

Looking for new physics with radio signatures

Andrea Caputo
UNDARK kick-off meeting

hep-ph/2405.13882 with Carl Beadle and Sebastian Ellis

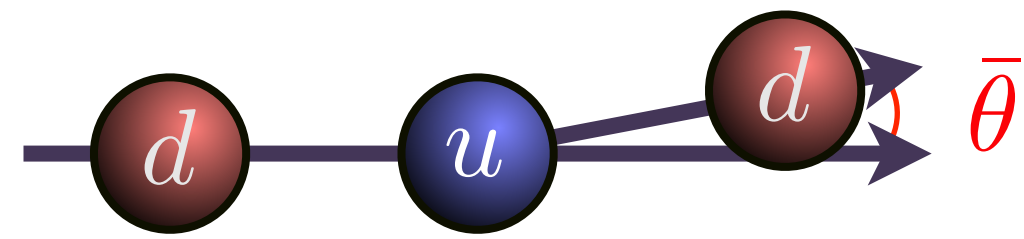


at some point in the future



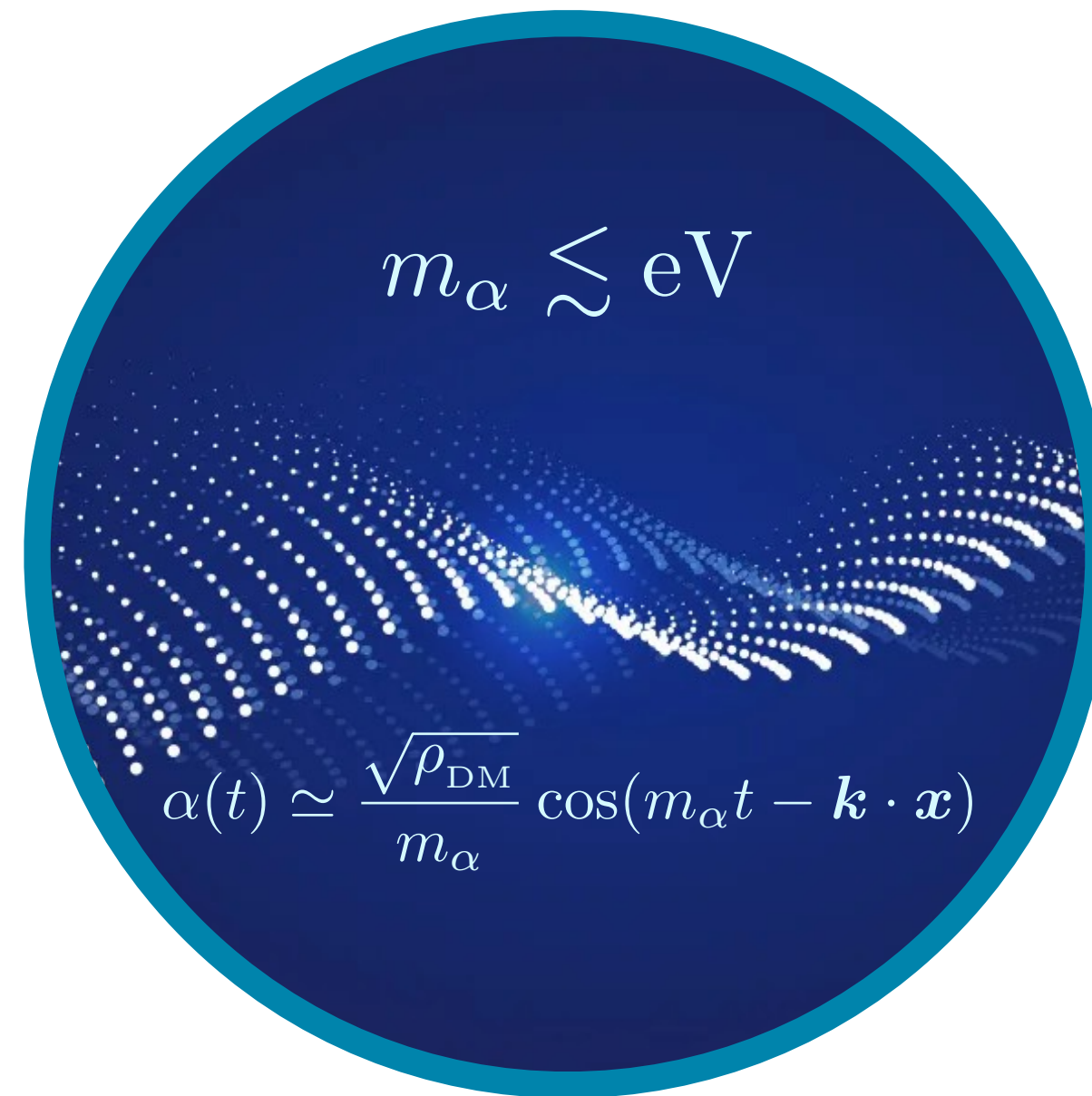
Two different targets (with some similarities)

Axions

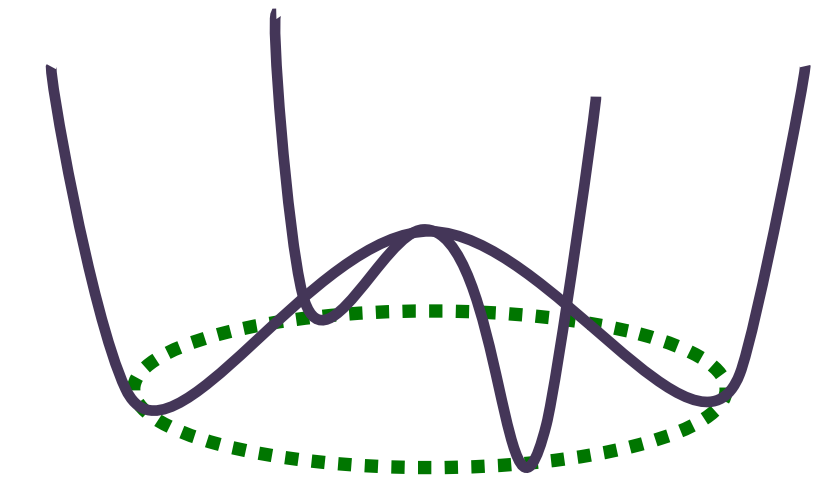


$$\mathcal{L} \supset \left(\frac{a}{f_a} + \bar{\theta} \right) \frac{g_s^2}{32\pi^2} G_{\mu\nu}^a \tilde{G}^{\mu\nu,a}$$

Peccei & Quinn (1977)
Weinberg (1978)
Wilczek (1978)



Massive Dark Photons



Proca (1936)
Stückelberg (1938)

- ▶ Misalignment
- ▶ Cosmic strings
- ▶ ...

Preskill et al, Abbott & Sikivie (1983)

Harari & Skive (1987), ..., Klaer & Moore (2017),
Gorghetto, Hardy & Villadoro (2018), Buschmann, Foster & Safdi (2019),
Gorghetto, Hardy & Villadoro (2020), Buschmann et al (2021)

- ▶ Misalignment
- ▶ Inflation
- ▶ ...

Nelson & Scholtz (2011)

Graham, Mardon & Rajendran (2015)

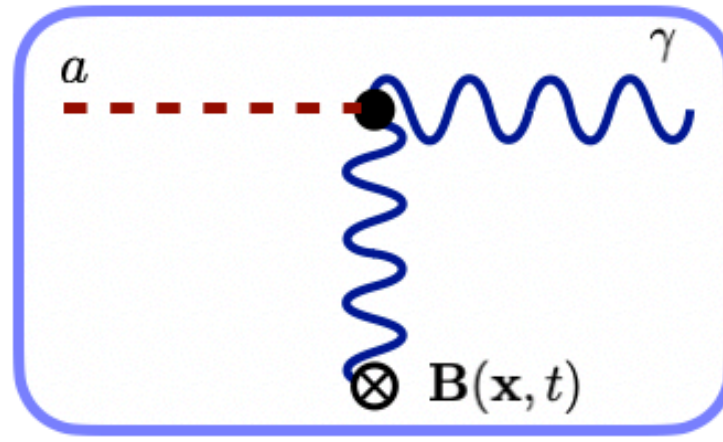
e.g. Agrawal et al, Co et al, Dror et al, Bastero-Gil et al (all 2018)

Two different targets (with some similarities)

Dark Photon:
$$\mathcal{L} \supset -\frac{1}{4} \left(F_{\mu\nu} F^{\mu\nu} - 2\epsilon F'_{\mu\nu} F^{\mu\nu} + F'_{\mu\nu} F'^{\mu\nu} \right) + \frac{1}{2} m_{A'}^2 A'_\mu A'^\mu - A_\mu \mathcal{J}^\mu$$

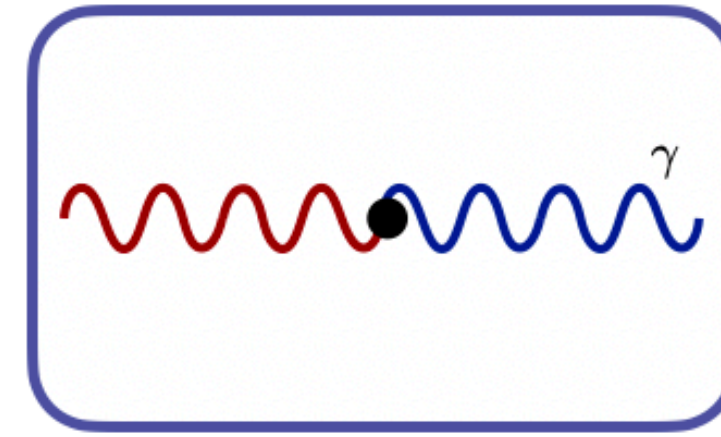
Axion:
$$\mathcal{L} \supset -\frac{1}{4} \left(F_{\mu\nu} F^{\mu\nu} - 2 \partial_\mu a \partial^\mu a + g_{a\gamma\gamma} a F_{\mu\nu} \tilde{F}^{\mu\nu} \right) - \frac{1}{2} m_a^2 a^2 - A_\mu \mathcal{J}^\mu$$

Axion:



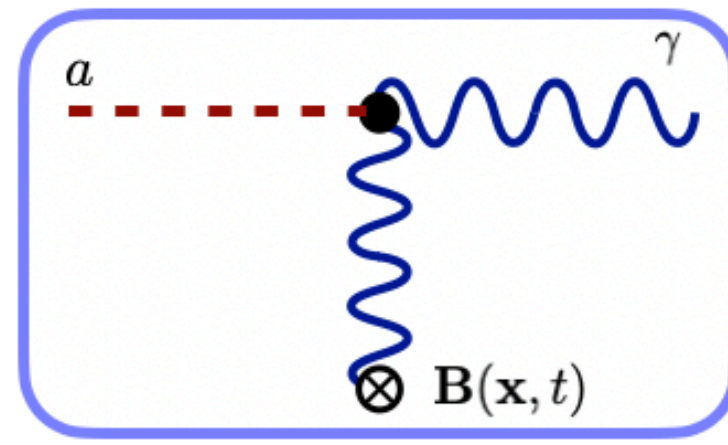
$$j_a \sim g_{a\gamma\gamma}(\partial a)F$$

Dark Photon:



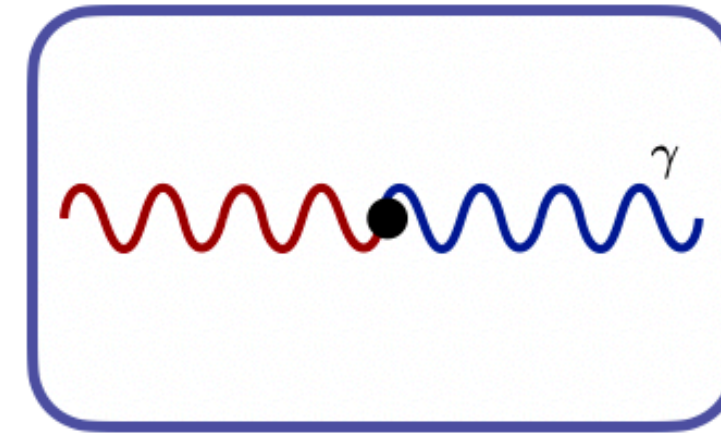
$$j_{A'} \sim -\epsilon m_{A'}^2 A'$$

Axion:



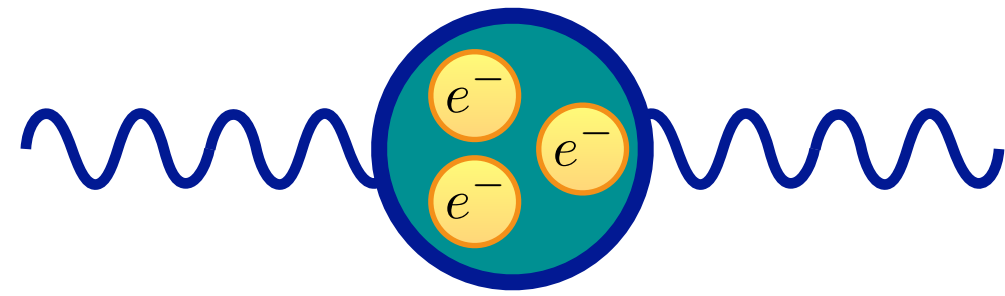
$$j_a \sim g_{a\gamma\gamma}(\partial a)F$$

Dark Photon:



$$j_{A'} \sim -\epsilon m_{A'}^2 A'$$

Free charges modify photon self-energy



$$\Pi^{\mu\nu} = \sum_p \Pi_p P^{\mu\nu}$$

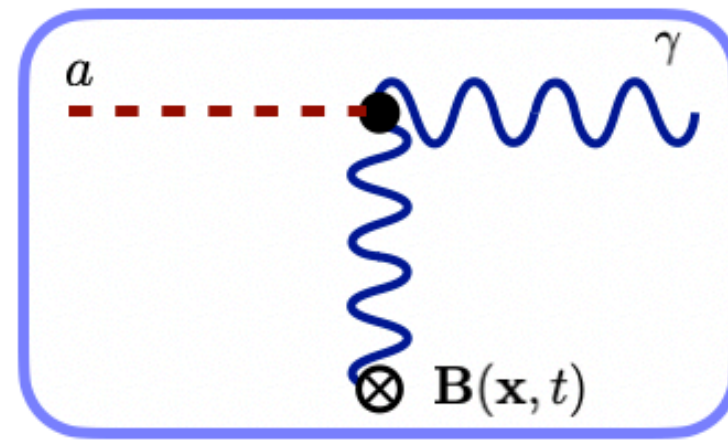
Transverse

$$\Pi_T \sim \omega_{\text{pl}}^2 \sim \frac{e^2 n_e}{m_e}$$

Longitudinal

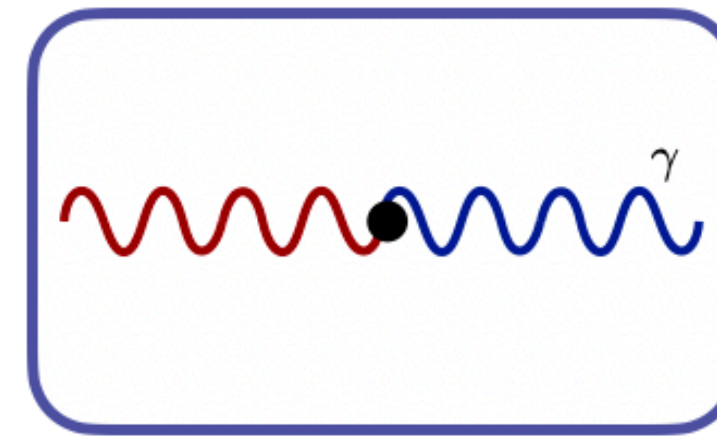
$$\Pi_L \sim \omega_{\text{pl}}^2 \left(1 - \frac{k^2}{\omega^2}\right)$$

Axion:



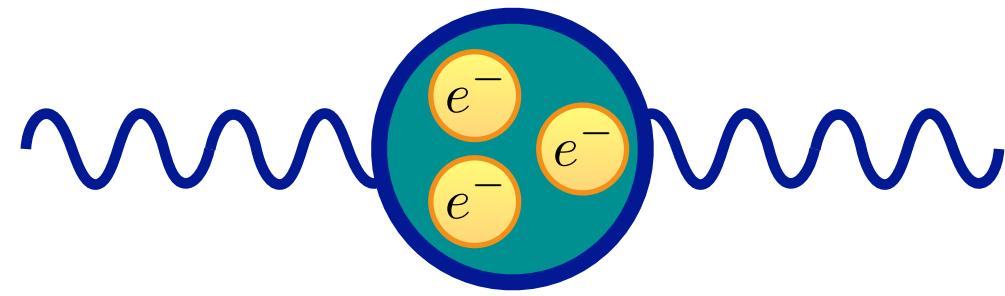
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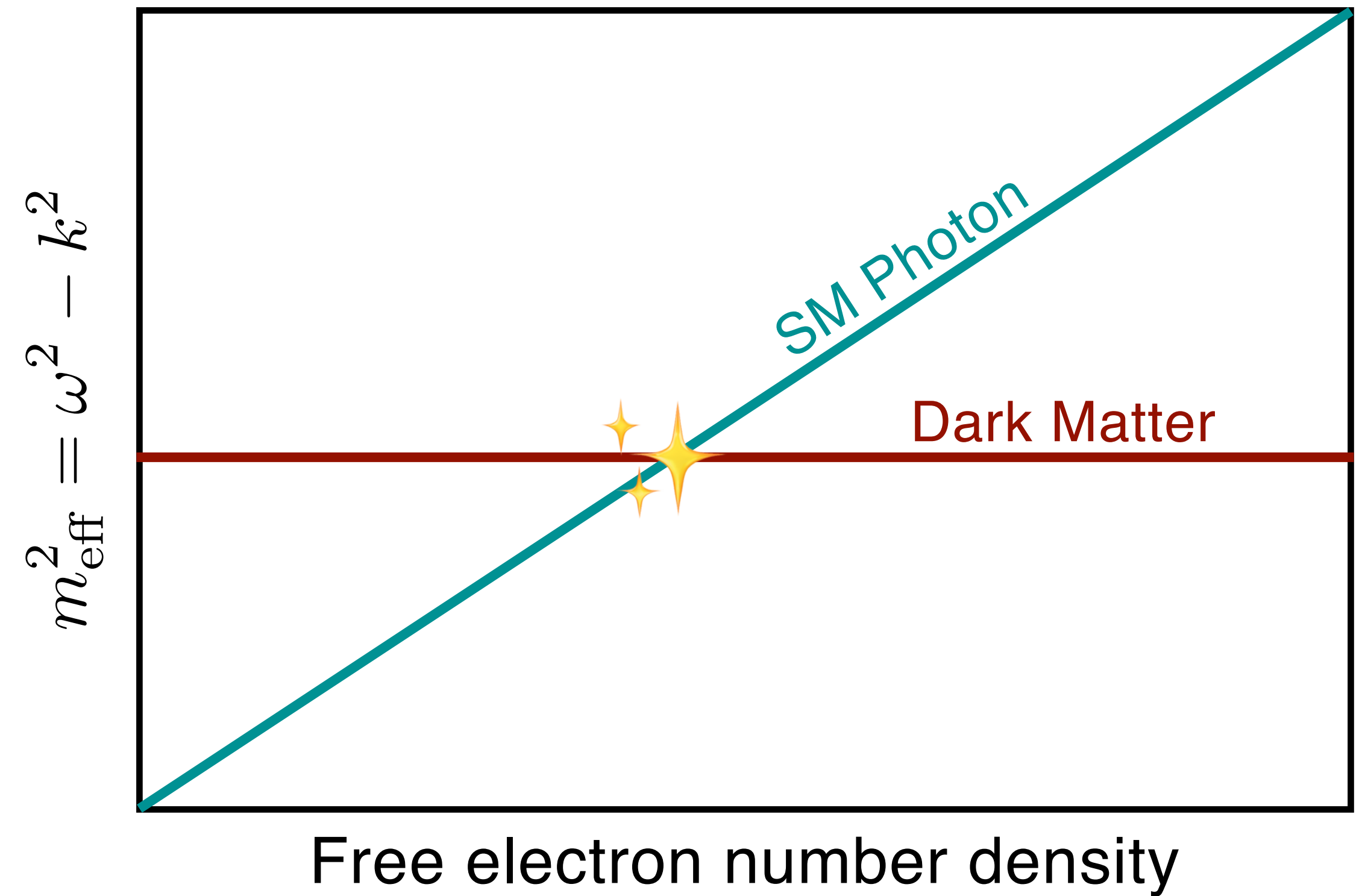
$$\Pi^{\mu\nu} = \sum_p \Pi_p P^{\mu\nu}$$

