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DEGLI STUDI  
DI PADOVA



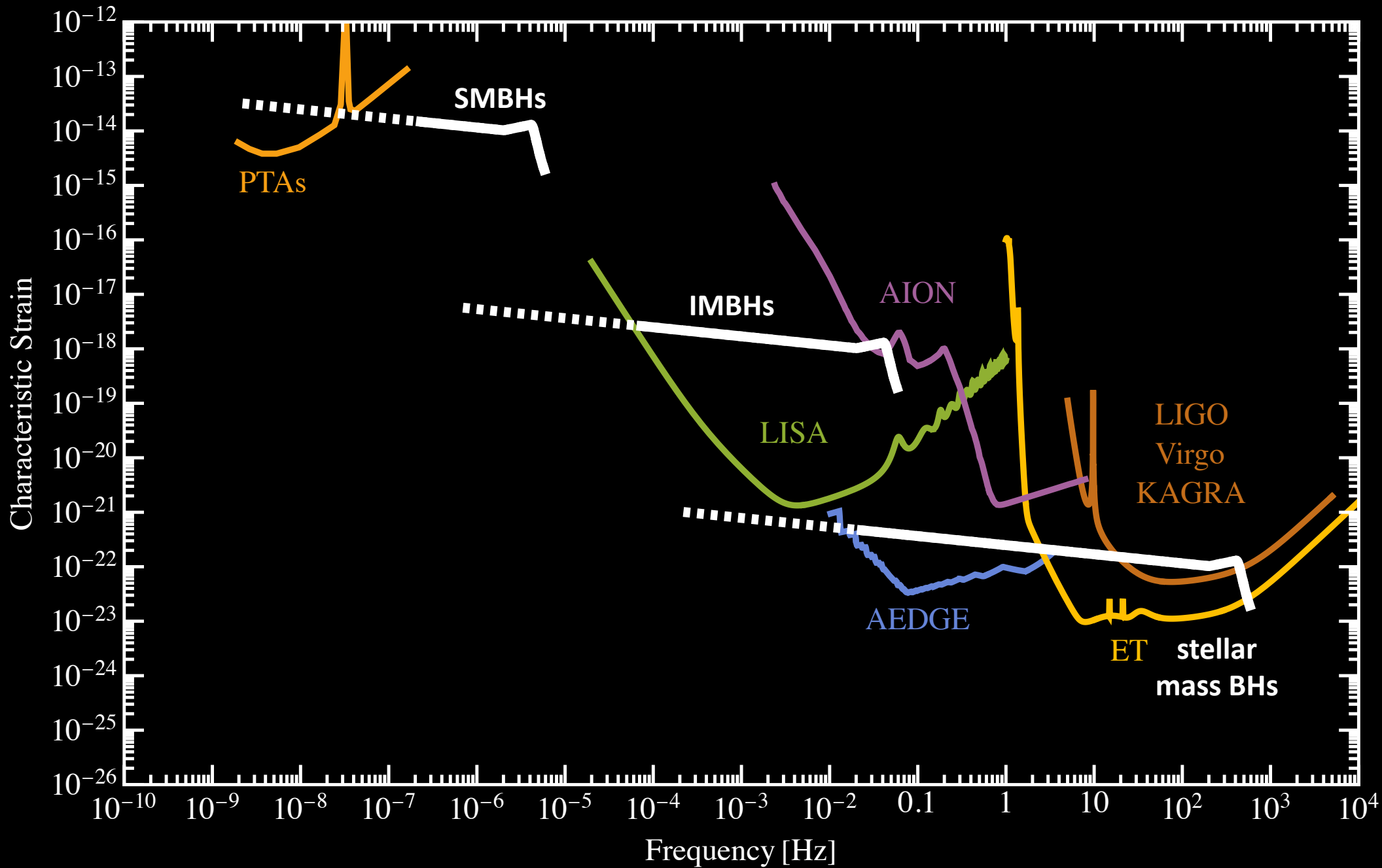
# Black holes and gravitational waves from slow and strongly supercooled phase transitions

Ville Vaskonen

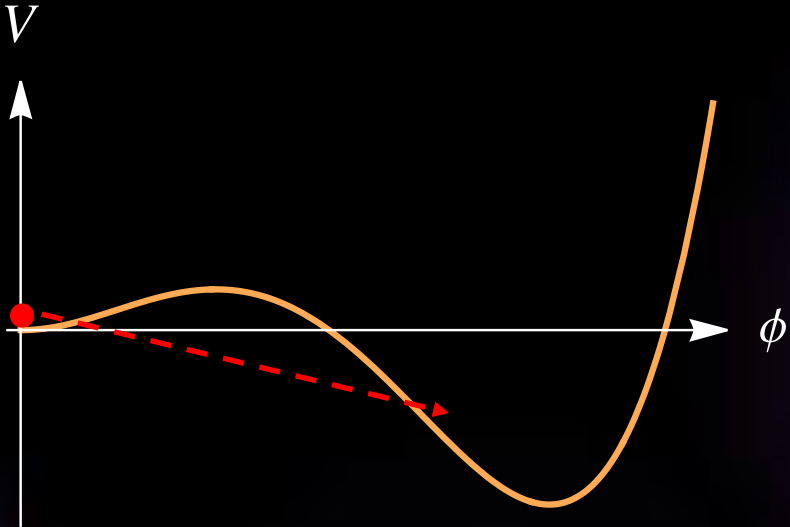


Co-funded by the  
European Union

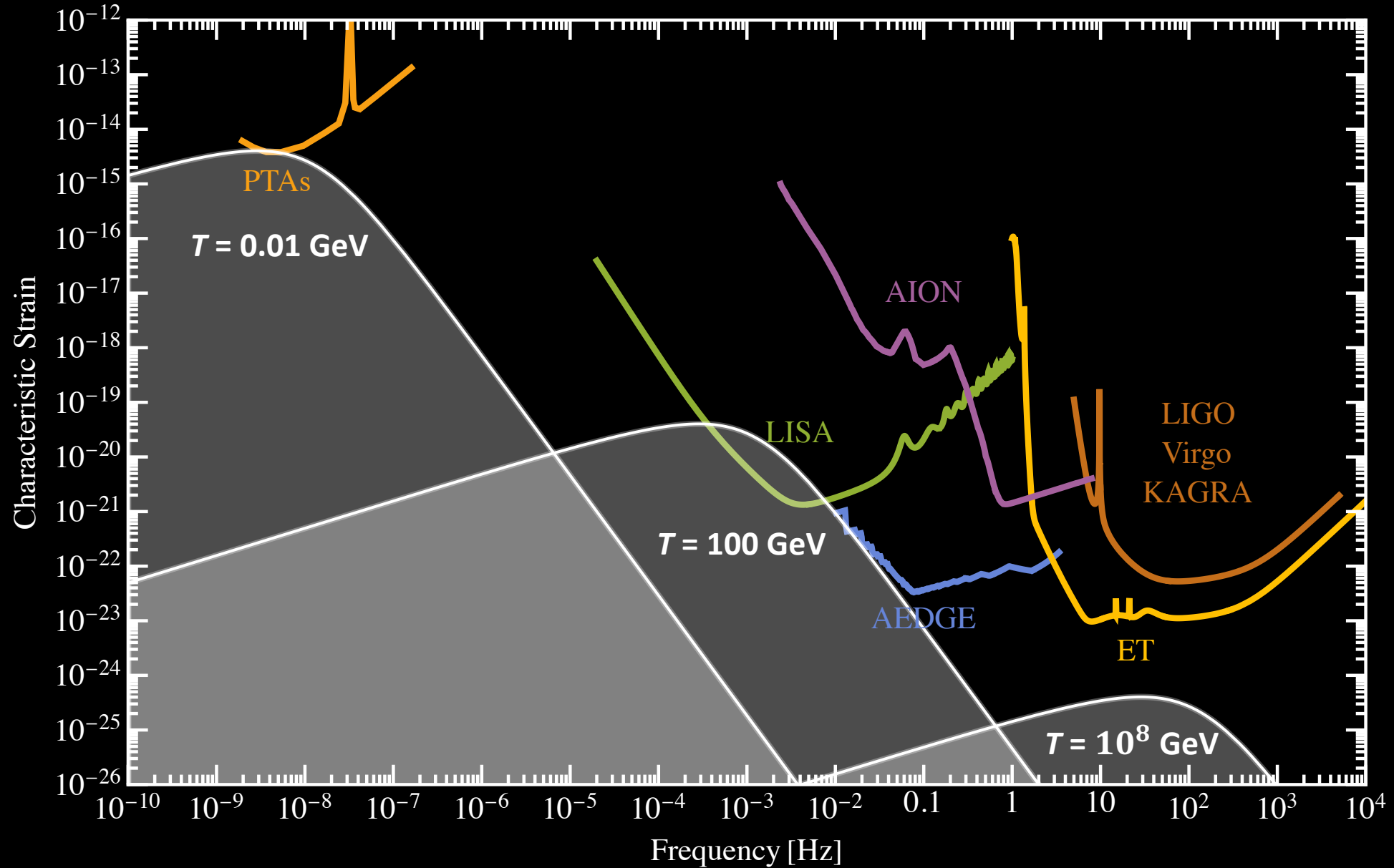
CERN, November 20, 2024.



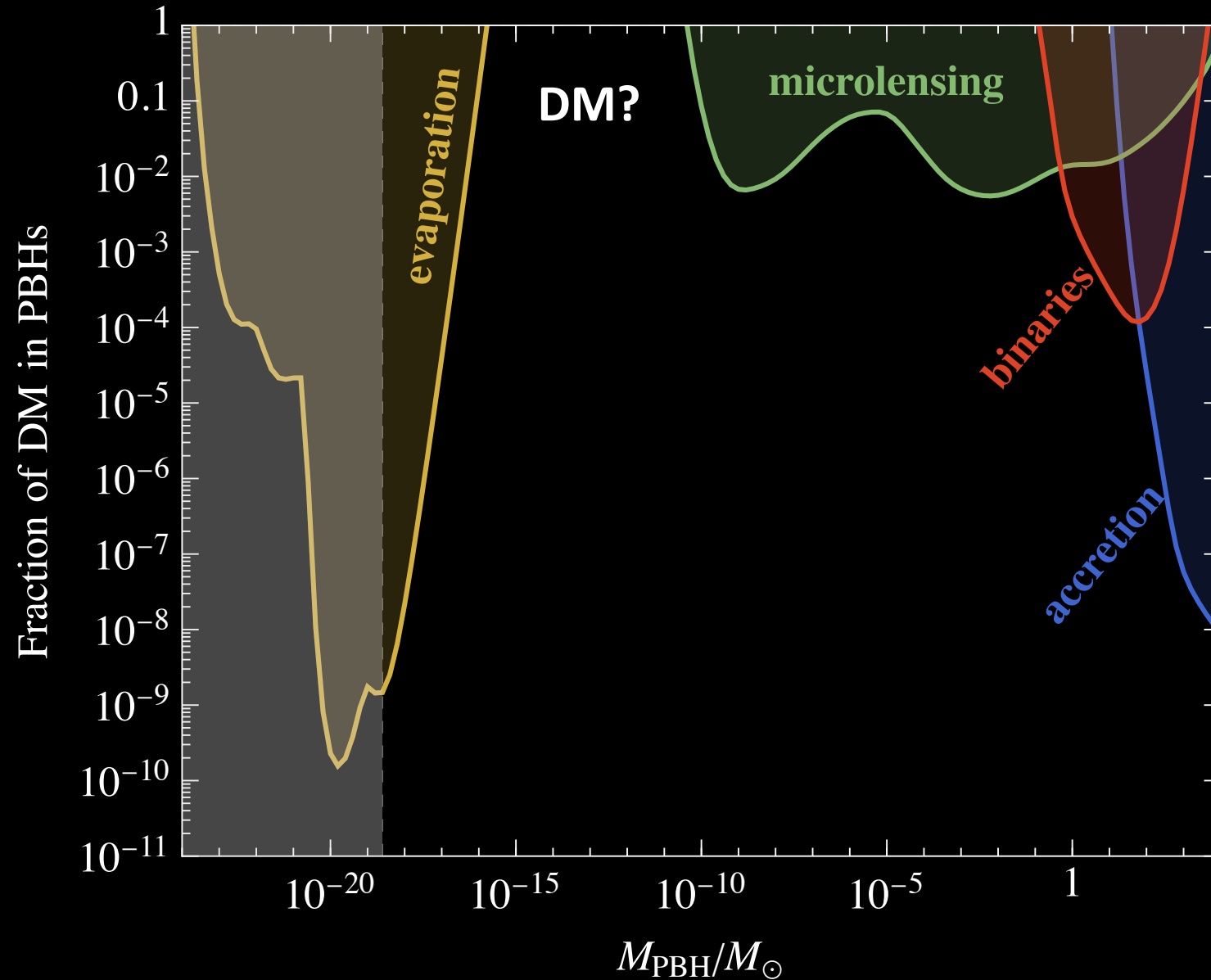
# First order phase transitions



# GWs from phase transitions

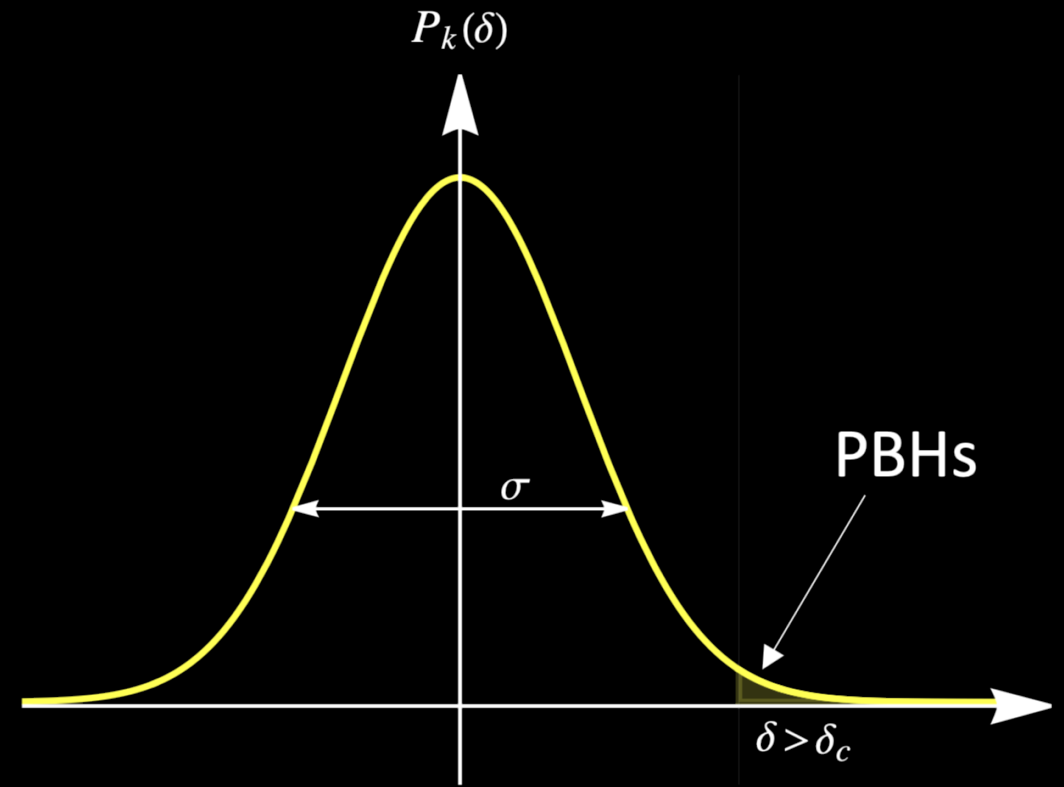
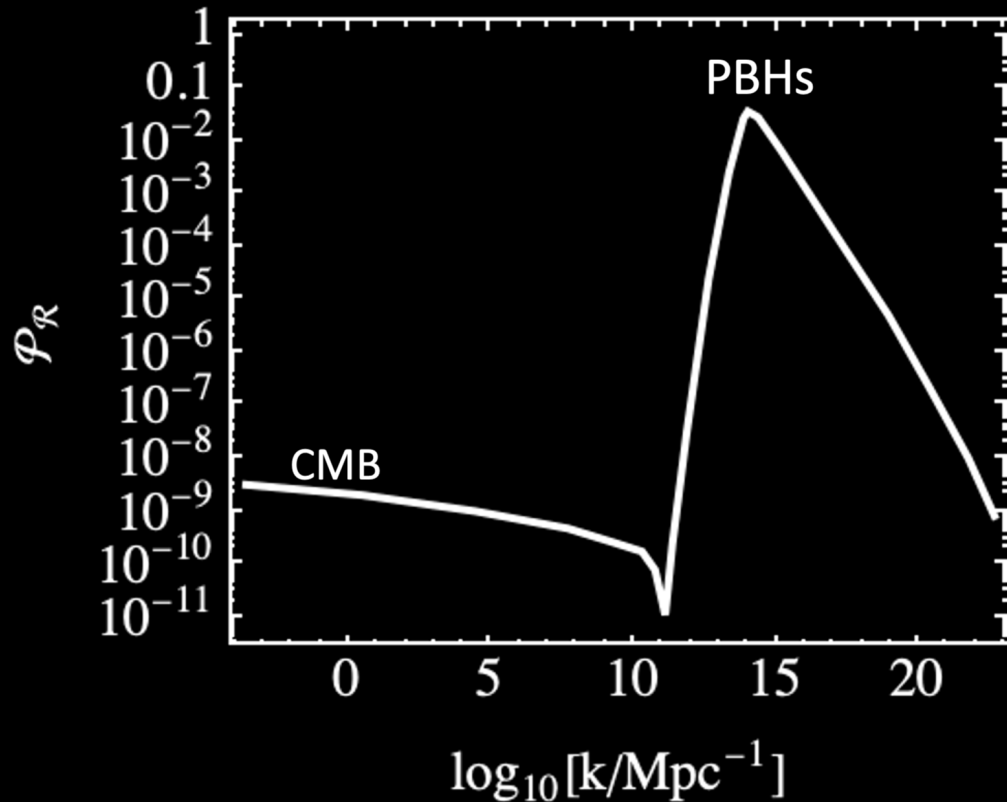


