Imaging the Deep Earth with Neutrinos

…..an emerging interdisciplinary field of research connecting neutrinos with geoscience

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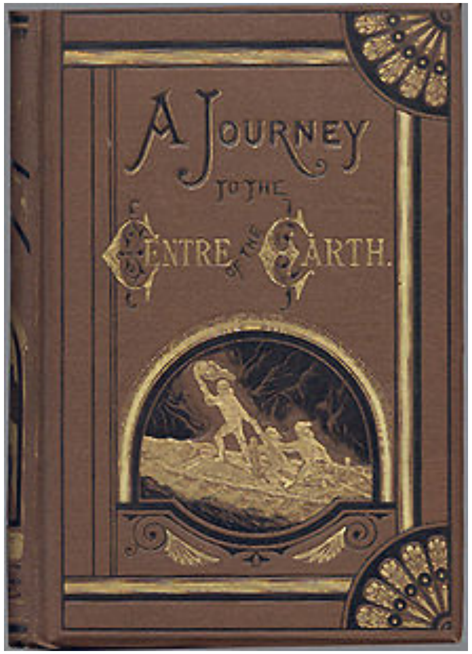
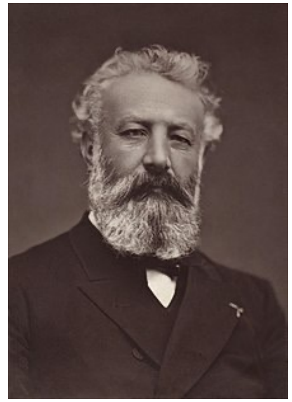
The Interior of Earth – A Great Mystery

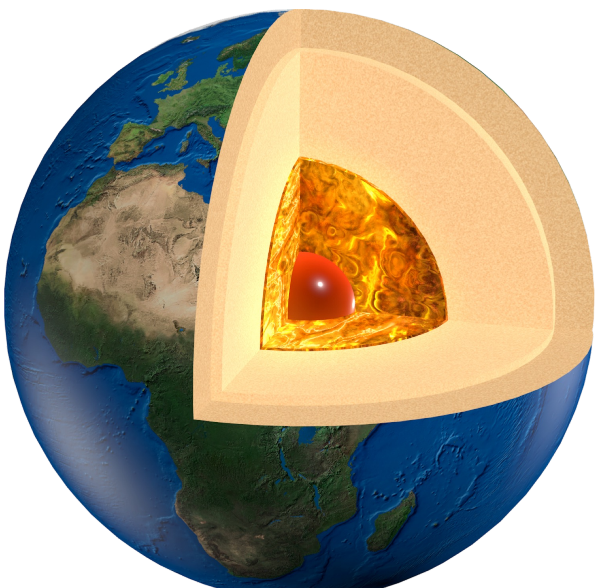




1828 - 1905

A classic science fiction by Jules Verne (French Novelist)

Front cover of an 1874 English translation



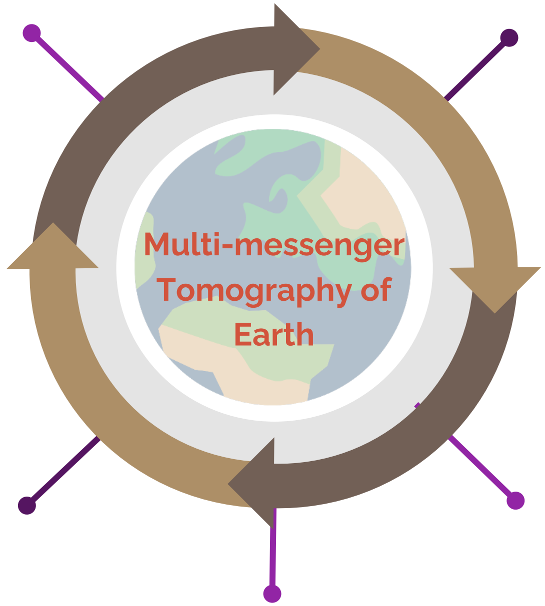
- What lies in the interior of Earth has been a long-standing puzzle and an active research is being carried out in this direction
- The regions deep below the Earth's surface are inaccessible due to large temperatures, pressures, and extreme environments
- There are several ways to look inside Earth scientifically!
  Novelists, poets, playwrights have their own imaginations!

Multi-Messenger Tomography of Earth

Upadhyay, Kumar, Agarwalla, Dighe, PRD 107 (2023) 11, 115030

Gravitational Measurements

- Gravitational interactions
- Information on Earth's total mass and moment of inertia



Seismic Studies

- Use seismic waves from earthquakes
- These are mechanical waves of acoustic energy that travel through or over the Earth
- Reveal the internal structures and identify large-scale variations in matter density and properties

Neutrino Absorption Tomography

- Weak interactions
- Absorption of high-energy (TeV-PeV) neutrinos

Neutrino Oscillation Tomography

- Weak interactions
- Coherent forward scattering of low-energy (MeV-GeV) electron neutrinos with ambient electrons
- Modify neutrino oscillation patterns

Geoneutrinos

- Brings crucial information about the mantle
- Disclose information on the radiogenic contribution to Earth's heat budget

Combine neutrino data with seismic and gravitational measurements

A new era of Multi-Messenger Tomography of Earth (MMTE)

Recent workshops: MMTE 2022 (Salt Lake City, Utah) and MMTE 2023 (APC, Paris)

Gravitational Measurements

Gravitational measurements exploit the gravitational interactions of matter inside Earth to provide information on mass and moment of inertia

Average density

- For given mass[1] ($\sim 5.97 \times 10^{24}$ kg) and radius of Earth ($\sim$ 6400 km), average density of Earth $\sim 5.5\ \mathrm{g/cm^3}$
- Density of ordinary rock $\sim 2.8\ \mathrm{g/cm^3}$, therefore, the density near the centre of Earth is higher than $5.5\mathrm{g/cm^3}$

Moment of inertia

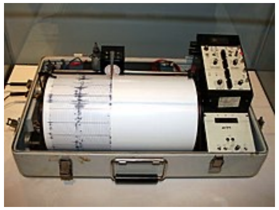
- For uniform sphere, $I = \frac{2}{5}MR^2 \Rightarrow \frac{I}{MR^2} = 0.4$
- Measured[2], $\frac{I}{MR^2} \sim 0.33$, [3]$I_\oplus \sim 8.017 \times 10^{37}$ kg m^2.
- Since $I_{\mathrm{measured}} < I_{\mathrm{expected}}$, more matter is concentrated near the axis of rotation

[1] B. Luzum et al., Celest. Mech. Dyn. Astron. 110, 293 (2011).

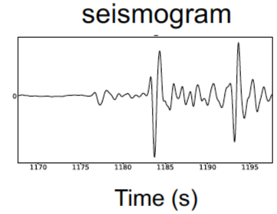
[2] Williams, James G. The Astronomical Journal. 108: 711 (1994)

[3] W. Chen, J. Ray, W. B. Shen, and C. L. Huang, J. Geod. 89, 179 (2015).

- Radius of Earth: (6371.23 ± 0.01) km
- Moment of inertia: (8.01736 ± 0.00097) x 10^{37} kg m^2
- Mass of Earth: (5.9722 ± 0.006) x 10^{24} kg

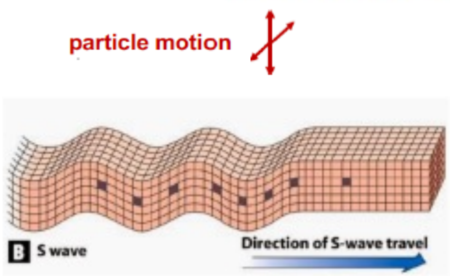


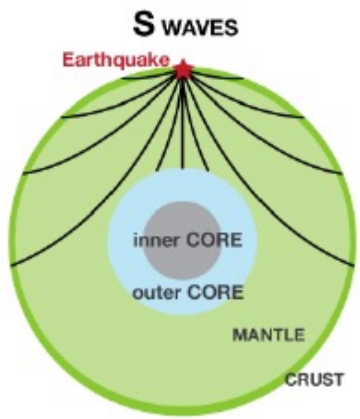
Geophysicists study seismic waves from earthquakes using seismograms recorded in seismometer



Seismic measurements exploits the electromagnetic interactions of matter inside Earth.

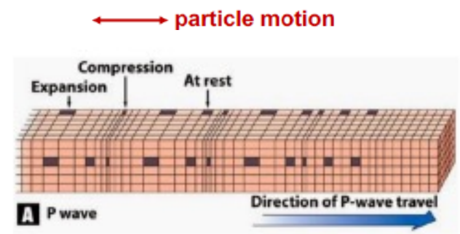
Body waves consisting of transverse vibrations are known as **S-waves**:

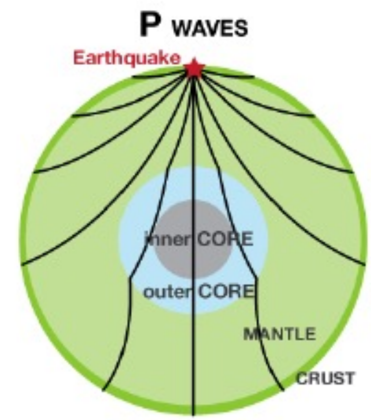




$$V_S = \sqrt{\frac{\mu}{\rho}}$$

Body waves consisting of longitudinal vibrations are known as **P-waves**:





$$V_P = \sqrt{\frac{\kappa + \frac{4}{3}\mu}{\rho}}$$

Velocities of seismic waves depend upon the elastic constants of the material, such as density (ρ), bulk modulus (κ), shear modulus (μ)

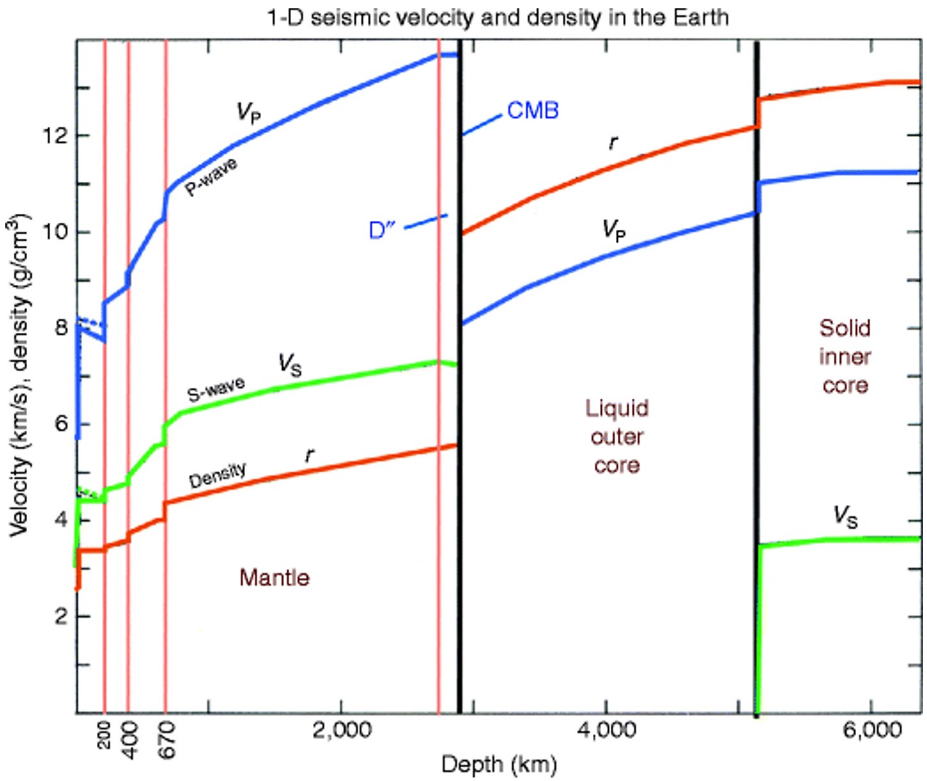
S-waves (slower): shear waves

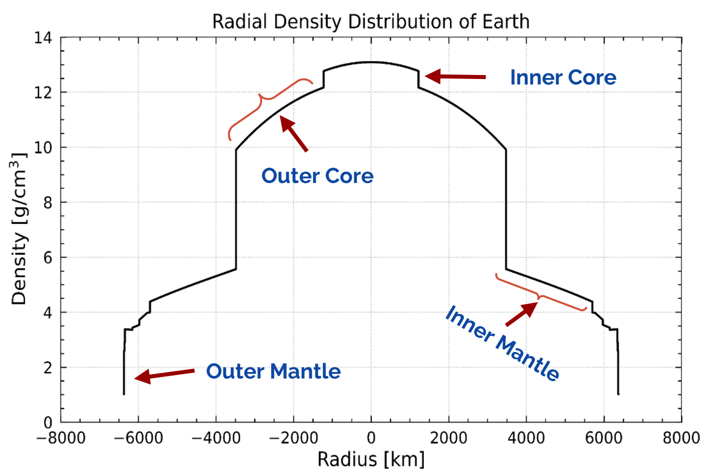
P-waves (fastest): pressure waves

S-wave cannot propagate through the liquid outer core!

- A.M. Dziewonski, D.L. Anderson, Phys.Earth Planet.Interiors 25 (1981) 297-356
- E. C. Robertson, The interior of the Earth, an elementary description, 1966

Preliminary Reference Earth Model





PREM is not a measured profile!

Inversion of seismic wave data + gravitational constraints on Earth's total mass and moment of inertia

↓

Radial profile of Earth matter density

↓

Preliminary Reference Earth Model (PREM)

"PREM": Dziewonski and Anderson (1981)

+ Study of meteorites and rock samples from crust & mantle + high-pressure experiments (deep-Earth mineralogy)

↓

chemical composition of mantle/core:

Upper mantle/Lower mantle:
Silicate minerals (Si, O + Fe, Mg, Mn….)
Benchmark composition: pyrolite (Z/A=0.496)

Outer core: Liquid (no S-waves)
Inner core: Solid
Benchmark composition: Fe-Ni alloy (Z/A=0.466)
+ light elements in outer core ? (Si, O, S, C, H)

Open Questions

+ Composition of the bulk silicate Earth (BSE) - Mg, Si, Fe, O

+ Mineralogy of the lower mantle (Mg/Si, amount of Th and U)

+ Density and composition of the core (H, C, O, Si, S,...?)