Transverse

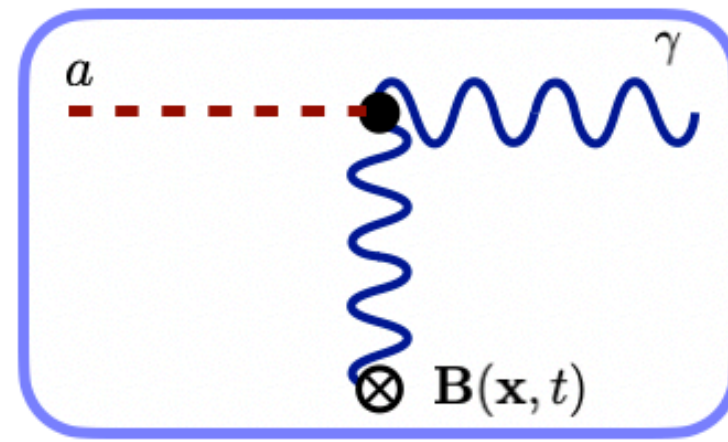
$$\Pi_T \sim \omega_{\text{pl}}^2 \sim \frac{e^2 n_e}{m_e}$$

Longitudinal

$$\Pi_L \sim \omega_{\text{pl}}^2 \left(1 - \frac{k^2}{\omega^2}\right)$$

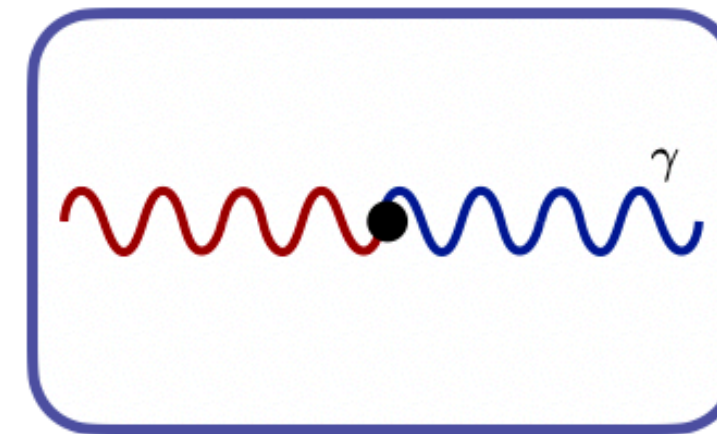


Axion:



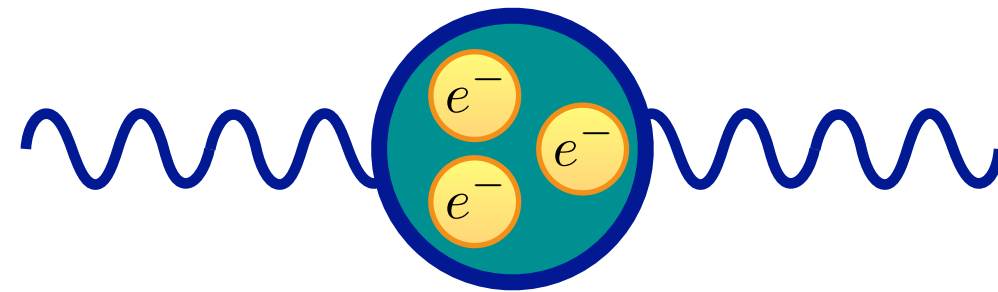
$$j_a \sim g_{a\gamma\gamma} (\partial a) F$$

Dark Photon:



$$j_{A'} \sim -\epsilon m_{A'}^2 A'$$

Free charges modify photon self-energy



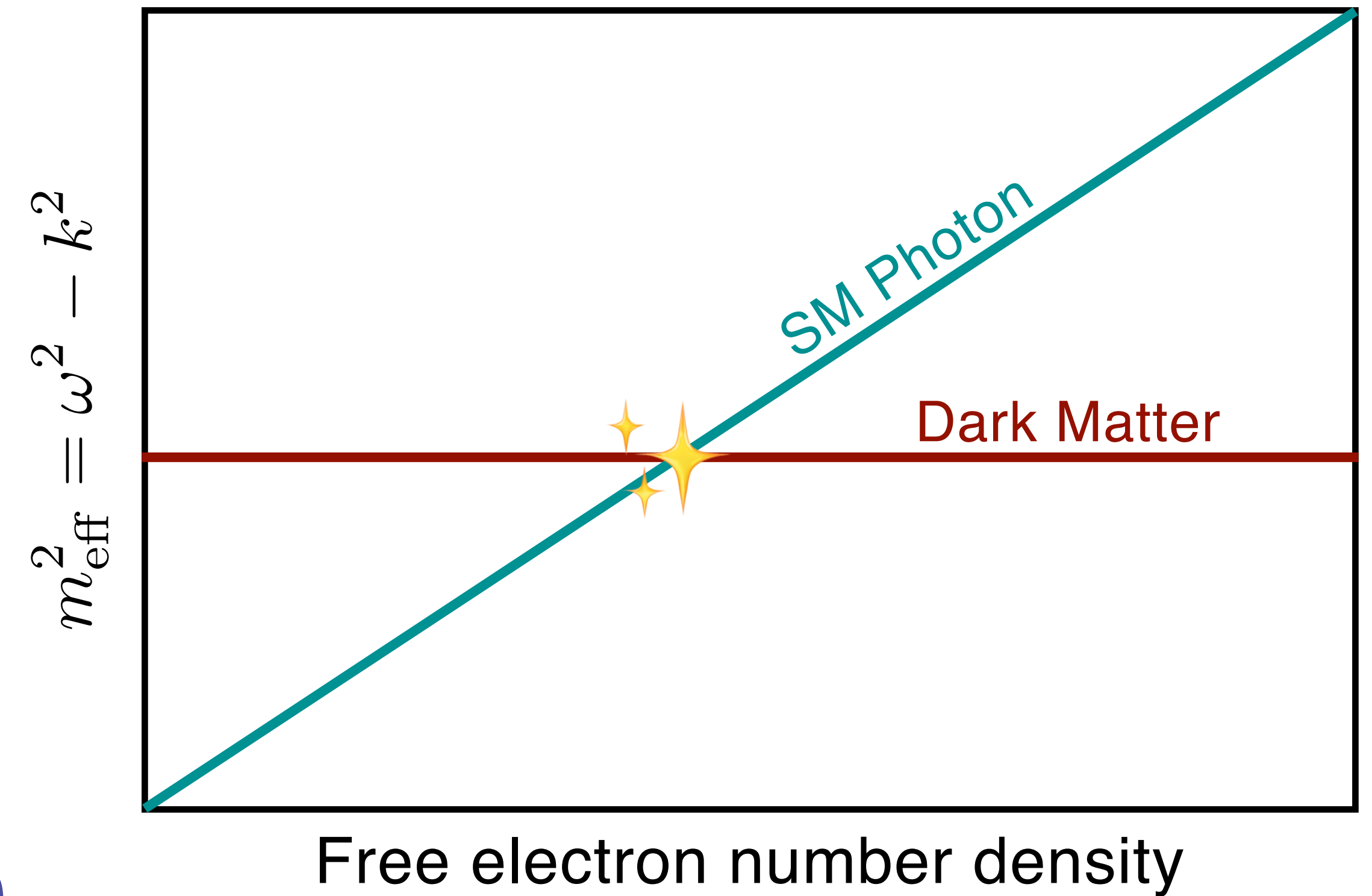
$$\Pi^{\mu\nu} = \sum_p \Pi_p P^{\mu\nu}$$

Transverse

$$\Pi_T \sim \omega_{\text{pl}}^2 \sim \frac{e^2 n_e}{m_e}$$

Longitudinal

$$\Pi_L \sim \omega_{\text{pl}}^2 \left(1 - \frac{k^2}{\omega^2} \right)$$



Resonance condition from two-level system

Resonant level crossing

Landau-Zener:
$$P_{\alpha \rightarrow \gamma} \simeq \frac{f_{\text{pol}} \pi}{v_r} g_{\text{eff}}^2 m_\alpha \left| \frac{\partial \ln \omega_{\text{pl}}^2}{\partial r} \right|_{r_c}^{-1}$$

Axion:
$$g_{\text{eff}} \rightarrow \frac{g_{a\gamma} |\mathbf{B}_T|}{m_a}$$

Dark Photon:
$$g_{\text{eff}} \rightarrow \epsilon$$

Resonant level crossing

Landau-Zener:
$$P_{\alpha \rightarrow \gamma} \simeq \frac{f_{\text{pol}} \pi}{v_r} g_{\text{eff}}^2 m_\alpha \left| \frac{\partial \ln \omega_{\text{pl}}^2}{\partial r} \right|_{r_c}^{-1}$$

Axion:
$$g_{\text{eff}} \rightarrow \frac{g_{a\gamma} |\mathbf{B}_T|}{m_a} \left(\omega^2 + \nabla^2 - \begin{pmatrix} \Pi_{\parallel} & -g_{a\gamma\gamma} B_T \omega \\ -g_{a\gamma\gamma} B_T \omega & m_a^2 \end{pmatrix} \right) \begin{pmatrix} A_{\parallel} \\ a \end{pmatrix} = 0$$

Dark Photon:
$$g_{\text{eff}} \rightarrow \epsilon \left(\omega^2 + \nabla^2 - \begin{pmatrix} \Pi_T & \epsilon m_{A'}^2 \\ \epsilon m_{A'}^2 & m_{A'}^2 \end{pmatrix} \right) \begin{pmatrix} \mathbf{A}_T \\ \mathbf{A}'_T \end{pmatrix} = 0$$

Couple system of equations, usually use WKB and solve linear PDE à la Raffelt-Stodolsky (1988)

Two-level system

WKB: $\omega^2 + \nabla^2 \simeq (\omega + k)(\omega - i\nabla)$

valid for $|\nabla^2 A| \ll k|\nabla A|, |\nabla^2 \alpha| \ll k|\nabla \alpha|$

$$\left(-i\nabla + \frac{1}{2k} \begin{pmatrix} \omega_{\text{pl}}^2 - m_\alpha^2 & \Pi_{A\alpha} \\ \Pi_{A\alpha} & 0 \end{pmatrix} \right) \begin{pmatrix} A \\ \alpha \end{pmatrix} = 0$$

Probability of conversion:

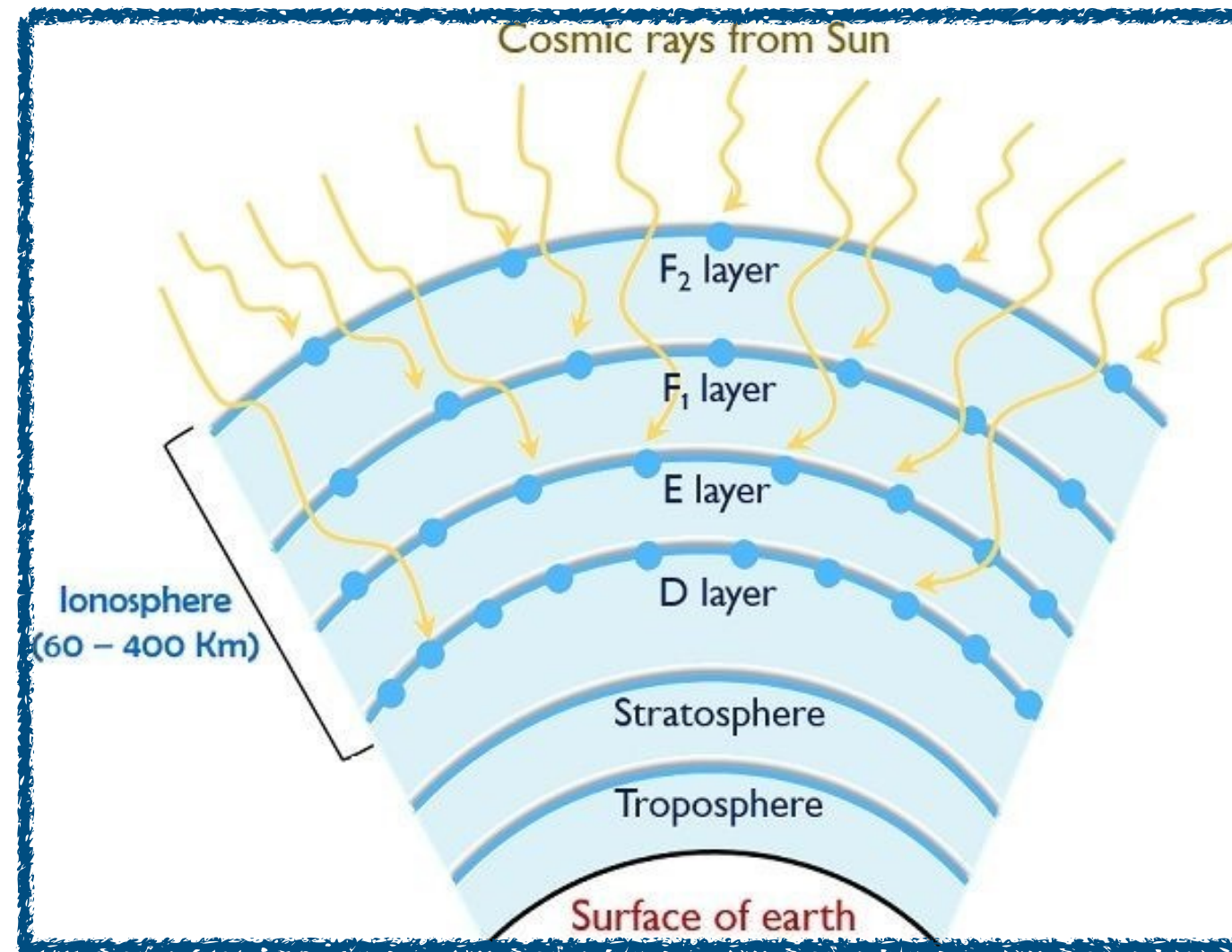
$$P_{\alpha \rightarrow A} \simeq \left| \int_0^\infty dz \frac{\Pi_{A\alpha}}{2k} e^{-i \int_0^z dz' (m_\alpha^2 - \omega_{\text{pl}}^2)/2k} \right|$$

Yields the Landau-Zener formula

Good approximation for many astrophysical environments, *e.g.*:
- *axions near NSs* — Hook et al (2018), Foster et al (2020), Foster et al (2022)
- *DPs near NSs* — Hardy & Song (2022)
- *DPs in the Solar corona* — An et al (2020, 2023)
+++

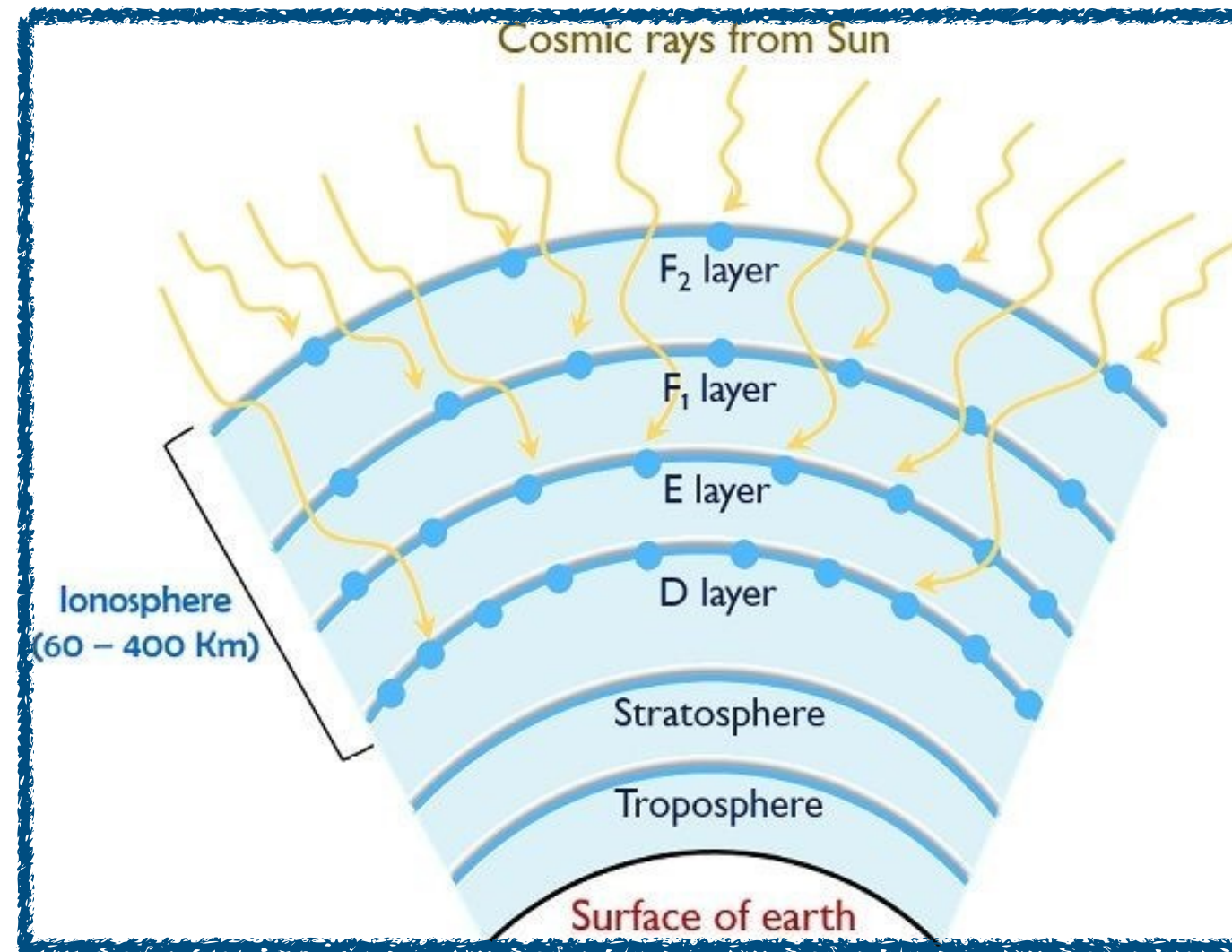
The Ionosphere

Created by ionising UV & X-ray radiation.