# Primordial black hole constraints



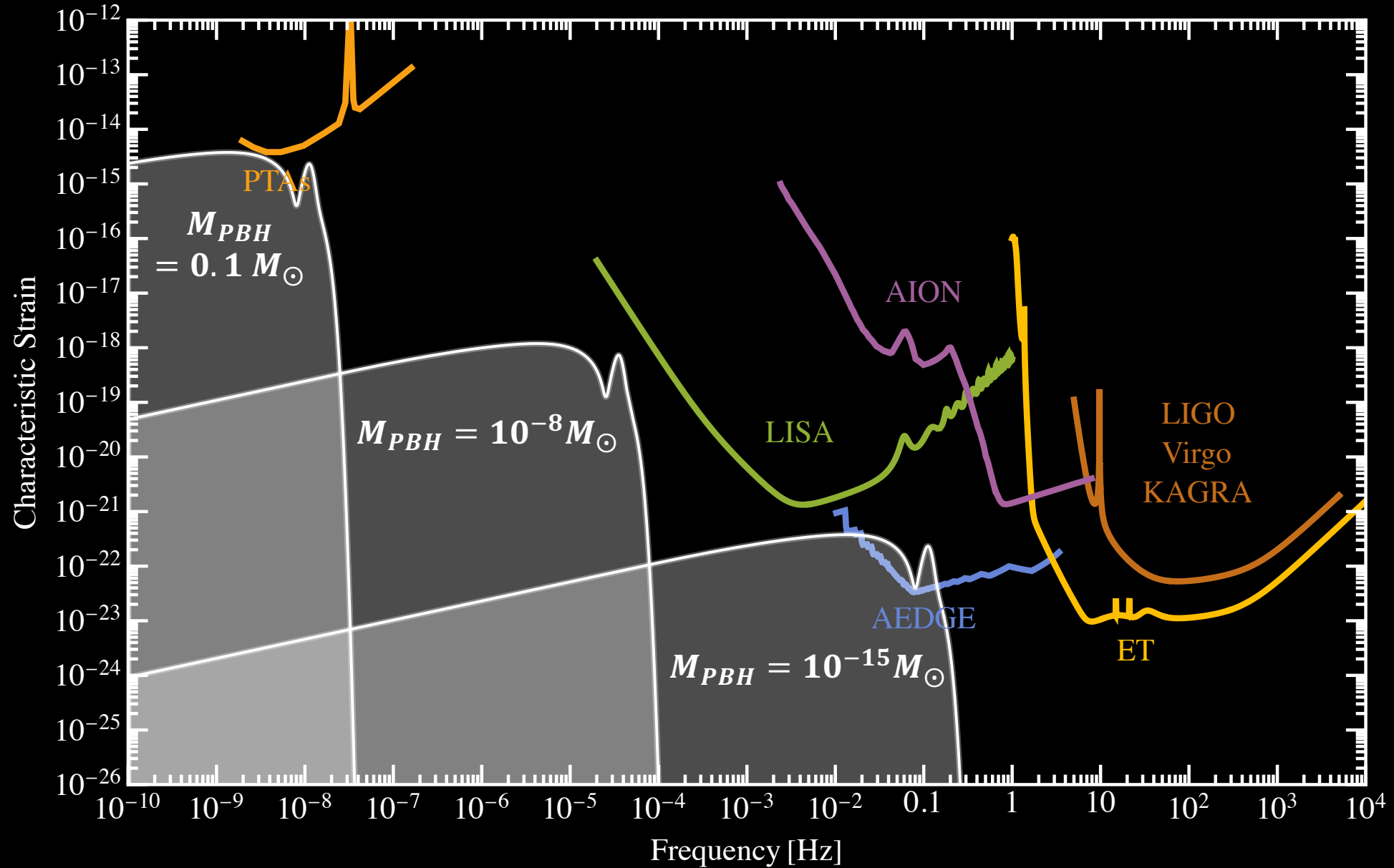
# Primordial black hole formation

Large fluctuations collapse against the fluid pressure to BHs at horizon reentry.



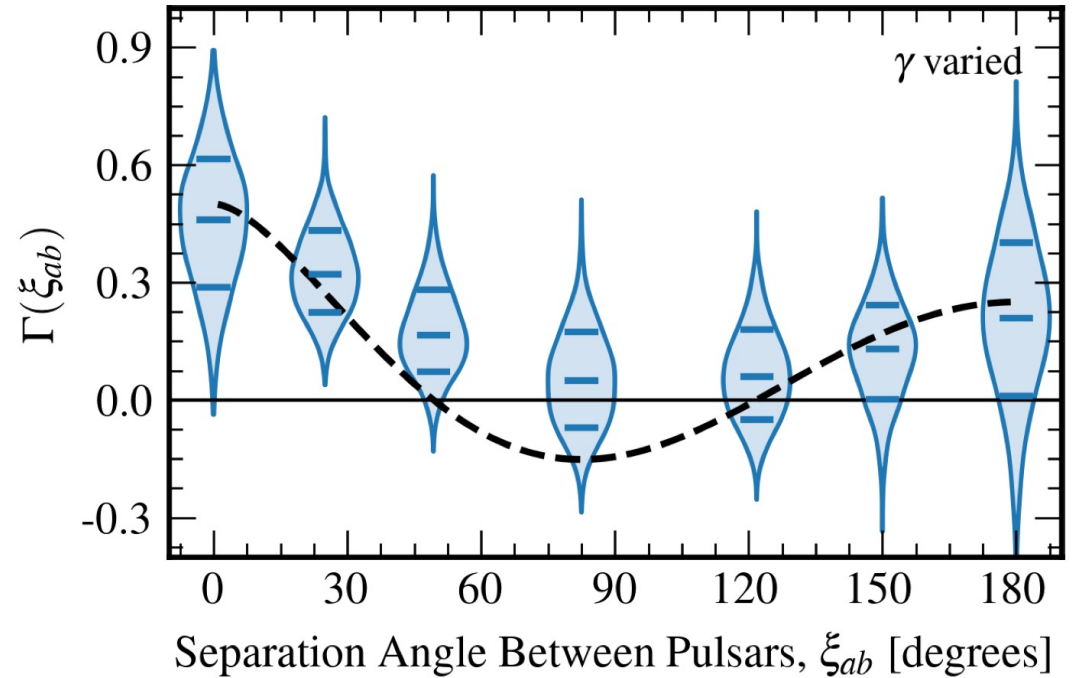
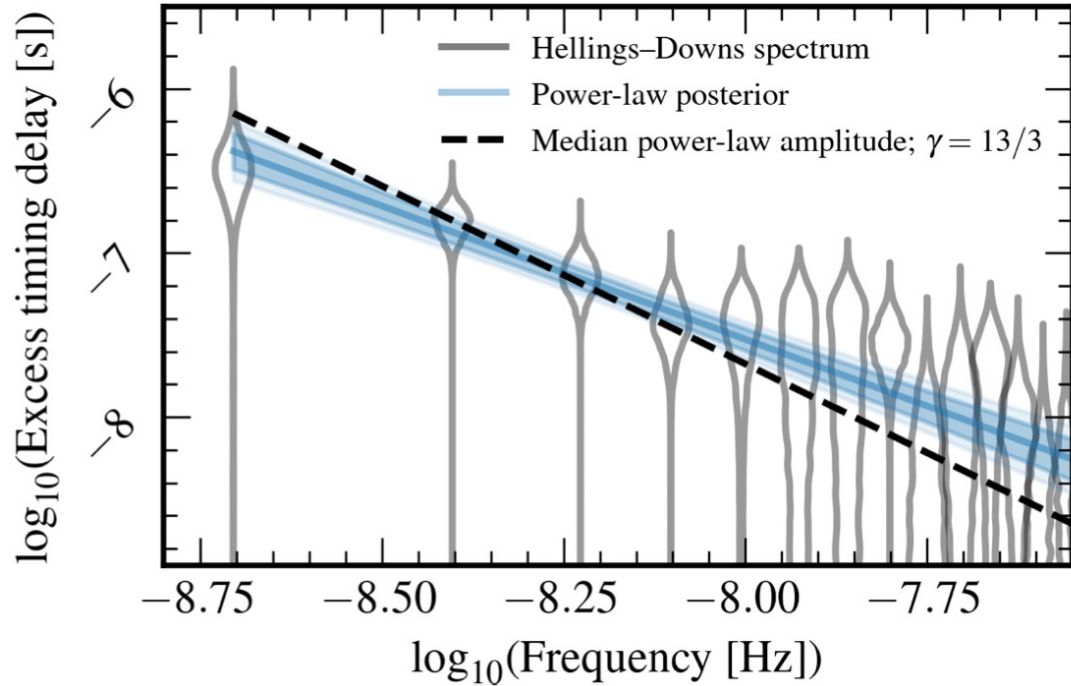
Large fluctuations source also scalar-induced GWs.

# Scalar-induced GWs



# Strong evidence for nHz GW background

NANOGrav, *ApJ. Lett.* 951 (2023), 2306.16213



**common spectrum noise**

**+**

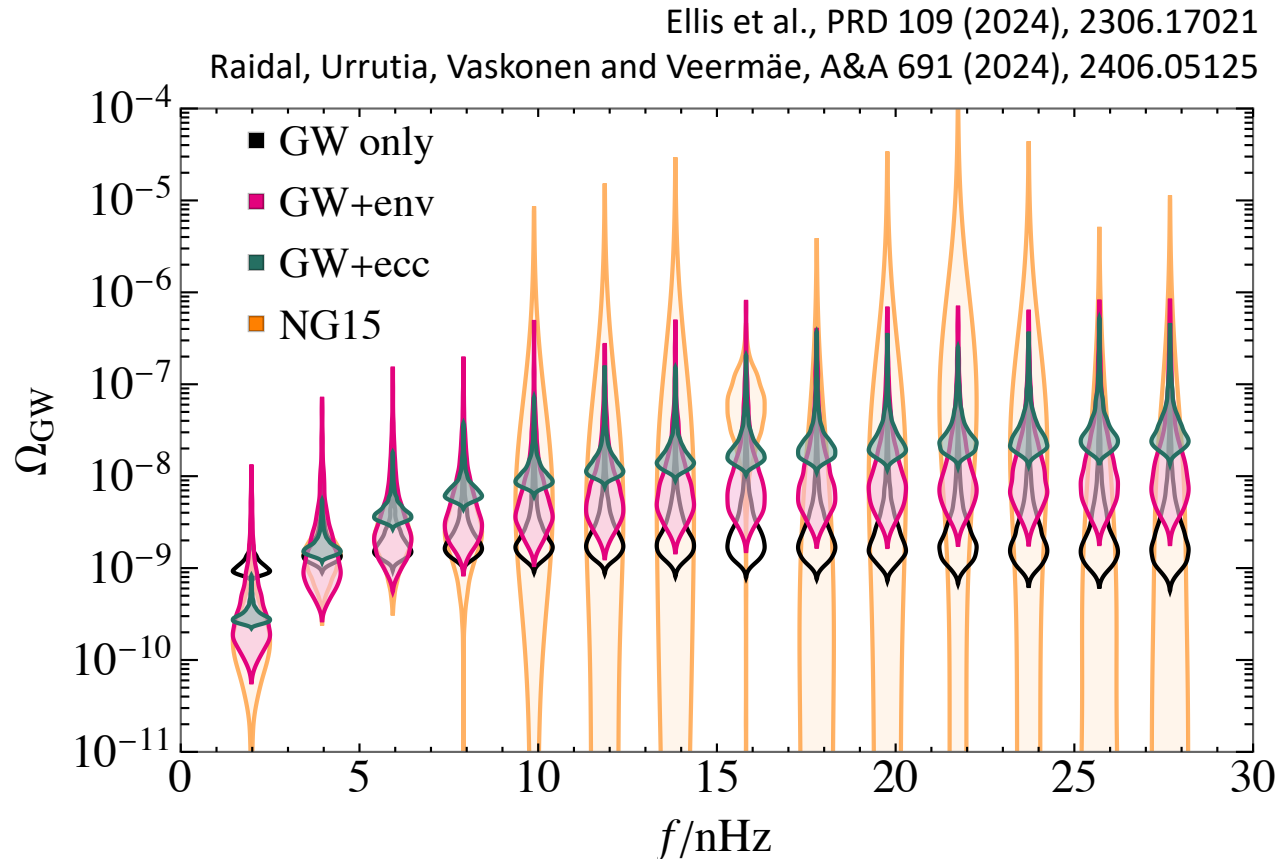
**Hellings-Downs angular correlation.**

Seen also by EPTA, InPTA, PPTA and CPTA.



# What is the source?

## SMBH fit:

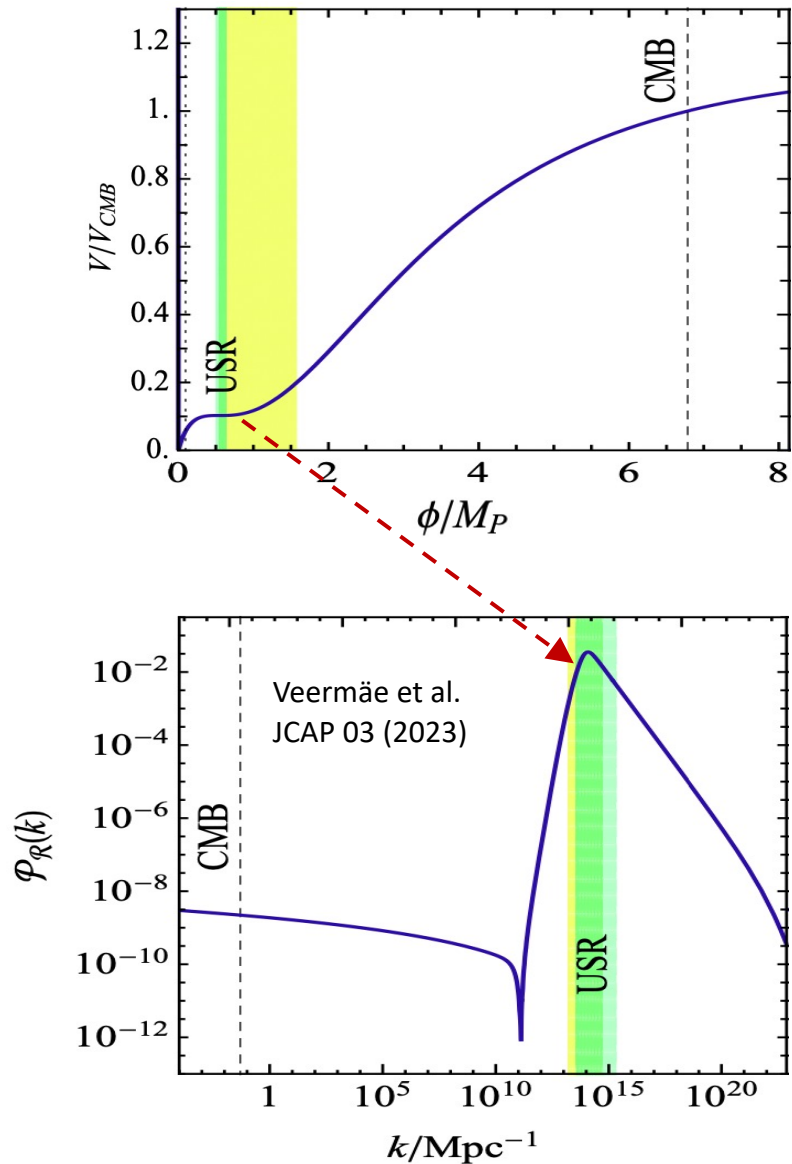


## MMA:

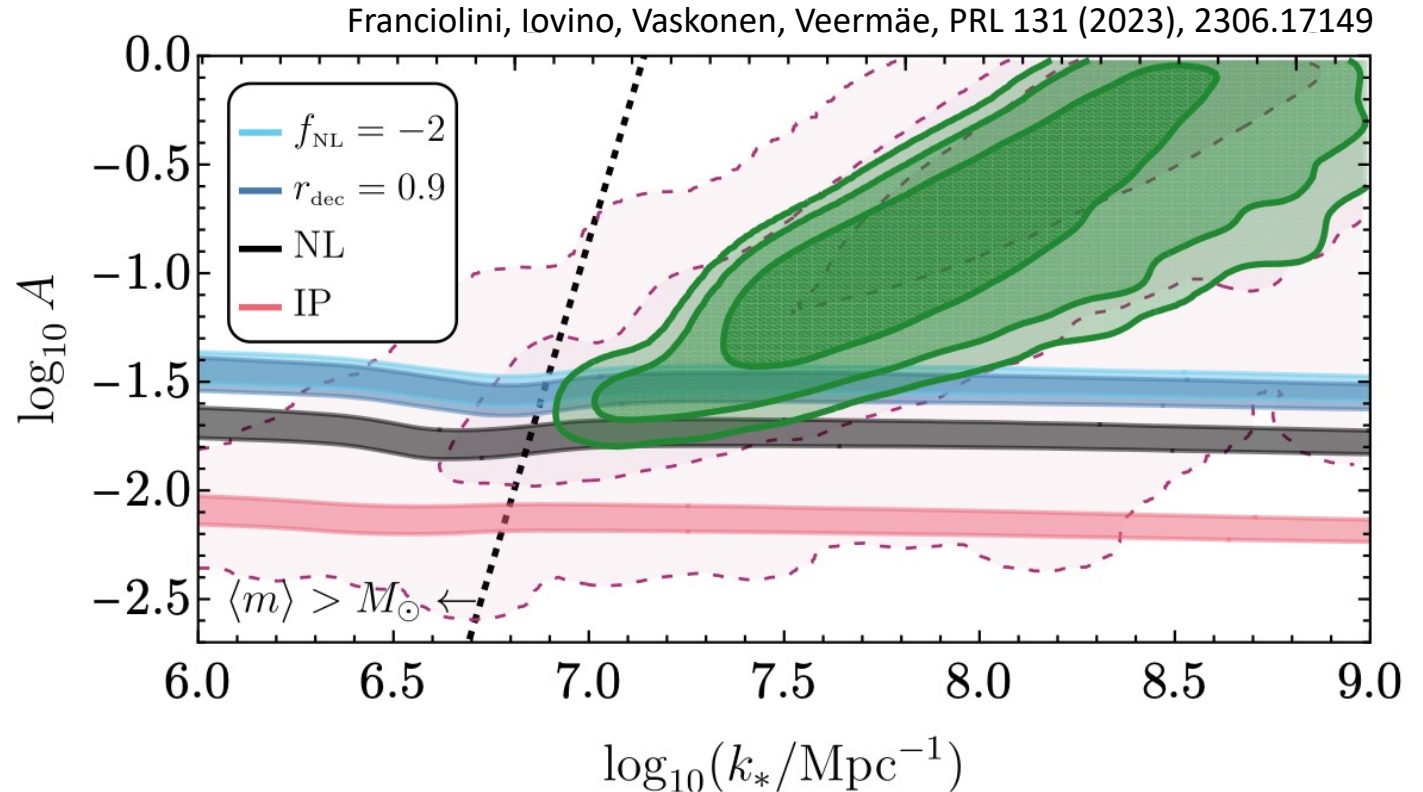
Ellis et al., PRD 109 (2024), 2308.08546

