Outlook for Theoretical Precision of the Luminosity at Future Lepton Colliders[a]

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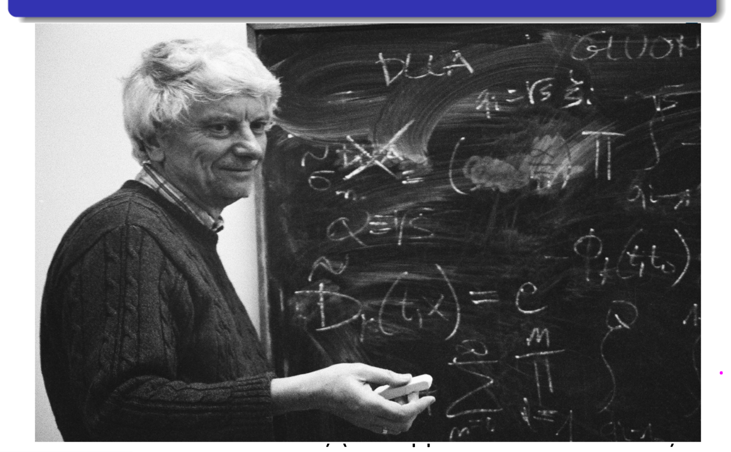
In Memoriam to Staszek Jadach



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Introduction, Lumi Basics

Bhabha cross sect. depends on detector acceptance angles

$$\sigma_{Bh} \simeq 4\pi\alpha^2\left(\frac{1}{t_{min}} - \frac{1}{t_{max}}\right) = 4\pi\alpha^2\left(\frac{t_{max} - t_{min}}{\bar{t}^2}\right), \quad \bar{t} = \sqrt{t_{min}t_{max}}$$

$\bar{t}$ is the characteristic scale of the process
$\bar{t}/s$ is the suppression factor between s- and t-channel contributions

Machine	$\theta_{min} \div \theta_{max}$ [mrad]	$\sqrt{s}$ [GeV]	$\bar{t}/s \simeq \bar{\theta}^2/4$	$\sqrt{\bar{t}}$ [GeV]
LEP	28÷50	M_Z	3.5×10^{-4}	1.70
FCCee	64÷86	M_Z	13.7×10^{-4}	3.37
FCCee	64÷86	240	13.7×10^{-4}	8.9
FCCee	64÷86	350	13.7×10^{-4}	13.0
ILC	31÷77	500	6.0×10^{-4}	12.2
ILC	31÷77	1000	6.0×10^{-4}	24.4
CLIC	39÷134	3000	13.0×10^{-4}	108
CEPC	26÷105	M_Z	6.8×10^{-4}	2.38
CEPC	26÷105	240	6.8×10^{-4}	6.27

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Current Situation, Related to LEP

LEP update 2018(2019)

Type of correction / Error	1999	Update 2018
(a) Photonic $O(L_e\alpha^2)$	0.027% [5]	0.027%
(b) Photonic $O(L_e^3\alpha^3)$	0.015% [6]	0.015%
(c) Vacuum polariz.	0.040% [7, 8]	0.013% (0.011%(JJ))
(d) Light pairs	0.030% [10]	0.010% [18, 19]
(e) s-channel Z-exchange	0.015% [11, 12]	0.015%
(f) Up-down interference	0.0014% [27]	0.0014%
(f) Technical Precision	–	(0.027)%
Total	0.061% [13]	0.038% (0.037%(JJ))

BAYLOR JJ <--> Janot and Jadach, Phys. Lett. B 803 (2019) 135319

Current Situation, Related to LEP

- Implied Upgrade of BHLUMI: 'still not urgent' in Review for US DOE OHEP Funding Program
- Recent Work by Banerjee *et al.*, PLB 820 (2021) 136547, T. Engel *et al.*, in arXiv:2203.12557, contacts

Type of correction / Error	Update 2018	FCCee forecast
(a) Photonic $O(L_e^4\alpha^4)$	0.027%	0.6×10^{-5}
(b) Photonic $O(L_e^2\alpha^3)$	0.015%	0.1×10^{-4}
(c) Vacuum polariz.	0.014% [25]	0.6×10^{-4}
(d) Light pairs	0.010% [18, 19]	0.5×10^{-4}
(e) Z and s-channel γ exchange	0.090% [11]	0.1×10^{-4}
(f) Up-down interference	0.009% [27]	0.1×10^{-4}
(f) Technical Precision	(0.027)%	0.1×10^{-4}
Total	0.097%	1.0×10^{-4}

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Current Situation, Related to LEP

References on Virtual Correction to Bremsstrahlung at $O(\alpha^2 L)$

1. S. Jadach et al., Phys. Lett. B 377 (1996) 168.
2. S. Jadach et al., Phys. Lett. B 450 (1999) 262.
3. S. Jadach et al., Comput. Phys. Commun. 70 (1992) 305.
4. S. Jadach et al., Comput. Phys. Commun. 102 (1997) 229.
5. S. Jadach et al., Acta Phys. Pol. B30 (1999) 1745.