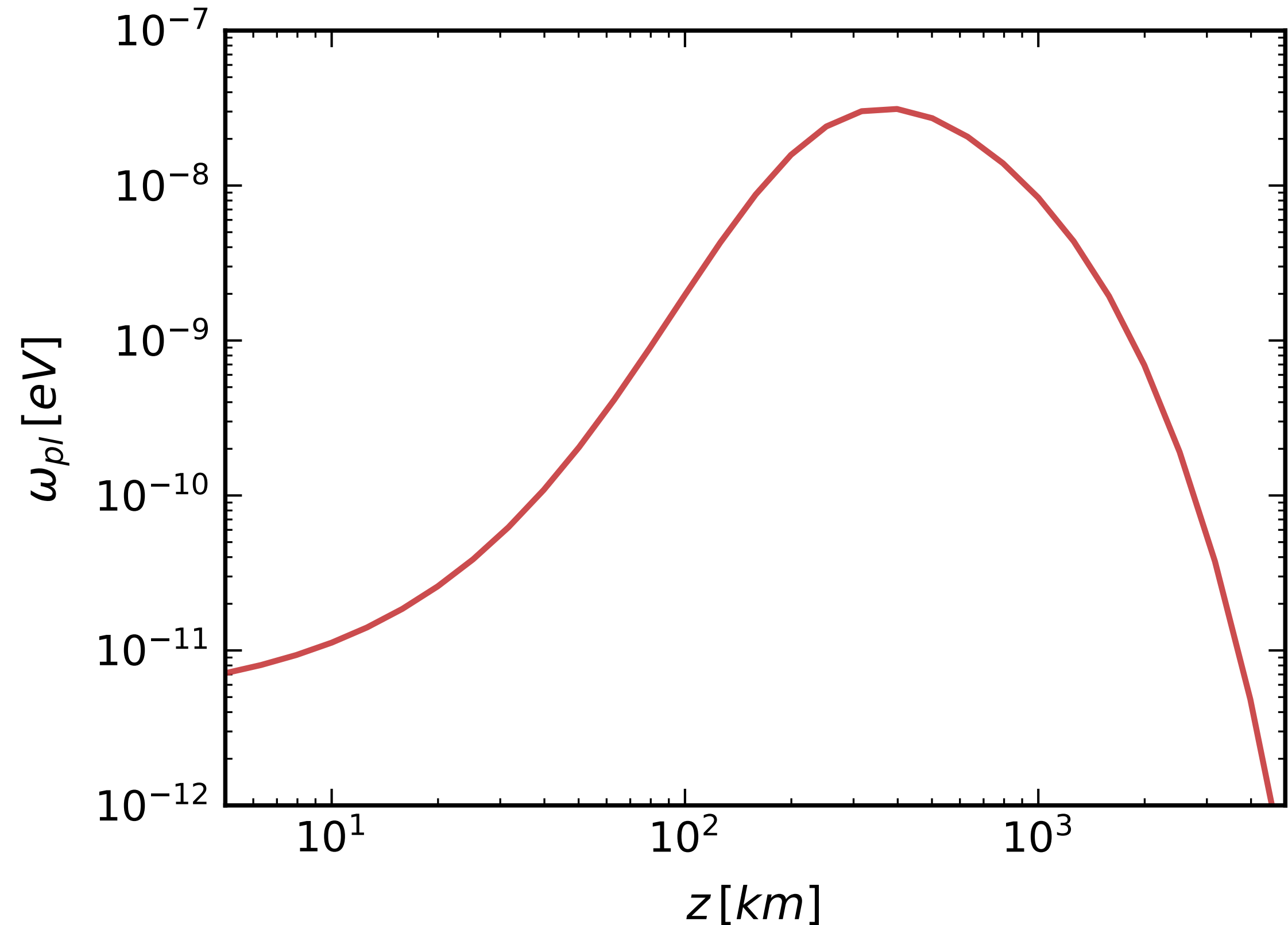
The Ionosphere

Created by ionising UV & X-ray radiation.



Chapman model:

- Scale height H
- Max. free electron n_e
- Max. height r_{\max}



Scale height sets variational length scale:

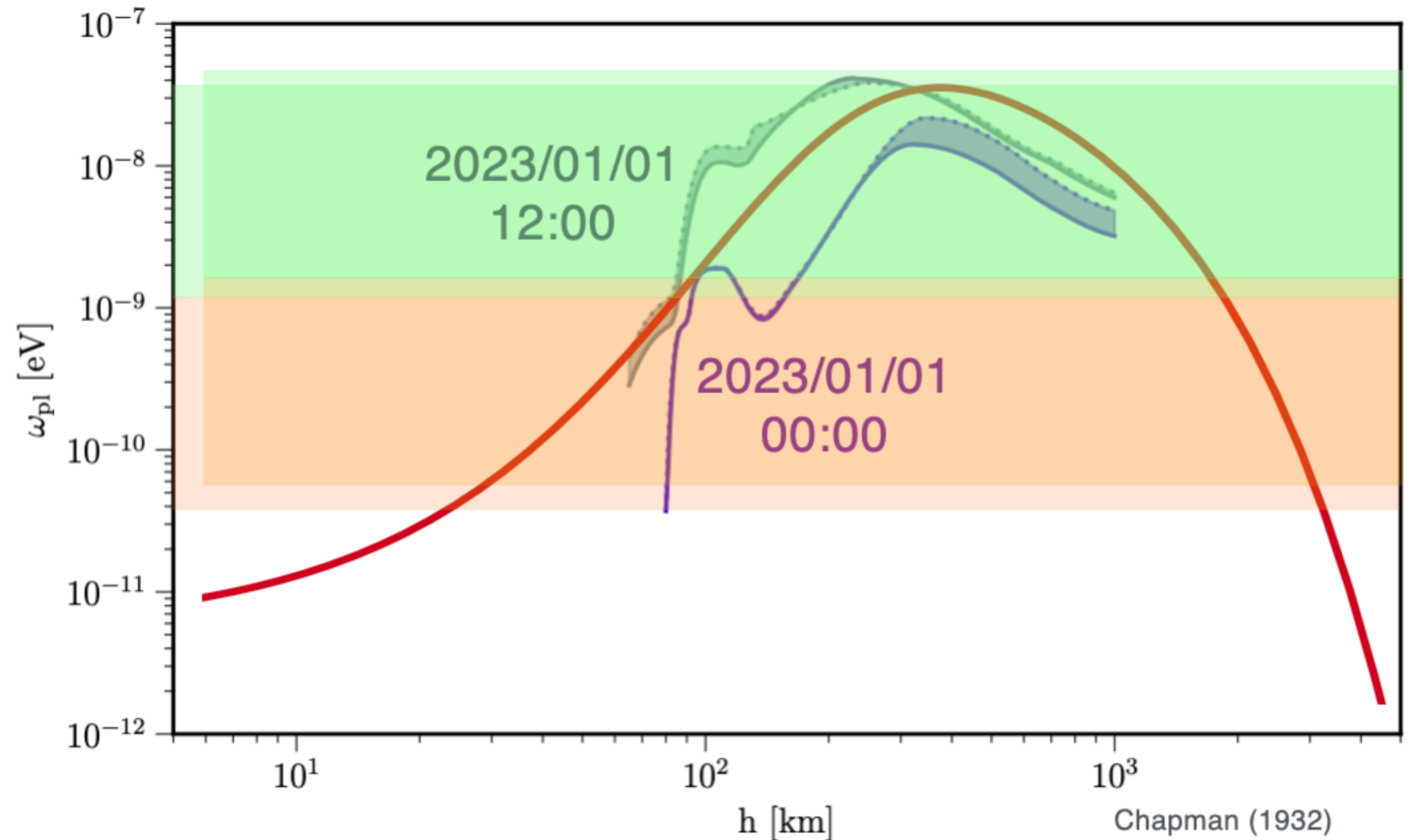
$$\left| \frac{\partial \ln \omega_{pl}^2}{\partial r} \right|_{r_c}^{-1} \gtrsim 2H$$

The *real* ionosphere

For decade in mass, Chapman is a reasonable middle ground

Low masses, poor approximation

Real ionosphere *steeper*

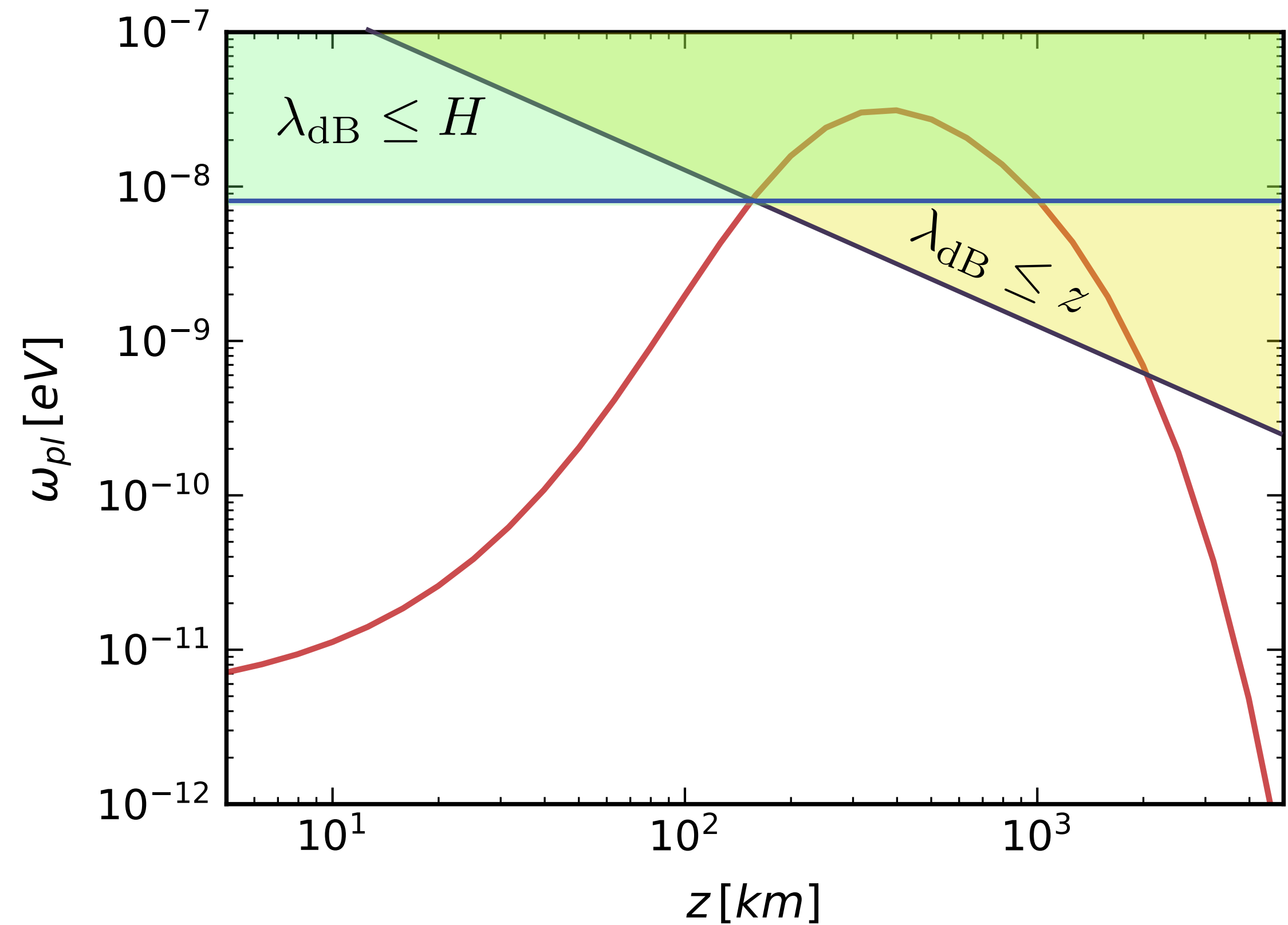


Data from International Ref. Ionosphere (IRI) <https://kauai.ccmc.gsfc.nasa.gov/instantrun/iri>

Chapman scale height $H \sim 100$ km

Over most parameter space, WKB
not valid

Presence of local maximum also problematic



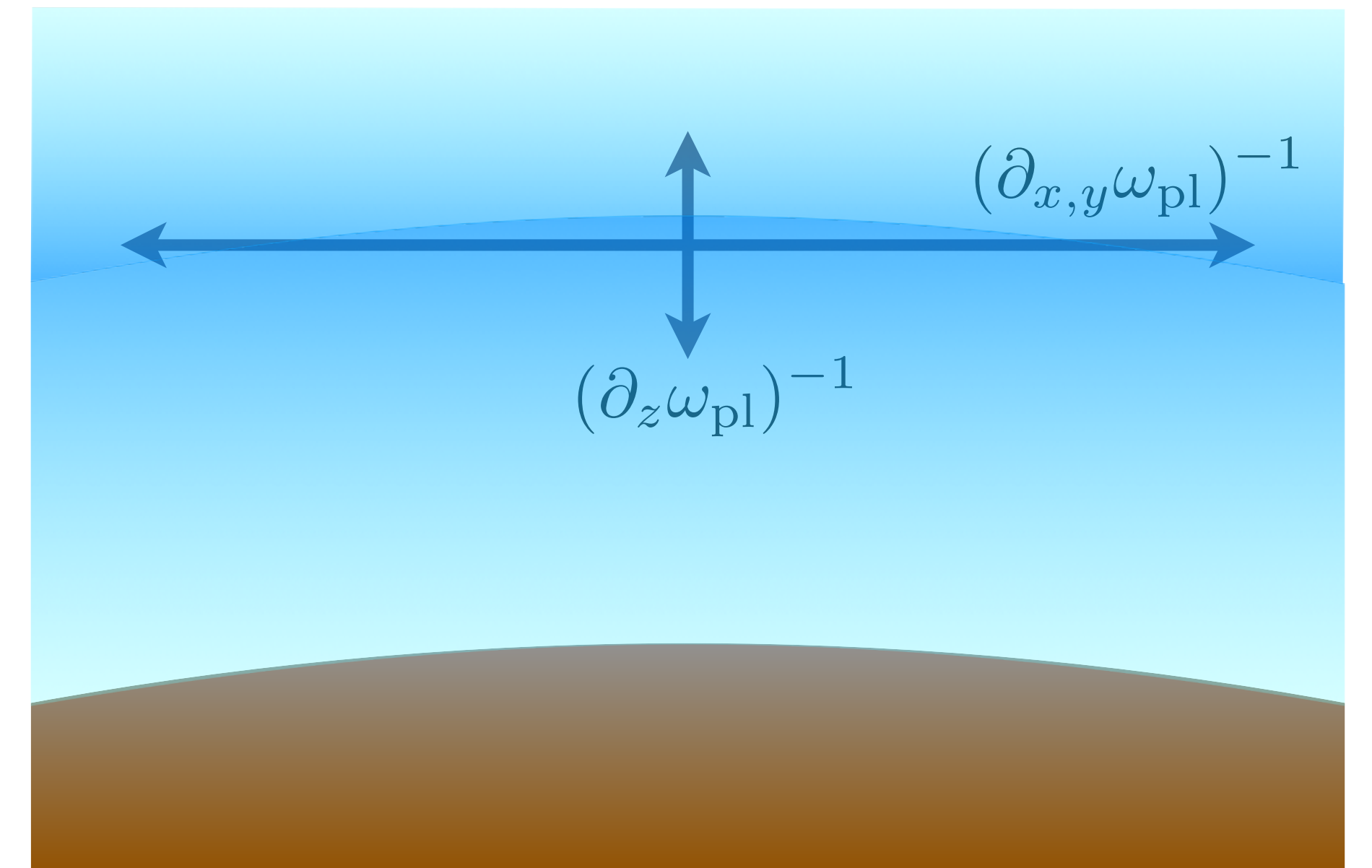
local extrema: Brahma et al (2023)

Back to initial equations

Need to solve full 2nd order equation $\left(\omega^2 + \nabla^2 - \begin{pmatrix} \Pi_{AA} & \Pi_{A\alpha} \\ \Pi_{A\alpha} & m_\alpha^2 \end{pmatrix} \right) \begin{pmatrix} A \\ \alpha \end{pmatrix} = 0$

Transverse variation length-scale long — model as 1D

$$\left[\partial_z^2 + \omega^2 - \frac{\omega^2}{\omega^2 + i\nu(z)\omega} \omega_{\text{pl}}^2(z) \right] \mathbf{E}_T(z) = \Pi_{A\alpha} \partial_t \alpha(z)$$