Scenario	Best-fit parameters	$\Delta\text{BIC}$
GW-driven SMBH binaries	$p_{\text{BH}} = 0.07$	6.0
GW + environment-driven SMBH binaries	$p_{\text{BH}} = 0.84$ $\alpha = 2.0$ $f_{\text{ref}} = 34 \text{ nHz}$	Baseline (BIC = 53.9)
Cosmic (super)strings (CS)	$G\mu = 2 \times 10^{-12}$ $p = 6.3 \times 10^{-3}$	-1.2 (4.6)
Phase transition (PT)	$T_* = 0.34 \text{ GeV}$ $\beta/H = 6.0$	-4.9 (2.9)
Domain walls (DWs)	$T_{\text{ann}} = 0.85 \text{ GeV}$ $\alpha_* = 0.11$	-5.7 (2.2)
Scalar-induced GWs (SIGWs)	$k_* = 10^{7.7} / \text{Mpc}$ $A = 0.06$ $\Delta = 0.21$	-2.1 (5.8)
First-order GWs (FOGWs)	$\log_{10} r = -14$ $n_t = 2.6$ $\log_{10} (T_{\text{rh}}/\text{GeV}) = -0.67$	-2.0 (6.0)
“Audible” axions	$m_a = 3.1 \times 10^{-11} \text{ eV}$ $f_a = 0.87 M_{\text{P}}$	-4.2 (3.7)

# PBHs from primordial inflation



SIGW fit of the PTA signal:



⇒ tension with PBH overproduction

**Slow and supercooled first-order  
phase transition**



**Large density fluctuations**



**PBHs and GWs**

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Liu et al. PRD 105 (2022), 2106.05637,  
Kawana, Kim, Lu, PRD 108 (2023), 2212.14037  
Gouttenoire, Volansky, PRD 110 (2024), 2305.04942

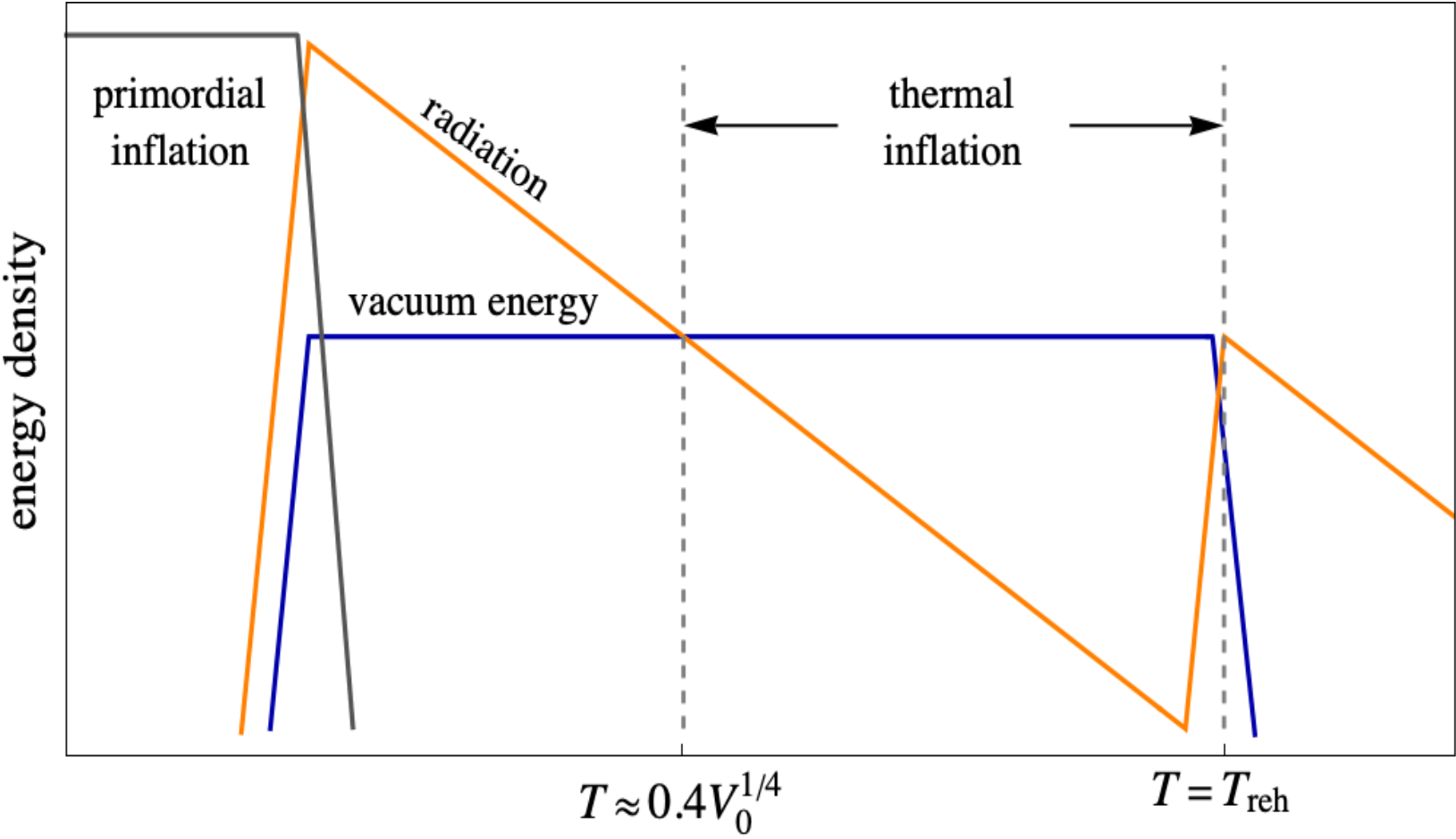
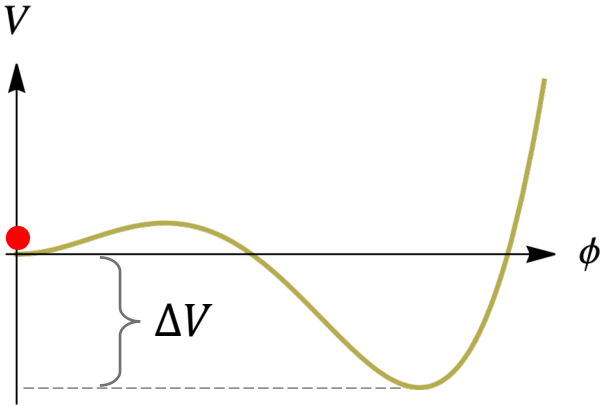
} PBHs

Lewicki, Toczek, Vaskonen, 2402.04158, accepted to PRL

PBHs + GWs

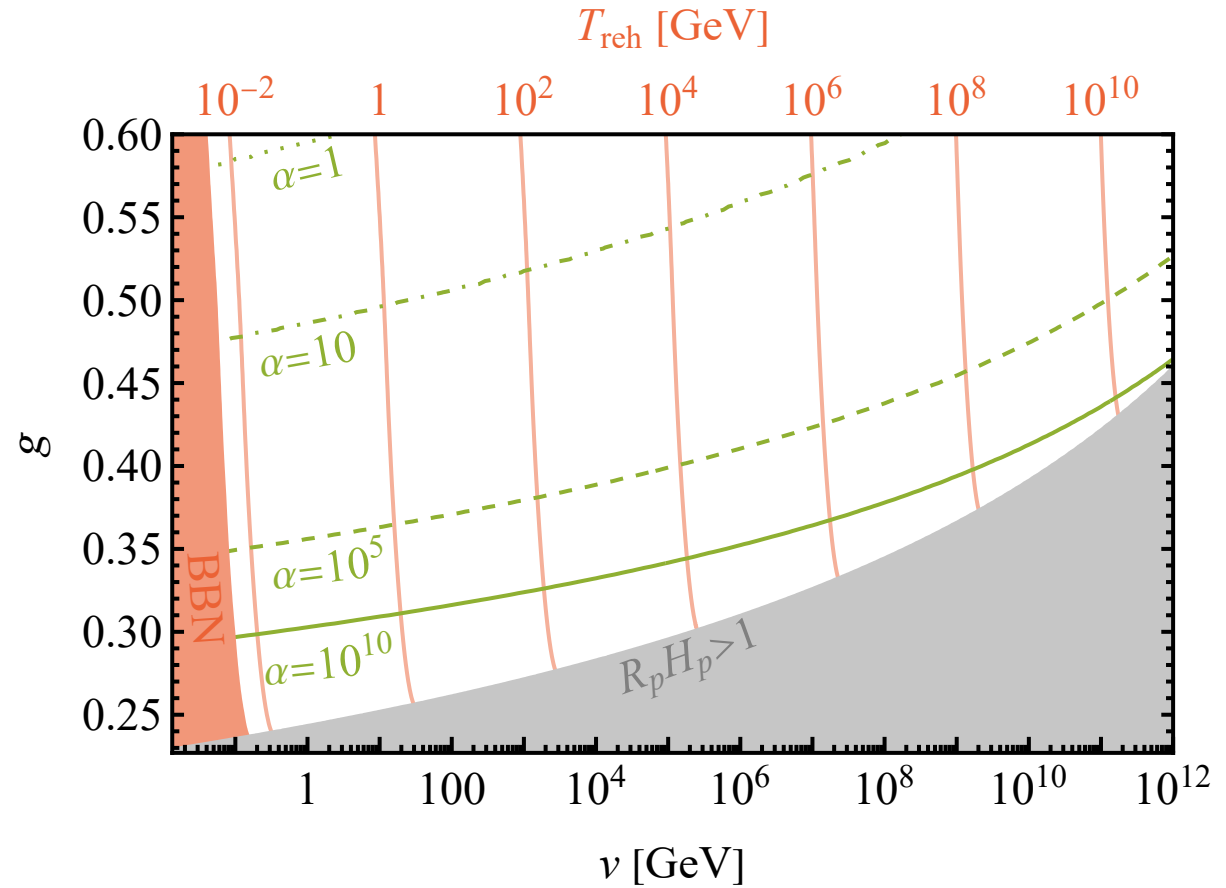
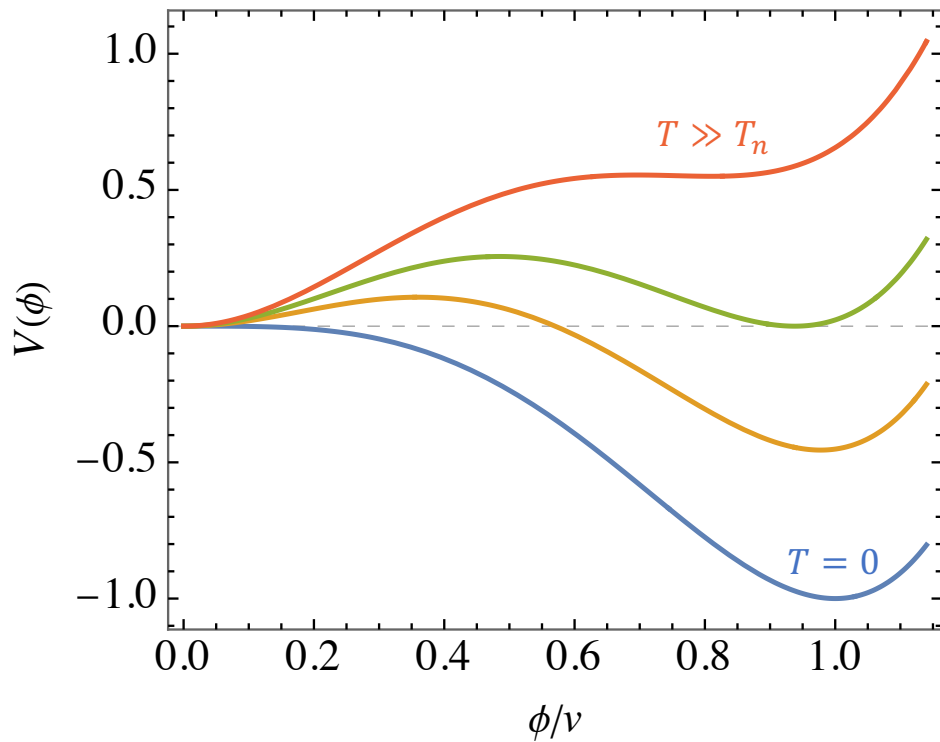
# Strong supercooling

Period of inflation before the transition happens,  $\Delta V > \rho_{\text{rad}}$ .



# Realistic model

$$V(\phi, T) = V_0 + \frac{3g^4}{4\pi^2} \phi^4 \left[ \ln \frac{\phi^2}{v^2} - \frac{1}{2} \right] + \frac{g^2 T^2}{2} \phi^2$$



$$N_{\text{TI}} \approx \ln(1/\alpha)$$

# Realistic model

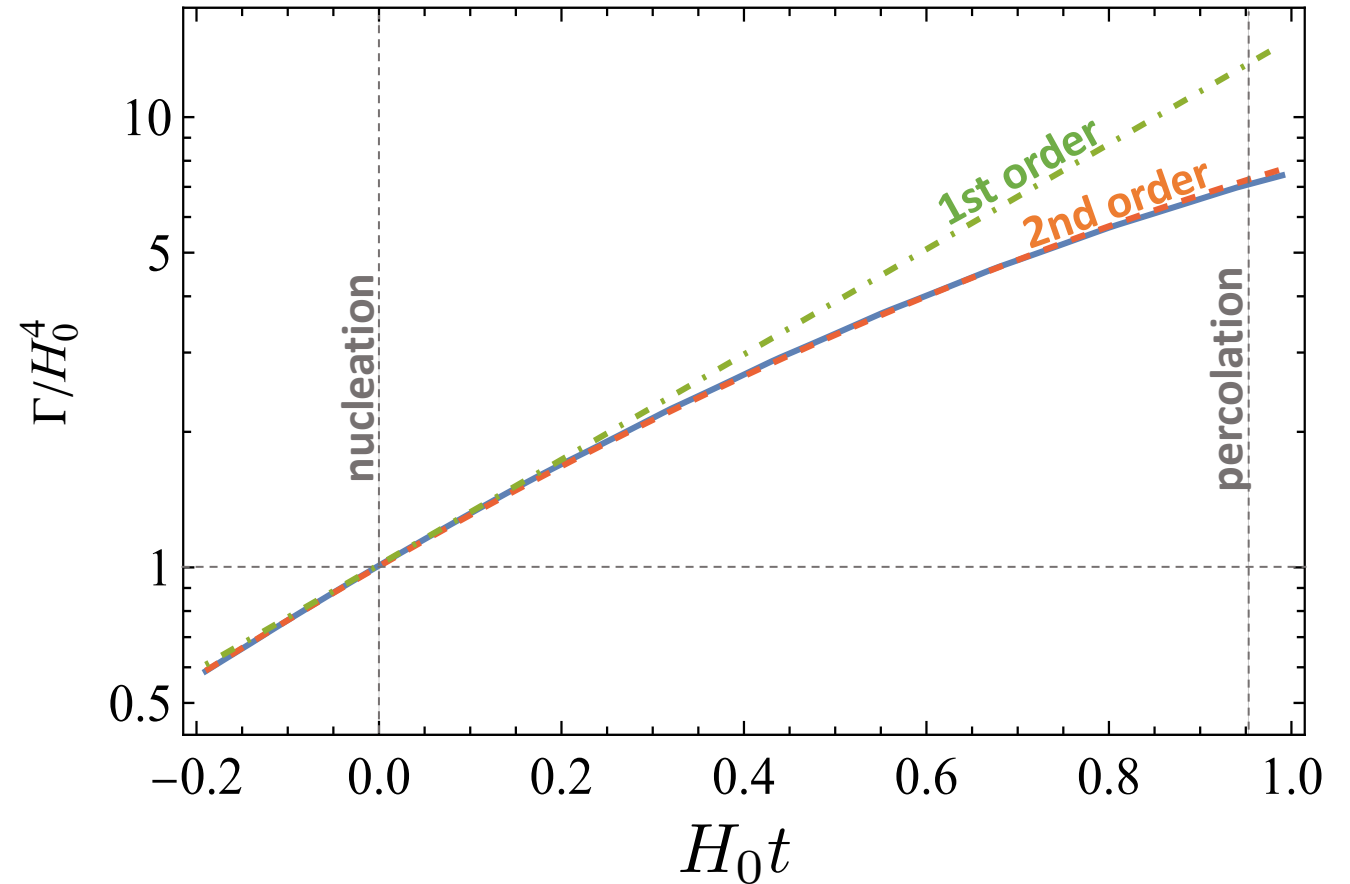
Nucleation rate:

$$\Gamma = \left( \frac{S_3}{2\pi T} \right)^{\frac{3}{2}} T^4 e^{-S_3/T} = \Lambda^4 e^{-S(T)}$$