Reference to $O(\alpha^3 L^3)$

1. S. Jadach and B.F.L. Ward, Phys. Lett. B 389 (1996) 129.

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$(\gamma_s + Z_s + \gamma_t + Z_t)^{\otimes 2}$ EW interferences

- Included in BHLUMI: $(\gamma_s + Z_s) \otimes \gamma_t$
- To be added:
  - complete Born – trivial
  - complete $\mathcal{O}(\alpha_{EW})$ – known, e.g. BHWIDE
- Error: $\mathcal{O}(\alpha_{EW}^2)$
  - estimated at FCCee(M_Z) based on analysis of S. Jadach *et.al.* Phys. Lett B790 (2019) 314 – from BHWIDE
  - estimated at other energies/angles based on analysis done with $\mathcal{O}(\alpha_{EW})$ DIZET/ZFITTER (by changing switch NPAR(2) from 2 to 3) M. Battaglia, S. Jadach, D. Bardin, *eConf* **C010630** (2001) E3015, http://www.slac.stanford.edu/econf/C010630/papers/E3015.PDF for the energies of 800 GeV and 3 TeV. Extrapolation from 800 to 350/240 GeV not done ⇒ error likely overestimated (factor of 2-3 ???)
  - Error at higher $\bar{t}/M_Z^2$ almost entirely from $\gamma_t \otimes Z_t$ interference
- Amplitude-level exponentiation (KKMC-style) needed to account for leading $\mathcal{O}(\alpha_{EW}^2)$ corrs.

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QED photonic up-down interference

- Missing in BHLUMI
  size at $\mathcal{O}(\alpha)$: $0.07 \times \bar{t}_{xx}/s$ – easy to include,
  $\bar{t}_{xx}/s$ depends only on angles
  LEP → FCCee: t/s grows 4 times (LEP → ILC: 2 times)
- Error: h.o.t. – suppressed by $(\alpha/\pi)\ln(\bar{t}_{xx}/m_e^2)$ times safety factor of 2 ($\mathcal{O}(\alpha_{QED}^2)$ calculations exist) – almost negligible

Vacuum polarisation

- Uncertainty due to vacuum polarisation:
  $\delta_{VP}\sigma/\sigma = 2\delta\alpha_{eff}(\bar{t})/\alpha_{eff}(\bar{t})$
  - $\delta\alpha_{eff}(\bar{t})$ from (based on R-ratio measured at low energies) F. Jegerlehner, *CERN Yellow Reports: Monographs* **3** (2020) 9–37
  - $\alpha_{eff}(\bar{t})$ from F. Jegerlehner, *Nucl. Phys. Proc. Suppl.* **162** (2006) 22–32
- By FCCee operation time factor of 2 improvement expected (F. Jegerlehner)

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Light pairs

- Current state of the art: BHLUMI + external four-fermion code + virtual semianalytical corrections
  – P. Janot and S. Jadach, *Phys. Lett. B* **803** (2020) 135319
- included components:
  - *ee*-pair, $\mu\mu$-pair, $\tau\tau$-pair, *qq*-pair with *s*-channel photonic emissions (FERMISV, KORALW)
  - result for LEP: $4 \times 10^{-4} \pm 1 \times 10^{-4}$
- future prospects for external 4*fermion* code scenario
  – error components:
  - $4f + \gamma$ (25% of 4f) – s vs. t mismatch ∼ 30%
    $\mathcal{O}(\alpha)$ 4*fermion* calculations exist for selected final states
  - $4f + 2\gamma$, $6f$
- future prospects for BHLUMI upgrade scenario
  – error components:
  - $4f + \gamma$ – absent – correct t-channel behavior (LL+soft), $\mathcal{O}(\alpha)$ 4*fermion* likely not needed
  - $4f + 2\gamma$ – included via exponentiation + LL,
  - $6f$

