

Opportunities of HFGW with Axion Detectors

Sung Mook Lee
CERN

JHEP 03 (2024) 128 (2306.03125)

with Valerie Domcke (CERN), Camilo Garcia-Cely (Valencia, IFIC), and Nicholas L. Rodd (LBNL)



**Funded by
the European Union**

Outline

- **HFGW – Why?**

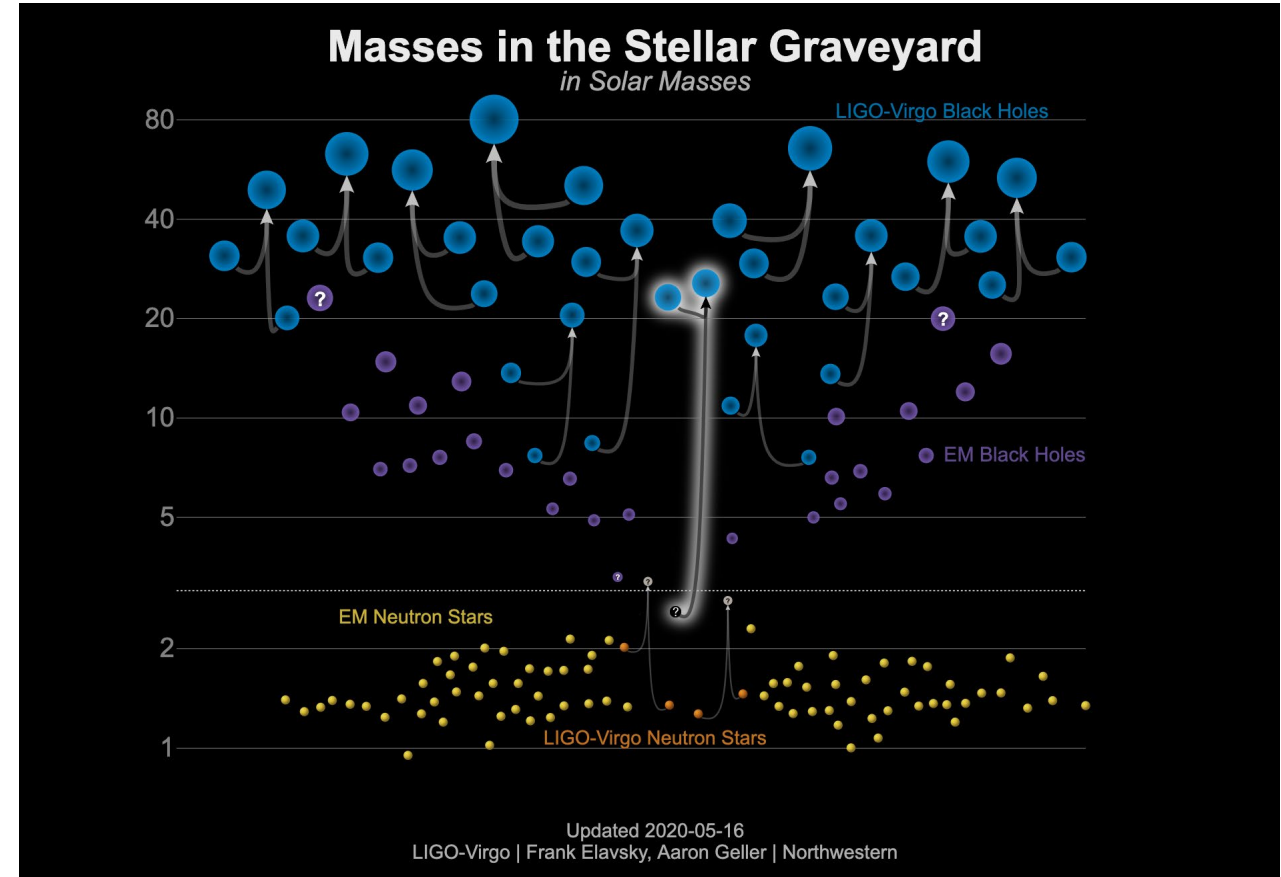
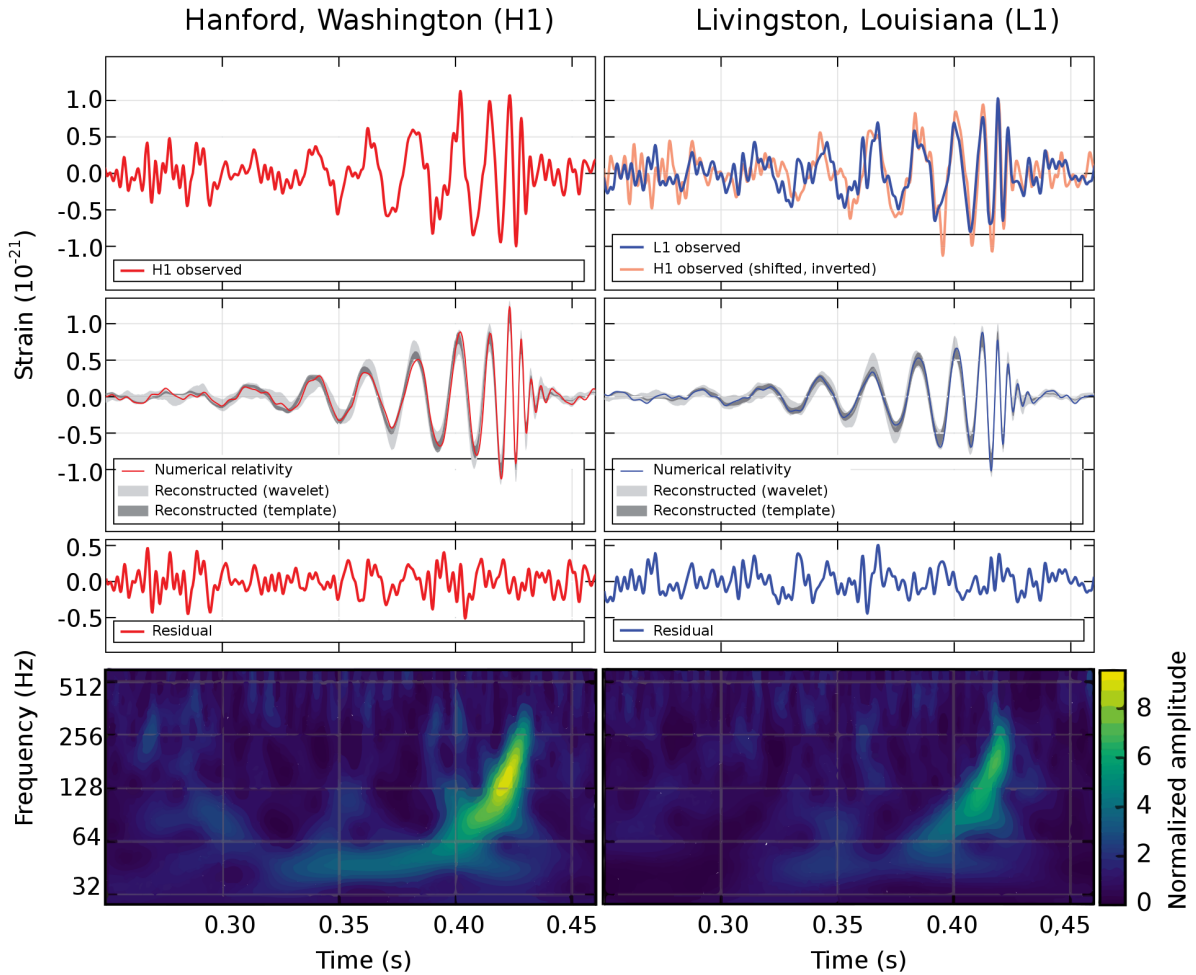
 - **Motivation / Source**