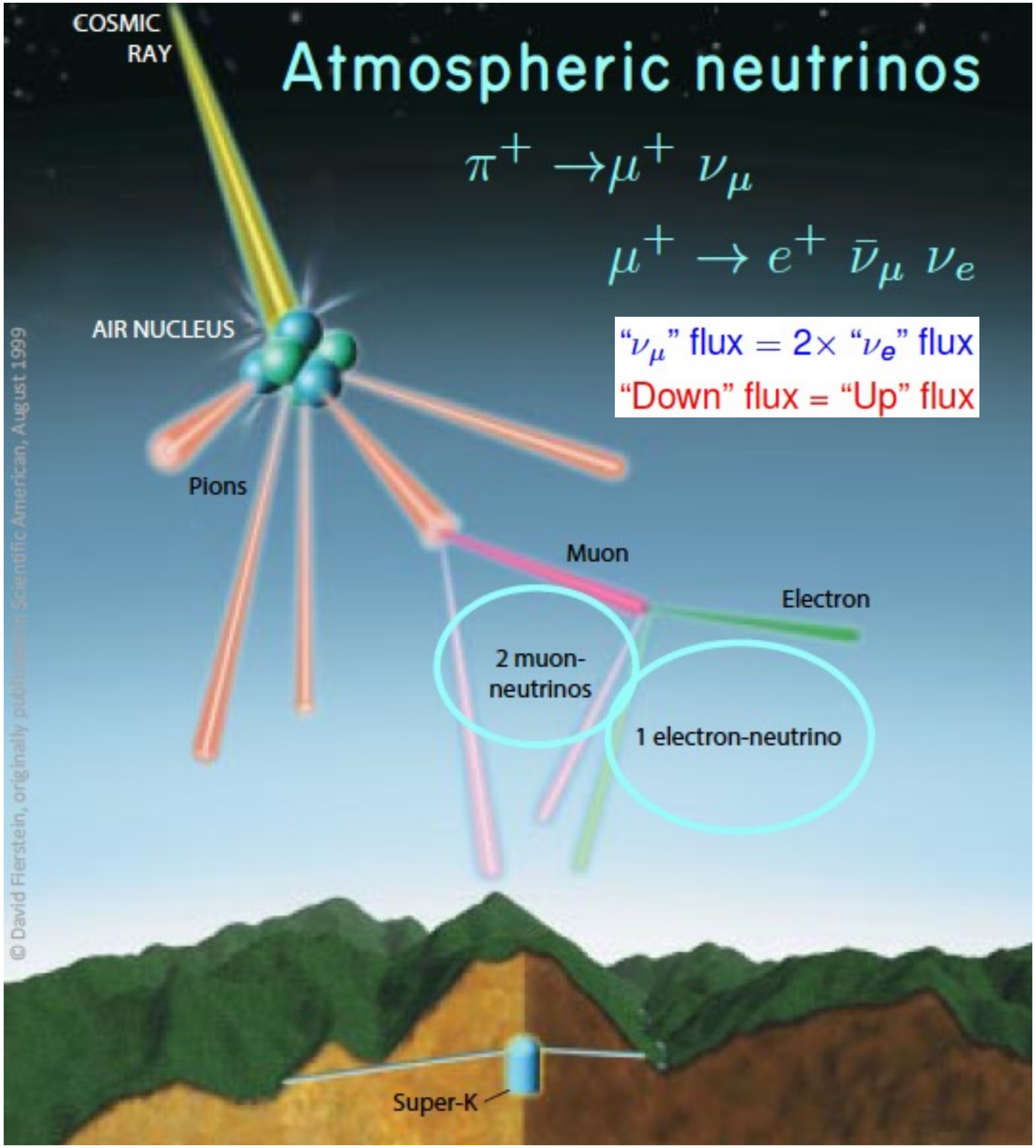
+ Presence of light elements inside Earth: amount of H_2O in the Mantle and H in the Core?

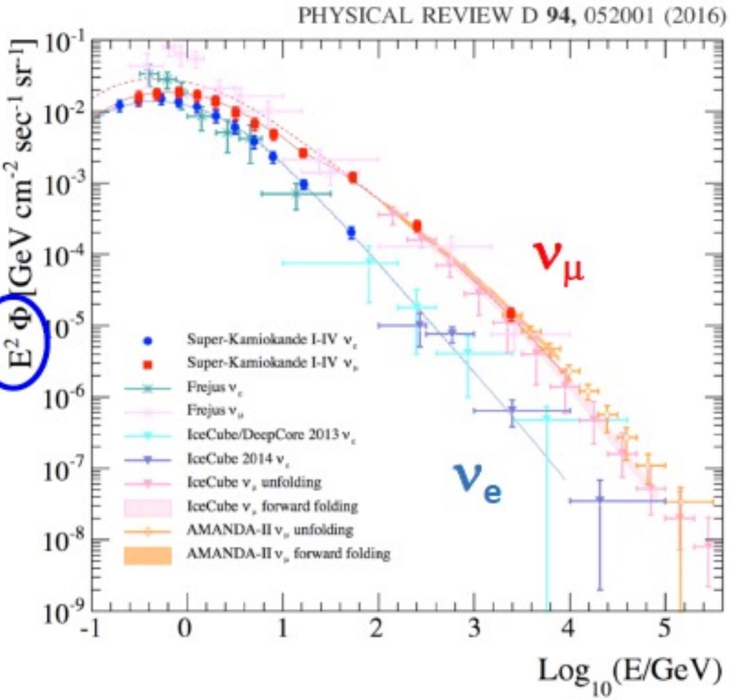
+ Precise locations of the core-mantle boundary (CMB) and the amount of density jump at the CMB

+ Many unexplained structures & heterogeneities observed in the lowermost mantle, beneath Africa and Pacific, that show lower-than-average seismic wave speeds - known as large low-shear-velocity provinces (LLSVPs) and ultra-low velocity zones (ULVZs) - nature and composition of the LLSVPs and ULVZs? (need complementary method to investigate the full 3D structure & composition of inner Earth)

+ Radioactive power in the mantle and core to understand the thermal dynamics inside Earth

Atmospheric Neutrinos: Ideal Candidate for Tomography of Earth





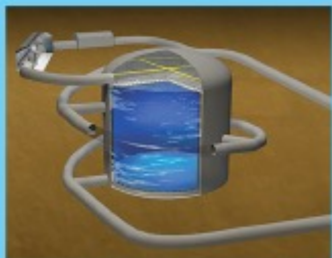
- **Almost isotropic flux (up-down symmetric)**
- **Known flavor composition (ν_e, ν_μ, & their antiparticles)**
- **Wide range of energies (GeV to PeV)**
- **Steeply falling power-law spectra (~ $E^{-2.7}$)**

Neutrino Detectors for Tomography of Earth

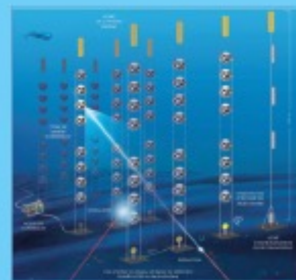
At low (GeV) energies: Neutrino oscillation tomography

(sub- or multi-)Megaton-scale detector

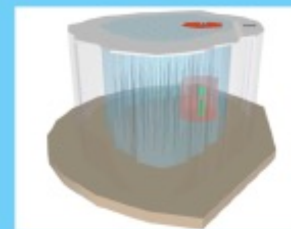
active
in construction
proposed/prototyping



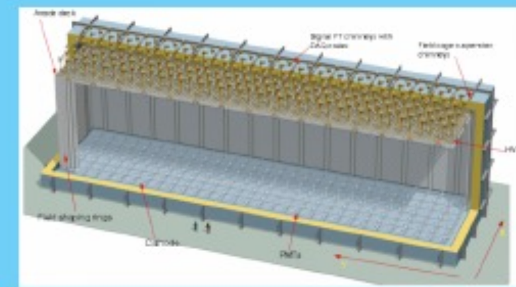
SuperK (50 kton)
HyperK (260 ktons)



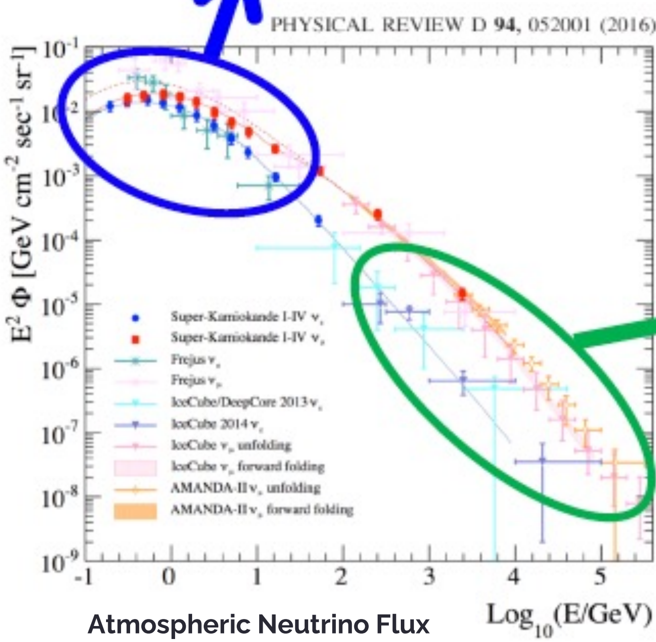
KM3NeT/ORCA (6 Mton)



DeepCore
PINGU (IceCube) (1-5 Mton)



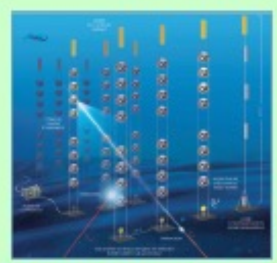
DUNE (40 kton)



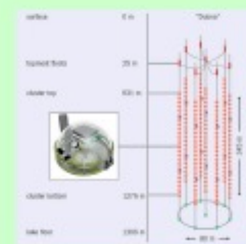
Atmospheric Neutrino Flux

At high (TeV-PeV) energies: Neutrino absorption tomography

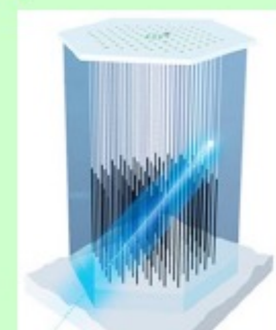
~ Gigaton-scale detectors



KM3NeT/ARCA (1 Gton)

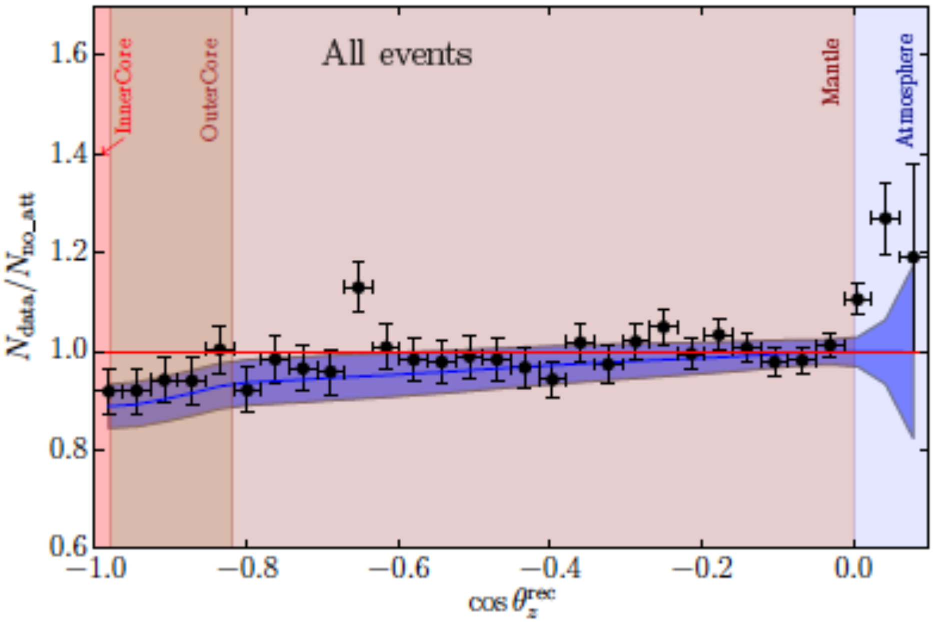


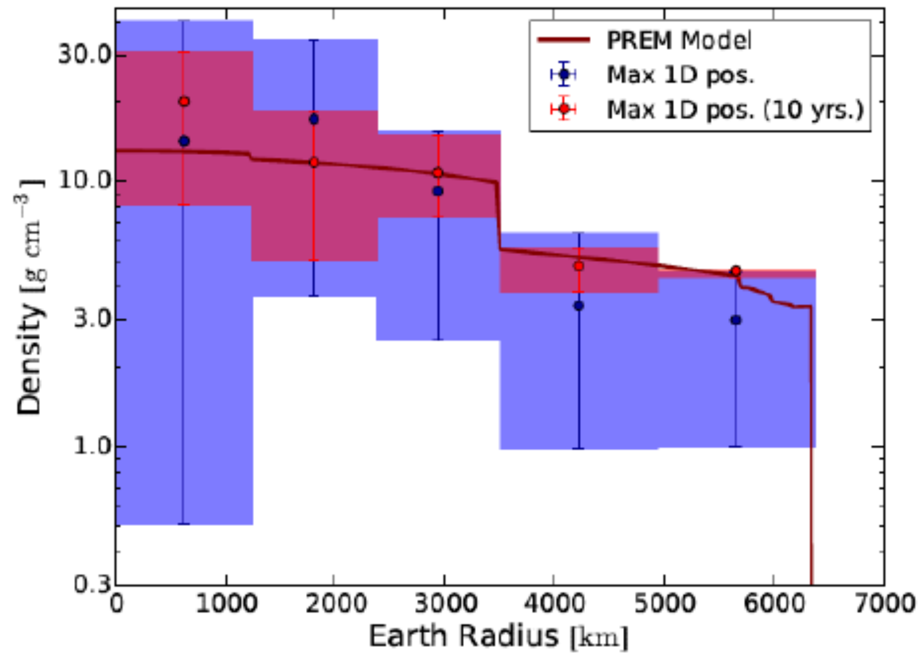
Baikal (1 Gton)

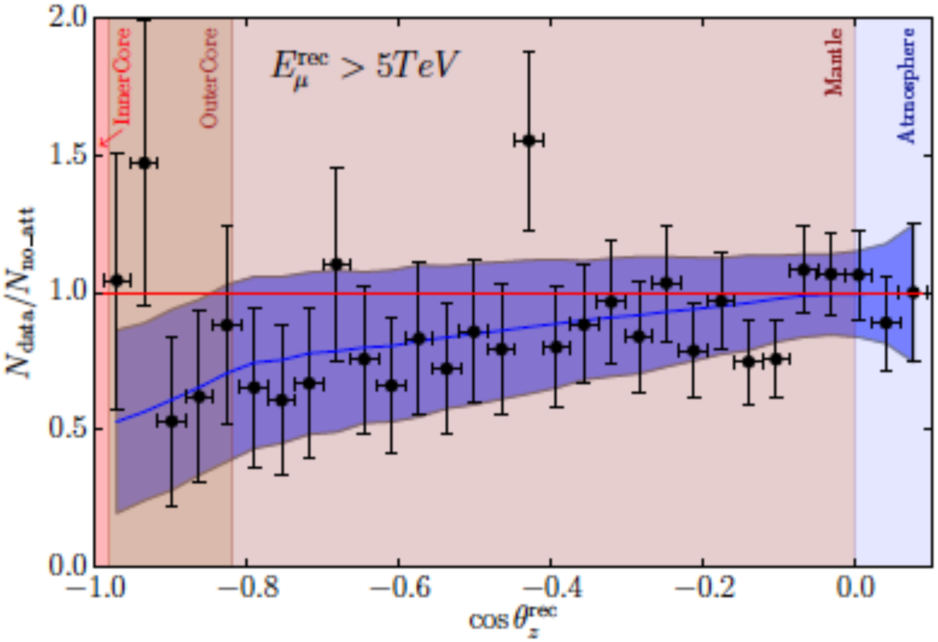


IceCube/Gen2 (1+ Gton)

