B2DK

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1. Formalism

Two mass eigenstates defined as:

$$
\begin{aligned}
|B^0_{s,L}\rangle &= p|B^0_s\rangle + q|\bar{B}^0_s\rangle \\
|B^0_{s,H}\rangle &= p|B^0_s\rangle - q|\bar{B}^0_s\rangle
\end{aligned} \tag{1}
$$

where the subscripts L [H] stands for light [heavy]. And their time evolution is:

$$
\begin{aligned}
|B^0_{s,L}(t)\rangle &= e^{-i(M_L-\frac{i}{2}\Gamma_L)t}|B^0_{s,L}(0)\rangle \\
|B^0_{s,H}(t)\rangle &= e^{-i(M_H-\frac{i}{2}\Gamma_H)t}|B^0_{s,H}(0)\rangle
\end{aligned} \tag{2}
$$

For an initially pure B^0_s beam the time evolution is obtain by inverting Eq.(1):

$$
\begin{aligned}
|B^0_s(t)\rangle &= \frac{1}{2p}(|B^0_{s,L}(t)\rangle + |B^0_{s,H}(t)\rangle) \\
&= \frac{1}{2p}(e^{-i(M_L-\frac{i}{2}\Gamma_L)t}|B^0_{s,L}(0)\rangle + e^{-i(M_H-\frac{i}{2}\Gamma_H)t}|B^0_{s,H}(0)\rangle) \\
&= f_+(t)|B^0_s\rangle + \frac{q}{p}f_-(t)|\bar{B}^0_s\rangle
\end{aligned} \tag{3}
$$

and likewise for the antiparticle

$$
\begin{aligned}
|\bar{B}^0_s(t)\rangle &= \frac{1}{2q}(|B^0_{s,L}(t)\rangle - |B^0_{s,H}(t)\rangle) \\
&= \frac{1}{2q}(e^{-i(M_L-\frac{i}{2}\Gamma_L)t}|B^0_{s,L}(0)\rangle - e^{-i(M_H-\frac{i}{2}\Gamma_H)t}|B^0_{s,H}(0)\rangle) \\
&= f_+(t)|\bar{B}^0_s\rangle + \frac{p}{q}f_-(t)|B^0_s\rangle
\end{aligned} \tag{4}
$$

with

$$
f_\pm(t) = \frac{1}{2}e^{-iMt}e^{-\frac{1}{2}\Gamma t}[e^{i\frac{\Delta m}{2}t}e^{-\frac{\Delta\Gamma}{4}t} \pm e^{-i\frac{\Delta m}{2}t}e^{\frac{\Delta\Gamma}{4}t}] \tag{5}
$$

where

$$M \equiv \frac{M_L+M_H}{2} \tag{6}$$

$$\Gamma \equiv \frac{\Gamma_L+\Gamma_H}{2} \tag{7}$$

$$\Delta m \equiv M_H - M_L \tag{8}$$

$$\Delta\Gamma \equiv \Gamma_L - \Gamma_H \tag{9}$$

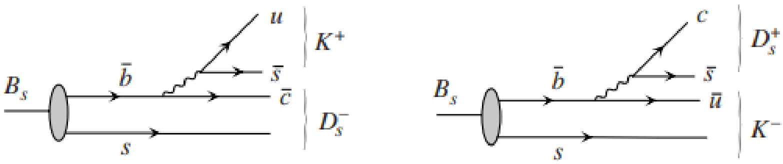

Figure 11.10 These two decay channels and their charge conjugate decays allow us to determine ϕ_3.

$$\begin{aligned}
A(\overline{B}_s \to D_s^+ K^-) &= e^{i\delta_-}\mathbf{V}_{cb}\mathbf{V}_{us}^*\mathcal{A}_- \\
A(\overline{B}_s \to D_s^- K^+) &= e^{i\delta_+}\mathbf{V}_{ub}\mathbf{V}_{cs}^*\mathcal{A}_+ \\
A(B_s \to D_s^- K^+) &= e^{i\delta_-}\mathbf{V}_{cb}^*\mathbf{V}_{us}\mathcal{A}_- \\
A(B_s \to D_s^+ K^-) &= e^{i\delta_+}\mathbf{V}_{ub}^*\mathbf{V}_{cs}\mathcal{A}_+ .
\end{aligned}$$

As $V_{ub} = |V_{ub}|e^{-i\gamma}$, let

$$A(B_s^0 \to D_s^- K^+) = A(\bar{B}_s^0 \to D_s^+ K^-) \equiv A \quad (10)$$

and

$$\begin{aligned}
A(B_s^0 \to D_s^+ K^-) &= rAe^{i\delta}e^{i\gamma} \quad (11) \\
A(\bar{B}_s^0 \to D_s^- K^+) &= rAe^{i\delta}e^{-i\gamma} \quad (12)
\end{aligned}$$

