left(\frac{\alpha}{\pi}\right)^5 + \dots$$

- For electrons, precision tests agrees up to 10 significant figures with the theory, which goes well beyond the LO term



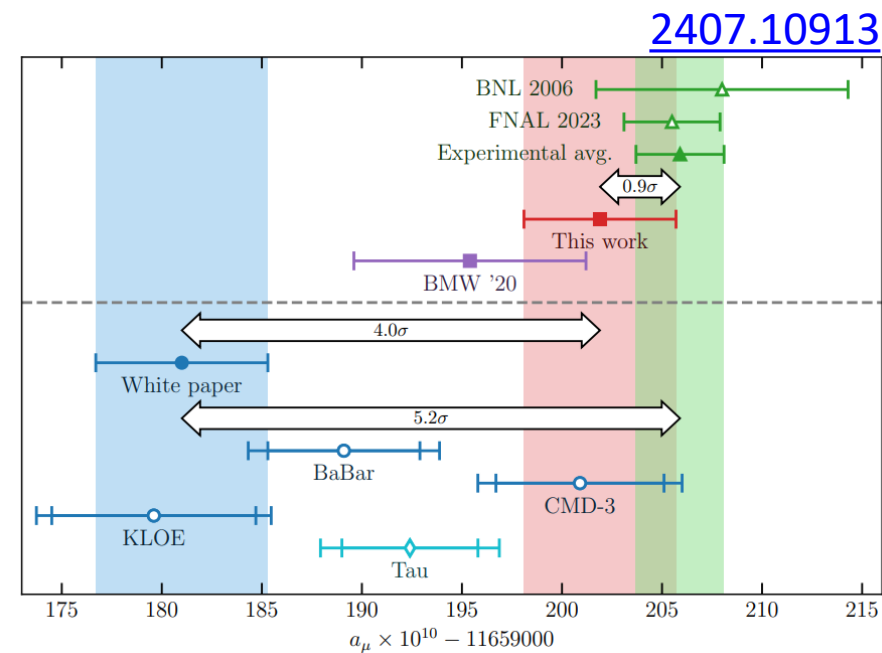
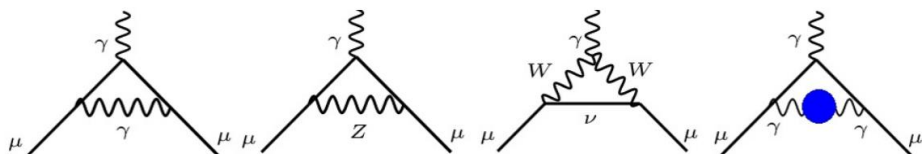
Photon lepton coupling

- Corrections to photon-lepton coupling



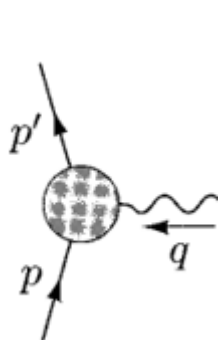
$$a_\mu = a_\mu(\text{QED}) + \overbrace{15.4 \times 10^{-10}}^{a_\mu(\text{EW})} + \overbrace{714 \times 10^{-10}}^{a_\mu(\text{HVP})} = 11659201 \times 10^{-10}$$

- For electrons, precision tests agrees up to 10 significant figures with the theory, which goes well beyond the LO term
- For muons, the measured precision ~ theoretical



Photon lepton coupling

- Corrections to photon-lepton coupling



$$a_\tau = a_\tau(\text{QED}) + \overbrace{a_\tau(\text{EW})}^{47 \times 10^{-8}} + \overbrace{a_\tau(\text{Hadron})}^{347 \times 10^{-8}} = \mathbf{117721(5)} \times 10^{-8}$$

- For electrons, precision tests agrees up to 10 significant figures with the theory, which goes well beyond the LO term
- For muons, the measured precision \sim theoretical
- For taus, the measurement of the a_τ is still a dream ([M. Perl, 5th WEIN Symposium 1998](#))



Past studies

Tau lepton $g-2$ at LEP

- The measurement of the tau – photon coupling, $\Gamma^\mu(p', p) = \gamma^\mu F_1(q^2) + \frac{i\sigma^{\mu\nu}q_\nu}{2m_\ell} F_2(q^2)$, and a constraint on a_τ was first performed at LEP

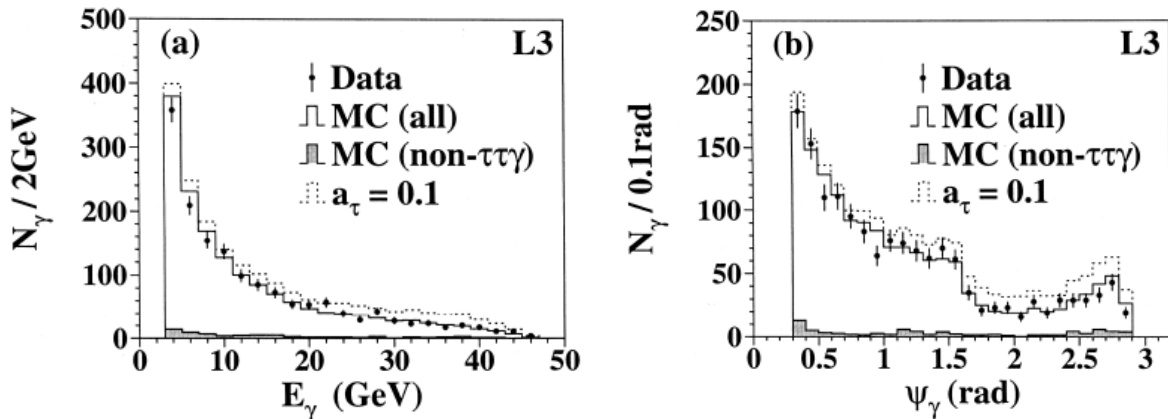
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[PLB 434 \(1998\) 169](#)

L3: $\sqrt{s} = m_Z$ and $\mathcal{L} = 100 \text{ pb}^{-1}$

Selection: $e^+e^- \rightarrow \tau^+\tau^-\gamma$



$$-0.052 < a_\tau < 0.058 \quad (\sim 50 \times a_\tau^{LO})$$

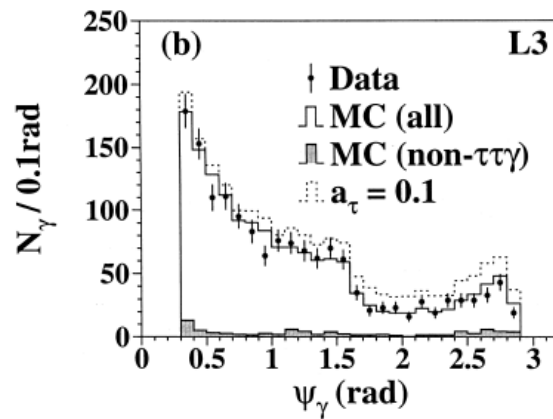
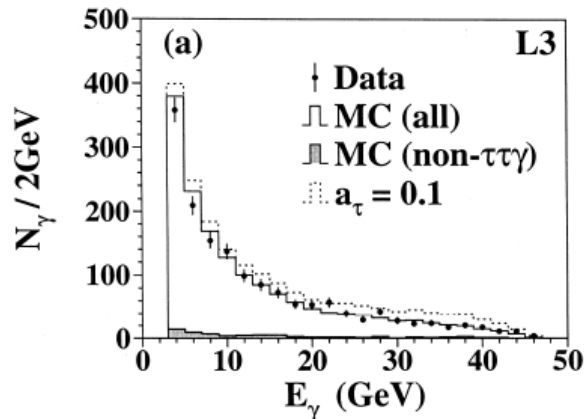
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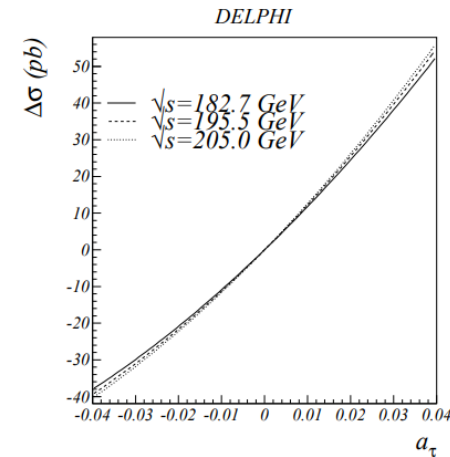


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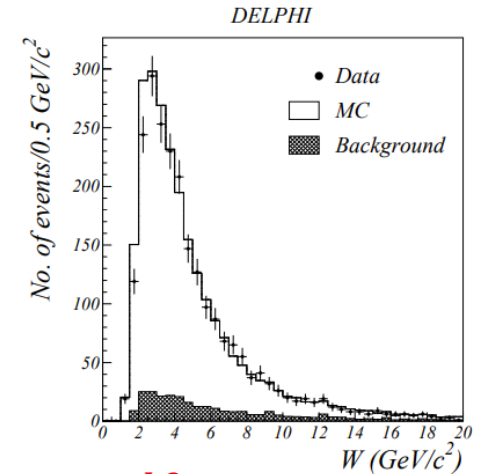
[EPJC 35 \(2004\) 159](#)