Neutrino Absorption Tomography: At High Energies

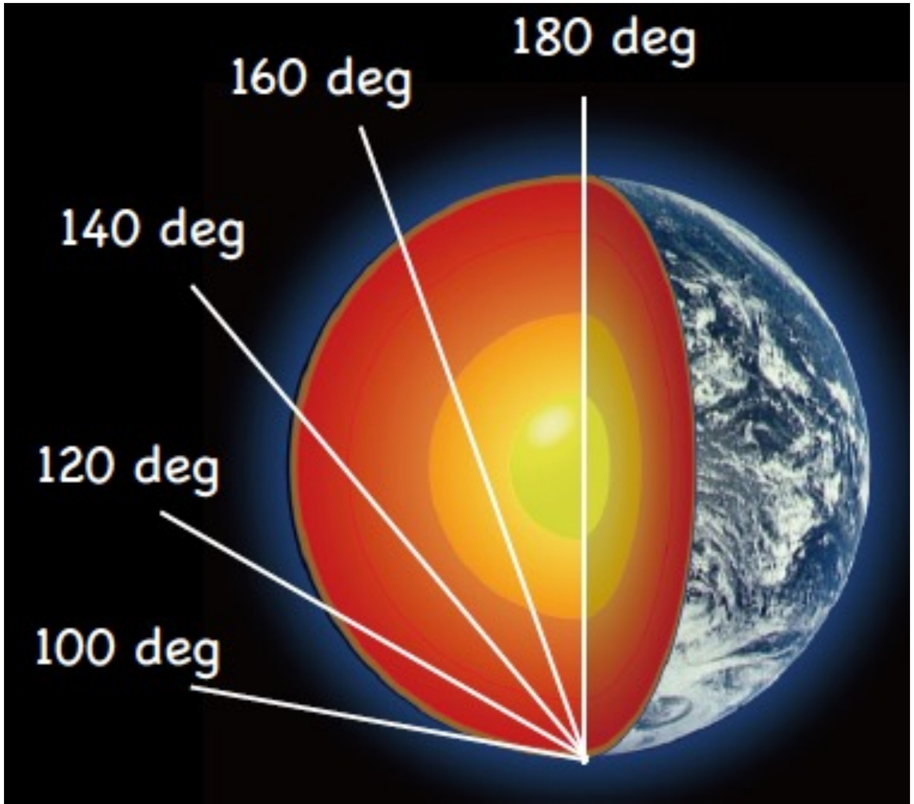




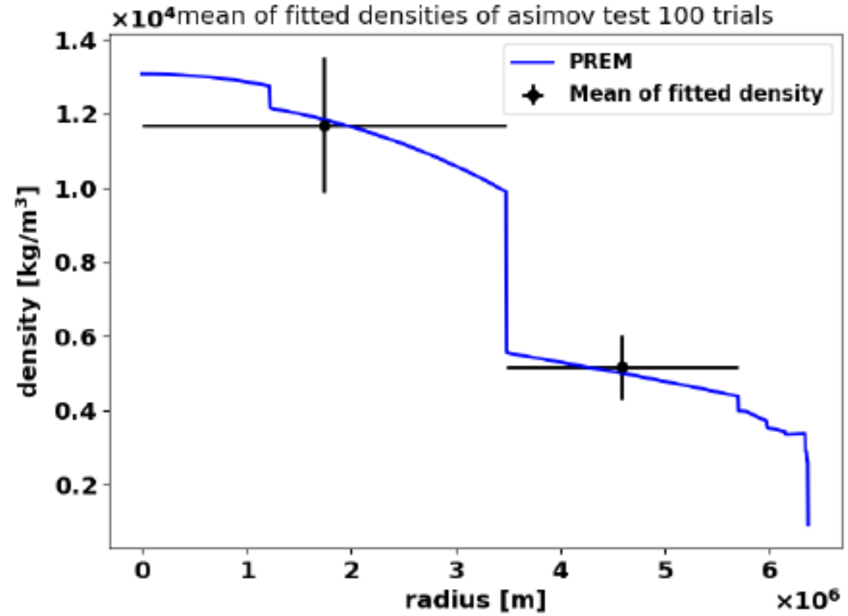


- **2018: first study with real IceCube data**
- **1 year sample (2011 - 2012) – upgoing ν_μ**
- **20145 reconstructed muons in the energy range of 400 GeV to 20 TeV**
- **Radial model with 5 layers of constant density**
- **Donini, Palomares-Ruiz, Salvado**
  Nature Phys. 15 (2019) 1, 37-40

Neutrino Absorption Tomography: At High Energies



	Energy*
100 deg	7.1 PeV
120 deg	480 TeV
140 deg	150 TeV
160 deg	49 TeV
180 deg	35 TeV



- **Results from 100 Asimov tests**
- **Scaled for 11 years of IceCube data**
- **Y-axis error bars represent standard deviations of fitted densities of 100 trials**

Kotoyo Hoshina (IceCube Collaboration) at MMTE 2022

Neutrino Oscillation Physics (1998 – 2024 & Beyond)

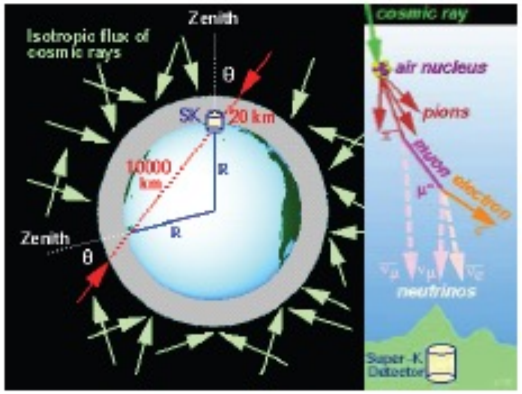
sun

Homestake, SAGE, GALLEX
SuperK, SNO, Borexino

reactors

KamLAND, CHOOZ
Double Chooz, Daya Bay, RENO

atmosphere



SuperKamiokande
IceCube, DeepCore

accelerators

K2K, MINOS, T2K
NOvA

Over the last two decades or so, marvelous data from world-class experiments

- Solar neutrinos (ν_e)
- Atmospheric neutrinos ($\nu_\mu, \bar{\nu}_\mu, \nu_e, \bar{\nu}_e$)
- Reactor anti-neutrinos ($\bar{\nu}_e$)
- Accelerator neutrinos ($\nu_\mu, \bar{\nu}_\mu$)

→ ***Data from various neutrino sources and vastly different energy and distance scales***

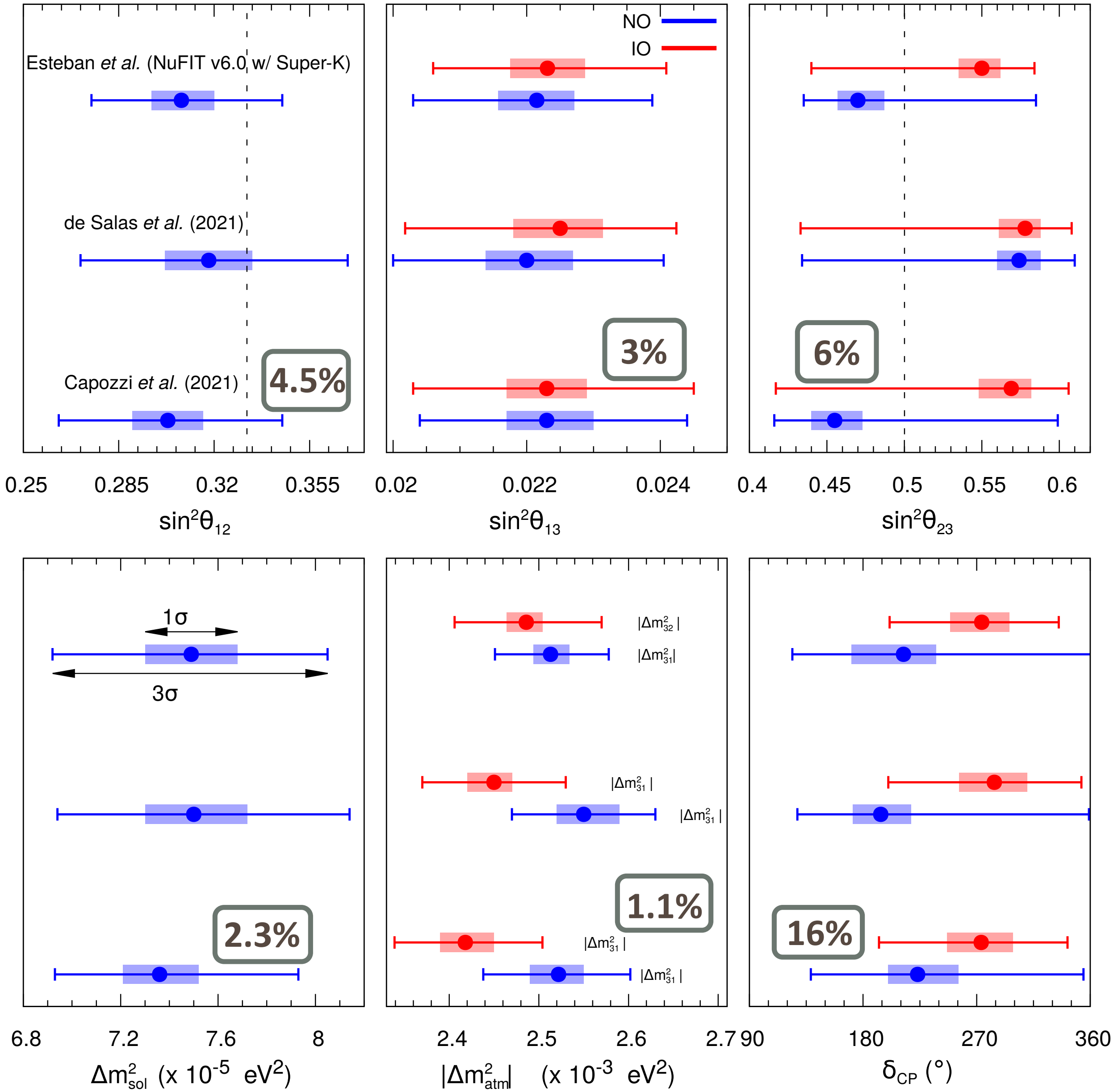
↓

Neutrinos change their flavor as they move in space and time

We have just started our journey in the mysterious world of neutrinos

Remarkable Precision on Neutrino Oscillation Parameters

Robust three-flavor neutrino oscillation paradigm

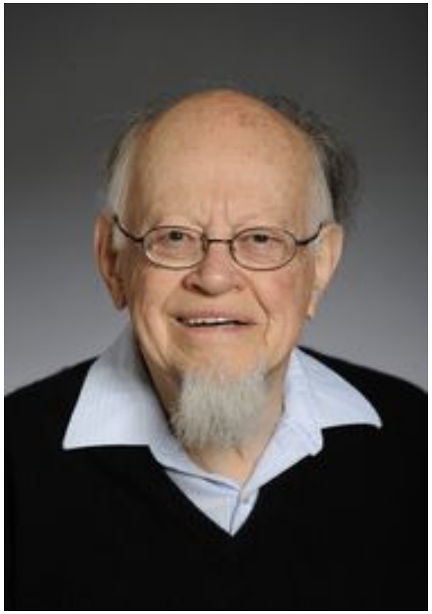


Huge boost for the discovery of NMO, CPV, and θ_{23} Octant

Agarwalla, Kundu, Prakash, Singh, JHEP 03 (2022) 206

Neutrino Oscillations in Matter: MSW Effect

- The MSW Effect (Wolfenstein 1978, Mikheyev and Smirnov 1985)
- Matter can change the pattern of neutrino oscillations significantly
- Resonant enhancement of oscillations and resonant flavor conversion possible
- Causes flavor conversion of solar neutrinos (LMA MSW solution established)



Lincoln Wolfenstein

Stanislav Mikheyev

Alexei Smirnov

Neutrino Oscillations in Matter: Parametric Resonance

- Oscillations of atmospheric neutrinos inside Earth can feel this resonance when neutrino trajectories cross the core of Earth

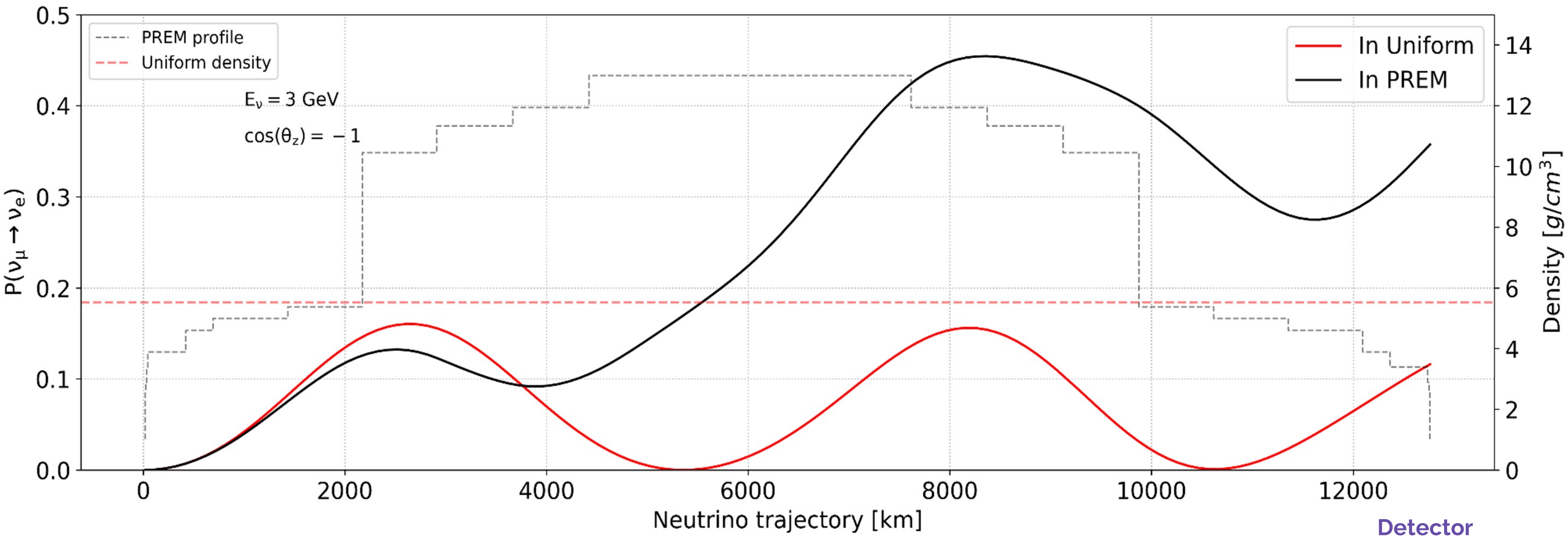
- The probabilities of neutrino flavor transitions can be strongly enhanced if the oscillation phase undergoes certain modification in matter

- This can happen if the variation in the matter density along the neutrino path is correlated in a certain way with the change in the oscillation phase

- This amplification of the neutrino oscillation probability in matter due to specific phase relationships can get accumulated if the matter density profile along the neutrino path repeats itself (periodic)

Petcov 1998, Liu, Mikheyev, and Smirnov 1998, Akhmedov 1998

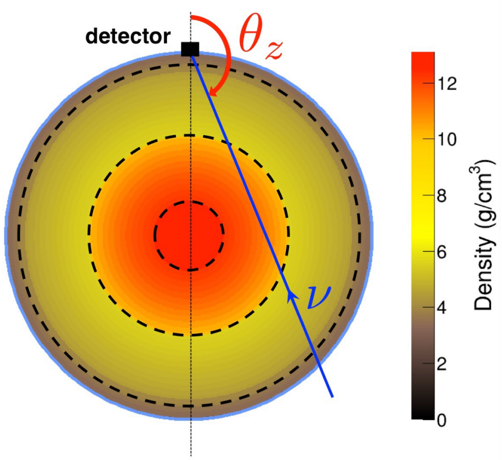
Parametric Resonance inside Earth



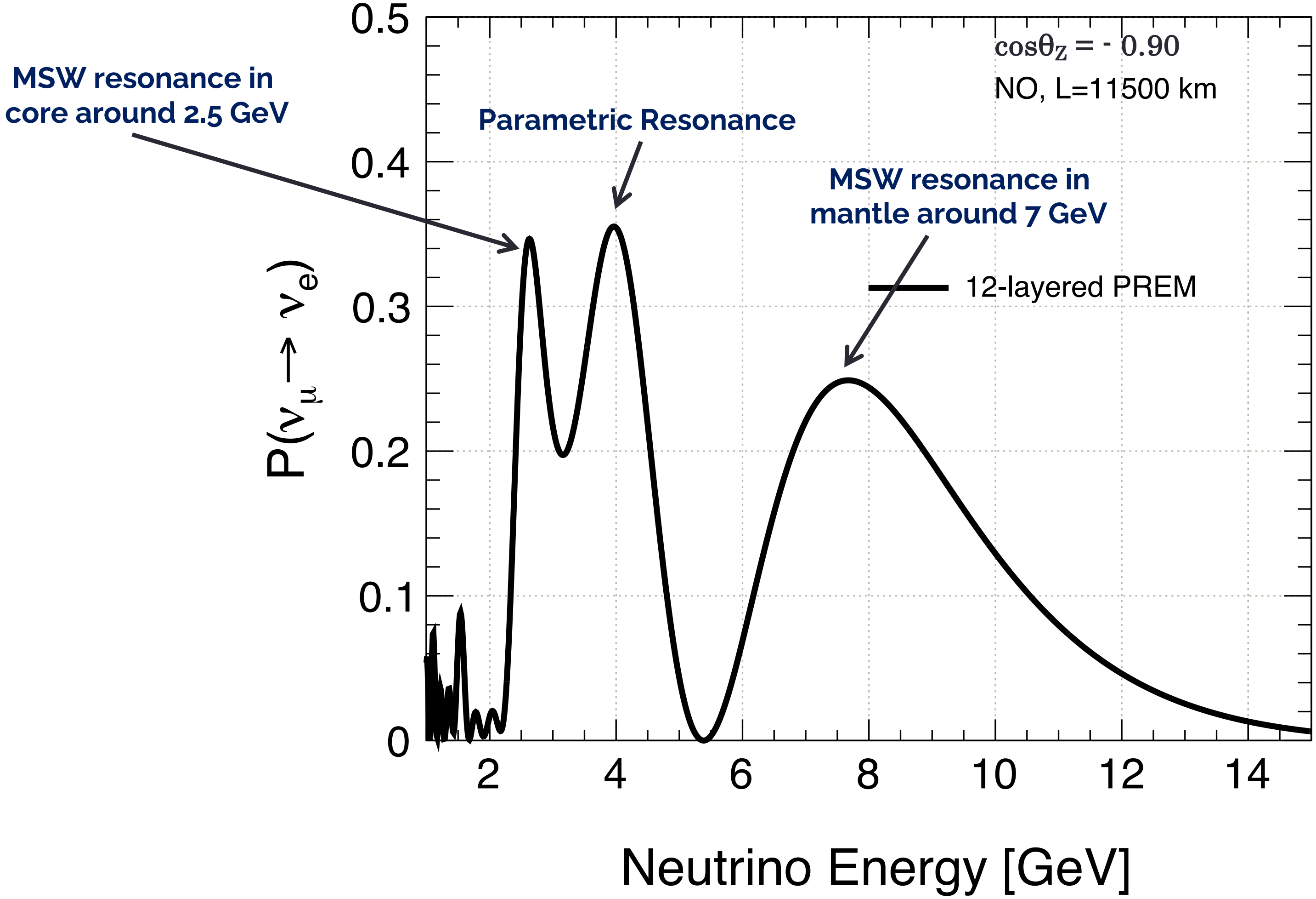
Detector

(Observable probability)

