## 1 Theory of very rare decays

The LHC is constructed to reveal the nature of new physics around the TeV scale, which is postulated to stabilise the electroweak scale. The production of new particles at the high energy frontier needs to be complemented by precision measurements which determine the couplings and mixing patterns of these new particles. To this end flavour-changing neutral current (FCNC) processes play a pivotal role, because they are highly suppressed in the Standard Model (SM) but not in its generic extensions. Clearly, the more suppression factors pile up in a given SM FCNC decay rate, the higher is the sensitivity to new physics. A particularly important class of very rare decays are the leptonic decays of a $B_d$ or a $B_s$ meson. In addition to the electroweak-loop suppression the corresponding decay rates are helicity suppressed by a factor of $m_\ell^2/m_B^2$, where $m_\ell$ and $M_B$ are the masses of lepton and $B$ meson, respectively. The SM decay rates were calculated in [1] at the next-to-leading-order (NLO) of QCD. The effective $|\Delta B| = |\Delta S| = 1$ hamiltonian, which describes $b \to s$ decays, already contains 17 different operators in the Standard Model, in a generic model-independent analysis of new physics this number will exceed 100. One virtue of purely leptonic $B_s$ decays is their dependence on a small number of operators, so that they are accessible to model-independent studies of new physics. These statements, of course, equally apply to $b \to d$ transitions and leptonic $B_d$ decays. While in the Standard Model all six $B_q \to \ell^+\ell^-$ decays (with $q = d$ or $s$ and $\ell = e, \mu$ or $\tau$) are related to each other in a simple way, this is not necessarily so in models of new physics. Therefore all six decay modes should be studied.

Other very rare decays to be mentioned in this context are non-hadronic $B$ decays which have multiple leton pairs in the final state or photons in addition to one or more lepton pairs. The theoretical information contained in these decays can be more easily obtained from other decays, for instance $B \to X_s\gamma$ or $B \to X_s\ell^+\ell^-$. Still they need to be considered as background processes to the leptonic decays of interest because of the finite resolution of the invariant mass of the lepton pair. Another related class of very rare B decays are lepton-flavour violating decays like $B \to e^-\mu^-$. In the Standard Model (meant here to include neutrino mass terms) they are suppressed by two powers of $m_\nu/M_W$, where $m_\nu$ denotes the largest neutrino mass. However, this suppression factor is absent in certain models of new physics.

### 1.1 $B_q \to \ell^+\ell^-$ in the Standard Model

Photonic penguins do not contribute to $B_q \to \ell^+\ell^-$, because a lepton pair with zero angular momentum has charge conjugation quantum number C=1, while the photon has C=-1. The dominant contribution stems from the Z-penguin diagram shown in Fig. 1. There is also a box diagram with two W bosons, which is suppressed by a factor of $M_W^2/m_t^2$ with respect to the Z-penguin diagram. These diagrams determine the Wilson coefficient $C_A$ of the operator

$$Q_A = \overline{b}_L\gamma^\mu q_L\,\overline{\ell}\gamma_\mu\gamma_5\ell. \tag{1}$$

We will further need operators with scalar and pseudoscalar couplings to the leptons:

$$Q_S = m_b\overline{b}_R q_L\,\overline{\ell}\ell, \qquad Q_P = m_b\overline{b}_R q_L\,\overline{\ell}\gamma_5\ell. \tag{2}$$

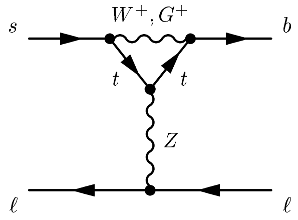


Fig. 1: Left: Z-penguin contribution to $B_s \to \ell^+\ell^-$.

