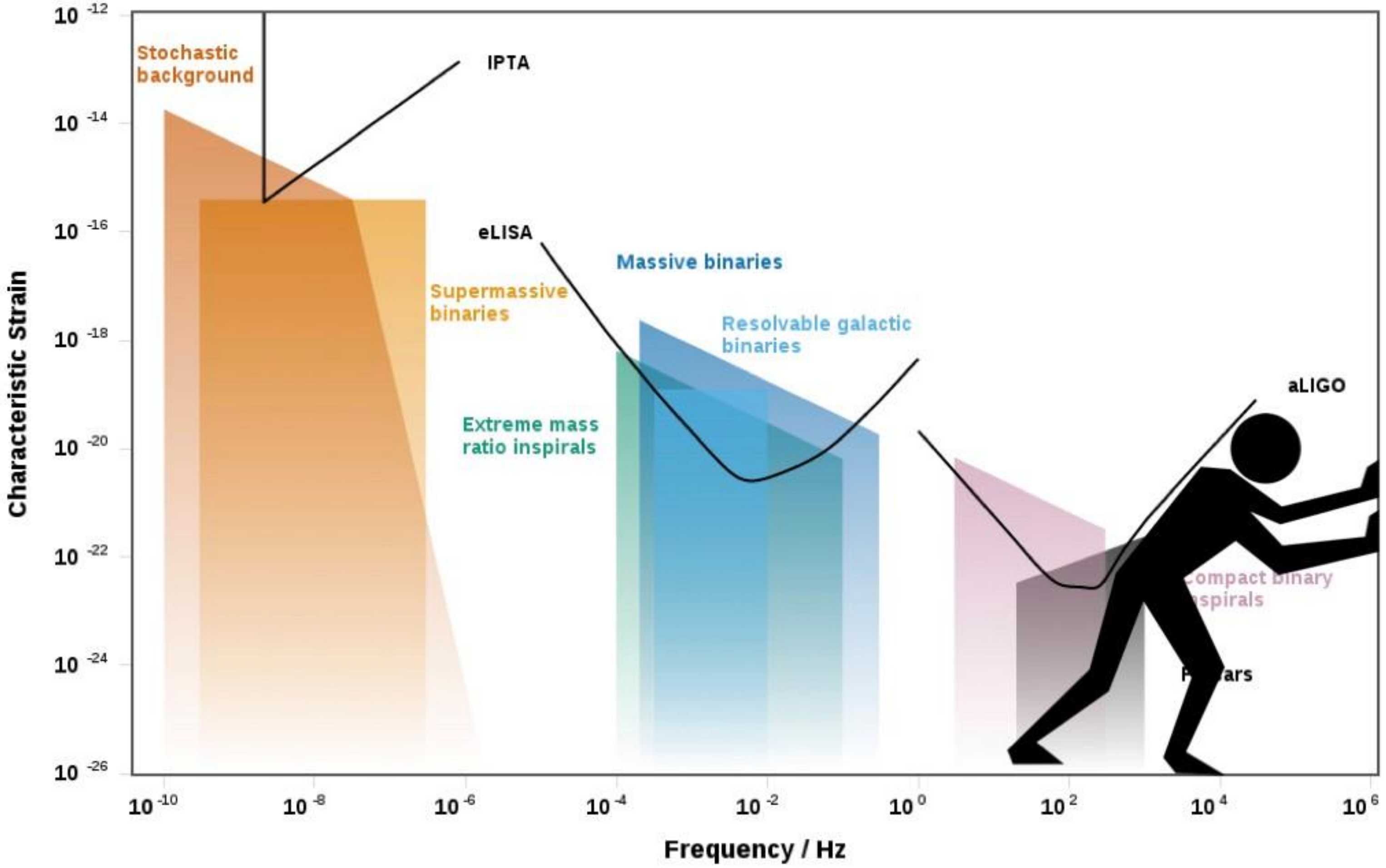
Astrophysical Probes of High Frequency Gravitational Waves

Jamie McDonald

University of Manchester

***work with Sebastian Ellis, Bjorn Garbrecht , Pete Millington**

Science Background



High-Frequency Gravitational Waves $(\nu > 10 \text{ kHz})$

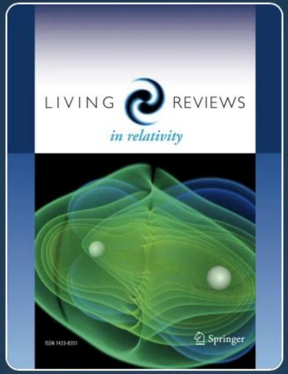
Image: https://www.ctc.cam.ac.uk/activities/UHF-GW.php

Challenges and opportunities of gravitational-wave searches at MHz to ~~GHz~~ frequencies

10^{17}GHz

Review Article | Open access | Published: 06 December 2021

Volume 24, article number 4, (2021) Cite this article



Living Reviews in Relativity

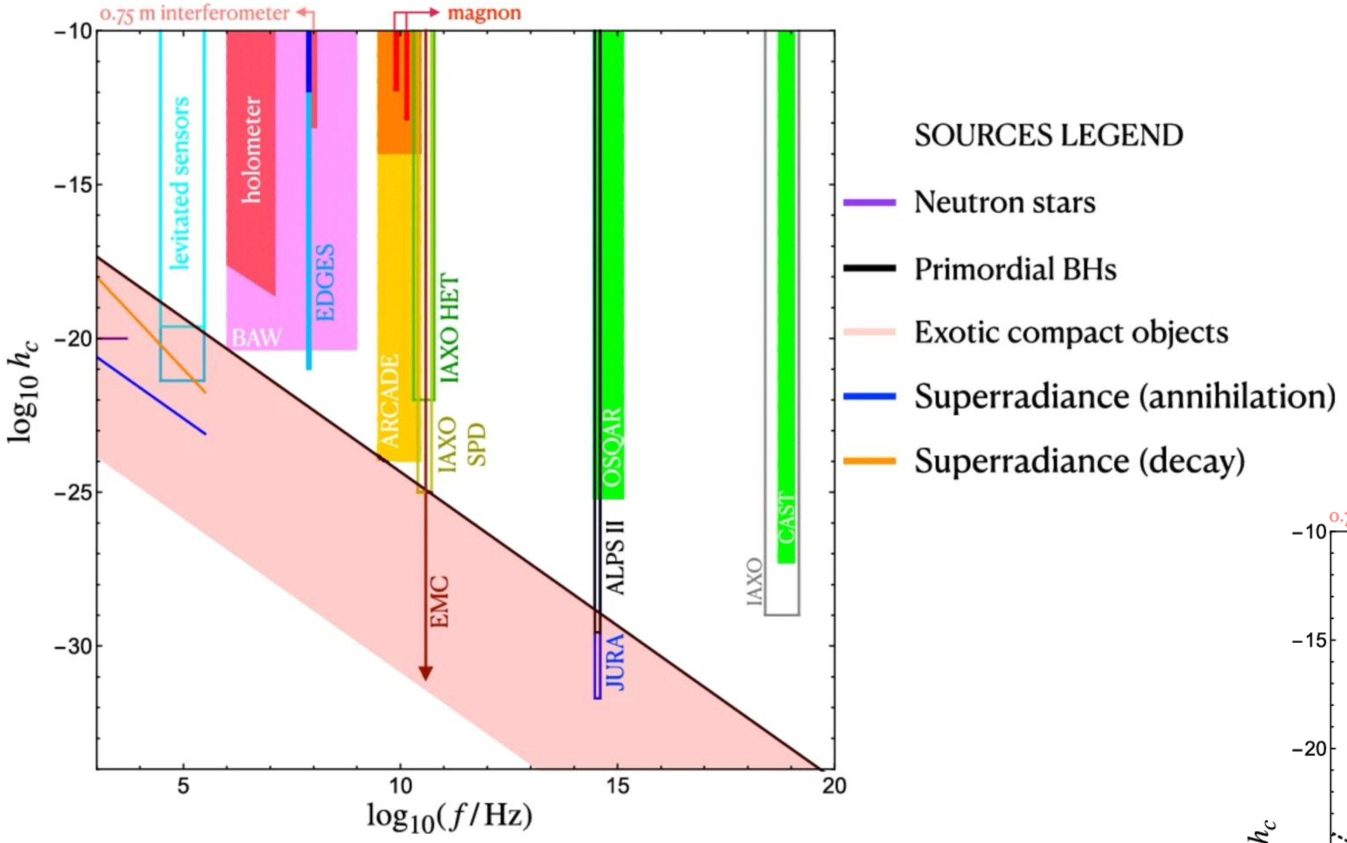
Aims and scope →

Nancy Aggarwal, Odylio D. Aguiar, Andreas Bauswein, Giancarlo Cella, Sebastian Clesse, Adrian Michael Cruise, Valerie Domcke, Daniel G. Figueroa, Andrew Geraci, Maxim Goryachev, Hartmut Grote, Mark Hindmarsh, Francesco Muia, Nikhil Mukund, David Ottaway, Marco Peloso, Fernando Quevedo, Angelo Ricciardone, Jessica Steinlechner, Sebastian Steinlechner, Sichun Sun, Michael E. Tobar, Francisco Torrenti, Caner Ünal & Graham White

Part of a collection:
Gravitational Waves

A little more on High frequency Gravitational Wave Sources

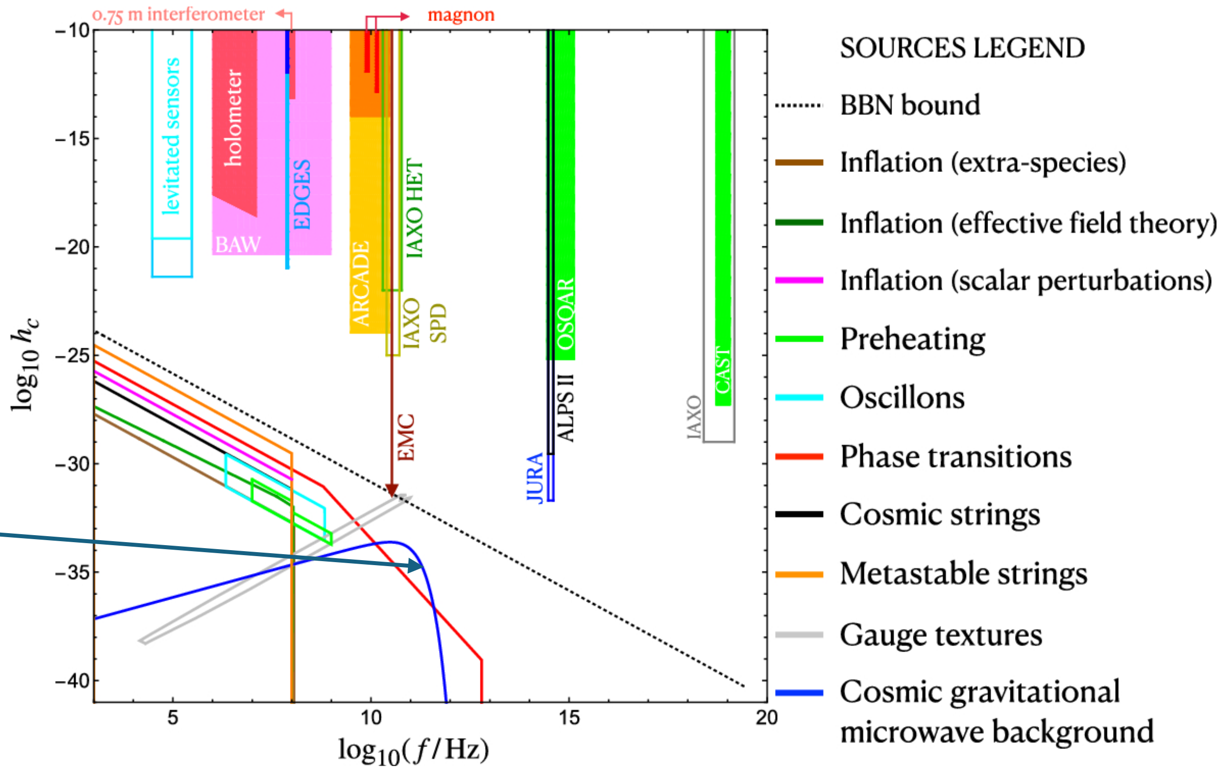
transient/continuous



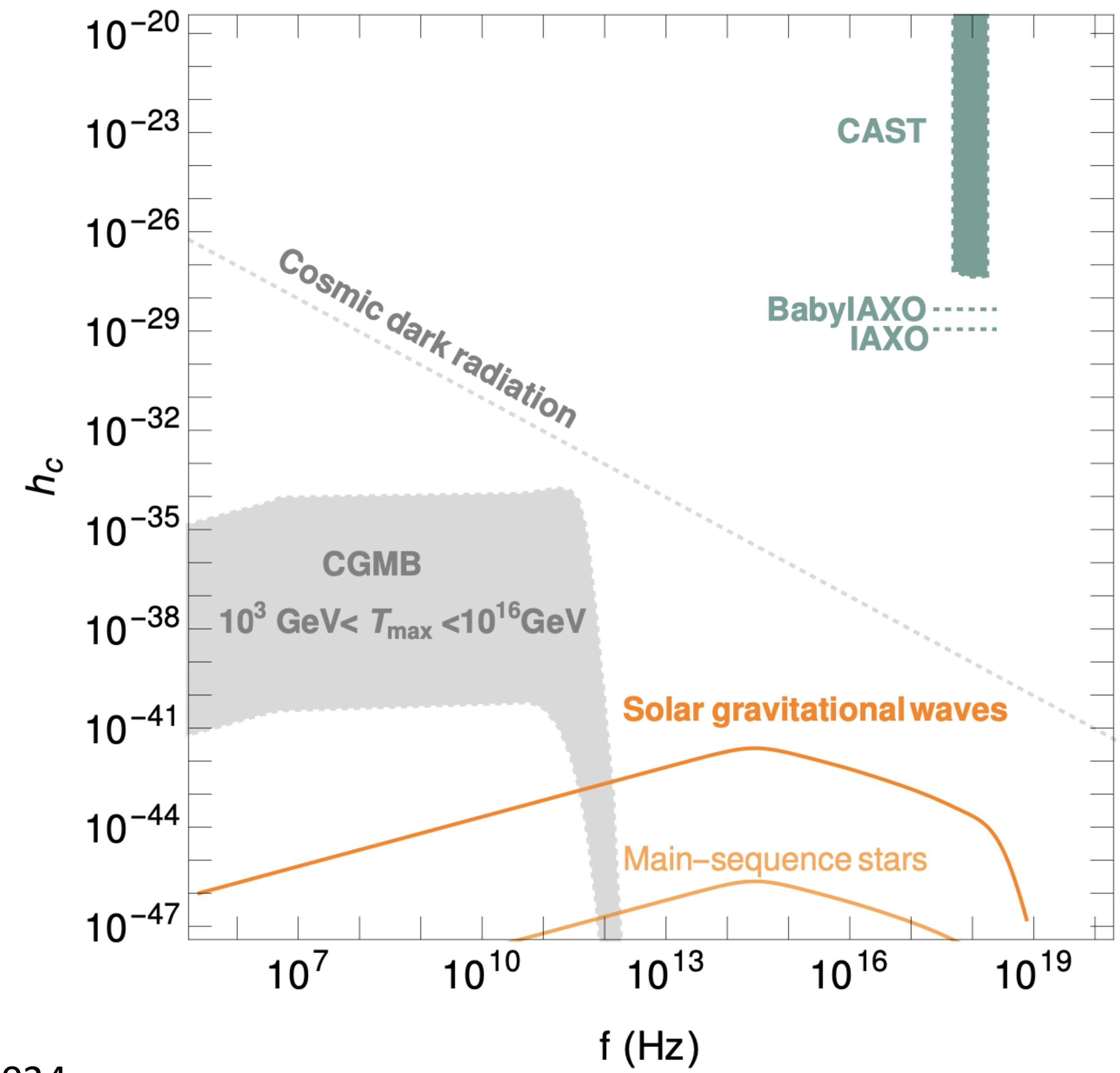
Large Review: 2011.12414

Stochastic

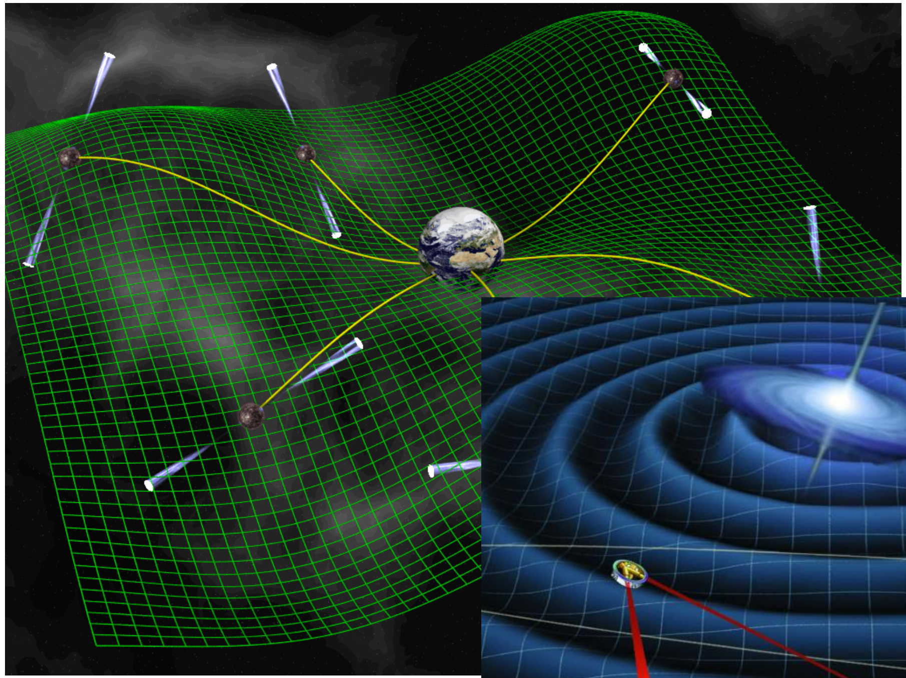
BBN Bound gives "no go theorem"