- **HFGW – How?**

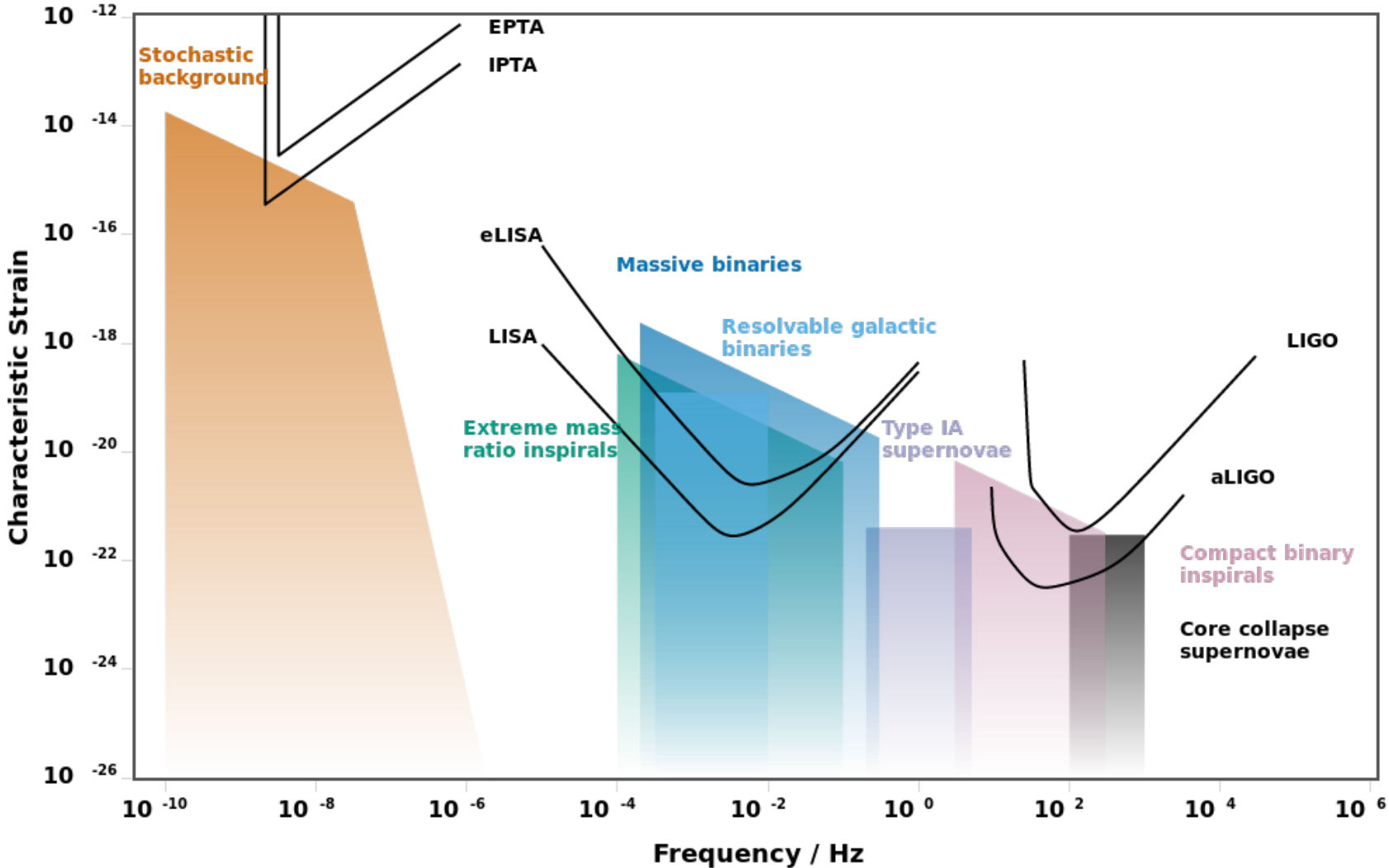
 - **On the table : Axion Haloscope**

 - **From the sky : Neutron Star**

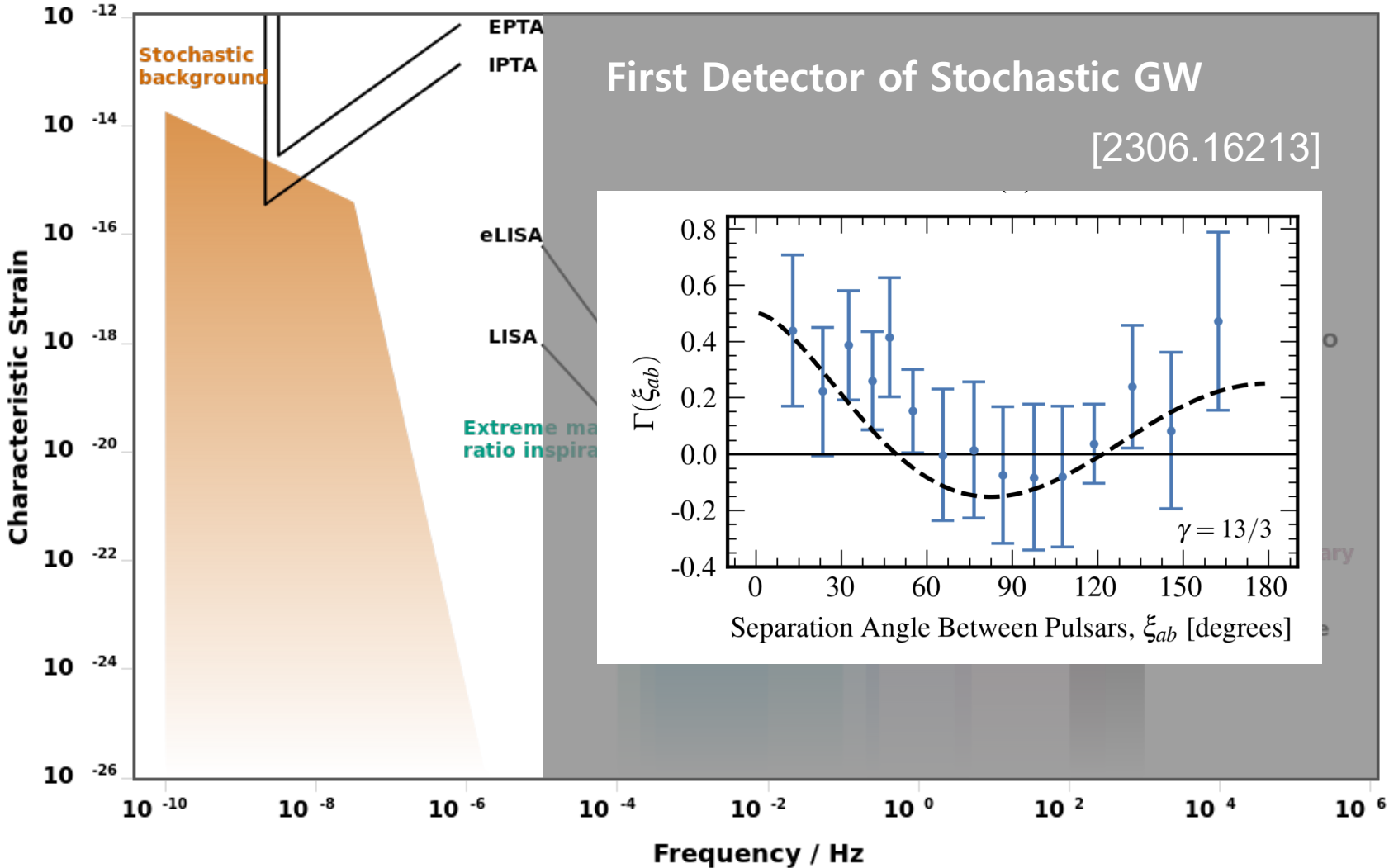
Detection of Gravitational Waves



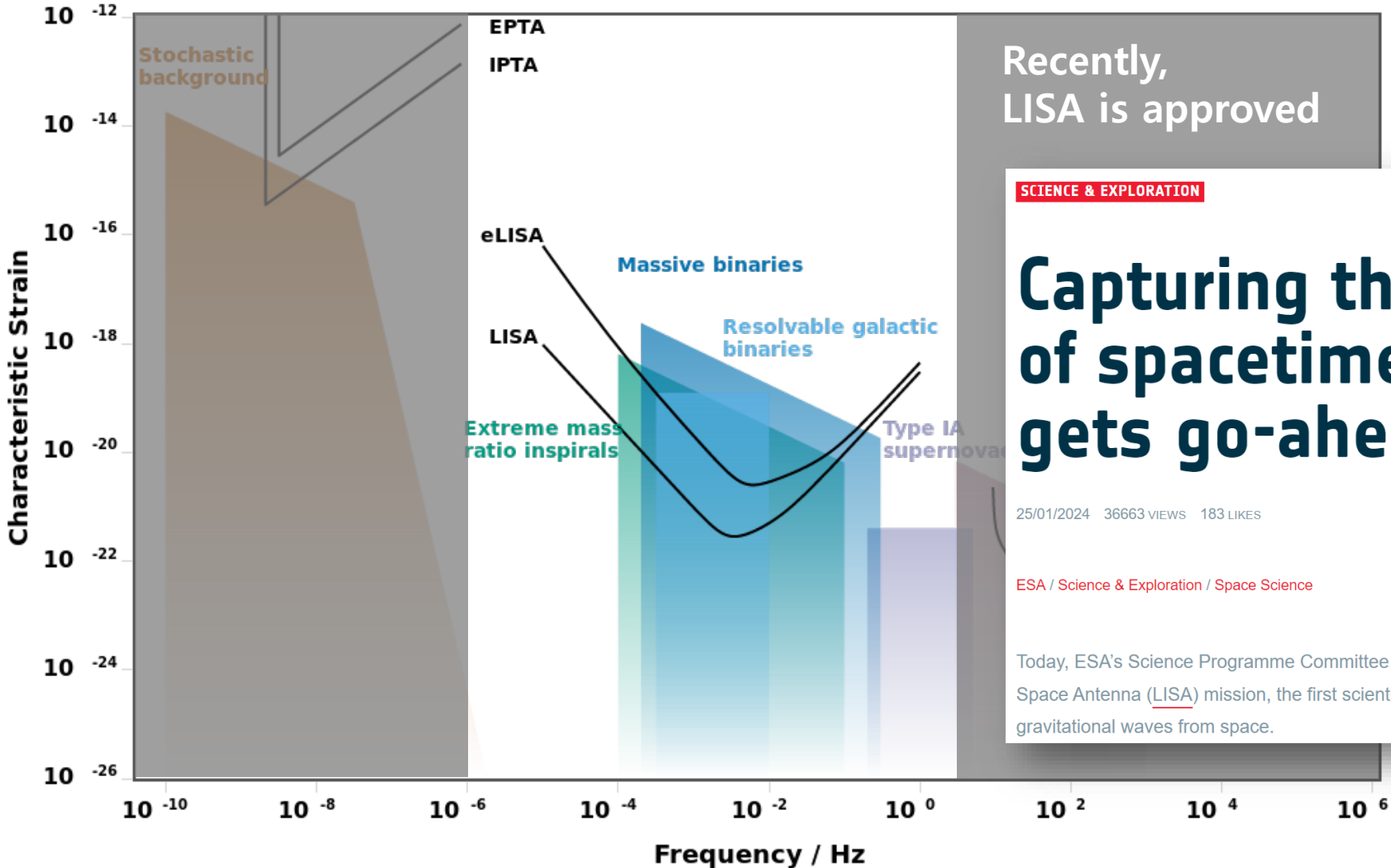
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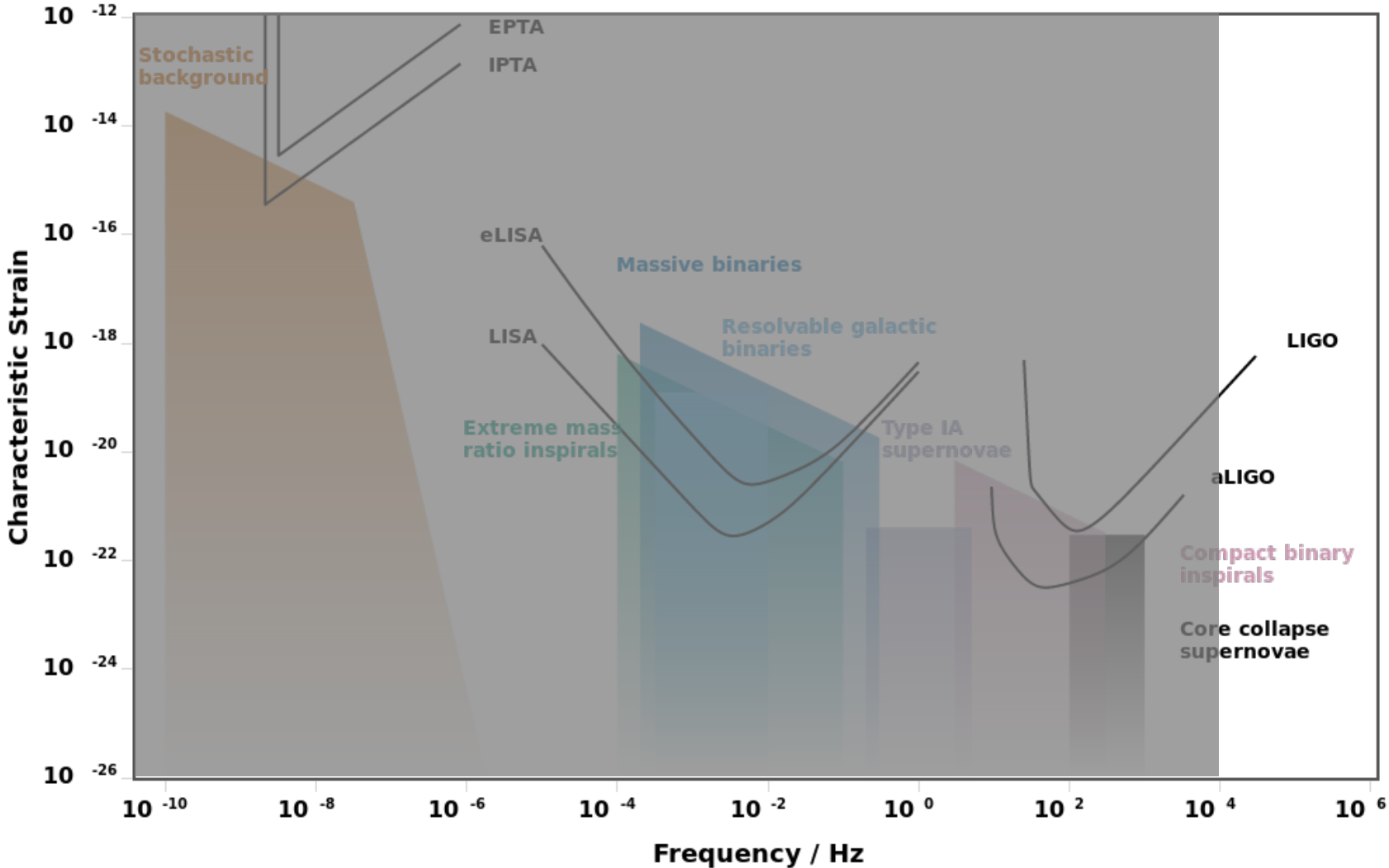
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Detection of Gravitational Waves

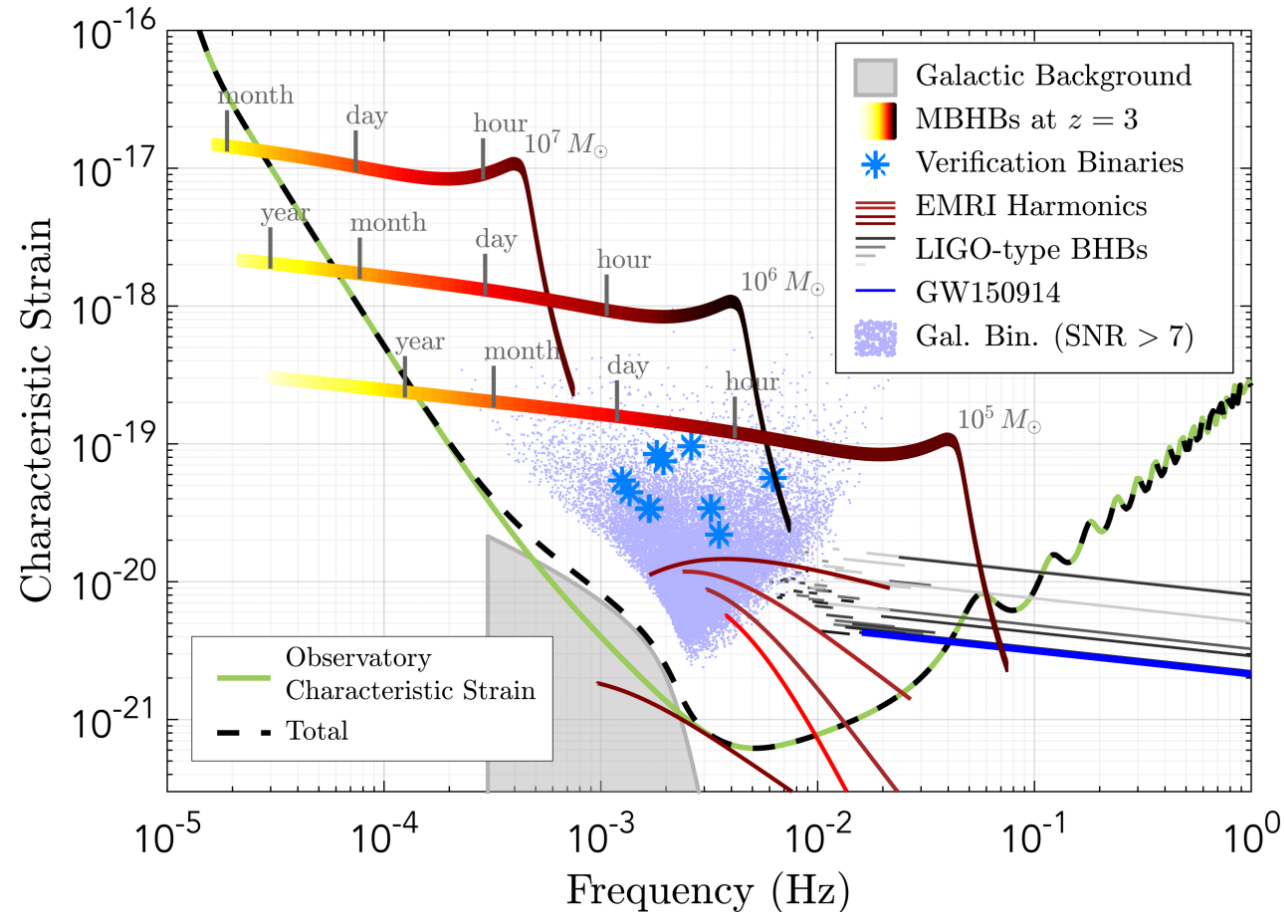


Detection of Gravitational Waves



Signals from BSM Physics?

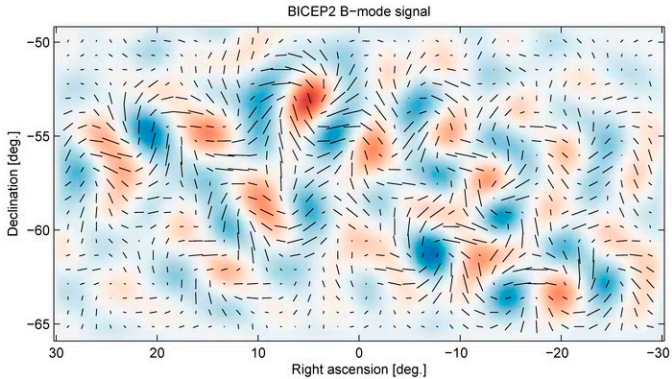
■ 'Background' from SMBH



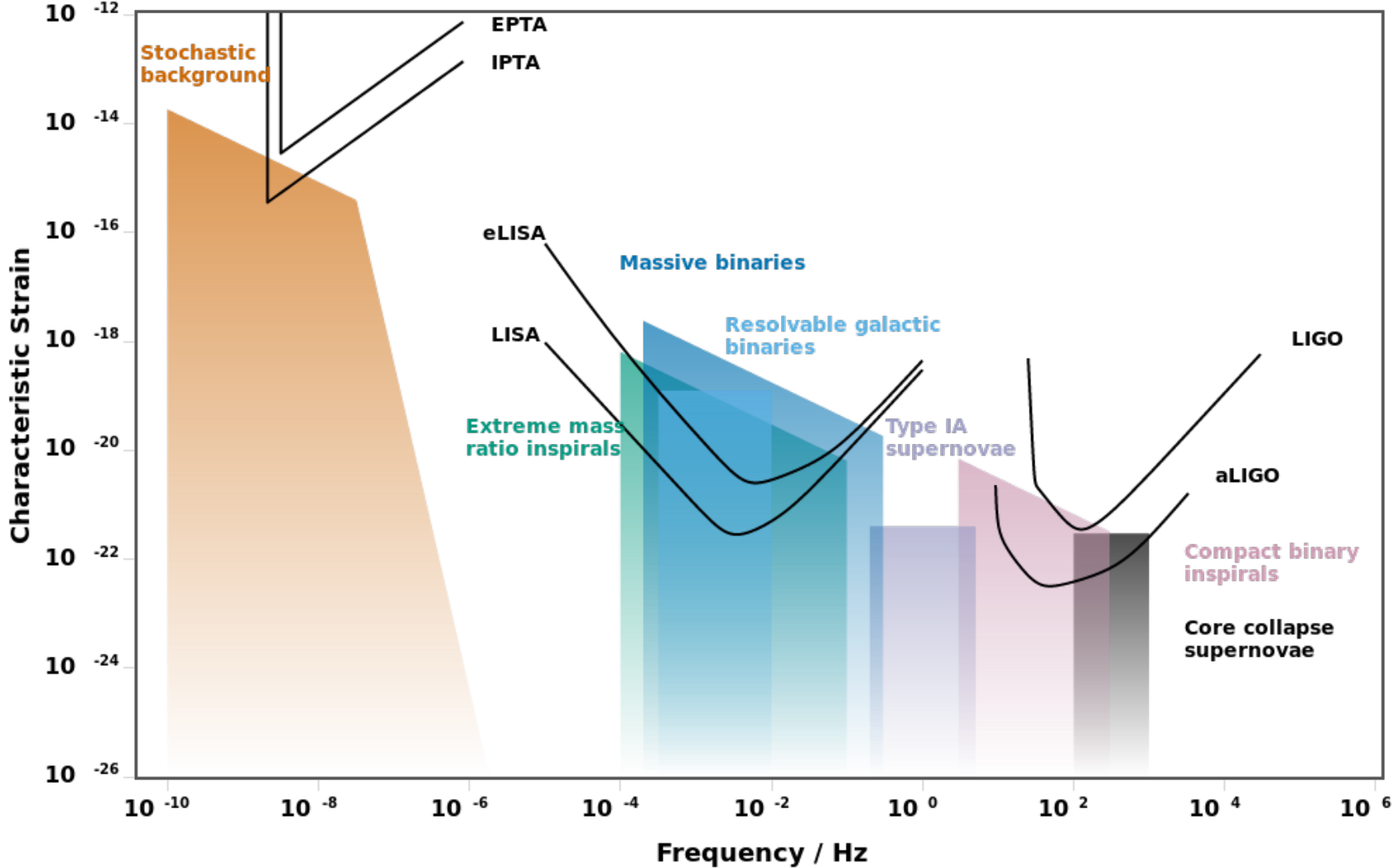
[LISA Proposal 1702.00786]

Detection of Gravitational Waves

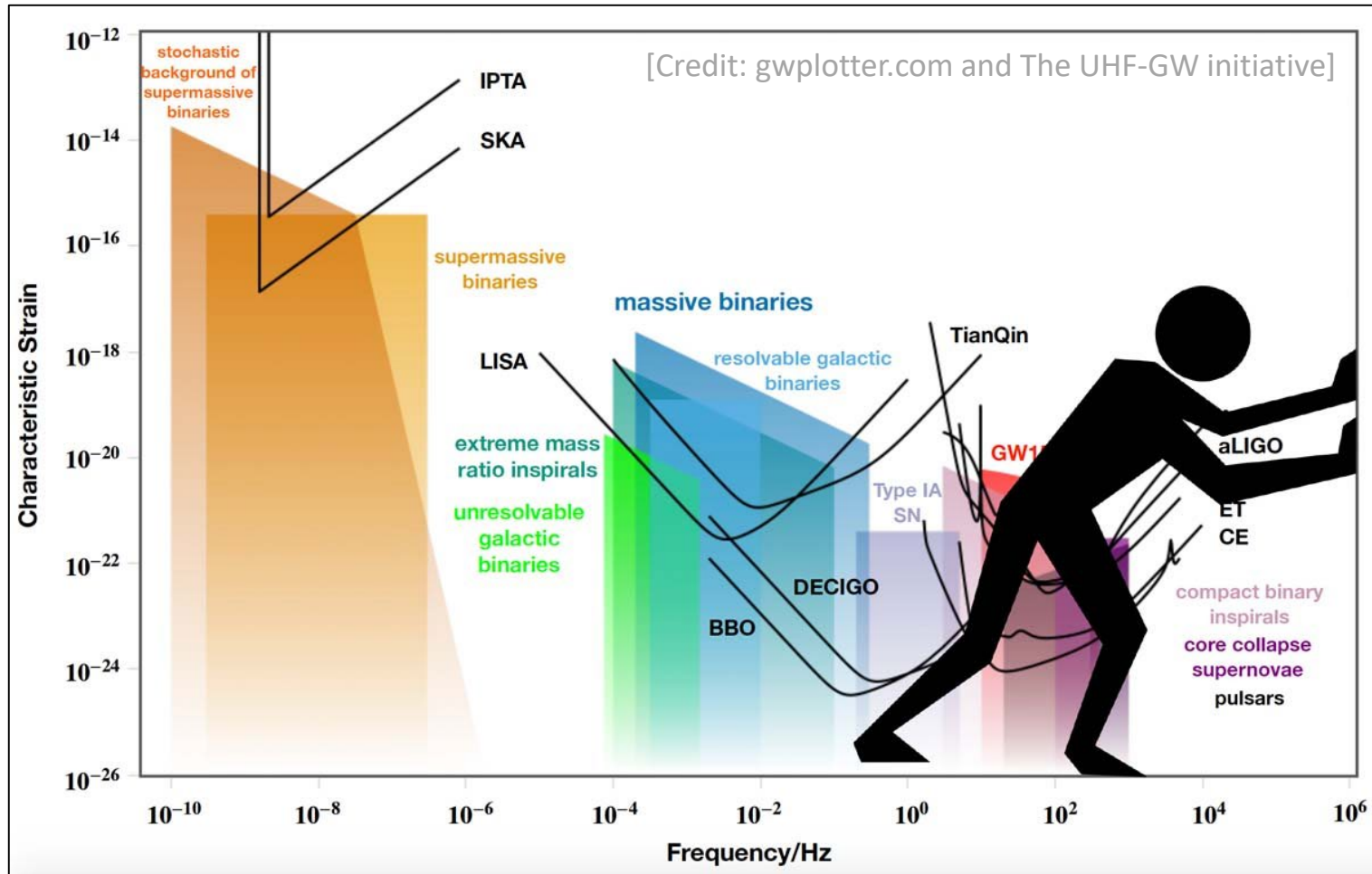
LOWER frequency



CMB B-mode Polarization



High Frequency Gravitational Waves



How can we detect GW
with $f > O(1\text{MHz})$?

