New Physics contamination in small-angle Bhabha scattering at FCC-ee

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Abstract

The accuracy of several key observables of the FCC-ee physics program will have a critical dependence on the knowledge of the absolute luminosity of the machine. In order to determine the luminosity with high precision one has to rely on some process which is in principle very well known theoretically, so that its cross section can be computed with a small theoretical error within the Standard Model. However, possible New Physics effect could contaminate such process, thus biasing the determination of the theoretical error associated with the luminosity measurement and, as a consequence, the precision of any absolute cross section measurement. We investigate the light and heavy New Physics contributions to the small-angle Bhabha scattering at FCC-ee and we propose possible strategies to overcome such potential additional uncertainty relying on observables that are independent of the luminosity. To this end, the Forward-backward asymmetry of the Large-angle Bhabha scattering is exploited, using the projected uncertainties of the FCC-ee Z-peak runs.

Precision Luminosity measurements at FCC-ee

At FCC-ee, the luminosity calibration will play a crucial role in the determination of M_W, Γ_W at the WW threshold, as well as HZZ couplings and Γ_H, though the measurement of $e^+e^- \to HZ$ cross section.

The precision of observed absolute cross sections $\sigma = (1/\epsilon)N/L$ is limited by the error on luminosity

$$\frac{\Delta\sigma^{\rm exp}}{\sigma^{\rm exp}} = \frac{\Delta N^{\rm exp}}{N^{\rm exp}} \oplus \frac{\Delta L}{L}$$

The time-integrated luminosity L is related to the cross section of some reference process through the relation

Luminosity

$$L = \int \mathscr{L}\,\mathrm{d}t = \frac{N_0^{\rm exp}}{\epsilon\,\sigma_0^{\rm th}}$$

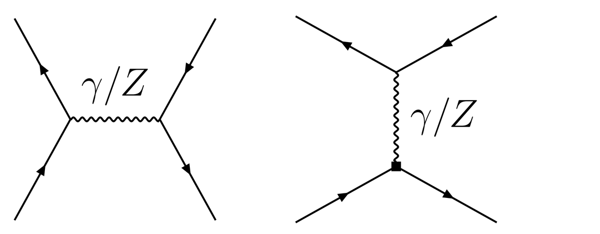
$\mathscr{L}$	Instantaneous luminosity
$N_0^{\rm exp}$	Number of events
$\sigma_0^{\rm th}$	Reference cross-section
ϵ	Experimental acceptance

in the hypothesis of high statistics, the theoretical error $\delta\sigma_0^{\rm th}$ dominates the error on luminosity: the reference process has to be very well known theoretically.

Small angle Bhabha scattering (SABS)

We focus on the process $e^+e^- \to e^+e^-$ at scattering angles $\theta \sim \mathcal{O}(30-100\text{mrad})$, proposed as one of the luminosity reference processes at FCCee.

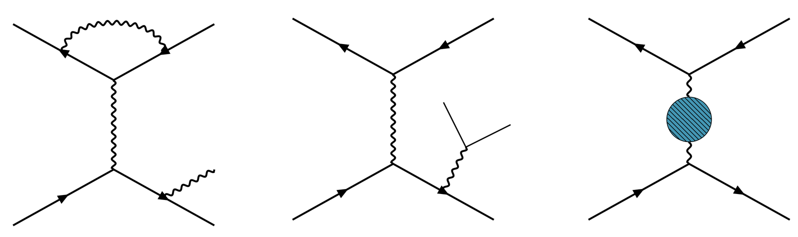
$$\sigma_{\rm SABS} \sim \frac{1}{\theta^2_{\rm min}}$$



The Standard Model (SM) cross section is largely dominated by the photon t-channel exchange $\mathcal{M}_\gamma(t)$.