Ionosphere as a driven cavity

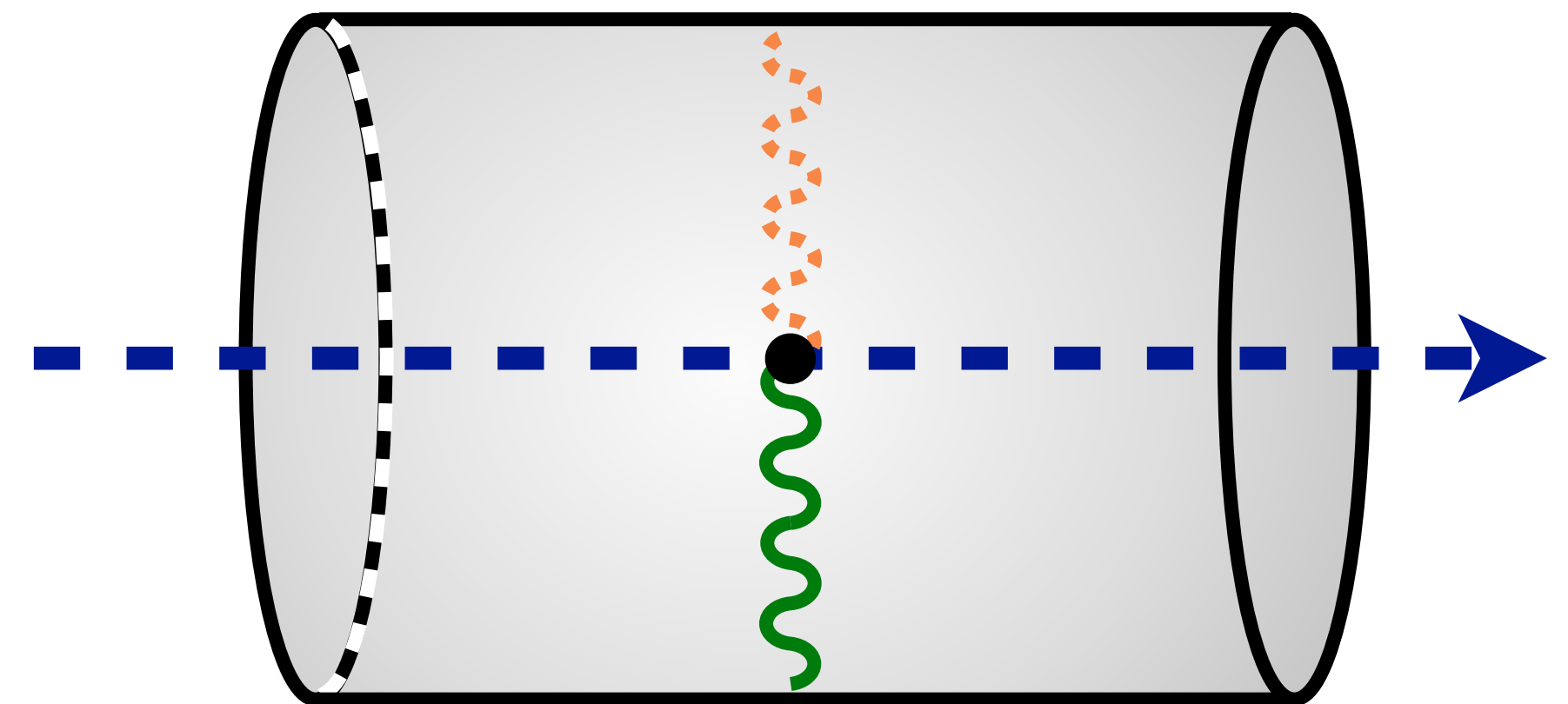
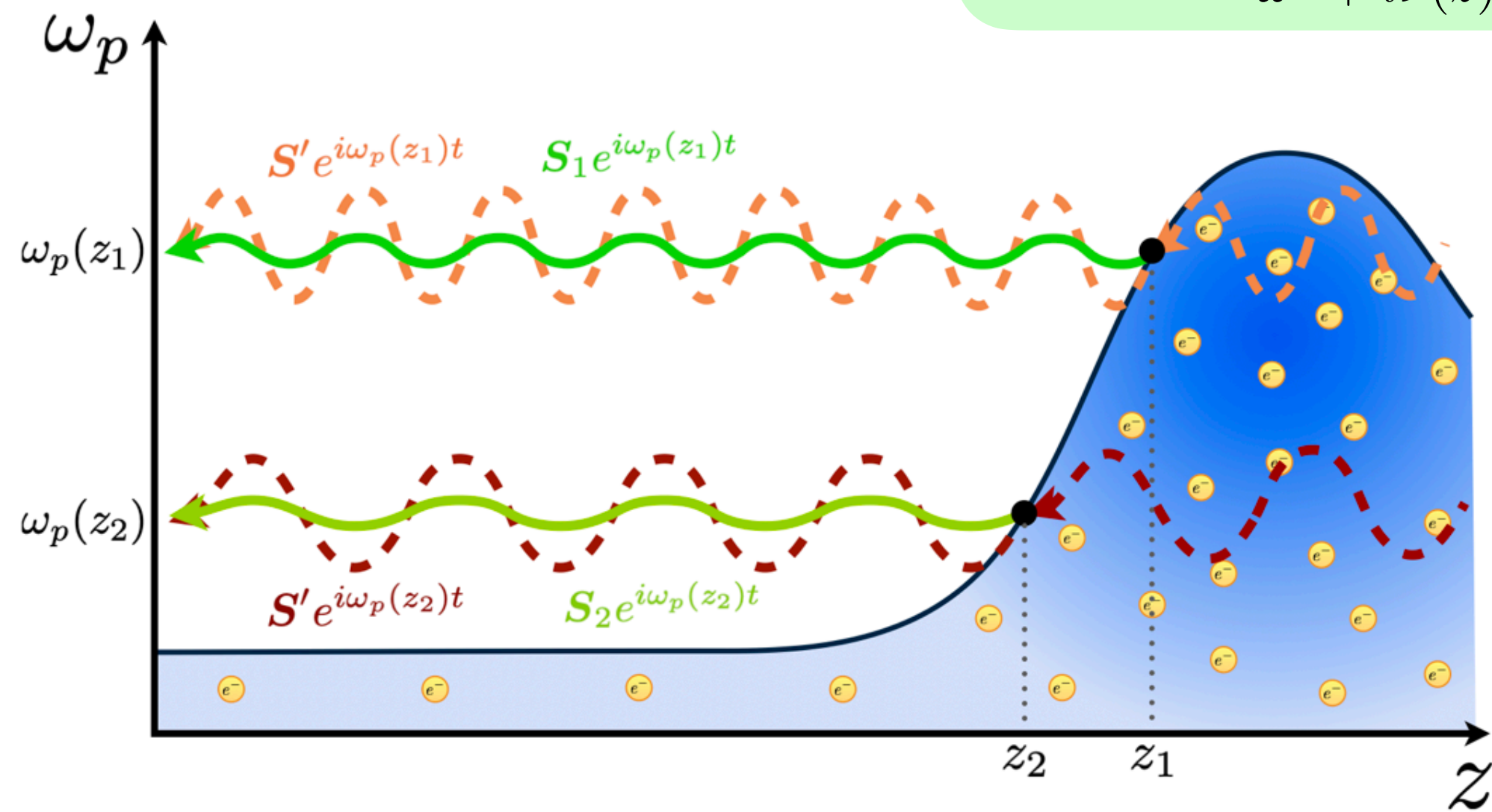
Ionosphere cavity:

Actual cavity:

$$\left[\nabla^2 + \omega^2 \left(1 - \frac{1}{\omega^2 + i\nu\omega} \omega_p^2 \right) \right] \mathbf{E}_T = i g_{\text{eff}} m_{\text{DM}}^2 \omega \mathbf{V}$$

$$(\nabla^2 - \mu \varepsilon \partial_t^2) \mathbf{E} = \mu \partial_t \mathbf{j}_\alpha$$

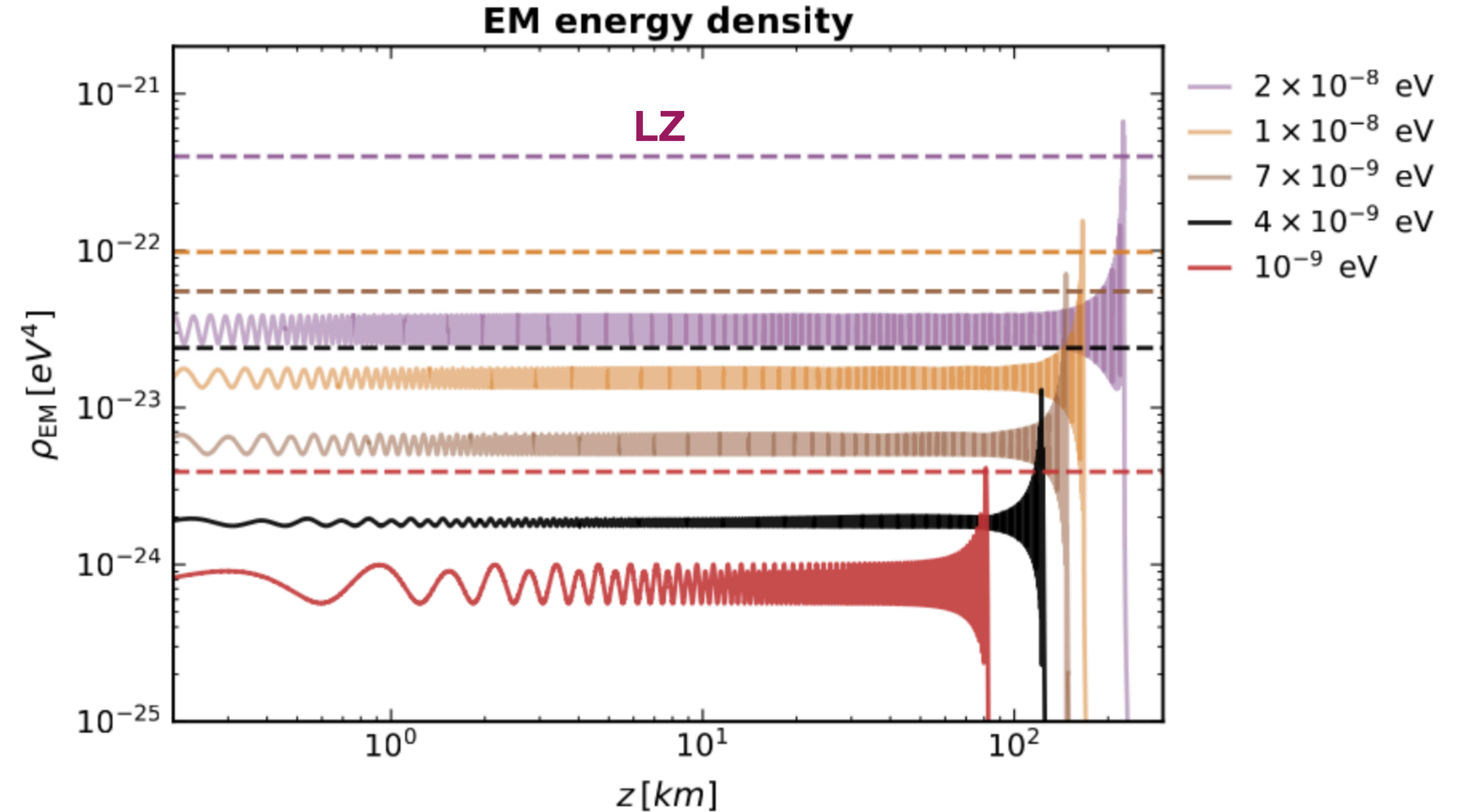
$$\varepsilon(z) \sim 1 - \frac{\omega_{\text{pl}}^2(z)}{\omega^2 + i\nu(z)\omega}$$



Our signal

Numerical solution w/ Thomas
Algorithm for tridiagonal matrix

```
def TDMASolveNew(a, b, c, d):  
    cprime = np.zeros(len(c), dtype = 'complex_')  
    dprime = np.zeros(len(d), dtype = 'complex_')  
  
    for i in range(len(c)-1):  
        if i == 0:  
            cprime[i] = c[i]/b[i]  
            dprime[i] = d[i]/b[i]  
        else:  
            cprime[i] = c[i]/(b[i]-a[i] * cprime[i-1])  
            dprime[i] = (d[i]-a[i]*dprime[i-1]) / (b[i] - a[i]*cprime[i-1])  
  
    Er = np.zeros(len(b), dtype = 'complex_')  
  
    for i in reversed(range(len(c)-1)):  
        if i == len(c)-1:  
            Er[i] = 0#Er[i] = dprime[i]  
        elif i == 0:  
            Er[i] = 0  
        else:  
            Er[i] = dprime[i] - cprime[i] * Er[i+1]  
  
    return Er
```



hep-ph/2405.13882 Carl Beadle, [AC](#) and Sebastian Ellis

$$\rho_{EM} \simeq \frac{3 \times 10^{-23} eV^4 \left(\frac{g_{eff}}{10^{-10}} \right)^2}{1 + \exp \left[- \left(\frac{m_\alpha}{2.3 \times 10^{-9} eV} - 3.8 \right) \right]}$$

Our noise

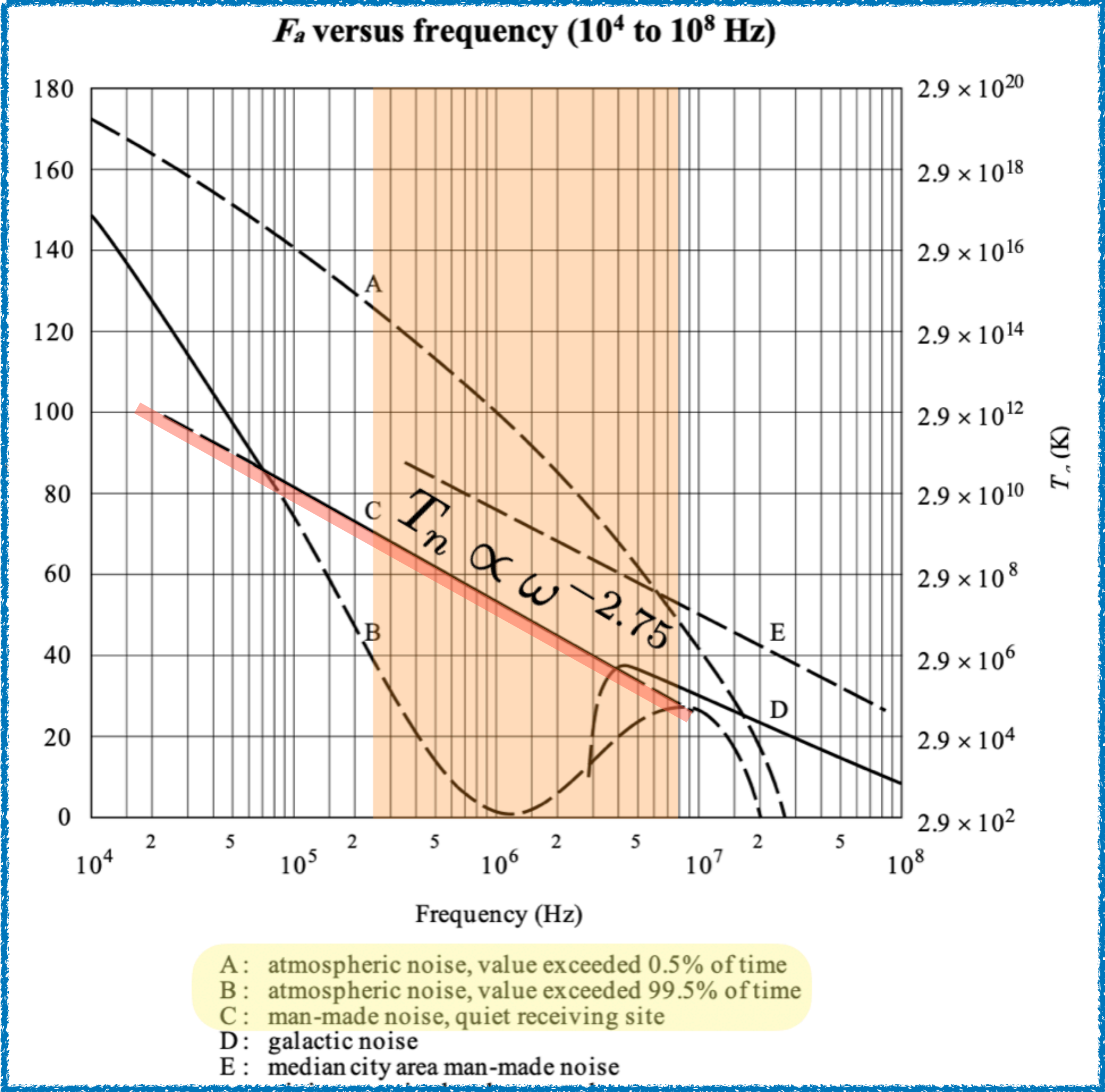


ITU provides us with estimates for our noise

$$10^5 \text{ K} \lesssim T_n \lesssim 10^9 \text{ K}$$

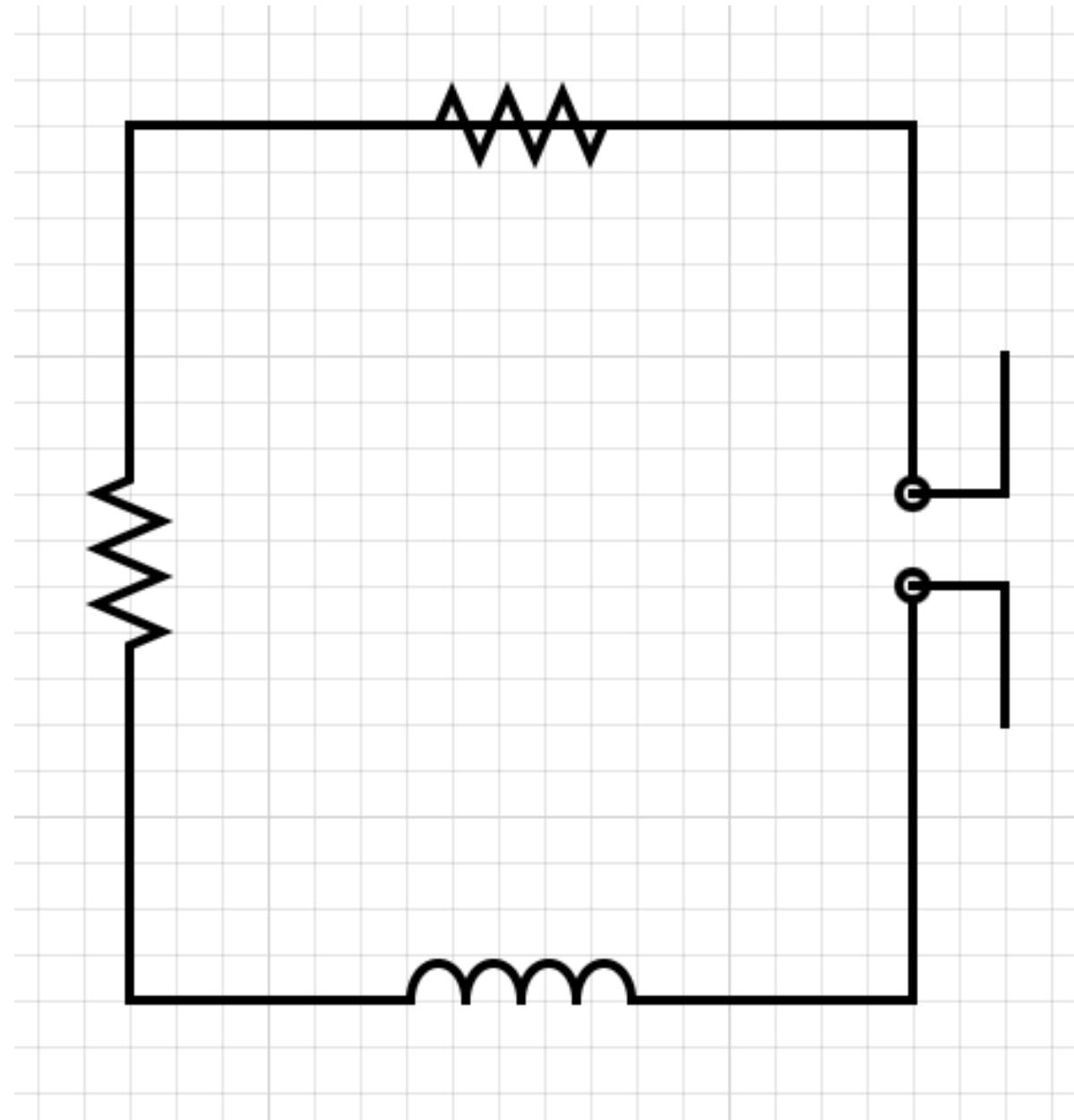
We can map from temperature to a noise PSD using:

$$S_n(\nu) \approx \frac{32}{3} \pi^2 \nu^2 T_n(\nu)$$



Antenna

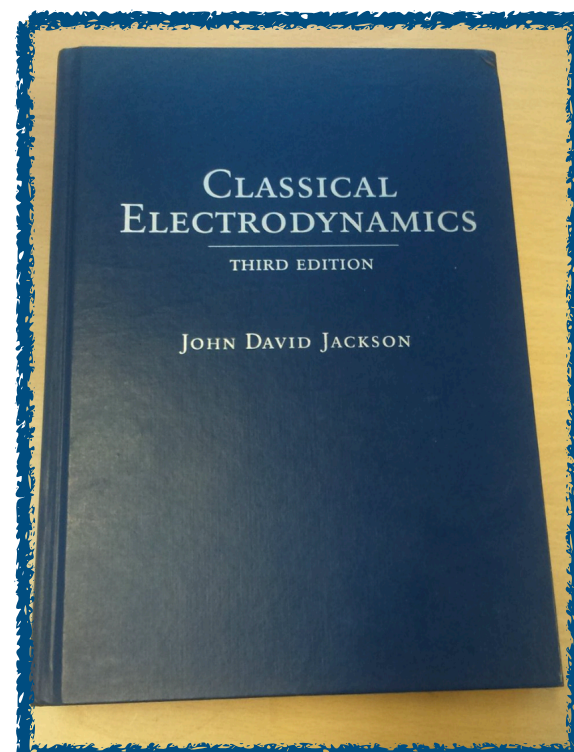
We model a prospective antenna and read-out as a simple RLC-circuit



$$P_L = \int d\omega \frac{\omega^2 h^2}{R_L L^2 \left[(\omega^2 - \omega_0^2)^2 + \omega^2 \Delta\nu^2 \right]} S_E(\omega)$$