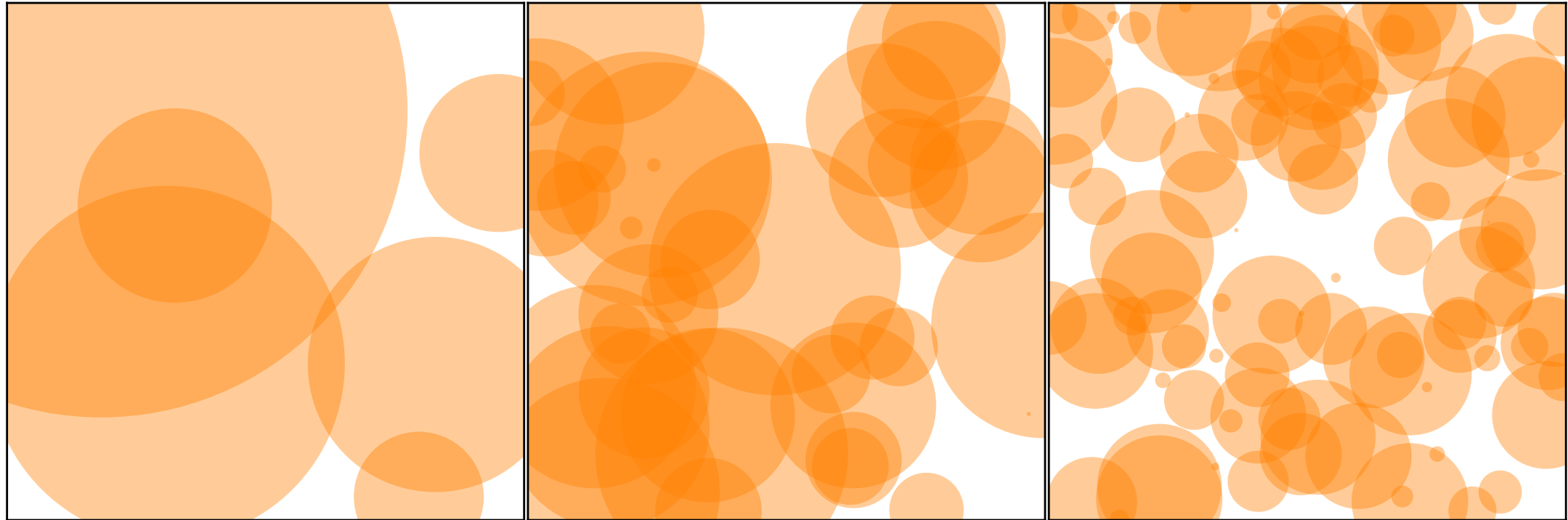
$$S_3 = 4\pi \int_0^\infty dr r^2 \left[ \frac{1}{2} \left( \frac{d\phi}{dr} \right)^2 + V(\phi, T) \right]$$

Approximation:

$$\Gamma \propto \exp \left[ \beta t - \frac{1}{2} \gamma^2 t^2 + \dots \right]$$



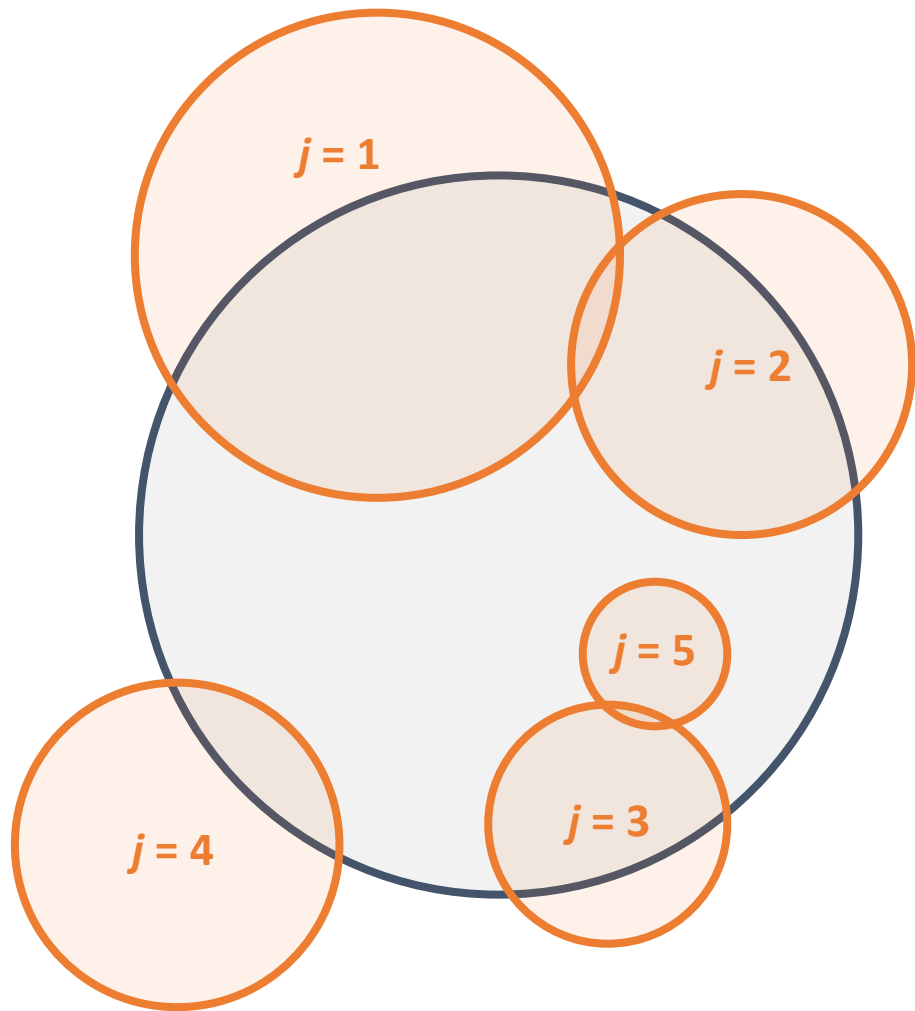
# Bubble nucleation



**slow:**  
few small bubbles  
small  $\beta$  or large  $\gamma$

**fast:**  
many small bubbles  
large  $\beta$  and small  $\gamma$

# Evolution of finite patches

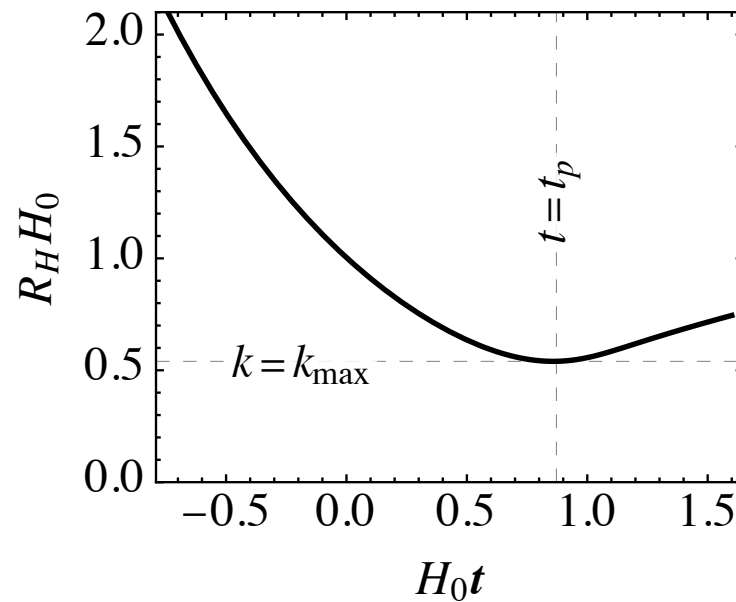


Bubbles convert vacuum energy into radiation:

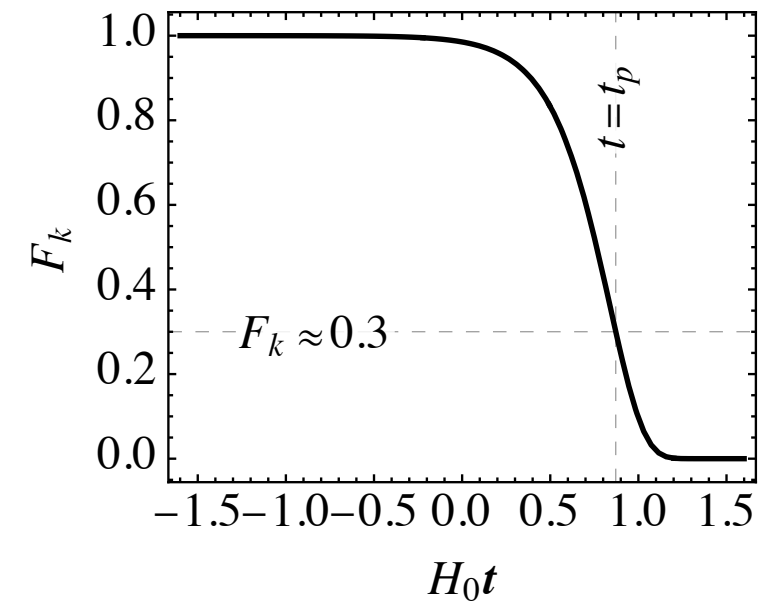
$$\dot{\rho}_r + 4H\rho_r = -\dot{\rho}_v$$

$$\rho_v = F_k(t)\Delta V$$

Comoving Hubble radius:



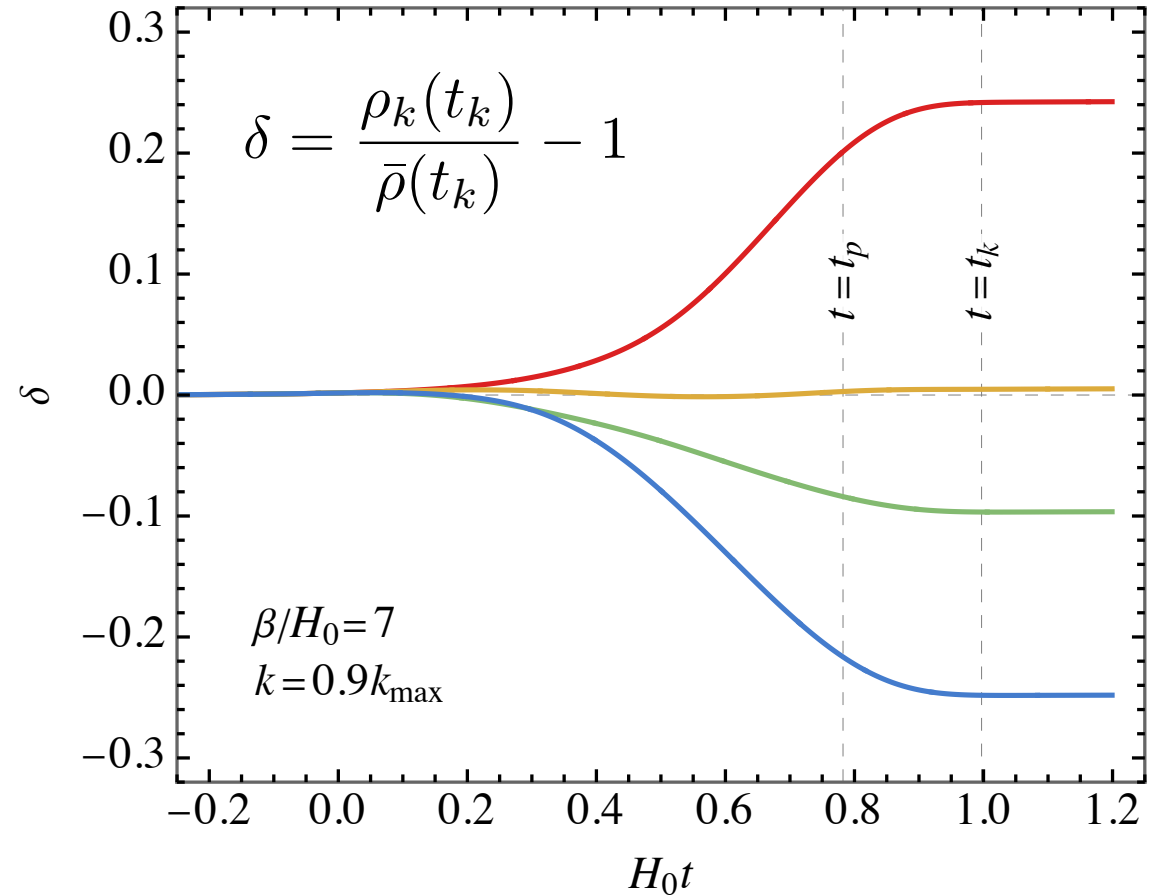
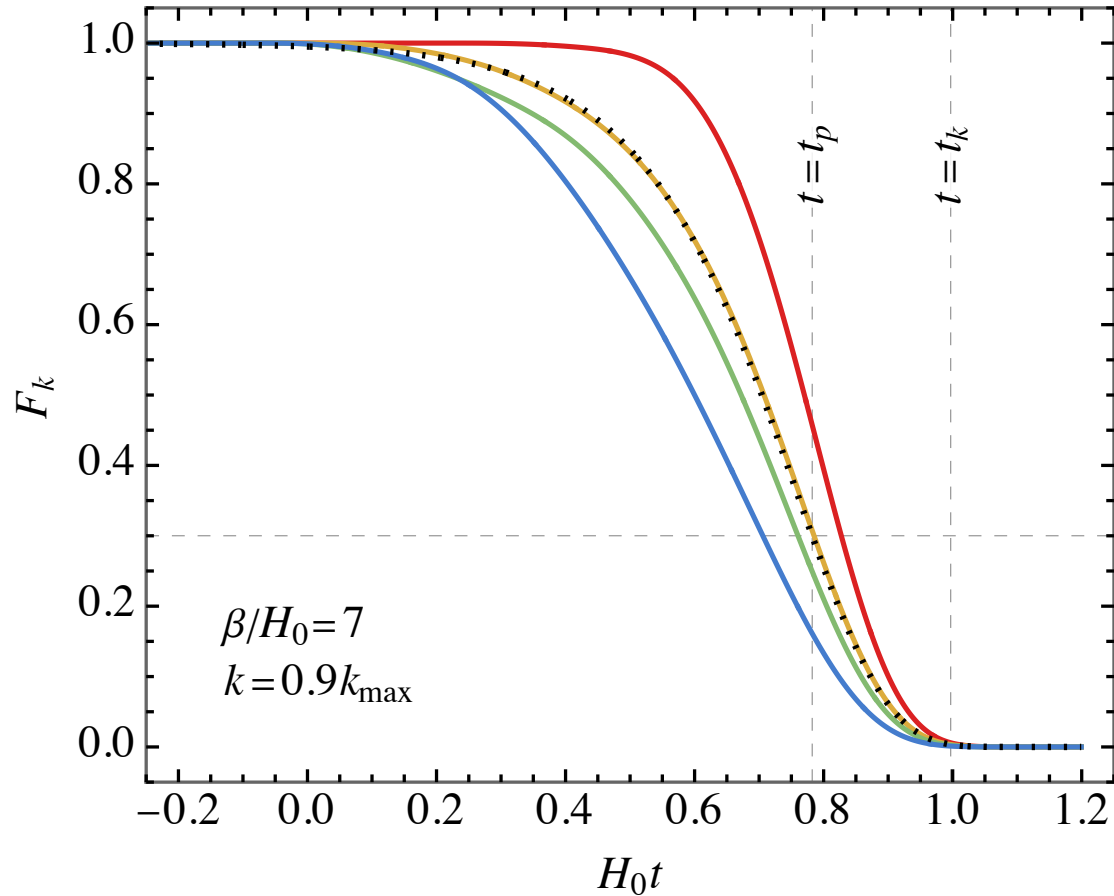
False vacuum fraction:





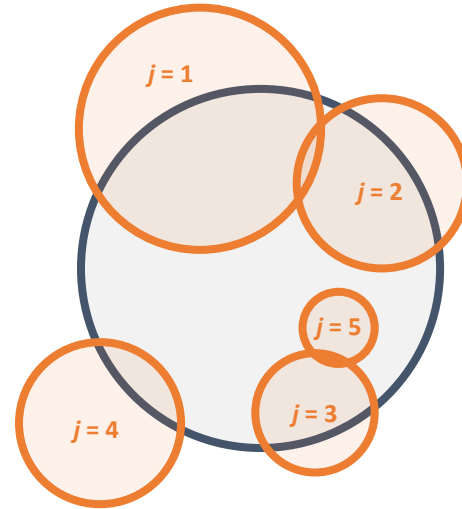
# Evolution of finite patches

Four examples:



# Computation

We want to generate  $\sim 10^6$  realizations.