DELPHI: $\sqrt{s} \cong 200 \text{ GeV}$ and $\mathcal{L} = 650 \text{ pb}^{-1}$

Selection: $\gamma\gamma \rightarrow \tau^+\tau^-$



$$-0.052 < a_\tau < 0.013 \quad (\sim 30 \times a_\tau^{LO})$$



Tau lepton $g-2$ at LHC

- The measurement of the tau – photon coupling, $\Gamma^\mu(p', p) = \gamma^\mu F_1(q^2) + \frac{i\sigma^{\mu\nu} q_\nu}{2m_\ell} F_2(q^2)$, and a constraint on a_τ was also performed at the LHC, first in PbPb UPC ($\gamma\gamma \rightarrow \tau\tau$)

[PRL 131 \(2023\) 151802](#)

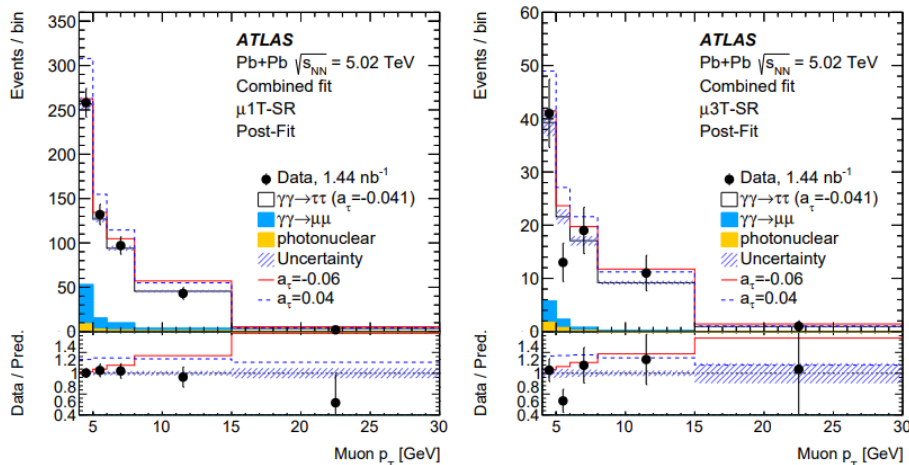
[CMS-PAS-HIN-24-0011](#)

ATLAS: $\sqrt{s_{NN}} = 5.02\text{TeV}$ and $\mathcal{L} = 1.44\text{nb}^{-1}$

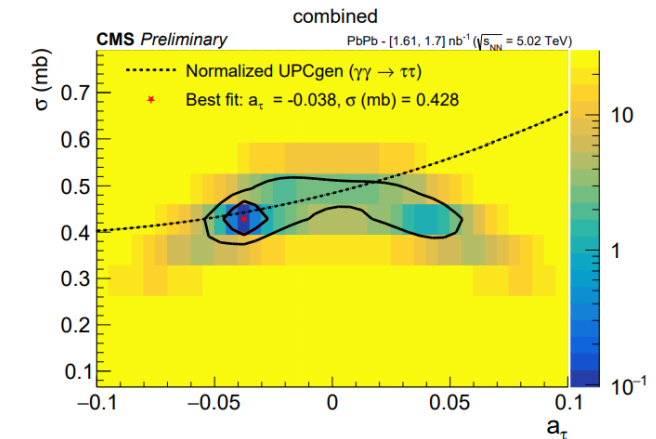
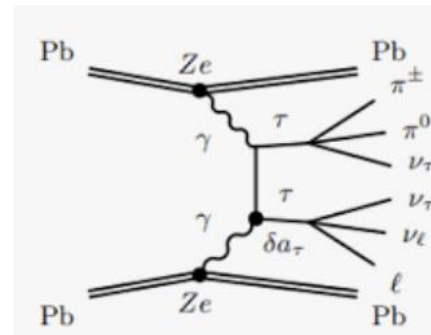
CMS: $\sqrt{s_{NN}} = 5.02\text{TeV}$ and $\mathcal{L} = 1.7\text{nb}^{-1}$

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Selection: $\gamma\gamma \rightarrow \tau^+\tau^-$



$-0.057 < a_\tau < 0.024$ ($\sim 70 \times a_\tau^{LO}$)



$-0.054 < a_\tau < 0.018$ ($\sim 60 \times a_\tau^{LO}$)

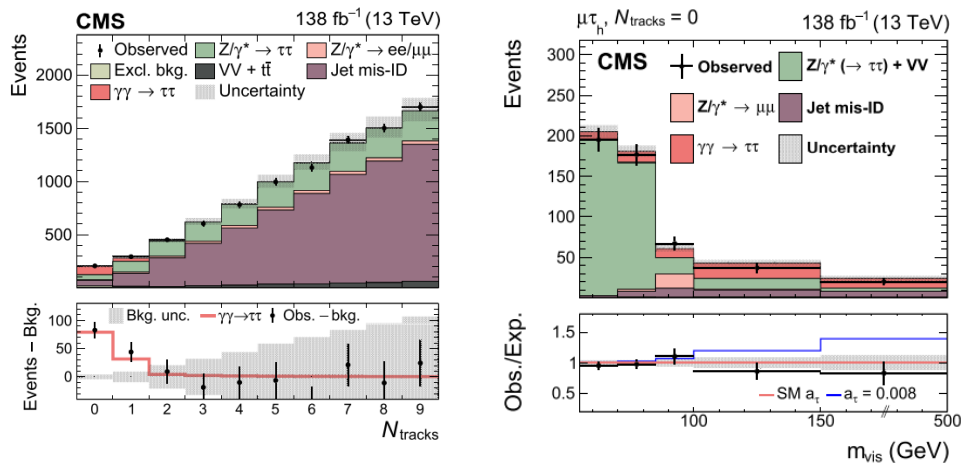
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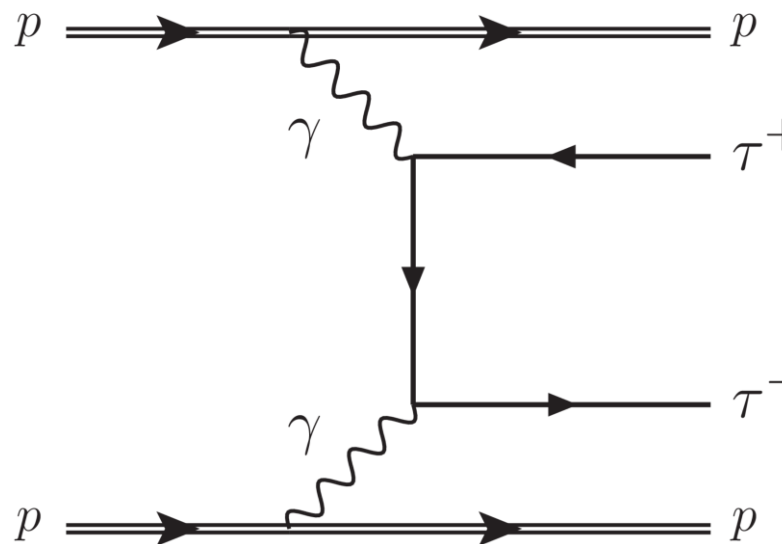
[ROPP 87 \(2027\) 107801](#)

CMS: $\sqrt{s} = 13\text{TeV}$ and $\mathcal{L} = 138\text{fb}^{-1}$

Selection: $\gamma\gamma \rightarrow \tau^+\tau^-$



$$-0.004 < a_\tau < 0.004 \quad (\sim 3.5 \times a_\tau^{L0})$$



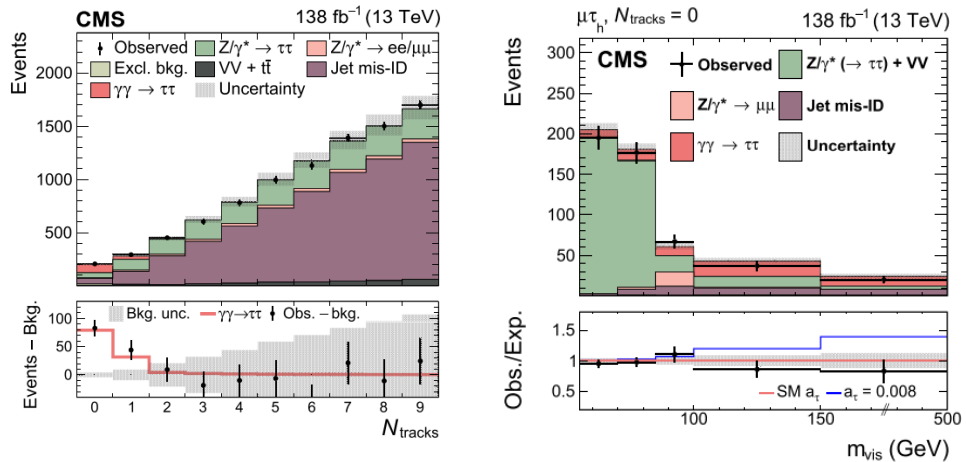
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CMS closes in on tau $g-2$