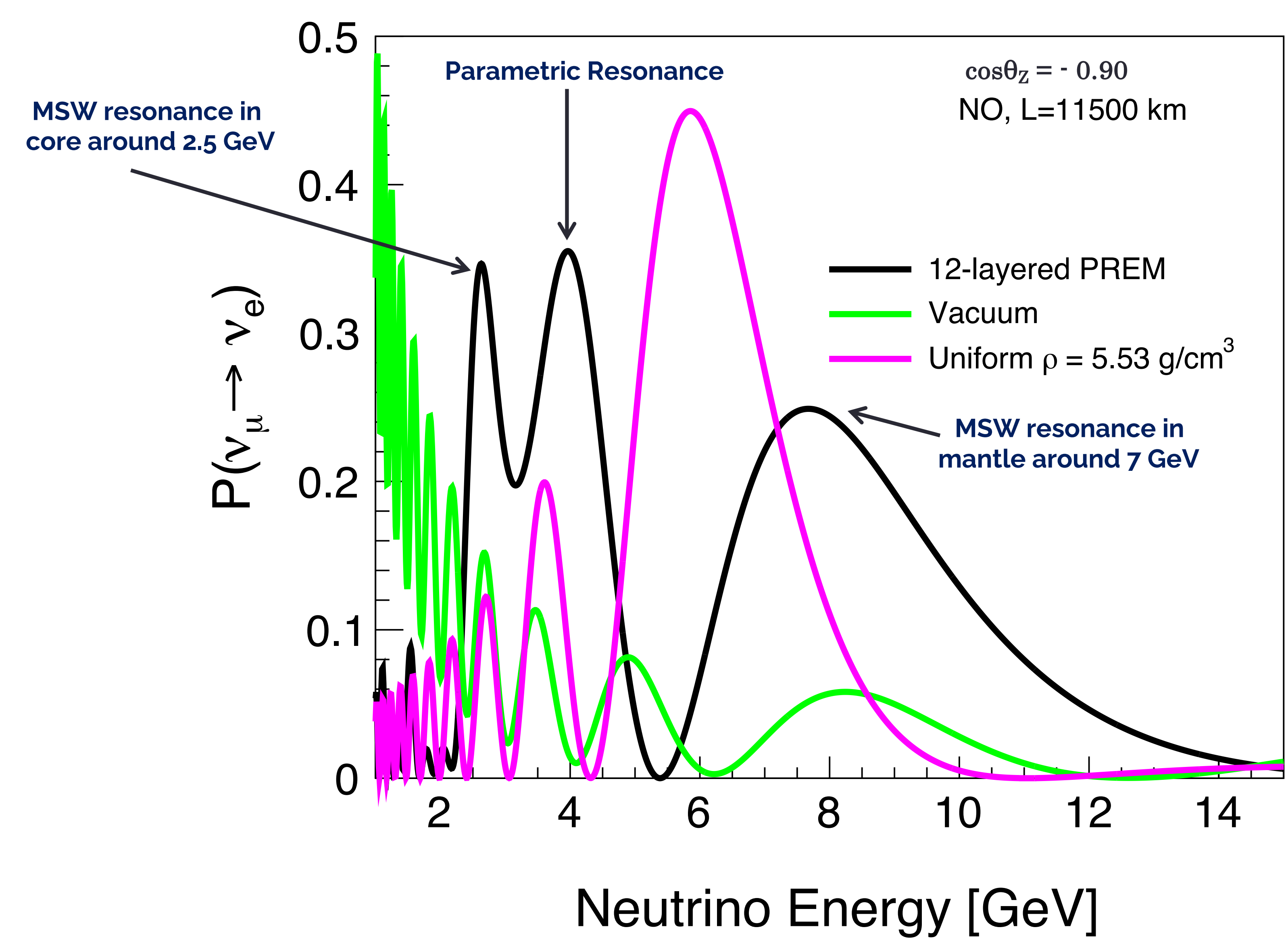
- Oscillation probability in PREM profile starts to differ from uniform density profile, once it sees the density jump in PREM at the core-mantle boundary (3480 km)
- Significant enhancement in probability in PREM is visible for core-passing trajectories due to parametric resonance



Matter Resonances inside Earth

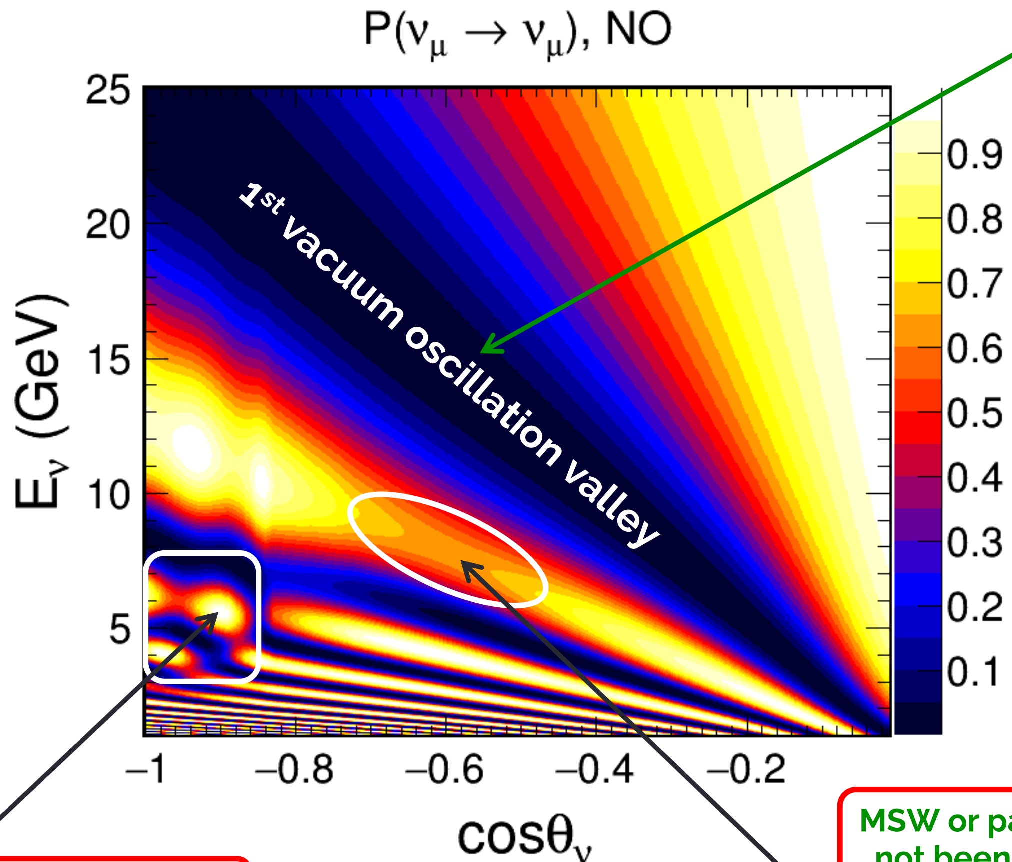


$$E_{\rm res} \equiv \frac{\Delta m^2_{31}\,\cos 2\theta_{13}}{2\sqrt{2}\,G_F\,N_e} \simeq 7\,{\rm GeV}\left(\frac{4.5\,{\rm g/cm^3}}{\rho}\right)\left(\frac{\Delta m^2_{31}}{2.4\times10^{-3}\,{\rm eV^2}}\right)\cos 2\theta_{13}$$



Similar oscillation patterns for antineutrinos with inverted mass ordering

Oscillograms for Muon Neutrino Survival Channel



Provides information on atmospheric neutrino oscillation parameters

MSW or parametric resonances have not been observed yet inside Earth!

Parametric resonance region

$\cos\theta_\nu < -0.8$
3 GeV $< E_\nu < 6$ GeV
reducing threshold helps

Useful to probe Earth's matter effect

MSW resonance region

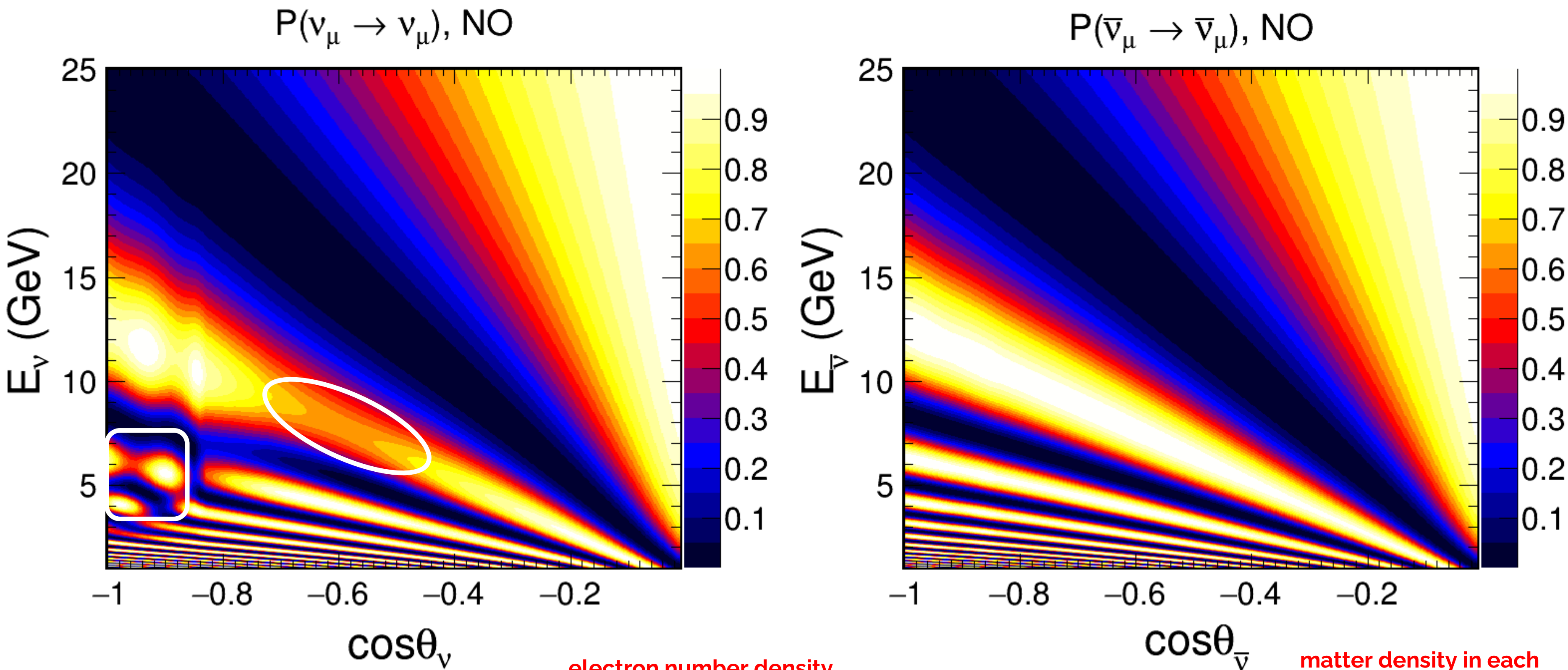
$-0.8 < \cos\theta_\nu < -0.5$
6 GeV $< E_\nu < 10$ GeV

Kumar, Khatun, Agarwalla, Dighe, EPJC 81 (2021) 2, 190

Neutrino vs. Antineutrino

Neutrinos (antineutrinos) feel Earth's matter effect for normal (inverted) mass ordering

normal ordering: $m_3 > m_2 > m_1$ or inverted ordering: $m_2 > m_1 > m_3$



electron number density

matter density in each layer inside Earth

$$V_{\rm CC} = \pm\sqrt{2}G_F N_e \approx \pm 7.6 \times Y_e \times 10^{-14} \left[\frac{\rho}{g/cm^3}\right] \mathrm{eV}$$

$$Y_e = N_e/(N_p + N_n)$$

relative electron number density

+1 (-1) for neutrino (antineutrino)

chemical composition of Earth

Neutrino Oscillation Tomography with Atmospheric Neutrinos

Atmospheric neutrinos have access to a wide range of energies and baselines passing through Earth's mantle and core

At multi-GeV energies, these atmospheric neutrinos can feel resonances in Earth matter effects inside mantle and core

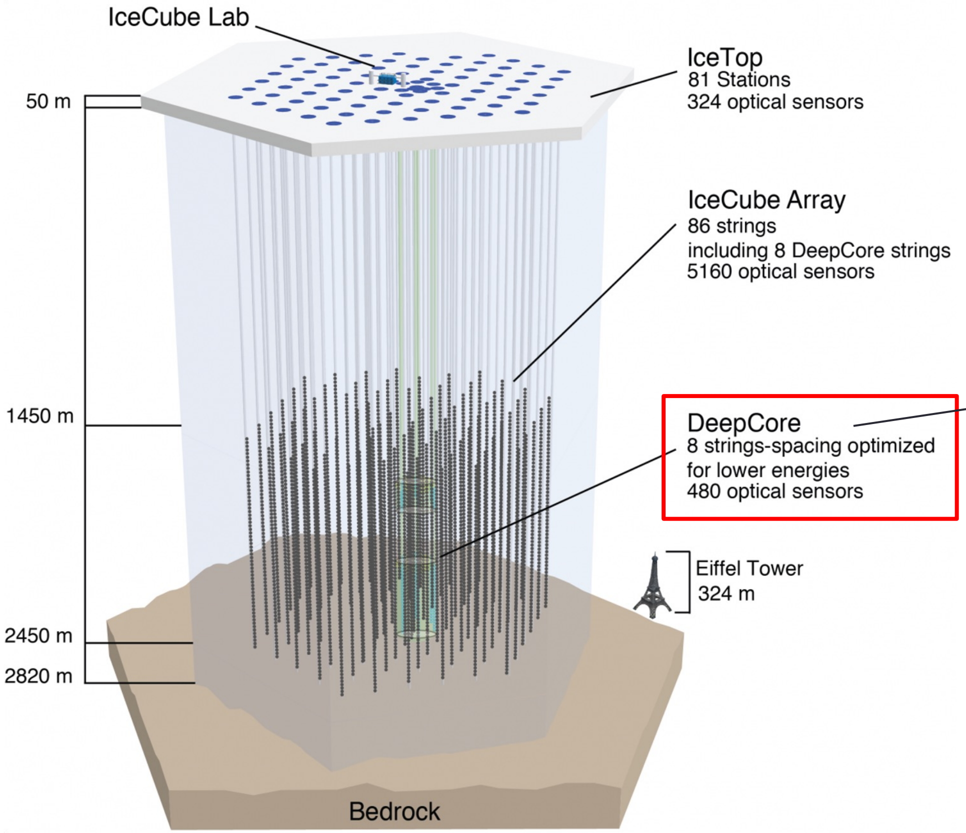
The recent advancement in the precision measurements of neutrino oscillation parameters has opened the avenue to perform a rich neutrino oscillation tomography using currently running and upcoming atmospheric neutrino experiments

