

Comment on mass increasing with energy

- IN NEWTONIAN PHYSICS: if $E \rightarrow 10E$, what happens to v ?

$$E = \frac{1}{2}mv^2 \Rightarrow v = \sqrt{\frac{2E}{m}}$$

$$E \rightarrow 10E \Rightarrow v \rightarrow \sqrt{10} \text{ & } v$$

- AT LEP:

$$v(100 \text{ GeV } e^-) = 0.999999999987c$$

$\downarrow \times 10$

$$v(1000 \text{ GeV } e^-) \quad \text{NOT MUCH MORE!}$$

Close to speed of light, Newtonian mechanics is not obeyed.

Response to force - not as expected

acceleration

as if mass was increasing
with speed

$$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

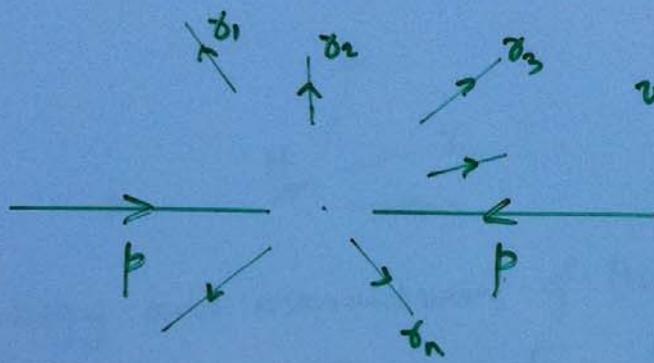
WHAT IS THE MASS OF THE HIGGS BOSON?

Typical collision produces many photons

Measure energies and momenta

$$E_i, \vec{p}_i$$

via EM calorimeter



Consider any pair, γ_1 and γ_2 say.

This pair has an "effective mass" given by

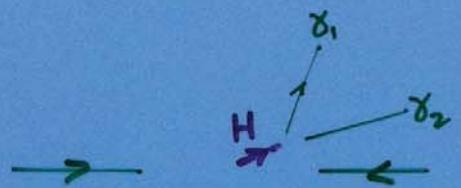
$$m(\gamma_1, \gamma_2) = \sqrt{\frac{(E_{\gamma_1} + E_{\gamma_2})^2 - (\vec{p}_{\gamma_1} + \vec{p}_{\gamma_2})^2 c^2}{c^4}}$$

Measure millions of such effective masses $M_{\gamma\gamma}$

Number with mass $M_{\gamma\gamma}$



What if a pair of γ s results from a Higgs decay?



If the energy and momentum of the Higgs are E_H and \vec{p}_H , then

$$E_H = E_{\gamma_1} + E_{\gamma_2}$$
$$\vec{p}_H = \vec{p}_{\gamma_1} + \vec{p}_{\gamma_2}$$

and

$$\sqrt{\frac{(E_{\gamma_1} + E_{\gamma_2})^2 - (\vec{p}_{\gamma_1} + \vec{p}_{\gamma_2})^2 c^2}{c^4}} = M_H$$



Problem Some introduce relativistic kinematics by defining E and p as follows:

$$E = \gamma mc^2 \text{ and } p = \gamma m v$$

Show that these follow from

$$E^2 = p^2 c^2 + m^2 c^4 \quad \dots \dots \text{I}$$

$$p = \frac{E}{c} v \quad \dots \dots \text{II}$$

Solution Substituting II into I

$$E^2 = \frac{E^2 v^2 c^2}{c^4} + m^2 c^4$$

$$\Rightarrow E^2 \left(1 - \frac{v^2}{c^2}\right) = m^2 c^4 \Rightarrow E = \frac{mc^2}{\sqrt{1 - \frac{v^2}{c^2}}} = \gamma mc^2$$

Substituting for E from II into I

$$\frac{p^2 c^4}{v^2} = p^2 c^2 + m^2 c^4$$

$$\Rightarrow p^2 c^2 (c^2 - v^2) = m^2 c^4 v^2$$

$$\Rightarrow p^2 c^4 \left(1 - \frac{v^2}{c^2}\right) = m^2 c^4 v^2$$

$$\Rightarrow p = \frac{mv}{\sqrt{1 - \frac{v^2}{c^2}}} = \gamma mv$$

Problem At the Large Electron Position collider (LEP) at CERN, electrons were accelerated to an energy E of 100 GeV. What is the electron speed?