19 March 2024

A report from the CMS experiment

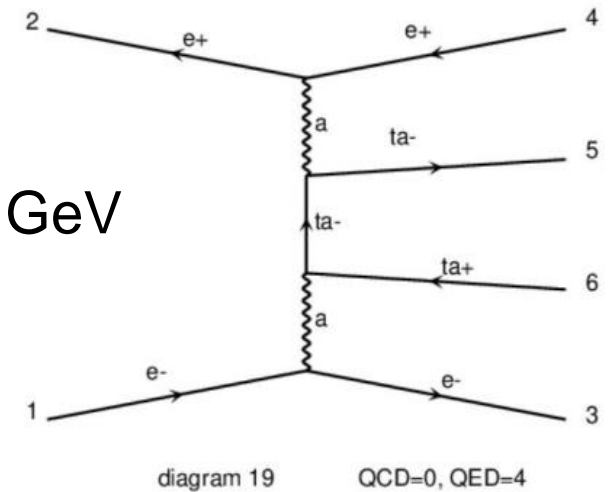


The CMS experiment. Credit: J Hosan / OPEN-PHO-LIFE-2019-022-5

Feasibility at the FCC-ee

Tau lepton $g-2$ at FCC-ee

- Processes were generated with Madgraph5 and processed with Pythia + Delphes using the FCCee framework.
- Event generator: MG5_aMC v3.5.3
 - For the 4 nominal runs: $\sqrt{s} = 91.2$ GeV, 160 GeV, 240 GeV, 365 GeV
 - Using full matrix element at LO (not the EPA photon flux)
 - Taus generated with $|\eta| < 3$
- Parton shower:
 - Pythia 8.311 with ISR + FSR
- Detector simulation:
 - Delphes with [card IDEA.tcl](#) and [edm4hep IDEA.tcl](#)



Tau lepton $g-2$ at FCC-ee

- Processes were generated with Madgraph5 and processed with Pythia + Delphes using the FCCee framework.
- BSM model – using EFT model for $(g - 2)_\tau$ extraction (approach valid only up to $a_\tau^{L^0}$ term)

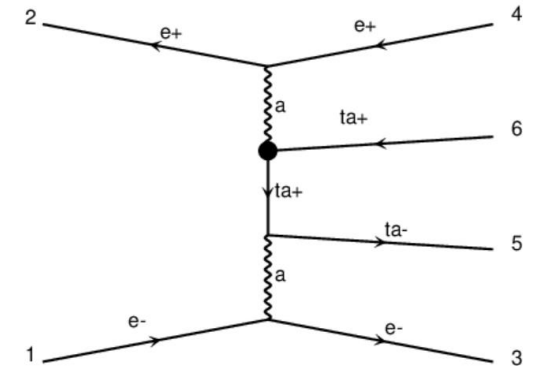
In the SM Lagrangian electromagnetic moments arise from:

$$\mathcal{L} \supset \frac{1}{2} \bar{\tau} \sigma^{\mu\nu} \left(a_\tau \frac{e}{2m_\tau} - id_\tau \gamma_5 \right) \tau_R F_{\mu\nu}$$

Deviation from the SM of a_τ can be parametrized in terms of dim-6 BSM

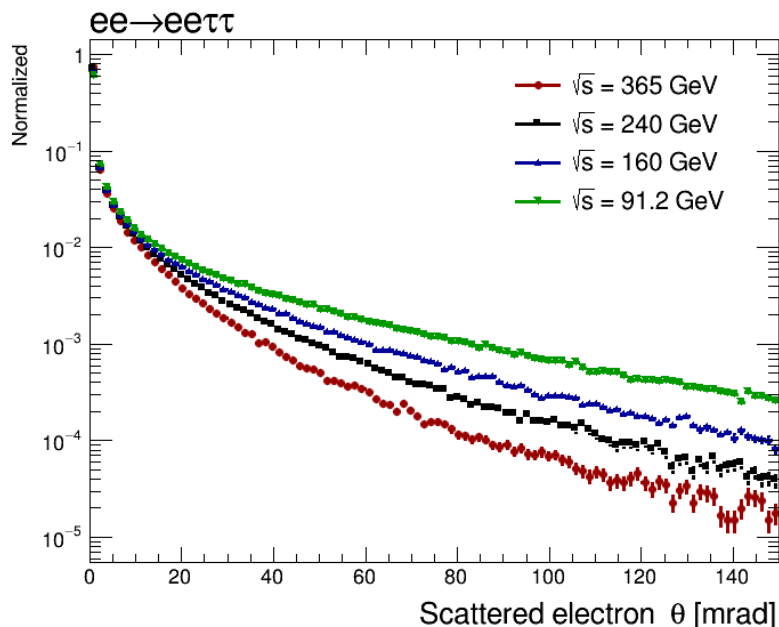
Lagrangian:

$$\mathcal{L}_{\text{BSM}} = \frac{C_{\tau B}}{\Lambda^2} \bar{L}_L \sigma^{\mu\nu} \tau_R H B_{\mu\nu} + \frac{C_{\tau W}}{\Lambda^2} \bar{L}_L \sigma^{\mu\nu} \tau_R \sigma^i H W_{\mu\nu}^i + \text{h.c.}, \quad (3)$$



Tau lepton $g-2$ at FCC-ee

- Electron / positron kinematics (GEN level)
 - Ideally, we need to tag very-forward e^{\pm} ($\theta \rightarrow 0$) to select processes from the fusion of (quasi)real photons (sensitive to $(g-2)_{\tau}$)
 - For increasing \sqrt{s} , less e^{\pm} are centrally scattered

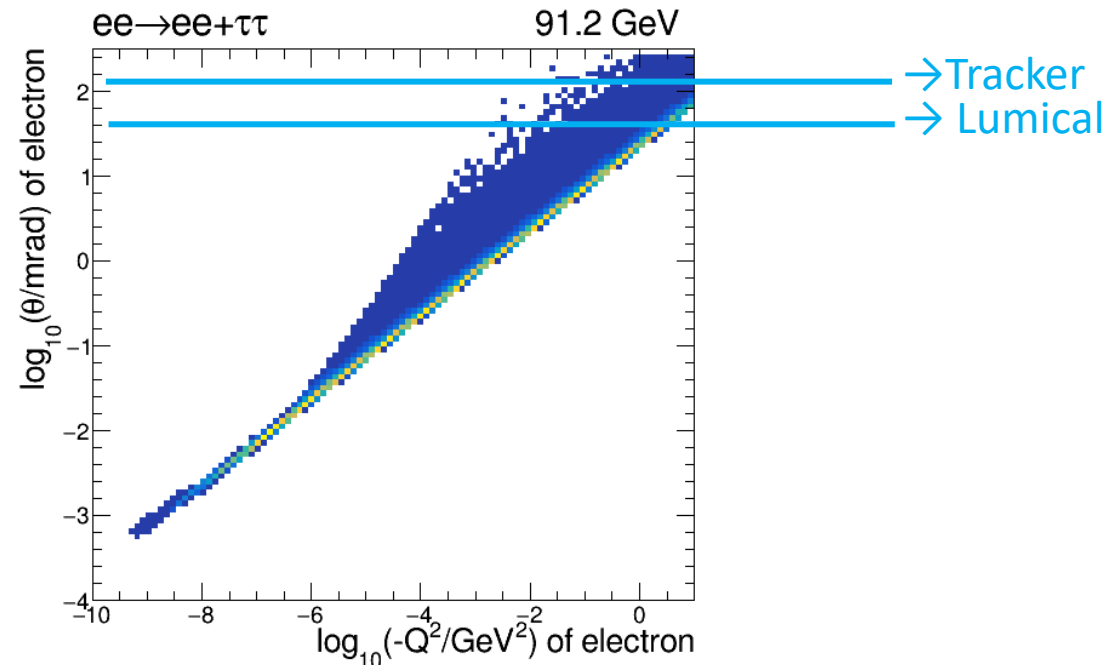
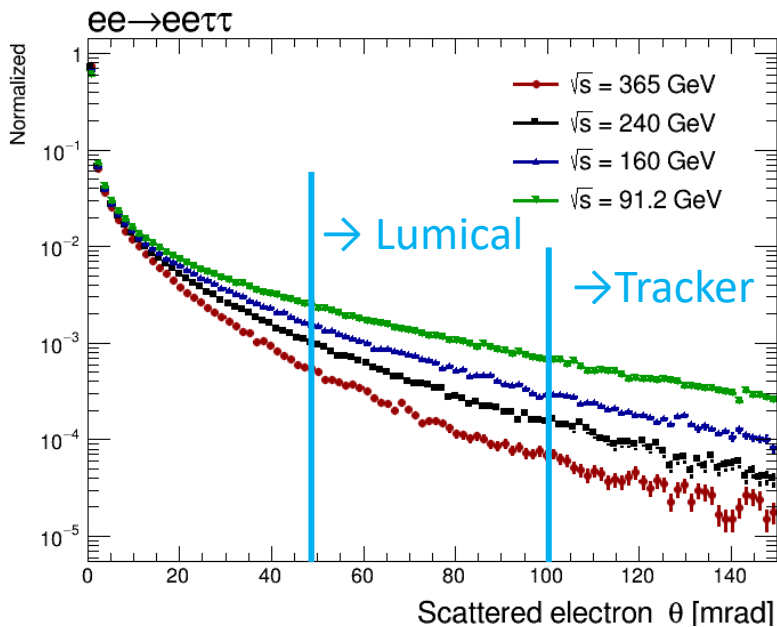