Their coefficients $C_S$ and $C_P$ are determined from penguin diagrams involving the Higgs or the neutral Goldstone boson, respectively. While $C_S$ and $C_P$ are tiny and can be safely neglected in the Standard Model, the situation changes dramatically in popular models of new physics discussed below. The effective hamiltonian reads

$$H = \frac{G_F}{\sqrt{2}} \frac{\alpha_{\rm QED}}{\pi \sin^2\theta_W} V_{tb}^* V_{tq} \left[ C_S Q_S + C_P Q_P + C_A Q_A \right]. \tag{3}$$

$C_A$ has been determined in the next-to-leading order (NLO) of QCD [1,2]. The NLO corrections are in the percent range and higher-order corrections play no role. $C_A$ is commonly expressed in terms of the $\overline{\rm MS}$ mass $\overline{m}_t$ of the top quark. A pole mass of $m_t^{\rm pole} = 171.4 \pm 2.1$ GeV corresponds to $\overline{m}_t = 163.8 \pm 2.0$ GeV. An excellent approximation to the NLO result for $C_A$, which holds with an accuracy of $5 \cdot 10^{-4}$ for 149 GeV $< \overline{m}_t <$ 179 GeV, is

$$C_A(\overline{m}_t) = 0.9636 \left[ \frac{80.4 \text{ GeV}}{M_W} \frac{\overline{m}_t}{164 \text{ GeV}} \right]^{1.52} \tag{4}$$

In the literature $C_A(\overline{m}_t)$ is often called $Y(\overline{m}_t^2/M_W^2)$. The exact expression can be found e.g. in Eqs. (16-18) of [2]. The branching fraction can be compactly expressed in terms of the Wilson coefficients $C_A$, $C_S$ and $C_P$:

$$\begin{aligned} \mathcal{B}\left(B_q \to \ell^+\ell^-\right) &= \frac{G_F^2\, \alpha_{\rm QED}^2}{64\,\pi^3 \sin^4\theta_W} \left|V_{tb}^* V_{tq}\right|^2 \frac{\tau_{B_q}}{\hbar} M_{B_q}^3 f_{B_q}^2 \sqrt{1 - \frac{4 m_\ell^2}{M_{B_q}^2}} \\ &\times \left[ \left(1 - \frac{4 m_\ell^2}{M_{B_q}^2}\right) M_{B_q}^2 C_S^2 + \left( M_{B_q} C_P - \frac{2 m_\ell}{M_{B_q}} C_A \right)^2 \right]. \end{aligned} \tag{5}$$

Here $f_{B_q}$ and $\tau_{B_q}$ are the decay constant and the lifetime of the $B_q$ meson, respectively, $\theta_W$ is the Weinberg angle and $\hbar = 6.582 \cdot 10^{-13}$ GeV ps. Since $B_q \to \ell^+\ell^-$ is a short-distance process, the appropriate value of $\alpha_{\rm QED}$ is $\alpha_{\rm QED}(M_Z) = 1/128$. With Eq. (4) and $C_S = C_P = 0$ Eq. (5) gives the following Standard Model predictions:

$$\mathcal{B}\left(B_s \to \tau^+\tau^-\right) = (8.20 \pm 0.31) \cdot 10^{-7} \times \frac{\tau_{B_s}}{1.527\,\text{ps}} \left[ \frac{|V_{ts}|}{0.0408} \right]^2 \left[ \frac{f_{B_s}}{240 \text{ MeV}} \right]^2 \tag{6}$$

$$\mathcal{B}\left(B_s \to \mu^+\mu^-\right) = (3.86 \pm 0.15) \cdot 10^{-9} \times \frac{\tau_{B_s}}{1.527\,\text{ps}} \left[ \frac{|V_{ts}|}{0.0408} \right]^2 \left[ \frac{f_{B_s}}{240 \text{ MeV}} \right]^2 \tag{7}$$