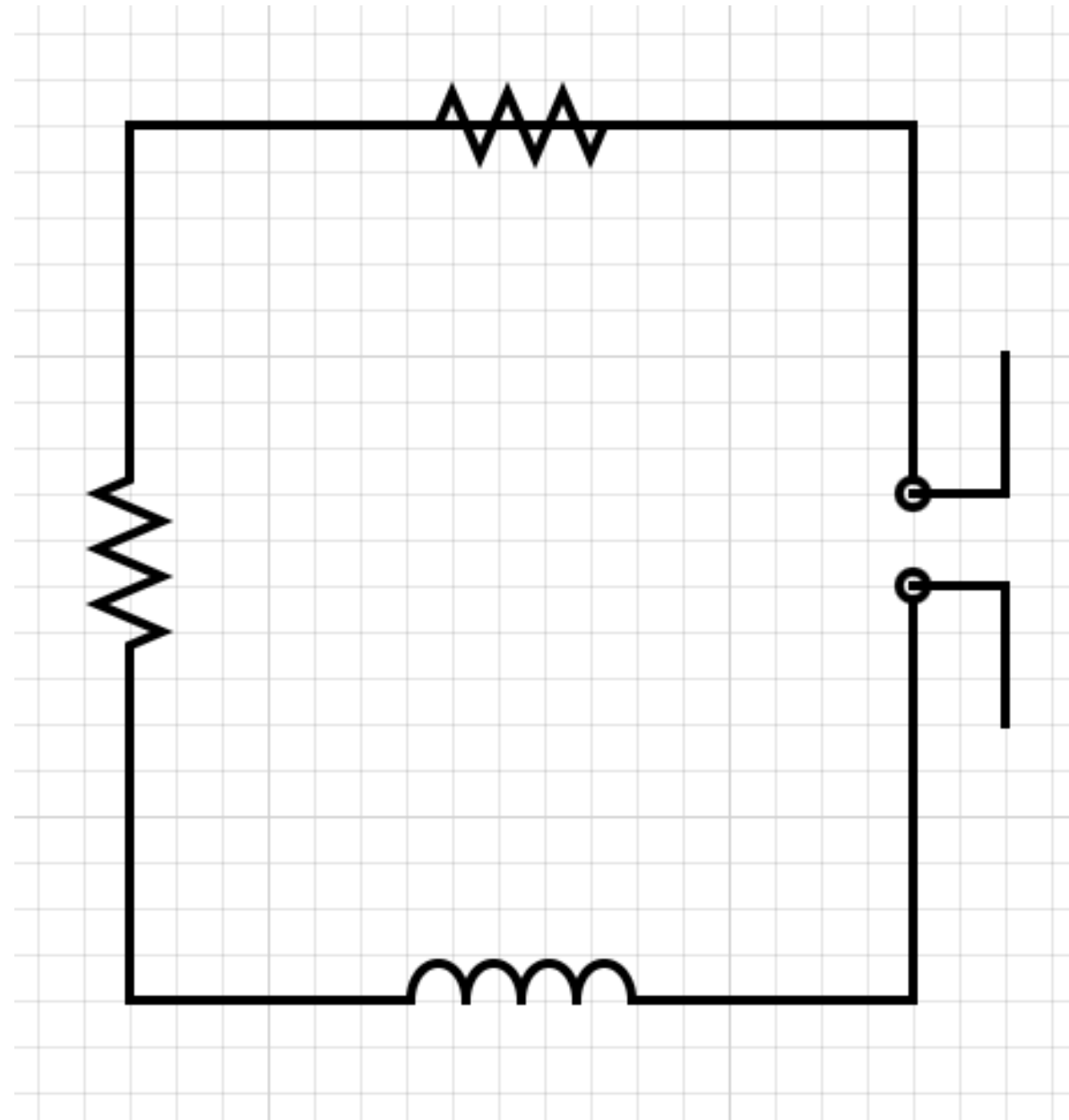
$$\omega_0^2 \equiv \frac{1}{C_A L}$$

$$\Delta\nu \equiv \frac{R_A + R_L}{L}$$



Antenna

We model a prospective antenna and read-out as a simple RLC-circuit

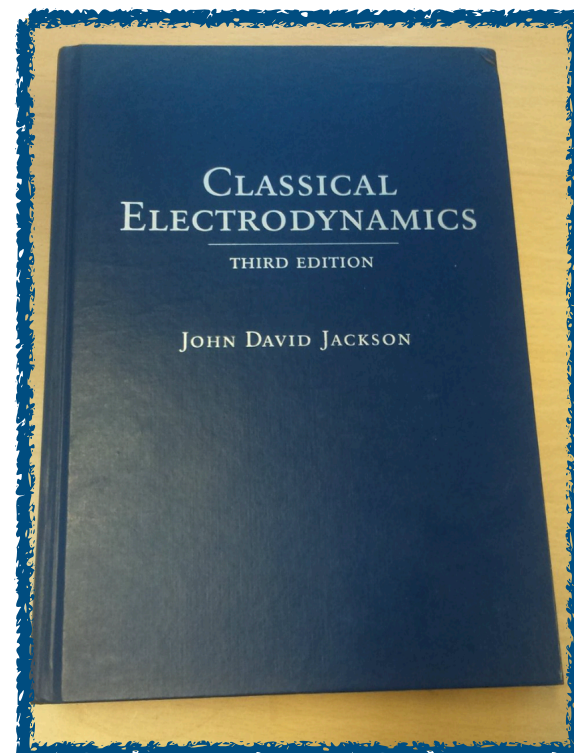


$$P_L = \int d\omega \frac{\omega^2 h^2}{R_L L^2 \left[(\omega^2 - \omega_0^2)^2 + \omega^2 \Delta\nu^2 \right]} S_E(\omega)$$

$$\omega_0^2 \equiv \frac{1}{C_A L}$$

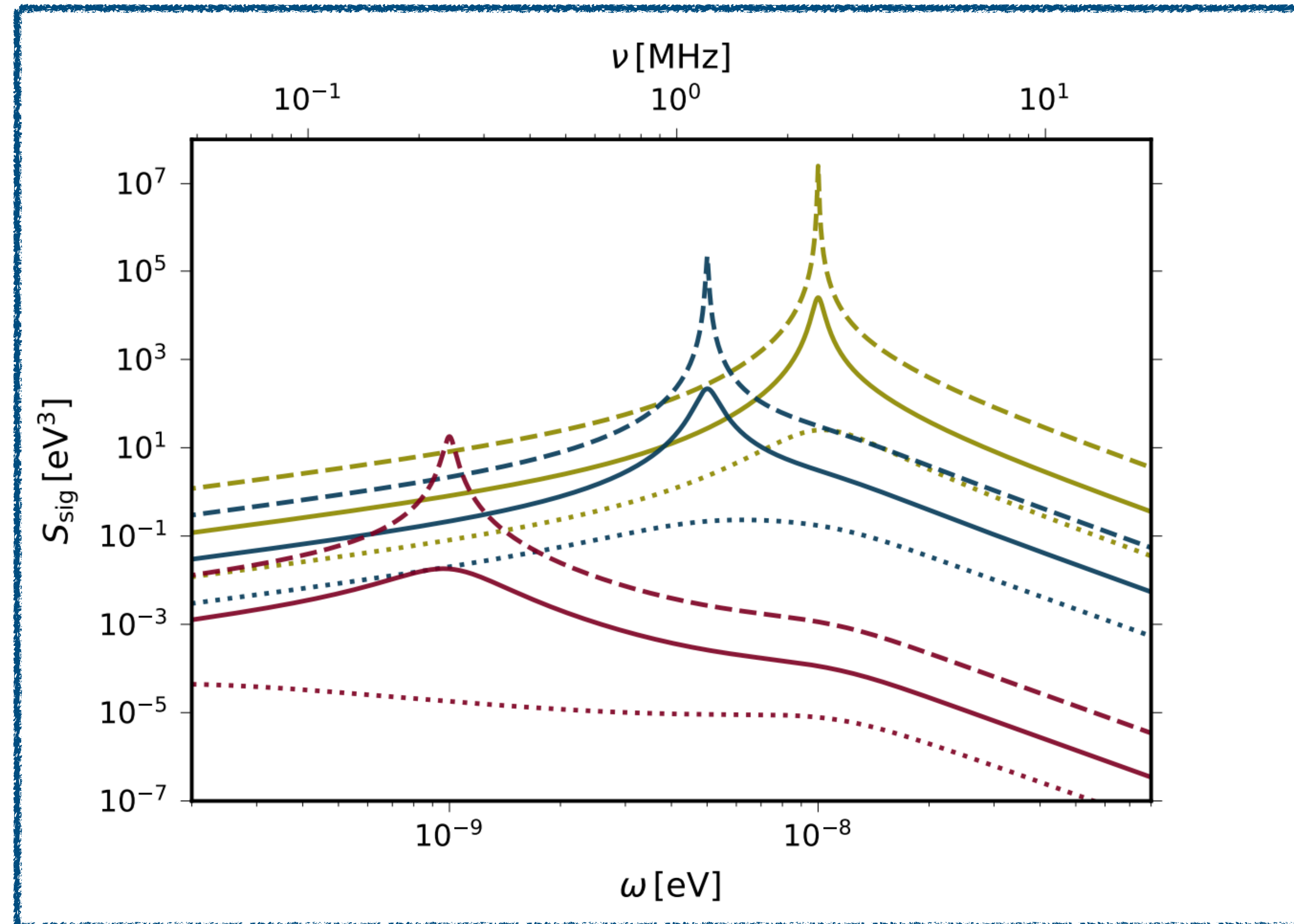
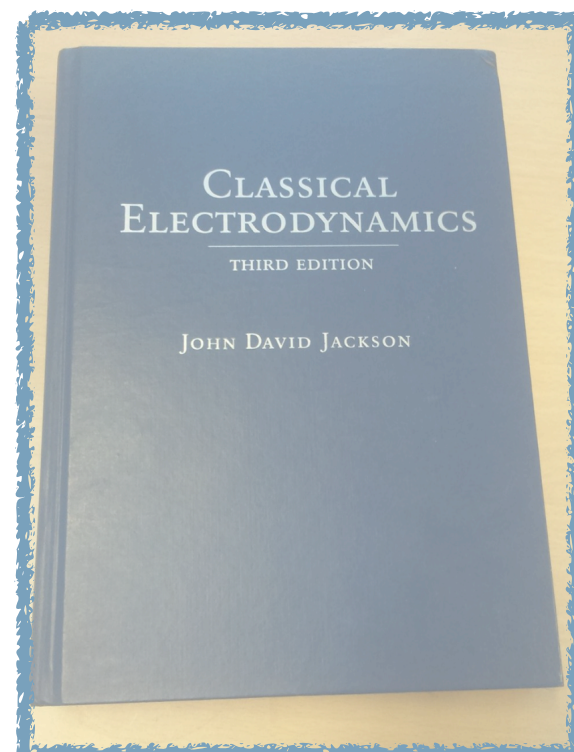
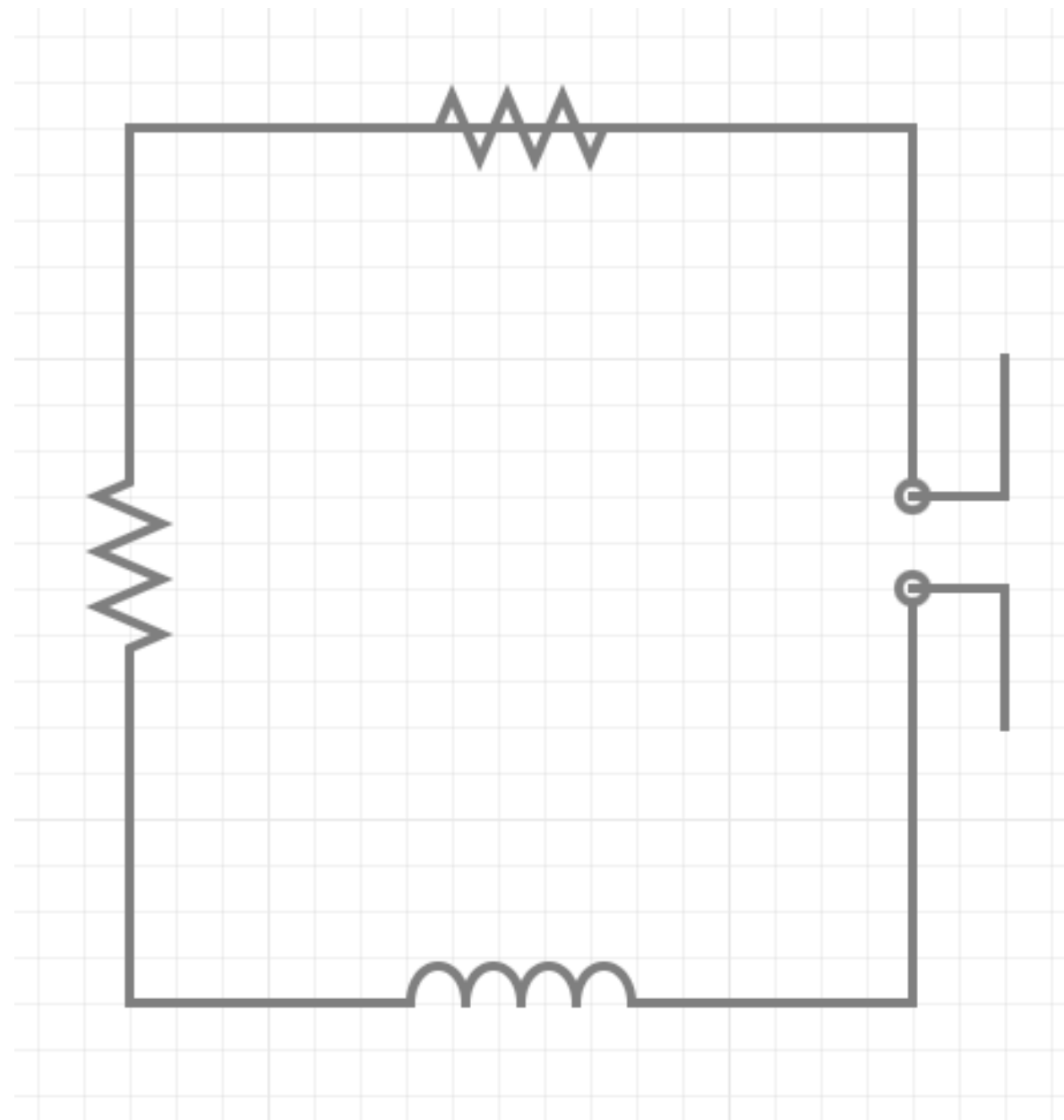
$$\Delta\nu \equiv \frac{R_A + R_L}{L}$$

$$\Delta\nu \sim 10 \text{ kHz} \times \left(\frac{h}{1 \text{ m}} \right) \left(\frac{m_\alpha}{10^{-8}} \right)^2 \left(\frac{R}{10 \text{ ohm}} \right)$$