Sources of HFGW

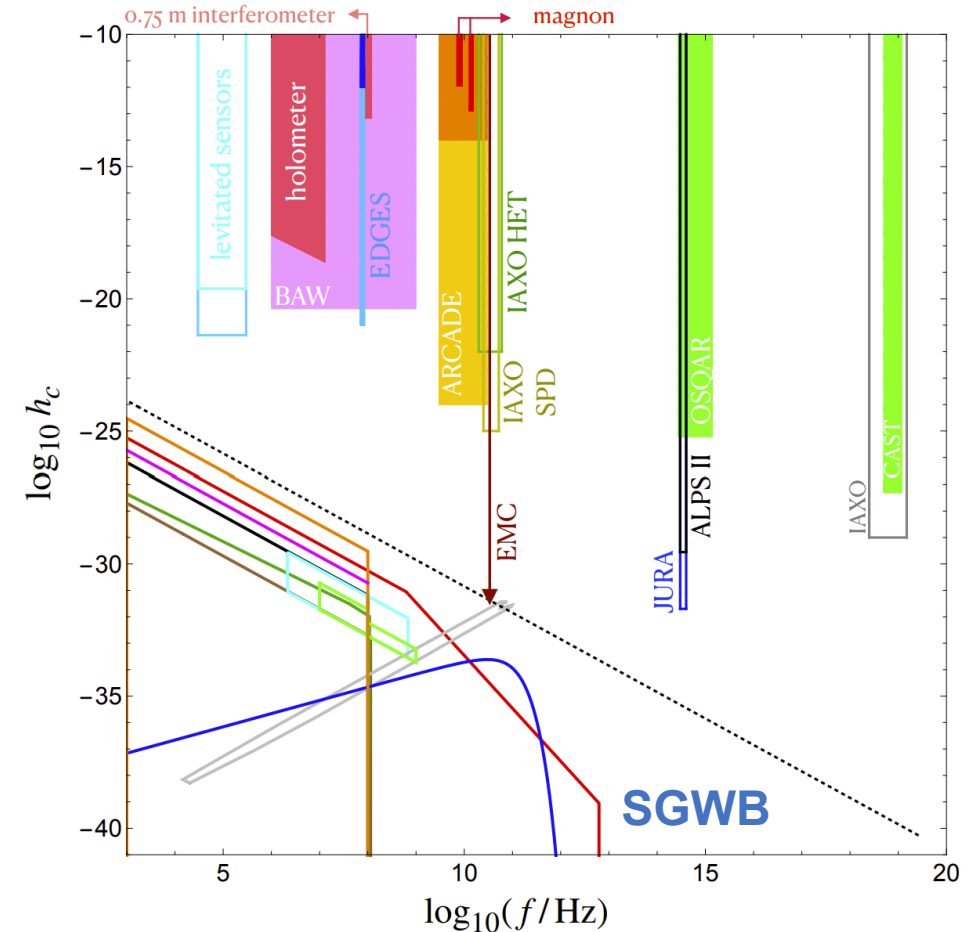
HFGW Sources

- Stochastic Gravitational Wave Background

- High frequency = Early universe

$$f_{\text{GW}} \gtrsim O(1) \text{ MHz} \left(\frac{T_*}{10^8 \text{ GeV}} \right)$$

[2011.12414 for review]



HFGW Sources

Stochastic Gravitational Wave Background

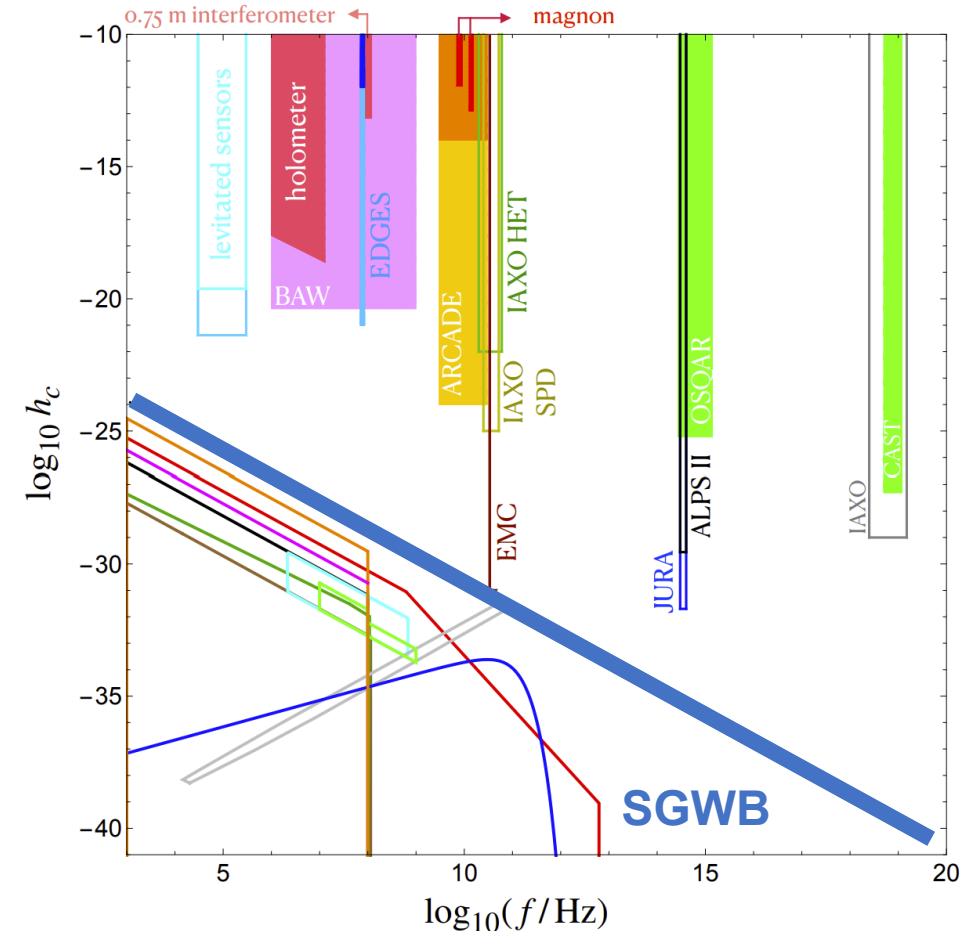
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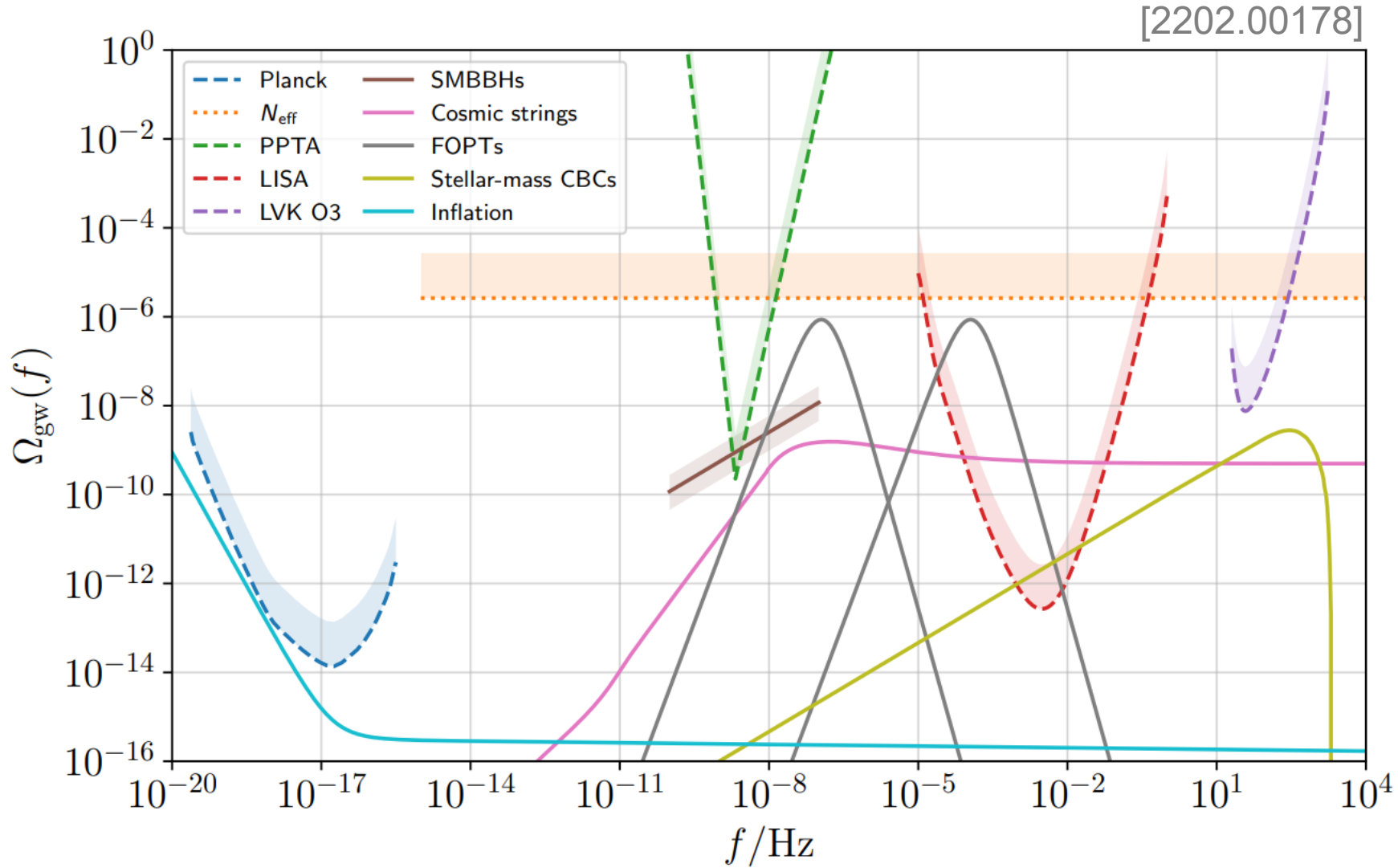
- Constraints from BBN/CMB on ΔN_{eff}

$$\left(\frac{\rho_{\text{GW}}}{\rho_\gamma} \right) \leq \frac{7}{8} \left(\frac{4}{11} \right)^{4/3} \Delta N_{\text{eff}} \lesssim 0.05$$

[2011.12414 for review]



HFGW Sources



HFGW Sources

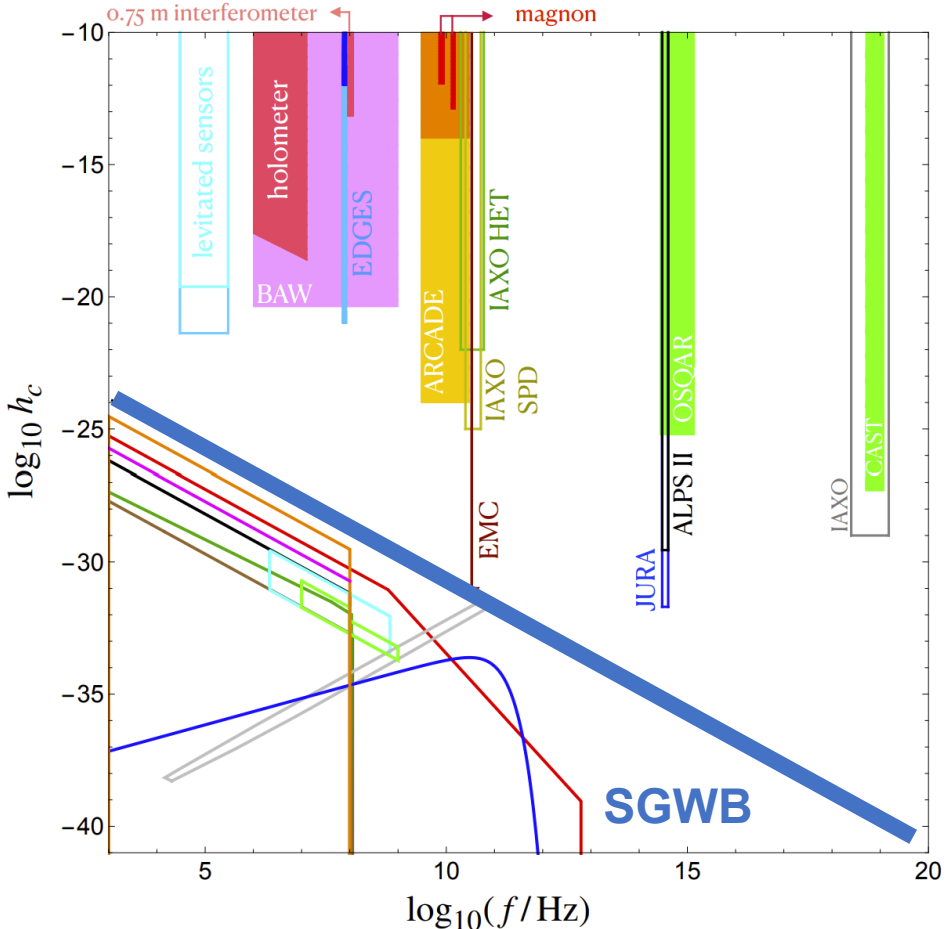
- **Localized Sources**

- PBH binaries / exotic compact objects

$$f \simeq 220 \text{ MHz} \left(\frac{10^{-5} M_{\odot}}{m_{\text{PBH}}} \right)$$

- Larger signals expected

[2011.12414 for review]



HFGW Sources

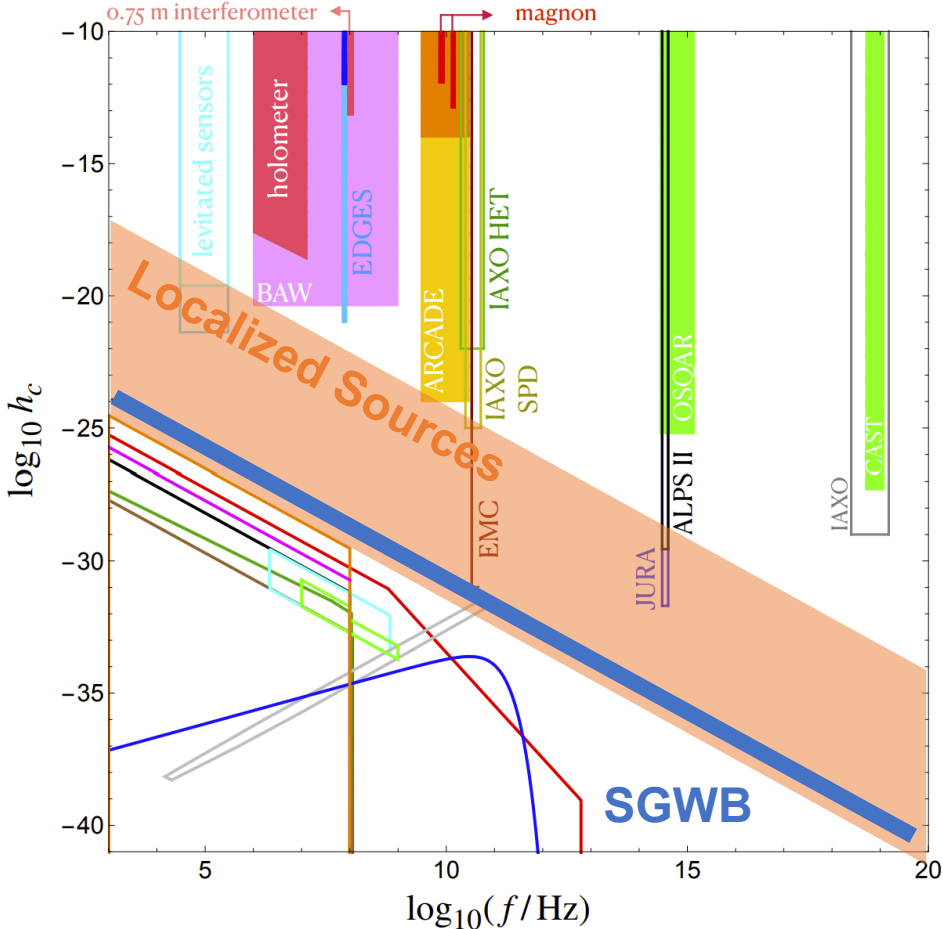
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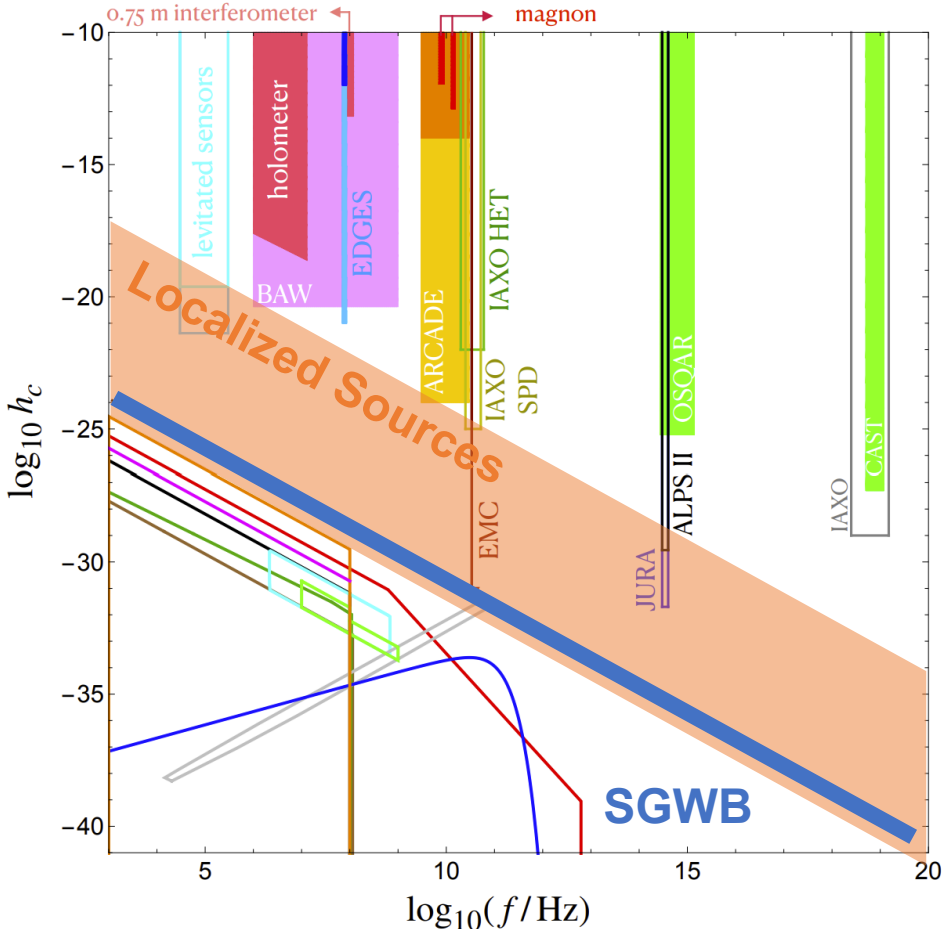
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*Detection of HFGW
= Smoking Gun of BSM Physics*

[2011.12414 for review]



Detection of GW

Electromagnetism in Curved Spacetime

[Gertsenshtein '62]

[Boccaletti, Sabbata, Fortini, Gualdi '70]

$$\partial_\nu F^{\mu\nu} = j^\mu$$

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↓ $\partial \rightarrow \nabla$

$$\nabla_\nu F^{\mu\nu} = \frac{1}{\sqrt{-g}} \partial_\nu (\sqrt{-g} F^{\mu\nu}) = j^\mu$$