Tau lepton $g-2$ at FCC-ee

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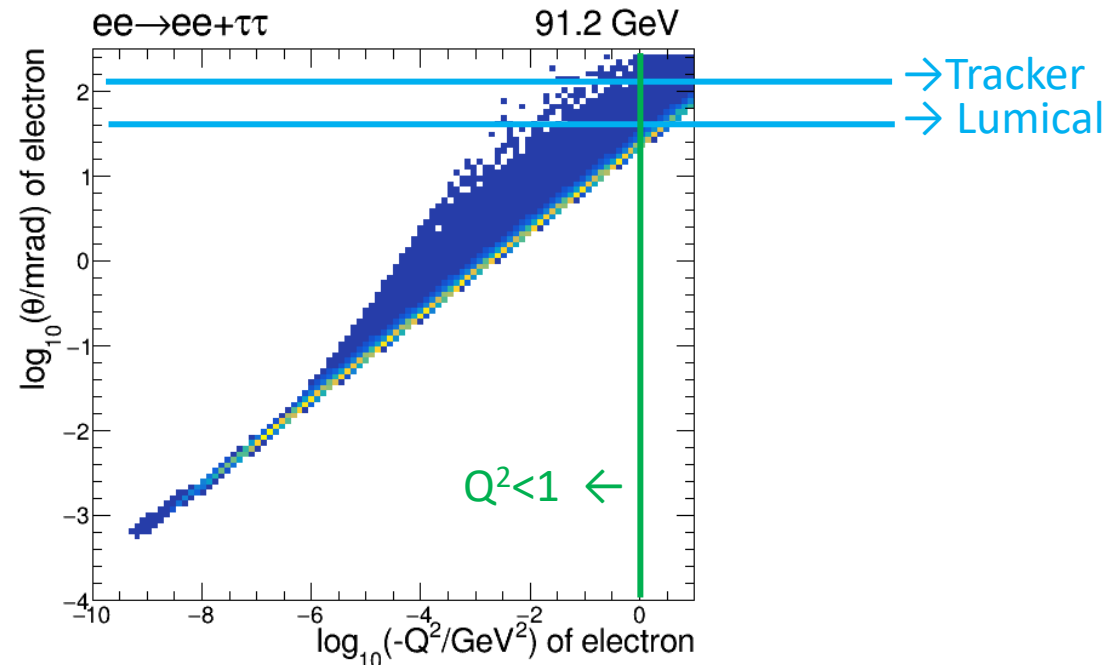
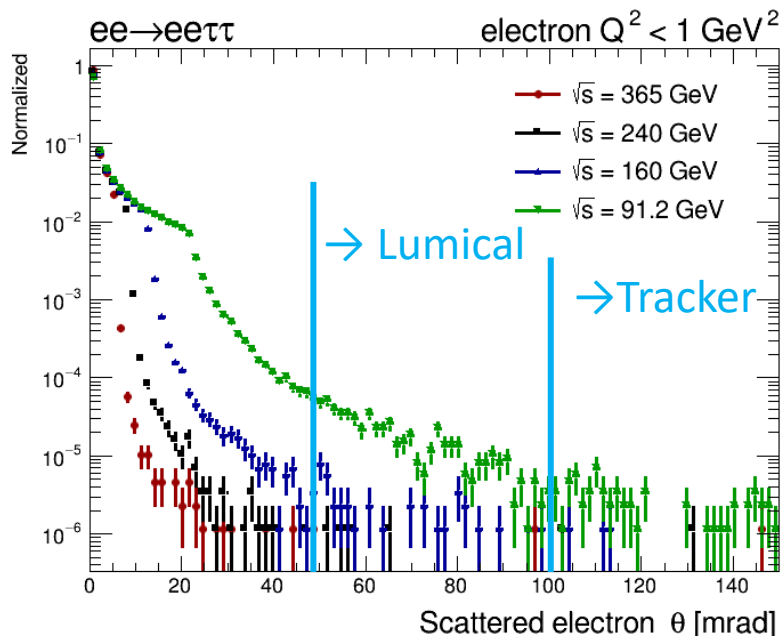
Elastic scattering $\theta_{MAX} = \sqrt{\frac{-q_{min}^2}{E_{BEAM}}}$



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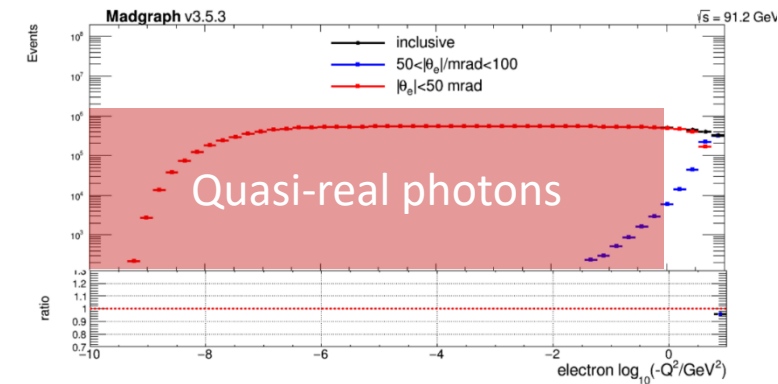
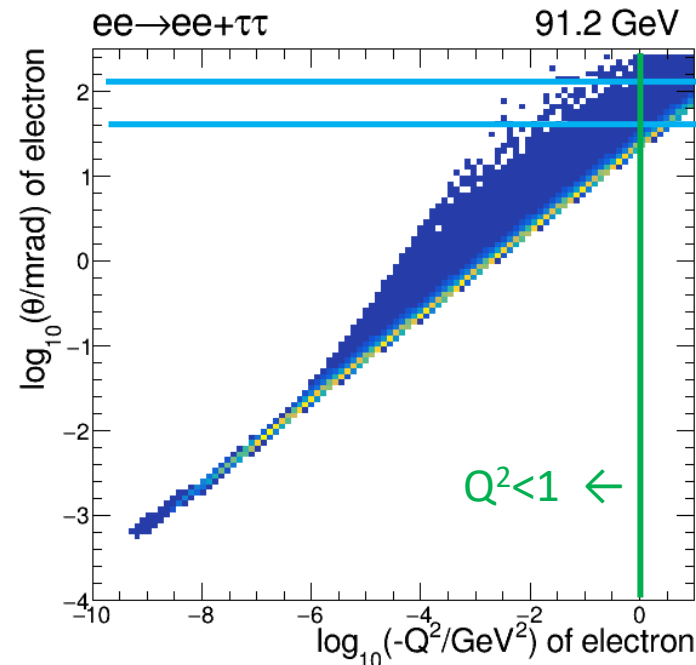
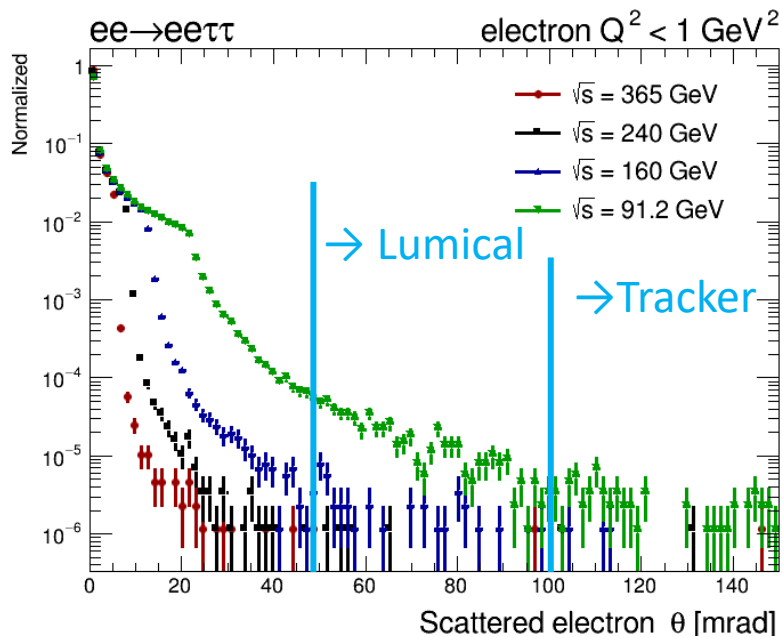