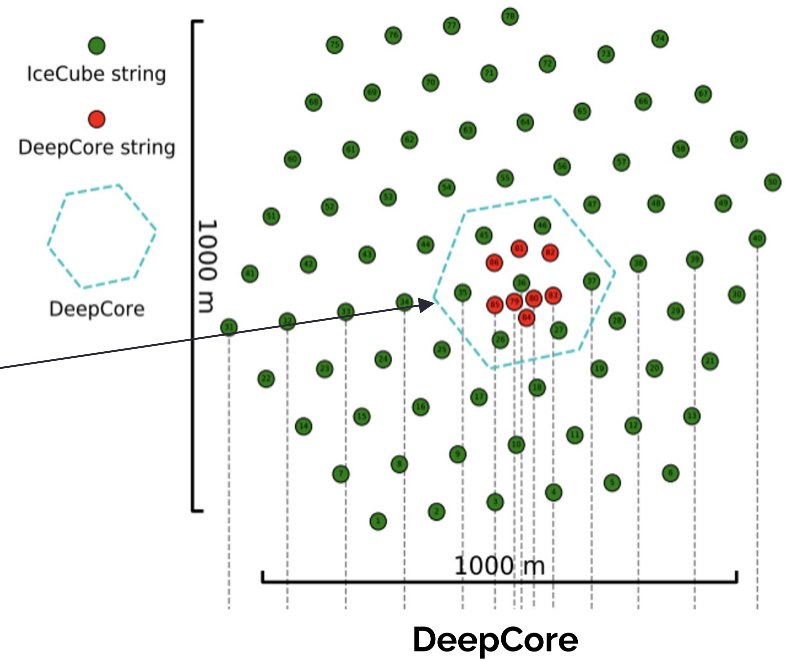
One can address the following important issues related to Earth:

- Observing the presence of Earth matter
- Ruling out the homogeneous matter
- Measuring the mass of Earth
- Validating the presence of core
- Locating the core-mantle boundary (CMB)
- Measuring the densities of core and mantle
- Chemical compositions of core and mantle

Goal is to provide an accurate 3-dimensional density distribution of electrons inside Earth!

IceCube DeepCore Detector





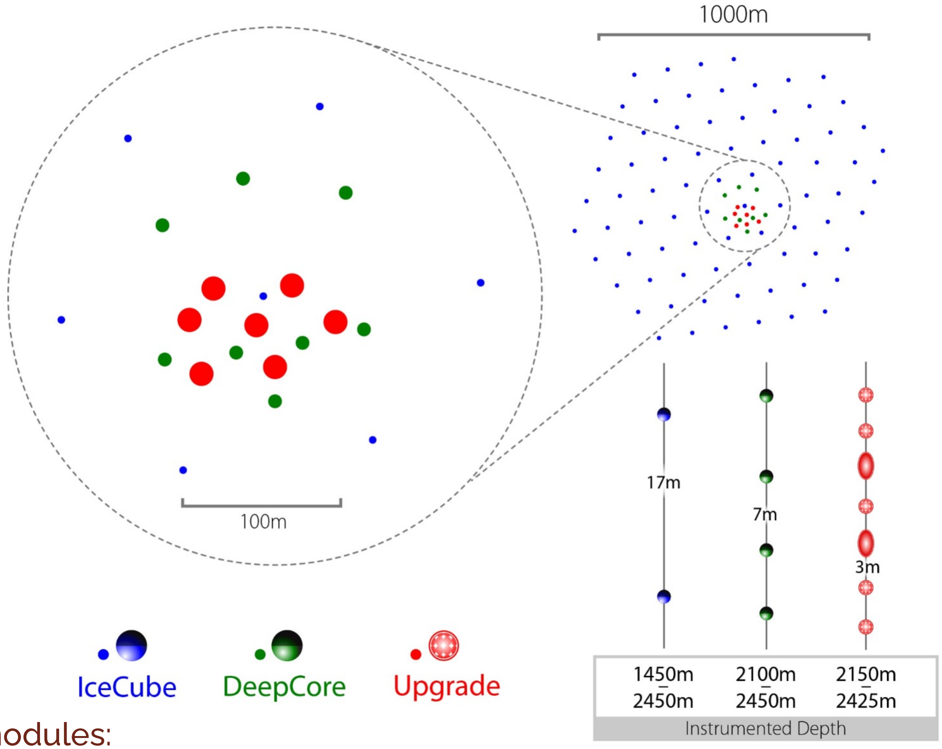
- 8 dedicated strings with denser spacing
- Optimized for GeV scale neutrinos
- Uses IceCube as VETO
- Fiducial volume ~ 10 Mton

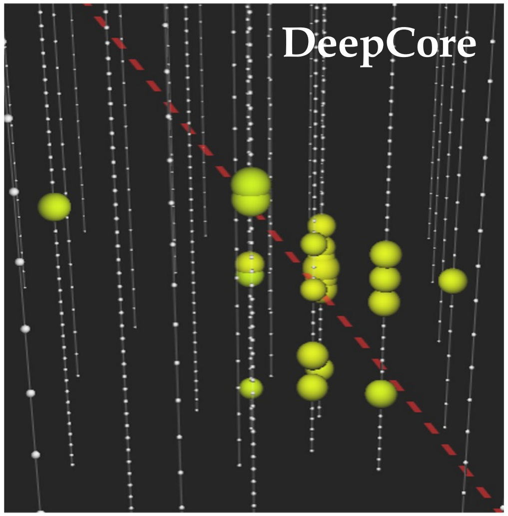
The design and performance of IceCube DeepCore (2012): Astroparticle Physics, 35(10), 615-624 (2012)

See talk by Mohamed Rameez for the IceCube Collaboration

A New Extension of DeepCore: IceCube Upgrade

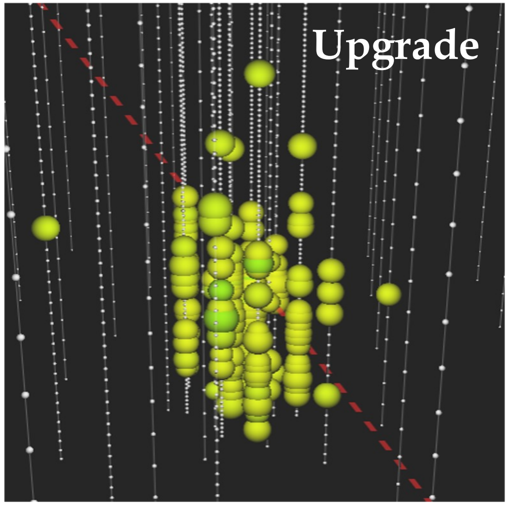
- **2 Mton of dense instrumentation for low-energy measurements**
- **7 new strings in the center region of detector: energy threshold ~ 1 GeV**
- **Higher event rate: 4 x DeepCore**
- **To be deployed in the Antarctic summer of 2025/26**

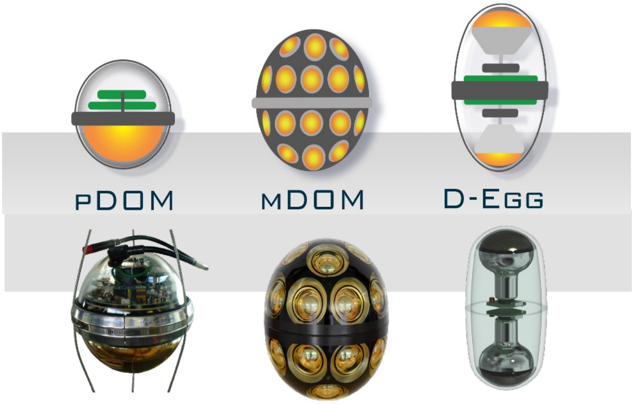




30 GeV Neutrino

Spacing between new modules:
20 m horizontally & 3 m vertically



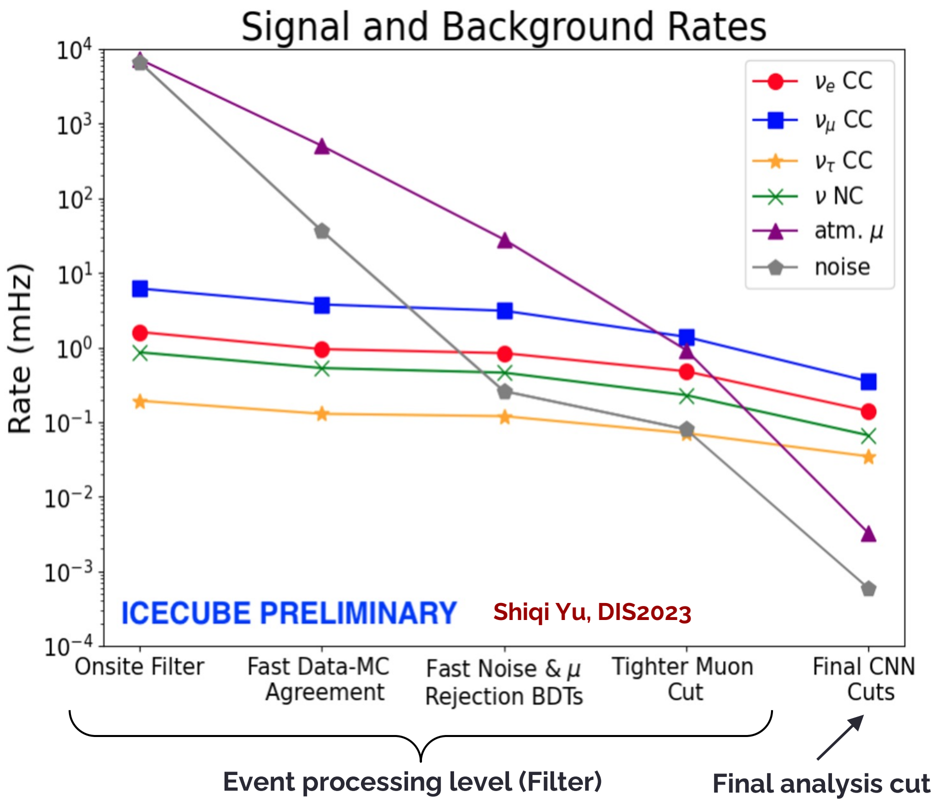


2 new types of optical modules w/ multi-PMT configurations

ICRC2019 arXiv:1908.09441 ICRC2023 arXiv:2307.15295

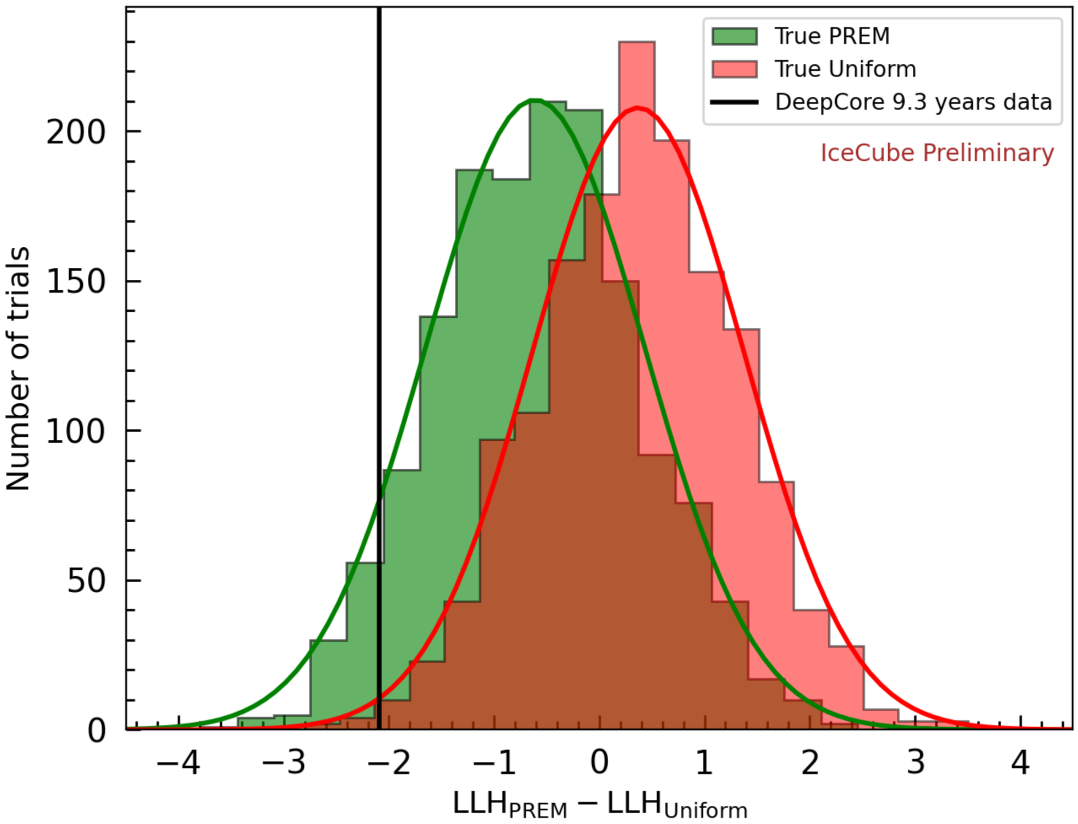
Simulated Neutrino Events

- Convolutional Neural Network (CNN) based reconstruction
- Monte Carlo (MC) simulation with 9.3 years of exposure (2012 - 2021)
- Huge statistics (~192k events)
- Neutrinos comprise 99.5% of sample
- High statistics in ν_μ CC channel
- Filters are applied to eliminate primary backgrounds: noise and atm. muon contamination (~0.5%)



Selection	Expected MC Events (9.3 yr)	% of Sample
$\nu_e + \bar{\nu}_e$ CC	48616	25.2
$\nu_\mu + \bar{\nu}_\mu$ CC	110656	57.5
$\nu_\tau + \bar{\nu}_\tau$ CC	10938	5.7
$\nu_{\text{all}} + \bar{\nu}_{\text{all}}$ NC	21412	11.1
μ_{atm}	973	0.5
All MC	192597	–

First Hint of Layered Structure Inside Earth using Neutrino Oscillation



P-value:

- **True PREM**:

94% (No. of trials right to the data line: 1406)

- **True Uniform**:

0.46% (No. of trials left to the data line: 7)