Antenna

We model a prospective antenna and read-out as a simple RLC-circuit

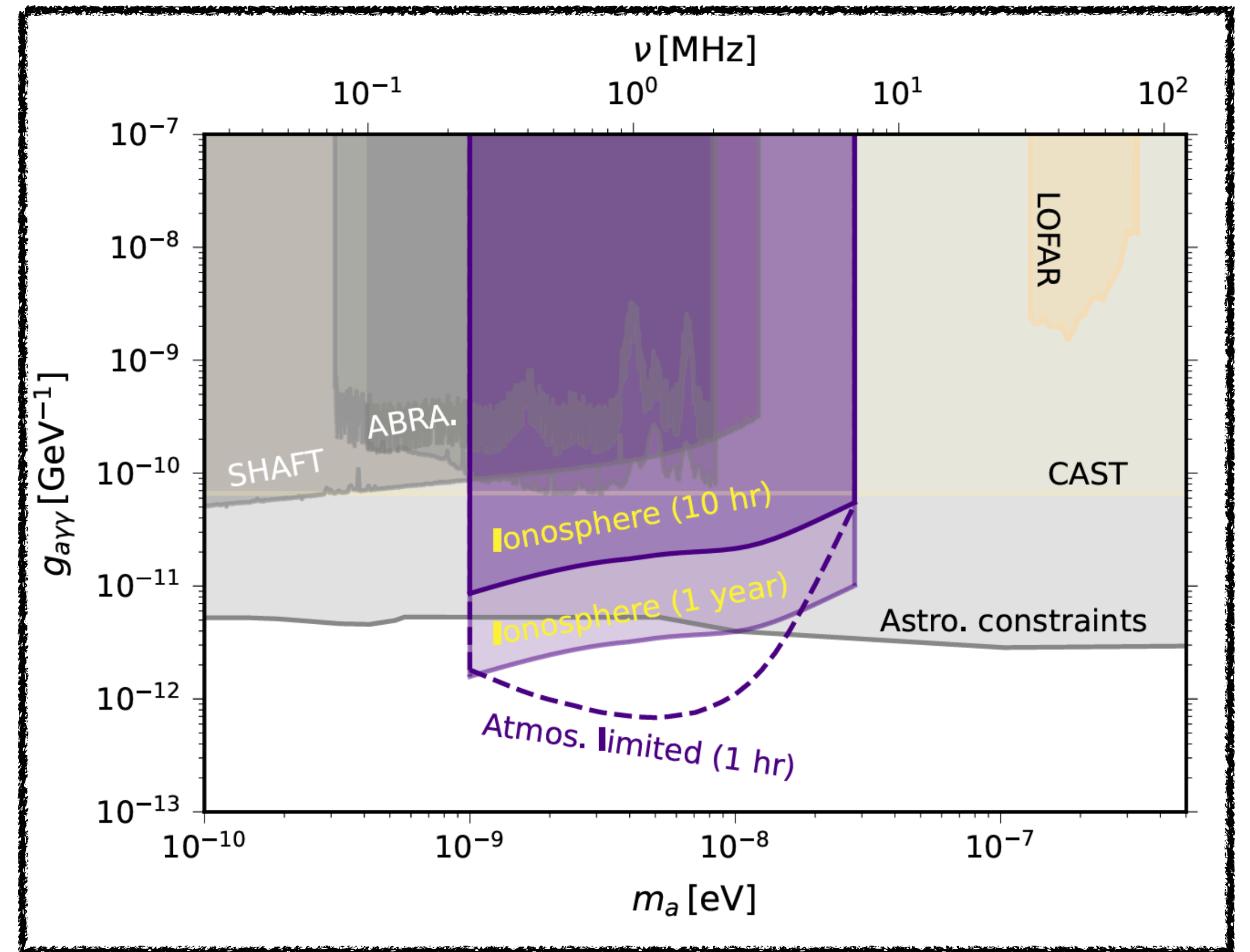
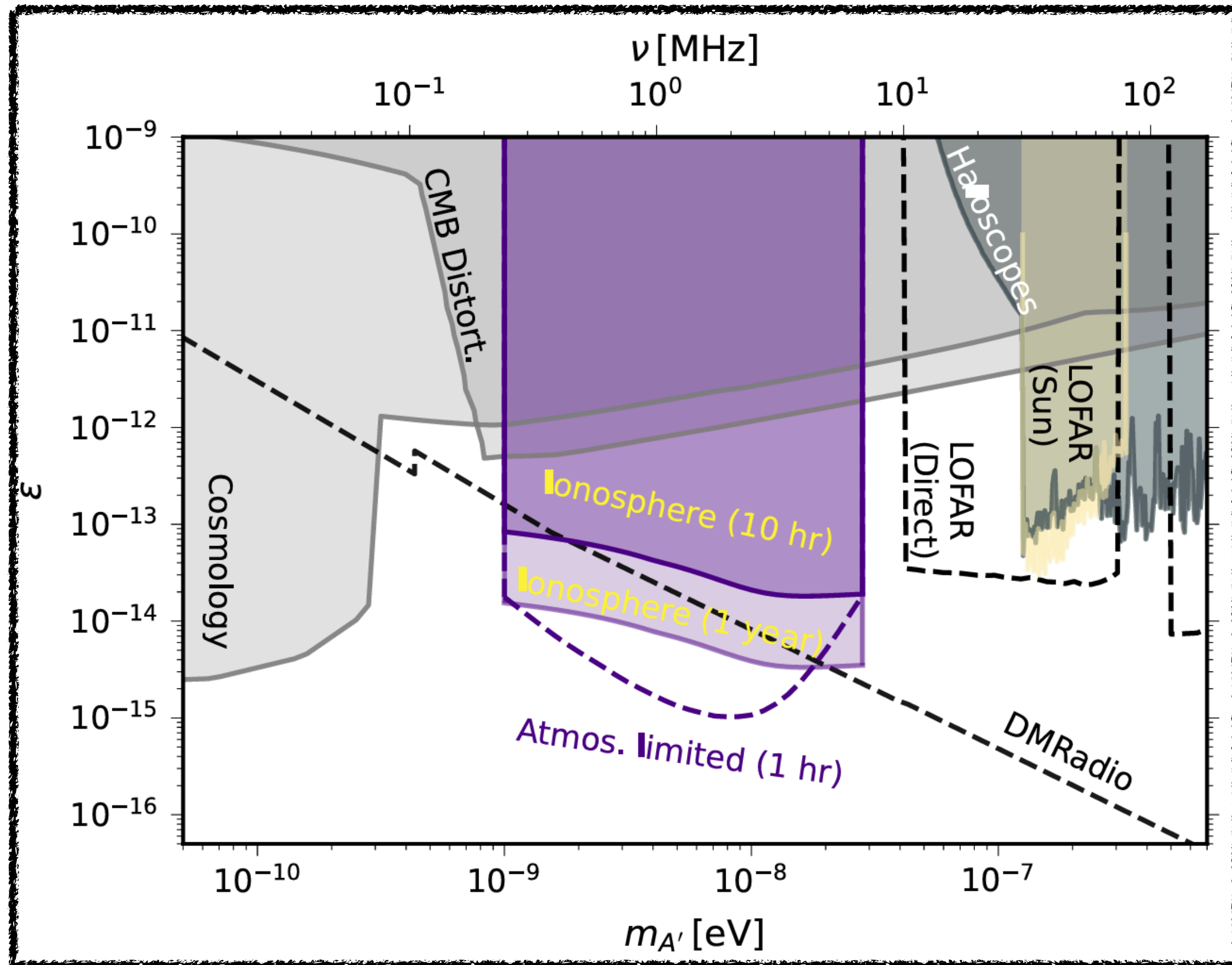


$$\left(\frac{R}{10 \text{ ohm}} \right)$$

Our results

$$\text{SNR}^2 \simeq t_{\text{int}} \int_0^\infty d\nu \left(\frac{\mathcal{S}_{\text{sig}}(\omega)}{\mathcal{S}_n(\omega)} \right)^2$$

$$\mathcal{S}_{\text{sig}}(\omega) \sim \rho_{\text{EM}} f(\omega)$$



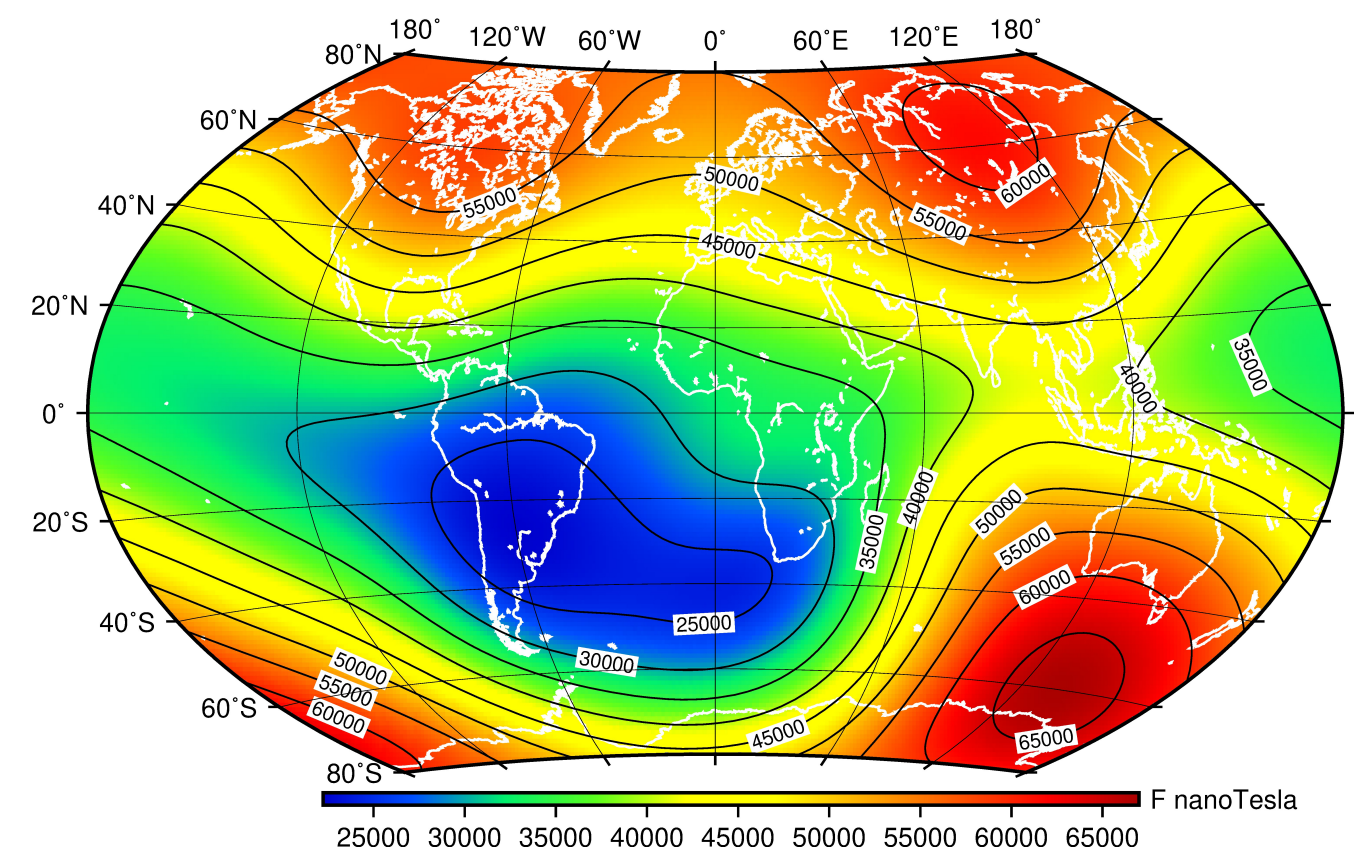
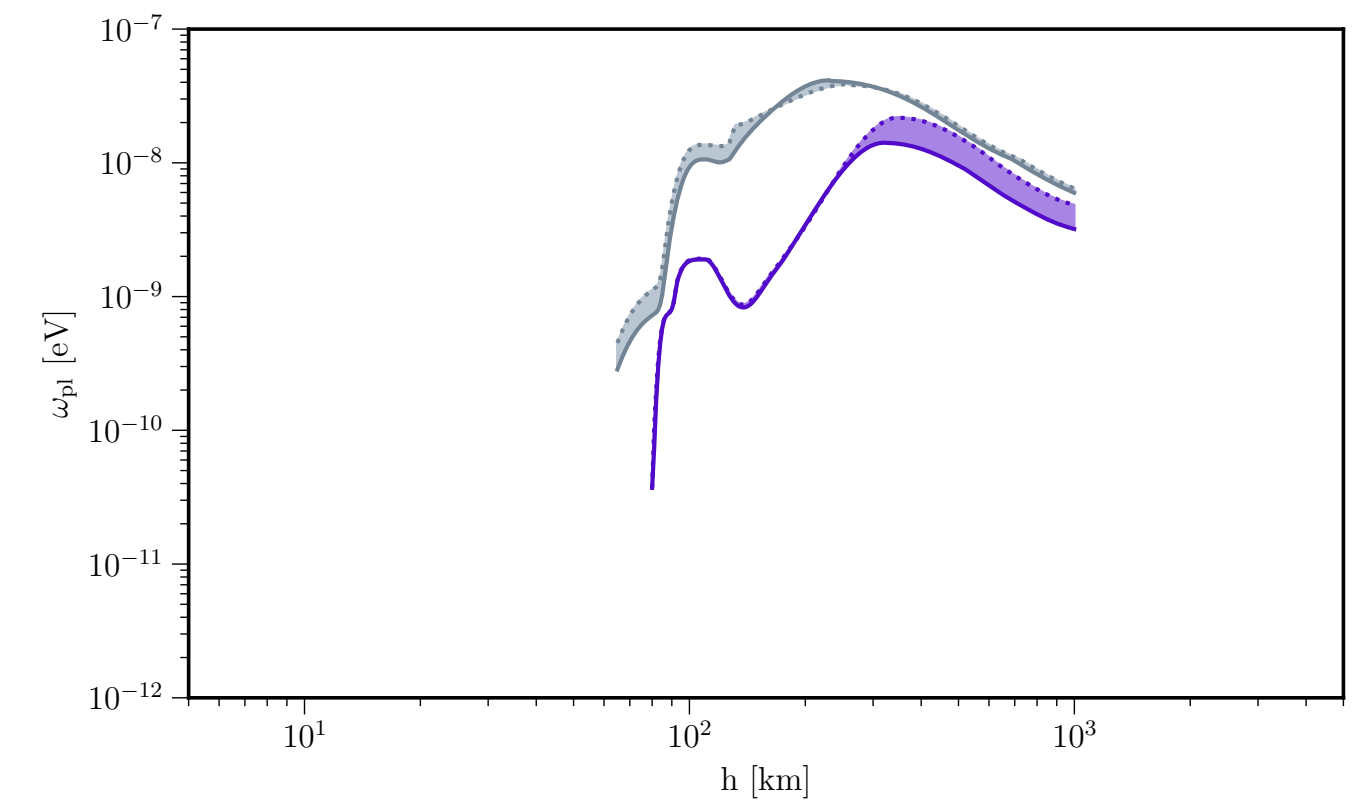
Next steps

Chapman can be replaced with real ionosphere data


Magnetic field modelling

Accounting for diurnal variation in both plasma and ***B***

Move on to operate a prototype or use some existing data [McGill and Stanford groups interested]



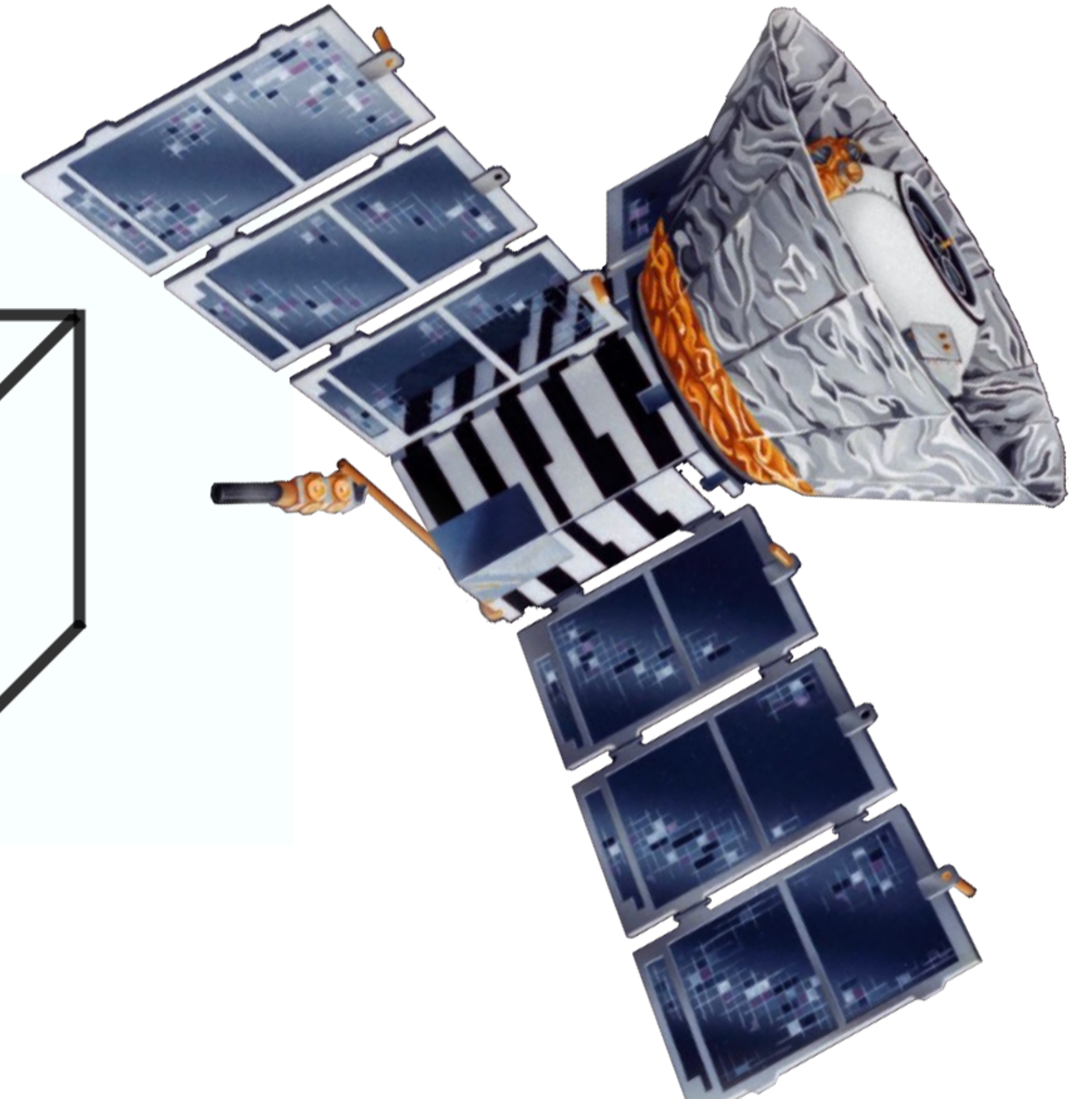
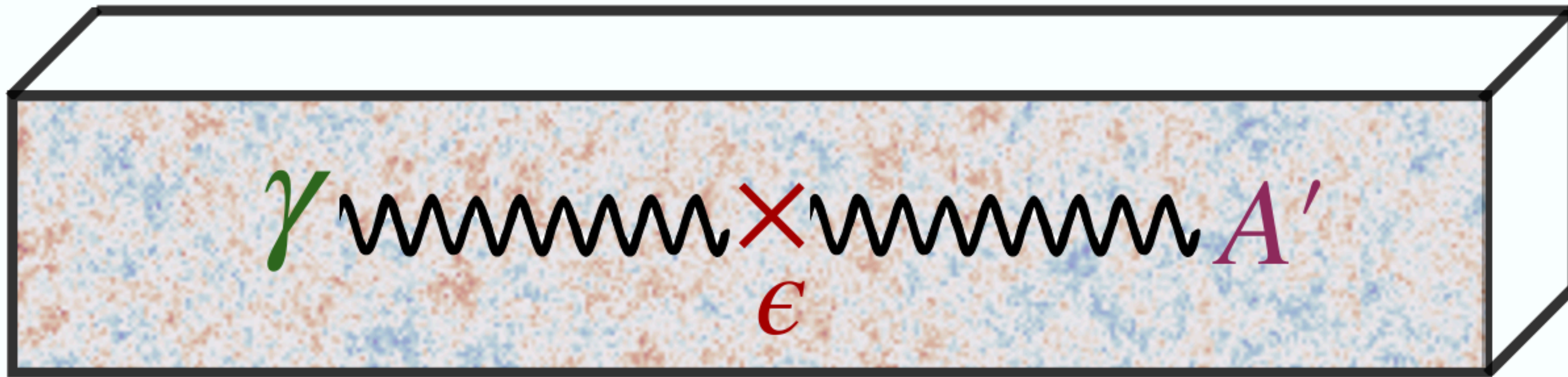
<https://geomag.bgs.ac.uk/education/earthmag.html>