Electromagnetism in Curved Spacetime

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↓ perturbation

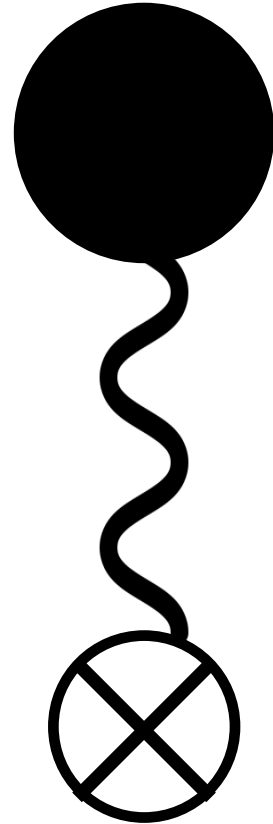
$$\partial_\nu F^{\mu\nu} = \left(1 + \frac{1}{2} h^\mu{}_\mu\right) j^\mu + \underbrace{\partial_\nu \left(-\frac{1}{2} h^\alpha{}_\alpha F^{\mu\nu} + F_\alpha{}^\nu h^{\alpha\mu} + F^\mu{}_\alpha h^{\alpha\nu} \right)}_{\text{'effective current'}} + O(h^2)$$

'effective current'

Electromagnetism in Curved Spacetime

[Gertsenshtein '62]

[Boccaletti, Sabbata, Fortini, Gualdi '70]



B_{ext}

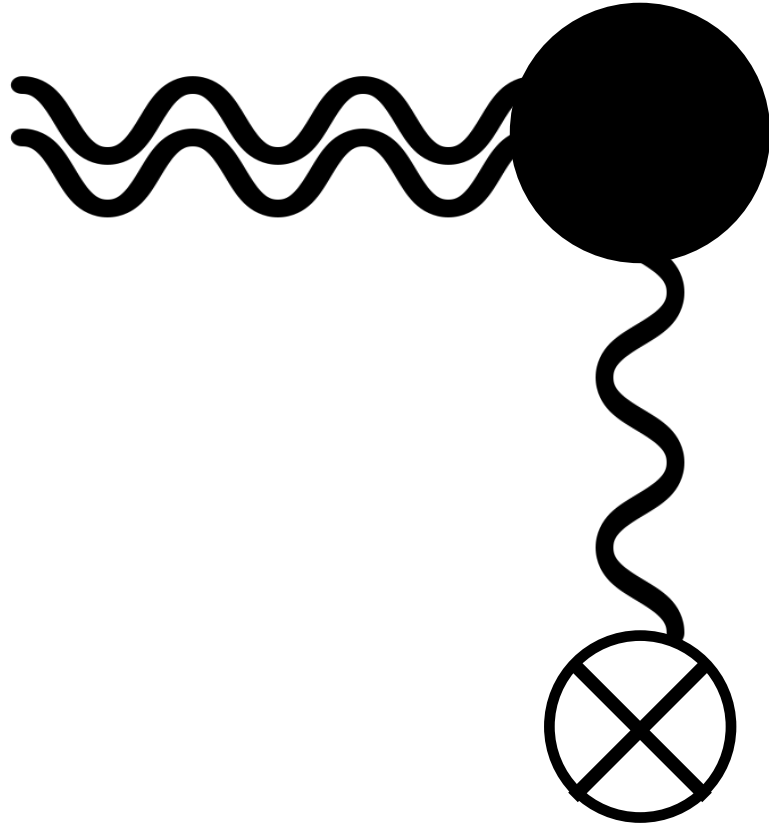
Electromagnetism in Curved Spacetime

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GW

h



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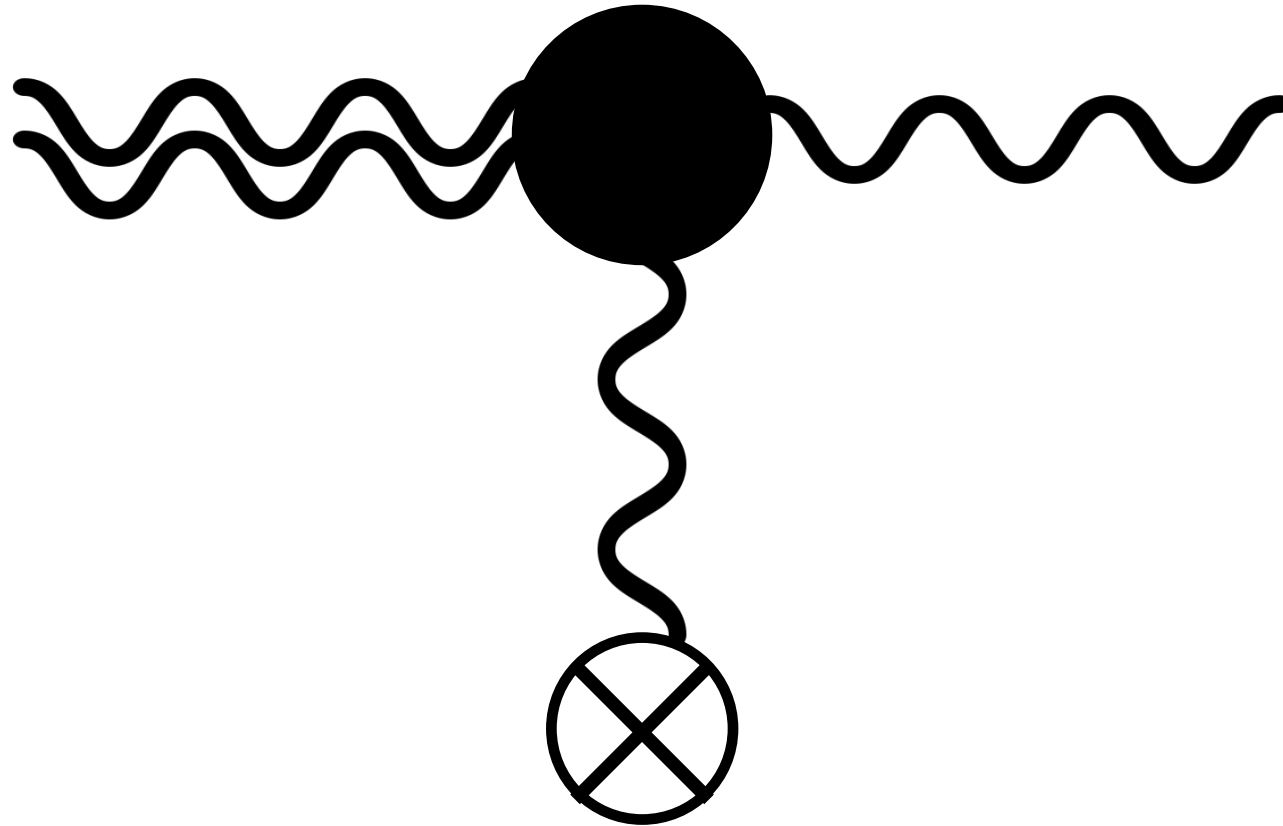
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GW

h



E/B_{ind}
 γ
EM Signal

B_{ext}

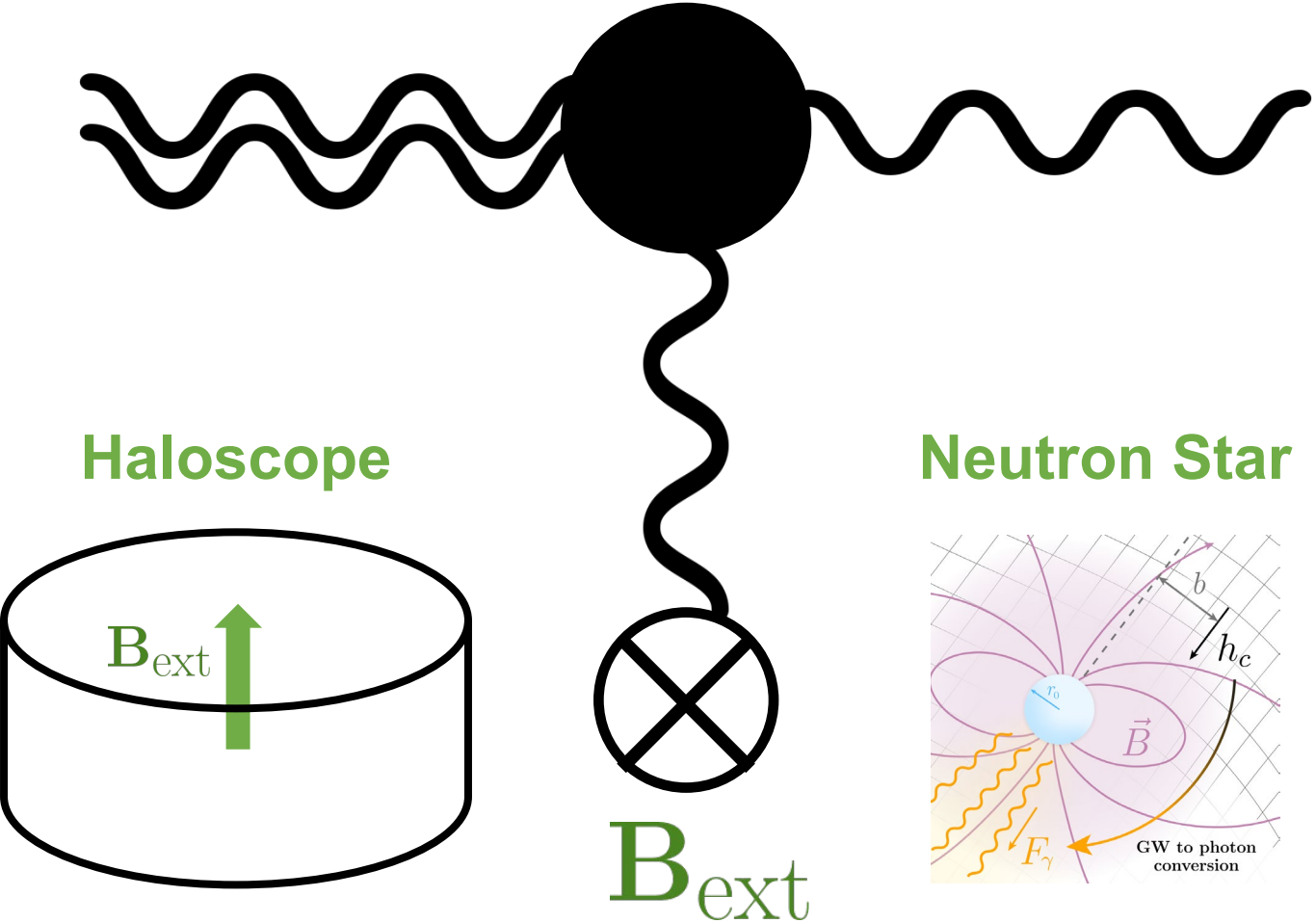
Electromagnetism in Curved Spacetime

[Gertsenshtein '62]

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(Inverse) Gertsenshtein effect

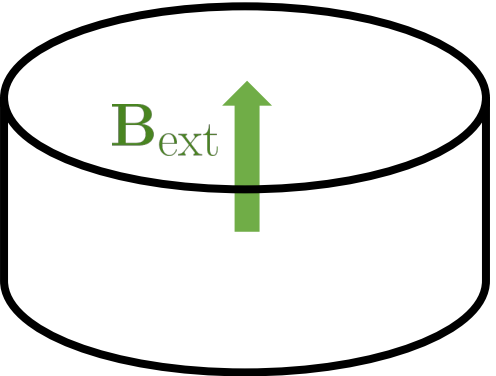
GW
h



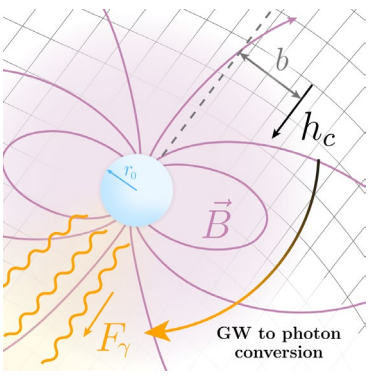
E/B_{ind}
 γ

EM Signal

Haloscope



Neutron Star



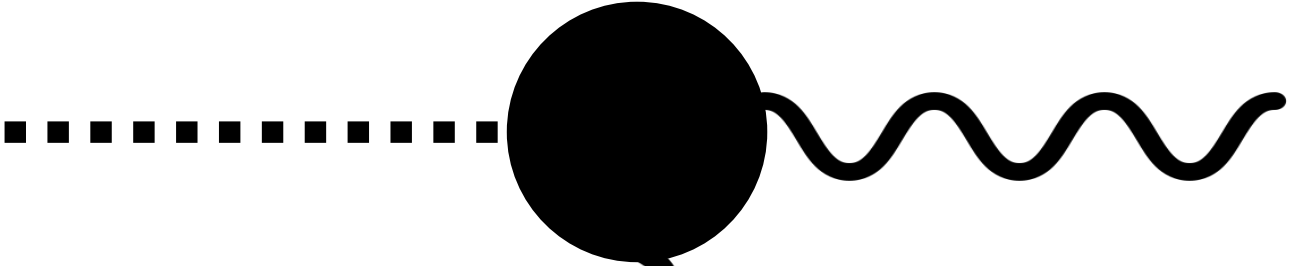
Electromagnetism in Curved Spacetime

[Raffelt '86]

(Inverse) Primakoff effect

Axion

a

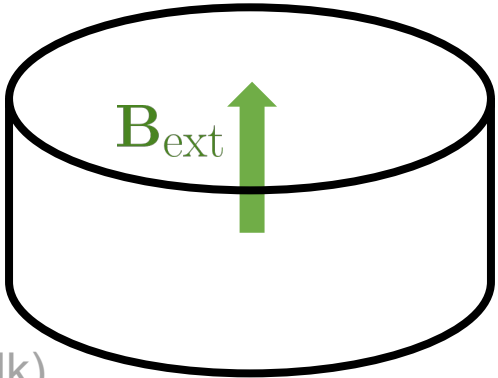


E/B_{ind}
 γ

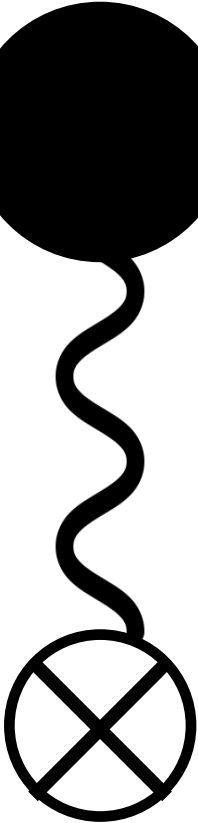
EM Signal

$$\mathbf{j}_{eff} = g_{a\gamma\gamma} \sqrt{2\rho_{DM}} \cos(m_a t) \mathbf{B}$$

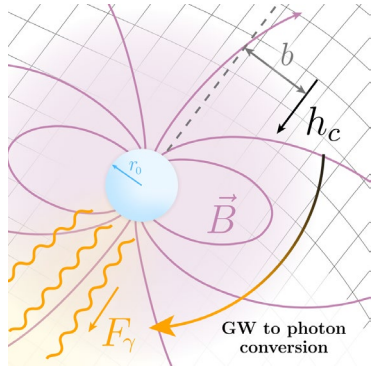
Haloscope



DALI?
(Javier De Miguel's Talk)