Solution With $E = 100 \text{ GeV}$

$$\text{and } m = 0.511 \frac{\text{MeV}}{\text{c}^2}$$

the mass term in $E^2 = p^2 c^2 + m^2 c^4$ is minuscule; so we anticipate that v will be very close to c .

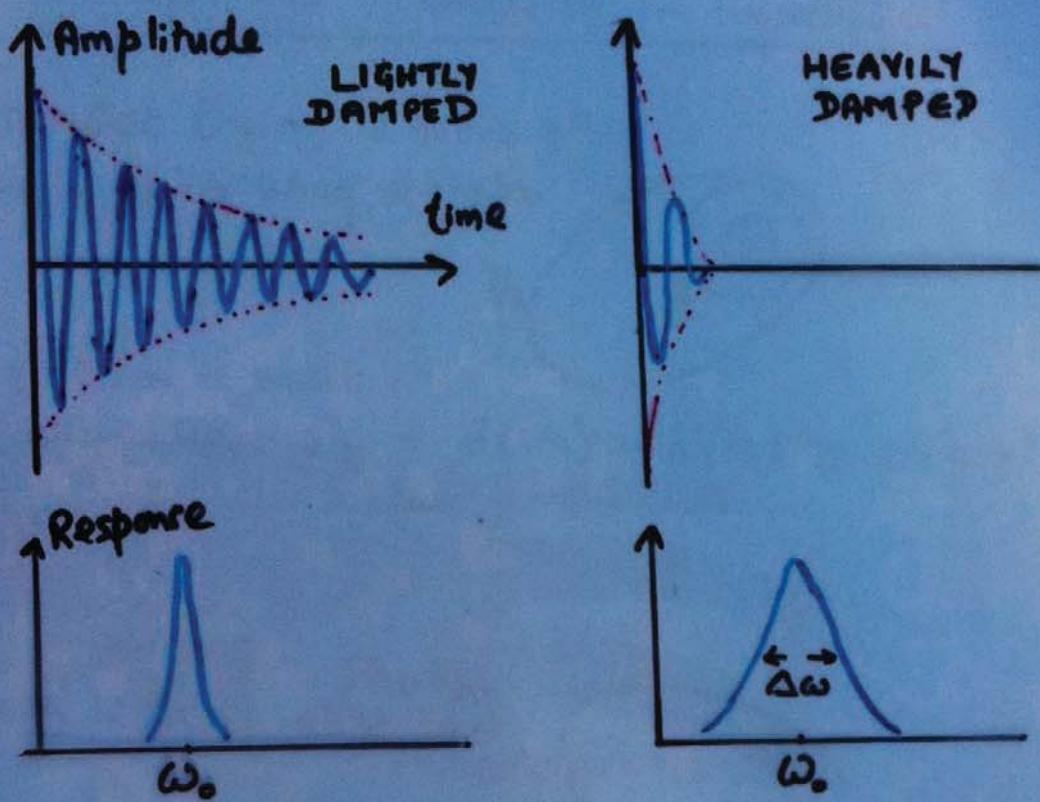
From $E = \frac{mc^2}{\sqrt{1-v^2/c^2}}$ we can write

$$E^2 = \frac{m^2 c^4}{(1-v/c)(1+v/c)} \approx \frac{m^2 c^4}{2(1-v/c)}$$

$$\begin{aligned} \Rightarrow (1-v/c) &\approx \frac{1}{2} \frac{(mc^2)^2}{E^2} \\ &= \frac{1}{2} \left(\frac{0.511 \times 10^6 \text{ eV}}{100 \times 10^9 \text{ eV}} \right)^2 \\ &= 0.000000000013 \end{aligned}$$