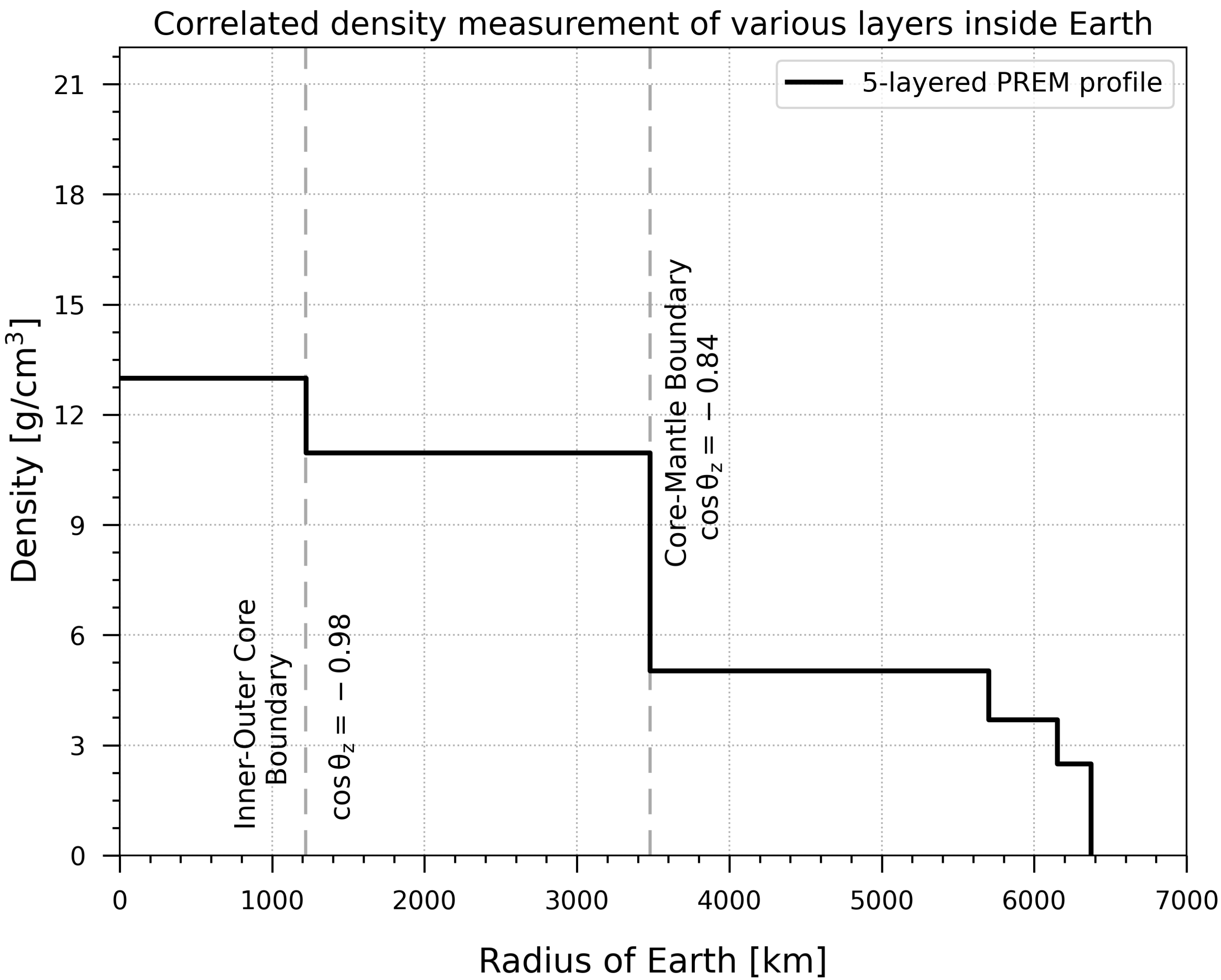
- **CLs** = (0.0046)/(1-0.94) = **7.6%**
- **CL to reject uniform hypothesis 92.4%**

$$\text{significane}\,(\eta_\sigma) = \sqrt{2}\,\text{erfc}^{-1}(2 \times CL_s)$$

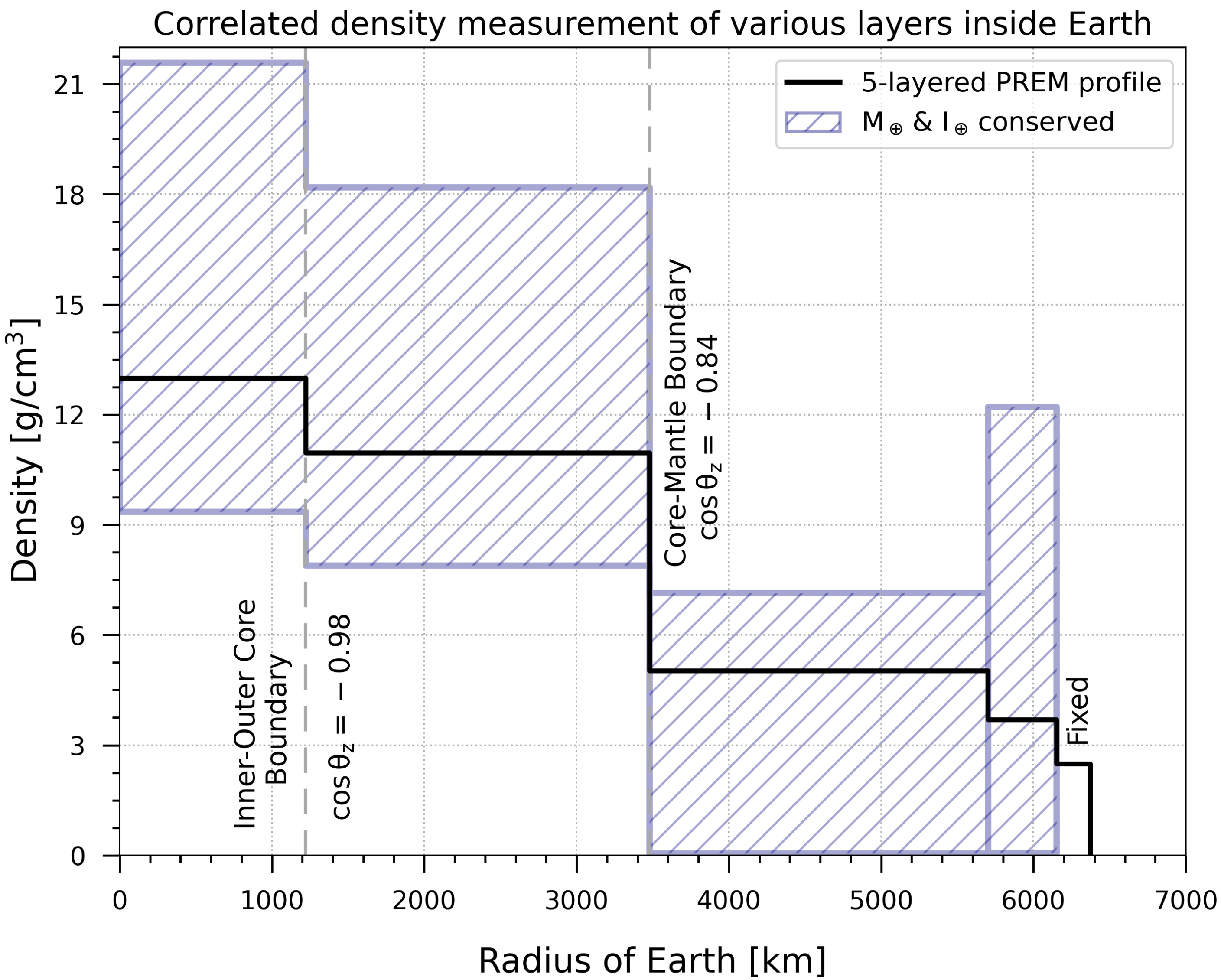
Significance to rule out homogeneous Earth is ~ 1.4σ

Talk by Krishnamoorthi J in PPC 2024 Conference, IIT Hyderabad, India

Five-layered PREM profile inside Earth

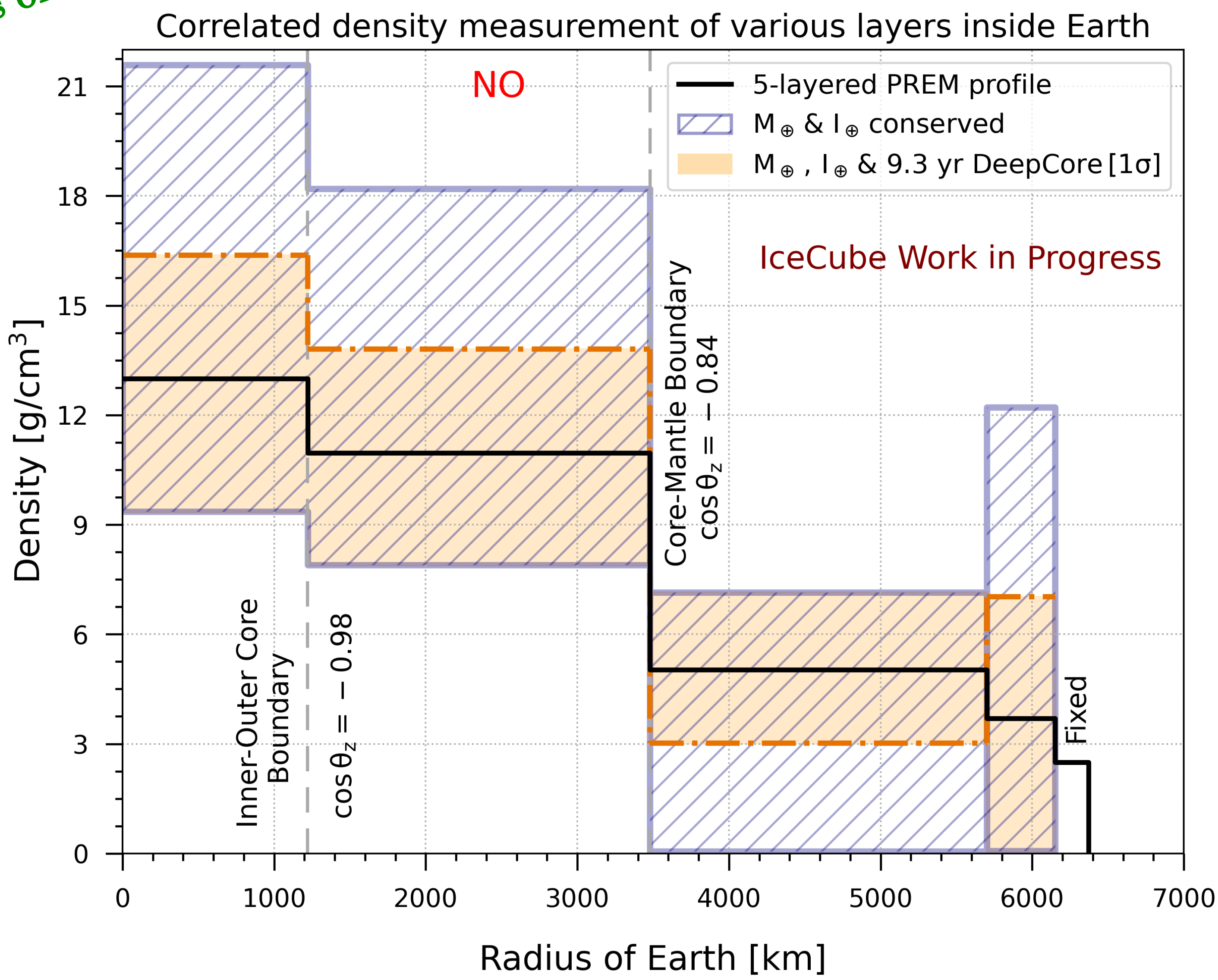


Constraints from Mass and Moment of Inertia of Earth



First Image of Earth using Atmospheric Neutrino Oscillation

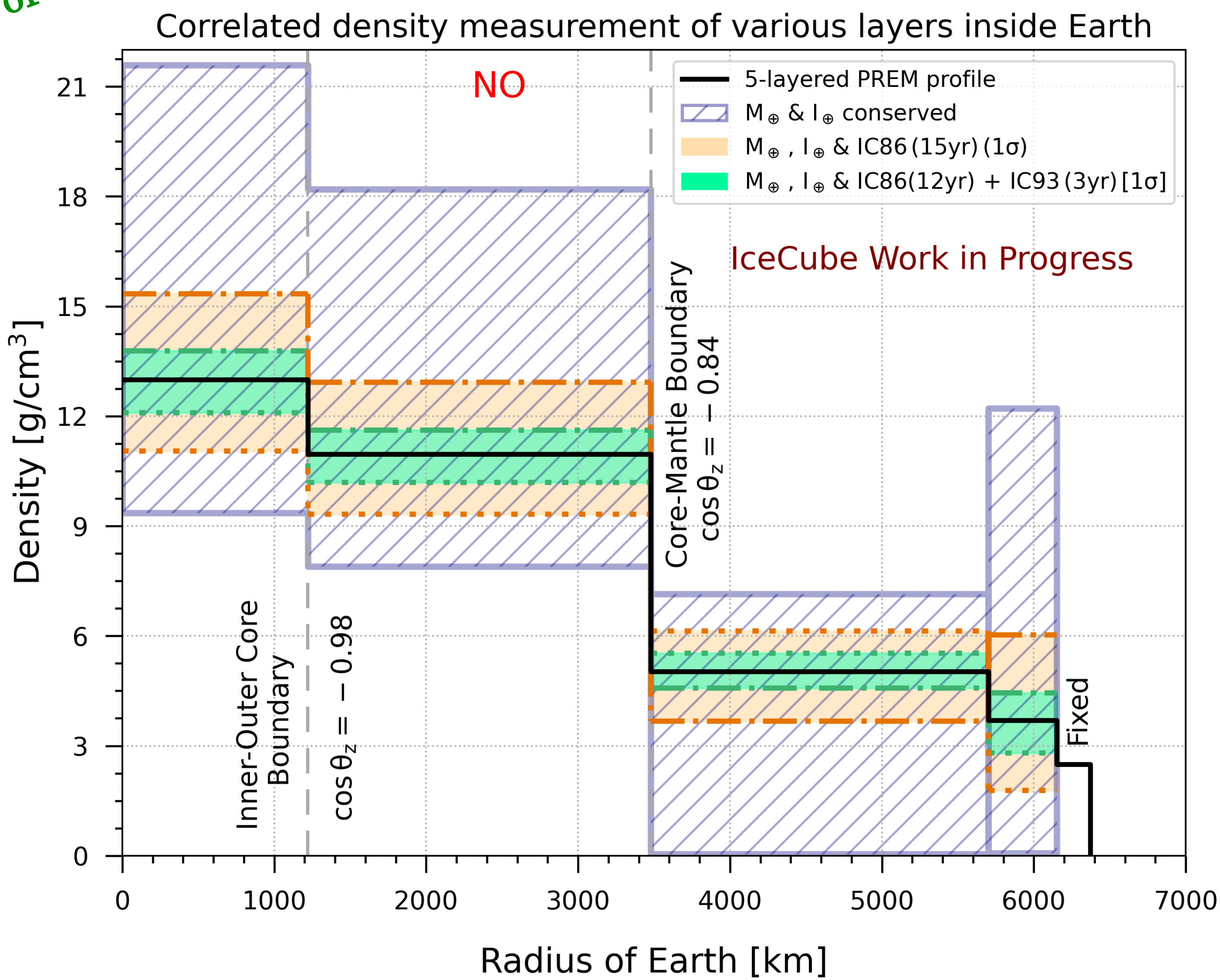
9.3 years of DeepCore



Talk by Sharmistha Chattopadhyay in PPC 2024 Conference, IIT Hyderabad, India

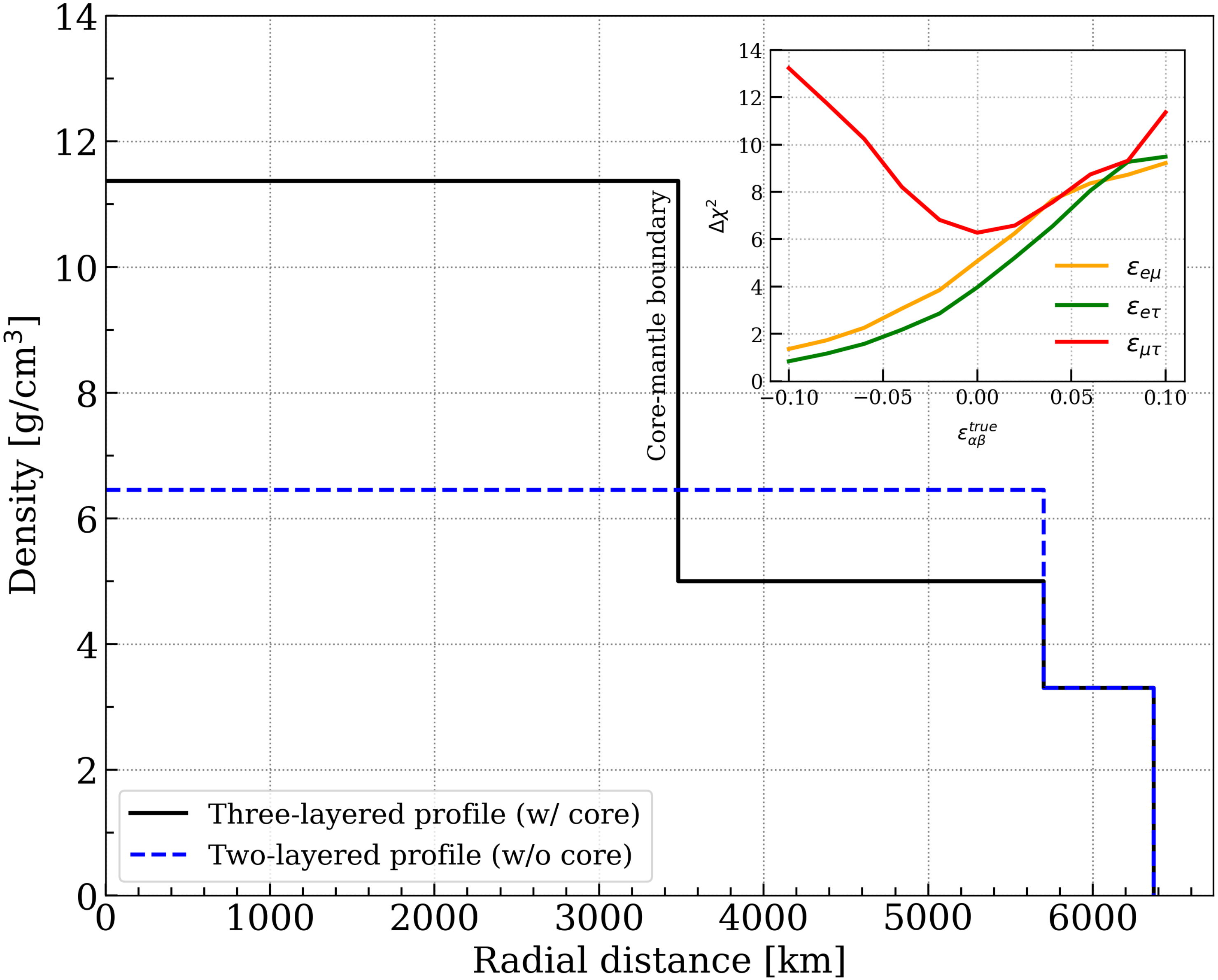
First Image of Earth using Atmospheric Neutrino Oscillation