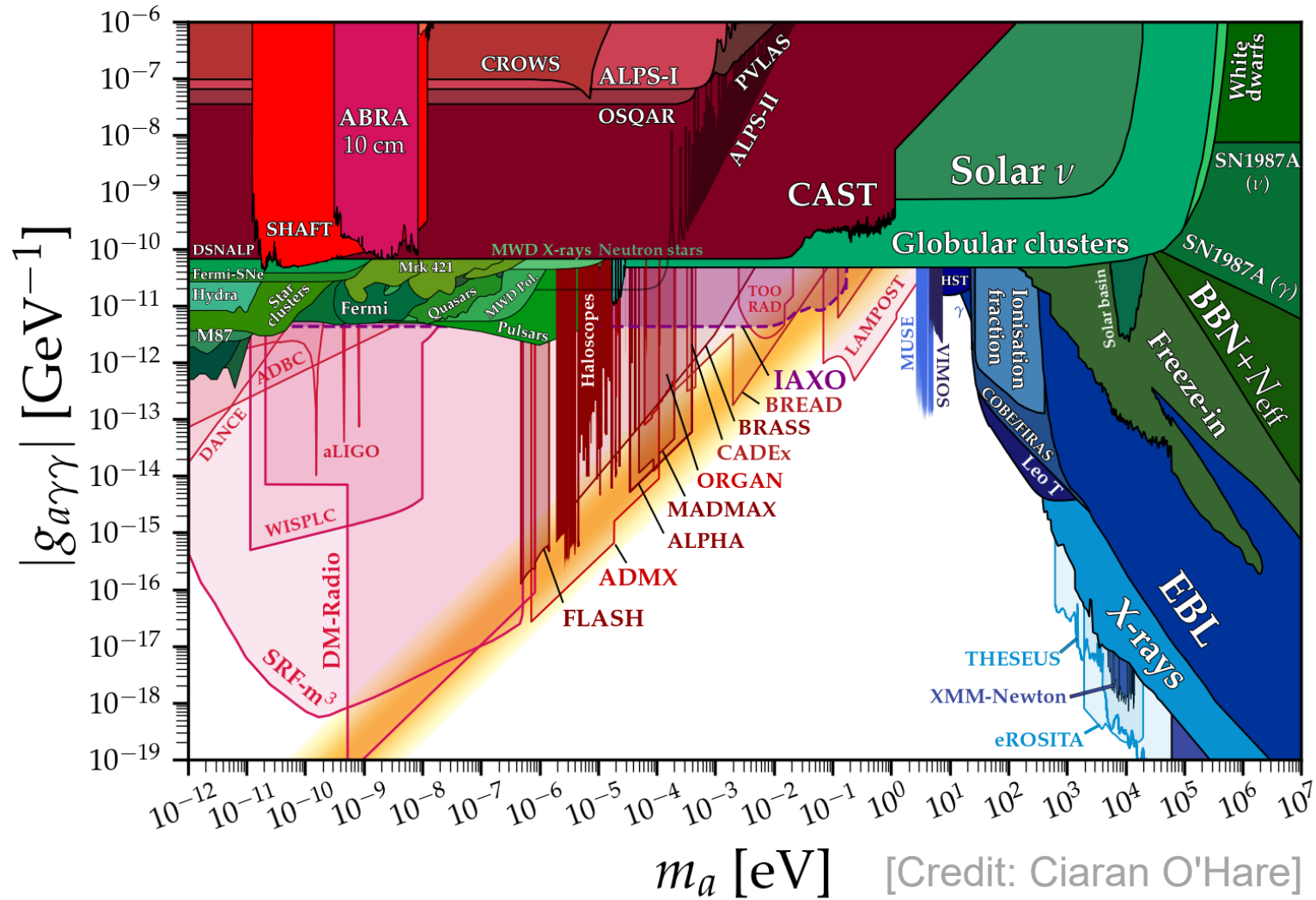


Neutron Star

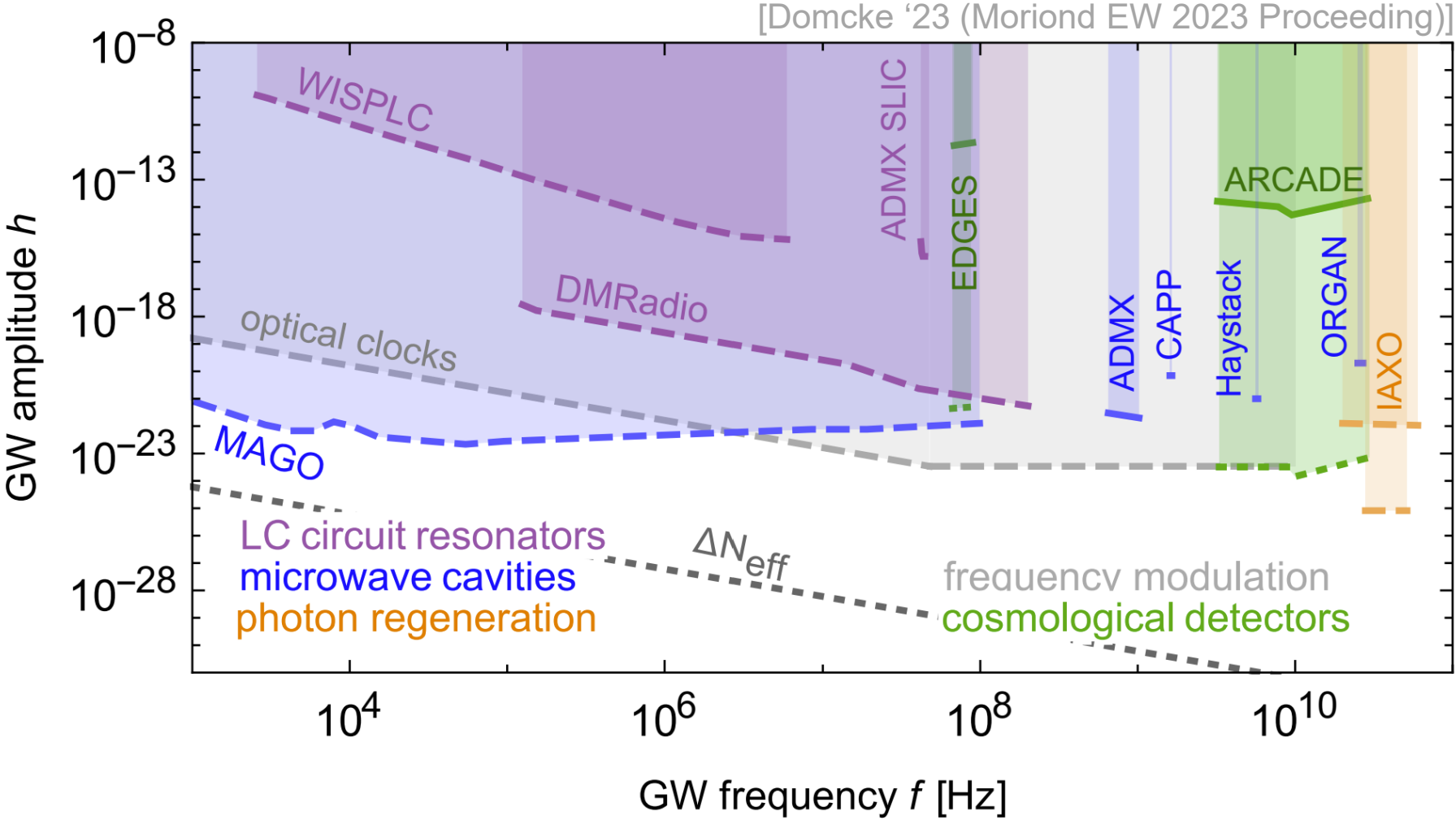


White dwarf?
(Ruben Zlatini' Talk)

Axion Experiment Zoo

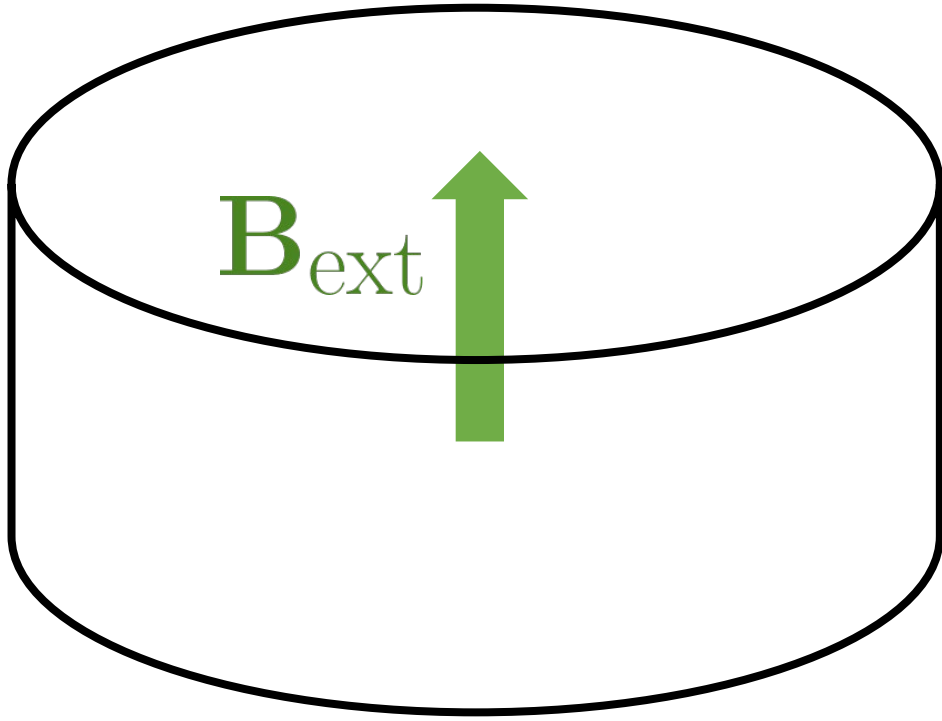


EM-HFGW Program



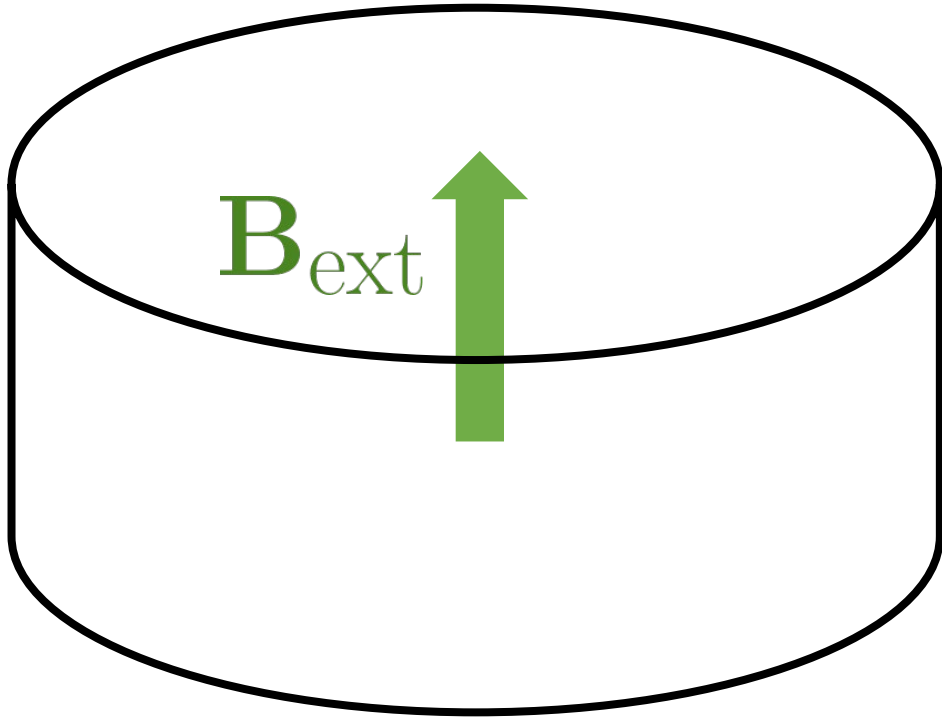
HFGW on the Table: Axion Haloscope

Axion Haloscope as GW Detector



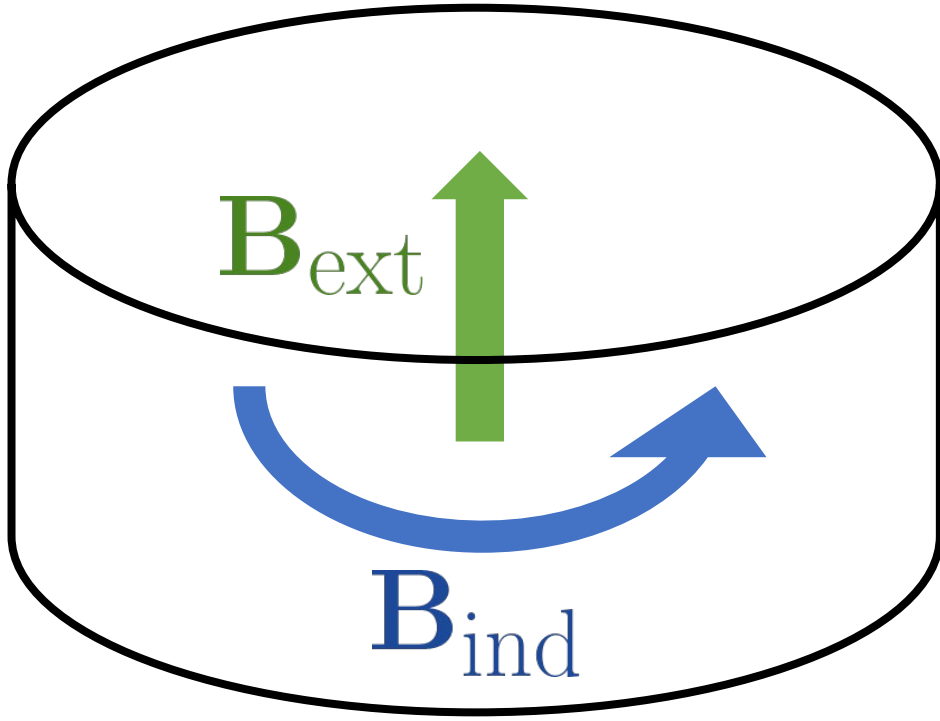
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 AXION

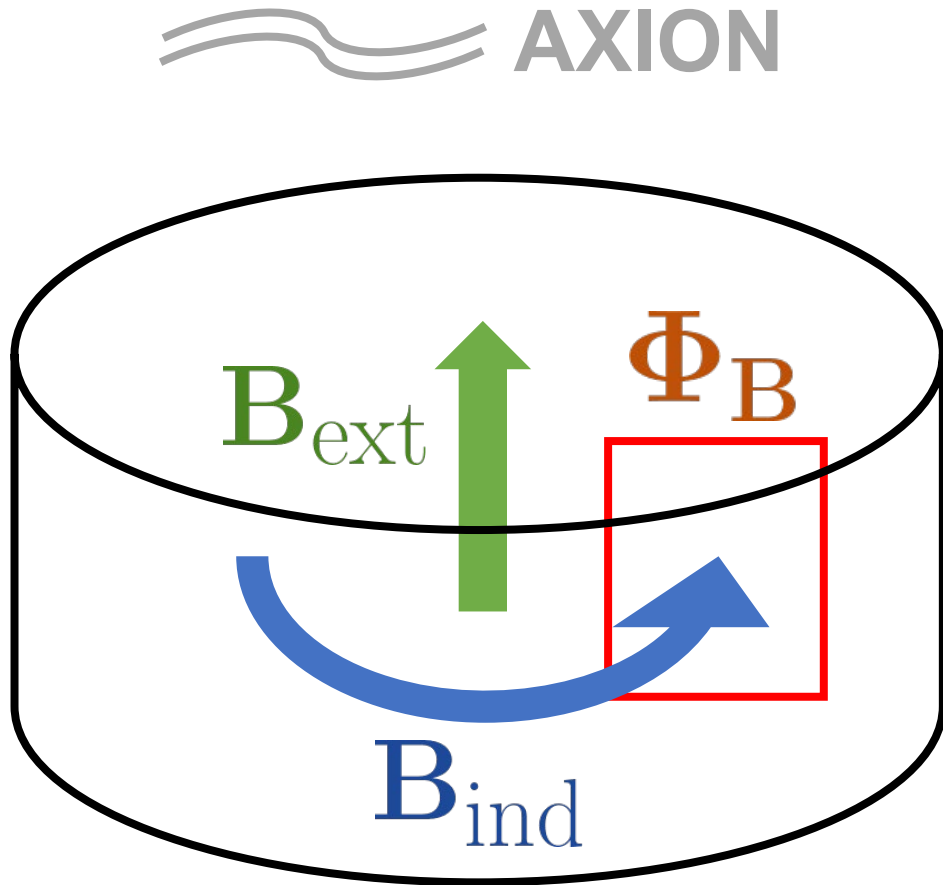


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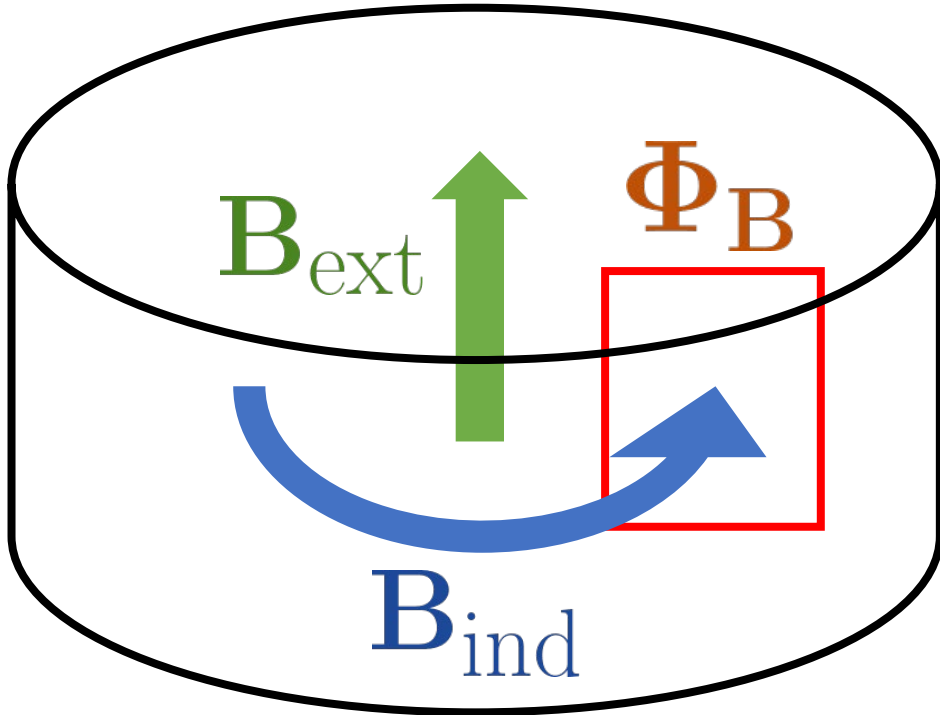


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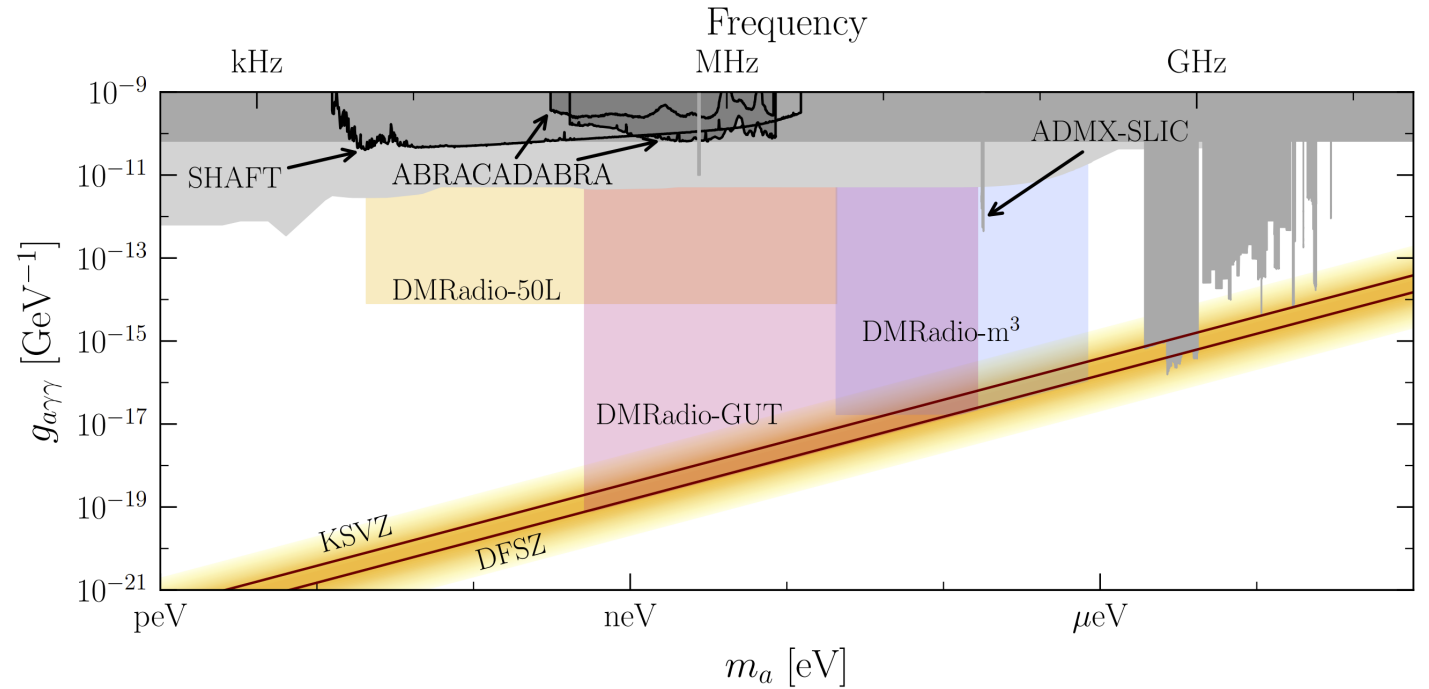