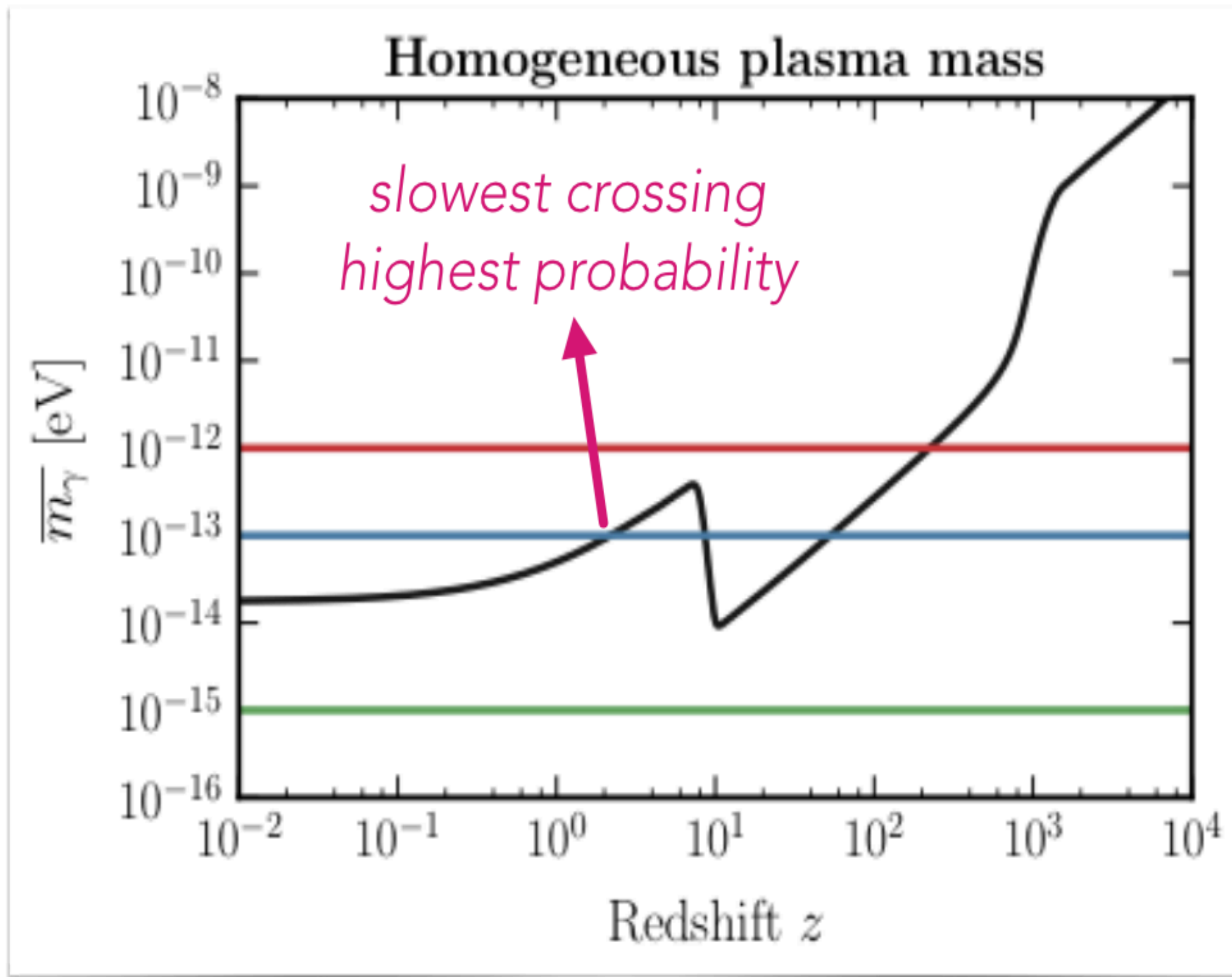


Gracias!

**What if Dark Photons [or
axions] are not dark matter?**

We can use the **inverse process**: instead of dark photons into photons, we can convert photons into dark photons [or axions]



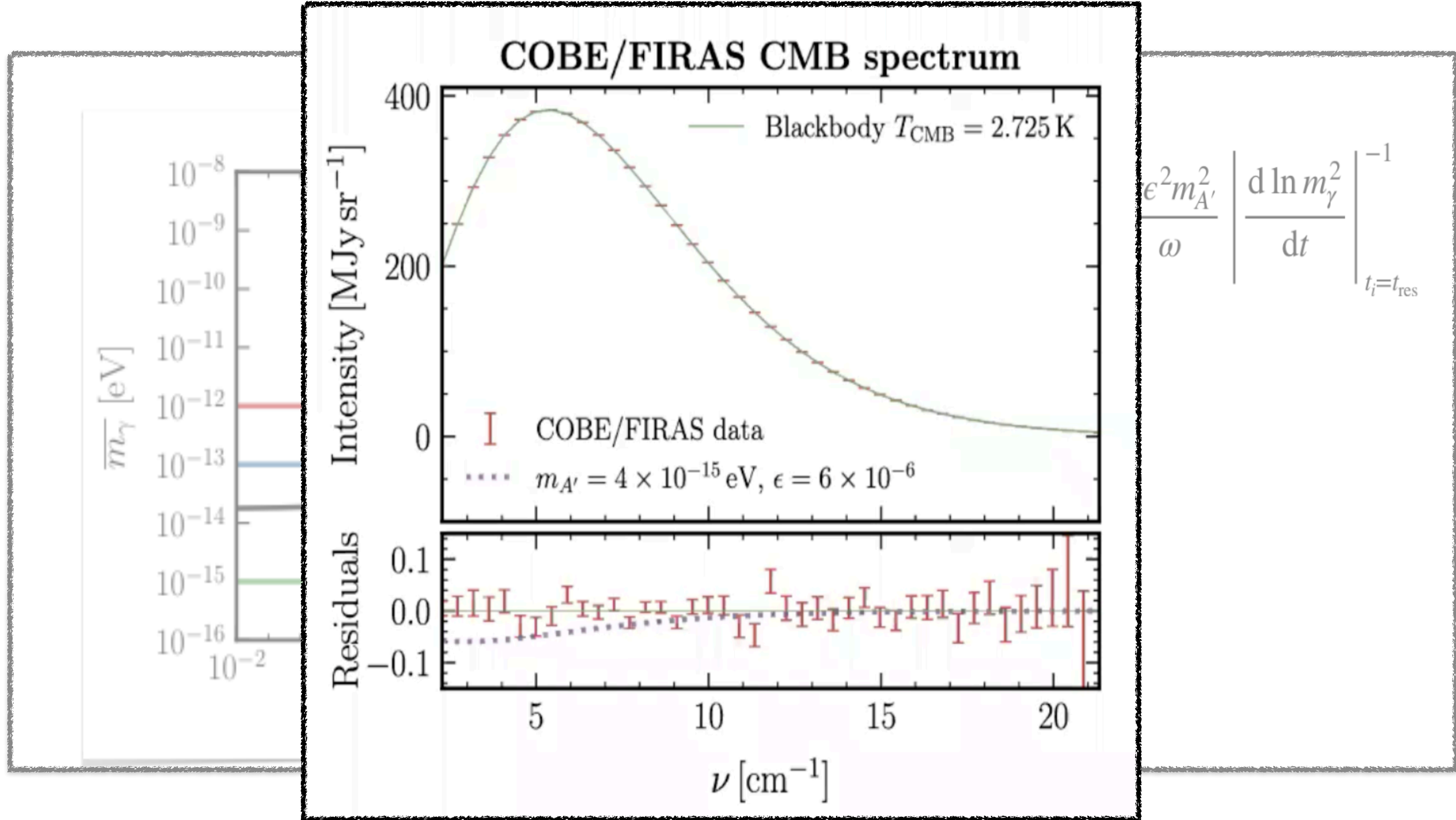


$$P_{\gamma \rightarrow A'} = \sum_i \frac{\pi \epsilon^2 m_{A'}^2}{\omega} \left| \frac{d \ln m_\gamma^2}{dt} \right|^{-1}_{t_i = t_{\text{res}}}$$

1 crossing

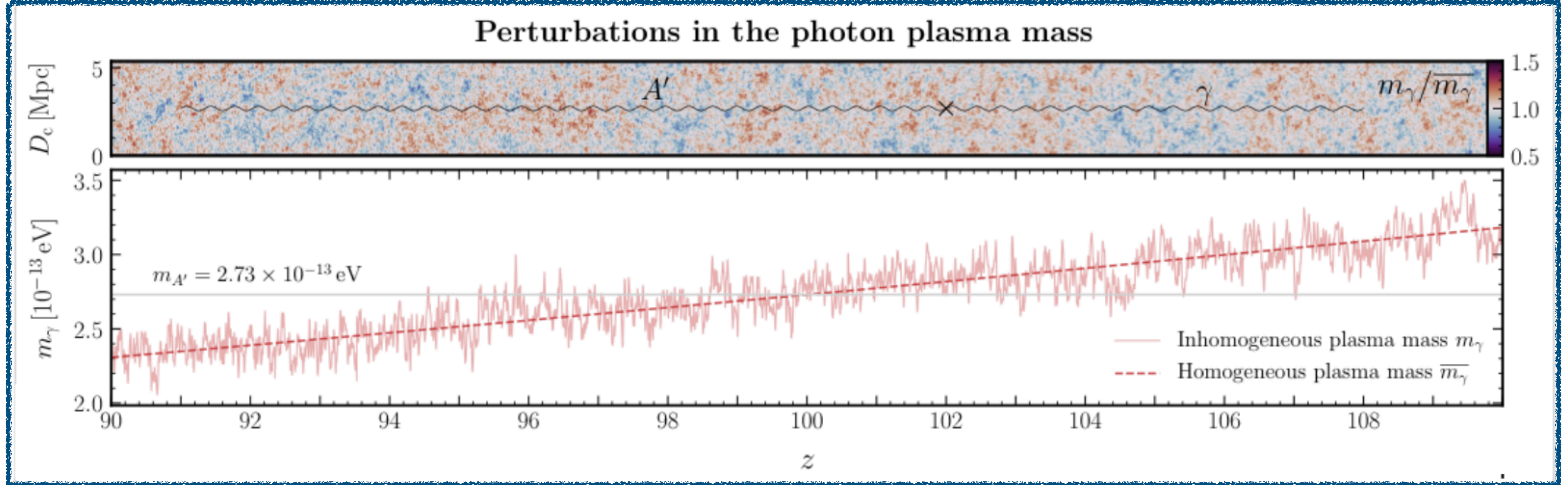
3 crossings

0 crossings



Use precise spectral measurements to look for new physics!

We developed a formalism to treat inhomogeneities in the plasma



AC, H. Liu, S. Mishra-Sharma & J. T. Ruderman

Phys.Rev.Lett. 125 (2020) 22, 221303

Phys.Rev.D 102 (2020) 10, 103533

$$\langle P_{\gamma \rightarrow A'} \rangle = \int dt f(m_\gamma^2 = m_{A'}^2; t) \frac{\pi \epsilon^2 m_{A'}^4}{\omega(t)}$$

Rice's Formula (1944)

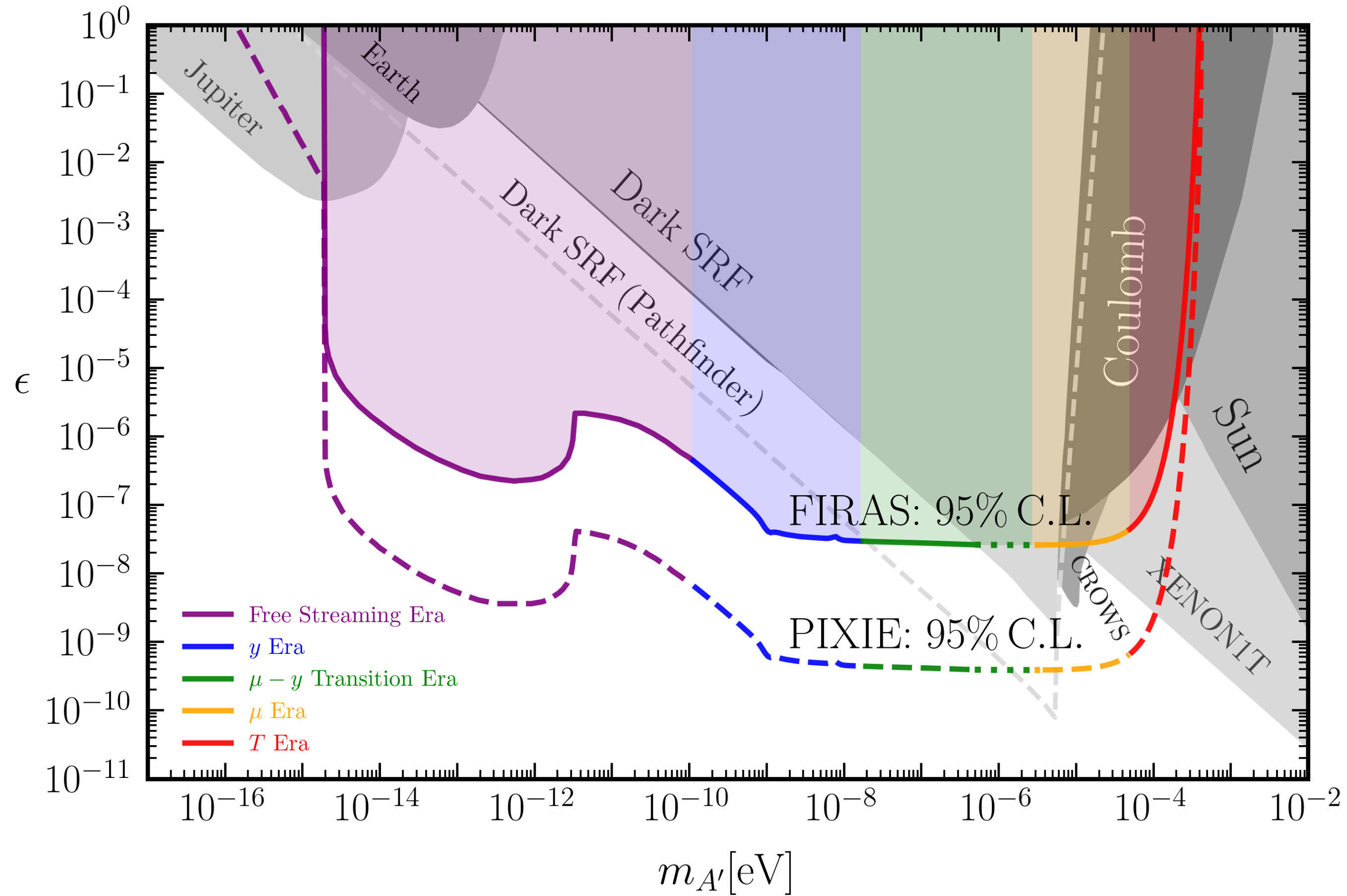
Mathematical Analysis of Random Noise

By S. O. RICE

INTRODUCTION

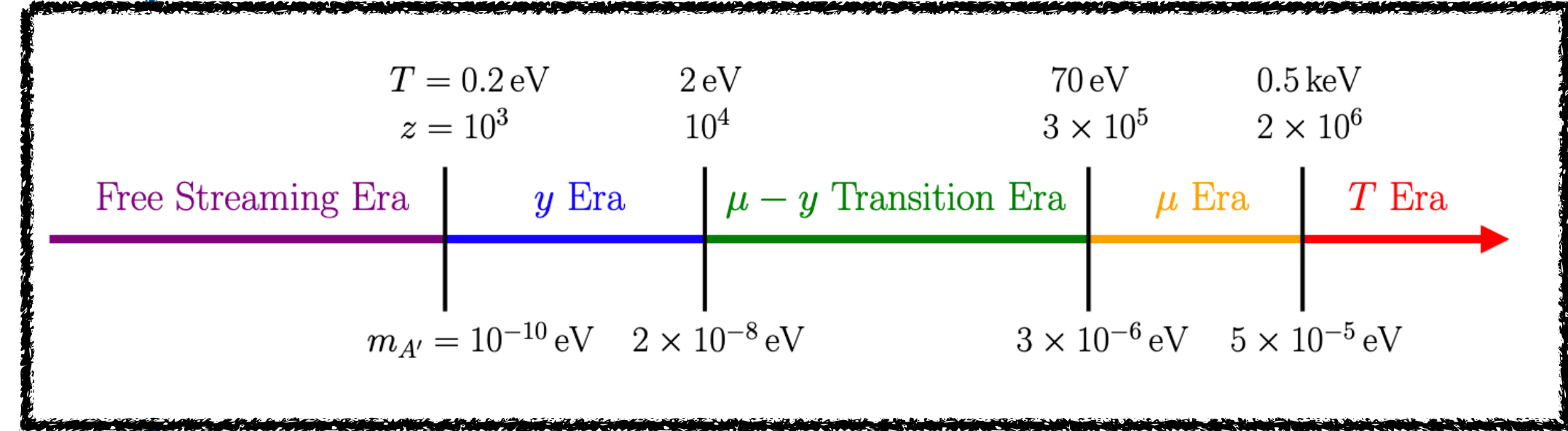
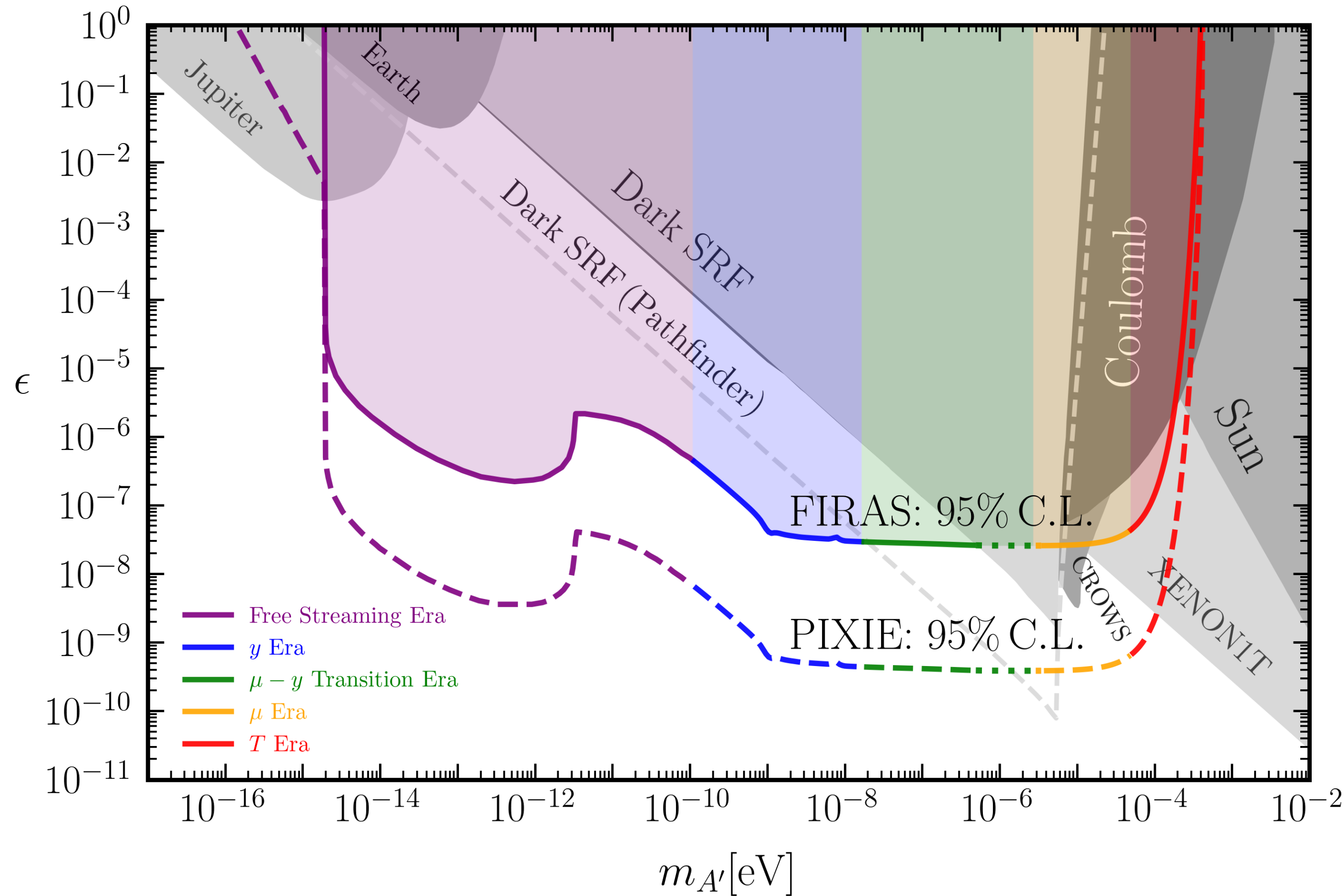
THIS paper deals with the mathematical analysis of noise obtained by passing random noise through physical devices. The random noise