$$\Rightarrow \underline{\underline{\frac{v}{c}}} = 0.999999999987$$

Excited state / unstable particle / resonance



If $\tau = \text{time for energy to drop to } \frac{1}{e}$,

$$\underline{\tau = \frac{1}{\Delta\omega} = \frac{1}{2\pi \Delta f}}$$

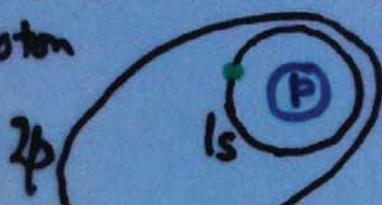
Multiply by $\frac{h/h}{h}$: $\underline{\tau = \frac{h}{2\pi \Delta E}} \quad (E=hf)$

Can be written: $\underline{\tau = \frac{h}{2\pi c^2 \Delta m}} \quad (E=mc^2)$

$$\left(\frac{h}{2\pi} = 1.05 \times 10^{-34} \text{ Js} = 6.582 \times 10^{-22} \text{ MeV s} \right)$$

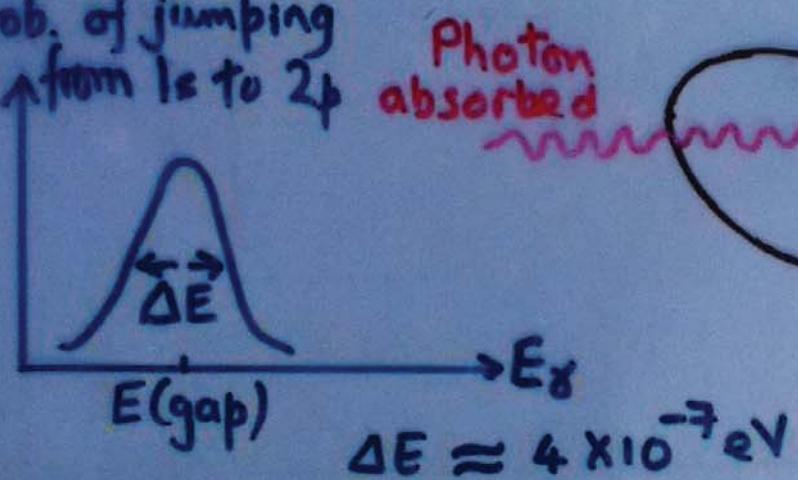
$2p \rightarrow 1s$ transition in hydrogen

Consider trying to move e^- from $1s$ to $2p$ using a photon

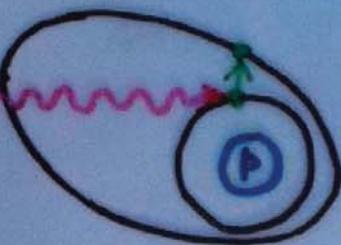


Absorption is most effective when $E_\gamma = E(2p) - E(1s) \equiv E(\text{gap})$