Radiative Corrections

On the SM side, to reach the precision goal radiative corrections to SABS have to be known at 10^{-4} level. This requires an unprecedented effor on the Monte Carlo (MC) side.



$$0.011\% \oplus 0.005\% \oplus 0.006\% \simeq 0.01\%$$

In the most recent LEP analysis, the most precise estimate $\Delta L/L = 0.037\%$ removed a long-standing tension on the number of light neutrino species N_ν.

A natural question:
is there any uncertainty induced by New Physics at 10^{-4} level?

New Physics in SABS

The NP contributions can be due to heavy new degrees of freedom (d.o.f.) or to light mediators with feeble couplings to the leptons or photons which have escaped detection until now.

Depending on whether the NP scale lies below or above the electroweak (EW) scale $\Lambda_{\rm EW}$, we use specific models or Effective Field Theories, respectively, to describe the NP interference with the SM.

We adopt the $\{\alpha(M_Z), G_\mu, M_Z\}$ input parameter scheme with $\alpha(M_Z) = 1/127.95, G_\mu = 1.16638\times10^{-5}\text{GeV}^{-2}$ and $M_Z = 91.1876\,\text{GeV}$.

The amplitudes have been generated with FeynArts and FeynCalc. For the SMEFT, we have SmeftFR, SMEFTSim, MG5_aMC@NLO, while light NP Lagrangians have been implemented in FeynRules.

The simulations have been performed with the BabaYaga@NLO Monte Carlo generator, for both light and heavy NP contributions.

Light New Physics scenarios

If the mass of NP is below the EW scale, i.e. $M_{\rm NP} \lesssim \Lambda_{\rm EW}$ we need to rely on specific models. We therefore specify the spin and the interaction of light d.o.f. The analytical estimates for the full MC simulations are given as

$$\delta_{\rm LNP} \simeq \frac{2\,\mathrm{Re}\left(\mathcal{M}_\gamma(t)^\dagger \mathcal{M}_{\rm NP}\right)}{|\mathcal{M}_\gamma(t)|^2}\,.$$

The interaction of a (pseudo)scalar axion-like particle (ALP) a of mass m_a with both the photon and the electron can be parameterised with the parity-violating Lagrangian

Axion-Like Particles

$$\mathscr{L}_{\rm ALP} = \frac{1}{2}\partial_\mu a\,\partial^\mu a - \frac{1}{2}m_a^2 a^2 + \frac{1}{4}g_{a\gamma\gamma}(F_{\mu\nu}\tilde{F}^{\mu\nu})\,a + g_{aee}(\bar{e}\,i\gamma_5\,e)\,a$$

λ	Value
$g_{a\gamma\gamma}$	$2\times10^{-4}\,\text{GeV}^{-1}$
g_{aee}	3×10^{-3}
m_a	$1\,\text{GeV}$

the scalar parity-conserving case is obtained with the substitutions $\tilde{F}^{\mu\nu} \to F^{\mu\nu}$, $i\gamma_5 \to \mathbb{I}$. In the limit of $m_e \simeq 0, \mathcal{M}^\dagger_\gamma(t)\mathcal{M}_a(t)$ vanishes yielding to

$$\delta_{aee} \simeq \frac{g^2_{aee}}{4\pi\alpha}\frac{s^2 t}{(s-m_a^2)\,(s^2+u^2)} \simeq -\frac{g^2_{aee}}{8\pi\alpha}(1-\cos\theta)\,,$$

which is suppressed at small angles, $\delta^{aee}_{\rm ALP} < 10^{-7}$.
Bhabha scattering could also be mediated by a dark vector boson V_μ associated with a new gauge group $U(1)'$

Dark Vectors

$$\mathscr{L}_{\rm Dark} = -\frac{1}{4}V^{\mu\nu}V_{\mu\nu} + \frac{1}{2}M_V^2 V_\mu V^\mu + g'_V(\bar{e}\gamma^\mu e)\,V_\mu + g'_A(\bar{e}\gamma^\mu\gamma_5\,e)\,V_\mu\,,$$

λ	Value
g'_V	3×10^{-4}
g'_A	3×10^{-4}
M_V	$1\,\text{GeV}$

The bulk of the deviation w.r.t. the SABS in the SM due to dark vectors is given as follows

$$\delta_{\rm Dark} \simeq \frac{t\left[{g'_V}^2\,(s^2+u^2) - {g'_A}^2\,(s^2-u^2)\right]}{2\pi\alpha\,(t-M_V^2)\,(s^2+u^2)}$$

yielding $\delta_{\rm Dark} \sim \mathcal{O}(10^{-6})$. We conclude that light NP do not contribute to the SABS uncertainty at 10^{-4} level.