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Light pairs

Extrapolation to other energies/angles

- use LEP result for *ff*: $4 \times 10^{-4} \pm 1 \times 10^{-4}$ and scale with $\ln^2(\bar{t}_{xx}/m_{yy}^2)/\ln^2(\bar{t}_{LEP}/m_{yy}^2)$ (pairs)
- use LEP result for $ff\gamma$ terms: $20\% \times 4 \times 10^{-4}$ (G. Montagna, M. Moretti, O. Nicrosini, A. Pallavicini, and F. Piccinini, *Nucl. Phys.* **B547** (1999) 39–59), and scale with $\ln(\bar{t}_{xx}/m_e^2)/\ln(\bar{t}_{LEP}/m_e^2)$ (photons)
- τ-pair (negligible at LEP) estimated relative to muon-pair as $\ln^2(\bar{t}_{xx}/m_\tau^2)/\ln^2(\bar{t}_{xx}/m_\mu^2)$
- *hadron*-pair estimated relative to muon-pair as $R_{had} \times \ln^2(\bar{t}_{xx}/(0.5GeV)^2)/\ln^2(\bar{t}_{xx}/m_\mu^2)$

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Lumi at FCCee – Forecast

Forecast			
Type of correction / Error	FCCee$_{M_Z}$[1]	FCCee$_{240}$[2]	FCCee$_{350}$[2]
(a) Photonic $\mathcal{O}(L_e^2\alpha^3)$	0.10×10^{-4}	0.10×10^{-4}	0.13×10^{-4}
(b) Photonic $\mathcal{O}(L_e^4\alpha^4)$	0.06×10^{-4}	$0.26 \times 10^{-4(a)}$	$0.27 \times 10^{-4(a)}$
(c) Vacuum polariz.	0.6×10^{-4}	1.0×10^{-4}	1.1×10^{-4}
(d) Light pairs	0.5×10^{-4}	0.4×10^{-4}	0.4×10^{-4}
(e) Z and s-channel γ exch.	$0.1 \times 10^{-4(\diamond)}$	$1.0 \times 10^{-4(*)}$	$1.0 \times 10^{-4(*)}$
(f) Up-down interference	0.1×10^{-4}	0.09×10^{-4}	0.1×10^{-4}
Total	1.0×10^{-4}	1.5×10^{-4}	1.6×10^{-4}

Numbers: (*) likely overestimated, (*a*) include safety factor 2. Technical error is not included

Precision dominated by:

- Vacuum polarisation (c) – seems irreducible.
- The EW $\mathcal{O}(\alpha^2)$ uncertainty (e): Numbers (*) are likely overestimated (taken from 800 GeV estimate) – factor 2 too big ? Number ($\diamond$) possibly underestimated (0.3×10^{-4} ?)

Precision loss at higher energies reasonable
factor of 2 loss w.r.t. M_Z

[1] S. Jadach, W. Płaczek, M. Skrzypek, B. F. L. Ward, S. A. Yost, *Phys. Lett. B* **790** (2019) 314
[2] S. Jadach, W. Płaczek, M. Skrzypek, B. F. L. Ward, Eur. Phys. J. C (2021) 81:1047

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Lumi at FCCee$_{M_Z}$ – Forecast study

Forecast study for FCCee$_{M_Z}$		
Type of correction / Error	Published [1]	Redone
(a) Photonic $\mathcal{O}(L_e^2\alpha^3)$	0.10×10^{-4}	0.10×10^{-4}
(b) Photonic $\mathcal{O}(L_e^4\alpha^4)$	0.06×10^{-4}	0.06×10^{-4}
(b') Photonic $\mathcal{O}(\alpha^2 L_e^0)$		0.17×10^{-4}
(c) Vacuum polariz.	0.6×10^{-4}	0.6×10^{-4}
(d) Light pairs	0.5×10^{-4}	0.27×10^{-4}
(e) Z and s-channel γ exch.	0.1×10^{-4}	0.1×10^{-4}
(f) Up-down interference	0.1×10^{-4}	0.08×10^{-4}
Total	1.0×10^{-4}	0.70×10^{-4}

- (d) light pairs are re-analysed w.r.t. [1] (safety factor 1.25 is removed; $ff\gamma$ non-leading contrib. less conservative: $z_{cut} \leq .5$ can help; *hadr*-pair uncertainty is set to few % as in [2])
- (f) value not rounded up is used as compared to Ref. [1]
- "Total" value not rounded up is used as compared to Ref. [1] (the above three entries corrected at 240 and 350 GeV as well)
- (b') missing non-logarithmic $\mathcal{O}(\alpha^2 L_e^0)$ correction added for completeness
- (e): size of $\mathcal{O}(\alpha^2)_{EW}$ corrs. to be revisited – available BHWIDE (conservative scaling 0.3×10^{-4}) & DIZET (switches, at higher energy) CEEX amplitude level exponentiation instrumental (KKMC style) ?

