New statistical analysis techniques in RooFit

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▶ hypothesis mass m_0 ▶ measured data with mass m_i

$$\mathcal{L}(m_0) = \prod_i p(m_i | m_0)$$

• Likelihood \mathcal{L}

- probability density function $\, p \,$





$$\mathcal{L}_{ratio}(m_0) = \prod_i \frac{p(m_i|m_0)}{p_{ref}(m_i)}$$

$$p(m_i|m_0) = \int dz_d \int dz_s \int dz_p p(m_i, z_d, z_s, z_p|m_0)$$

- Integrals cannot evaluated directly
 - Detector simulations take time

Morphing = Way of interpolating between different template Histograms



3×10²

2×10³ 3×10³

m_{ee} [GeV]

6



Morphing



- based on a linear combination of input templates (<u>Moment Morphing</u>)
- Morphing was already implemented in RooFit, but no documentation
 Lots of forum questions
- ► <u>Tutorial</u>

SBI = Use of ML to learn the likelihood function



- train parameterized classifier to
 - discriminate between samples $m_i \sim p(m_i|m_0)$ $m_i \sim p_{ref}(m_i)$
 - transform output of classifier ŝ(m_i|m₀) to estimator of likelihood ratio function by

$$\mathcal{L}_{ratio}(m_0) = \prod_i \frac{1 - \hat{s}(m_i | m_0)}{\hat{s}(m_i | m_0)}$$



Tutorial



Learned vs analytical likelihhood function



- compared to morphing, really easy to implement (don't care of binnings, ...)
- higher accuracy due to no binning effects
- reduction of necessary data samples i.e. detector simulations, especially in higher dimensions
- "trivial" sampling of MC Data (cont. sampling possible, less interpolation error)
- but: beware of overtraining
- no sophisticated method to deal with uncertainties



Mixture Models

• in some cases, the pdf is a sum of different pdfs $p(m|\mu) = \sum_{c} w_{c}(\mu)p_{c}(m|\mu)$

$$\frac{p(m|\mu=0)}{p(m|\mu)} = \frac{\sum_{c} w_{c}(0) p_{c}(m|0)}{\sum_{c'} w_{c'}(\mu) p_{c'}(m|\mu)}$$
$$= \sum_{c} \left[\sum_{c'} \frac{w_{c'}(\mu)}{w_{c}(0)} \frac{p_{c'}}{p_{c}} \right]^{-1}$$
$$= \sum_{c} \left[\sum_{c'} \frac{w_{c'}(\mu)}{w_{c}(0)} \frac{p_{c'}(\hat{s}_{c,c'})}{p_{c}(\hat{s}_{c,c'})} \right]^{-1}$$

 μ = signal strength $c, c' \in \{zz, higgs\}$

- Allows the classifier to focus on simpler subproblems (unparameterized classifier) (<u>Mixture models</u>)
- ► <u>Tutorial</u>



Mixture Models

NLL



Conclusion



- First RooFit tutorial based on open data, directly connecting RDF tutorial
- Generalized RooFit interface for wrapping python functions to arbitrary number of inputs
- Total: rf615, rf616, rf617, rf618

Next Steps



- Integration of the mixture model formular as a RooFit object
- improve support inside RooFit for external likelihoods (correct plotting, ...)
- Inverse functionality: pythonic wrapper to use RooFit functions in other scientific python code
- Prepare CHEP conference presentation

Thank you! In particular Jonas for your guidance and patience

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