Prob. of jumping
from $1s$ to $2p$

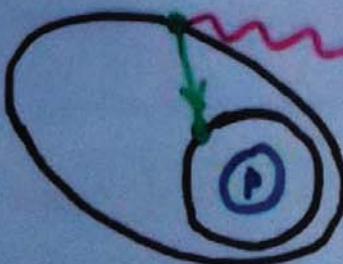


Photon absorbed



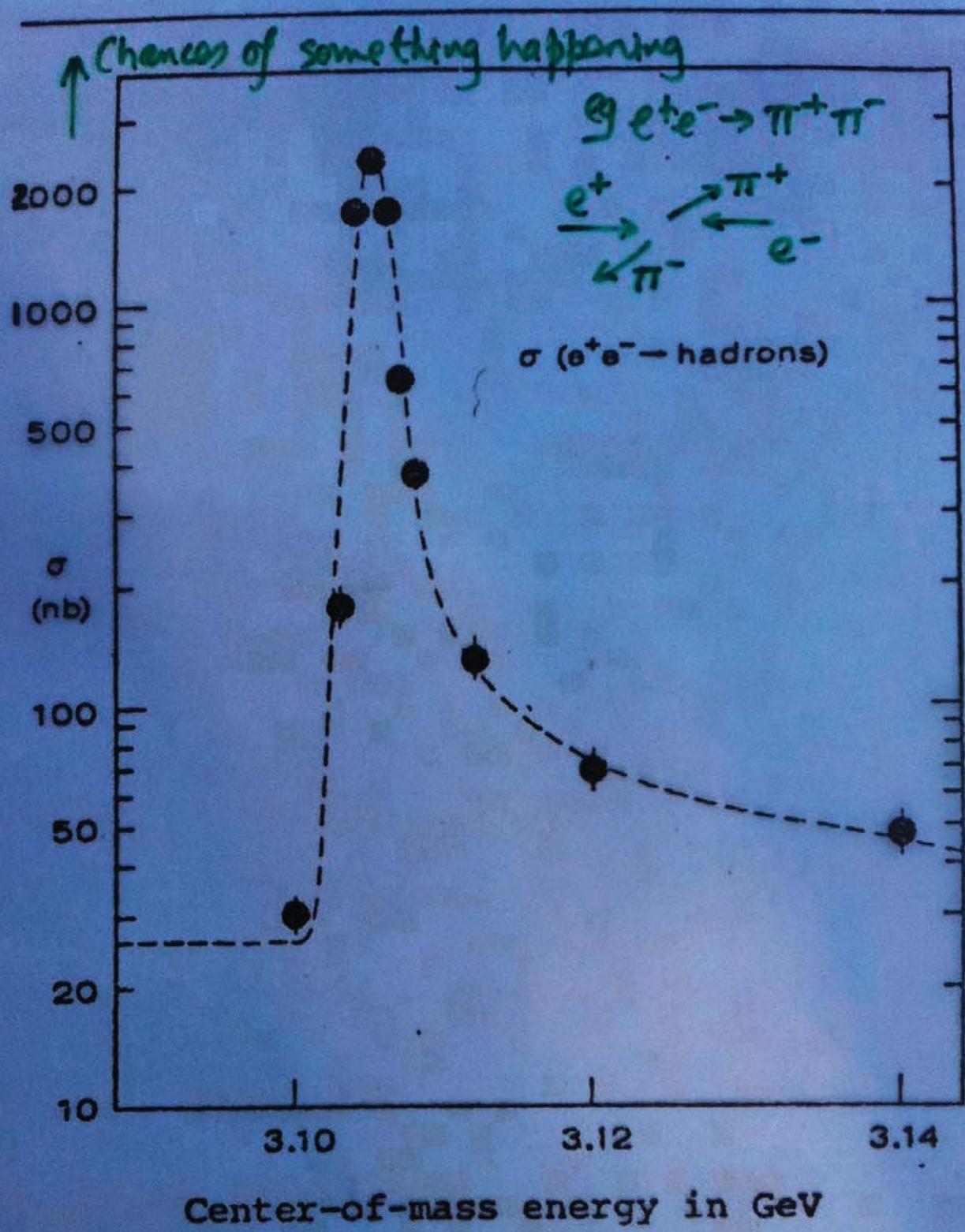
$$\Delta E \approx 4 \times 10^{-7} \text{ eV}$$

Photon emitted



Lifetime
of $2p$ state

$$= \frac{h}{2\pi \Delta E} \sim 1.6 \text{ ns}$$



$$\Delta E \sim 2 \text{ MeV}$$

$$\Delta E \Delta t \sim \hbar \quad \text{where } \hbar = 6.6 \times 10^{-34} \text{ MeV sec}$$

$$\text{So } \Delta t \sim \frac{\hbar}{\Delta E} = \frac{6.6 \times 10^{-34} \text{ MeV sec}}{2 \text{ MeV}} \sim 3 \times 10^{-32} \text{ sec}$$

(Actually much longer)