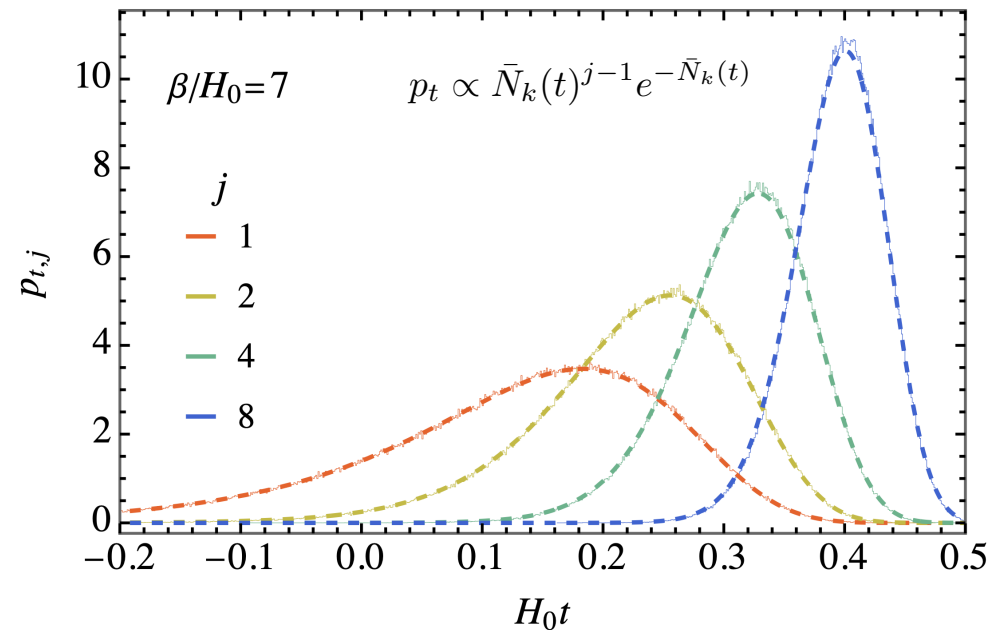
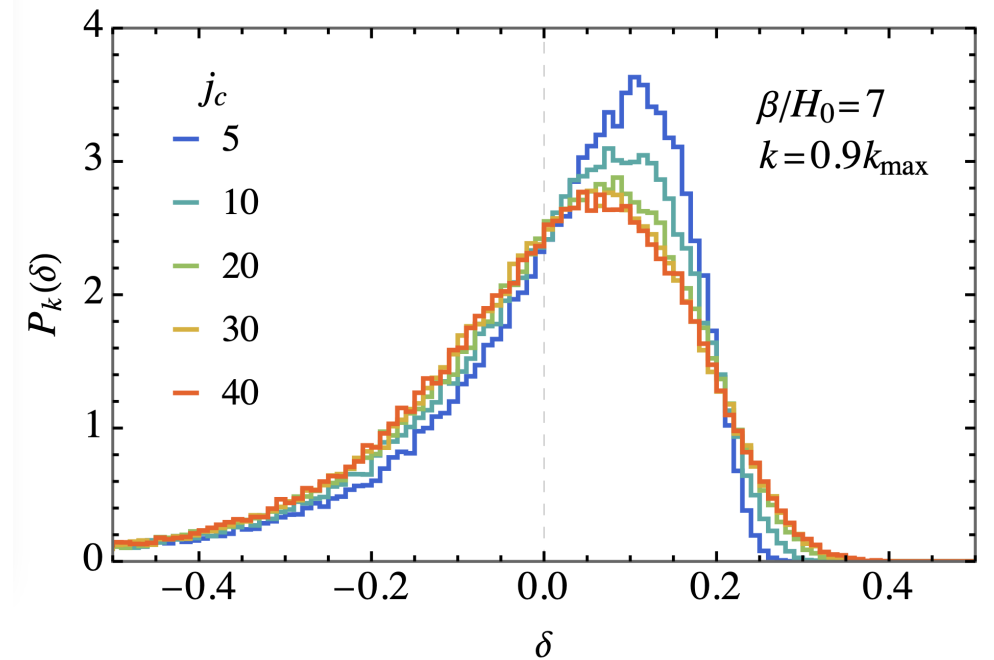
$$F_k(t) = F_k^{(j \leq j_c)}(t) F_k^{(j > j_c)}(t)$$

$$F_k^{(j \leq j_c)}(t) \approx \prod_{j=1}^{j_c} [1 - f(t; t_j, d_j, k)]$$

$$F_k^{(j > j_c)}(t) \approx \exp \left[ -\frac{4\pi}{3} \int dt' \theta(\bar{N}_k(t') - j_c) \Gamma(t') a(t')^3 R(t; t')^3 \right]$$



Code available at  
[github.com/vianvask/deltaPT](https://github.com/vianvask/deltaPT)

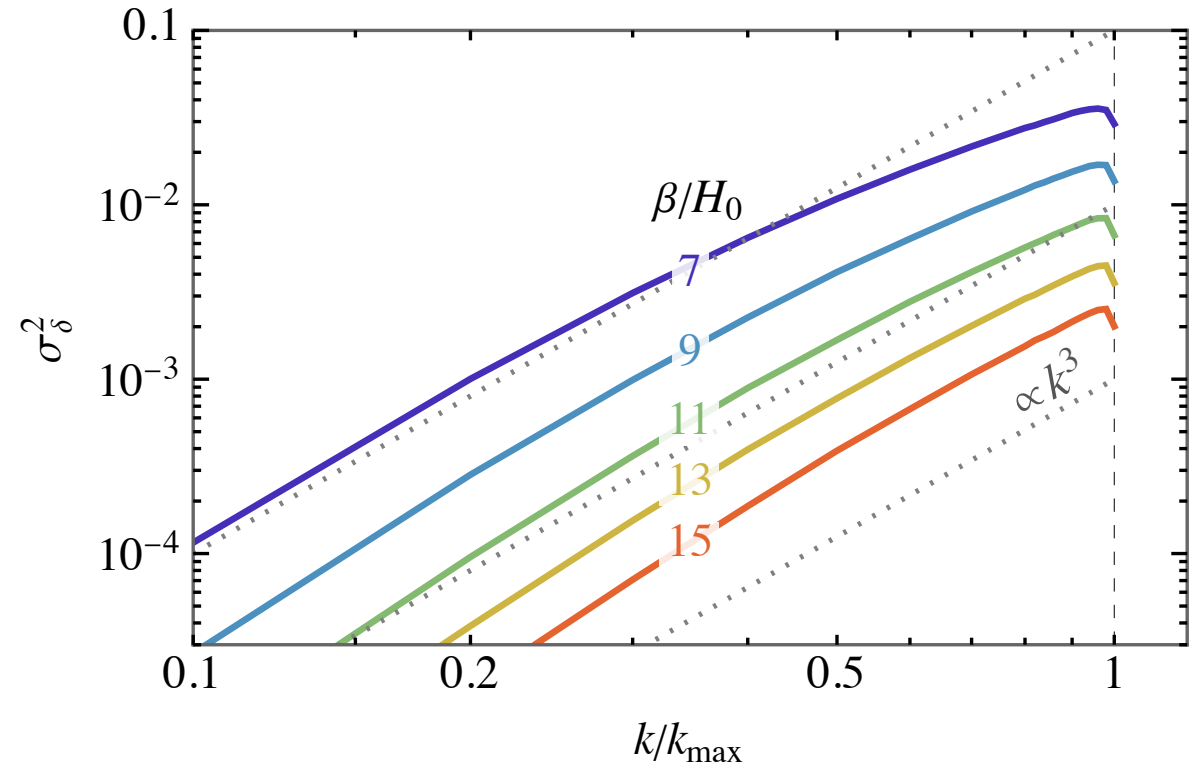
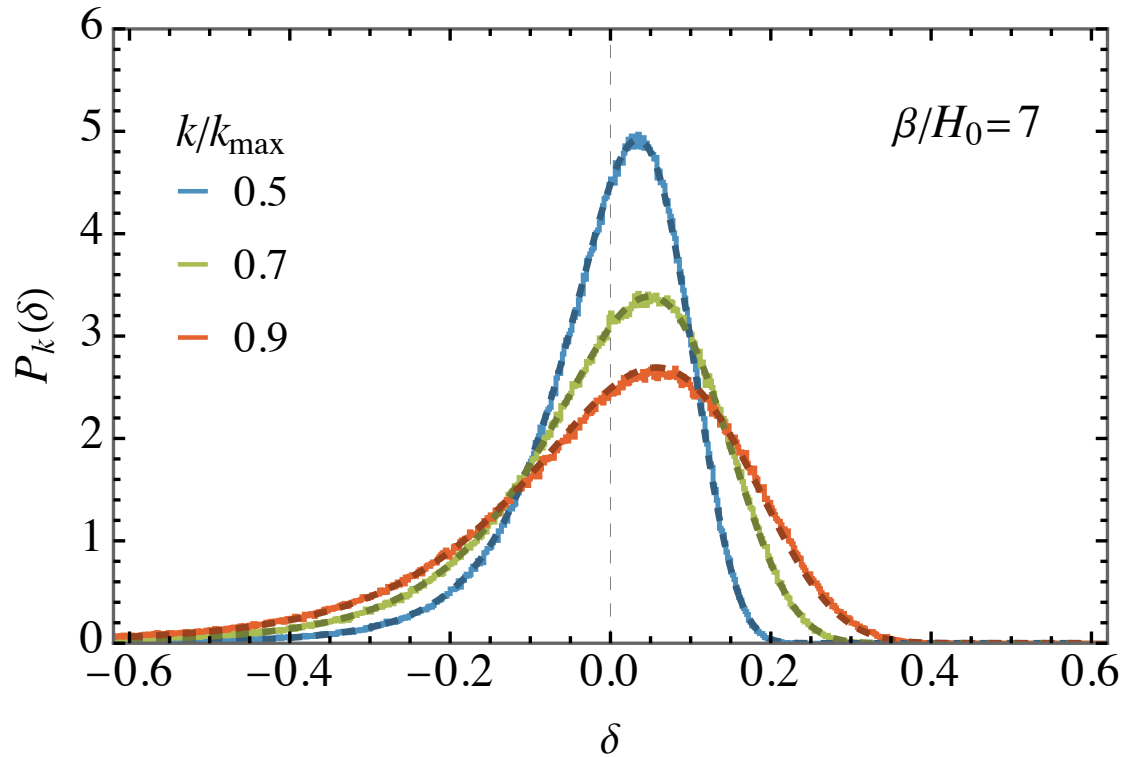


# Distribution of density contrast

Lewicki, Toczek, Vaskonen, 2402.04158

Fitting function:

$$P_k(\delta) \propto \exp \left[ \frac{\epsilon}{2}(\delta - \mu) - \frac{2}{\epsilon^2 \sigma^2} \left( 1 - e^{\frac{\epsilon}{2}(\delta - \mu)} \right)^2 \right]$$



- distribution of the fluctuations has **negative non-Gaussianity**
- small  $\beta/H_0 \Rightarrow$  slow transition  $\Rightarrow$  large variance of  $\delta$

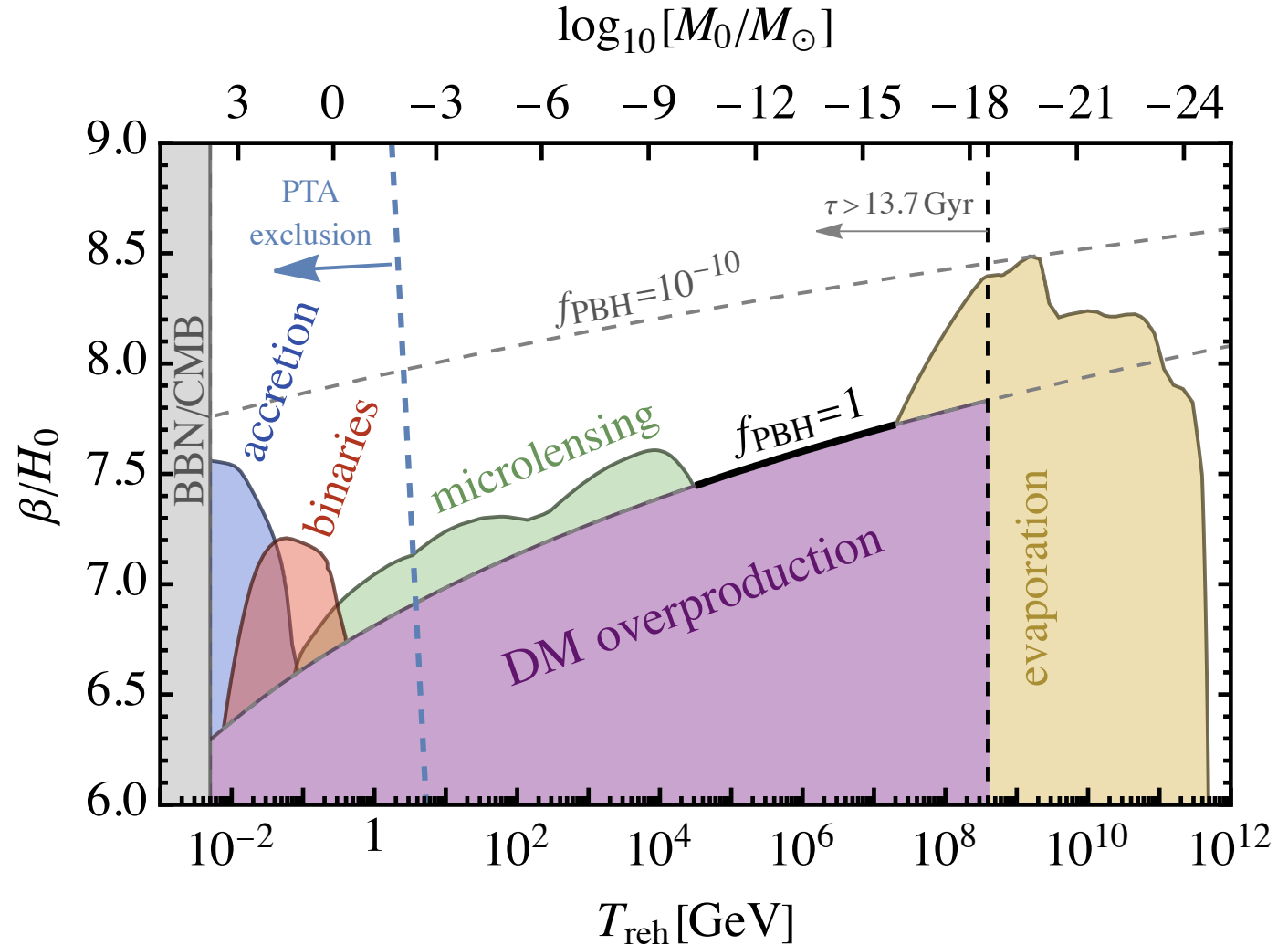
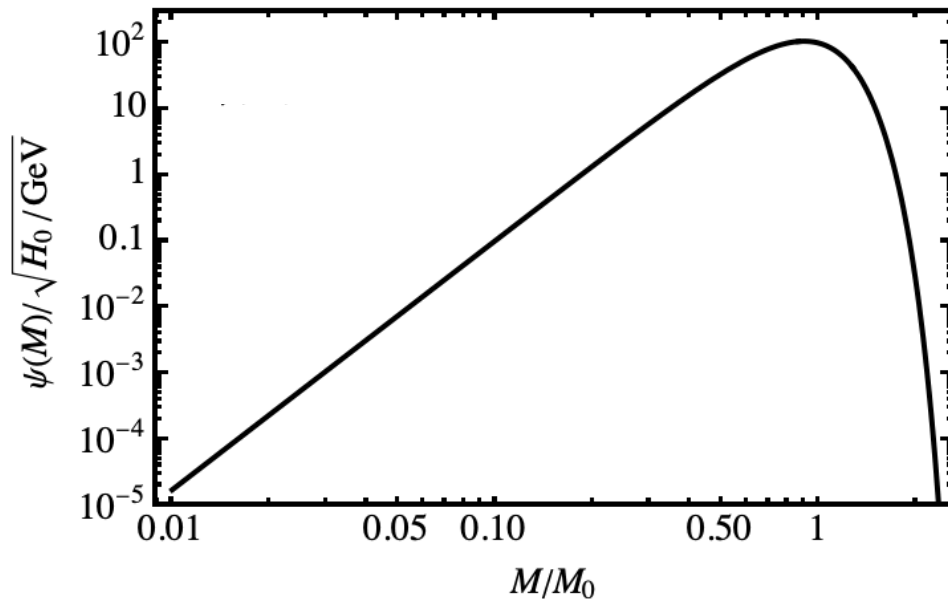
# Primordial black holes