Known Standard Model Sources (as of 2024)

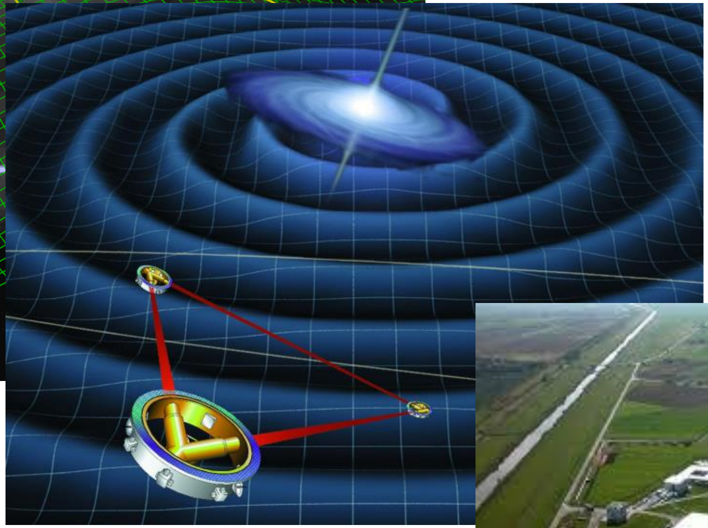


Garcia-Cely, Ringwald 2024



$10^{-9} Hz$

10 kpc ~ 10^{20} m



$10^{-2} Hz$

0.01 Au ~ 10^{9} m

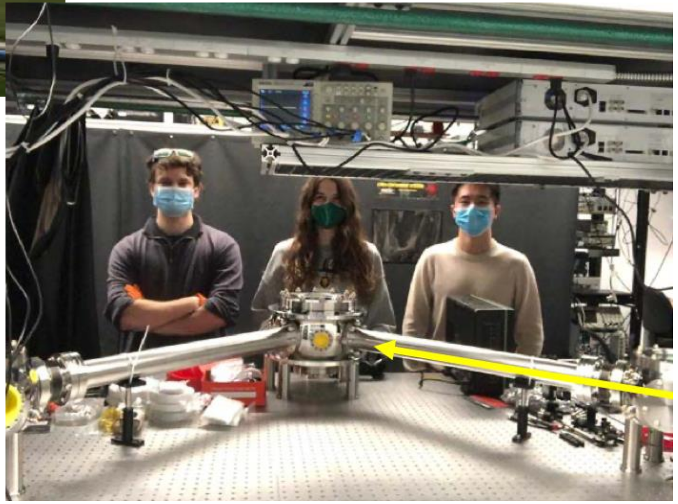
LISA

$100\ Hz$

1 km ~ 10^{3} m

Ultra-High Frequency GWs

$10^{4} Hz$ and above

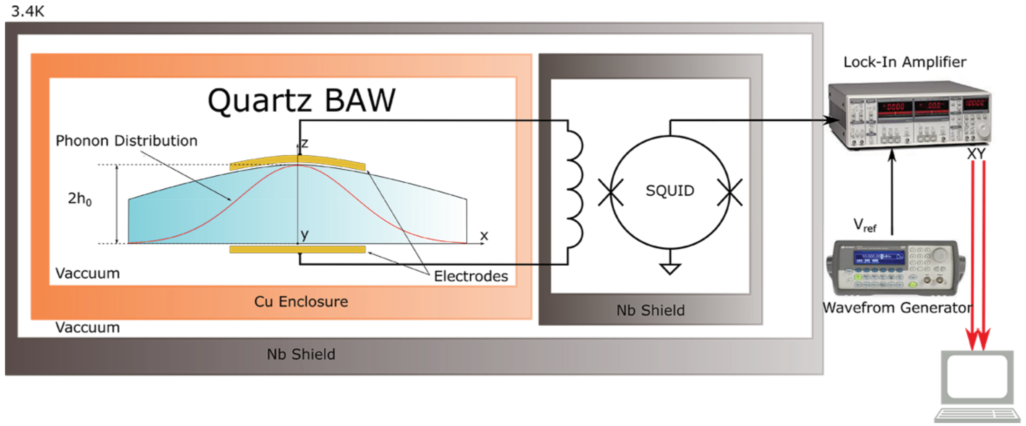


Tabletop ~ m

(+ astrophysical!)

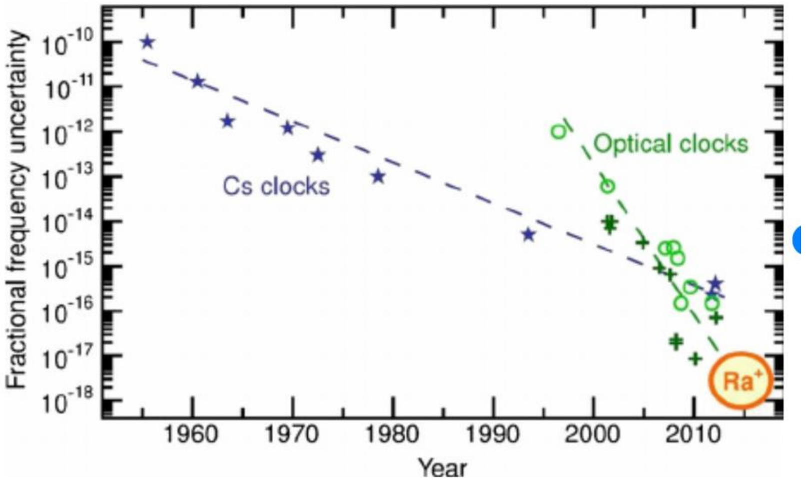
High Frequency Gravitational Waves

Bulk Acoustic Wave Resonators - 2307.00715

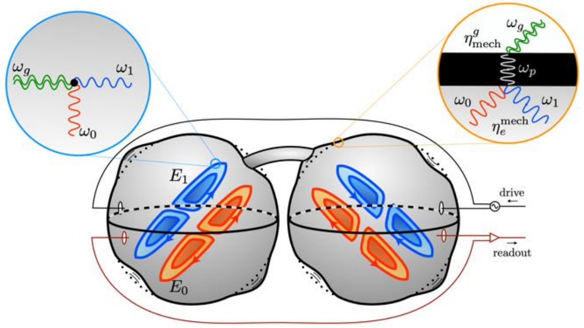


High-Frequency Gravitational Wave Detection via Optical Frequency Modulation - 2304.10579

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$
$$p^\mu = (\omega_0, \omega_0, 0, 0) + \delta p^\mu$$
$$u^\mu = (1, 0, 0, 0) + \delta u^\mu \, ,$$

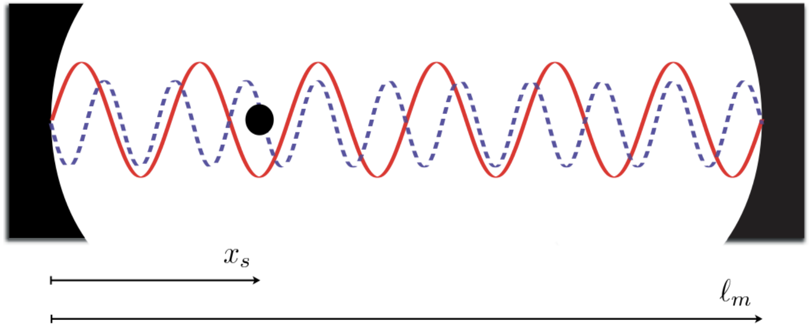


Cavities - 2303.01518



(Ballantini et al., arXiv:gr-qc/0203024 (2005))

Quantum Levitated Sensors - 1207.5320

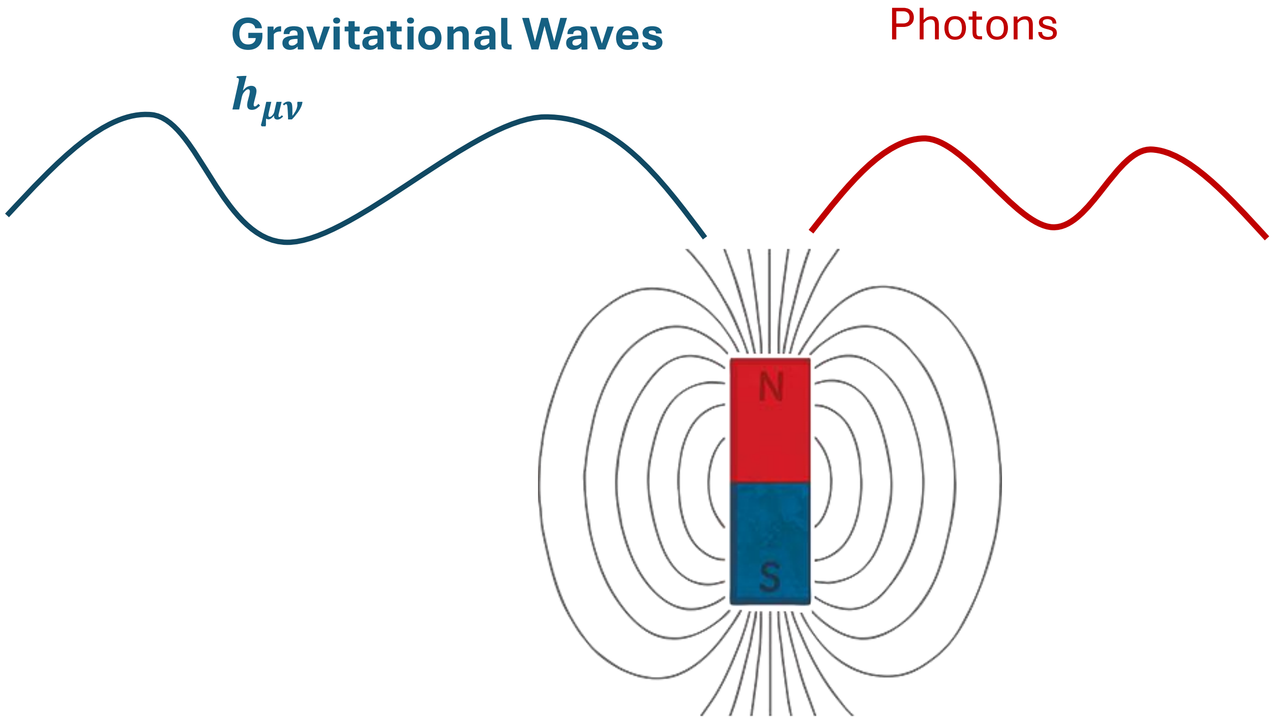


See also work by Valerie, Sebastian, Camilo, Joachim, Nick on co-opting axion experiments, 2202.00695, 2408.01483, 2409.06462

[Ultra-high frequency gravitational waves: where to next ?](#)

Dec 2023 CERN

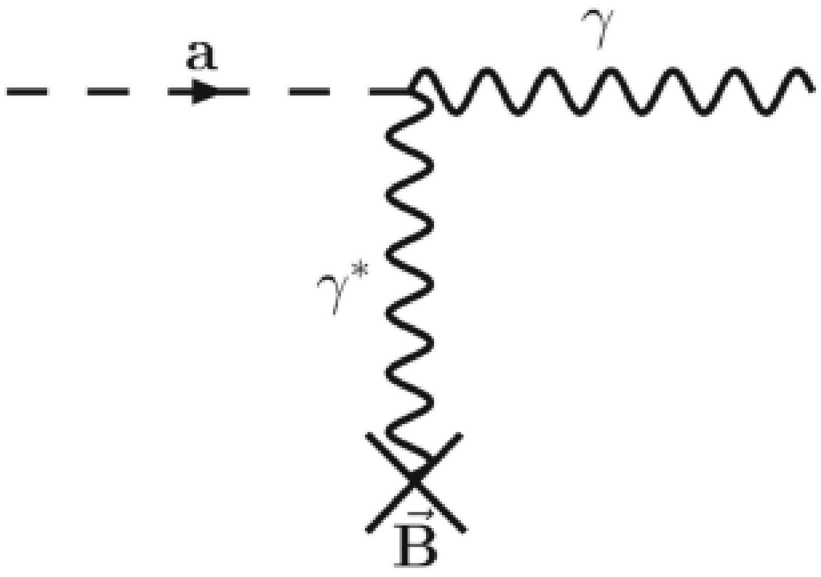
Astrophysical Detection



Astrophysical Detection

Axions

$$\mathcal{L}_{\rm int} = -\frac{g_{\phi\gamma}}{4}\phi F_{\mu\nu}\tilde{F}^{\mu\nu} + \frac{g_{\phi N}}{2m_N}\partial_\mu\phi(\bar{N}\gamma^\mu\gamma_5 N) + \frac{g_{\phi e}}{2m_e}\partial_\mu\phi(\bar{e}\gamma^\mu\gamma_5 e)$$



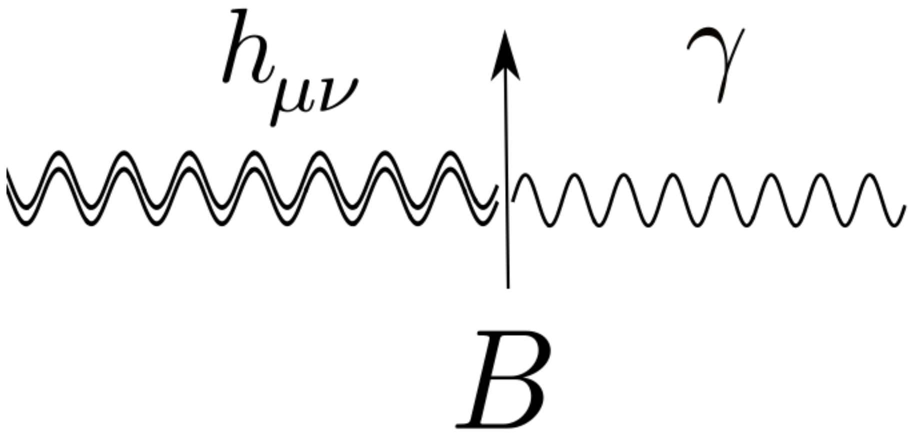
Inverse Primakoff

High-Frequency Gravitational Waves

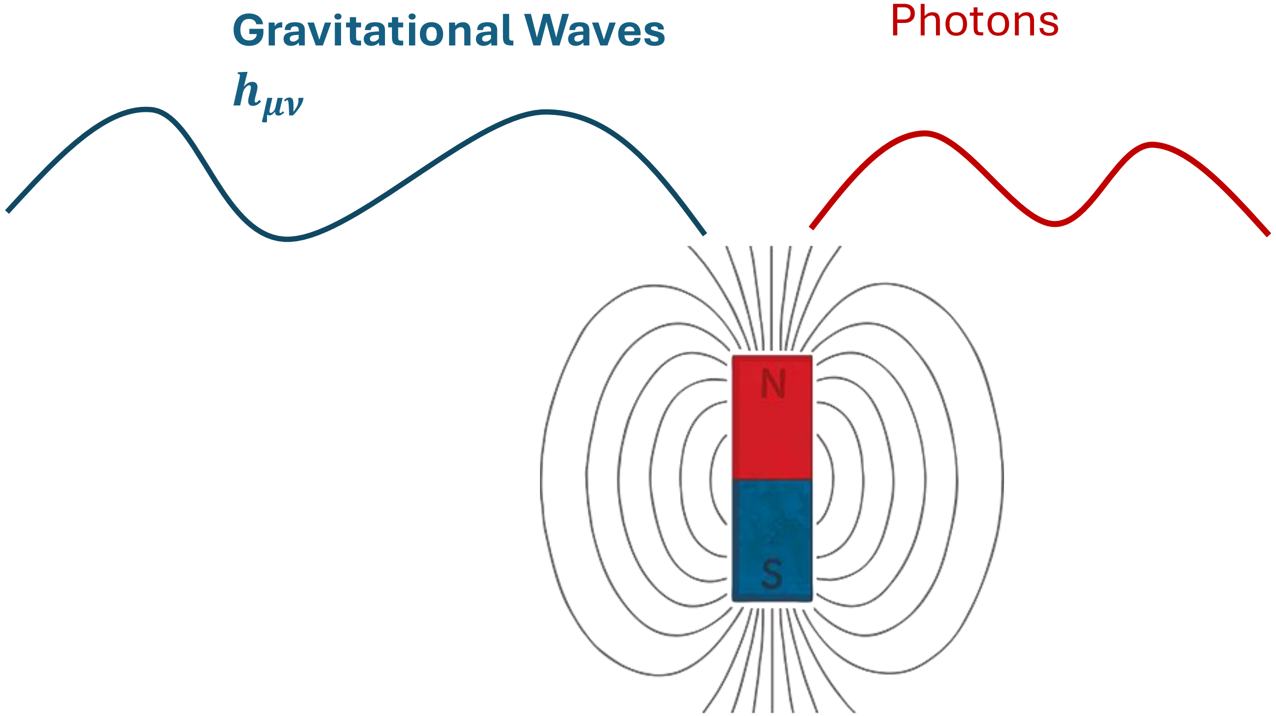
$$S = \int d^4x\, \sqrt{-g}\left(-\frac{1}{4}\, g^{\mu\alpha}\, g^{\nu\beta}\, F_{\mu\nu}\, F_{\alpha\beta}\right),$$