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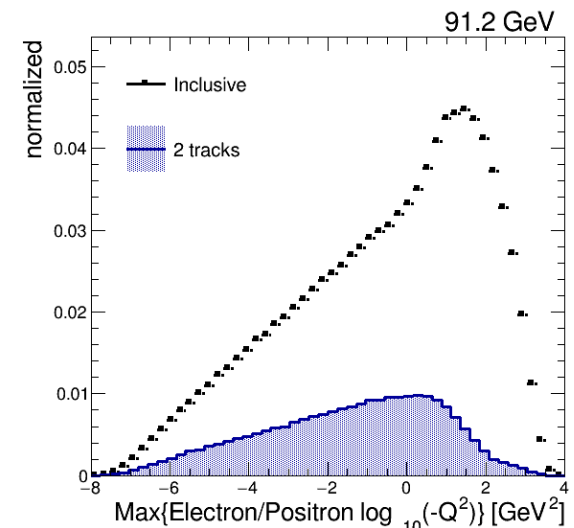
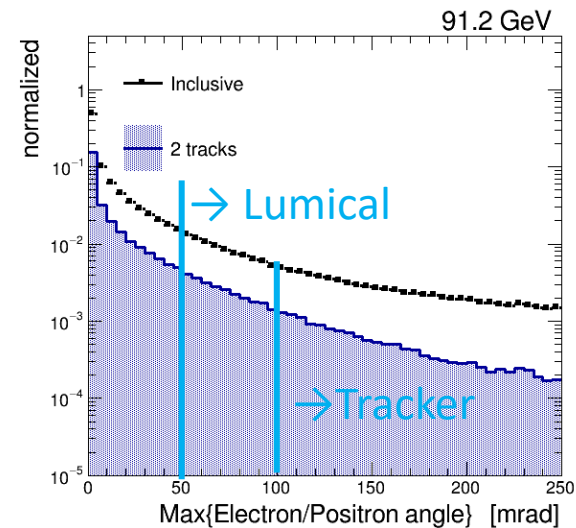


Analysis strategy (LEP style)

Cut efficiency

23.17%

- Event selection:
 - Exactly two tracks ($\tau \rightarrow \pi\nu + n\pi^0$ decays only for now) with $p > 300$ MeV

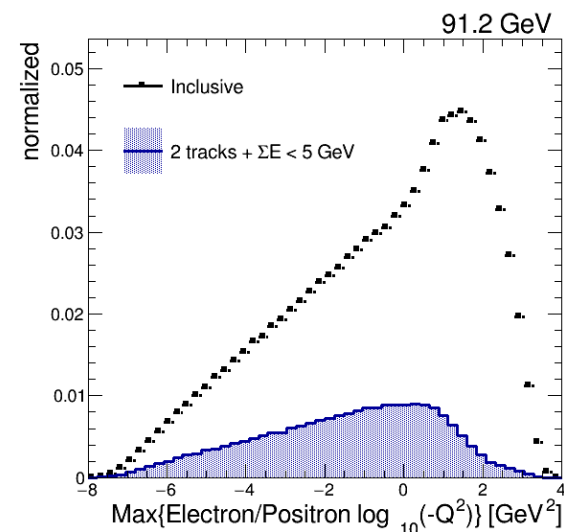
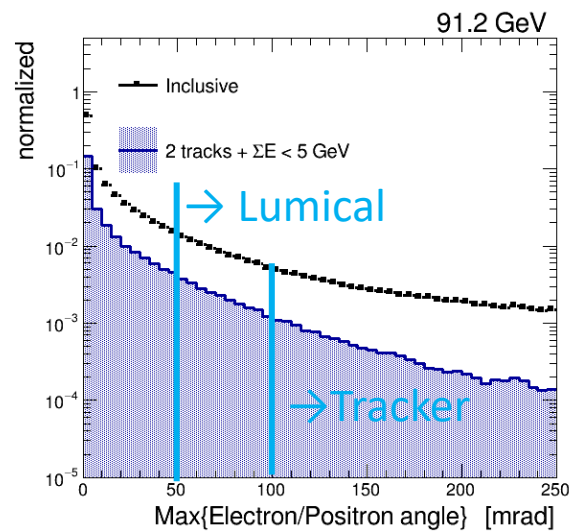
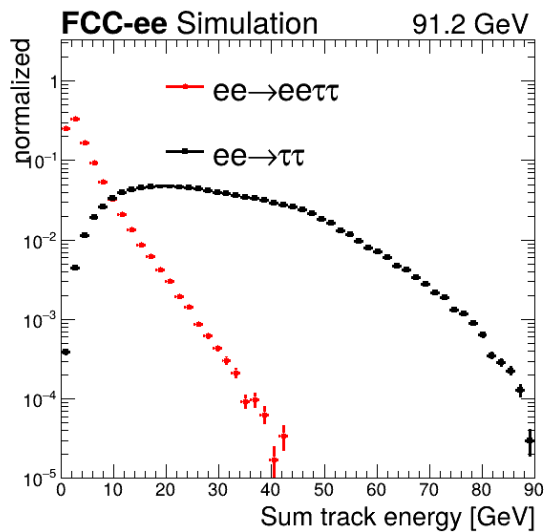


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- To suppress $ee \rightarrow \ell\ell$ choose $\Sigma E_{\text{TRK}} < 5$ GeV

Cut efficiency	
	23.17%
	16.90%

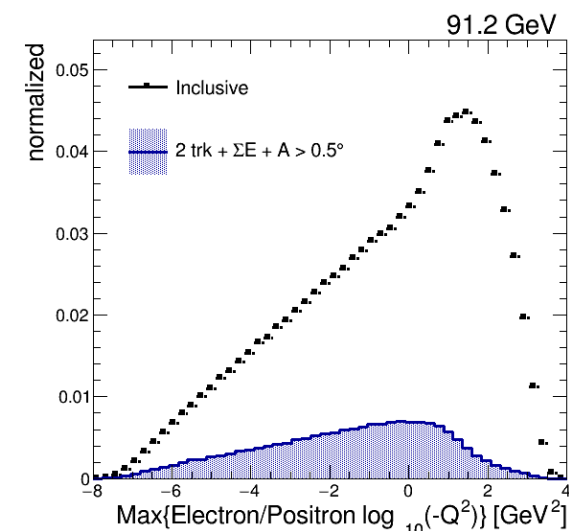
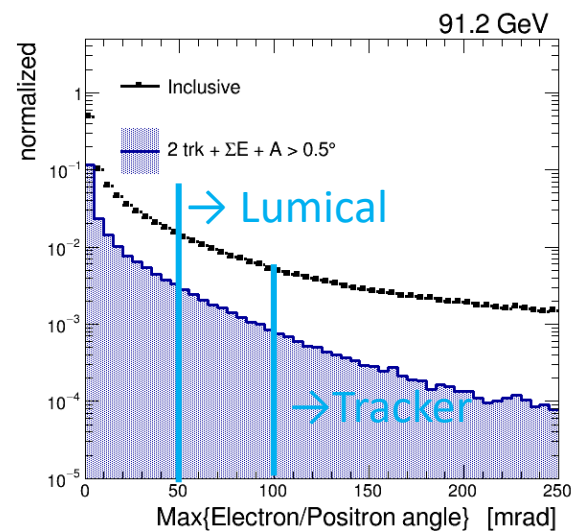
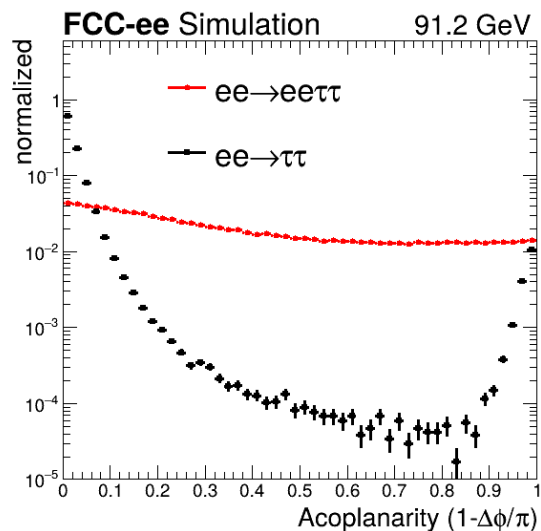
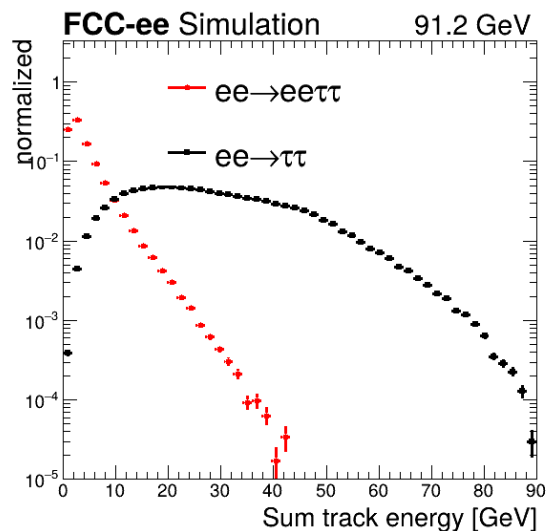