Heavy New Physics scenarios

Under the assumption that the NP scale lies far above the electroweak scale, i.e. $\Lambda_{\rm NP} \gtrsim \mathcal{O}(\text{TeV})$, one can study deviations from the SM using the SMEFT framework at dimension six.
The effective Lagrangian is expanded about the SM

SMEFT Lagrangian

$$\mathscr{L}_{\rm SMEFT} = \mathscr{L}_{\rm SM} + \sum_i \frac{C_i\hat{O}_i^{(6)}}{\Lambda^2_{\rm NP}} + \mathcal{O}\left(\frac{1}{\Lambda^4_{\rm NP}}\right)$$

where the gauge invariant operators $\hat{O}_i^{(6)}$ are built from the same d.o.f. of the SM and C_i are the associated Wilson Coefficients (WCs). At dimension six, the leading contribution to SABS cross section is given by four-fermion contact interactions, reading

$$\mathscr{L}^{4f}_{\rm SMEFT} = \frac{1}{2}\frac{C_{ll}}{\Lambda^2_{\rm NP}}(\bar{e}_L\gamma^\mu e_L)(\bar{e}_L\gamma_\mu e_L) + \frac{C_{le}}{\Lambda^2_{\rm NP}}(\bar{e}_L\gamma^\mu e_L)(\bar{e}_R\gamma_\mu e_R) + \frac{1}{2}\frac{C_{ee}}{\Lambda^2_{\rm NP}}(\bar{e}_R\gamma^\mu e_R)(\bar{e}_R\gamma_\mu e_R)\,.$$

At LO SMEFT, the prediction for the Bhabha cross section is

$$\sigma_{\rm SMEFT} = \sigma_{\rm SM} + \sigma^{(6)} = \sigma_{\rm SM} + \sum_{i=1}^{n}\frac{C_i}{\Lambda^2_{\rm NP}}\sigma_i^{(6)}\,,$$

where $\sigma_i^{(6)} = 2\,\mathrm{Re}\,\mathcal{M}^\dagger_{\rm SM}\mathcal{M}^{(6)}_{{\rm SMEFT},i}$ is the interference between the SM and SMEFT amplitude due to the i-th coefficient. We neglect NLO corrections whose effect is of $\mathcal{O}(10\%)$ w.r.t the LO SMEFT. We define the deviation from SM (differential) cross sections due to heavy NP

$$(\delta \pm \Delta\delta)_{\rm SMEFT} = \frac{1}{\sigma_{\rm SM}}\left(\sigma^{(6)} \pm \sqrt{\sum_{ij}\sigma_i^{(6)}V_{ij}\sigma_j^{(6)}}\right),$$

where the covariance matrix $V_{ij} = \Delta C_i\rho_{ij}\Delta C_j$ takes into account the correlation between fitted coefficients and their errors.
The numerical inputs, taken from a global fit, are