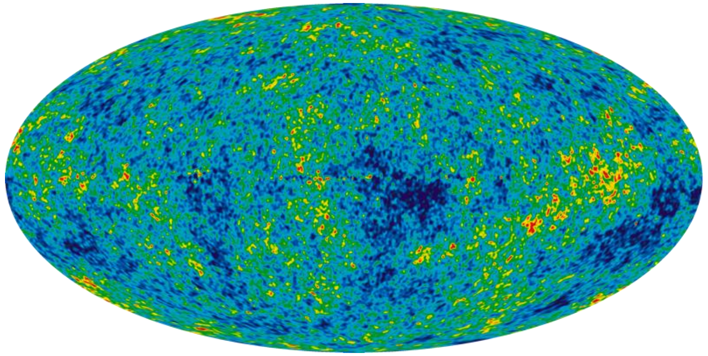
$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

(GWs)



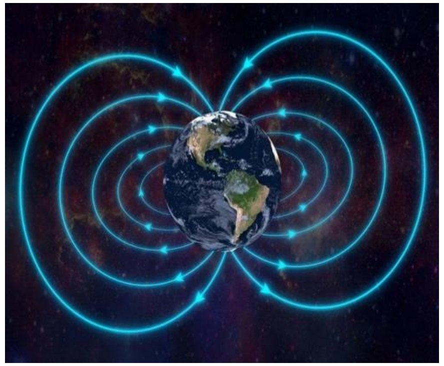
Inverse Gertsenshtein



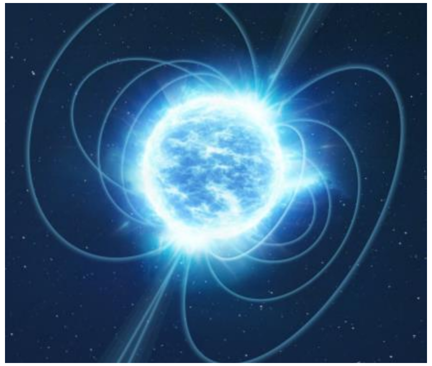


CMB (spectral distortions)

Galaxies

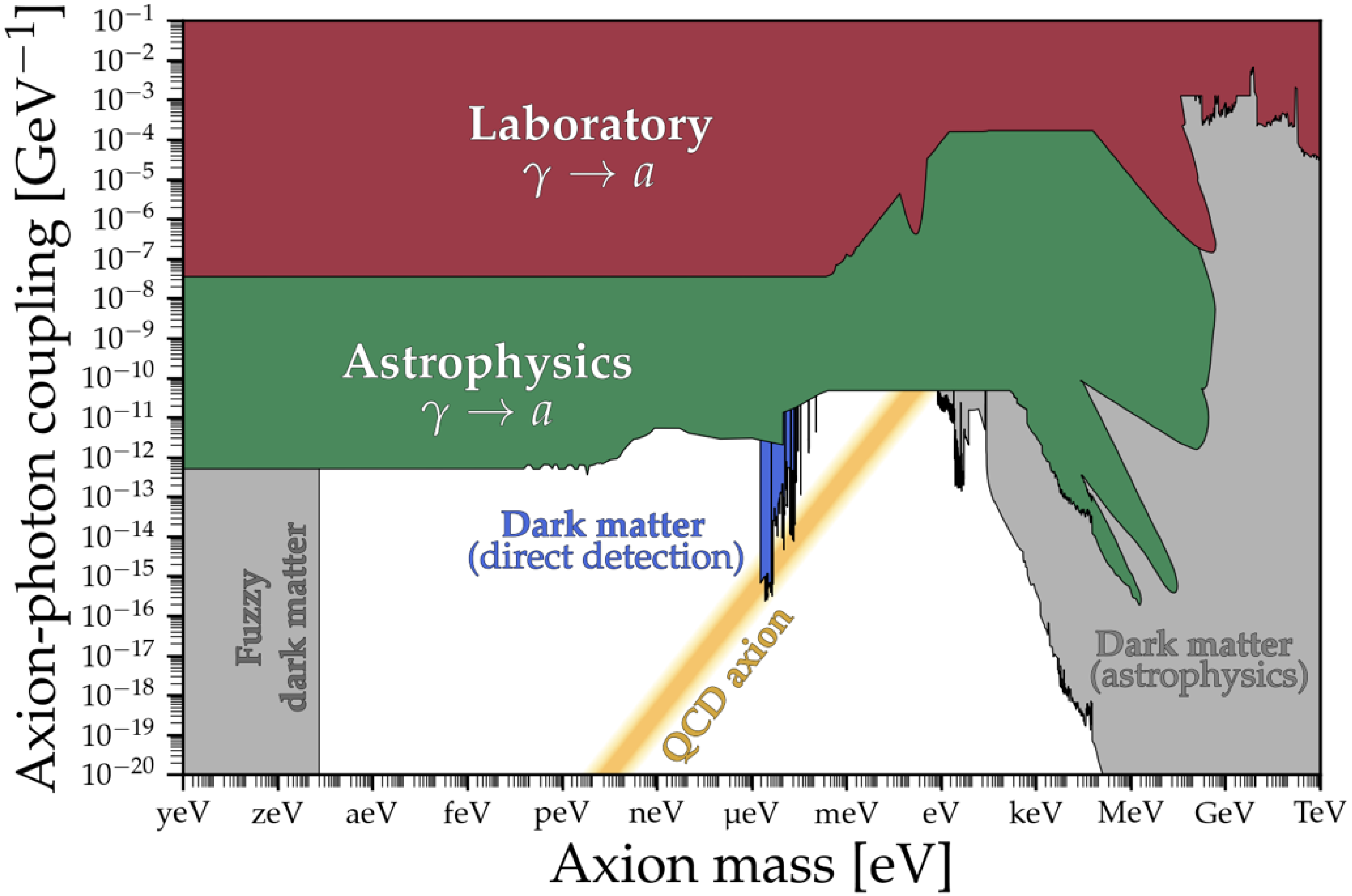


Earth



Neutron Stars

Rich array of astrophysical constraints on axions



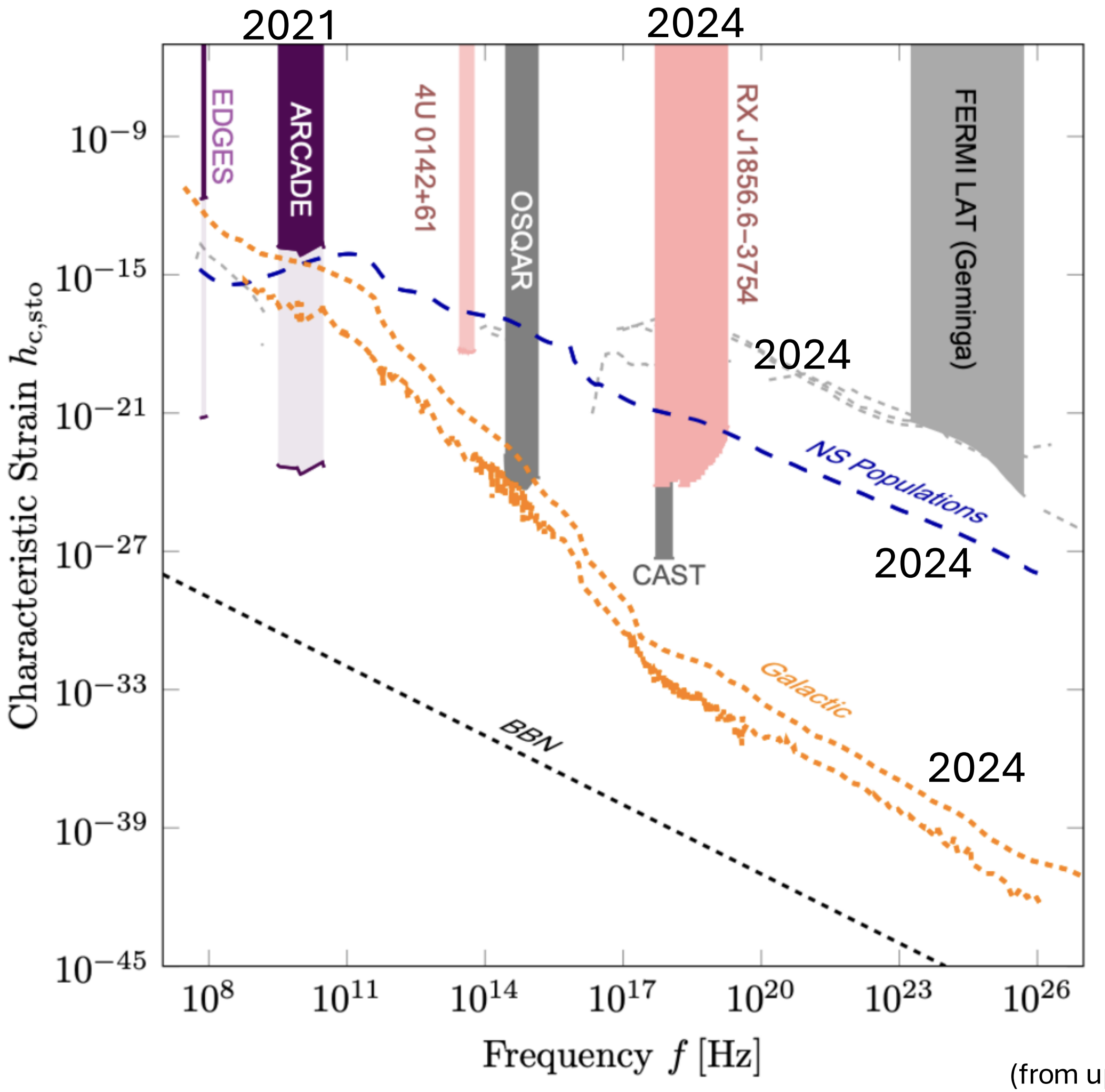
https://cajohare.github.io/AxionLimits/docs/ap.html

Astrophysical Constraints on High-Frequency Gravitational Waves

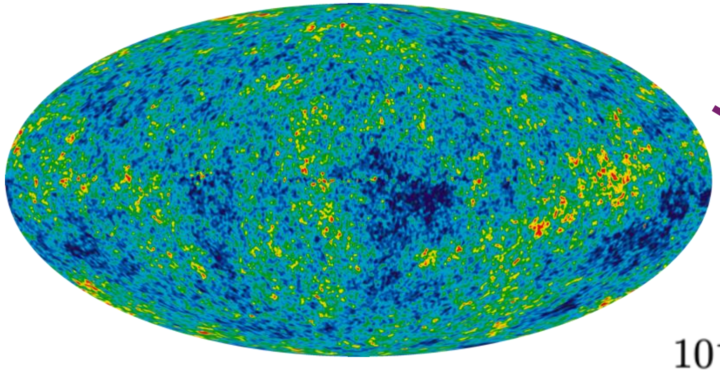
A Few Years Ago:

Astrophysical Constraints on High-Frequency Gravitational Waves

Today :



(from update to *Living Rev. Rel.* 24 (2021) 1, 4 – in prep)

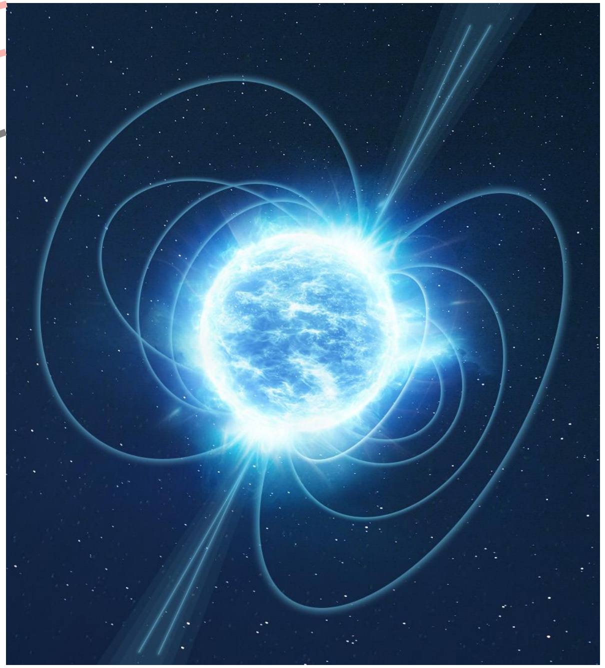


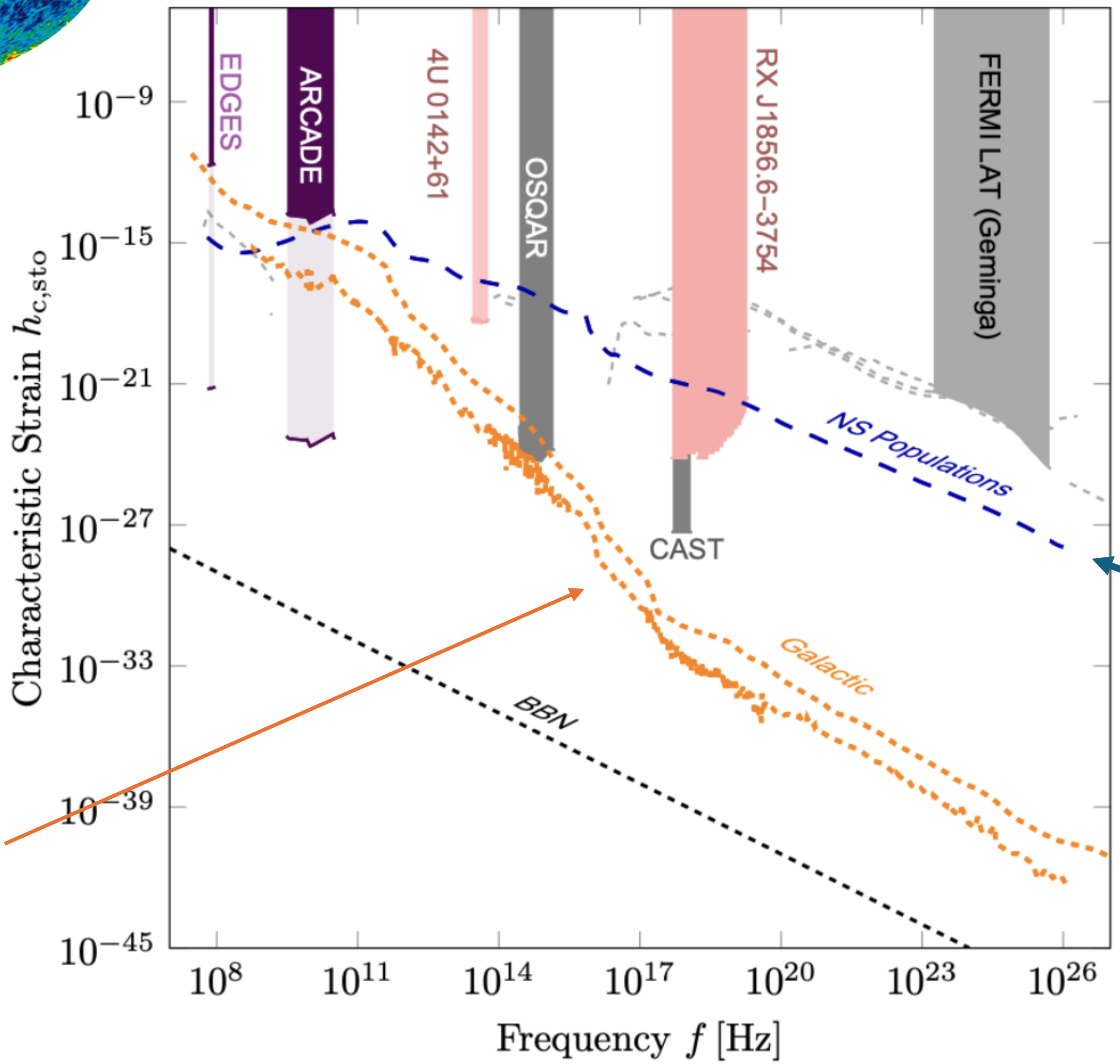
CMB

Domcke, Garcia-Cely 2021

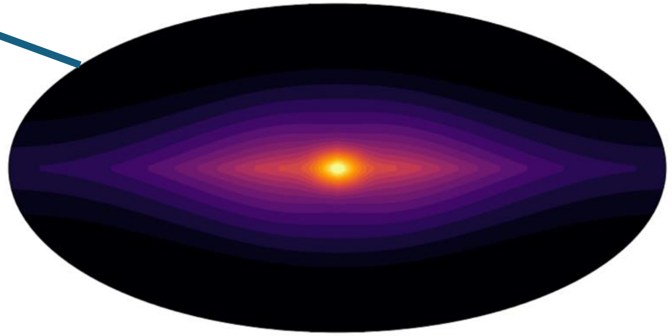
JM, Ellis (2024)

Ito, Kohri, Nakayama (2024)