$$\mathcal{B}\left(B_s \to e^+e^-\right) = (9.05 \pm 0.34) \cdot 10^{-14} \times \frac{\tau_{B_s}}{1.527\,\text{ps}} \left[ \frac{|V_{ts}|}{0.0408} \right]^2 \left[ \frac{f_{B_s}}{240 \text{ MeV}} \right]^2 \tag{8}$$

$$\mathcal{B}\left(B_d \to \tau^+\tau^-\right) = (2.23 \pm 0.08) \cdot 10^{-8} \times \frac{\tau_{B_d}}{1.527\,\mathrm{ps}} \left[\frac{|V_{td}|}{0.0082}\right]^2 \left[\frac{f_{B_d}}{200\ \mathrm{MeV}}\right]^2 \quad (9)$$

$$\mathcal{B}\left(B_d \to \mu^+\mu^-\right) = (1.06 \pm 0.04) \cdot 10^{-10} \times \frac{\tau_{B_d}}{1.527\,\mathrm{ps}} \left[\frac{|V_{td}|}{0.0082}\right]^2 \left[\frac{f_{B_d}}{200\ \mathrm{MeV}}\right]^2 \quad (10)$$

$$\mathcal{B}\left(B_d \to e^+e^-\right) = (2.49 \pm 0.09) \cdot 10^{-15} \times \frac{\tau_{B_d}}{1.527\,\mathrm{ps}} \left[\frac{|V_{td}|}{0.0082}\right]^2 \left[\frac{f_{B_d}}{200\ \mathrm{MeV}}\right]^2 \quad (11)$$

The dependences on the decay constants, which have sizable theoretical uncertainties, and on the relevant CKM factors have been factored out. While $|V_{ts}|$ is well-determined through the precisely measured $|V_{cb}|$, the determination of $|V_{td}|$ involves the global fit to the unitarity triangle and suffers from larger uncertainties. The residual uncertainty in Eqs. (6–11) stems from the 2 GeV error in $\overline{m}_t$.

## 1.2 $B_q \to \ell^+\ell^-$ and new physics

### *1.2.1 Additional Higgs bosons*

The helicity suppression factor of $m_\ell/M_{B_q}$ in front of $C_A$ in Eq. (5) makes $\mathcal{B}(B_q \to \ell^+\ell^-)$ sensitive to physics with new scalar or pseudoscalar interactions, which contribute to $C_S$ and $C_P$. This feature renders $B_q \to \ell^+\ell^-$ highly interesting to probe models with an extended Higgs sector. Practically all weakly coupled extensions of the Standard Model contain extra Higgs multiplets, which puts $\mathcal{B}(B_q \to \ell^+\ell^-)$ on the center stage of indirect new physics searches. Higgs bosons couple to fermions with Yukawa couplings $y_f$. In the Standard Model $y_b \propto m_b/M_W$ and $y_\ell \propto m_\ell/M_W$ are so small that Higgs penguin diagrams, in which the Z-boson of Fig. 1 is replaced by a Higgs boson, play no role. In extended Higgs sectors the situation can be dramatically different. Models with two or more Higgs multiplets can not only accomodate Yukawa couplings of order one, they also generically contain tree-level FCNC couplings of neutral Higgs bosons. In simple two–Higgs–doublet models these unwanted FCNC couplings are usually switched off in an ad-hoc way by imposing a discrete symmetry on the Higgs and fermion fields, which leads to the celebrated two-Higgs-doublet models of type I and type II. Here we only discuss the latter model, in which one Higgs doublet $H_u$ only couples to up-type fermions while the other one, $H_d$, solely couples to down-type fermions [3]. The parameter controling the size of the down–type Yukawa coupling is $\tan\beta = v_u/v_d$, the ratio of the vacuum expectation values acquired by $H_u$ and $H_d$. The Yukawa coupling of $H_d$ to the fermion $f$ is $y_f \sin\beta = m_f \tan\beta/v$ with $v = \sqrt{v_u^2 + v_d^2} = 174$ GeV. Hence $y_b \approx 1$ for $\tan\beta \approx 50$. The dominant contributions to $C_S$ and $C_P$ for large $\tan\beta$ involve charged and neutral Higgs bosons, but the final result can be solely expressed in terms of $\tan\beta$ and the charged Higgs boson mass $M_{H^+}$ [4]:

$$C_S = C_P = \frac{m_\ell}{2M_W^2} \tan^2\beta \, \frac{\ln r}{r-1} \qquad \text{with} \quad r = \frac{M_{H^+}^2}{\overline{m}_t^2}. \quad (12)$$