Dark Photon Constraints



hep-ph/2405.XXXXX, Giorgi Arsenadze, AC, Xucheng Gan,
Hongwan Liu and Josh Ruderman

Dark Photon Constraints



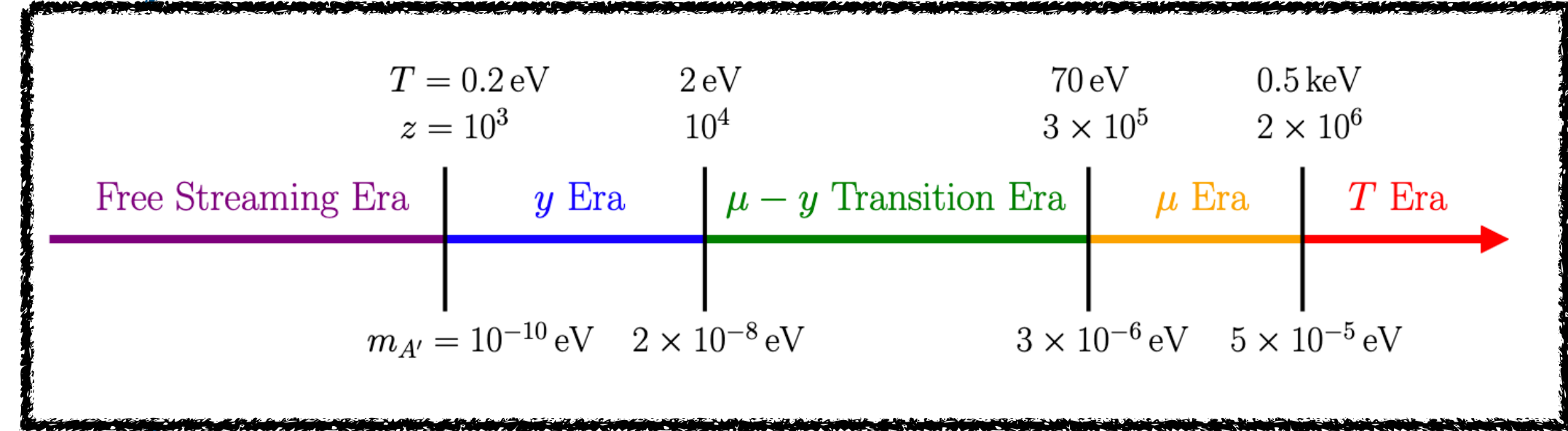
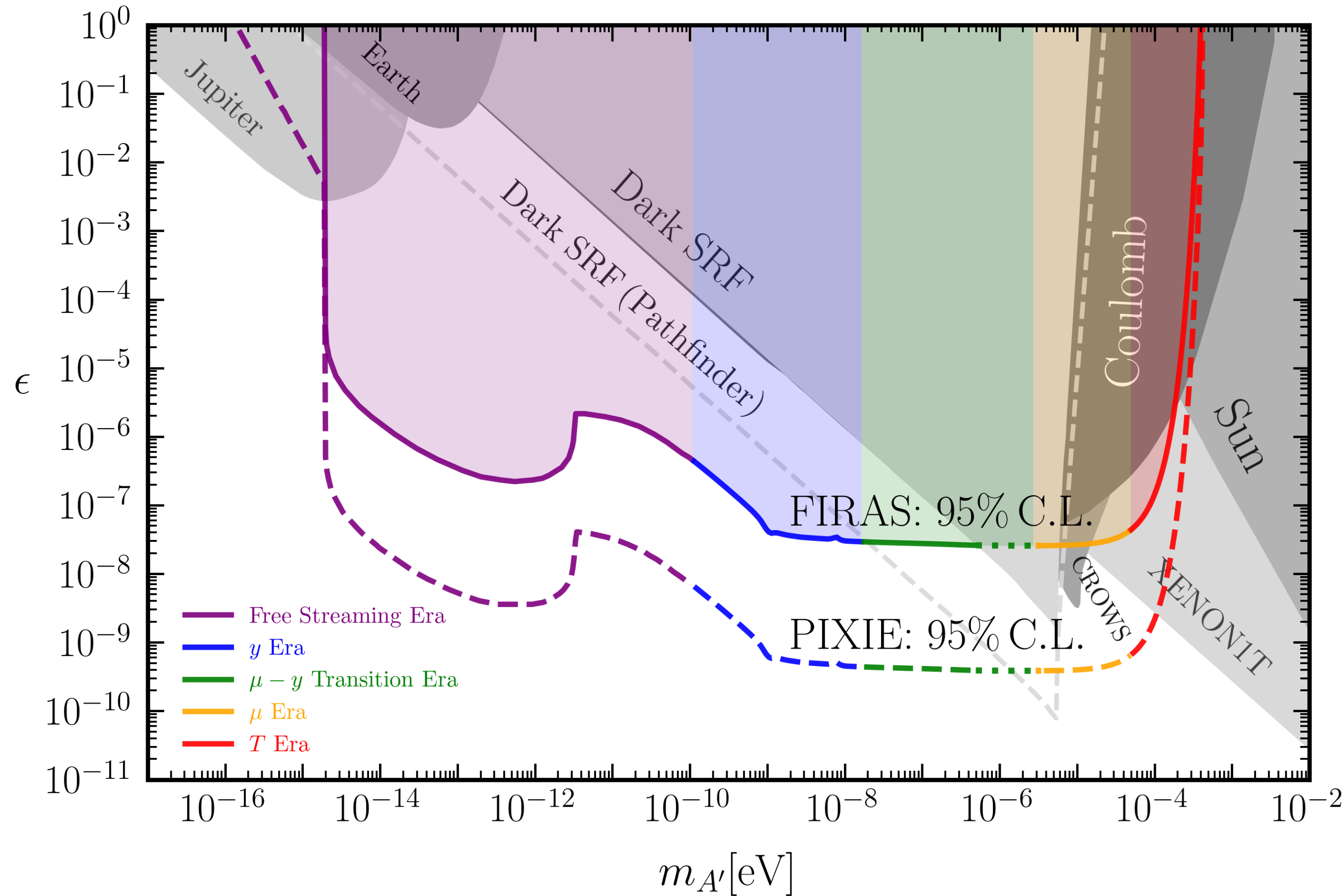
- Compton Scattering (CS): $e^- + \gamma \leftrightarrow e^- + \gamma$,
- Double Compton Scattering (DCS): $e^- + \gamma \leftrightarrow e^- + \gamma + \gamma$,
- Bremsstrahlung (BR): $e^- + X \leftrightarrow e^- + X + \gamma$.

The efficiency of these processes determine the type of distortions we can induce

$$f_\gamma(x) = \bar{f}_\gamma(x) + \Delta f_\gamma(x)$$

hep-ph/2405.XXXXX, Giorgi Arsenadze, AC, Xucheng Gan, Hongwan Liu and Josh Ruderman

Dark Photon Constraints



- Compton Scattering (CS): $e^- + \gamma \leftrightarrow e^- + \gamma$,
- Double Compton Scattering (DCS): $e^- + \gamma \leftrightarrow e^- + \gamma + \gamma$,
- Bremsstrahlung (BR): $e^- + X \leftrightarrow e^- + X + \gamma$.

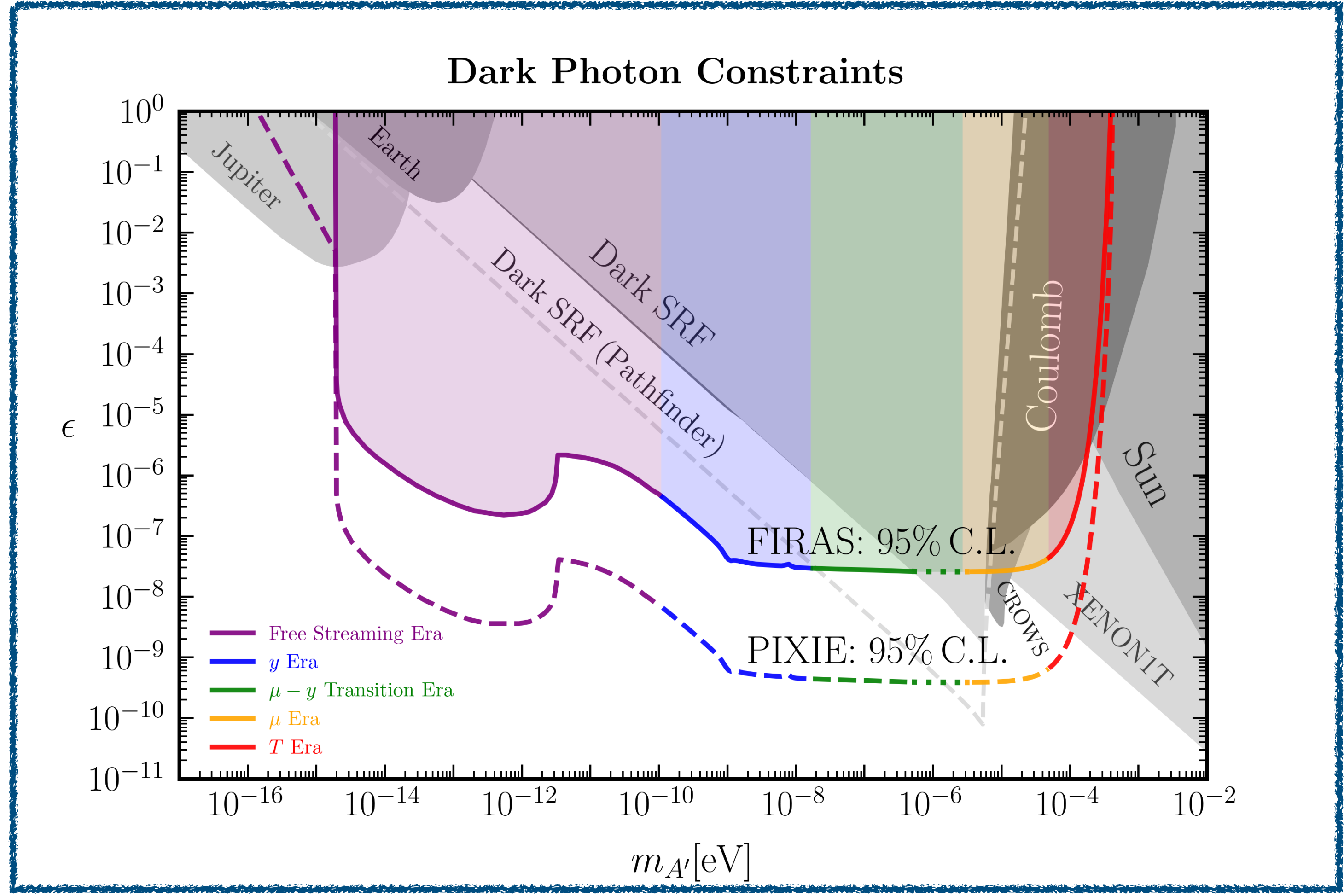
The efficiency of these processes determine the type of distortions we can induce

$$f_\gamma(x) = \bar{f}_\gamma(x) + \Delta f_\gamma(x)$$

hep-ph/2405.XXXXX, Giorgi Arsenadze, AC, Xucheng Gan, Honqwan Liu and Josh Ruderman

Green's function

$$I_\gamma(\omega_0; T_0) = \frac{dP_\gamma}{dA d\Omega d\nu_0} = \frac{\omega_0^3}{2\pi^2} f_\gamma(\omega_0, T_0) \quad \Delta I_\gamma(x; T_0) = - \int dx' \frac{1}{\bar{n}_\gamma} \frac{d\bar{n}_\gamma}{dx'} P_{\gamma \rightarrow A'} G(x; x', z'_{\text{res}}; T_0)$$



With the intensity variation at hands, we can compare our predictions with data

hep-ph/2405.XXXXX, Giorgi Arsenadze, AC, Xucheng Gan, Hongwan Liu and Josh Ruderman

$$\log \mathcal{L}(\text{data}|m_{A'}, \epsilon) = \max_{T_0} \left\{ -\frac{1}{2} [\Delta \mathbf{I}_\gamma(m_{A'}, \epsilon; T_0) - \mathcal{R}]^T \cdot \mathbf{C}^{-1} \cdot [\Delta \mathbf{I}_\gamma(m_{A'}, \epsilon; T_0) - \mathcal{R}] \right\}$$

$$\text{TS}(m_{A'}, \epsilon) = 2 \left[\log \mathcal{L}(\text{data}|m_{A'}, \epsilon) - \min_{\epsilon} \log \mathcal{L}(\text{data}|m_{A'}, \epsilon) \right]$$

The Cosmic Microwave Background Spectrum from the Full COBE* FIRAS Data Set

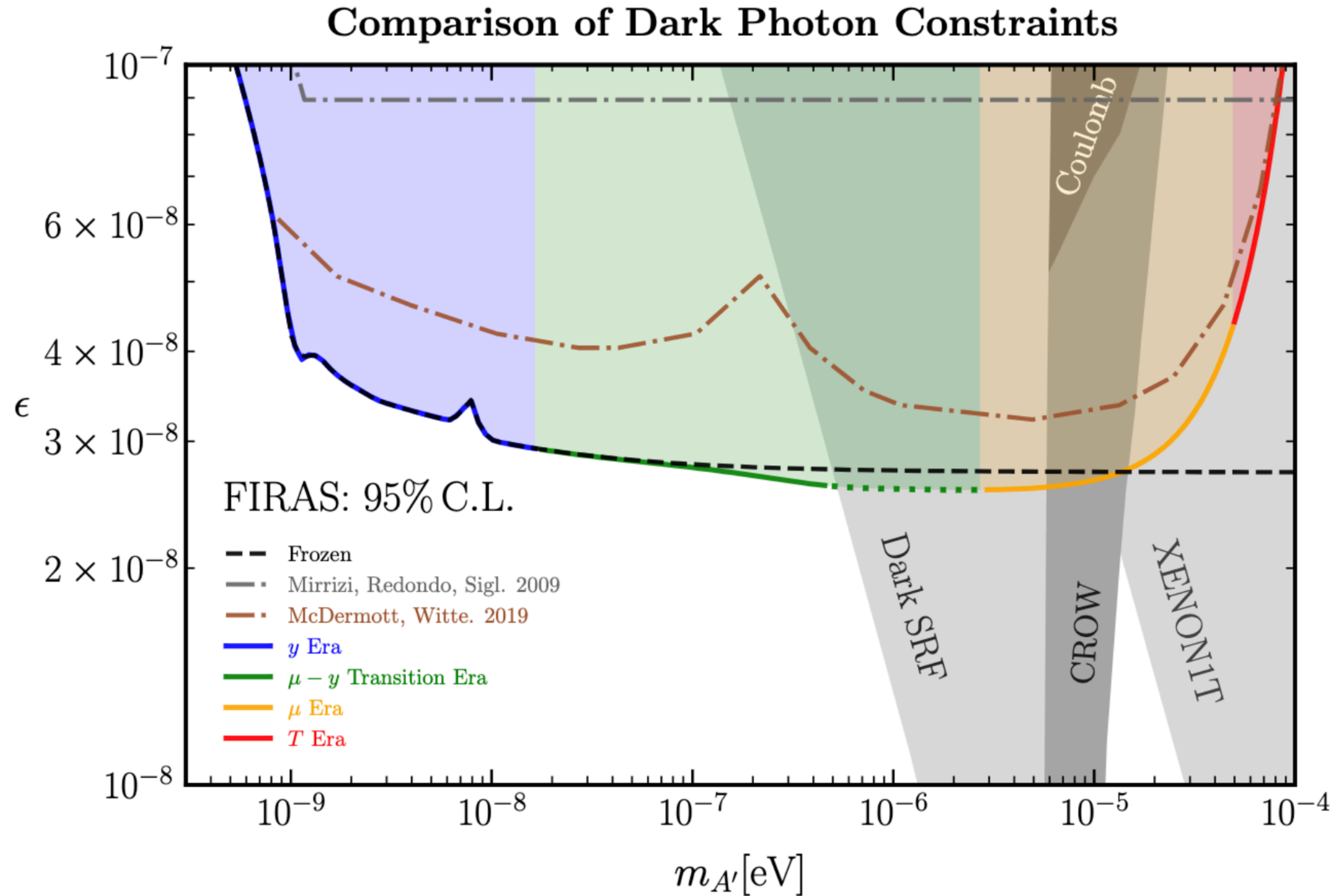
D. J. Fixsen¹, E. S. Cheng², J. M. Gales¹, J. C. Mather², R. A. Shafer², and E. L. Wright³
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We are not the first ones to do this analysis, but the most accurate up to date



Possible future directions

Axion case, modelling [possible] extra-galactic magnetic fields

Look at anisotropies and use cross-correlation with other maps
[see also McCarthy et al. (2024), Pirvu et al. (2023)]

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