Neutron Stars

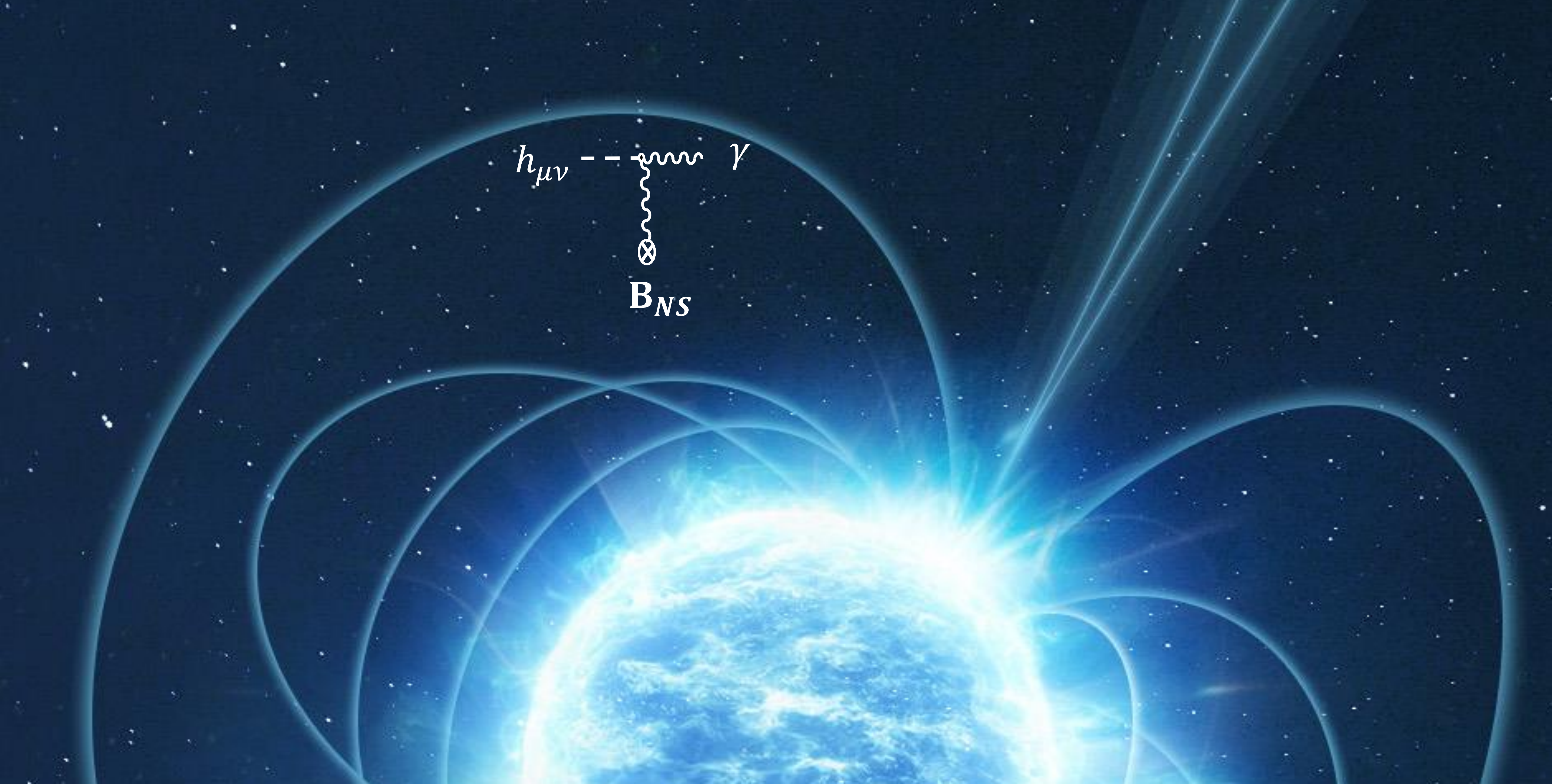


Dandoy, Bertolez-Martinez, Costa (2024)

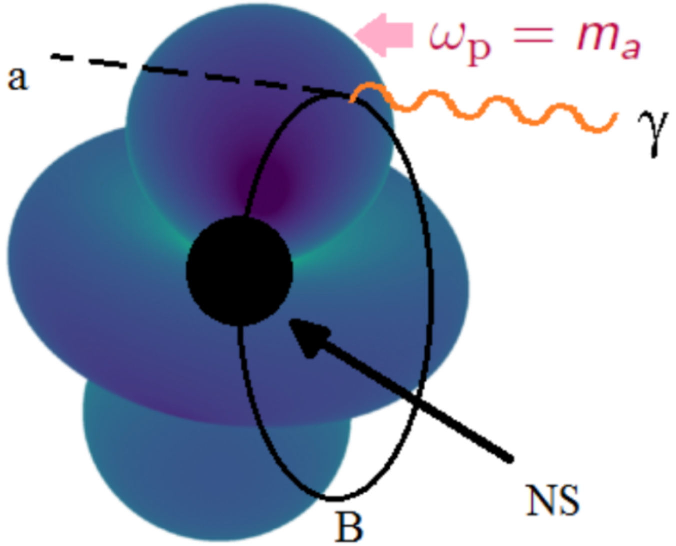
Galactic

Lella, Calore, Carenza, Mirizzi (2024)

Probing High-Frequency Gravitational Waves with Neutron Stars



Resonant Axion DM Conversion Around Neutron Stars



$$P_{a\to\gamma} \sim \frac{g_{a\gamma\gamma}^2 B^2}{\frac{d}{dz}(\omega_p(x_{res}))}$$

A. Hook, Y. Kahn B. Safdi, Z. Sun Phys. Rev. Lett. 121 (2018) 24, 241102

F. P. Huang, K. Kadota, T. Sekiguchi, H. Tashiro Phys.Rev.D 97 (2018) 12, 123001

M.S. Pshirkov, S.B. Popov J. Exp. Theor. Phys. 108 (2009) 384-388 (Original Proposal!)

Observations

Stellar Populations

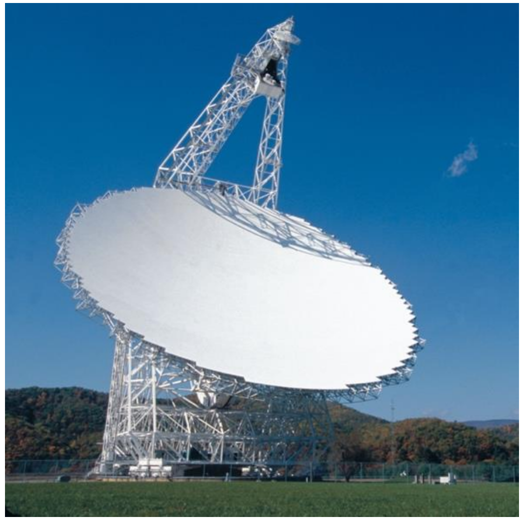
Foster et al, Phys. Rev. Lett. 129, 251102 (2022)
[GBT, Galactic Centre]

Foster et al *Phys. Rev. Lett.* 125 (2020) 17, 171301]
[GBT, Effelsberg, Galactic Centre + Isolated NSs]
Battye, Bhura, JM, Srinivasan (2407.19028 in press JCAP)

Single Objects

Darling [*Phys. Rev. Lett.* 125 (2020) 12, 121103]
Battye, Darling, **JM**, Srinivasan
[Phys. Rev. D 105 (2022) 2, L021305]
[Galactic Centre Magnetar, VLA, PSR J1745−2900]

Battye, Keith, **JM**, Srinivasan, Stappers, Weltevrede
[Phys. Rev. D 108 (2023) 6, 063001]
[MeerKAT, **matched-filter/time-domain search** PSR J2144-3933]



Green Bank Telescope (GBT) - USA

Effelsberg - Germany

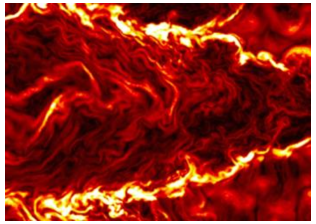
Very Large Array (VLA) - USA

MeerKAT – S. Africa

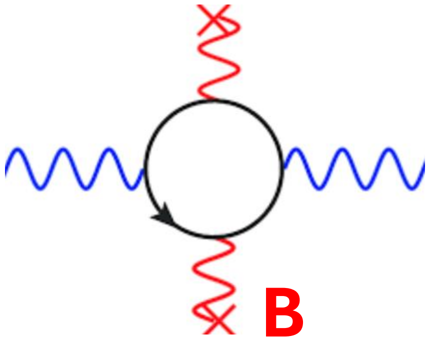
Resonant Gravitational Wave Photon Conversion

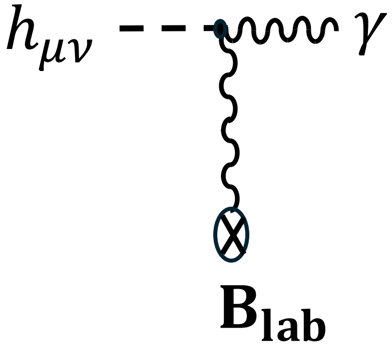
$$n^2 = 1 - \frac{\omega_p^2}{\omega^2} + \frac{16\,\alpha^2 B^2}{45\, m_e^4}$$

Plasma



Vacuum Birefringence

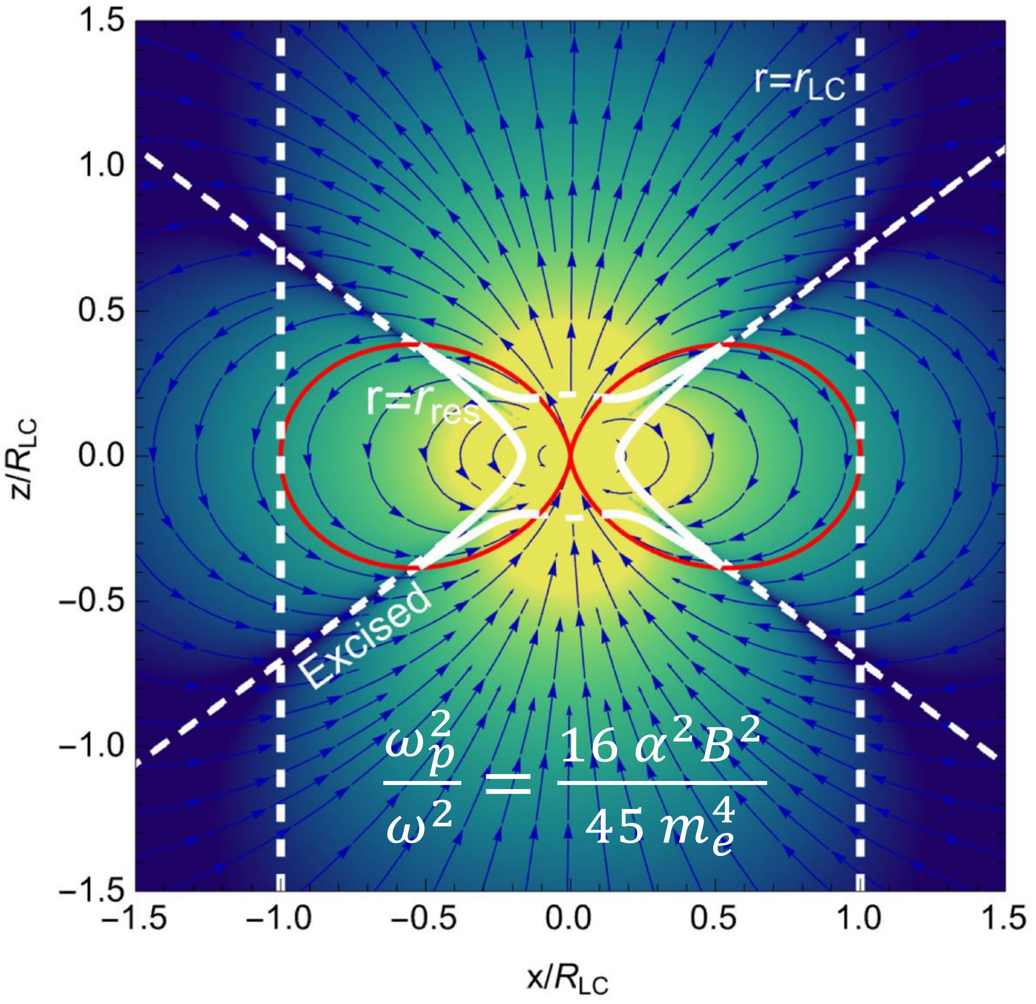




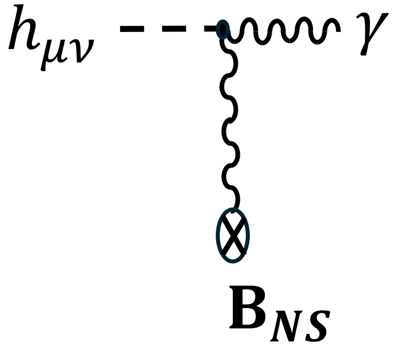
Resonance Occurs When : $\frac{\omega_p^2}{\omega^2} = \frac{16\,\alpha^2 B^2}{45\, m_e^4}$

Applications on High Frequency Gravitational Waves

JM, S. Ellis - PRD



(Goldreich Julian Model 1969)



Axion-Photon Conversion in 3D Plasmas

$$\nabla \cdot \mathbf{D} = -g_{\mathrm{a}\gamma\gamma}\mathbf{B} \cdot \nabla a$$

$$\nabla \times \mathbf{B} - \dot{\mathbf{D}} = g_{\mathrm{a}\gamma\gamma}\dot{a}\mathbf{B} - g_{\mathrm{a}\gamma\gamma}\mathbf{E} \times \nabla a$$

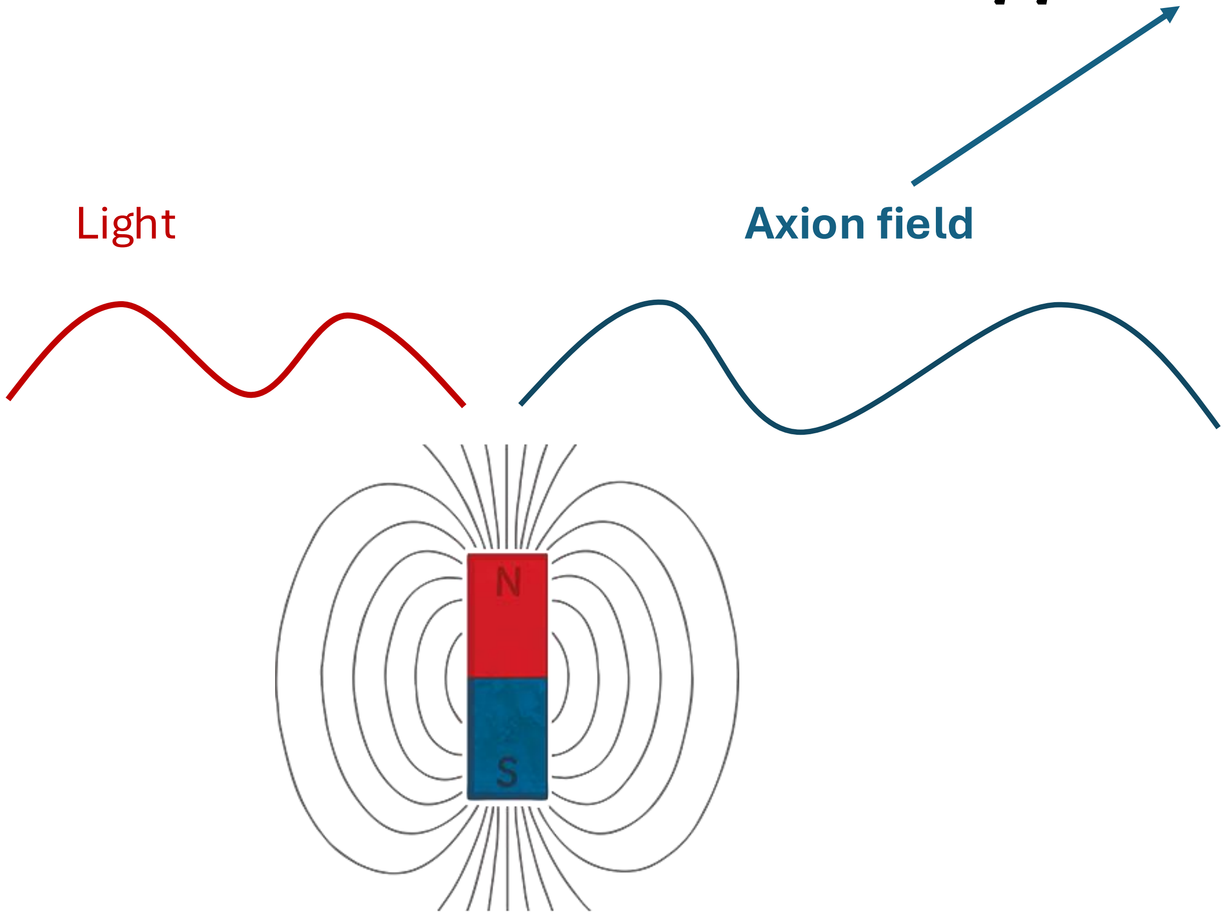
$$\nabla \cdot \mathbf{B} = 0$$

$$\dot{\mathbf{B}} + \nabla \times \mathbf{E} = 0$$

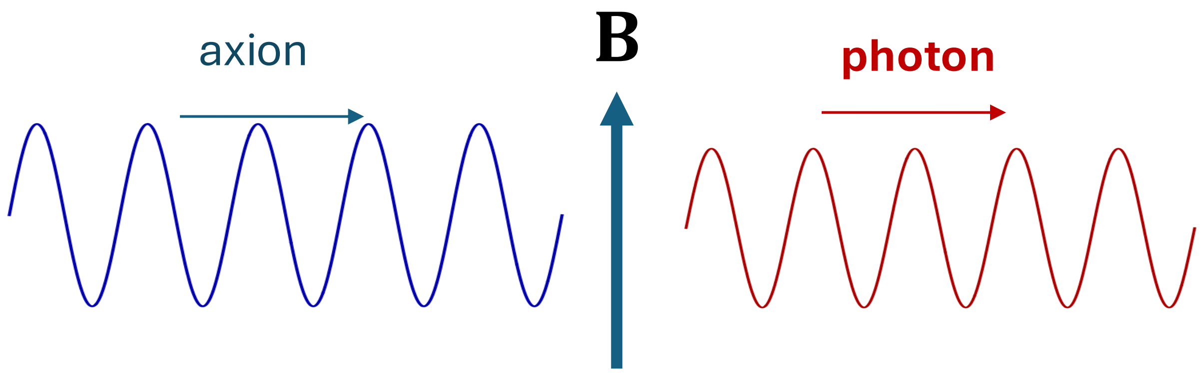
Maxwell's Equations

Axion-Photon Conversion in 3D Plasmas

$$-\boldsymbol{\nabla}^2 \mathbf{E} + \boldsymbol{\nabla}(\boldsymbol{\nabla} \cdot \boldsymbol{E}) - \boldsymbol{\omega}^2 \varepsilon . \boldsymbol{E} = \boldsymbol{g}_{a\,\gamma\gamma}\boldsymbol{\omega}^2 \boldsymbol{a}\, \boldsymbol{B}$$



Solve a Toy Problem in 1D ?