Lewicki, Toczek, Vaskonen, 2402.04158

$$f_{\text{PBH}} \sim \int d \ln k \int_{\delta_c} d\delta P_k(\delta) \dots$$

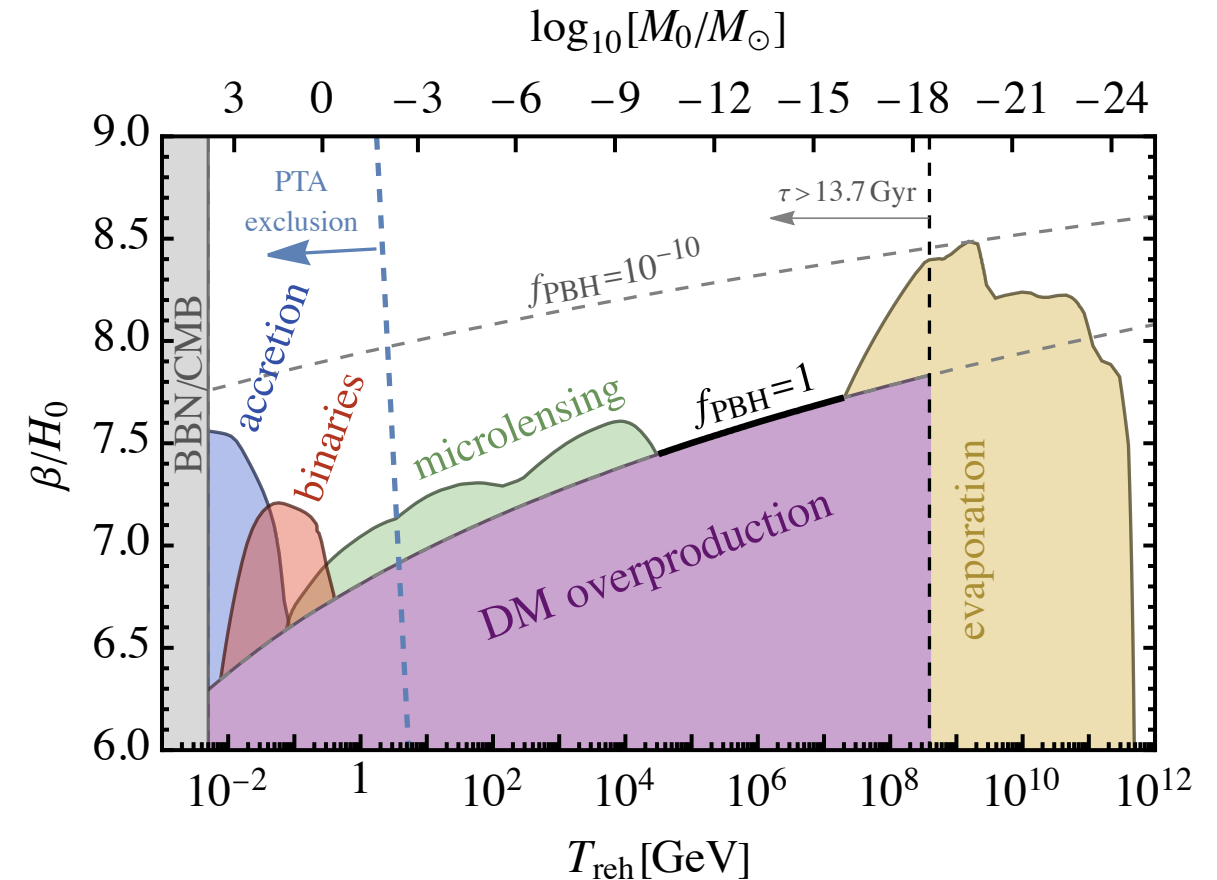
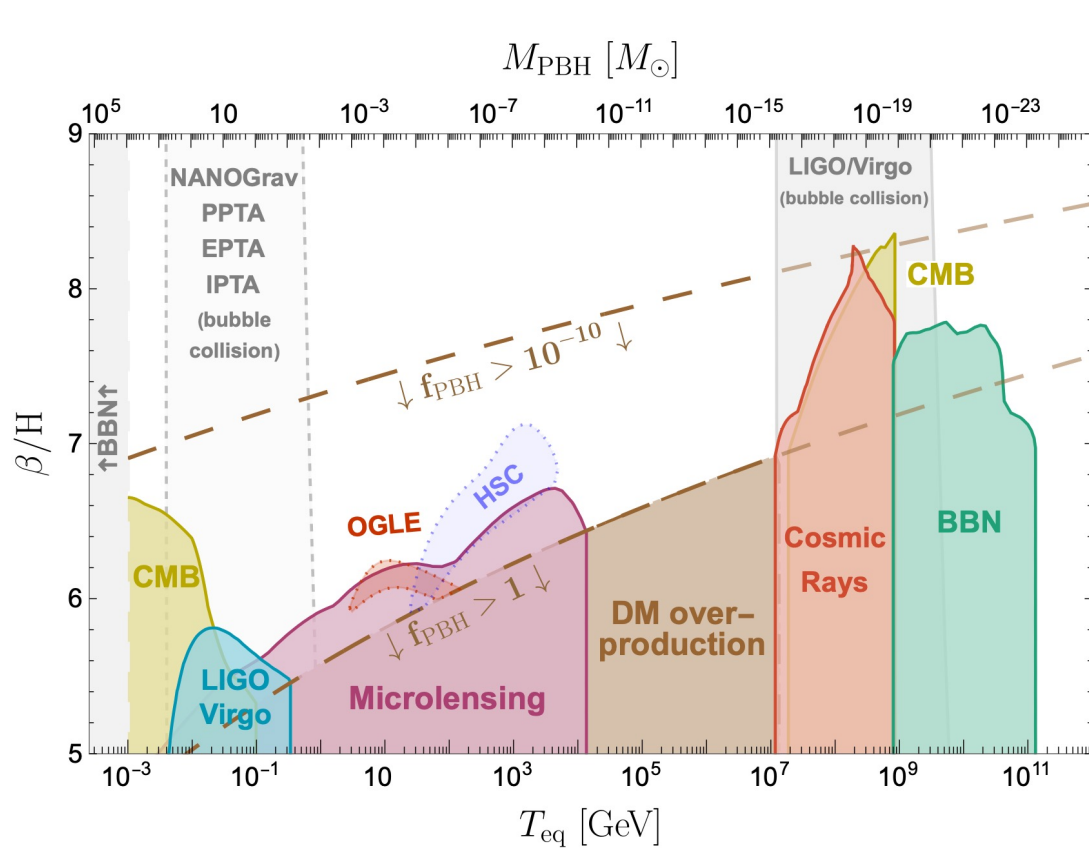
$$M(\delta) = \kappa M_k (\delta - \delta_c)^\gamma$$

$$\gamma = 0.38, \quad \kappa = 4.2, \quad \delta_c = 0.55$$



# Comparison with Gouttenoire & Volansky 2305.04942

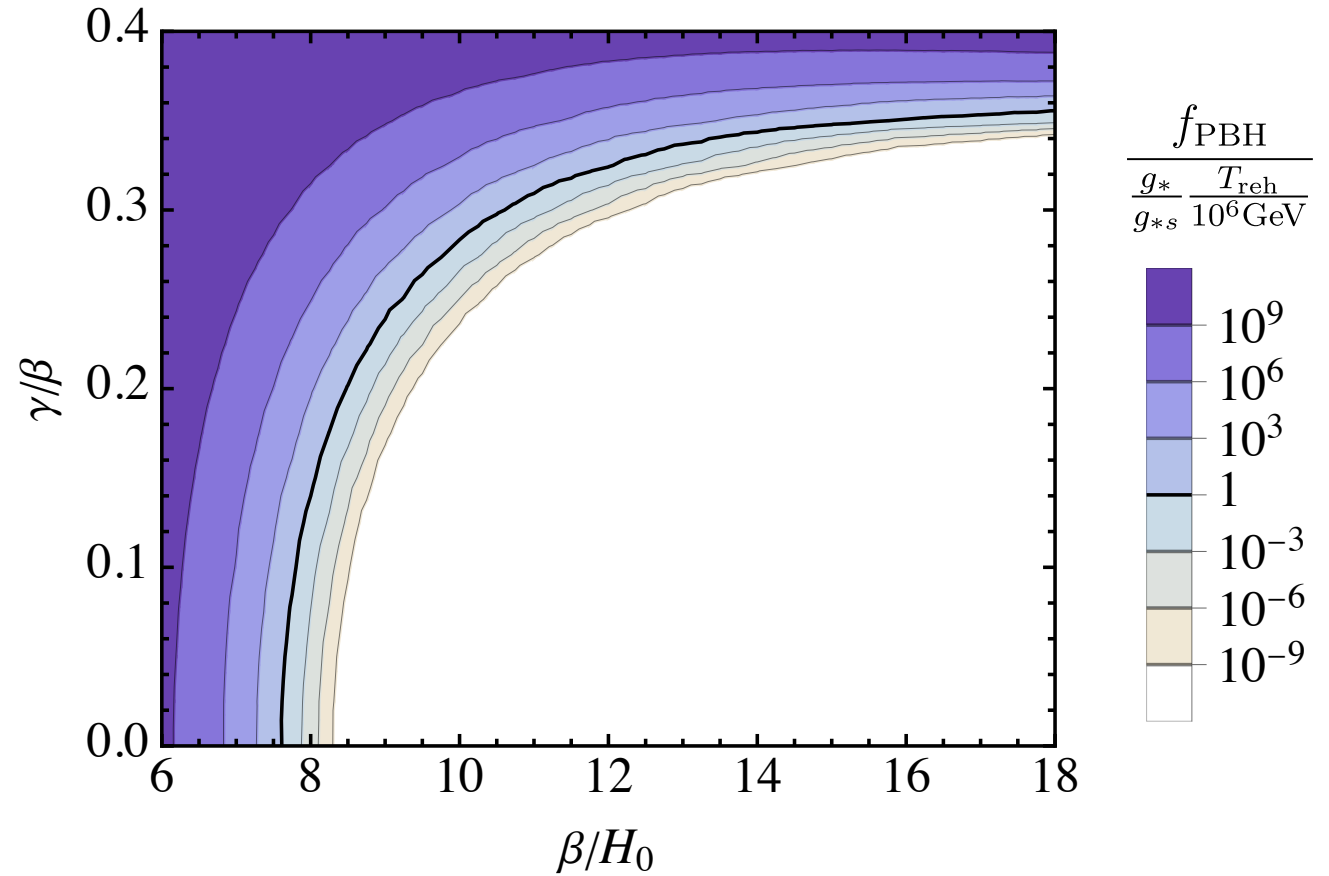
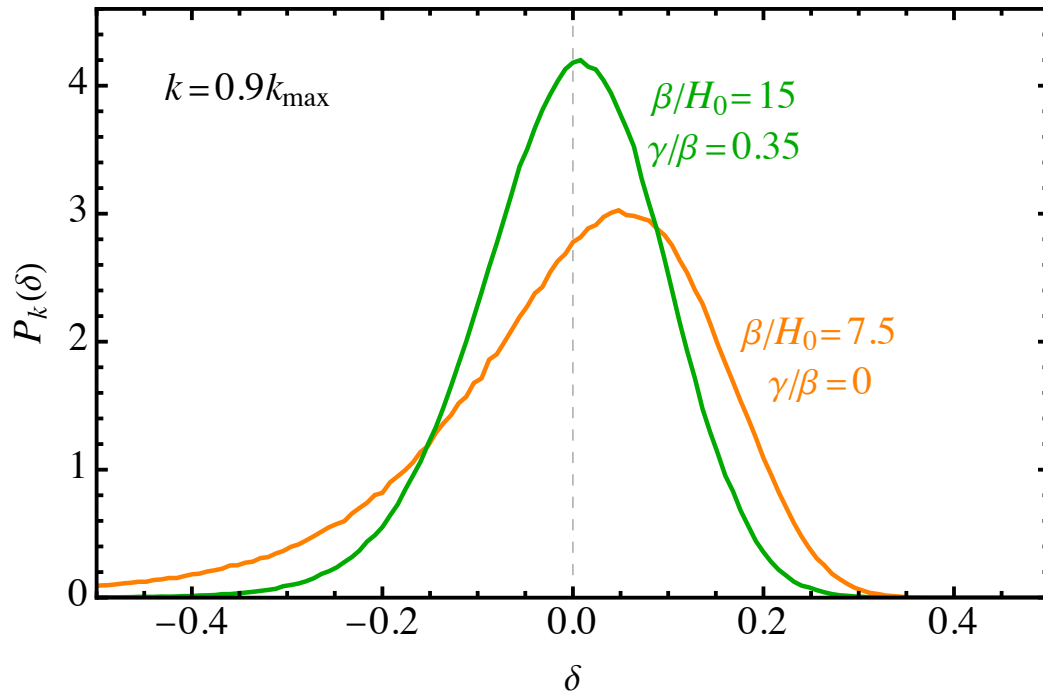
1. We account for fluctuations in the nucleation times of **several bubbles**, GV23 only in the time when nucleation started.
2. We compute the **distribution of the fluctuations** at  $k < k_{\max}$ , GV23 only at the scale of the largest fluctuations.



# 2nd order term in the nucleation rate

Lewicki, Toczek, Vaskonen, in preparation.

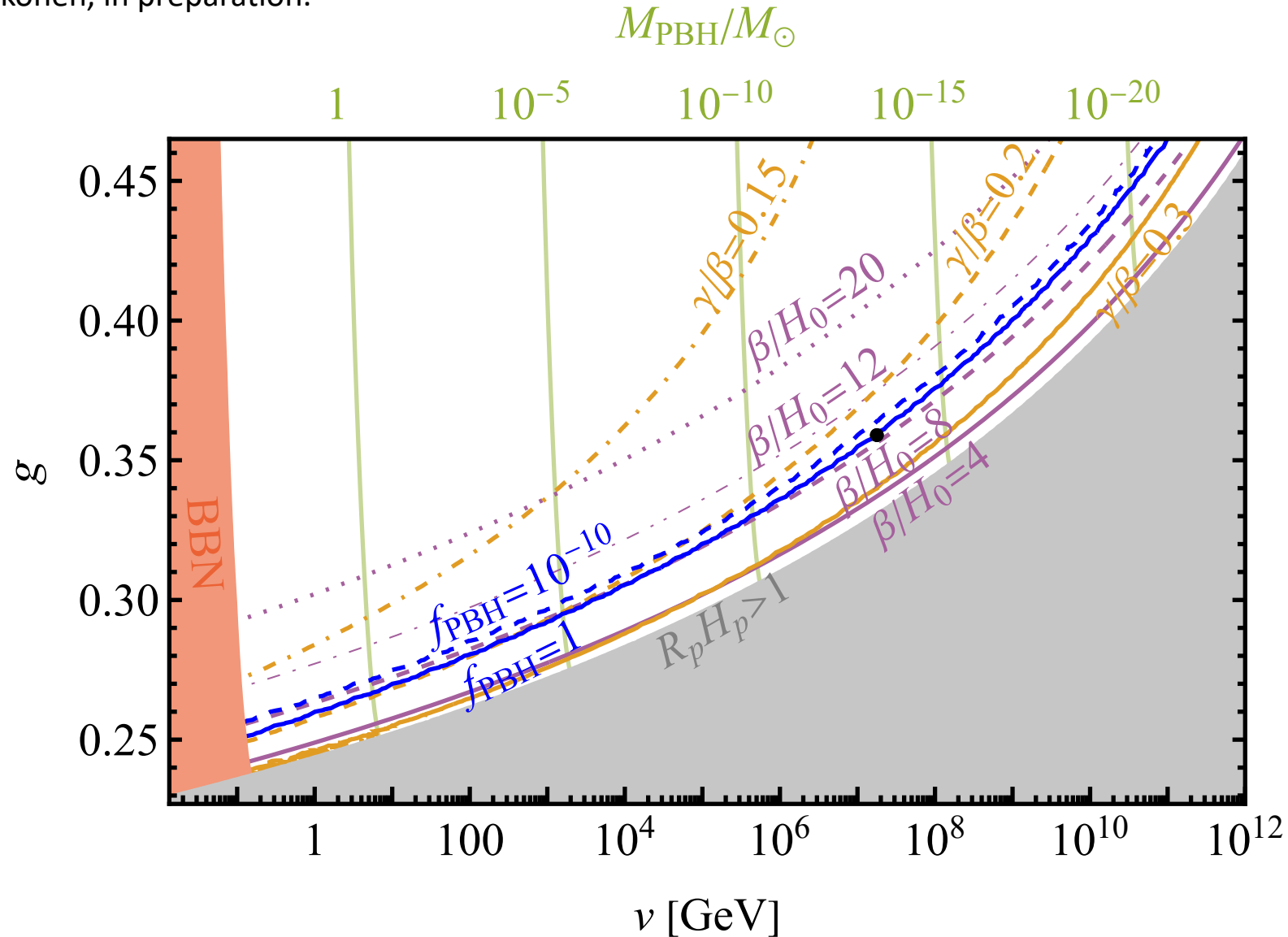
$$\Gamma \propto \exp \left[ \beta t - \frac{1}{2} \gamma^2 t^2 + \dots \right]$$



# Results in the CSI model

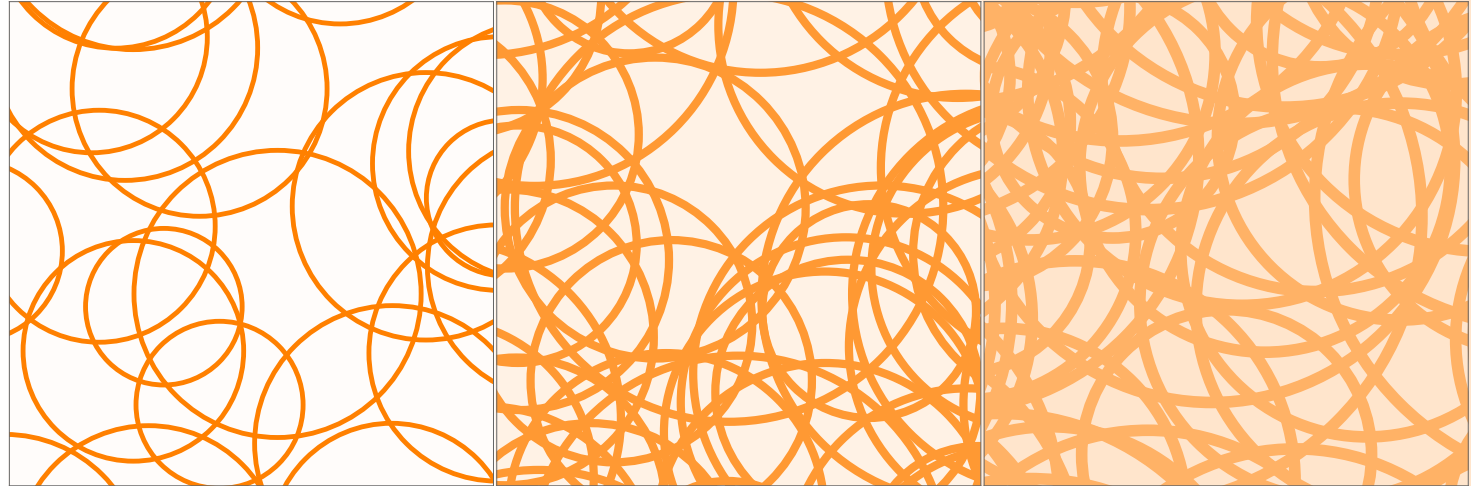
$$V(\phi, T) = V_0 + \frac{3g^4}{4\pi^2} \phi^4 \left[ \ln \frac{\phi^2}{v^2} - \frac{1}{2} \right] + \frac{g^2 T^2}{2} \phi^2$$

Lewicki, Toczek, Vaskonen, in preparation.



