

## The Concept of Mass

- Mass is one of the most fundamental concepts in physics

When a new particle is discovered (e.g. the elusive 'Higgs boson' at the LHC), the first question a particle physicist will ask is '**What is its mass?**'

### Classical physics (vac)

$$T = \frac{1}{2}mv^2 \Rightarrow m = \frac{2T}{v^2}$$

$$p = mv \Rightarrow m = p/v$$

$$T = \frac{p^2}{2m} \Rightarrow m = \frac{p^2}{2T}$$

• Any 2 of  $T, p, v$  gives  $m$ .

• Same true in relativity, but need generalized formulae that take 'speed limit'  $c$  into account.

## Einstein's equation

$$E_0 = mc^2$$

$$E = mc^2$$

$$E_0 = m_0c^2$$

$$E = m_0c^2$$

where  $c$  = velocity of light

$E$  = total energy of free body

$E_0$  = rest energy of free body

$m_0$  = rest mass

$m$  = mass

Q1 Which equation most rationally follows from special relativity and expresses one of its main consequences and predictions?

Q2 Which was first written by Einstein and was considered by him to be a consequence of special relativity?

"This choice is caused by confusing terminology widely used in popular science literature and many textbooks. According to this terminology, the body at rest has a "proper mass" or "rest mass"  $m_0$ , whereas a body moving with velocity  $v$  has a "relativistic mass" or "mass"  $m$  given by

$$m = \frac{m_0}{\sqrt{1 - v^2/c^2}}$$

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... this terminology had some historical justification at the start of our century, but it has no justification today."

"Today, particle physicists only use the term "mass".

According to this rational terminology, the terms "rest mass" and "relativistic mass" are redundant and misleading.

There is only one mass in physics,  $m$ , which does not depend on the reference frame.

As soon as you reject the "relativistic mass" there is no need to call the other mass "rest mass" and to mark it with the subscript 0. "

OKUN

## The 2 fundamental equations

- Conservation of energy and momentum are close to the heart of physics.

(They are related to 2 deep symmetries of nature. Discuss.)

All this is looked after in special relativity if we define energy and momentum by the following:

$$E^2 - p^2 c^2 = m^2 c^4 \quad \dots \dots \quad ①$$

$$\vec{p} = \vec{v} \frac{E}{c^2} \quad \dots \dots \quad ②$$

where

$E$  = total energy

$\vec{p}$  = momentum

$\vec{v}$  = velocity

$m$  = ordinary mass as  
in Newtonian mechanics

### Special case 1

Let us call the energy of an object when it is at rest ( $v=0$ )  $E_0$ .

$$\text{Consider } E^2 - p^2 c^2 = m^2 c^4.$$

When  $\vec{v} = 0$ ,  $\vec{p} = 0$ ,  $E = E_0$ ; so

$$\underline{E_0 = mc^2}$$

Rest energy is one of the great discoveries of relativity

### Special case 2

One could be wondering:

"Why write  $m$  for mass here, not  $m_0$ ?"

To answer, let us consider  $v \ll c$ .

$$\text{Then } \vec{p} \approx \vec{v} \frac{E_0}{c^2} = m\vec{v}$$

$$\text{and } E = E_0 + T = \sqrt{p^2 c^2 + m^2 c^4}$$

$$= mc^2 \left( 1 + \frac{p^2 c^2}{m^2 c^4} \right)^{1/2}$$

$$\approx mc^2 \left( 1 + \frac{1}{2} \frac{p^2 c^2}{m^2 c^4} + \dots \right)$$

$$= mc^2 + \frac{p^2}{2m} + \dots$$

Thus, in the non-relativistic limit,  
our two fundamental equations  
reduce to the Newtonian equations  
for momentum and KE.

This means that the  $m$  in ①  
is the ordinary Newtonian mass.

If we'd used  $m_0$  in ① our  
relativistic and non-relativistic  
notations would not have matched.

Special case 3 (extreme "anti-Newtonian",  $m=0$ )

$$\text{If } m=0, \quad p = \frac{vE}{c^2} = \frac{v\sqrt{p^2 c^2}}{c^2} = \frac{vp}{c}$$

$$\Rightarrow \underline{v=c}$$

Such bodies have no rest frame.

Also,  $m=0 \Rightarrow \underline{E=pc}$

Massless bodies have no rest energy, just KE.

e.g. photon, graviton ...

Summary  $E^2 = p^2 c^2 + m^2 c^4$  and  $p = \frac{vE}{c}$

describe the kinematics of a free body for all velocities from 0 to  $c$ .

- Also,  $E_0 = mc^2$  follows from them.