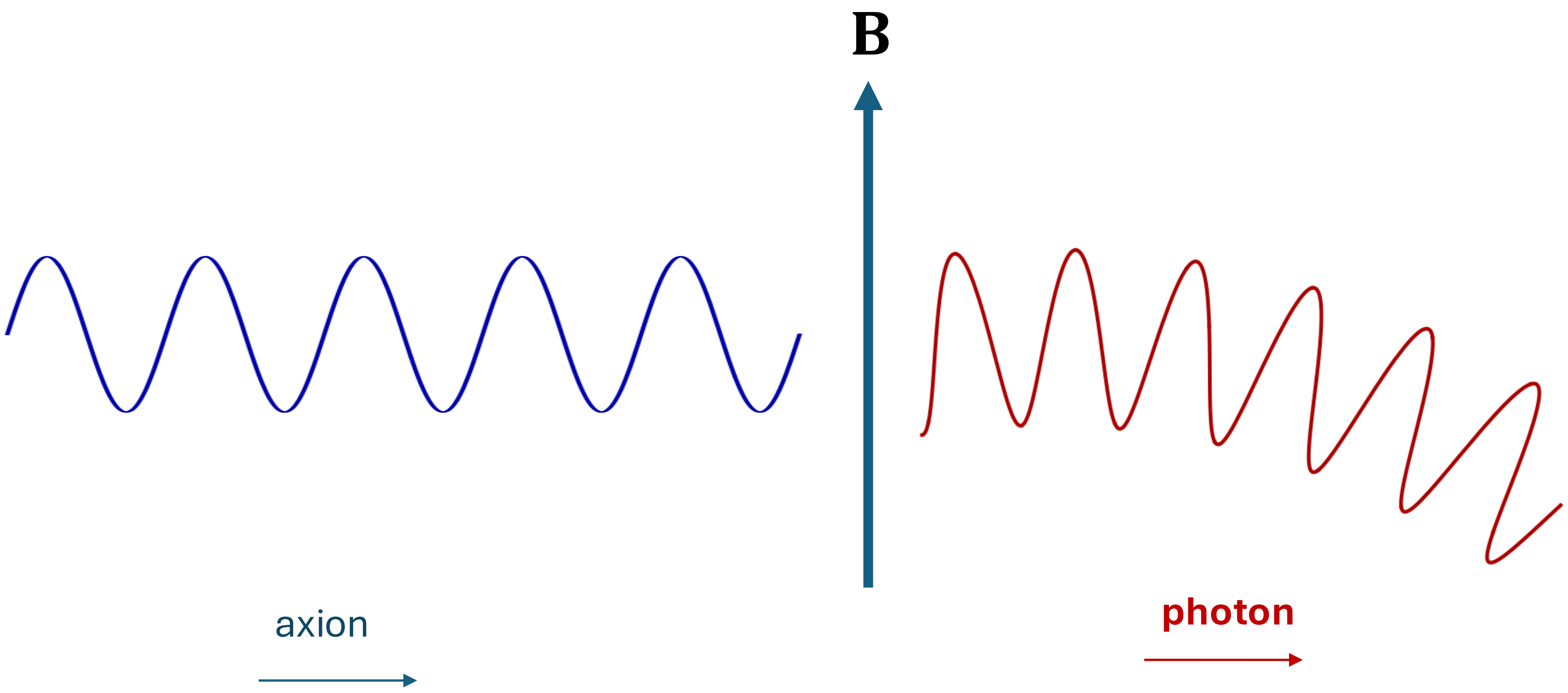
$$\left[\omega + \begin{bmatrix}\Delta_\perp & 0 & 0\\ 0 & \Delta_\parallel & \Delta_M \\ 0 & \Delta_M & \Delta_a\end{bmatrix} - i\partial_z\right]\begin{bmatrix}A_\perp\\ A_\parallel\\ a\end{bmatrix}=0 \,,$$

☑ dilute plasmas ☑ relativistic limit

- Magnetized plasmas: eigenmodes are polarized both parallel and perpendicular to **B**
- Not valid in 3D
- Longitudinal effects ($\nabla . E \neq 0$) dense plasmas have longitudinal excitations
- Conductivity in an anisotropic medium can mix different components (⊥, ||)
- Axions and photons have different worldlines: ∂_z

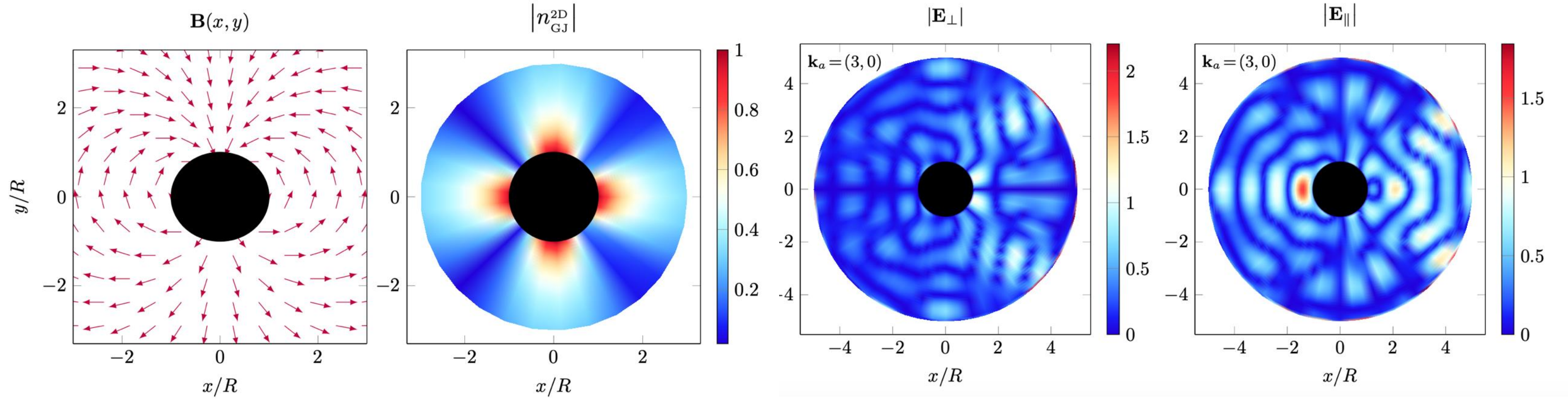
Raffelt, Stodolsky Phys. Rev. D 37 (1988) 1237

Solve a Toy Problem in 1D ?



Solve Numerically ?

$$-\nabla^2 \mathbf{E} + \nabla(\nabla \cdot E) - \omega^2 \varepsilon \cdot E = g_{a\gamma\gamma}\omega^2 a\,\mathbf{B}$$



Huge Numerical Hierarchies !

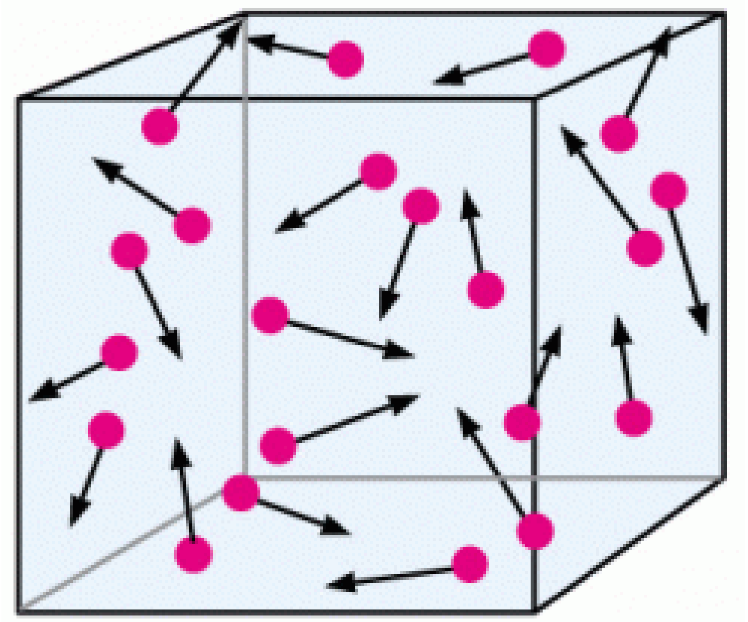
Battye, Garbrecht, … et al Phys. Rev. D 102 (2020) 2, 023504

See also - Gines, Noordhuis, Weniger, Witte [hep-ph/Xiv:2405.08865]

Analytical Solutions : Two Approaches

Kinetic Theory

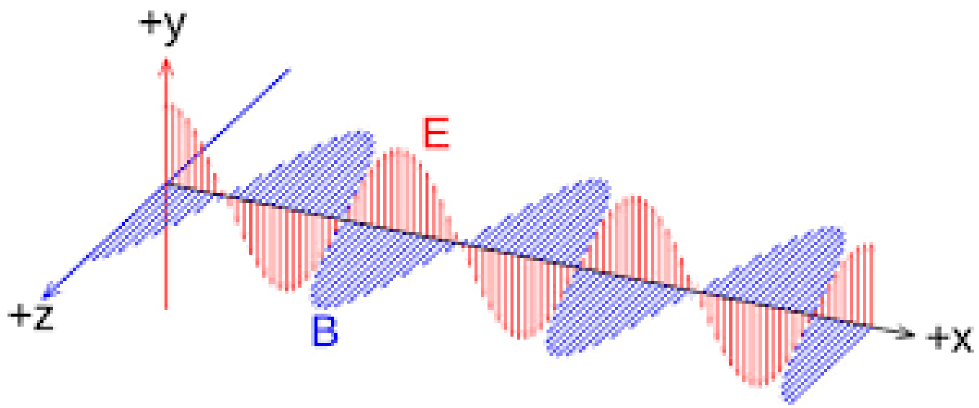
[**JM**, *Millington, Garbrecht, JCAP* 12 (2023) 031]



Photon Phase Space

Wave Equations

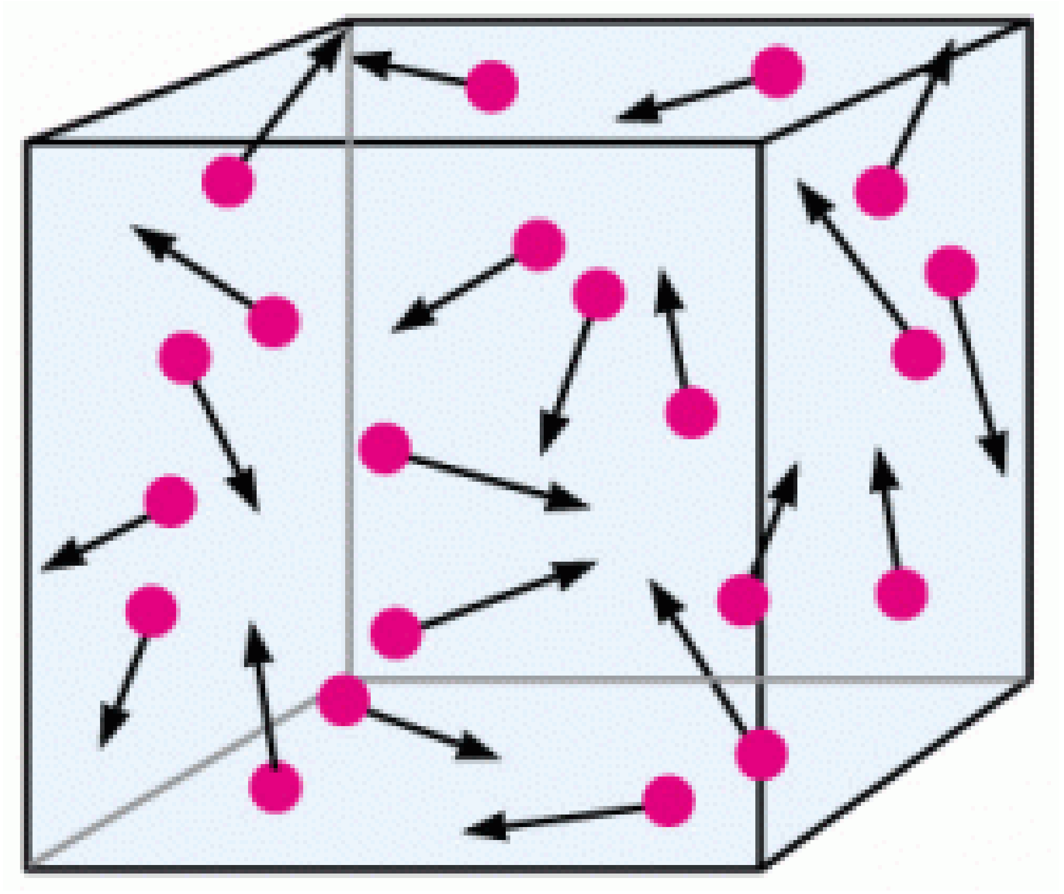
JM, Millington JCAP 09 (2024) 072



$$P^{3D}_{a\to\gamma}$$

Kinetic Theory

$$f_\gamma = f_\gamma(\mathbf{k}\,,\mathbf{x})$$



Photons

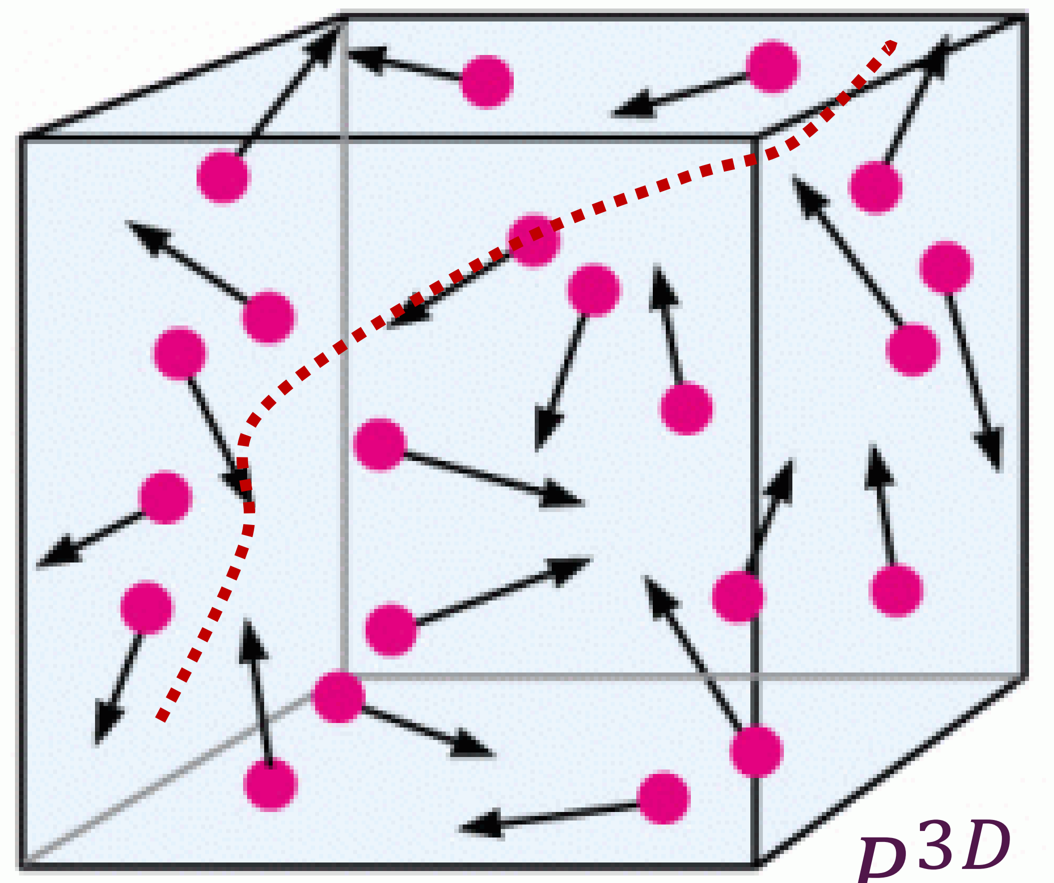
$$\partial_t\, f_\gamma + \boldsymbol{v}_{\boldsymbol{\gamma}} \cdot \nabla_{\mathbf{x}}\, f_\gamma + F_\gamma\,.\nabla_{\boldsymbol{k}} f_\gamma = C_a\, f_a$$

$$C_a \sim\ g_{a\gamma\gamma}^2\,\delta\left(E_\gamma^2 - E_a^2\right)E_\gamma\left|\boldsymbol{\epsilon}_\gamma.\boldsymbol{B}\right|^2$$