# Suppression

Energy is not distributed smoothly right after the phase transition.

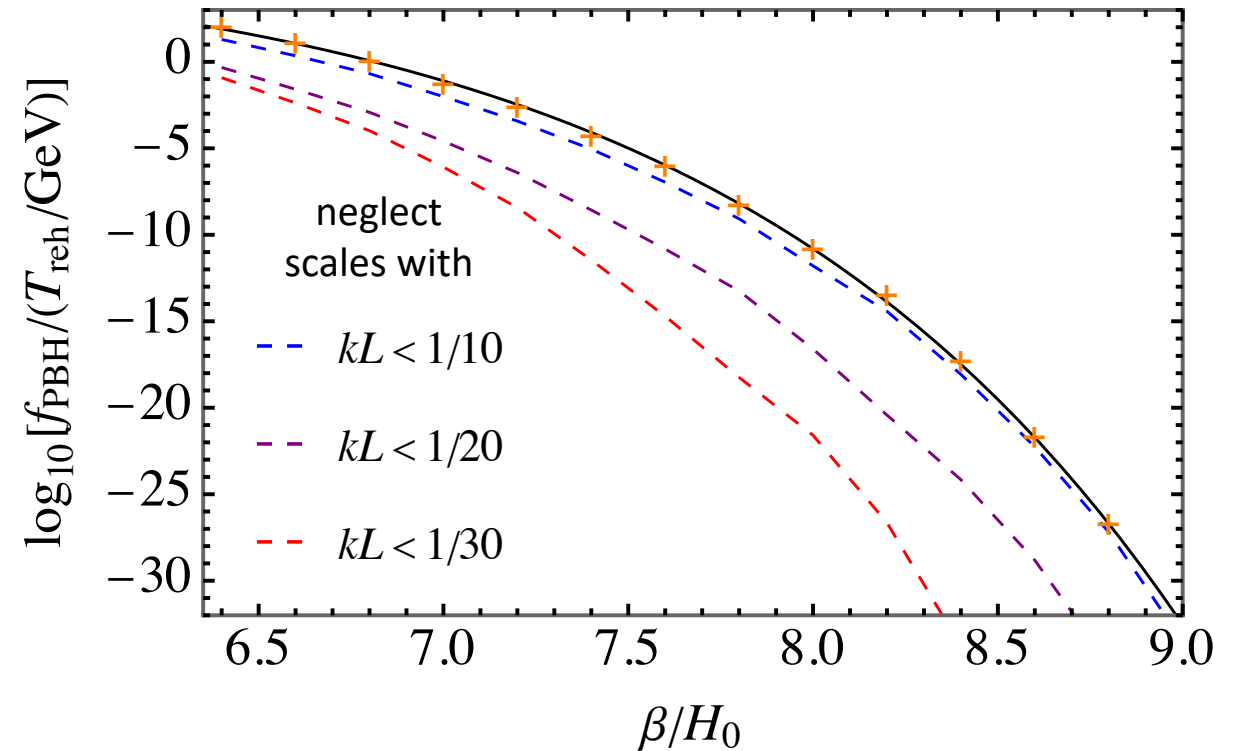


The area of bubble surfaces inside a given volume increases.

⇒ The length scale of the subhorizon fluctuations

$$L = 3V_k / S_{\text{tot}}$$

decreases.





# Gravitational waves

Lewicki, Toczek, Vaskonen, 2402.04158

## 1. Primary GWs from bubble collisions:

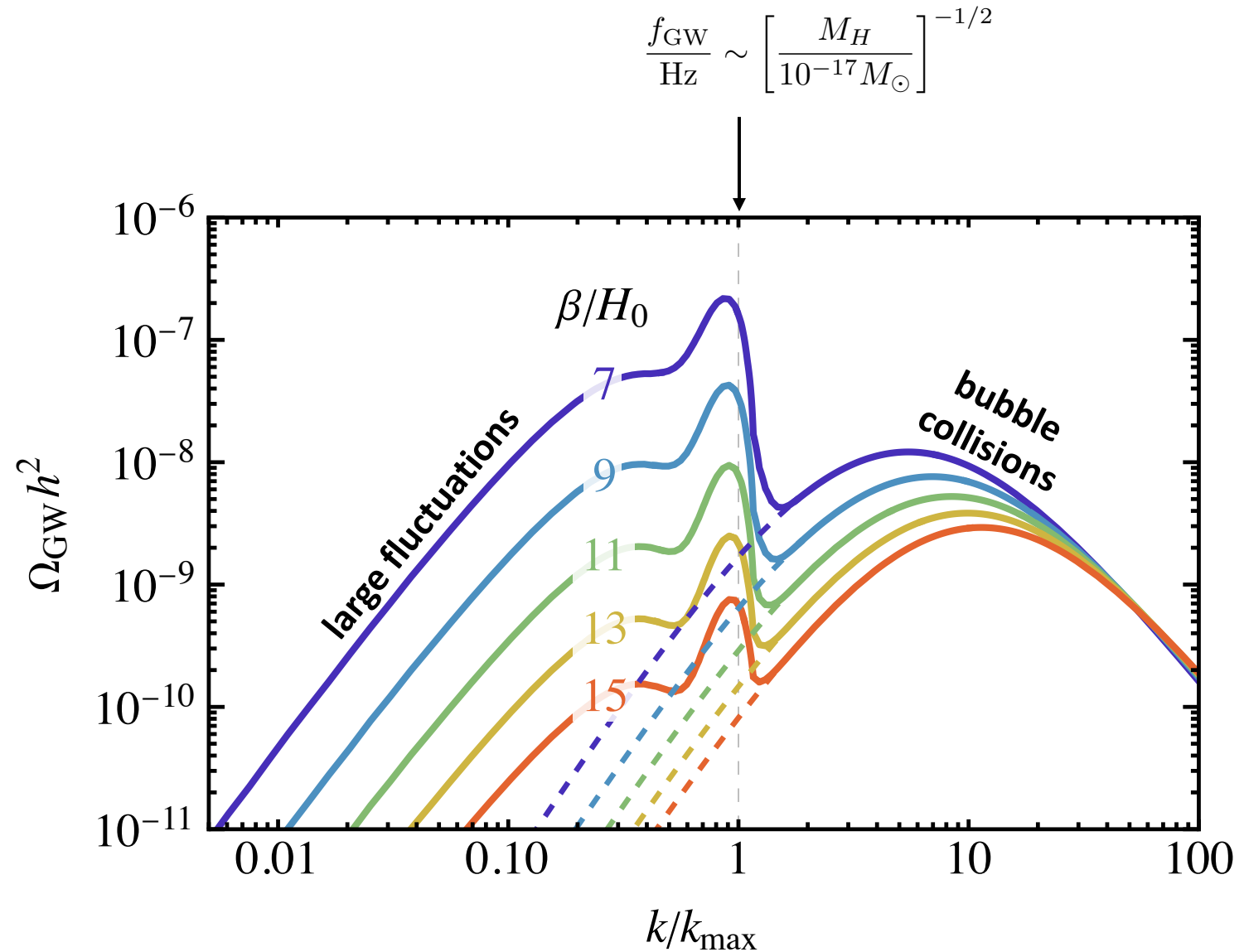
$$k_{\text{peak}} \approx k_{\text{max}} \beta / H_0$$

$$\Omega_{\text{PGW}} h^2 \propto \left( \frac{\beta}{H_0} \right)^{-2}$$

## 2. Secondary GWs induced by the large fluctuations:

$$k_{\text{peak}} \approx k_{\text{max}}$$

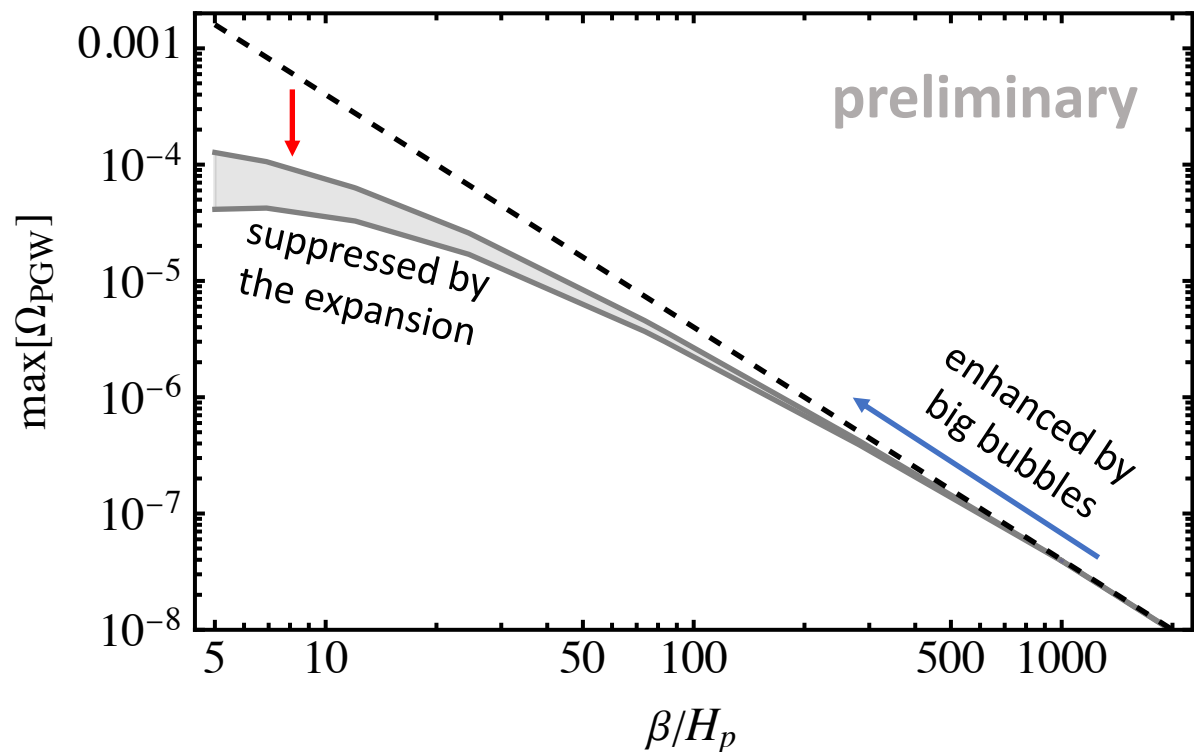
$$\Omega_{\text{SGW}} h^2 \propto e^{-\beta/H_0}$$



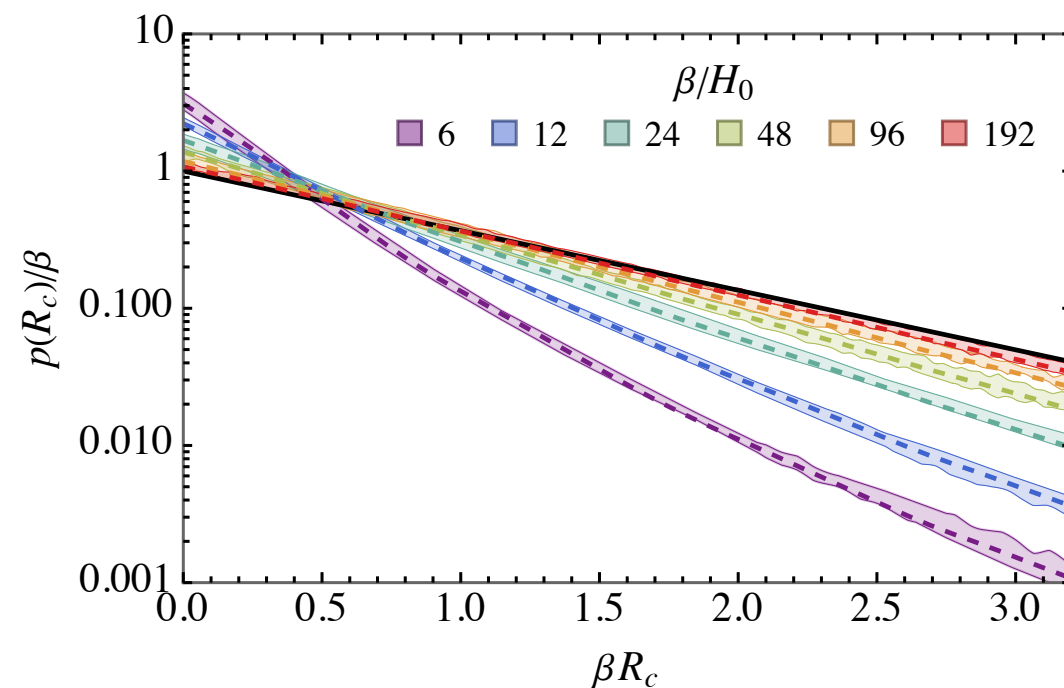
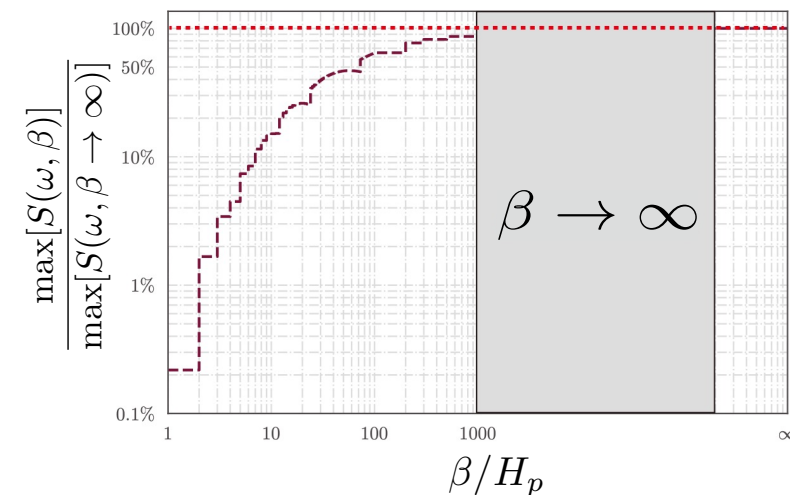
# Suppression of the primary GWs

$$\Omega_{\text{PGW}} \propto \left[ \frac{H_p}{\beta} \right]^2 S(\omega, \beta)$$

Suppression for strongly supercooled transitions  
[Lewicki, Vaskonen, in preparation]:



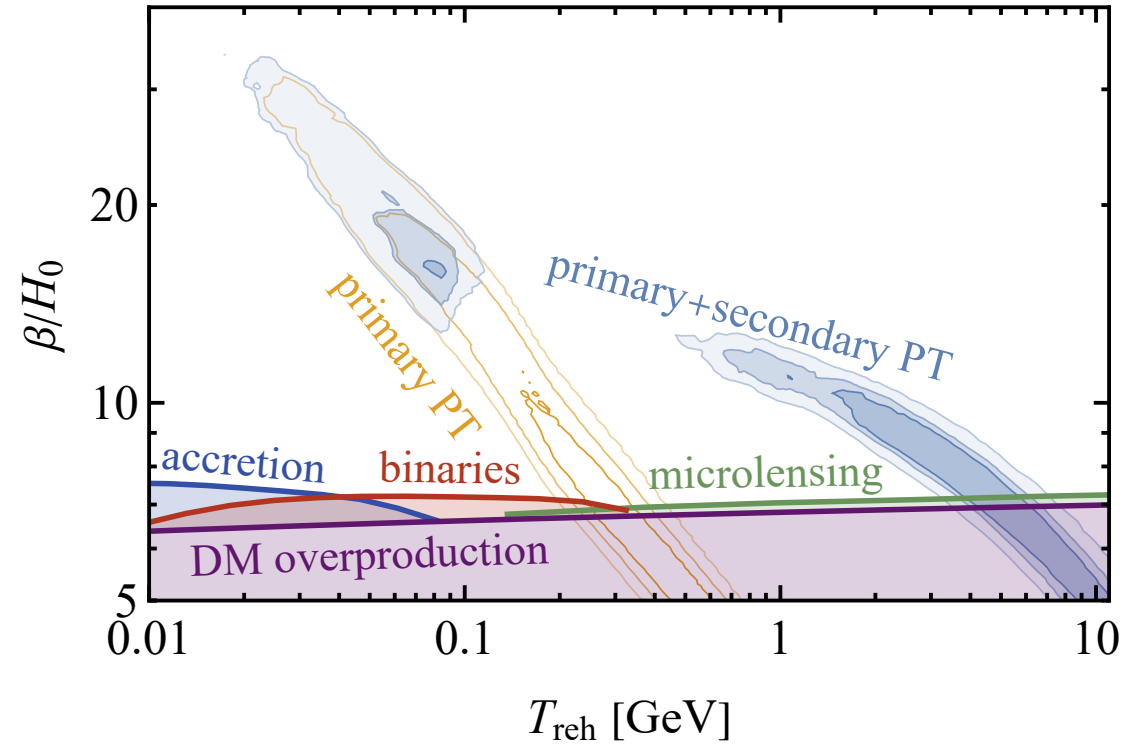
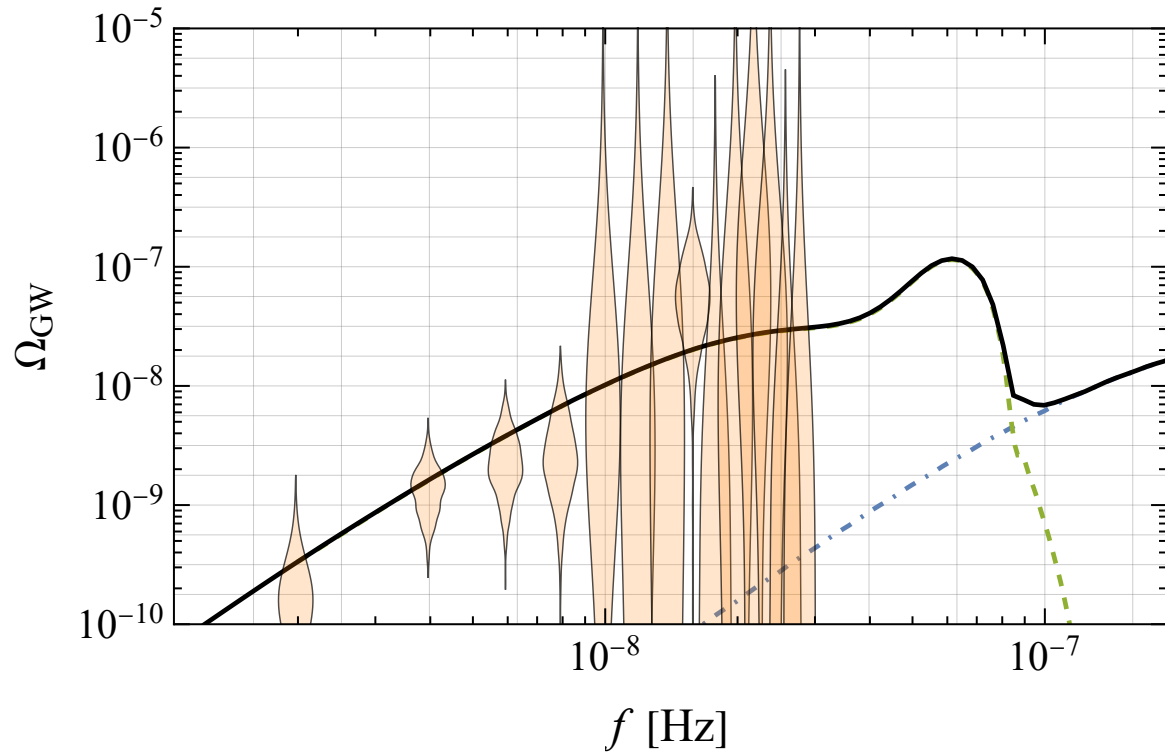
Suppression factor in RD  
[Zhong, Gong, Qiu, JHEP 02 (2022)]:



# PTA fit

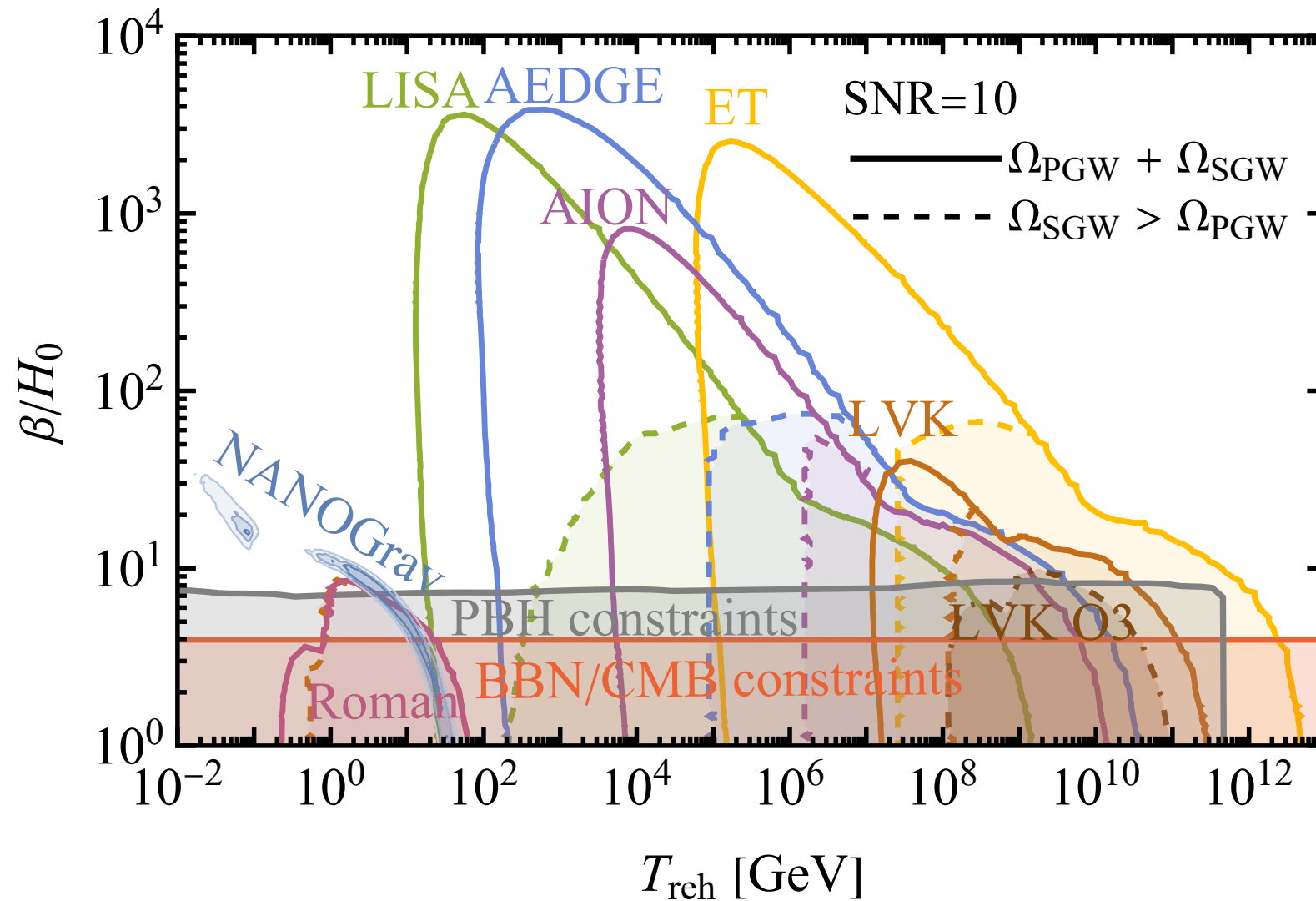
Lewicki, Toczek, Vaskonen, 2402.04158

**Negative non-Gaussianity**  $\Rightarrow$  PTA fit is not in tension with PBH production



# Prospects

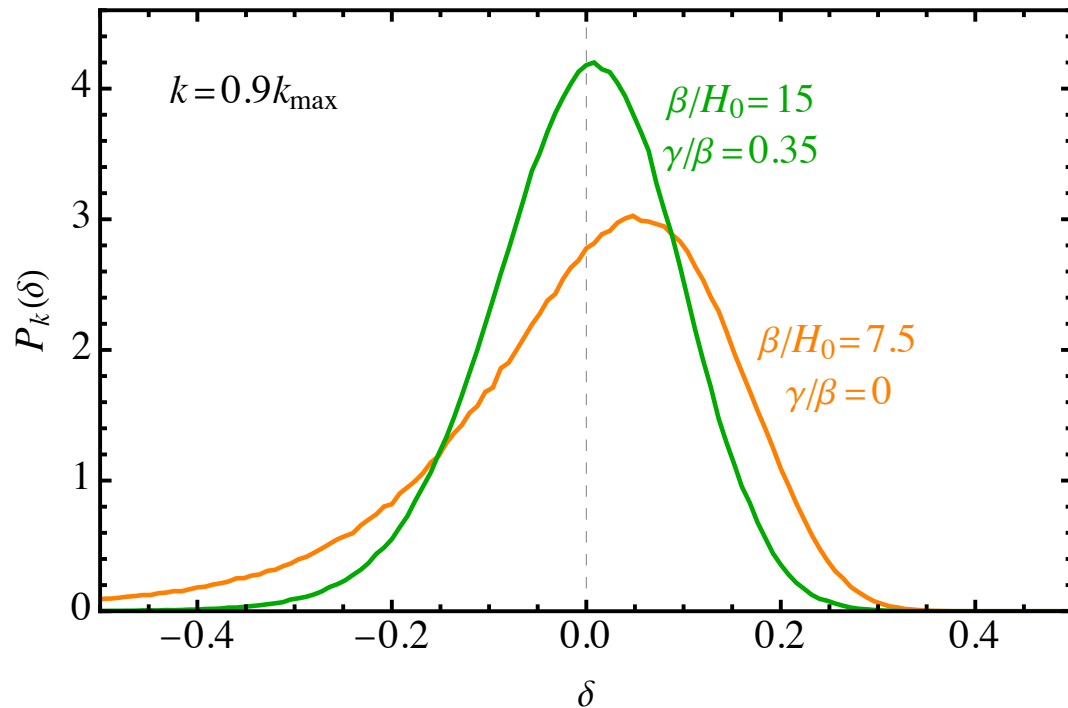
Lewicki, Toczek, Vaskonen, 2402.04158



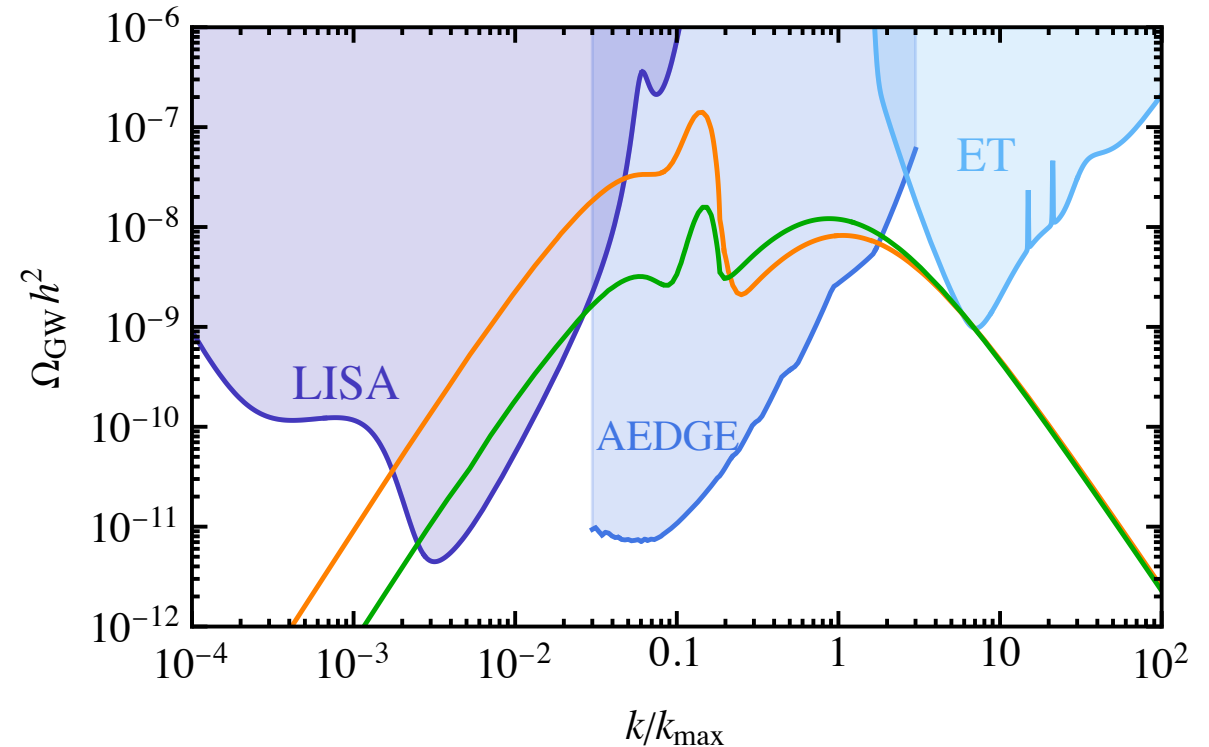
# 2nd order term in the nucleation rate

Lewicki, Toczek, Vaskonen, in preparation.

$$\Gamma \propto \exp \left[ \beta t - \frac{1}{2} \gamma^2 t^2 + \dots \right]$$



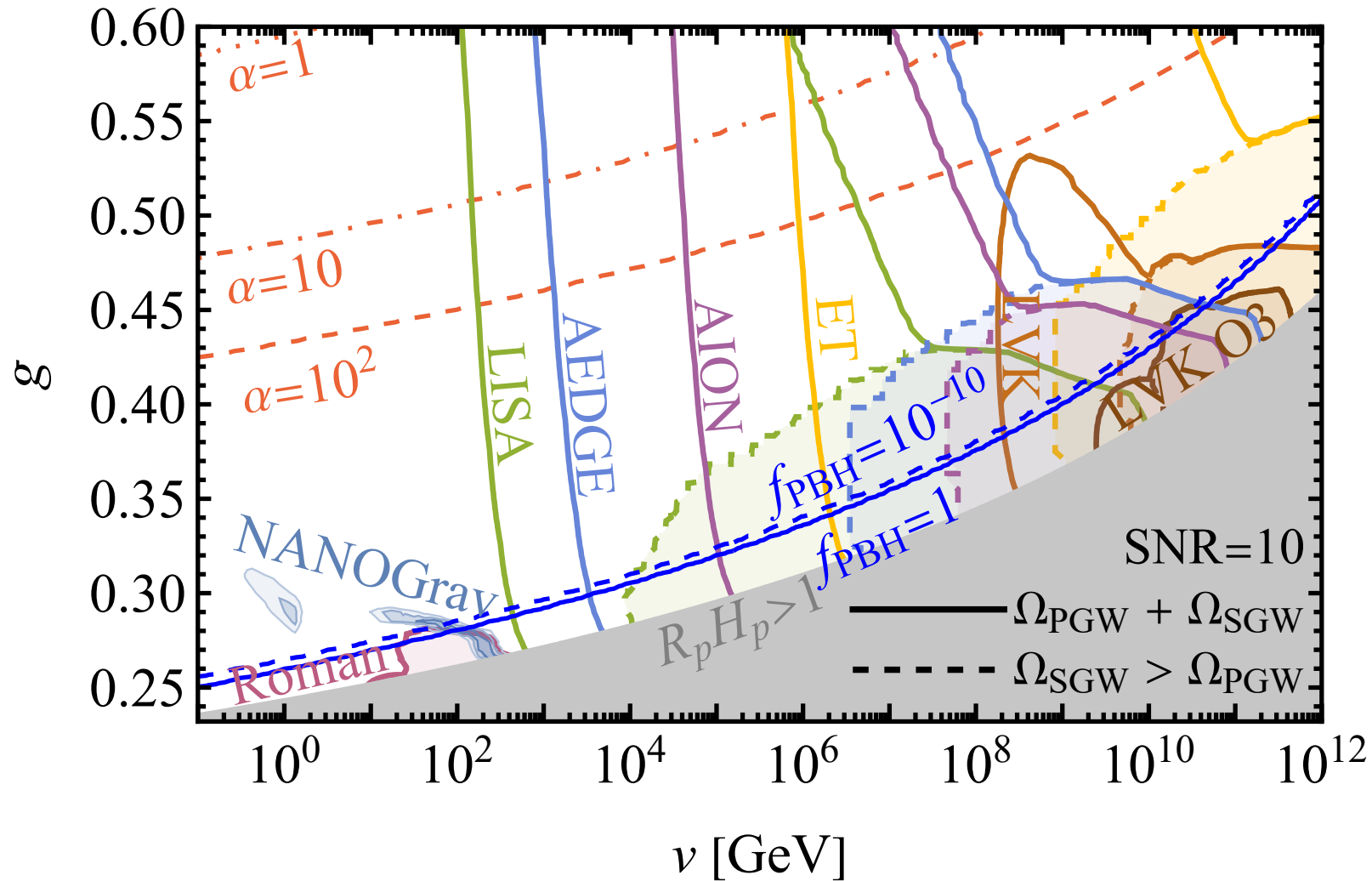
Same PBH abundance,  
different GW spectra:



# Results in the CSI model

Lewicki, Toczek, Vaskonen, in preparation.

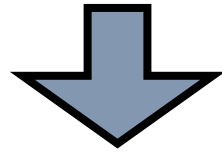
$$V(\phi, T) = V_0 + \frac{3g^4}{4\pi^2} \phi^4 \left[ \ln \frac{\phi^2}{v^2} - \frac{1}{2} \right] + \frac{g^2 T^2}{2} \phi^2$$



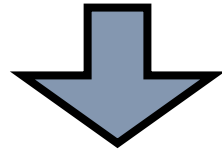
# Summary

## Slow and supercooled transition

e.g. in CSI models



## Large density fluctuations



## PBHs and scalar induced GWs

- Negative non-Gaussianity suppresses PBH formation.
- 2nd order term in the nucleation rate is relevant.
- Primary GWs are suppressed for slow transitions.