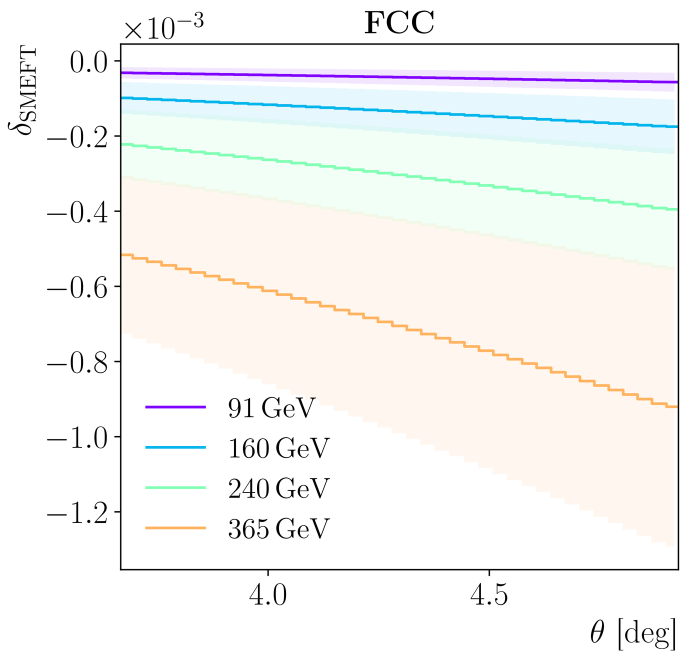
C_i	$C_i \pm \Delta(C_i)$
Δg_L^{Ze}	-0.0038 ± 0.0046
Δg_R^{Ze}	-0.0054 ± 0.0045
C_{ll}	0.17 ± 0.06
C_{le}	-0.037 ± 0.036
C_{ee}	0.034 ± 0.062

$$\rho = \begin{pmatrix} 1 & & & & \\ 0.15 & 1 & & & \\ -0.09 & -0.08 & 1 & & \\ 0.04 & -0.05 & -0.54 & 1 & \\ 0.08 & 0.08 & -0.04 & -0.54 & 1 \end{pmatrix}.$$

The following table summarises the results for the SABS in the FCC setup, showing non-negligible contribution at 10^{-4} level.

Exp.	$[\theta_{\rm min}, \theta_{\rm max}]$	$\sqrt{s}$ [GeV]	$(\delta \pm \Delta\delta)_{\rm SMEFT}$	$\Delta L/L$
FCC	$[3.7°, 4.9°]$	91	$(-4.2 \pm 1.7)\times10^{-5}$	$< 10^{-4}$
		160	$(-1.3 \pm 0.5)\times10^{-4}$	10^{-4}
		240	$(-2.9 \pm 1.2)\times10^{-4}$	
		365	$(-6.7 \pm 2.7)\times10^{-4}$	

Table 1. Heavy NP contamination to SABS at FCC-ee. $\Delta L/L$ represents the luminosity target precision.



The figure shows the relative differential deviation in the FCCee setup. It can be seen that the deviation grows with the scattering angle.

Constraining Contact interactions with LABS

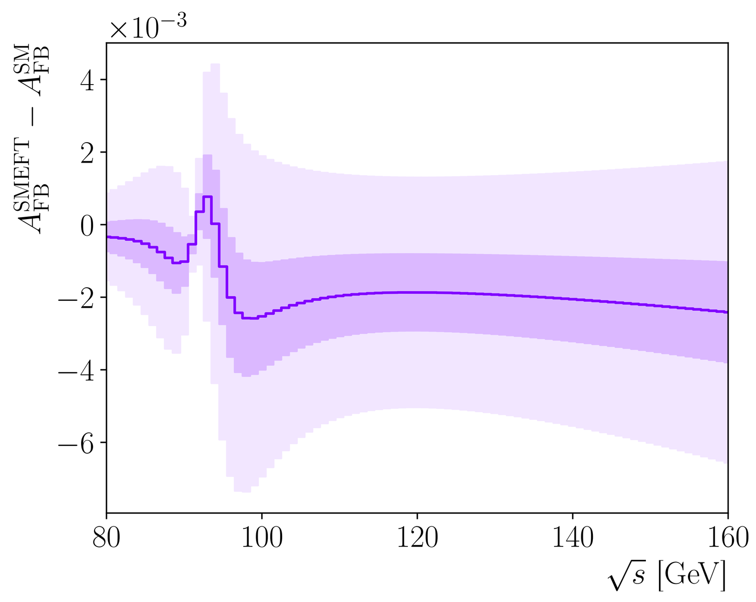
In the worst-case scenario of no significant improvement on WCs bounds by the timeline of the start of future colliders, we explore the possibility of constraining such coefficients using asymmetries in the range $\theta \in [40°, 140°]$