[**JM**, *Millington, Garbrecht, JCAP* 12 (2023) 031]

Kinetic Theory

$f_\gamma = f_\gamma(\mathrm{k}\,,\,\mathrm{x})$



$$\frac{d\boldsymbol{x}_\gamma}{d\lambda} = \boldsymbol{v}_g, \qquad \frac{d\boldsymbol{k}_\gamma}{d\lambda} = -\nabla_x E_\gamma:$$

$$C_a \sim g_{a\gamma\gamma}^2\, \delta\left(E_\gamma^2 - E_a^2\right) E_\gamma \left|\boldsymbol{\epsilon}_\gamma . \boldsymbol{B}\right|^2$$

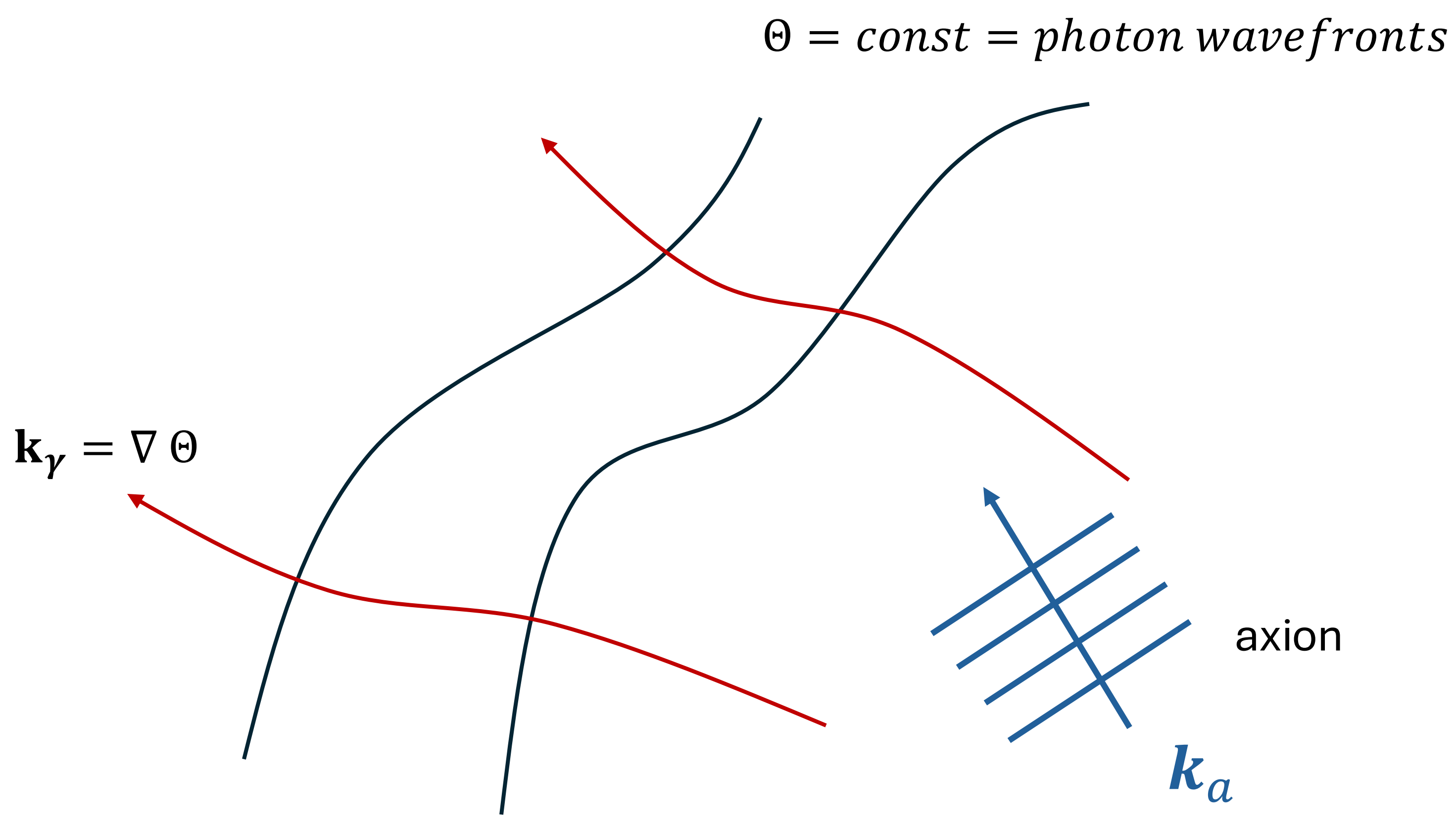
$$\frac{\mathrm{d}f_\gamma}{d\lambda} = C_a f_a$$

$$P_{a\to\gamma}^{3D} = f_\gamma / f_\mathrm{a} \sim \frac{g_{a\gamma\gamma}^2 E_\gamma \left|\boldsymbol{B}\cdot\boldsymbol{\epsilon}_\gamma\right|^2}{\left|\boldsymbol{k}\,\cdot\,\nabla_{\boldsymbol{x}} E_\gamma\right|}$$

[**JM**, *Millington, Garbrecht, JCAP* 12 (2023) 031]

Solutions via Classical Wave Equations $\mathbf{E} = \mathcal{A}\,\hat{\boldsymbol{\epsilon}}\,e^{i\Theta(\mathbf{x})}$

$$-\boldsymbol{\nabla}^2\,\mathbf{E} + \boldsymbol{\nabla}(\boldsymbol{\nabla}\cdot\boldsymbol{E}) - \boldsymbol{\omega}^2\boldsymbol{\varepsilon}\cdot\boldsymbol{E} = \boldsymbol{g}_{a\,\gamma\gamma}\boldsymbol{\omega}^2 a_0 e^{i\,\mathbf{k}_a\,.\,\mathbf{x}}\,\mathbf{B}$$



JM, Millington JCAP 09 (2024) 072

Solutions via Classical Wave Equations $\mathbf{E} = \mathcal{A}\,\hat{\boldsymbol{\epsilon}}\,e^{i\Theta(\mathbf{x})}$

Transport Field Amplitude

$$\mathbf{v}_g^\gamma \cdot \nabla\mathcal{A} + \chi\mathcal{A} = g_{a\gamma\gamma}\omega a_0 \left(\mathbf{B}_{\text{ext.}} \cdot \hat{\boldsymbol{\epsilon}}^*\right) \frac{U_E}{U_\gamma} e^{i(\mathbf{k}_a \cdot \mathbf{x} - \Theta)}$$

Transport Energy Density

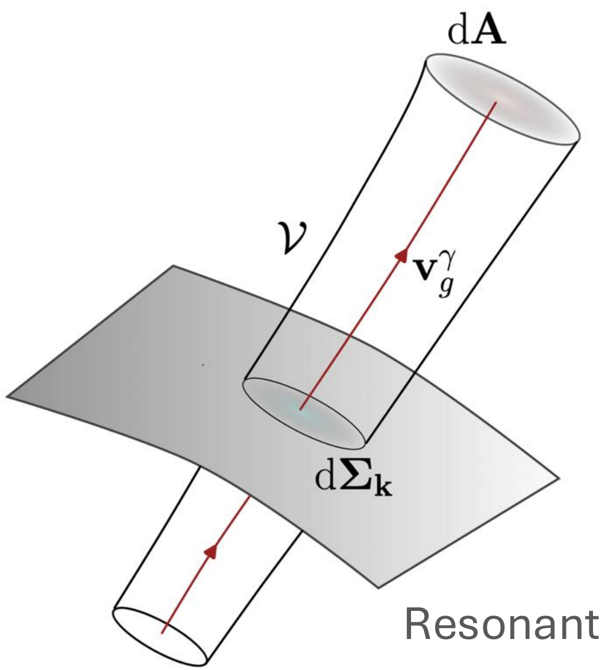
$$\mathbf{v}_g^\gamma \cdot \nabla U_\gamma + \left(\nabla \cdot \mathbf{v}_g^\gamma\right) U_\gamma = \frac{1}{4} g_{a\gamma\gamma}\omega^2 a_0 \left(\mathbf{B}_{\text{ext.}} \cdot \hat{\boldsymbol{\epsilon}}^*\right) e^{i(\mathbf{k}_a \cdot \mathbf{x} - \Theta)} \mathcal{A}^* + \text{H.c.}$$

Poynting/axion Flux

$$\mathbf{S}_\gamma = \mathbf{v}_g^\gamma U_\gamma \qquad \mathbf{S}_a = \mathbf{v}_g^a U_a$$

$$\int \mathrm{d}\mathbf{A} \cdot \mathbf{S}_\gamma = \int \mathrm{d}\Sigma_{\mathbf{k}} \cdot \mathbf{S}_a P_{a\gamma}$$

$$P_{a\gamma} = \frac{\pi g_{a\gamma\gamma}^2 \left|\mathbf{B}_{\text{ext.}} \cdot \hat{\boldsymbol{\epsilon}}\right|^2}{\left|\mathbf{v}_g^a \cdot \nabla E_\gamma\right|} \frac{U_E}{U_\gamma}$$



Resonant Surface

JM, Millington JCAP 09 (2024) 072

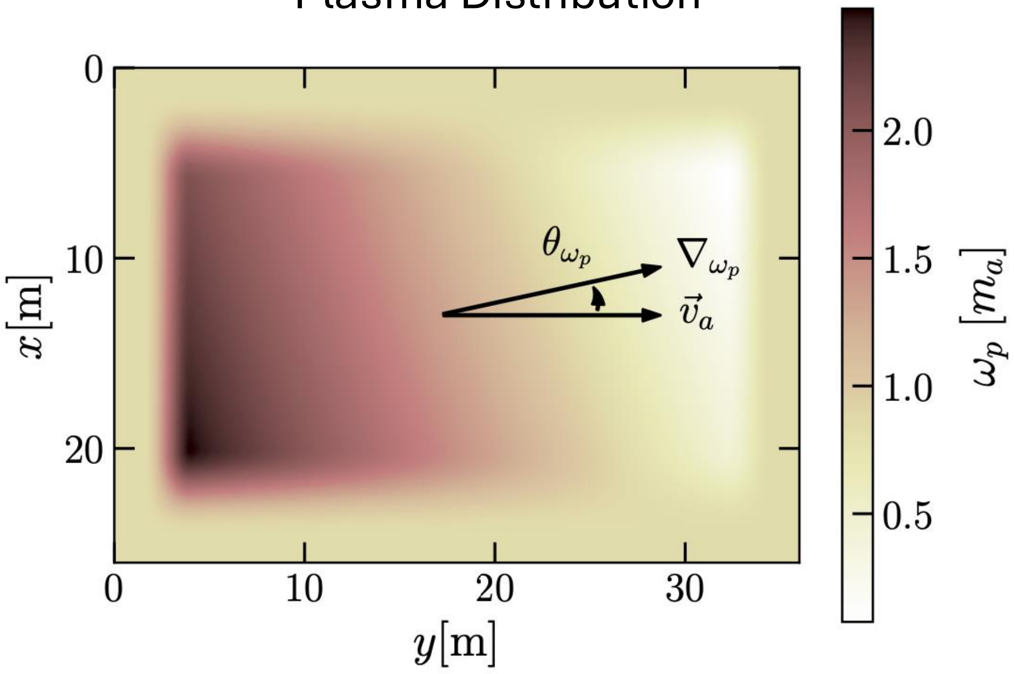
Universal Form of Resonant Conversion Probability

$$P^{3D}_{X\to\gamma} \sim \frac{\pi |M_{X\to\gamma}|^2}{E_\gamma |\boldsymbol{k} \cdot \nabla_{\boldsymbol{x}} E_\gamma|}$$

- ✓ Easily adaptable for arbitrary particles/spins (dark photons, gravitons etc)
- ✓ Incorporates photon refraction
- ✓ No "dephasing" (photon bending seems not to suppress conversion)
- ✓ Valid for any medium, any polarization/dispersion relation (for k in WKB regime)
- ✓ Divergence free! $power \sim \int d^3k \int dA.k\, P_{a\to\gamma} f_a$ $dA \parallel \nabla E_\gamma$
- ✓ Direct link with ray-tracing

Comparison With Numerical Studies

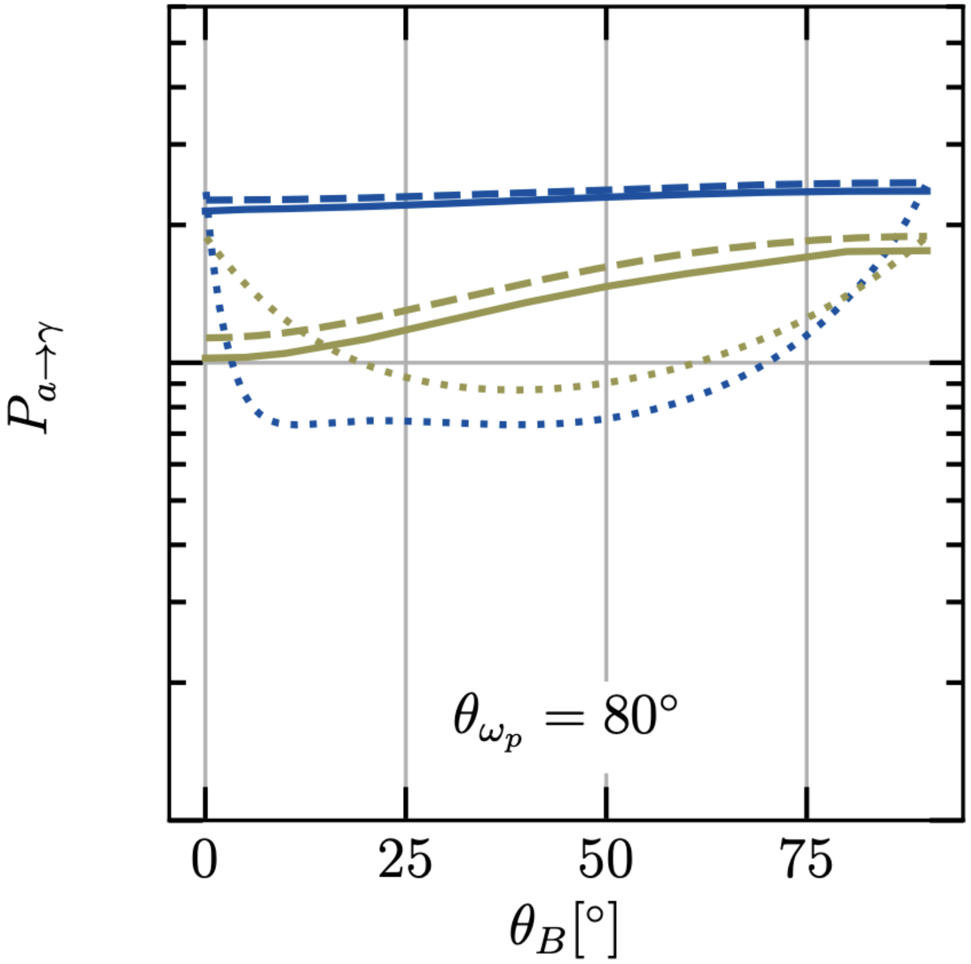
Plasma Distribution



$$-\nabla^2\vec{E} + \nabla(\nabla\cdot\vec{E}) - \omega^2\boldsymbol{\epsilon}\vec{E} = \omega^2 g_{a\gamma\gamma}\vec{B}_0 a\,.$$