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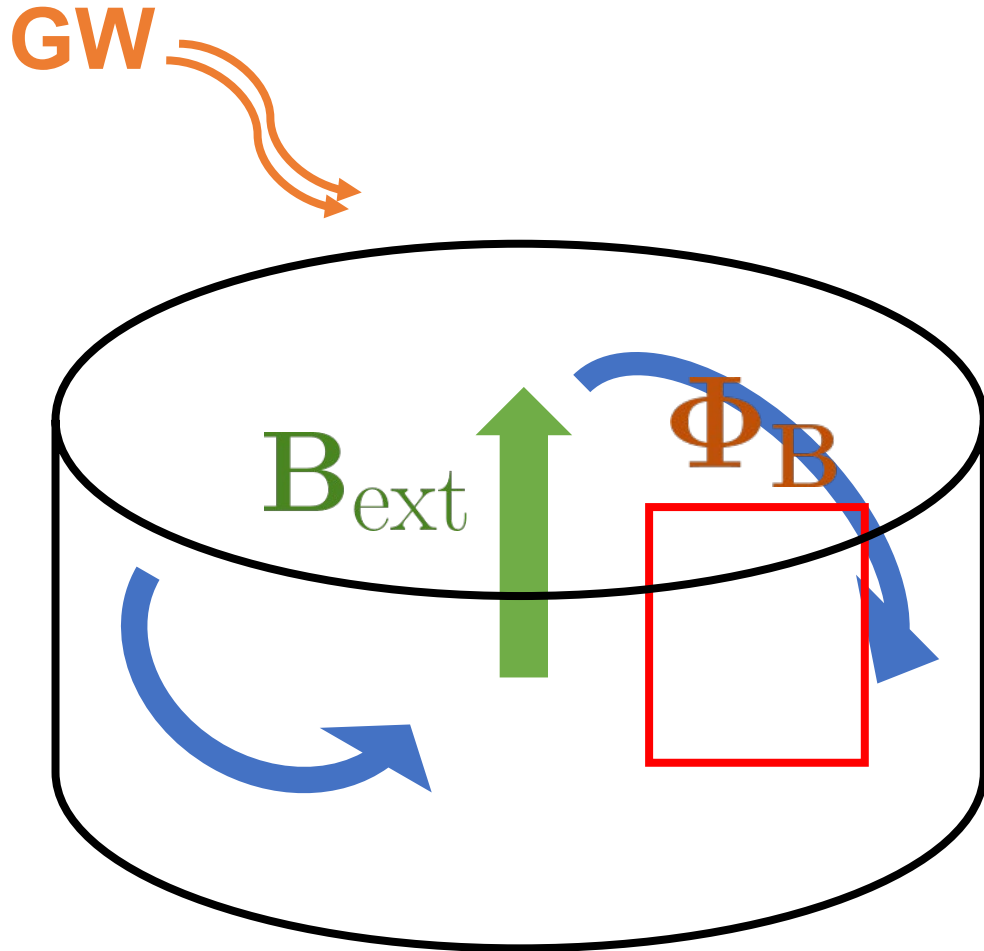
 AXION



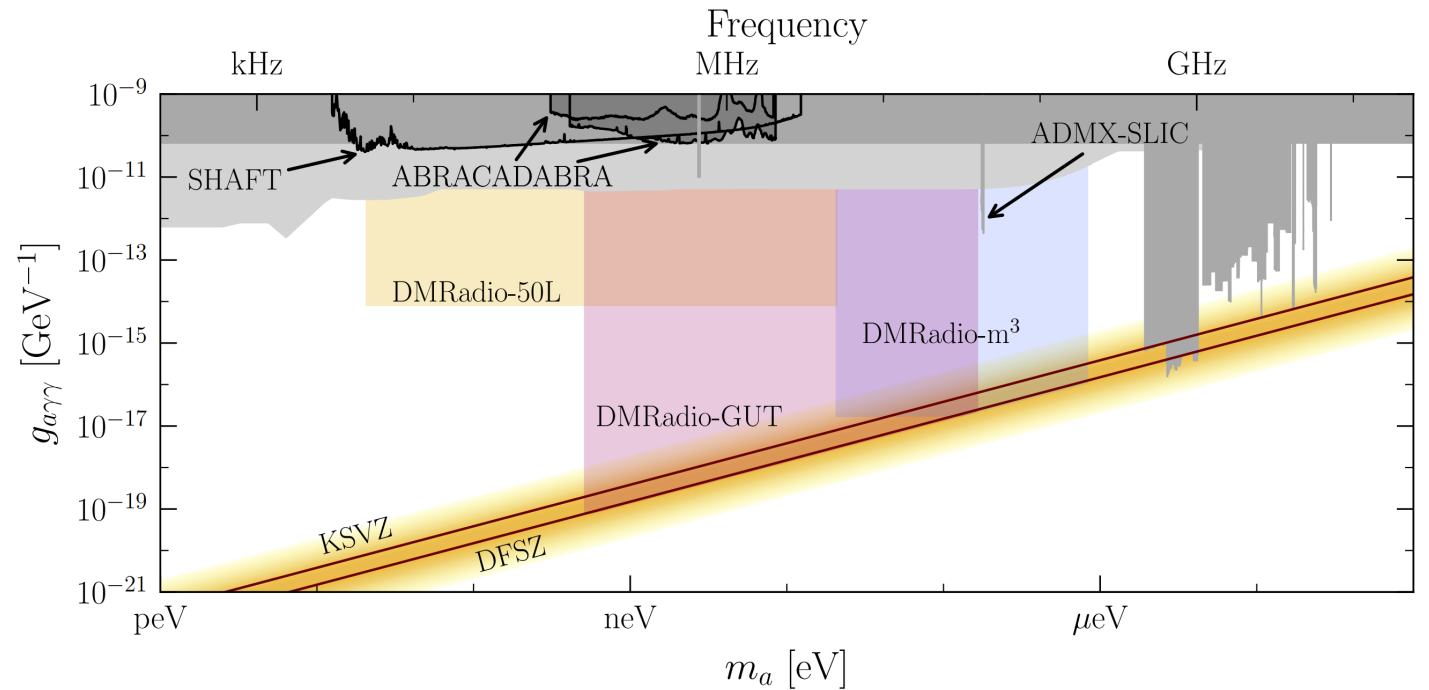
$$\Phi_{\text{Axion,Solenoid}} = \frac{1}{4} g_{a\gamma\gamma} \dot{a} B_0 l R^2$$



Axion Haloscope as GW Detector



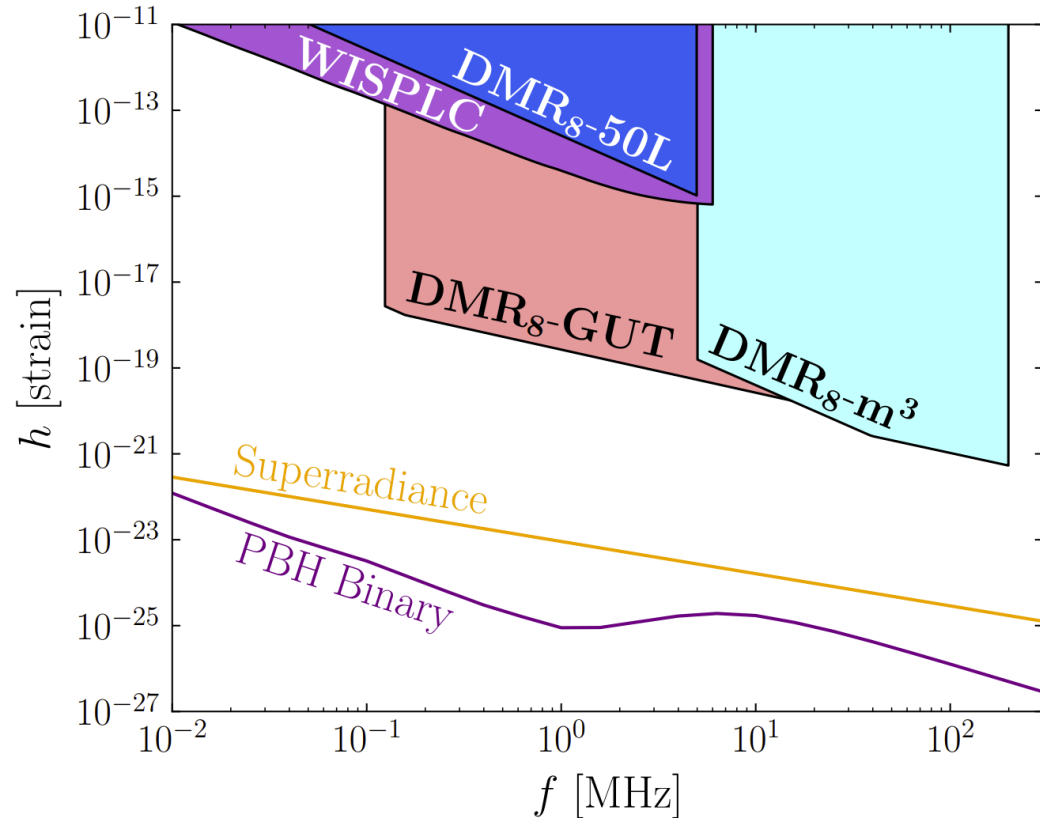
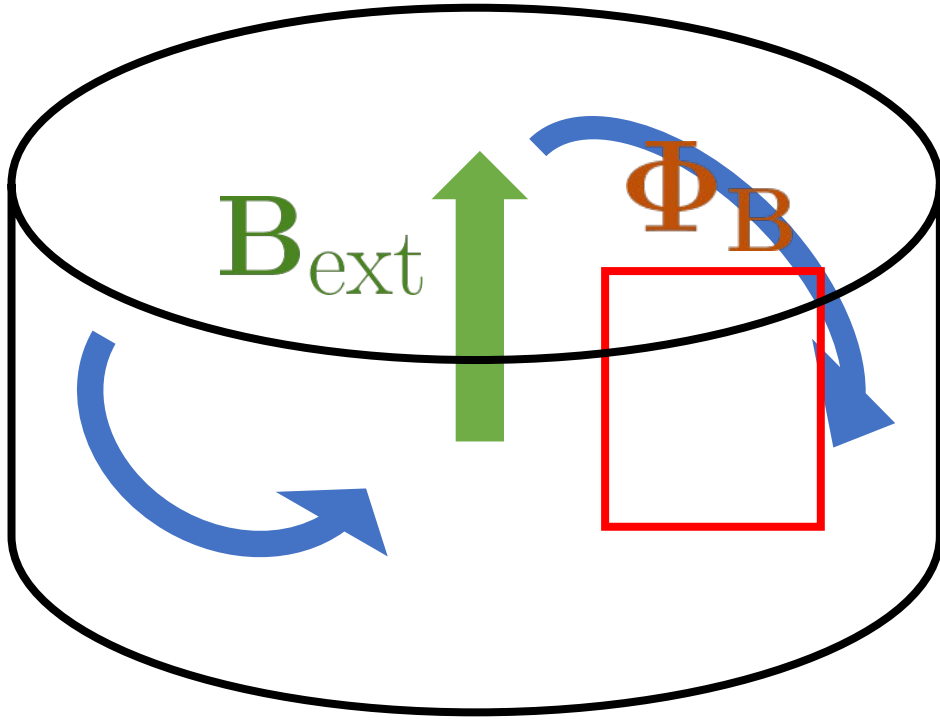
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Axion Haloscope as GW Detector

[Domcke, Carcia-Cely, SML, Rodd '23]

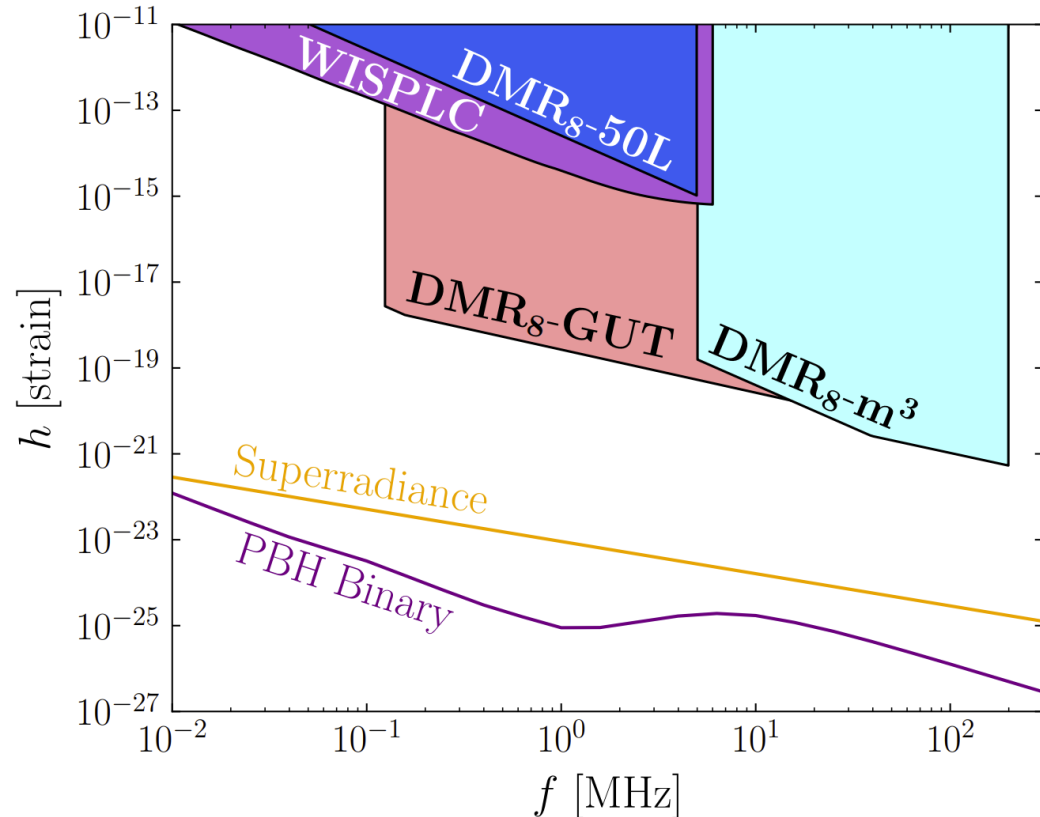
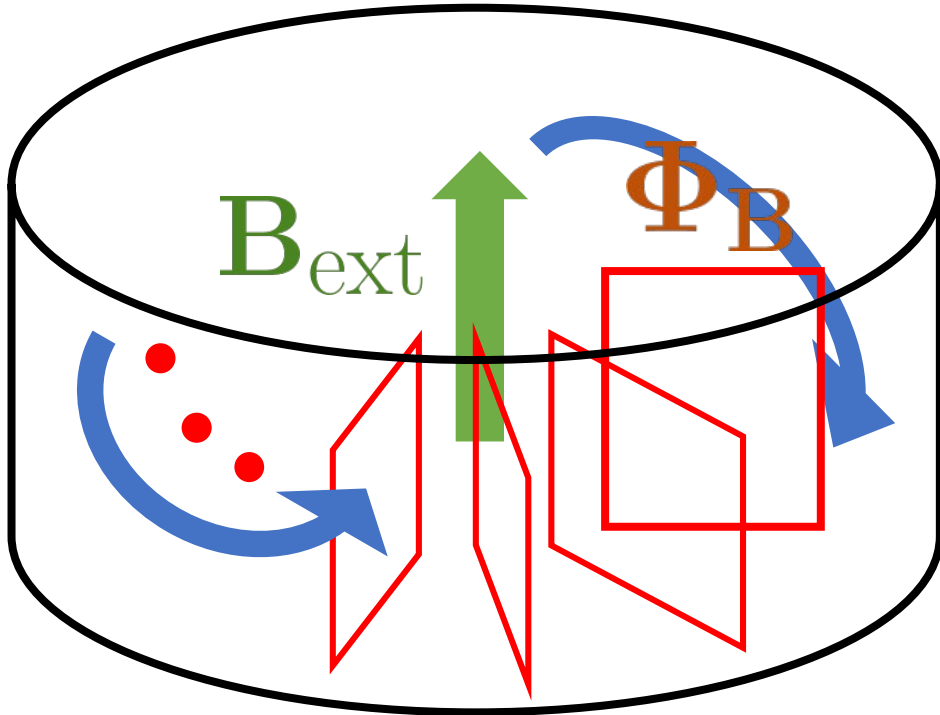
$$\Phi_{\text{GW}} = \frac{e^{-i\omega t}}{144} \omega^2 B_z l r (30R^2 - 13r^2) \sin \theta_h (h^+ \cos \theta_h \sin \phi_L + h^\times \cos \phi_L) + \mathcal{O}[(\omega L)^3]$$



Axion Haloscope as GW Detector

[Domcke, Carcia-Cely, SML, Rodd '23]