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- To suppress $ee \rightarrow \ell\ell$ choose $\Sigma E_{\text{TRK}} < 5$ GeV
- To enhance $ee \rightarrow ee + \tau\tau$ select acoplanarity $A = (1 - \Delta\phi/\pi) > 0.5$ deg

Cut efficiency	
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	16.90%
	16.66%

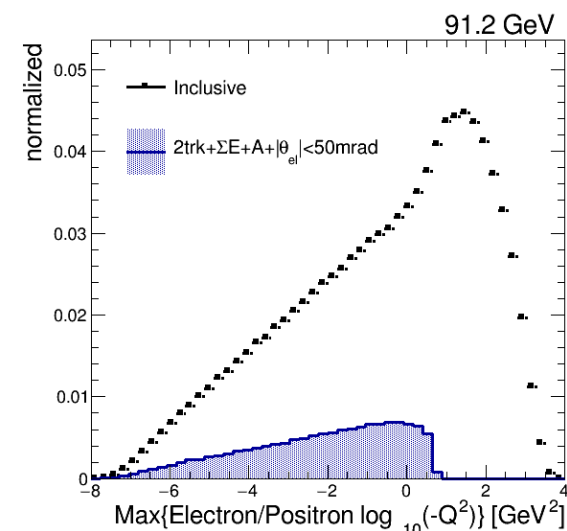
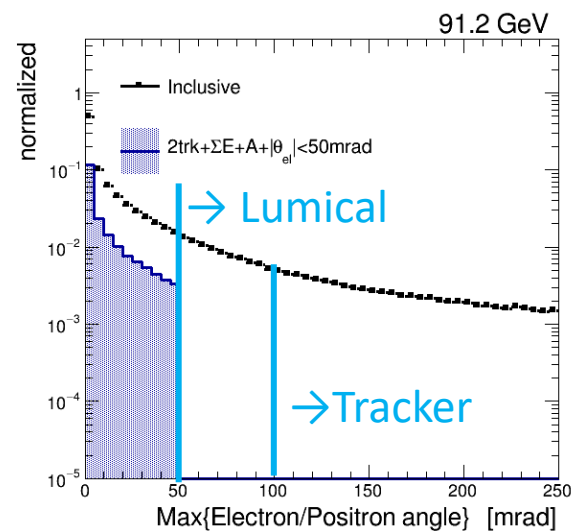
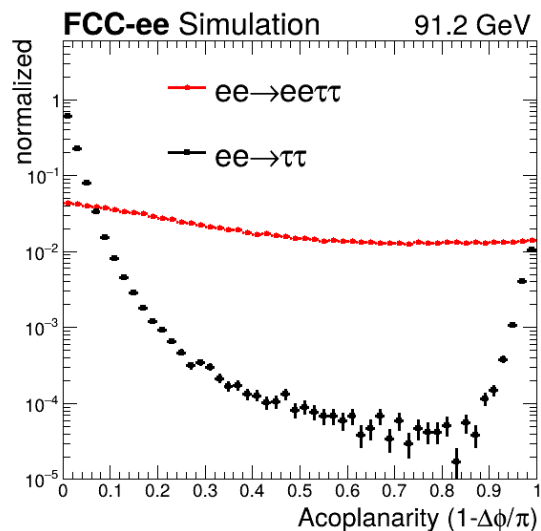
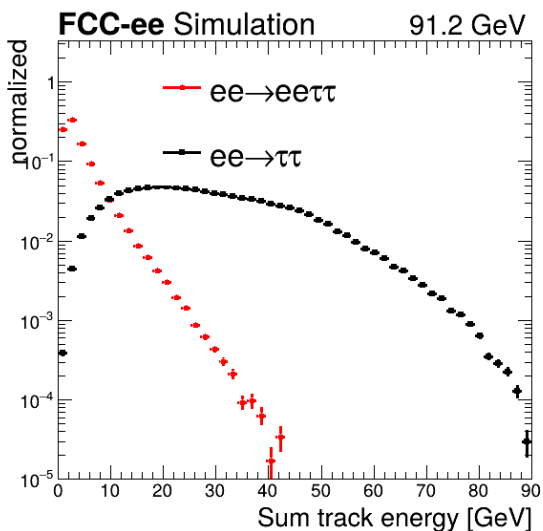


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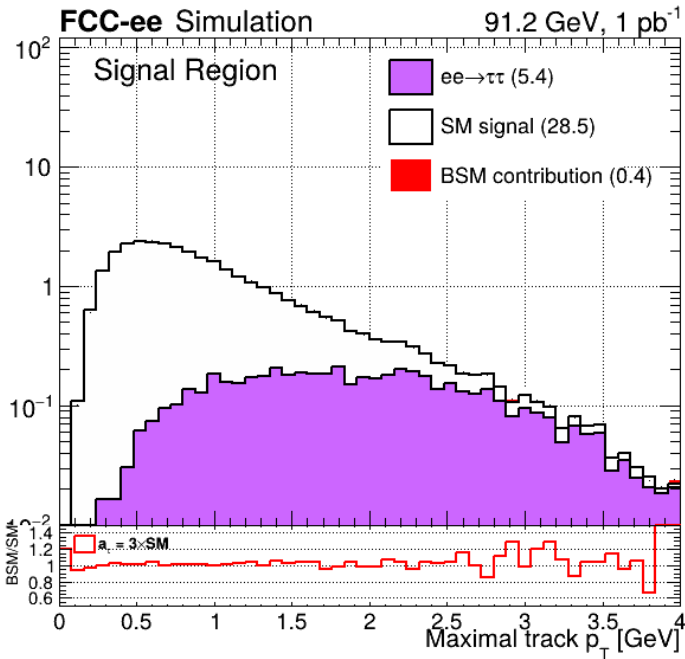
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- To suppress $ee \rightarrow \ell\ell$ choose $\Sigma E_{\text{TRK}} < 5$ GeV
- To enhance $ee \rightarrow ee + \tau\tau$ select acoplanarity $A = (1 - \Delta\phi/\pi) > 0.5$ deg
- Veto beam $e\pm$ in “lumical+tracker” ($|\theta_e| > 50$ mrad at truth level)

Cut efficiency	
	23.17%
	16.90%
	16.66%
	13.49%



Tau lepton $g-2$ at FCC-ee

- Energy dependence and $g-2$
 - BSM contributions are enhanced at high photon energies
 - Backgrounds are suppressed at high \sqrt{s}

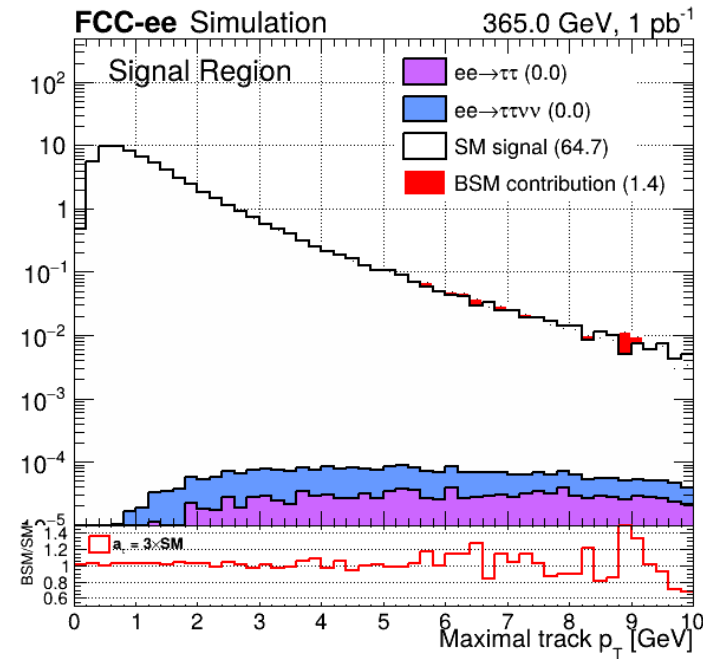


BSM/SM $\sim 1.5\%$ @ m_Z

2e4 events \rightarrow CMS limit

With $N \sim 30$ / pb a single experiment supersede CMS limit with $< 1 \text{ fb}^{-1}$

(few hours at FCC-ee assume no systematics)