While for very large values of $\tan\beta/M_{H^+}$ the branching fraction can be enhanced, the contributions in Eq. (12) typically reduce $\mathcal{B}(B_q \to \ell^+\ell^-)$ with respect to the Standard Model value. The decoupling for $M_{H^+} \to \infty$ is slow, e.g. for $\tan\beta = 60$ and $M_{H^+} = 500\,$GeV the new Higgs contributions reduce $\mathcal{B}(B_q \to \ell^+\ell^-)$ by 50%!

### *1.2.2 Supersymmetry*

The generic Minimal Supersymmetric Standard Model (MSSM) contains many new sources of flavour violation in addition to the Yukawa couplings. These new flavour violating parameters stem from the supersymmetry–breaking terms and their effects could easily exceed those of the CKM mechanism. In view of the success of the CKM description of flavour–changing transitions one usually supplements the MSSM with the hypothesis of *Minimal Flavour Violation (MFV)*, which can be justified by symmetry arguments [5]. In the MFV–MSSM the only sources of flavour violation are the Yukawa couplings, just as in the Standard Model. In this section the MSSM is always understood to be supplied with the assumption of MFV. While in MFV scenarios the contributions from virtual supersymmetric particles to FCNC processes are normally smaller than the Standard Model contribution, the situation is very different for $B_q \to \ell^+\ell^-$.

The MSSM has two Higgs doublets. At tree-level the couplings are as in the two-Higgs-doublet model of type II, because the holomorphy of the superpotential forbids the coupling of $H_u$ to down-type fermions and that of $H_d$ to up-type fermions. At the one-loop level, however, the situation is different, and both doublets couple to all fermions. The loop-induced couplings are proportional to the product of a supersymmetry-breaking term and the $\mu$ parameter. If $\tan\beta$ is large, the loop-induced coupling of $H_u^*$ and the tree-level coupling of $H_d$ give similar contributions to the masses of the down-type fermions, because the loop suppression is compensated by a factor of $\tan\beta$ [6]. In this scenario the Higgs sector is that of a *general* two-Higgs-doublet model, which involves FCNC Yukawa couplings of the heavy neutral Higgs bosons $A^0$ and $H^0$ [7]. The Wilson coefficients $C_S$ and $C_P$ differ from those in Eq. (12) in two important aspects: they involve three rather than two powers of $\tan\beta$ and they depend on the mass $M_{A^0} \sim M_{H^0}$ instead of the charged Higgs boson mass. The branching ratios scale as

$$Br^{\rm SUSY}(B_q \to \ell^+\ell^-) \;\; \propto \;\; \frac{m_b^2 m_\ell^2 \, \tan^6\beta}{M_{A^0}^4}$$

and could, in principle, exceed the Standard Model results in Eqs. (6–11) by a factor of 1000 [8]. Thus the experimental upper limit on $Br(B_s \to \mu^+\mu^-)$ from the Tevatron, which is larger than $Br^{\rm SM}(B_s \to \mu^+\mu^-)$ in Eq. (7) by a factor of 25, already severly cuts into the parameter space of the MSSM. $Br(B_s \to \mu^+\mu^-)$ in MSSM scenarios with large $\tan\beta$ has been studied extensively [8–11].