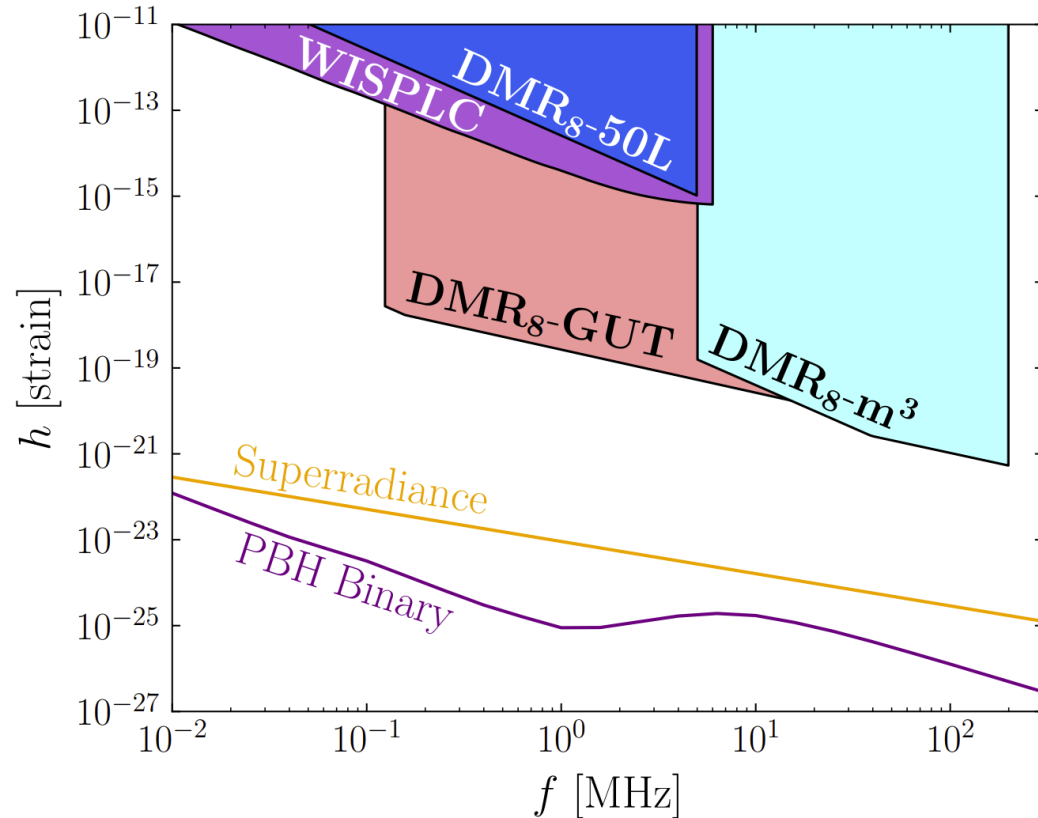
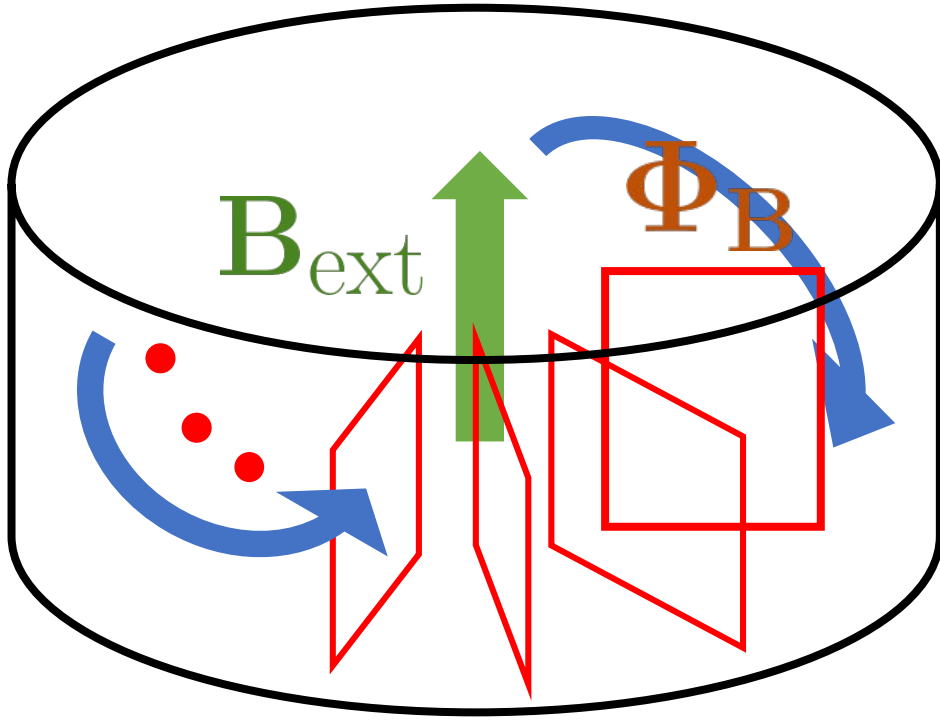
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Axion Haloscope as GW Detector

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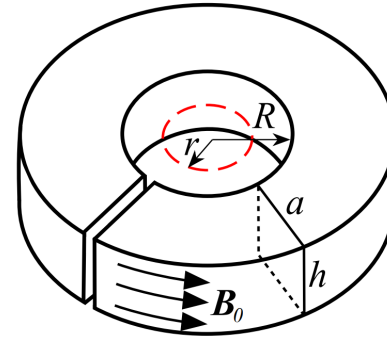


Good for Axion, Bad for GW

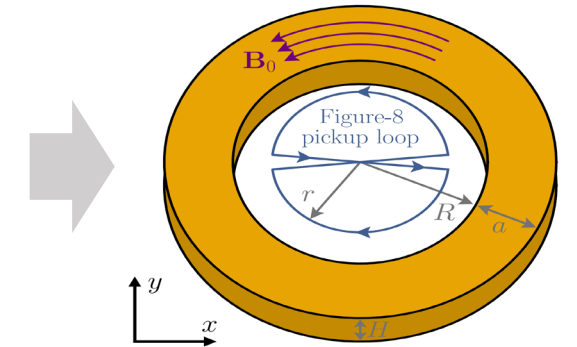
- **Cylindrical symmetry**
 - cancels leading order of GW signal

[Domcke, Carcia-Cely, SML, Rodd '23]

ABRACADABRA



[Kahn, Safdi, Thaler '16]



[Domcke, Carcia-Cely, Rodd '22]

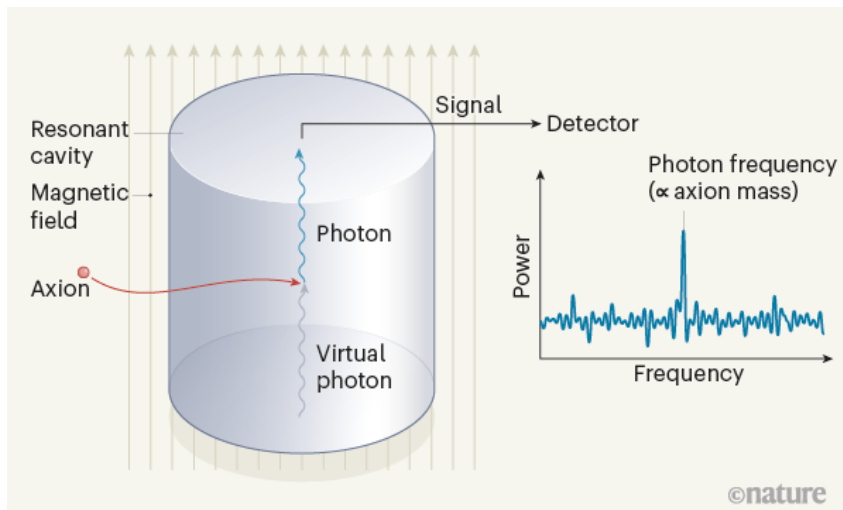
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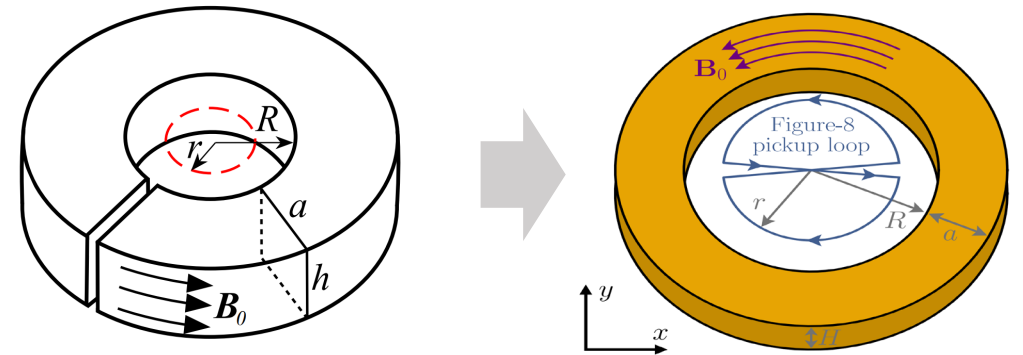
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[Domcke, Carcia-Cely, SML, Rodd '23]

- **Resonant vs Broadband**

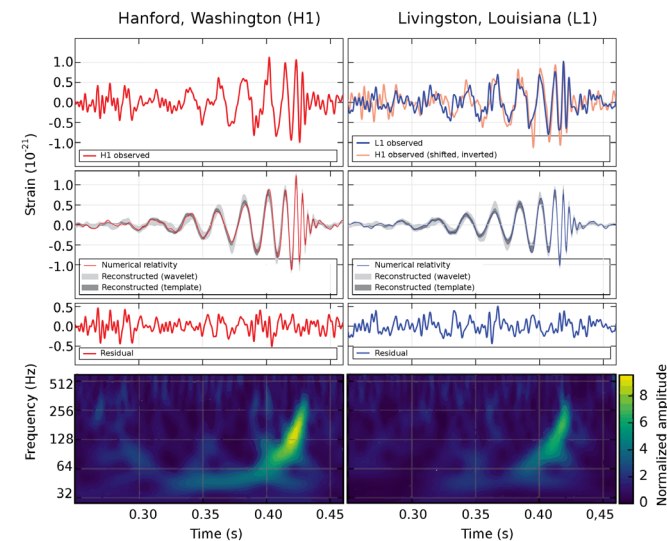


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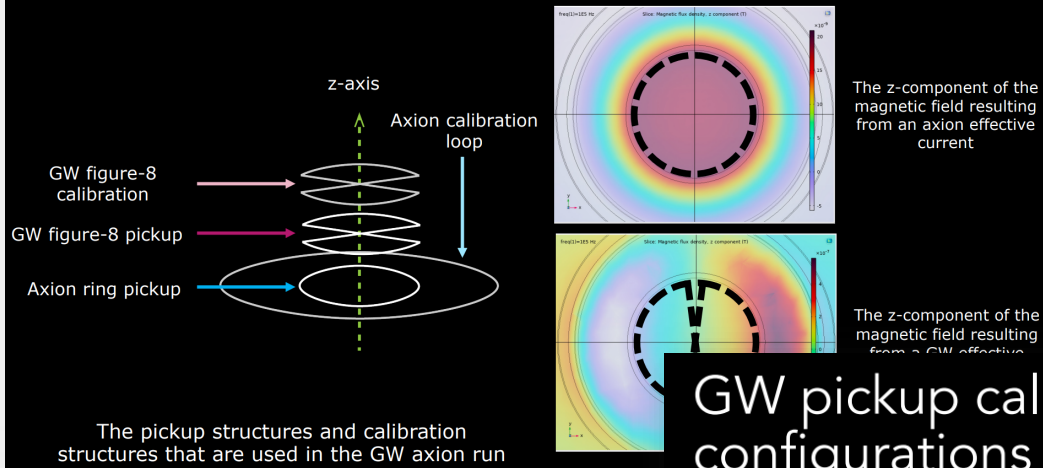
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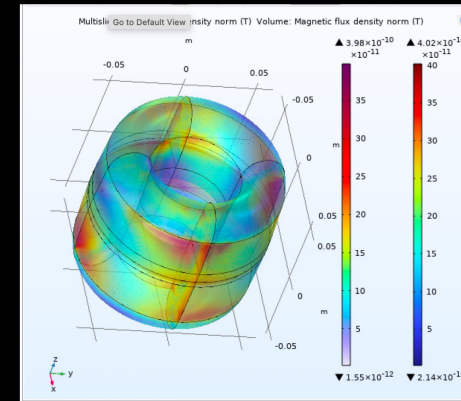


REAL experimental efforts is going on!

Experimental Setup



Signal modeling in detector

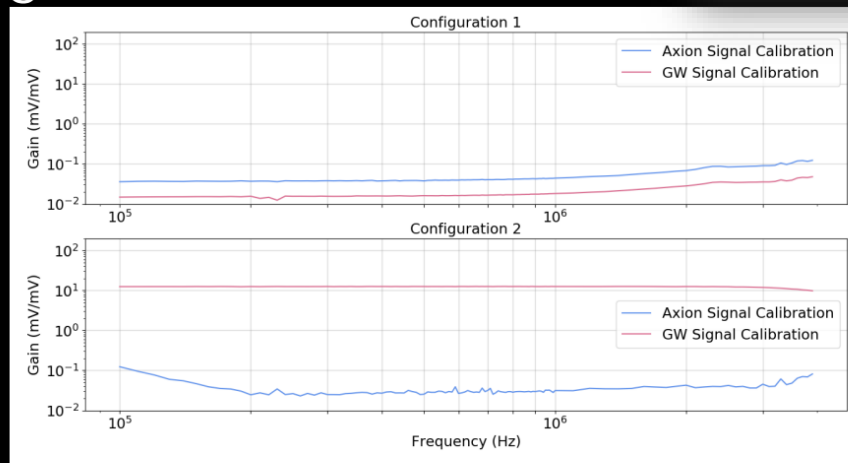


COMSOL is used to model the signal in the ABRA magnetic volume

Using the equations from Valerie Domcke, Camilo Garcia-Cely, Sung Mook Lee, Nicholas L. Rodd 2306.03125

Equations E.4–6 for effective current in a toroidal magnet

GW pickup calibration results from configurations 1 & 2

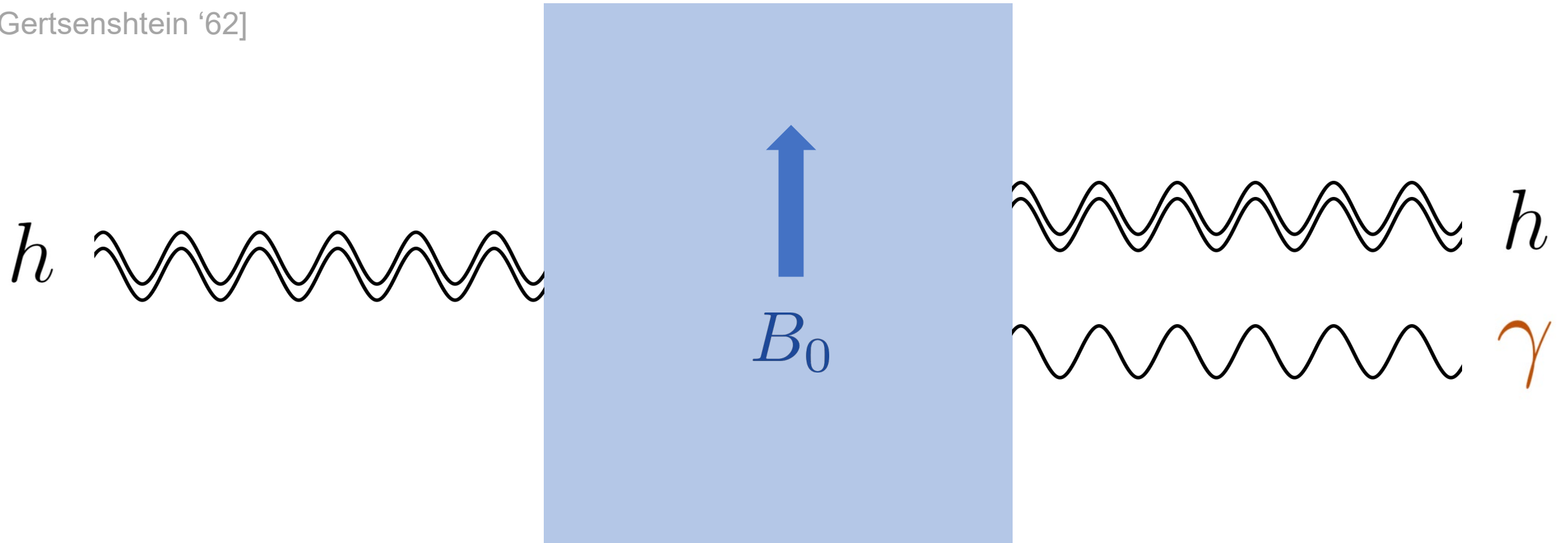


[Kaliroë Pappas's Slides @ 'UHF-GW: where to next?' workshop at CERN (Dec, 2023)]

HFGW from the Sky: Neutron Stars

Neutron Star as GW Detector

[Gertsenshtein '62]



$$P_{h \rightarrow \gamma} = \sin^2 \left(\frac{L}{\sqrt{2} M_P / B_0} \right) \simeq \frac{2L^2 B_0^2}{M_P^2}$$

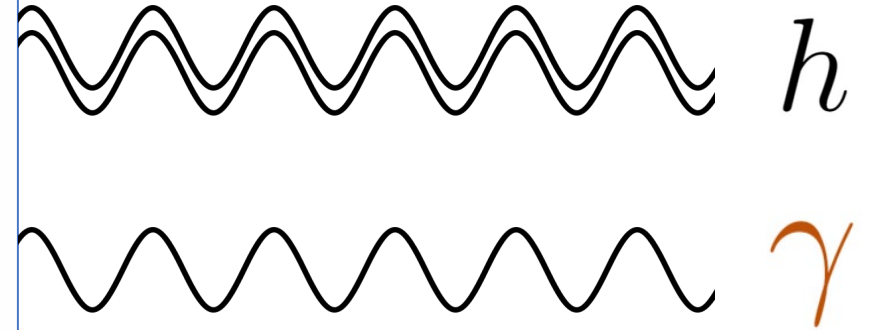
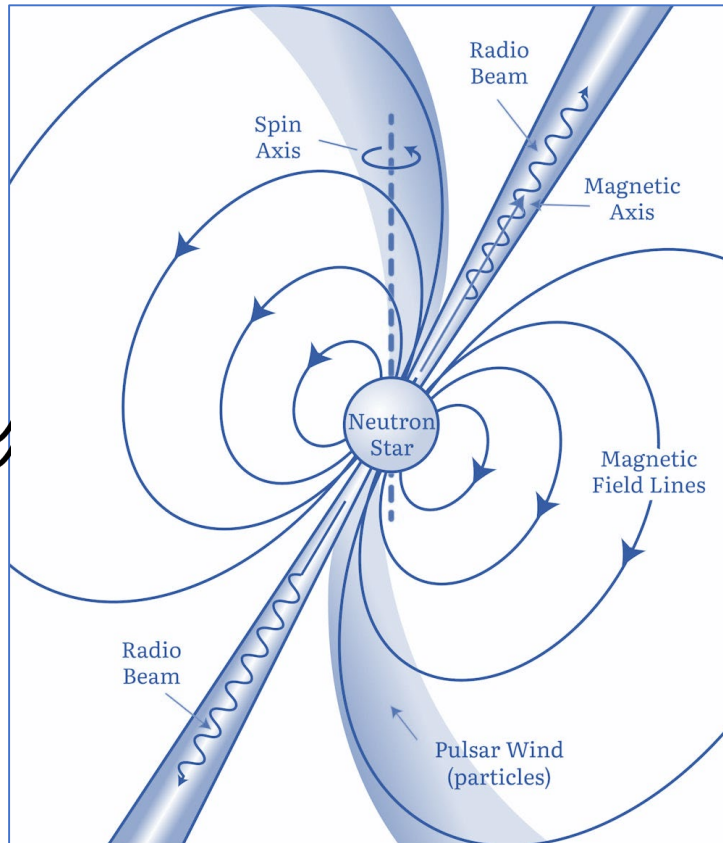
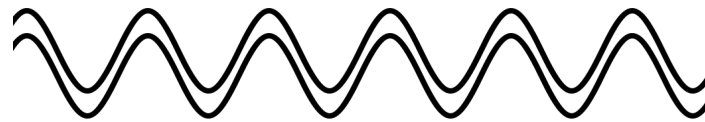
Neutron Star as GW Detector