BSM/SM $\sim 2.1\%$ @ m_{tt}

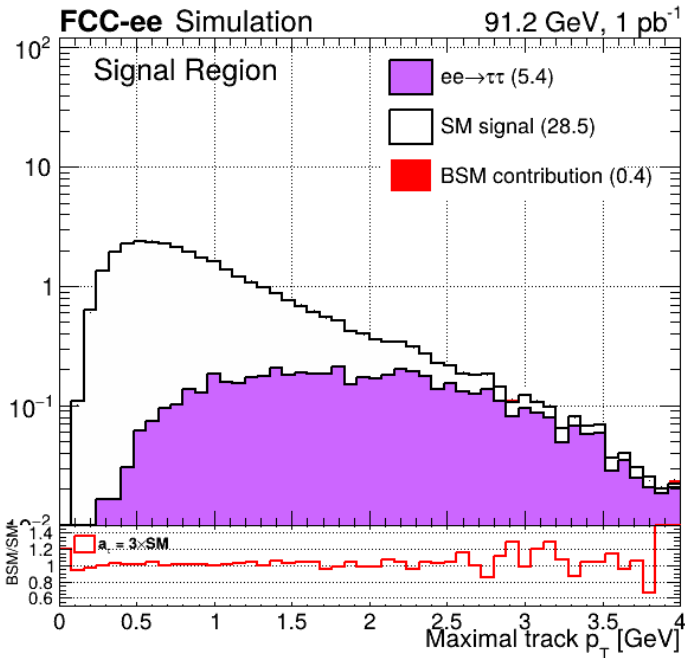
1e4 events \rightarrow CMS limit

With $N \sim 60$ / pb a single experiment supersede CMS limit with $< 1 \text{ fb}^{-1}$

(few days at FCC-ee assume no systematics)

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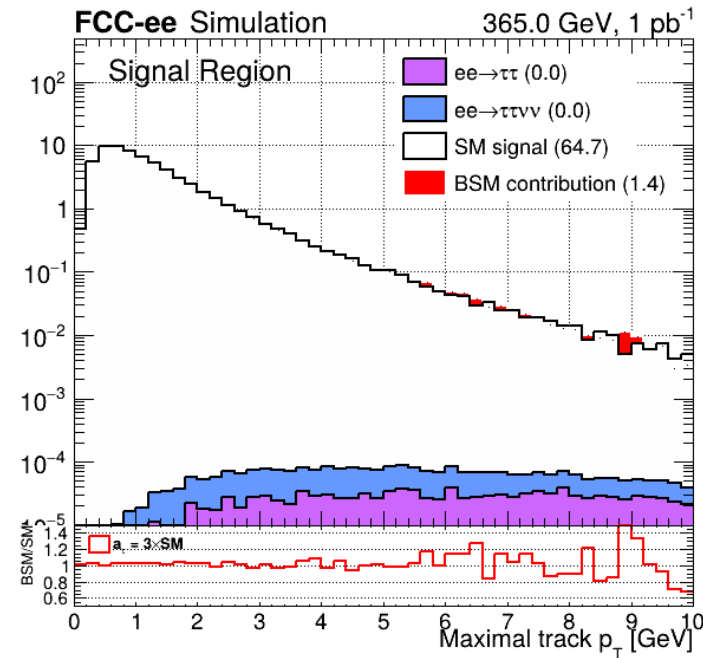


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(few hours at FCC-ee assume no systematics)



BSM/SM $\sim 2.1\%$ @ m_{tt}

$1e4$ events \rightarrow CMS limit

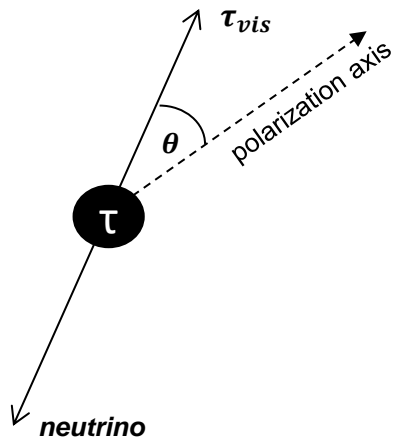
With $N \sim 60$ / pb a single experiment supersede CMS limit with $< 1 \text{ fb}^{-1}$

(few days at FCC-ee assume no systematics)

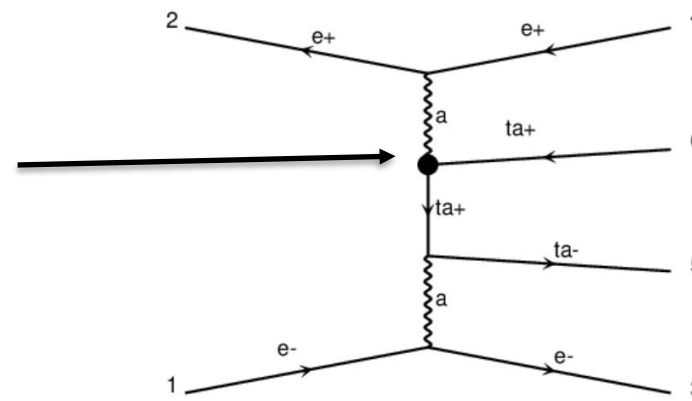
NOTE: EFT is just a preliminary approach (as it does not allow to extract a_τ exactly), a complete extraction will be provided in the upcoming studies.

Tau lepton $g-2$ at FCC-ee

- Tau polarization and $g-2$
 - In hadronic tau decays polarization variables can be used to discriminate dipole moment:



$$\mathcal{L} \sim \bar{\tau} \sigma^{\mu\nu} \left(a_\tau \frac{e}{2m_\tau} \right) \tau_R F_{\mu\nu}$$



Modification of the strength of the magnetic moment will impact the spin correlations:

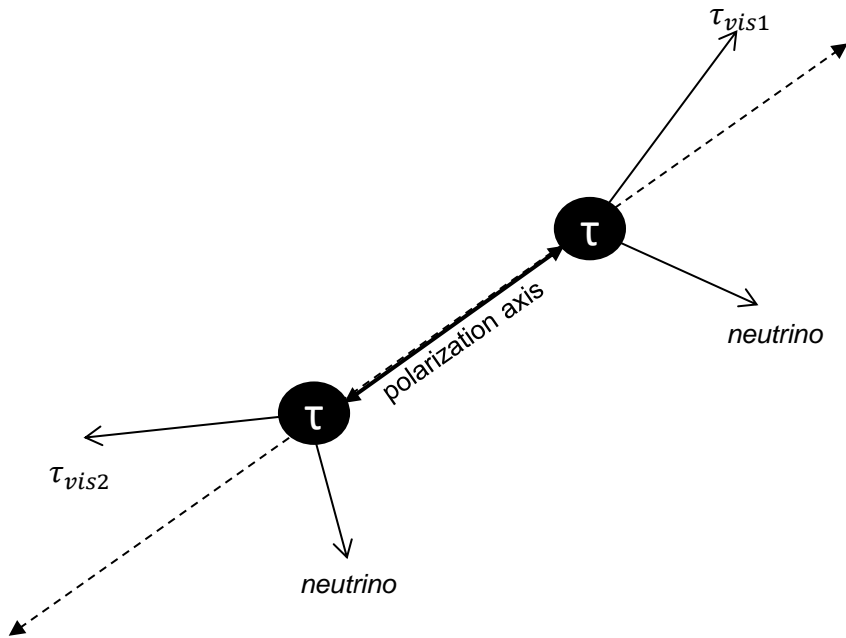
➤ More at high p_T region (2.5% \rightarrow 10%)

Tau polarization	SM [pb]	BSM [pb]
(+,+) or (-,-)	155.0	158.8
(+,-) or (+,-)	175.4	179.3
Apply high p_T selection: Average τ $p_T > 10$ GeV		
(+,+) or (-,-)	0.98	1.12
(+,-) or (+,-)	11.0	11.0

Tau lepton $g-2$ at FCC-ee

- Tau polarization and $g-2$

For spin correlations, the polarization axis can be defined along the τ boost, from $\tau\tau$ boost frame ($\tau\tau$ CoM):



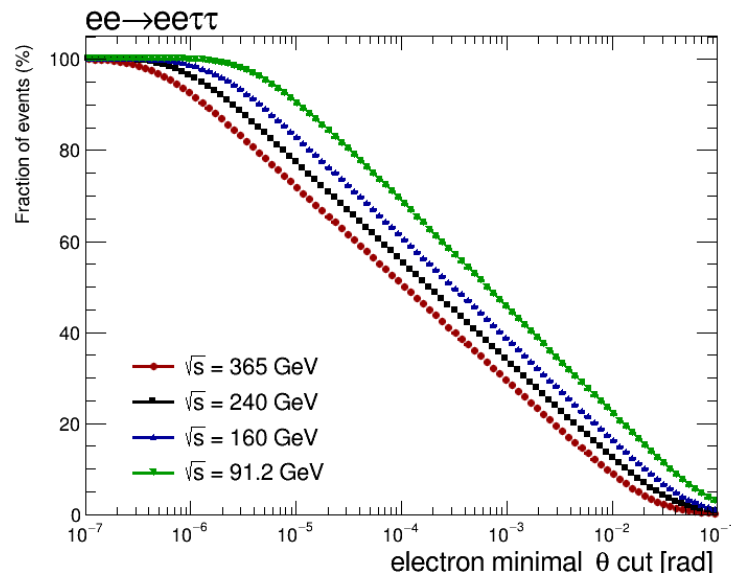
The decay angle with respect to the τ boost from $\tau\tau$ rest frame ($\cos\theta$), can be expressed in terms of variables measured in $\tau\tau$ rest frame:

$$\cos\theta = \frac{E_{vis} - E_{\nu}}{P_W} = \frac{2E_{vis} - \gamma m_{\tau}}{\gamma\beta m_{\tau}}$$