Very popular special cases of the MSSM are the minimal supergravity model (mSUGRA) [12] and its slight variant, the Constrained Minimal Supersymmetric Standard Model (CMSSM). While the MSSM contains more than 100 parameters, mSUGRA involves only 5 parameters and is therefore much more predictive. In particular correlations between $Br(B_s \to \mu^+\mu^-)$ and other observables emerge, for example with the anomalous magnetic moment of the muon and the mass of the lightest neutral Higgs boson [11]. Other well-motivated variants of the MSSM incorporate the parameter constraints from grand unified theories (GUTs). $Br(B_s \to \mu^+\mu^-)$ is especially interesting in GUTs based on the symmetry group SO(10) [11, 14]. In the minimal SO(10) GUT the top and bottom Yukawa couplings $y_b$ and $y_t$ unify at a high scale implying that $\tan\beta$ is of order 50. While realistic SO(10) models contain a non–minimal Higgs sector, any experimental information on the deviation of $y_b/y_t$ from 1 is very desirable, as it probes the Higgs sectors of GUT theories. In conjunction with other observables like the mass difference in the $\mathrm{B_s}-\overline{\mathrm{B}}_\mathrm{s}$ system [10] or $Br(B^+ \to \tau^+\nu_\tau)$ [15], which depend in different ways on $\tan\beta$ and the masses of the non-Standard Higgs bosons and the supersymmetric particles, the measurement of $Br(B_s \to \mu^+\mu^-)$ at the LHC will, within the MSSM, answer the question whether the top and bottom Yukawa couplings unify at high energies.

### 1.3 Other very rare decays

The decays $B_q \to \ell^+\ell^-\gamma$ and $B_q \to \ell^+\ell^-\ell'^+\ell'^-$ are of little interest from a theoretical point of view. First, they are difficult to calculate, since they involve photon couplings to quarks and are thereby sensitive to soft hadron dynamics. Second, they are not helicity–suppressed, because the (real or virtual) photon can recoil against a lepton pair in a $J = 1$ state. This implies that they probe operators of the effective hamiltonian which can more easily be studied from $B_q \to X\gamma$ and $B \to X\ell^-\ell^-$ decays. However, the absence of a helicity suppression makes $B_q \to \ell^+\ell^-\gamma$ a dangerous background to $B_q \to \ell^+\ell^-$, if the experimental resolution of the invariant mass of the lepton pair is limited. A naive estimate gives $Br(B_s \to \mu^+\mu^-\gamma) \sim (m_B^2/m_\mu^2)\, \alpha_{\rm QED}/(4\pi) \sim Br(B_s \to \mu^+\mu^-)$, while a more detailed analysis even finds $Br(B_s \to \mu^+\mu^-\gamma) > Br(B_s \to \mu^+\mu^-)$ [16].

Lepton-flavour violating (LFV) decays like $B_q \to \ell^\pm\mu^\mp$ are negligibly small in the Standard Model. In supersymmetric theories with R parity (such as the MSSM) their branching ratios are smaller than those of the corresponding lepton-flavour conserving decay, e.g. $B_q \to \mu^+\mu^-$. Large effects, however, are possible in models which contain LFV tree-level couplings or leptoquarks. Here supersymmetric theories without R parity and the Pati-Salam model should mentioned. Supersymmetry without R parity involves a plethora of new couplings, which are different for all combinations of quark and lepton flavour involved, so that no other experimental constraints prevent large effects in $B_q \to \ell^\pm\mu^\mp$. Flavour physics in the Pati-Salam model has been studied in [17].

### References


[1] G. Buchalla and A. J. Buras, Nucl. Phys. B **400** (1993) 225. M. Misiak and J. Urban, Phys. Lett. B **451** (1999) 161 [arXiv:hep-ph/9901278].

[2] G. Buchalla and A. J. Buras, Nucl. Phys. B **548** (1999) 309 [arXiv:hep-ph/9901288].