Gines, Noordhuis, Weniger, Witte, *PRD* 110 (2024) 8, 083007

Comparison With Numerical Studies



Kinetic Theory (2023) = Classical (2024) **JM**, Millington, Garbrecht

Numerical (2024) Gines, Noordhuis, Weniger, Witte

(2021) Millar, Baum, Lawson, Marsh JCAP 11 (2021) 013

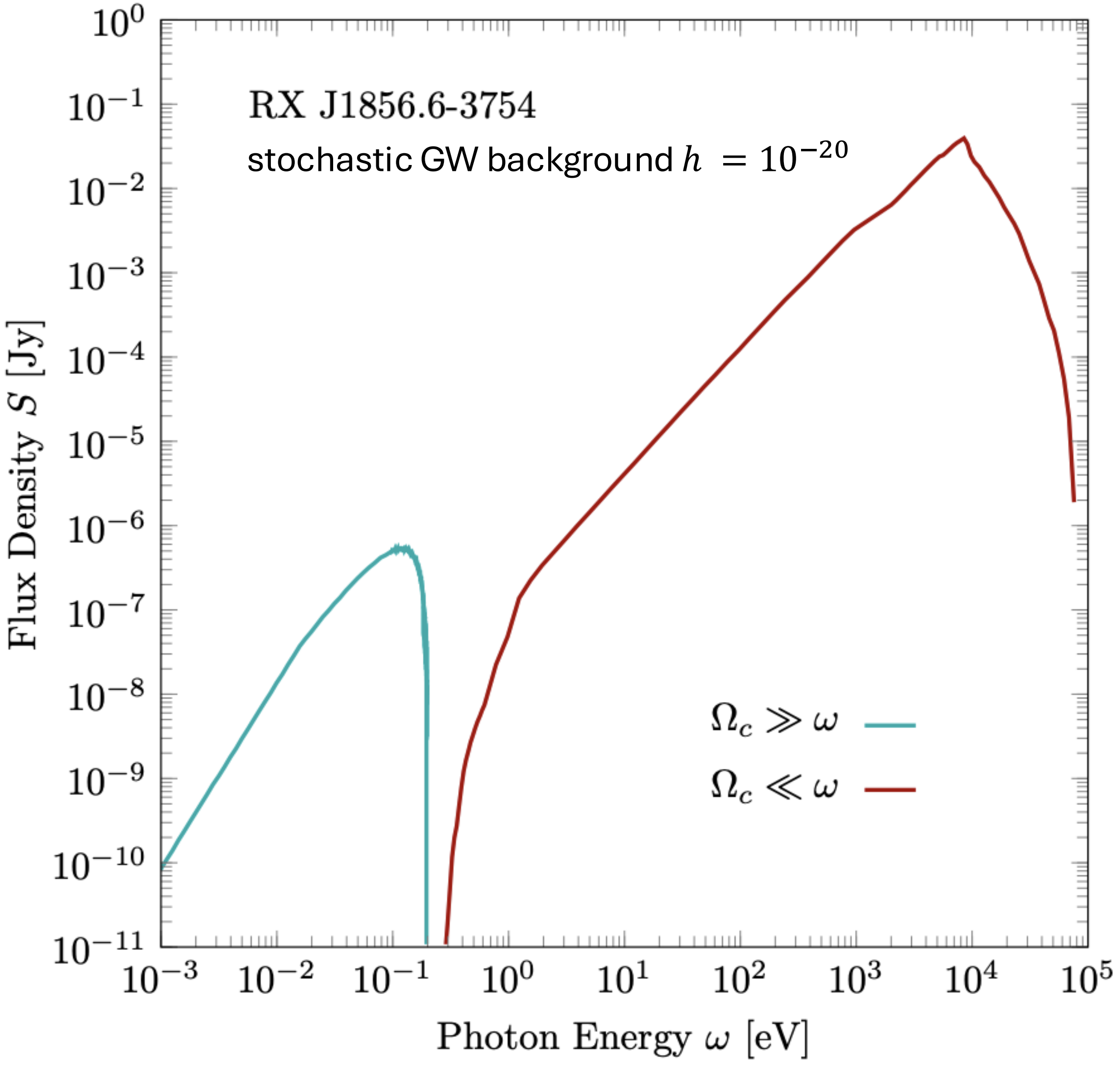
Repeat These Steps with Gravitons

$$P_{h\to\gamma}^{3D} \sim \frac{\pi |M_{h\to\gamma}|^2}{E_\gamma |\boldsymbol{k} \cdot \nabla_{\boldsymbol{x}} E_\gamma|}$$

↓

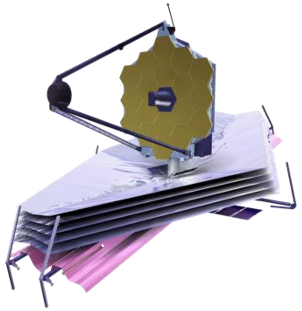
$$P_{h\to\gamma_{\perp,\parallel}}^{+,\times} = \pi \frac{\sin^2\theta_B |\mathbf{B}|^2}{|\mathbf{k}\cdot\nabla E_{\perp,\parallel}|} \frac{\omega}{m_p^2}$$

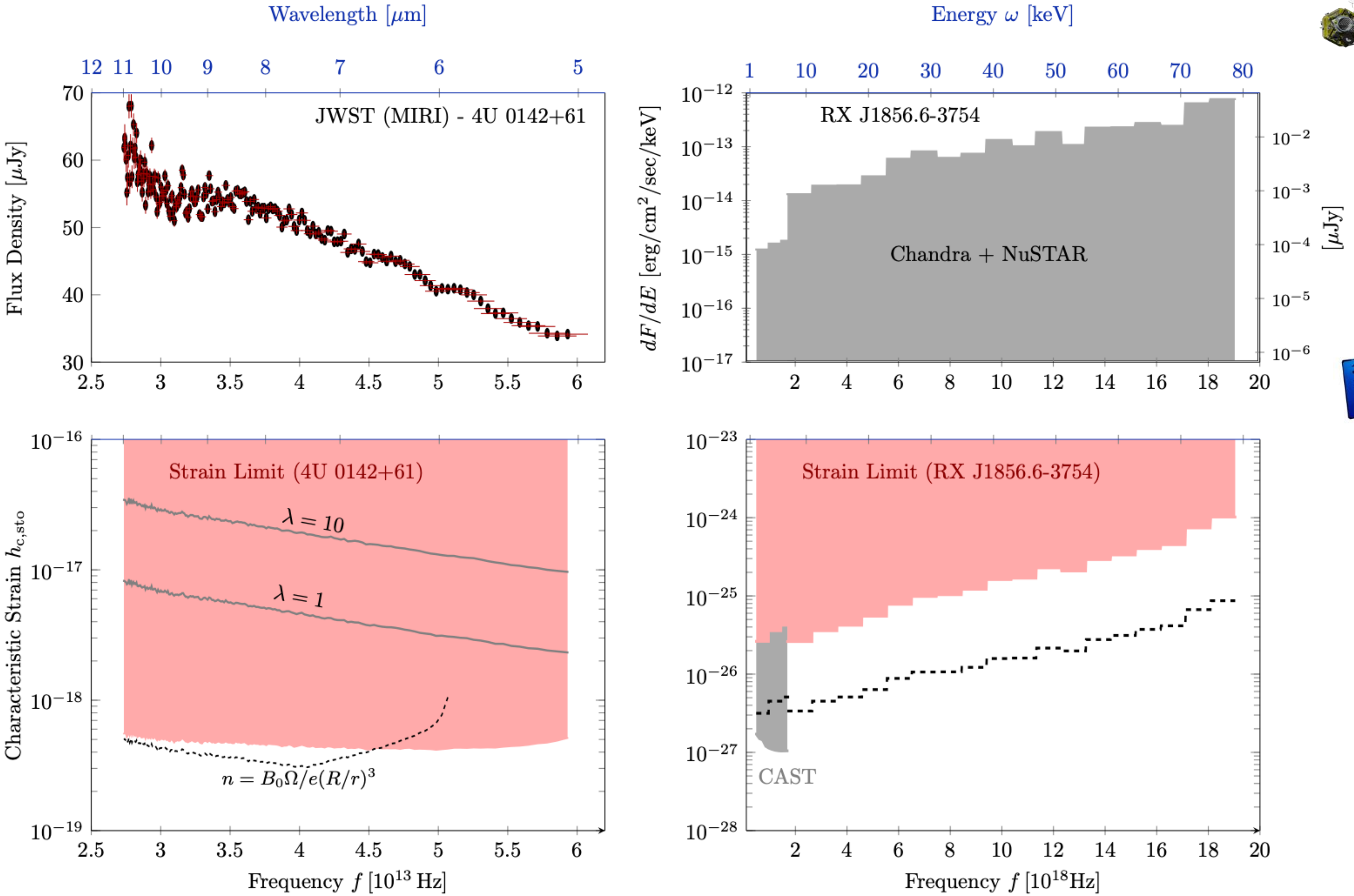
Flux From Resonant GW Conversion

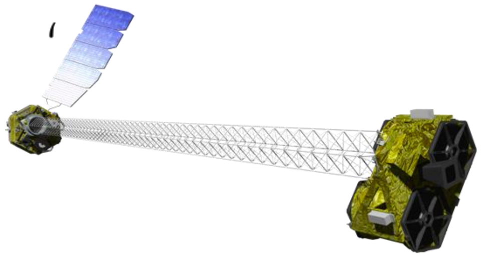


$$S = \frac{1}{4\pi d^2} \int d\Omega_{\mathbf{k}} \int d\mathbf{\Sigma}_{\mathbf{k}} \cdot \mathbf{v}_p P_{h\to\gamma} \frac{\omega}{16\pi G} h^2_{\mathrm{c,sto}}$$

JM, Ellis PRD 110 (2024) 10, 103003



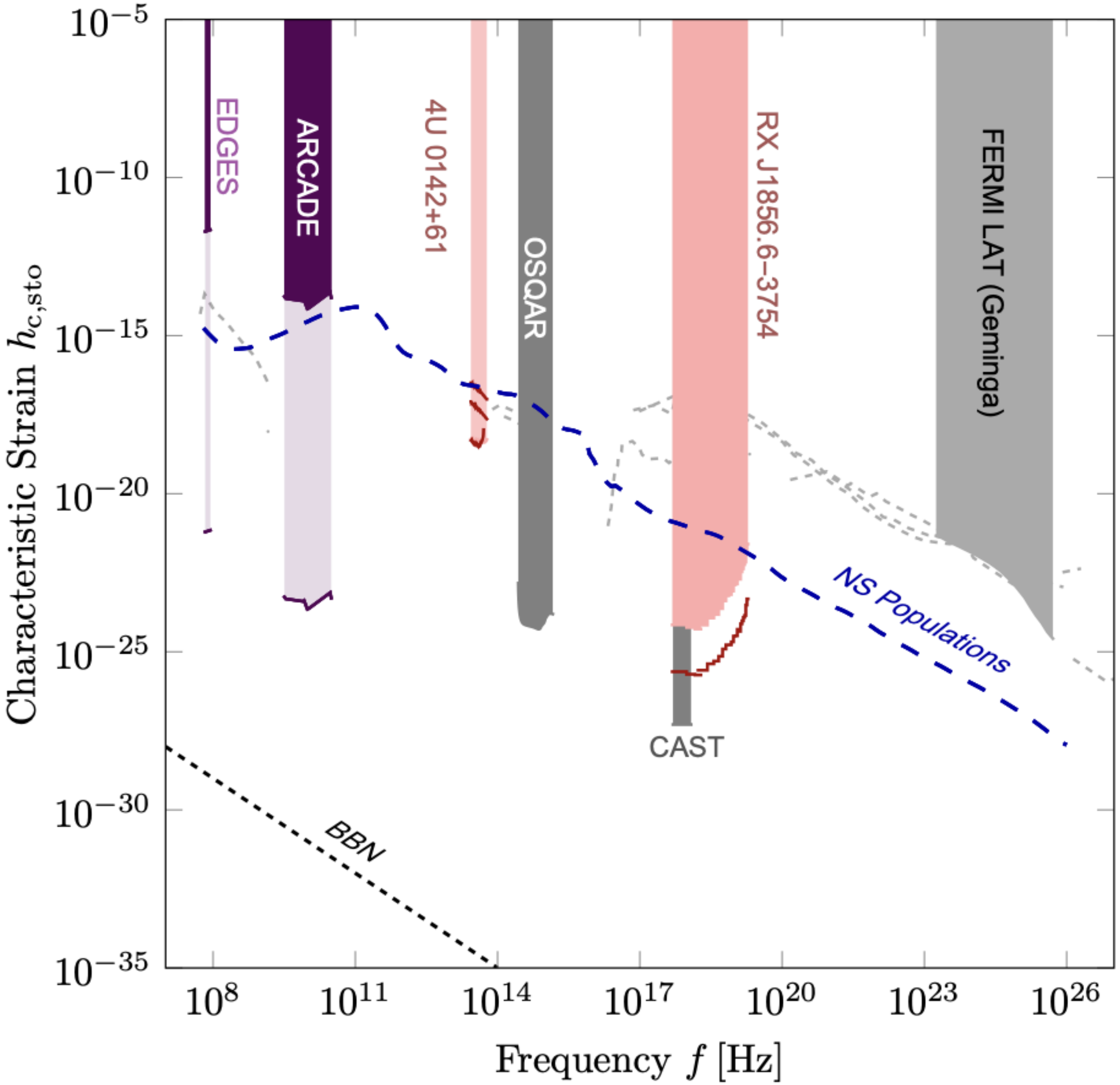




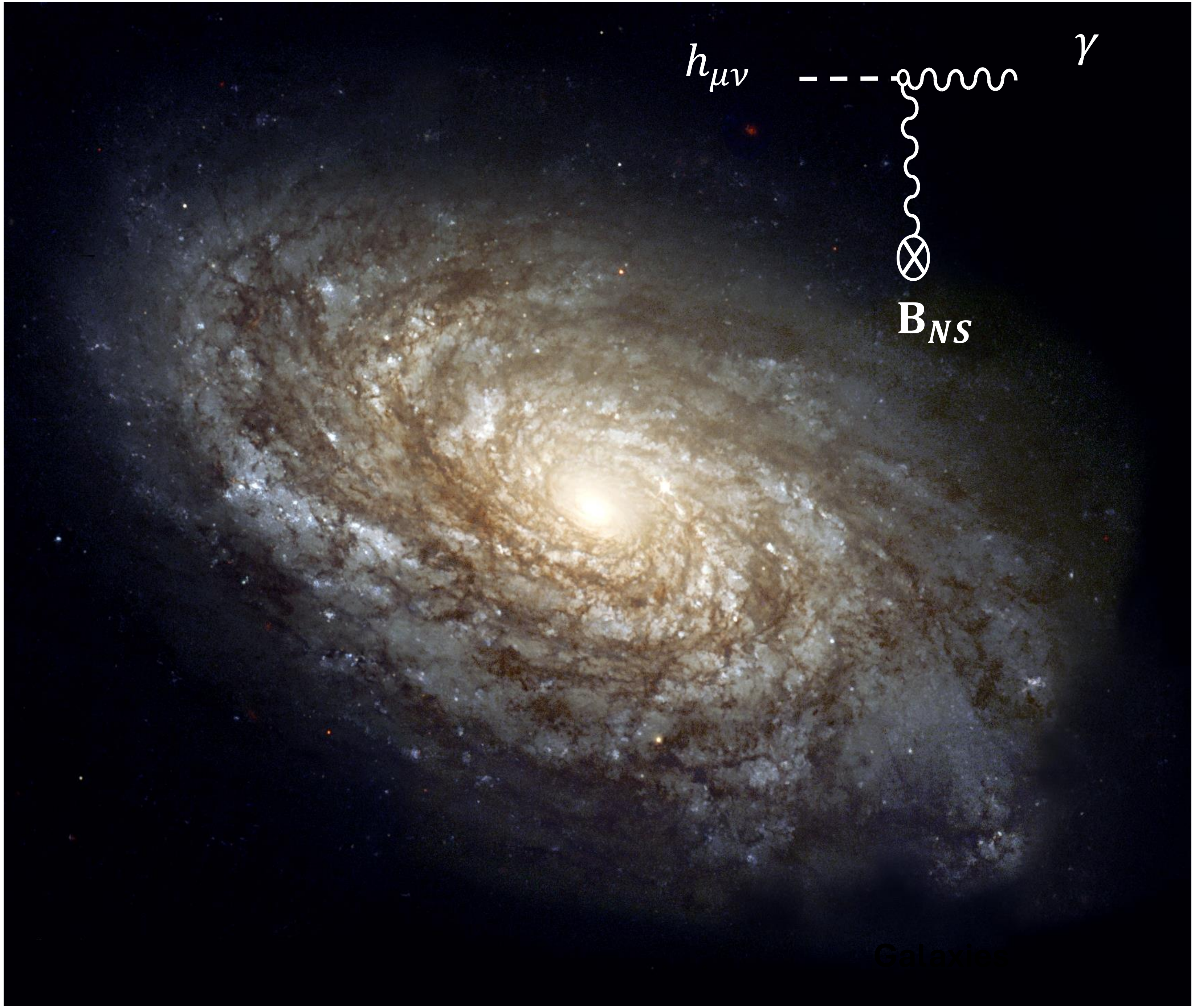
Thanks - Jeremy Hare, George Pavlov, Bettina Posselt, Oleg Kargaltsev, Tea Temim and StevenChen for JWST data – (2024)