The theoretical prediction for the forward-backward asymmetry $A_{\rm FB}$ in the SMEFT can be written as

$$A_{\rm FB}^{\rm th} = A_{\rm FB}^{\rm SM}\left\{1 + \frac{(\sigma_{\rm F} - \sigma{\rm B})^{(6)}}{(\sigma_{\rm F} - \sigma_{\rm B})_{\rm SM}} - \frac{(\sigma_{\rm F} + \sigma_{\rm B})^{(6)}}{(\sigma_{\rm F} + \sigma_{\rm B})_{\rm SM}}\right\},$$

where $\sigma^{(6)}_{\rm F,B} = \sum_i C_i/\Lambda^2_{\rm NP}\,\sigma^{(6),i}_{\rm F,B}$, $\sigma_{\rm F} = \int_0^{c_{\rm max}}\mathrm{d}c(\frac{\mathrm{d}\sigma}{\mathrm{d}c})$ and $\sigma_{\rm B} = \int_{-c_{\rm max}}^0\mathrm{d}c(\frac{\mathrm{d}\sigma}{\mathrm{d}c})$ are the forward/backward cross sections, in which $c = \cos\theta$ and $c_{\rm max} = 0.77$ is a realistic cut for the LABS.

If one wants to fit $\vec{C}_{4f} = (C_{ll}, C_{le}, C_{ee})$, one can measure $A_{\rm FB}$ at three different values of $\sqrt{s}$, determined by the maximal sensitivity to the WCs, calculated as $A_{\rm FB}^{\rm SMEFT} - A_{\rm FB}^{\rm SM}$



We write the relative deviation of $A_{\rm FB}(\sqrt{s_\alpha})$ in the SMEFT as

Forward-backward asymmetry fit

$$\sum_{i\in 4f}\frac{C_i}{\Lambda^2_{\rm NP}}\left[\frac{(\sigma_{\rm F} - \sigma_{\rm B})_i^{(6)}}{(\sigma_{\rm F} - \sigma_{\rm B})_{\rm SM}} - \frac{(\sigma_{\rm F} + \sigma_{\rm B})_i^{(6)}}{(\sigma_{\rm F} + \sigma_{\rm B})_{\rm SM}}\right]_\alpha = \frac{\Delta A^0_{{\rm FB},\alpha}}{A^0_{{\rm FB},\alpha}},$$

where $\alpha = \{1,2,3\}$ labels the energy point. We solve the system by generating MC replicas of the experimental value $A^0_{{\rm FB},\alpha} \sim g\big(A^{\rm SM}_{\rm FB}, \Delta A^0_{\rm FB}\big)_\alpha$ as a Gaussian centered about the SM tree-level prediction.

Considering one year of run (10^7s) for each $\sqrt{s_\alpha}$ with $\mathcal{L}_{\rm FCC} = 1.4\times10^{36}\text{cm}^{-2}\text{s}^{-1}$, we find $\Delta A^0_{{\rm FB},\alpha} \lesssim 2\times10^{-5}$.

The 1σ uncertainty on four-electrons coefficients is reduced to $\Delta C_{ll/ee} \lesssim 10^{-2}$ and $\Delta C_{le} \lesssim 10^{-3}$ yielding to $\delta_{\rm SMEFT} \sim 5\times10^{-6}$ on the Z-peak luminosity at FCC, below the precision goal. This constraint would be enough also for future e^+e^- colliders runs at higher

References

[1] Mauro Chiesa, Clara L. Del Pio, Guido Montagna, Oreste Nicrosini, Fulvio Piccinini, and Francesco P. Ucci.
New physics contamination to precision luminosity measurements at future e^+e^- colliders, 2025.

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