[3] J.F. Gunion, H.E. Haber, G. Kane and S. Dawson, *The Higgs' Hunter's Guide*, (Addison-Wesley, Reading, MA, USA, 1990); errata arXiv:hep-ph/9302272.

[4] H. E. Logan and U. Nierste, Nucl. Phys. B **586** (2000) 39 [arXiv:hep-ph/0004139].

[5] G. D'Ambrosio, G. F. Giudice, G. Isidori and A. Strumia, Nucl. Phys. B **645** (2002) 155 [arXiv:hep-ph/0207036].

[6] L. J. Hall, R. Rattazzi and U. Sarid, Phys. Rev. D **50** (1994) 7048 [arXiv:hep-ph/9306309].

[7] C. Hamzaoui, M. Pospelov and M. Toharia, Phys. Rev. D **59** (1999) 095005 [arXiv:hep-ph/9807350].

[8] K. S. Babu and C. F. Kolda, Phys. Rev. Lett. **84** (2000) 228 [arXiv:hep-ph/9909476].

[9] P. H. Chankowski and L. Slawianowska, Phys. Rev. D **63** (2001) 054012 [arXiv:hep-ph/0008046]. G. Isidori and A. Retico, JHEP **0111** (2001) 001 [arXiv:hep-ph/0110121]. G. Isidori and A. Retico, JHEP **0209** (2002) 063 [arXiv:hep-ph/0208159]. A. J. Buras,

P. H. Chankowski, J. Rosiek and L. Slawianowska, Nucl. Phys. B **659** (2003) 3 [arXiv:hep-ph/0210145]. C. Bobeth, T. Ewerth, F. Kruger and J. Urban, Phys. Rev. D **64** (2001) 074014 [arXiv:hep-ph/0104284].

[10] A. J. Buras, P. H. Chankowski, J. Rosiek and L. Slawianowska, Phys. Lett. B **546** (2002) 96 [arXiv:hep-ph/0207241].

[11] A. Dedes, H. K. Dreiner and U. Nierste, Phys. Rev. Lett. **87** (2001) 251804 [arXiv:hep-ph/0108037].

[12] H. P. Nilles, Phys. Lett. B **115** (1982) 193; Nucl. Phys. B **217** (1983) 366; A. Chamseddine, R. Arnowitt, P. Nath, Phys. Rev. Lett. **49** (1982) 970; R. Barbieri, S. Ferrara, C. Savoy, Phys. Lett. B **119** (1982) 343; L. Hall, J. Lykken, S. Weinberg, Phys. Rev. D **27** (1983) 2359; S. K. Soni, H. A. Weldon, Phys. Lett. B **126** (1983) 215.

[13] J. R. Ellis, K. A. Olive and V. C. Spanos, Phys. Lett. B **624** (2005) 47 [arXiv:hep-ph/0504196]. J. R. Ellis, K. A. Olive, Y. Santoso and V. C. Spanos, JHEP **0605** (2006) 063 [arXiv:hep-ph/0603136].

[14] R. Dermisek, S. Raby, L. Roszkowski and R. Ruiz De Austri, JHEP **0304** (2003) 037 [arXiv:hep-ph/0304101]. R. Dermisek, S. Raby, L. Roszkowski and R. Ruiz de Austri, JHEP **0509** (2005) 029 [arXiv:hep-ph/0507233].

[15] W. S. Hou, Phys. Rev. D **48** (1993) 2342. A. G. Akeroyd and S. Recksiegel, J. Phys. G **29** (2003) 2311 [arXiv:hep-ph/0306037]. G. Isidori and P. Paradisi, Phys. Lett. B **639** (2006) 499 [arXiv:hep-ph/0605012].

[16] D. Melikhov and N. Nikitin, Phys. Rev. D **70** (2004) 114028 [arXiv:hep-ph/0410146].

[17] G. Valencia and S. Willenbrock, Phys. Rev. D **50**, 6843 (1994) [arXiv:hep-ph/9409201].