The $\tau\tau$ rest frame can be reconstructed from outgoing electron/positron kinematics

Tau lepton $g-2$ at FCC-ee

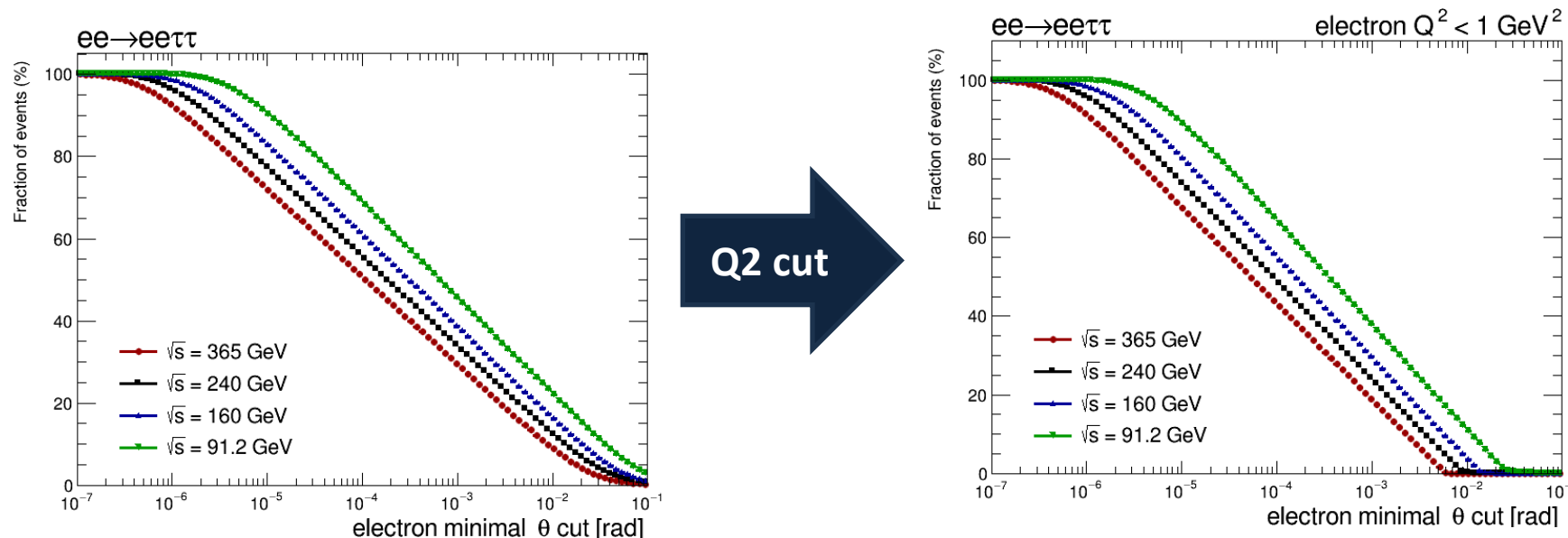
- Electron / Positron tagging
 - Electron/Positron acceptance depends on the lepton transport and the location of the detectors (distance from the IP and distance from the beam center)



Tau lepton $g-2$ at FCC-ee

- Electron / Positron tagging

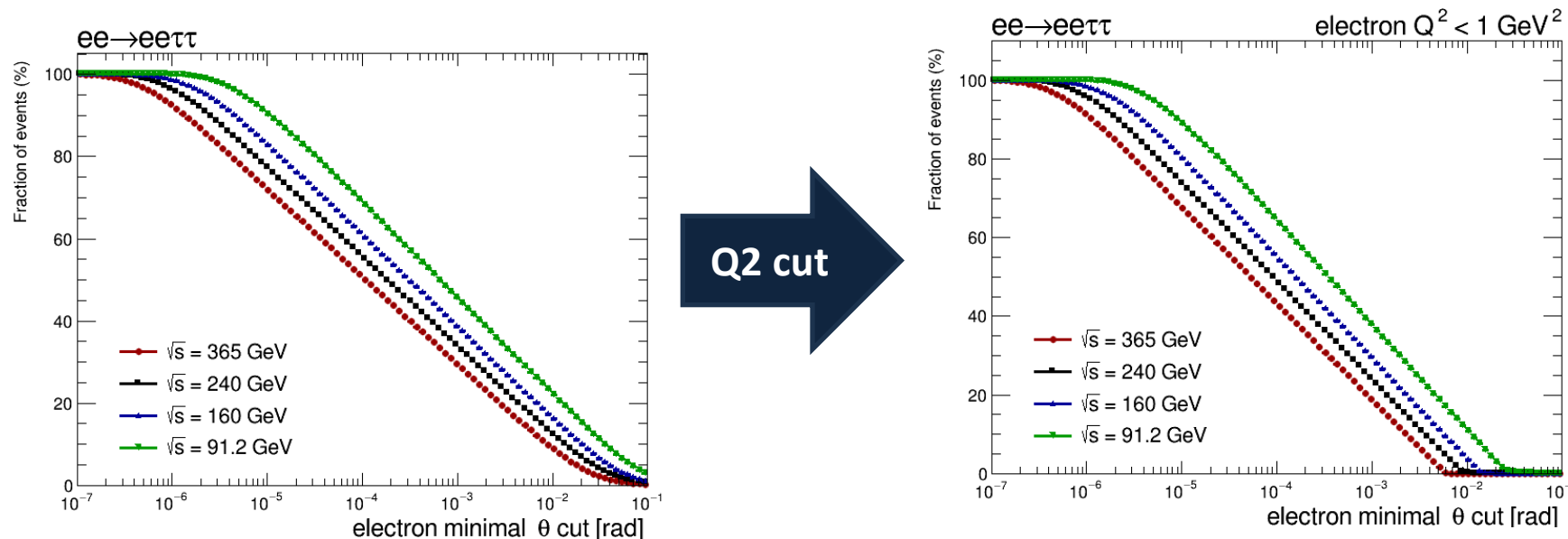
- Electron/Positron acceptance depends on the lepton transport and the location of the detectors (distance from the IP and distance from the beam center)
- To reach few mrad we would need to install e \pm - taggers inside the beampipe.



Tau lepton $g-2$ at FCC-ee

- Electron / Positron tagging

- Electron/Positron acceptance depends on the lepton transport and the location of the detectors (distance from the IP and distance from the beam center)
- To reach few mrad we would need to install e \pm - taggers inside the beampipe.
- The discussions with MDI is starting.



Summary

Conclusions

- Magnetic moment ($g-2$) of the tau lepton can be measured at the FCC-ee:
 - First exploratory studies indicate that one can measure the LO quantum correction (Schwinger term) with FCC-ee data at $\sqrt{s} = 90$ (and 365) GeV as it starts.
 - Already with a few days of running, we can achieve the CMS upper limits of $3 \times \text{SM}$.
- More precise determination of a_τ require e^\pm tagging inside the beampipe
 - This would benefit all photon-photon BSM/SM studies (ALPs, Missing mass, dipole moment of quarks, light resonances and hadronic states)

Upcoming studies

- Change EFT-approach by a study that allows to derive the measurable a_τ value.
- Study the possible locations for e^-/e^+ taggers. Need also to understand electron transport through the FCC-ee magnetic lattice.

Backup

Parameter	Z	WW	H (ZH)	ttbar
beam energy [GeV]	45.6	80	120	182.5
beam current [mA]	1270	137	26.7	4.9
number bunches/beam	11200	1780	440	60
bunch intensity [10^{11}]	2.14	1.45	1.15	1.55
SR energy loss / turn [GeV]	0.0394	0.374	1.89	10.4
total RF voltage 400/800 MHz [GV]	0.120/0	1.0/0	2.1/0	2.1/9.4
long. damping time [turns]	1158	215	64	18
horizontal beta* [m]	0.11	0.2	0.24	1.0
vertical beta* [mm]	0.7	1.0	1.0	1.6
horizontal geometric emittance [nm]	0.71	2.17	0.71	1.59
vertical geom. emittance [pm]	1.9	2.2	1.4	1.6
vertical rms IP spot size [nm]	36	47	40	51
beam-beam parameter ξ_x / ξ_y	0.002/0.0973	0.013/0.128	0.010/0.088	0.073/0.134
rms bunch length with SR / BS [mm]	5.6 / 15.5	3.5 / 5.4	3.4 / 4.7	1.8 / 2.2
luminosity per IP [$10^{34} \text{ cm}^{-2}\text{s}^{-1}$]	140	20	≥ 5.0	1.25
total integrated luminosity / IP / year [ab^{-1}/yr]	17	2.4	0.6	0.15
beam lifetime rad Bhabha + BS [min]	15	12	12	11

4 years
 5×10^{12} Z
 LEP $\times 10^5$

2 years
 $> 10^8$ WW
 LEP $\times 10^4$

3 years
 2×10^6 H

5 years
 2×10^6 tt pairs