3 years of Upgrade



Talk by Sharmistha Chattopadhyay in PPC 2024 Conference, IIT Hyderabad, India

Validating Earth's Core in the presence of Non-Standard Interactions



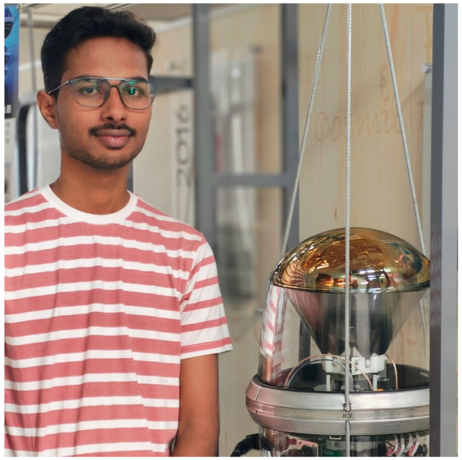
Krishnamoorthi J, Anuj Kumar Upadhyay, Anil Kumar, Sanjib Kumar Agarwalla, in preparation

The Earth Tomography Team

Amol Dighe, TIFR

Sanjib Kumar Agarwalla, IOP

Anil Kumar, Postdoc
DESY, Zeuthen



Krishnamoorthi J
Ph.D. Student, IOP

Anuj Kumar Upadhyay
Ph.D. Student, IOP

Sharmistha Chattopadhyay
Ph.D. Student, IOP

Concluding Remarks

Neutrino Tomography of the Earth is one of the most interesting, important, and not yet fully explored fields of research in today's Neutrino Physics

Two very different methods: neutrino absorption tomography and neutrino oscillation tomography

At present, neutrino oscillation tomography of the Earth with atmospheric neutrino seems to be the most feasible option (in view of the existing and upcoming oscillation experiments)

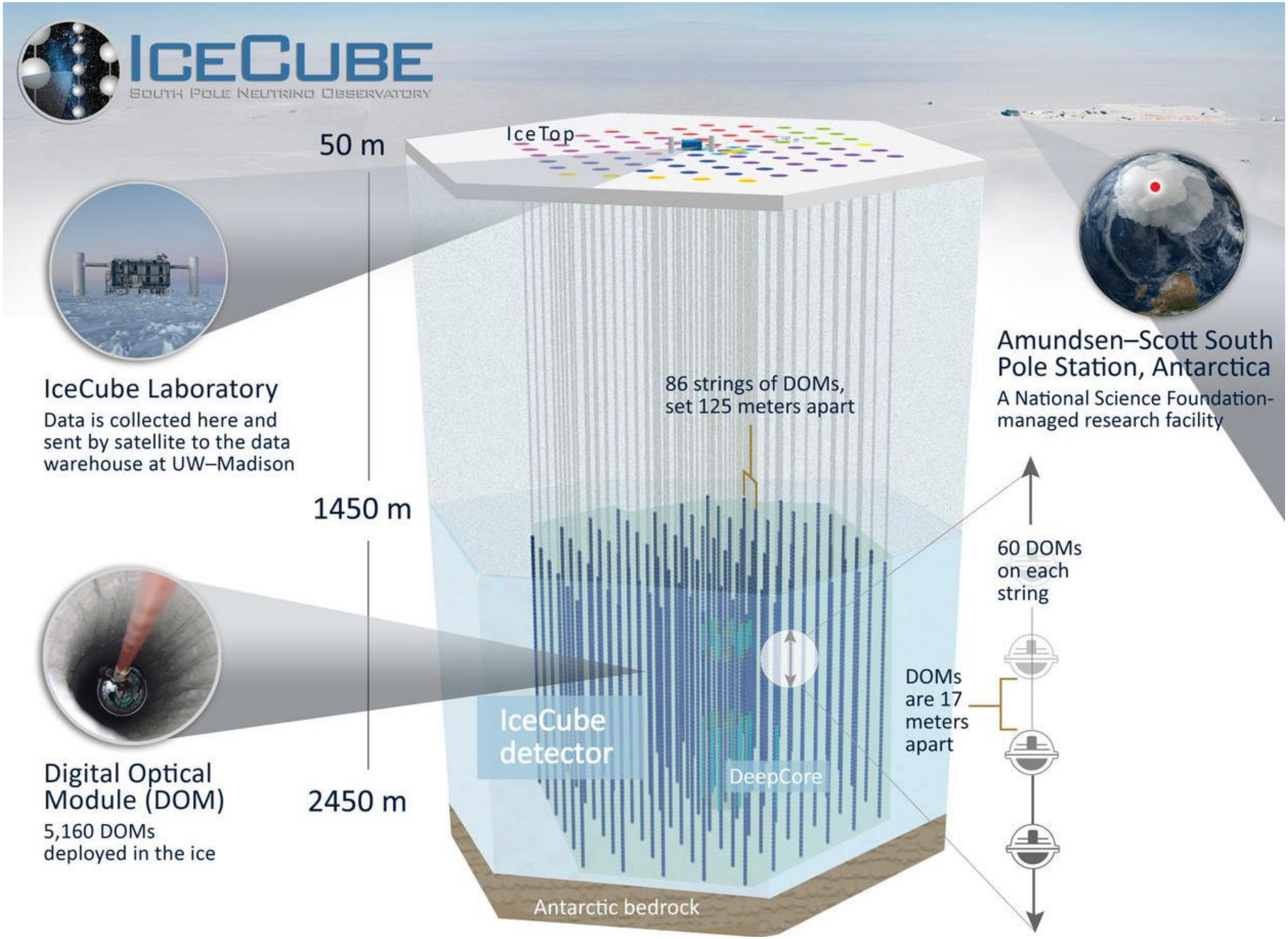
The neutrino tomography of the Earth is a promising powerful alternative method of obtaining valuable information about the Earth's interior. It is at initial stage of development. The neutrino tomography of the Earth can improve or even push the boundaries of our understanding of the Earth's interior, which can have far-reaching fundamental implications

The physics potential of the neutrino tomography of Earth should be fully explored!

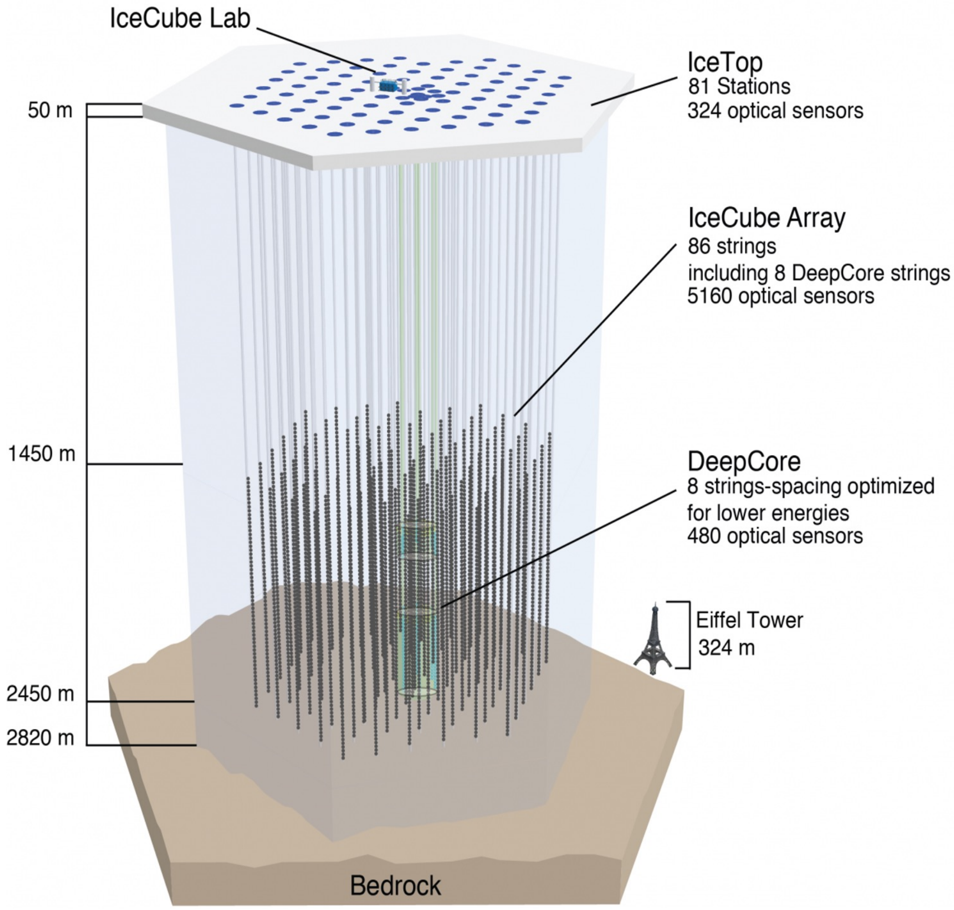
For the success of this endeavour, an active collaboration with the geophysicists/geochemists is crucial!


Thank you!
MMTE 2022 (Salt Lake City, Utah, USA, July 2022)

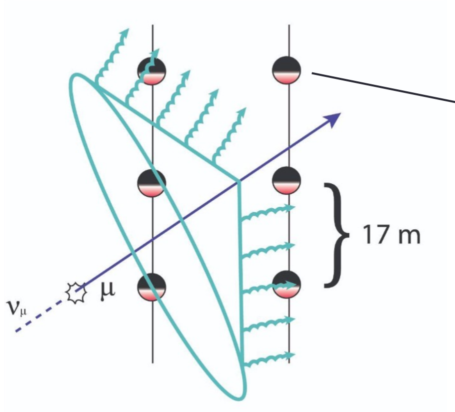
IceCube Neutrino Telescope at South Pole



IceCube Neutrino Telescope



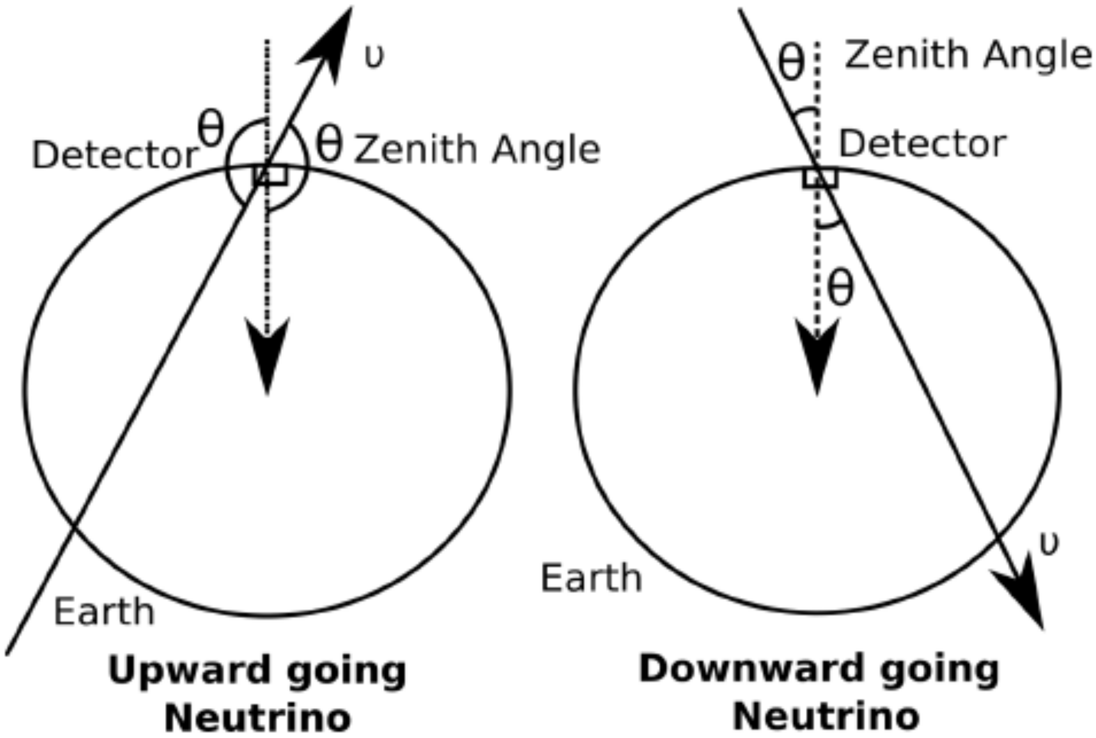
- 1 km³ **ν** detector deep under ice at South Pole
- 3 components: IceTop, IceCube, and DeepCore
- 5160 DOMs across 86 strings
- Optimized for TeV-PeV



DOM

- Neutrino interacts with ice and produces charged lepton
- Lepton direction closely aligned with neutrino
- Charged leptons emit Cherenkov radiation, when they travel faster than light in a medium
- Radiation detected by DOM (Digital Optical Modules)

