Operational Framework for a Quantum Database

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Introduction and motivation for Quantum Databases

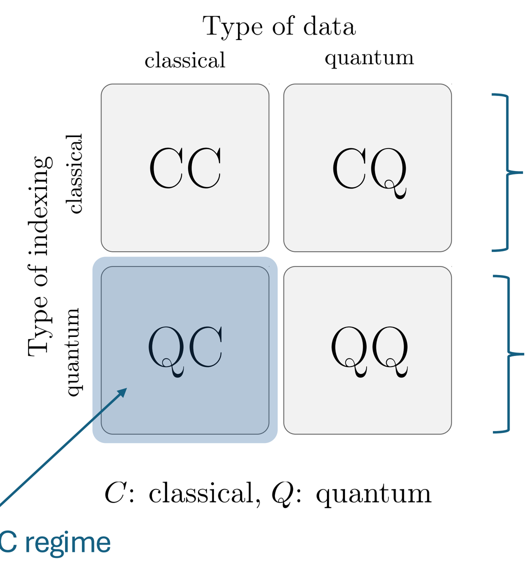
Presenting a framework for defining a **set of efficient and essential database operations in correspondence with their classical counterpart**, highlighting the main similarities and differences.

- Algorithms relevant for Quantum Databases: Pattern recognition [2,3], collision finding [4], Quantum search [5], etc.
- The Quantum Database structure is aligned with the quantum random access memory (QRAM) model introduced in [6].
- Specific algorithms are also relevant in the context of general state preparation algorithms.

Databases in the broader context of *classical/quantum* data and indexing

classical data: States drawn from a set of orthogonal states with known unitary transformation to the computational basis

quantum data: States that can't be cloned exactly (drawn from a set of non-orthogonal states or the aforementioned unitary is unknown)



$\bigotimes_{i=0}^{p} |d_i\rangle_D \in (\mathcal{H}_D)^{\otimes p}$

Classical indexing **scales linearly** wrt. the qubit number

Quantum databases allow for **superpositions of index states** and thus allow for **exponential compression**

$|\mathrm{QDB}^{(k)}\rangle = \sum_{j=0}^{k-1} \psi_j |j\rangle_I |d_j\rangle_D \in \mathcal{H}_I \otimes \mathcal{H}_D$

A Quantum Database State - *in detail*

$$|QDB^{(k)}\rangle = \underbrace{\frac{1}{\sqrt{k}} |0\rangle_I |0\rangle_D}_{\text{Use this reference state as probability reservoir}} + \underbrace{\frac{1}{\sqrt{k}} \sum_{j=1}^{k-1} |j\rangle_I |d_j\rangle_D}_{\text{Superposition of indexed data states}} \in \mathcal{H}_I \otimes \mathcal{H}_D$$

Superposition state contains (k-1) indexed data elements d_j

Hilbert space of dimension $2^{\lceil \log_2(k) \rceil}$

Hilbert space of dimension $2^{\tilde{m}}$

Preparation of a general Quantum Database State

$$\mathcal{H}_I \otimes \mathcal{H}_D \rightarrow \mathcal{H}_I \otimes \mathcal{H}_D$$
$$|0\rangle_{I\otimes D} \xmapsto{P(k)} \frac{1}{\sqrt{k}} \sum_{j=0}^{k-1} |j\rangle_I |0\rangle_D$$

This trivially results in **Walsh-Hadamard transformation** on index register I if $\log_2(k) \in \mathbb{N}_{>0}$ with circuit depth $\mathcal{O}(1)$

Efficient preparation of uniform superposition states [7] on the index register I if $\log_2(k) \notin \mathbb{N}_{>0}$ with circuit depth $\mathcal{O}(\log_2(k))$

We present a protocol for **non-trivial preparation operation** on index register I with $\log_2(k) \notin \mathbb{N}_{>0}$ (including preparation of a *probability reservoir*) with a circuit-depth scaling as $\mathcal{O}((\log_2(k))^2)$