[1] S. Jadach, W. Płaczek, M. Skrzypek, B. F. L. Ward, S. A. Yost, *Phys. Lett. B* **790** (2019) 314
[2] ALEPH Collaboration, D. Buskulic *et al.*, *Z. Phys. C* **66** (1995) 3–18

Possible precision ∼ 0.7×10^{-4} within the reach ??

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Lumi via gamma-gamma

Another option for Lumi measurement: $e^+e^- \to \gamma\gamma$ [1],[2] (KLOE, CLEO, BESIII)

- statistical error $\sim 2 \times 10^{-5}$
- not sensitive to hadronic corrections (vac. pol. contributes at NNLO) $\leq 10^{-5}$
- not sensitive to detector geometry (measurement at wide angles), low angle Bhabha needs μ-meter alignment
- huge background due to large angle Bhabhas (∼ 100 times bigger than signal)
- channel sensitive to new physics, must be well controled
- precision below 10^{-4} requires full $\mathcal{O}(\alpha^2)$ QED and EW corrections

[1] Electroweak corrections to $e^+e^- \to \gamma\gamma$ as a luminosity process at FCC-ee Carlo M. Carloni Calame, Mauro Chiesa, Guido Montagna Oreste Nicrosinia, Fulvio Piccinini, arXiv:1906.08056
[2] Precision studies of quantum electrodynamics at future e^+e^- colliders J. Alcaraz Maestre, arXiv:2206.07564

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Bhabha lumi: 0.1×10^{-4} at Z-peak?

Vacuum polarisation

Note: Lattice methods with Jegerlehner's results allow, in principle, (c) -> (c)/6

$$\Delta\alpha_{had}(t) = \Delta\alpha_{had}(-Q_0^2)|_{lat} + [\Delta\alpha_{had}(t) - \Delta\alpha_{had}(-Q_0^2)]|_{pQCDAdler}$$

Lattice results are mainly limited now by statistics (?), so if enoug computing resources are available, the 0.1×10^{-4} precision at –few GeV^2 may be feasible.

The above is more optimistic than the 3.5σ tension with estimates based on exp. data of R-ratio reported in arXiv: 2203.08676, 2211.11401 [hep-lat] for $\Delta\alpha_{had}^{(5)}(-Q^2)$, $Q^2 = 3 \div 7$ GeV^2.

The precision of lattice results given in the above papers is $\Delta\alpha_{had}(-5GeV^2) = 0.00716 \pm 0.9 \times 10^{-4}$ – on par with R-ratio method.

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Bhabha lumi: 0.1×10^{-4} at Z-peak?

EW corrections

In *S. Jadach, W. Płaczek, M. Skrzypek, B. F. L. Ward, and S. A. Yost, Phys. Lett. B 790 (2019) 314–321* we estimated

- $\mathcal{O}(\alpha_{EW}^2)$ uncertainties in BHWIDE at Z-peak: Conservatively estimated as $\frac{\alpha}{\pi}\ln\frac{\bar{t}}{m_e^2} \times \mathcal{O}(\alpha^2_{from\ exponentiation})$ times safety factor of 2.
  This gives 0.7×10^{-4} for QED part and 0.3×10^{-4} for EW part.
  Added linearly one obtains 1×10^{-4}.
  But we are interested only in EW part !
- More aggresive estimate (no safety factor, added in quadratures) would give 0.4×10^{-4} for total and 0.15×10^{-4} for EW part.

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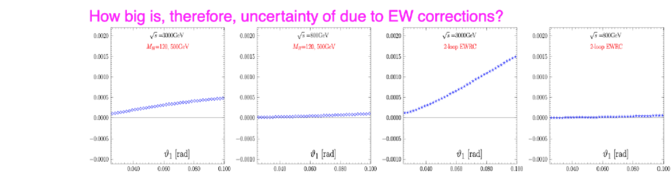
Bhabha lumi: 0.1×10^{-4} at Z-peak?

DIZET analysis of EW corrs. done above Z-peak. At the peak different graphs contribute ($\gamma_t \otimes Z_s$ vs $\gamma_t \otimes Z_t$), but rough idea could be valid?
M. Battaglia, S. Jadach, and D. Bardin, eConf C010630 (2001) E3015
S. Jadach, "MC tools for extracting luminosity spectra. What do we need?". https://jadach.web.cern.ch/jadach/public/LumLCslac.pdf, 2002

How big is, therefore, uncertainty of due to EW corrections?