Upward and Downward Directions for Atmospheric Neutrinos



Upward-going neutrinos:

$$\pi/2 < \theta < \pi$$

$$-1 < \cos\theta < 0$$

$$L_\nu \approx 2R\cos\theta_\nu$$

Downward-going neutrinos:

$$0 < \theta < \pi/2$$

$$0 < \cos\theta < 1$$

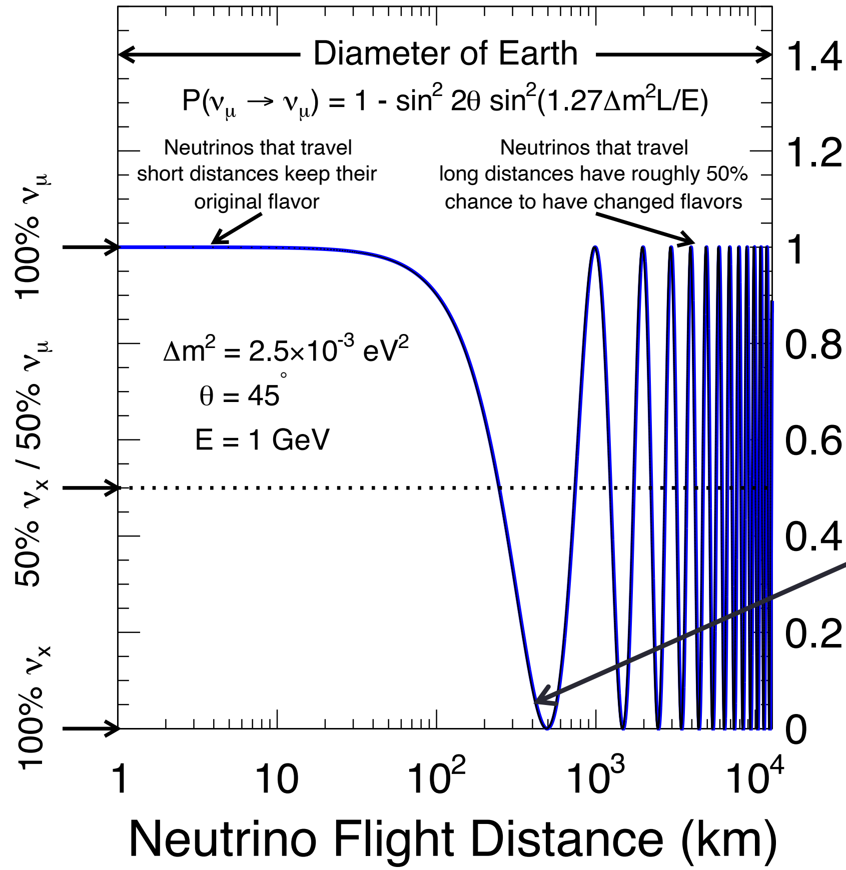
$$L_\nu \approx 0$$

$$L_\nu = \sqrt{(R+h)^2 - (R-d)^2 \sin^2\theta_\nu} - (R-d)\cos\theta_\nu$$

R = radius of Earth (6371 km)
h = average ν production height from surface (~ 10 to 25 km)
d = depth of the detector underground (~ 100 m to 2 km)

Upward-going neutrinos may interact with ambient matter inside Earth – key for neutrino tomography of Earth

Neutrino oscillation as a function of distance travelled



1st osc. dip

Simple two-flavor neutrino oscillation as seen by Super-K

location of 1st oscillation dip → consider muon survival probability in 2-flavor oscillations

$$P(\nu_\mu \to \nu_\mu) = 1 - \sin^2 2\theta_{23} \cdot \sin^2\left(1.27 \cdot |\Delta m^2_{32}|\,(\mathrm{eV}^2) \cdot \frac{L_\nu\,(\mathrm{km})}{E_\nu\,(\mathrm{GeV})}\right)$$

$\theta = 45^\circ$

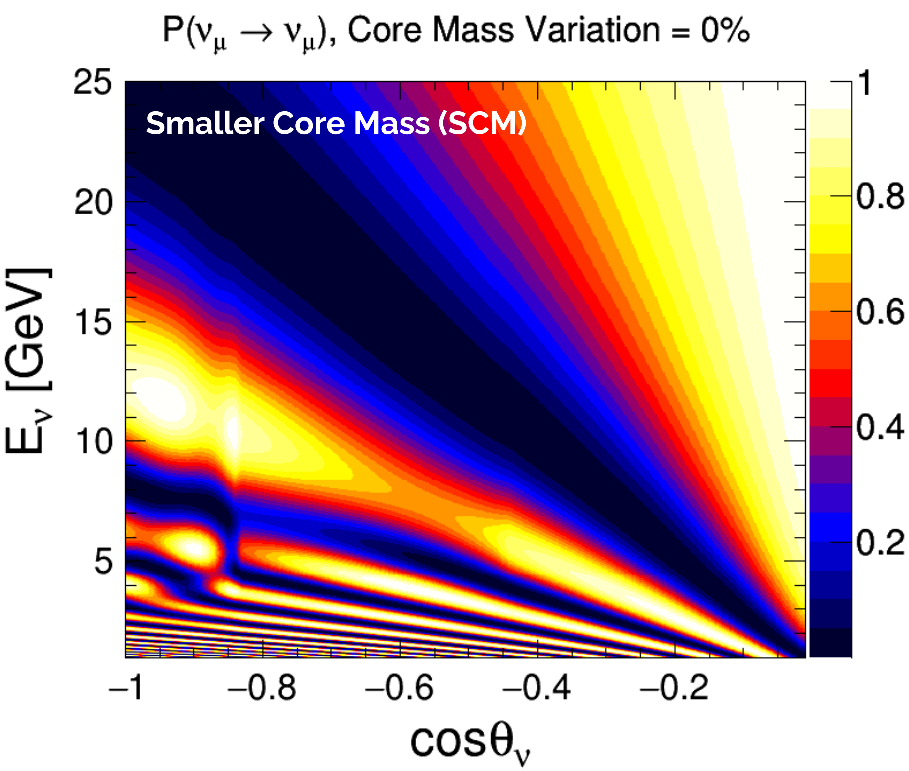
$\Delta m^2 = 2.4\text{e-}03\ \mathrm{eV}^2$

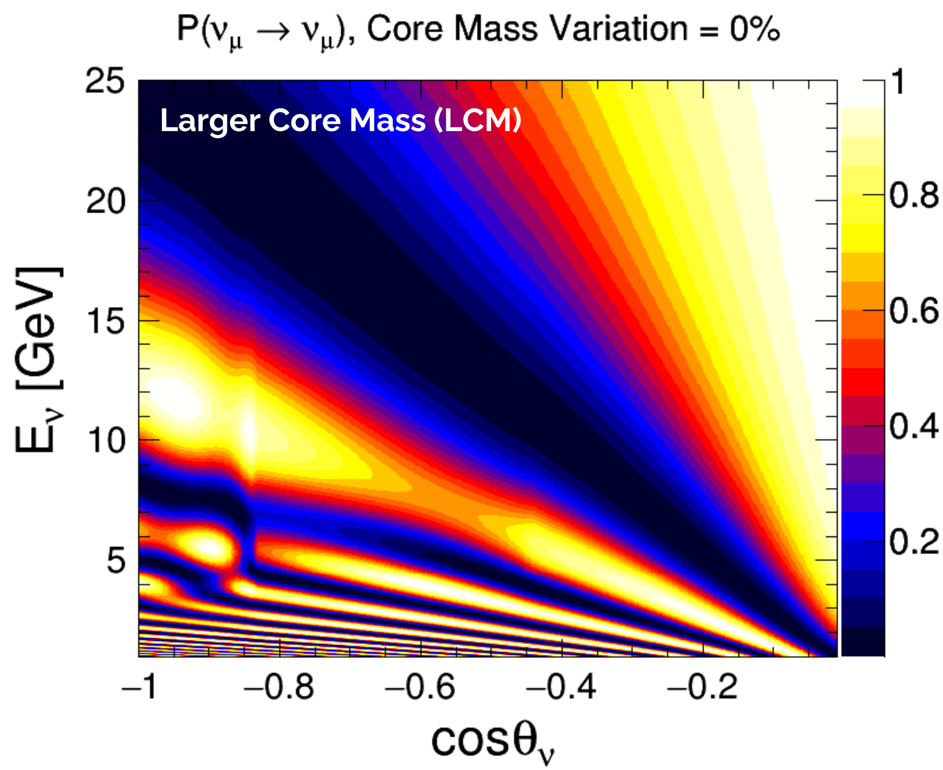
$\frac{1.27\Delta m^2 L}{E} = \frac{\pi}{2}$

$\frac{L}{E} = \frac{\pi}{2\times 1.27\times\Delta m^2} = 515.35$

$\log_{10}\left(\frac{L}{E}\right) = 2.71$

Varying the mass of core in 3-layered PREM profile





Total mass & moment of inertia of Earth fixed

Upadhyay, Kumar, Agarwalla, Dighe, PRD 107 (2023) 11, 115030

Hydrostatic equilibrium condition preserved

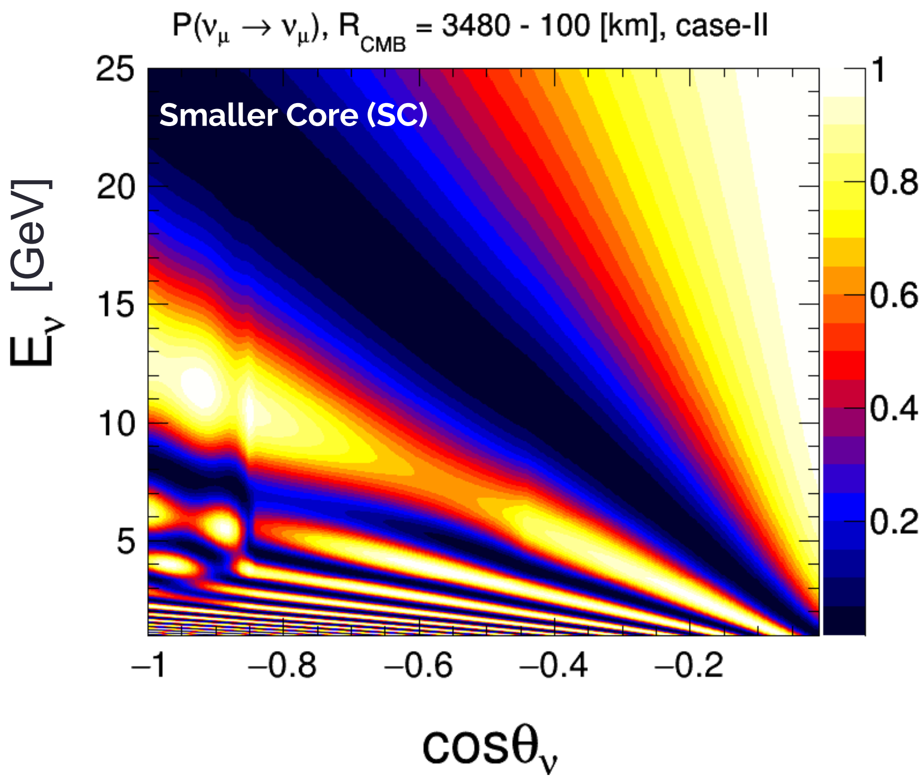
$\rho_{\text{inner-layer}} > \rho_{\text{outer-layer}}$

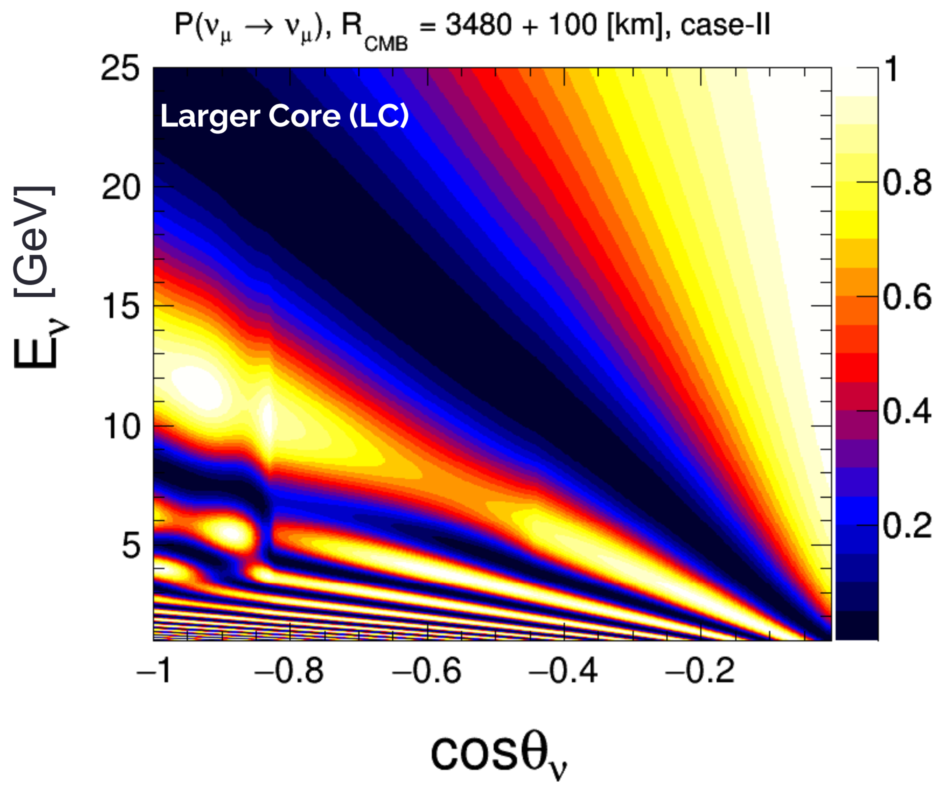
- **For SCM:**
  - **Parametric resonance regions shift to right and patterns shrink**
  - **MSW patterns stretch to left**

- **For LCM:**
  - **Parametric resonance regions shift to left and patterns broaden**
  - **MSW patterns shrink to right**

Opposite modifications in MSW and parametric resonance regions for SCM and LCM scenarios

Varying the core-mantle boundary in 3-layered PREM profile





Upadhyay, Kumar, Agarwalla, Dighe, JHEP 04 (2023) 068

Densities of inner and outer mantle fixed

Core density varies to keep $M_\oplus$ invariant

$\rho_{\text{inner-layer}} > \rho_{\text{outer-layer}}$

- **For SC:**
  - **Parametric resonance regions shift to left and patterns shrink**
  - **MSW patterns remains same**
- **For LC:**
  - **Parametric resonance regions shift to right and patterns broaden**
  - **MSW patterns shrink to right**

Opposite modifications in parametric resonance regions for SC and LC scenarios

Measuring density of various layers inside Earth

- External constraints :
    - Total mass of the Earth
    - Moment of Inertia of the Earth

$$M = \frac{4\pi}{3}[\rho_{\mathrm{IC}} R_{\mathrm{IC}}^3 + \rho_{\mathrm{OC}}(R_{\mathrm{OC}}^3 - R_{\mathrm{IC}}^3) + \rho_{\mathrm{IM}}(R_{\mathrm{IM}}^3 - R_{\mathrm{OC}}^3) + \rho_{\mathrm{MM}}(R_{\mathrm{MM}}^3 - R_{\mathrm{IM}}^3) + \rho_{\mathrm{OM}}(R_{\oplus}^3 - R_{\mathrm{MM}}^3)]$$

$$I = \frac{8\pi}{15}[\rho_{\mathrm{IC}} R_{\mathrm{IC}}^5 + \rho_{\mathrm{OC}}(R_{\mathrm{OC}}^5 - R_{\mathrm{IC}}^5) + \rho_{\mathrm{IM}}(R_{\mathrm{IM}}^5 - R_{\mathrm{OC}}^5) + \rho_{\mathrm{MM}}(R_{\mathrm{MM}}^5 - R_{\mathrm{IM}}^5) + \rho_{\mathrm{OM}}(R_{\oplus}^5 - R_{\mathrm{MM}}^5)]$$

- Inner core and outer core scaled by same scaling factor (say α) (by our choice)
- Inner mantle and middle mantle scaled by separate scaling factors (say β and γ respectively)
- Outer mantle not scaled (as it is assumed to be known)
- Radial boundaries kept fixed

Measuring density of various layers inside Earth

- External constraints :
  - Total mass of the Earth
  - Moment of Inertia of the Earth

$$M = \frac{4\pi}{3}[\alpha(\rho_{IC}R_{IC}^3+\rho_{OC}(R_{OC}^3-R_{IC}^3))+\beta(\rho_{IM}(R_{IM}^3-R_{OC}^3))+\gamma(\rho_{MM}(R_{MM}^3-R_{IM}^3))+\rho_{OM}(R_{\oplus}^3-R_{MM}^3)]$$

$$I = \frac{8\pi}{15}[\alpha(\rho_{IC}R_{IC}^5+\rho_{OC}(R_{OC}^5-R_{IC}^5))+\beta(\rho_{IM}(R_{IM}^5-R_{OC}^5))+\gamma(\rho_{MM}(R_{MM}^5-R_{IM}^5))+\rho_{OM}(R_{\oplus}^5-R_{MM}^5)]$$

- Scaling factor for core - core_density_scale (α) is given as an independent input (this is our choice)
- The equations are then solved for β and γ in terms of α
- For every value of core_density_scale (α), there will be a unique value of β and γ
- The observable in this analysis is core_density_scale, which will be fitted to the real data