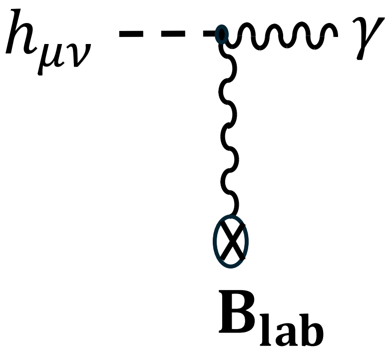
JM, Ellis PRD 110 (2024) 10, 103003

Constraints on Stochastic GWs from Neutron Stars

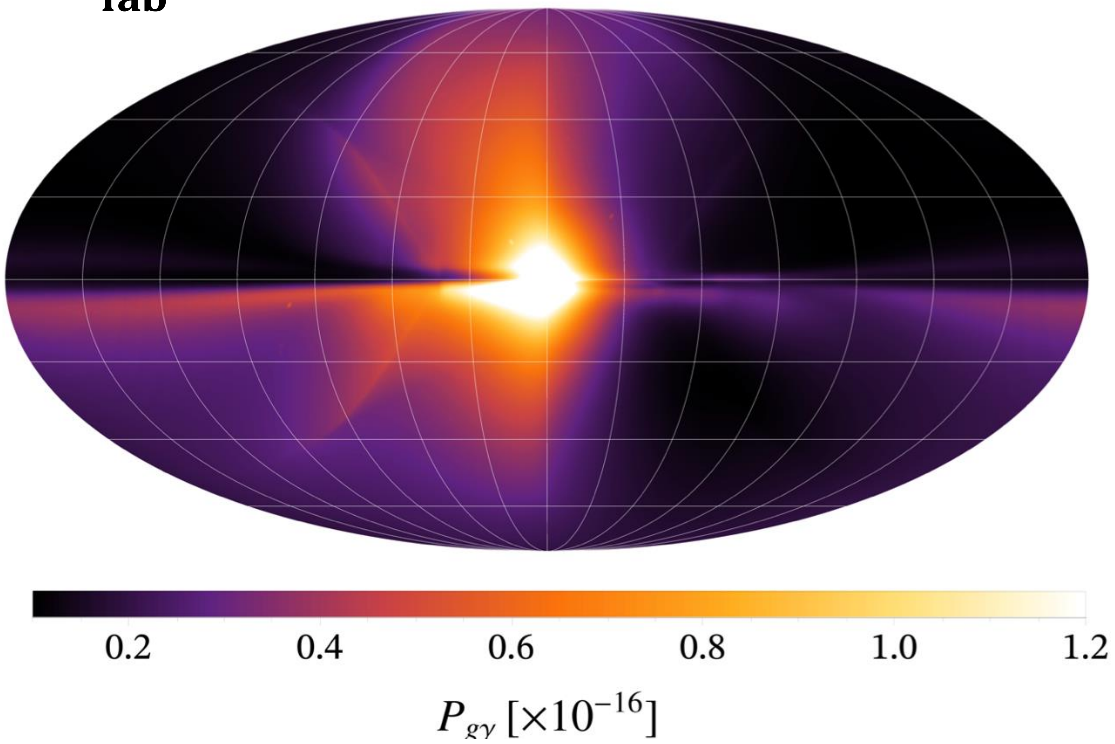


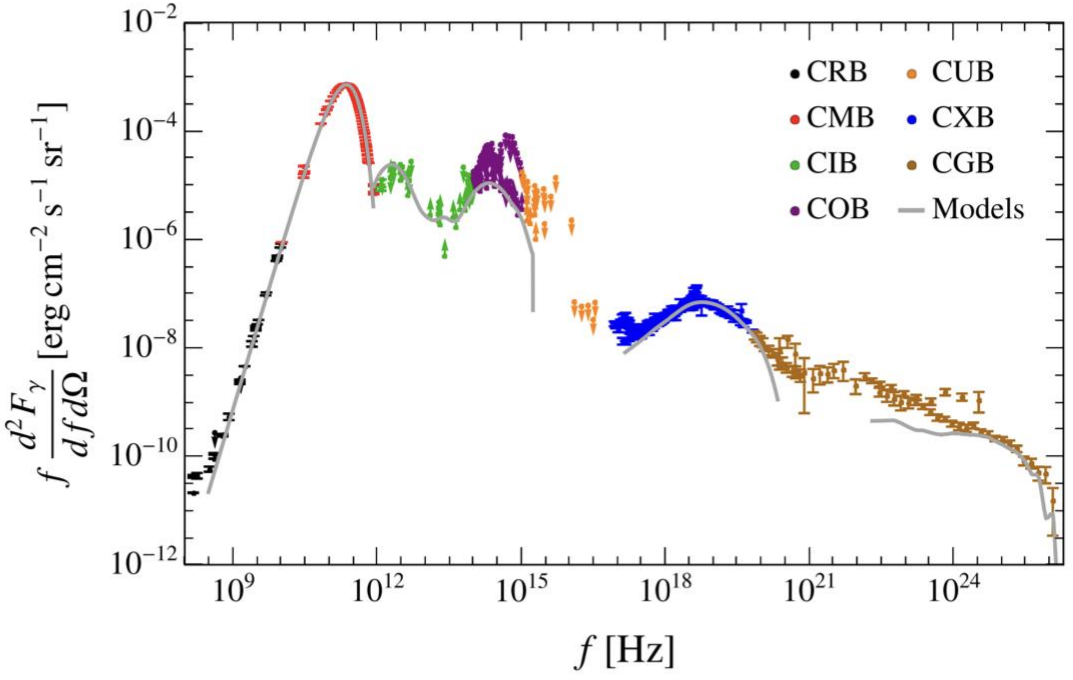
JM, Ellis PRD 110 (2024) 10, 103003


$h_{\mu\nu}$
γ
$\mathbf{B}_{NS}$



Galactic Magnetic Fields





Constraining gravitational-wave backgrounds from conversions into photons in the Galactic magnetic field

Alessandro Lella,[1,2,*] Francesca Calore,[3,4,†] Pierluca Carenza,[5,‡] and Alessandro Mirizzi[1,2,§]

[1] *Dipartimento Interateneo di Fisica "Michelangelo Merlin", Via Amendola 173, 70126 Bari, Italy*
[2] *Istituto Nazionale di Fisica Nucleare - Sezione di Bari, Via Orabona 4, 70126 Bari, Italy*

Summary

- High-frequency gravitational waves are an exciting emerging field

- Lots more to do on indirect astrophysical probes (as with axions)

- Lots of interesting R&D to do in the lab to increase sensitivity

Backup

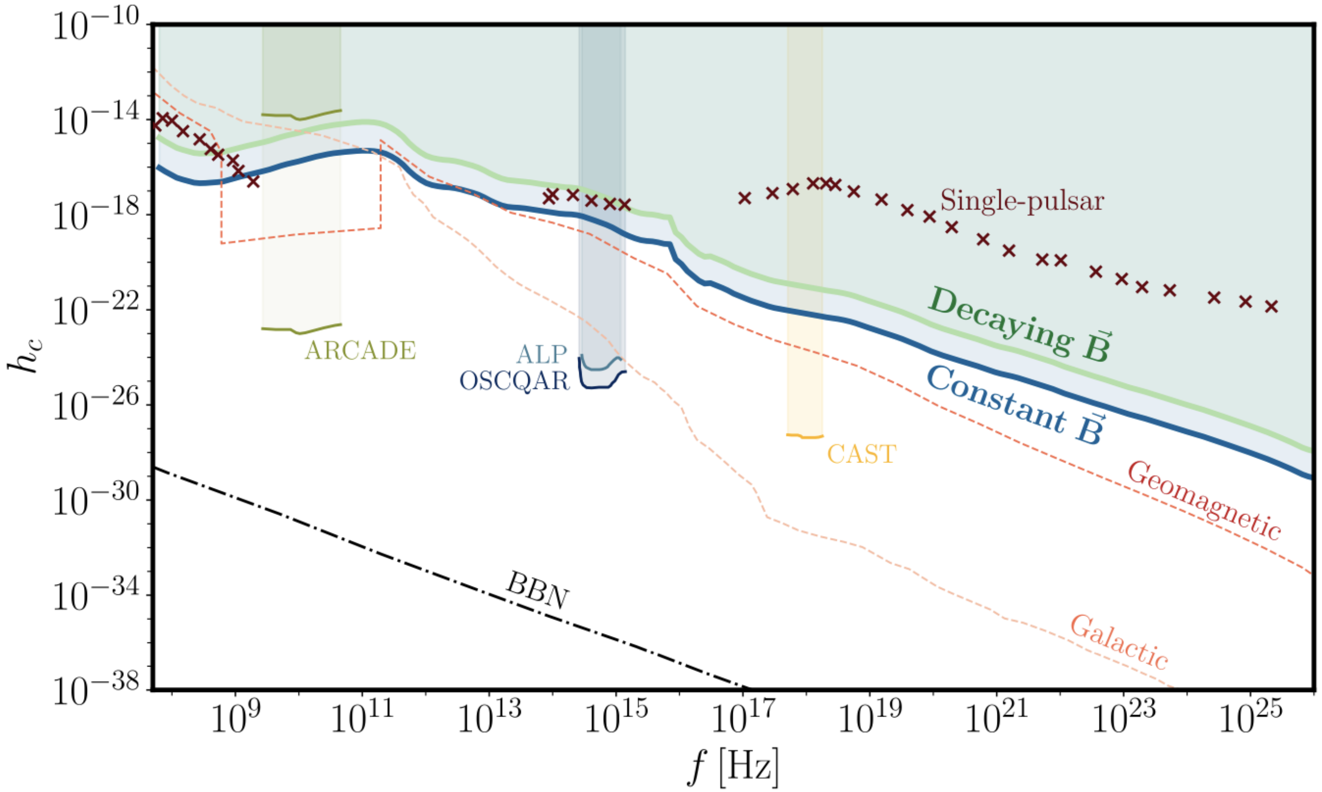
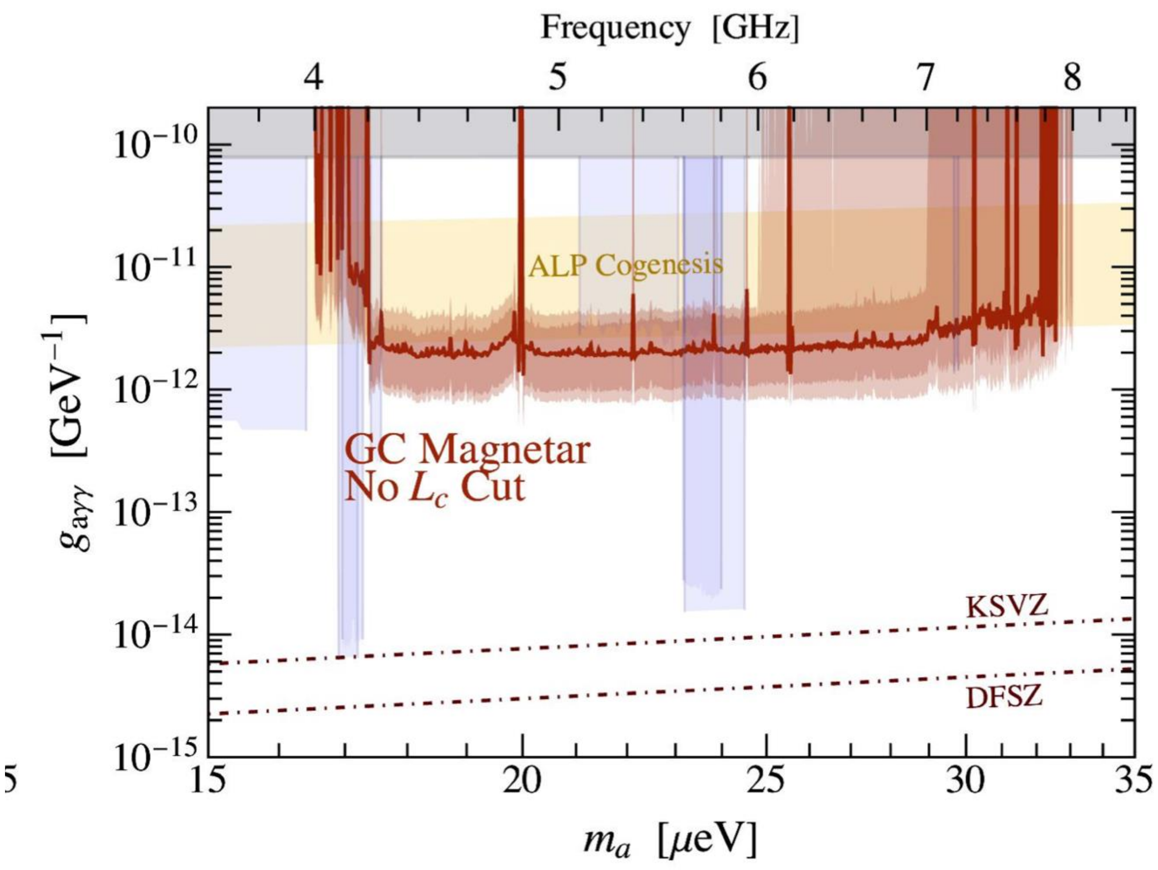


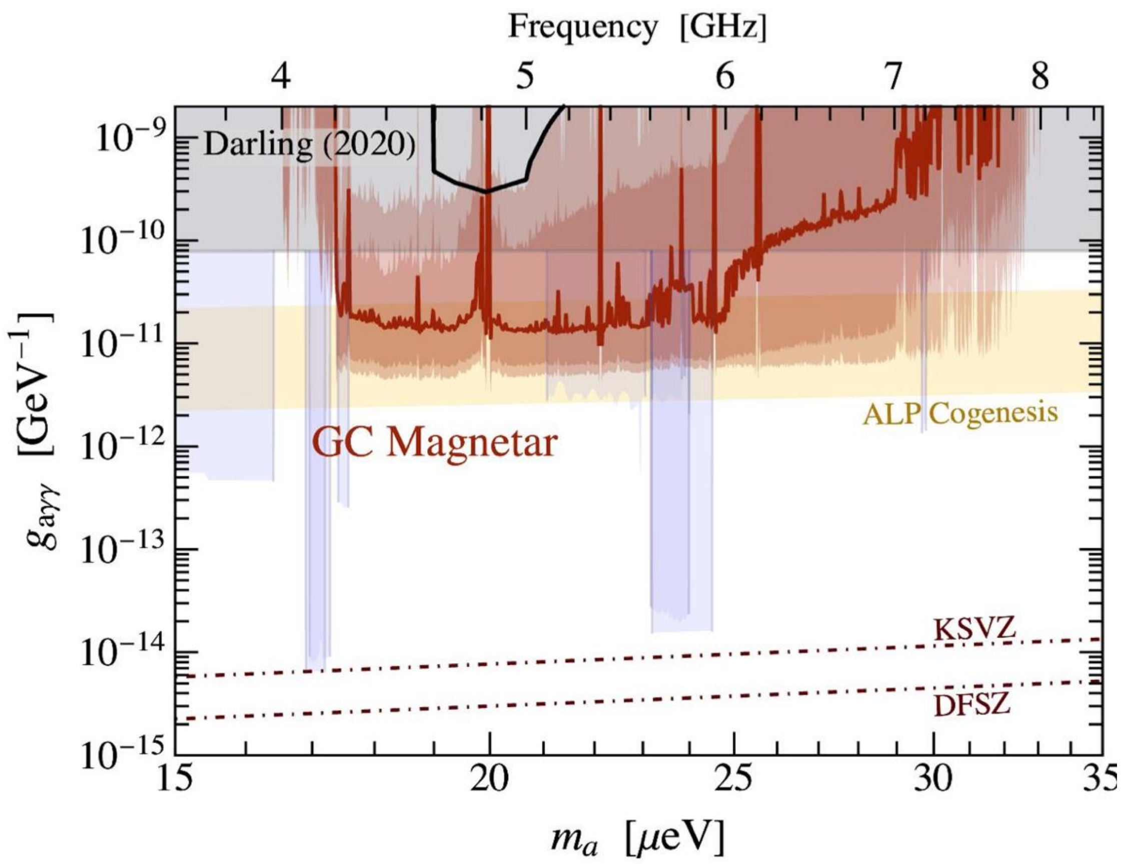
Figure 6: Constraints on the characteristic strain h_c from different experiments. Blue and green solid lines represent our bounds from an induced photon flux in the magnetosphere of the galactic NSs, for NS models with a constant and decaying $\vec{B}$, respectively. We also show limits from OSQAR, ALP and CAST [38] as well as the one from ARCADE (the upper and lower lines corresponding to uncertainty on the cosmic magnetic field) [43] and Big Bang Nucleosynthesis [61] (black dashed-dotted lines). The red and orange dashed curves represent the limits from conversion in the geomagnetic field and galactic magnetic field respectively [47].

1D Calculation

19

Conjectured effects of photon refraction





$$\chi = \frac{1}{\partial_\omega \mathcal{H}} \left[\hat{\boldsymbol{\varepsilon}} \cdot (\partial_{\mathbf{k}} \mathcal{D}) \cdot \nabla_{\mathbf{x}} \hat{\boldsymbol{\varepsilon}} \, + \, \frac{1}{2} \hat{\boldsymbol{\varepsilon}}_i \left(\partial_{\mathbf{k}_l} \partial_{\mathbf{k}_{l'}} \mathcal{D}_{ij} \right) \hat{\boldsymbol{\varepsilon}}_j \nabla_l \mathbf{k}_{l'} \right]$$

$$\mathcal{D}_{ij} = - \left| \mathbf{k} \right|^2 \delta_{ij} + \mathbf{k}_i \mathbf{k}_j + \omega^2 \varepsilon_{ij}(\omega, \mathbf{x})$$

3.2.3 Exotic compact objects

Beyond the very well-known astrophysical compact objects, namely BHs and neutron stars, there are several candidates for stable (or long-lived) exotic compact objects that are composed of beyond the Standard Model particles [51]. For instance, they can be composed of beyond the Standard Model fermions, such as the gravitino in supergravity theories, giving rise to gravitino stars [52]. Exotic compact objects can also be composed of bosons, such as moduli in string compactifications and supersymmetric theories [53]. Depending on the mechanism that makes the compact object stable (or long-lived), scalar field exotic compact objects have specific names such as Q-balls, boson stars, oscillatons, oscillons. There are also more exotic possibilities, such as gravastars [54]. Exotic compact objects can form binaries and emit GWs in the same way as BH and neutron star binaries do. During the early inspiral phase, the frequency of the emitted GWs is twice the orbital frequency. At the ISCO, the frequency for a binary system of two exotic compact objects with mass M and radius R is given by [51]

$$f_{\rm ISCO} = \frac{1}{6\sqrt{3}\pi}\frac{C^{3/2}}{GM} \simeq C^{3/2}\left(\frac{6\times 10^{-3}\,M_\odot}{M}\right) 10^6\,{\rm Hz}\,, \tag{29}$$