DIZET: Varied M_H =120→500GeV, M_t =165→185GeV and NPAR(2)=3→NPAR(2)=4 which manipulates non-leading 2-loop EW corrections $\mathcal{O}(G_F^2 M_t^2 M_Z^2)$, Degrassi et.al., keeping 2-loop EW corrections $\mathcal{O}(G_F^2 M_t^4)$.

At 800 GeV $\mathcal{O}(\alpha_{EW}^2)$ contributes below 0.4×10^{-4} and decreases with energy decrease ($15. \times 10^{-4}$ at 3 TeV).

Bottom line: leading α_{EW}^2 contribs may be needed

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Bhabha lumi: 0.1×10^{-4} at Z-peak?

Fermion pairs

One will probably need $\mathcal{O}(\alpha)$ corrections to four fermion final state.

- Calculations of Denner et.al. *(PLB 612(2005) 223)* exist for charged current final states. Claimed physical precision (due to higher orders) at WW threshold is few×0.1% of the 4f Born.
- The whole pair contribution to Bhabha is $\sim 4 \times 10^{-4}$. Assuming precision of 1% for NC final states we are well below 0.1×10^{-4} target, provided t-channel multiphotons are properly resummed.

Note, that above ∼ 500 GeV Sudakov logs must be resummed.

Bottom line
0.1×10^{-4} precision *a priori* not excluded

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Lumi forecast at ILC and CLIC

Forecast			
Type of correction / Error	ILC$_{500}$	ILC$_{1000}$	CLIC$_{3000}$
(a) Photonic $\mathcal{O}(L_e^2\alpha^3)$	0.13×10^{-4}	0.15×10^{-4}	0.20×10^{-4}
(b) Photonic $\mathcal{O}(L_e^4\alpha^4)$	0.27×10^{-4}	0.37×10^{-4}	0.63×10^{-4}
(c) Vacuum polariz.	1.1×10^{-4}	1.1×10^{-4}	1.2×10^{-4}
(d) Light pairs	0.4×10^{-4}	0.5×10^{-4}	0.7×10^{-4}
(e) Z and s-channel γ exch.	$1.0 \times 10^{-4(*)}$	2.4×10^{-4}	16×10^{-4}
(f) Up-down interference	$< 0.1 \times 10^{-4}$	$< 0.1 \times 10^{-4}$	0.1×10^{-4}
Total	1.6×10^{-4}	2.7×10^{-4}	16×10^{-4}

Number (*) is somewhat overestimated (taken from 800 GeV estimate)

- Precision at high energies totally due to the EW $\mathcal{O}(\alpha^2)$ hard process uncertainty (e).
- EW interferences are dominated by $\gamma_t \otimes Z_t$ (15% of $\gamma_t \otimes \gamma_t$ at CLIC) and $Z_t \otimes Z_t$ (2% of $\gamma_t \otimes \gamma_t$ at CLIC)
  – usefull for $\mathcal{O}(\alpha_{EW}^2)$ calculation ?
  CEEX amplitude level exponentiation mandatory ?

At 3 TeV loss of precision is dramatic, dominant $\mathcal{O}(\alpha_{EW}^2)$ and CEEX are a must!

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Lumi forecast at CEPC

Forecast		
Type of correction / Error	CEPC$_{M_Z}$	CEPC$_{240}$
(a) Photonic $\mathcal{O}(L_e^2\alpha^3)$	0.08×10^{-4}	0.10×10^{-4}
(b) Photonic $\mathcal{O}(L_e^4\alpha^4)$	0.14×10^{-4}	0.21×10^{-4}
(c) Vacuum polariz.	0.6×10^{-4}	1.2×10^{-4}
(d) Light pairs	0.24×10^{-4}	0.34×10^{-4}
(e) Z and s-channel γ exch.	0.5×10^{-4}	$1.0 \times 10^{-4(*)}$
(f) Up-down interference	0.03×10^{-4}	0.04×10^{-4}
Total	0.83×10^{-4}	1.62×10^{-4}

Number (*) is likely overestimated.

- The total error is summed in quadrature. A technical error is not included.
- In the lines (d), (f) and "Total" of the column "CEPC$_{M_Z}$" safety factors were removed as compared to [1].
- In line (e) estimate based on BHWIDE in [1] is used with a 1/2 factor applied due to reduced transfer. That number differs from the 0.1×10^{-4} used for $FCCee_{M_Z}$.

[1] S. Jadach, W. Płaczek, M. Skrzypek, B. F. L. Ward, S. A. Yost, *Phys. Lett. B* **790** (2019) 314

BAYLOR SUMMARY: With proper support, lumi error needs can be met.