Extending a general Quantum Database State

$$\mathcal{H}_I \otimes \mathcal{H}_D \rightarrow \mathcal{H}_{I'} \otimes \mathcal{H}_D \text{ with } \mathcal{H}_{I'} := \mathcal{H}_0 \otimes \mathcal{H}_I$$
$$|QDB^{(k)}\rangle = \frac{1}{\sqrt{k}} \sum_{j=0}^{k-1} |j\rangle_I |d_j\rangle_D \xmapsto{E(l)} \frac{1}{\sqrt{k+l}} \sum_{j=0}^{k+l-1} |j\rangle_{I'} |d_j\rangle_D$$
$$\text{with } |d_j\rangle_D = |0\rangle_D \quad \forall j > (k-1) \text{ and } j = 0$$

The general extension operation is a **non-unitary transformation.** We extend a database containing k data elements by l new indices correlated to the empty data string by adding ancilla qubits in state $|0\rangle$.

Proof: See [1].

Empty data state

Preparation of an *empty* QDB state with $\log_2(k) \notin \mathbb{N}_{>0}$

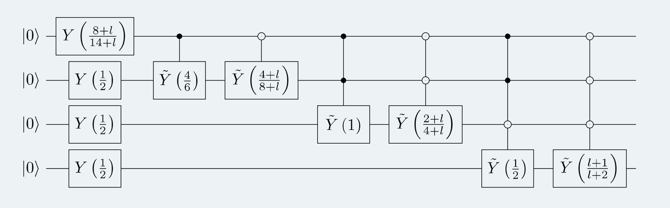
Prepare the following state:

$$\left(\underbrace{\sqrt{\frac{l+1}{k+l}} |0\rangle_I}_{\text{Initialize probability reservoir}} + \underbrace{\sqrt{\frac{1}{k+l}} \sum_{j=1}^{k-1} |j\rangle_I}_{\text{Balanced superposition of indices}} \right) |0\rangle_D$$

Using the following definition:

$$Y(p) = \begin{pmatrix} \sqrt{p} & -\sqrt{1-p} \\ \sqrt{1-p} & \sqrt{p} \end{pmatrix}, \quad 0 \leq p \leq 1$$
$$\tilde{Y}(p) := Y(p) \cdot Y(1/2)^{-1}$$

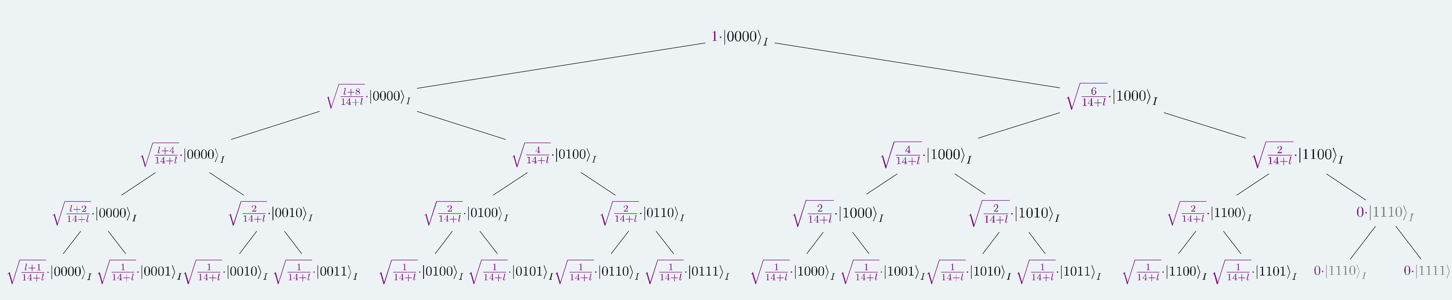
Example case for $k=14$ and general l:



State preparation of the following state:

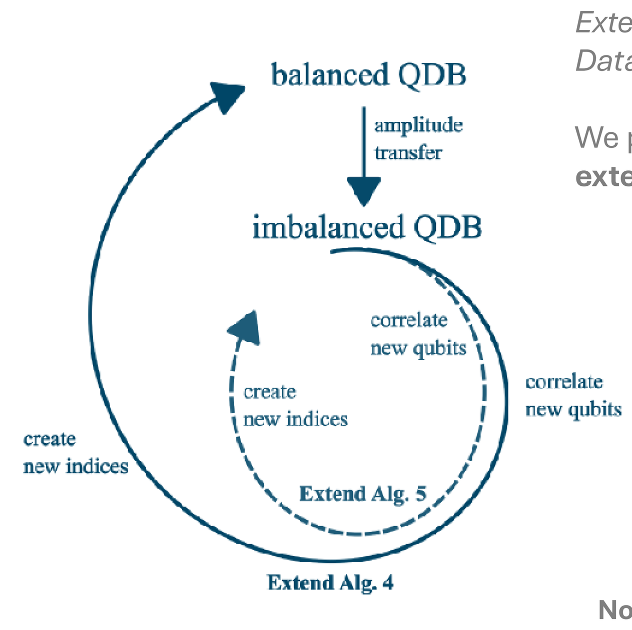
$$\sqrt{\frac{l+1}{14+l}} |0\rangle + \frac{1}{\sqrt{14+l}} \sum_{j=1}^{13} |j\rangle$$

Quantum circuit implementation of specific state preparation operation generating the state with $k=14$.



Visualization of preparation operation as a binary tree. Each level of branching denotes the application of a (controlled) single-qubit operation corresponding to the quantum circuit given above.

Extending a general QDB State: two different protocols



*Extending a Quantum Database is **non-trivial:***

We present **two extension protocols**

Algorithm 4: Grover-like approach for amplitude transfer and unfolding step, *Problem:* uses unitary creating QDB state

Algorithm 5: Start in an imbalanced QDB state and create new indices correlated to an empty data state based on the unfolding step

Note: a general unfolding step includes the state preparation unitary defined above $PREPARE(k, l)$

Outlook on the regime with quantum indexing and data (QQ)

Algorithms that are also valid in QQ-regime: *E.g. Prepare (Alg. 1) and Extend (Alg. 5) in [1]*

Main difficulty: Restrictions arising due to the *no-cloning theorem* [9]

QQ: Utilize approximate methods

Conclusion and Outlook

Presented a **set of algorithms** that operate within a framework of a specifically designed quantum database model focusing on **quantum indices in a uniform superposition state and "classical" data**. Within this framework individual algorithms can be extended and used to **study the feasibility** when aiming to use a given quantum database operation.

Further transformations implemented by Quantum Database Operations (QDB with "Classical" Data, QC-regime)

- **Write operation:** $|QDB^{(k)}_{\neq f}\rangle + \frac{1}{\sqrt{k}} |f\rangle_I |0\rangle_D \xmapsto{W(f)} |QDB^{(k)}_{\neq f}\rangle + \frac{1}{\sqrt{k}} |f\rangle_I |d_f\rangle_D$
- **Read-out operation** (or consider a projective mmt.): $|QDB^{(k)}\rangle |0\rangle_A \xmapsto{G(f)} |QDB^{(k)}_{\neq f}\rangle |0\rangle_A + \frac{1}{\sqrt{k}} |f\rangle_I |d_f\rangle_D |d_f\rangle_A$
- **Remove operation:** $|QDB^{(k)}\rangle = \frac{1}{\sqrt{k}} \sum_{j=0}^{k-1} |j\rangle_I |d_j\rangle_D \xmapsto{R(f)} \frac{1}{\sqrt{k-1}} \sum_{j=0, j\neq f}^{k-1} |j\rangle_I |d_j\rangle_D$
- **Permute operation:** $|QDB^{(k)}\rangle \xmapsto{P_\pi} \frac{1}{\sqrt{k}} \sum_{j=0}^{k-1} |\pi(j)\rangle_I |d_j\rangle_D = \frac{1}{\sqrt{k}} \sum_{j=0}^{k-1} |j\rangle_I |d_{\pi^{-1}(j)}\rangle_D$

Consider **entanglement** creation between the QDB state and possible ancilla qubits; restrictions occur due to **no-cloning** and the **non-deletion** theorem following thereof.

Using the following definition:

$$|QDB^{(k)}_{\neq f}\rangle := \frac{1}{\sqrt{k}} \sum_{j=0, j\neq f}^{k-1} |j\rangle_I |d_j\rangle_D$$

Find all the details in our paper here

References: [1] Carla Rieger, Michele Grossi, Gian Giacomo Guerreschi, Sofia Vallecorsa, & Martin Werner, *Operational Framework for a Quantum Database*, arXiv:2405.14947
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