Wolfenstein Parameterization: λ, A, ρ, η; ($\lambda \approx 0.22$)

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

$$= \begin{pmatrix} 1-\frac{\lambda^2}{2} & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4)$$

Only very small complexe contributions, up to third order in λ ($\sim$ 0.5%) only in V_{ub} and V_{td}

then,

$$\begin{aligned}
P_{++} &\equiv P(B_s^0 \to D_s^+ K^-) \\
&= |\langle D_s^+ K^- | H_w | B_s^0(t) \rangle|^2 \\
&= |f_+(t)\langle D_s^+ K^- | H_w | B_s^0 \rangle + \frac{q}{p} f_-(t) \langle D_s^+ K^- | H_w | \bar{B}_s^0 \rangle|^2 \\
&= |f_+(t) A(B_s^0 \to D_s^+ K^-) + \frac{q}{p} f_-(t) A(\bar{B}_s^0 \to D_s^+ K^-)|^2 \\
&= |f_+(t) r A e^{i\delta} e^{i\gamma} + \frac{q}{p} f_-(t) A|^2
\end{aligned} \tag{13}$$

one knows $\frac{q}{p} = e^{2i\beta_s}$ is a excellent approximation, so

$$\begin{aligned}
P_{++} &= |f_+(t) r A e^{i\delta} e^{i\gamma} + \frac{q}{p} f_-(t) A|^2 \\
&\propto |f_+(t) r e^{i\delta} e^{i\gamma} + e^{2i\beta_s} f_-(t)|^2 \\
&= |f_+(t) r e^{i(\delta+(\gamma-2\beta_s))} + f_-(t)|^2
\end{aligned} \tag{14}$$

finally, all four decay rates are:

$$P_{++} \propto |f_+(t) r e^{i(\delta+(\gamma-2\beta_s))} + f_-(t)|^2 \tag{15}$$
$$P_{+-} \propto |f_+(t) + f_-(t) r e^{i(\delta-(\gamma-2\beta_s))}|^2 \tag{16}$$
$$P_{-+} \propto |f_+(t) + f_-(t) r e^{i(\delta+(\gamma-2\beta_s))}|^2 \tag{17}$$
$$P_{--} \propto |f_+(t) r e^{i(\delta-(\gamma-2\beta_s))} + f_-(t)|^2 \tag{18}$$

$$P_{++} \propto \mathrm{e}^{-\Gamma t}\left(\cosh\left(\frac{\Delta\Gamma}{2}t\right) - C\cos(\Delta m t) + \mathrm{D}_{\bar{\mathrm{f}}}\sinh\left(\frac{\Delta\Gamma}{2}t\right) - \mathrm{S}_{\bar{\mathrm{f}}}\sin(\Delta m t)\right) \tag{19}$$
$$P_{+-} \propto \mathrm{e}^{-\Gamma t}\left(\cosh\left(\frac{\Delta\Gamma}{2}t\right) + C\cos(\Delta m t) + \mathrm{D}_{\mathrm{f}}\sinh\left(\frac{\Delta\Gamma}{2}t\right) - \mathrm{S}_{\mathrm{f}}\sin(\Delta m t)\right) \tag{20}$$
$$P_{-+} \propto \mathrm{e}^{-\Gamma t}\left(\cosh\left(\frac{\Delta\Gamma}{2}t\right) + C\cos(\Delta m t) + \mathrm{D}_{\bar{\mathrm{f}}}\sinh\left(\frac{\Delta\Gamma}{2}t\right) + \mathrm{S}_{\bar{\mathrm{f}}}\sin(\Delta m t)\right) \tag{21}$$
$$P_{--} \propto \mathrm{e}^{-\Gamma t}\left(\cosh\left(\frac{\Delta\Gamma}{2}t\right) - C\cos(\Delta m t) + \mathrm{D}_{\mathrm{f}}\sinh\left(\frac{\Delta\Gamma}{2}t\right) + \mathrm{S}_{\mathrm{f}}\sin(\Delta m t)\right) \tag{22}$$

$$C = \frac{1-r^2}{1+r^2}, \tag{23}$$

$$D_f = \frac{-2r\cos(\delta-(\gamma-2\beta_s))}{1+r^2}, \qquad D_{\bar{f}} = \frac{-2r\cos(\delta+(\gamma-2\beta_s))}{1+r^2}, \tag{24}$$

$$S_f = \frac{2r\sin(\delta-(\gamma-2\beta_s))}{1+r^2}, \qquad S_{\bar{f}} = \frac{-2r\sin(\delta+(\gamma-2\beta_s))}{1+r^2}. \tag{25}$$

$$\gamma - 2\beta_s = \frac{1}{2}\arg[(D_f + iS_f)(D_{\bar{f}} + iS_{\bar{f}})] \tag{26}$$