[Ito, Kohri, Nakayama '23]

[Dandoy, Bertolez-Martinez, Costa '24]

[McDonald, Ellis '24]

h



$$P_{h \rightarrow \gamma} = \sin^2 \left(\frac{L}{\sqrt{2} M_P / B_0} \right) \simeq \frac{2L^2 B_0^2}{M_P^2}$$

Neutron Star as GW Detector

[Raffelt, Stodolsky '88]

▪ Effective photon mass

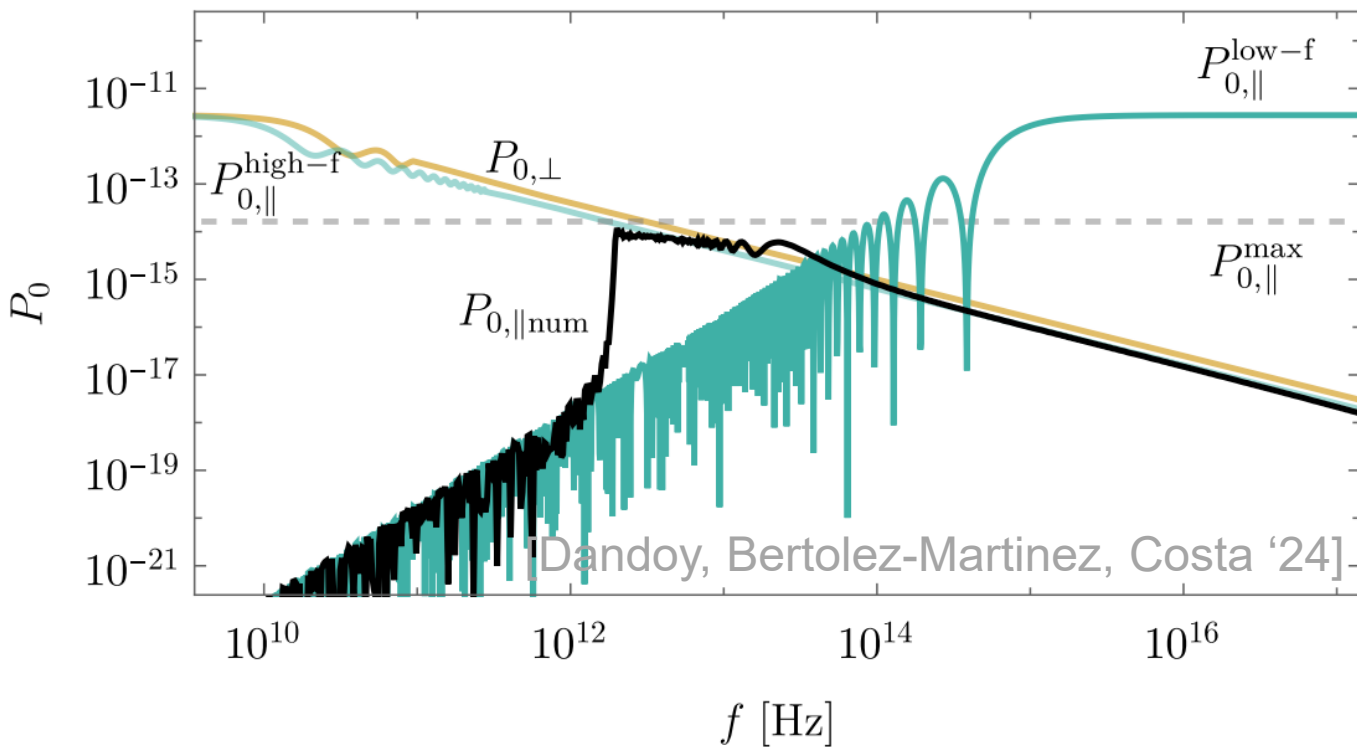
$$\left(\omega - i\partial_n + \begin{pmatrix} \Delta_{\parallel(\perp)} & \kappa B_{\parallel}/2 \\ \kappa B_{\parallel}/2 & 0 \end{pmatrix} \begin{pmatrix} A_{\parallel(\perp)} \\ h_{\times(+)} \end{pmatrix} \right) = 0$$

$$\Delta_{\parallel} = \Delta_{\text{pla},\parallel} + \Delta_{\text{vac},\parallel} \begin{cases} \Delta_{\text{pla},\parallel} = -\frac{m_{\text{pla}}^2}{2\omega} & \propto -\omega^{-1} \\ \Delta_{\text{vac},\parallel} = \frac{7\alpha\omega}{90\pi} \left(\frac{eB_{\parallel}}{m_e^2} \right)^2 & \propto \omega \end{cases} \quad m_{\text{pla}}^2 \simeq \frac{4\pi\alpha n_e}{m_e}$$

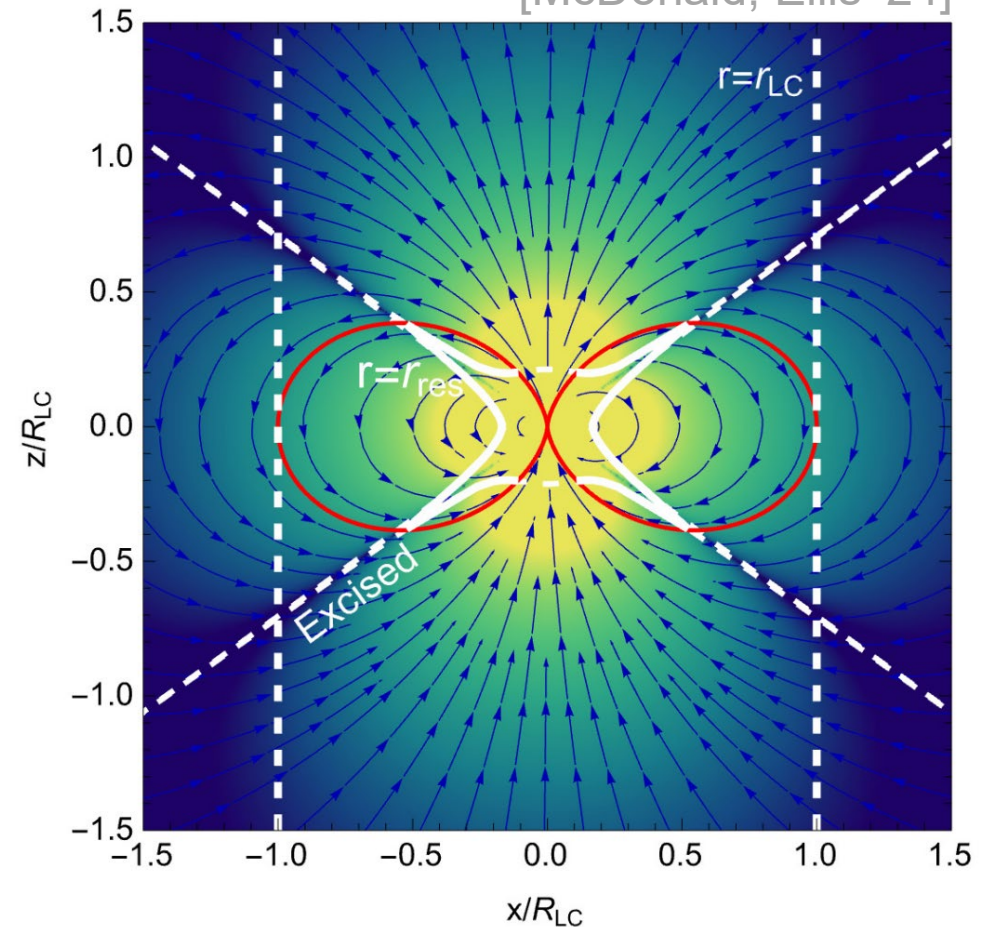
Neutron Star as GW Detector

[Raffelt, Stodolsky '88]

$$P_{\parallel(\perp)}(f) = \left| \int_{\ell_0}^{\ell_1} d\ell \Delta_M(\ell) \exp \left\{ -i \int_{\ell_0}^{\ell} d\ell' \Delta_{\parallel(\perp)}(\ell') \right\} \right|^2$$

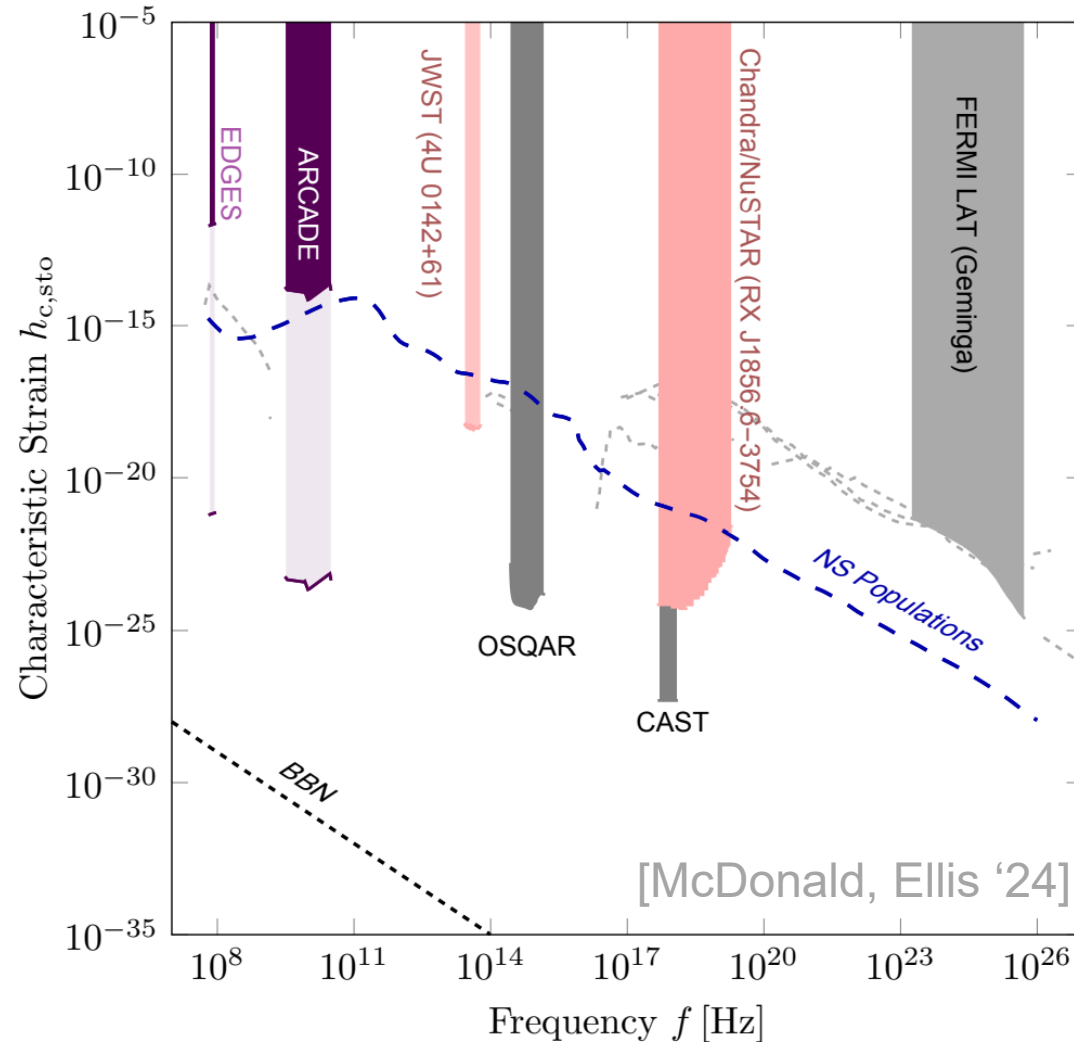


[McDonald, Ellis '24]



$$S = \frac{1}{4\pi d^2} \int d\Omega_{\mathbf{k}} \int d\Sigma_{\mathbf{k}} \cdot \mathbf{v}_p P_{h \rightarrow \gamma} \frac{\omega}{16\pi G} h_{c,sto}^2$$

Neutron Star as GW Detector



Conclusion

- Detection of HFGW is a smoking gun of BSM
 - new opportunities, interesting theoretical questions, and experimental challenges

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 - Different sources, different strategies (e.g. geometries, resonant/broadband)

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- Strong interplay to axion experiments
 - Different sources, different strategies (e.g. geometries, resonant/broadband)
- A lot more opportunities from cosmology/astrophysics
 - New ideas are exploding!

Back Ups

Selection Rules of Cylindrical Detector

Selection Rule 1: For an instrument with azimuthal symmetry, $\Phi_h \propto h^+$ at $\mathcal{O}[(\omega L)^2]$.¹⁶

Selection Rule 2: For an instrument with azimuthal symmetry, the flux is proportional to either h^+ or h^\times , but not both. This holds to all orders in (ωL) .¹⁷

$$\Phi \sim h^\times \omega^3 L^5$$

Selection Rule 3: For an instrument with full cylindrical symmetry, Φ_h will contain only even or odd powers of ω .

Selection Rules of Cylindrical Detector

Leading Orders

[Domcke, Garcia-Cely, SML, Rodd '23]

		\hat{n}'		
		\hat{e}_z	\hat{e}_ϕ	\hat{e}_ρ
B	\hat{e}_z (Sol)	h_+ , even : $O[(\omega L)^2]$	h_\times , odd : $O[(\omega L)^3]$ BASE	h_+ , odd : $O[(\omega L)^3]$ off-center: $O[(\omega L)^2]$
	\hat{e}_ϕ (Toro)	h_\times , odd : $O[(\omega L)^3]$ ABRA	h_+ , even : $O[(\omega L)^{\cancel{3}4}]$	h_\times , even : $O[(\omega L)^4]$ off-center: $O[(\omega L)^3]$

Optimal axion detection forbids optimal GW detection

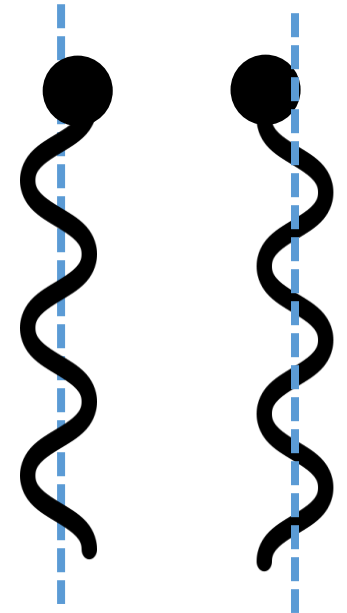
Note on Frame

▪ **TT frame**
$$h_{ij}^{TT} = (h^+ e_{ij}^+(\phi_h, \theta_h) + h^\times e_{ij}^\times(\phi_h, \theta_h)) e^{i(\mathbf{k}\cdot\mathbf{r} - \omega t)}$$

- Coordinates fixed by geodesic of freely falling test masses

- GW takes the simple form
$$h_{0\mu} = 0, h_i^i = 0, \partial_j h^{ij} = 0$$

- Detector (rigid body) looks oscillating in presence of GWs
→ makes the experimental setup & observables obscure



Note on Frame

[Berlin, Blas, Tito D'Agnolo, Ellis, Harnik, Kahn, Schutte-Engel '21]
[Domcke, Carcia-Cely, Rodd '22]

■ Proper detector frame

- Coordinates fixed by laboratory frame
- More involved form

$$h_{00} = \omega^2 e^{-i\omega t} F(\mathbf{k} \cdot \mathbf{r}) r_m r_n \sum_{A=+, \times} h^A e_{mn}^A(\hat{\mathbf{k}}), \quad F(\xi) = (e^{i\xi} - 1 - i\xi)/\xi^2$$

$$h_{0i} = \frac{1}{2} \omega^2 e^{-i\omega t} [F(\mathbf{k} \cdot \mathbf{r}) - iF'(\mathbf{k} \cdot \mathbf{r})] [\hat{\mathbf{k}} \cdot \mathbf{r} r_m \delta_{ni} - r_m r_n \hat{k}_i] \sum_{A=+, \times} h^A e_{mn}^A(\hat{\mathbf{k}}),$$

$$h_{ij} = -i\omega^2 e^{-i\omega t} F'(\mathbf{k} \cdot \mathbf{r}) [|\mathbf{r}|^2 \delta_{im} \delta_{jn} + r_m r_n \delta_{ij} - r_n r_j \delta_{im} - r_m r_i \delta_{jn}] \sum_{A=+, \times} h^A e_{mn}^A(\hat{\mathbf{k}})$$

- Description of the experimental setup and observables is straightforward

Note on Frame

[Berlin, Blas, Tito D'Agnolo, Ellis, Harnik, Kahn, Schutte-Engel '21]
 [Domcke, Carcia-Cely, Rodd '22]

■ Proper detector frame

- Coordinates fixed by laboratory frame

- More involved form

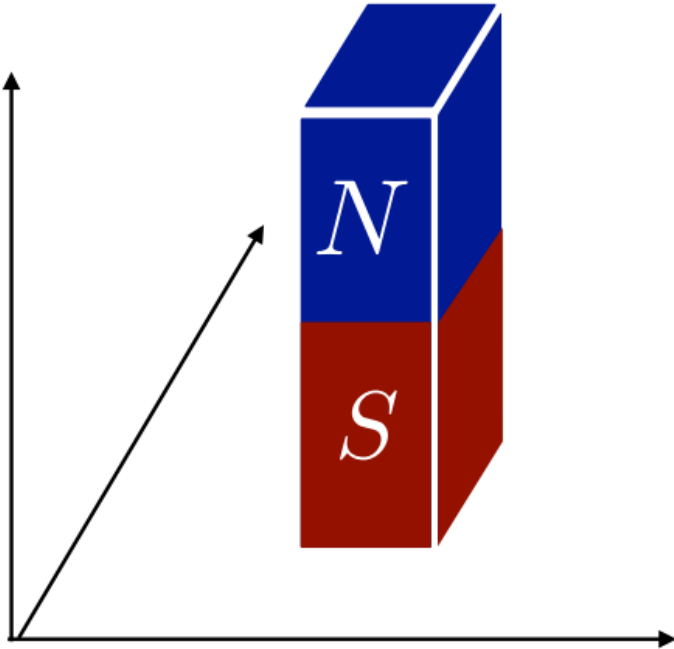
Leading order : $O(\omega^2 L^2)$

$$\begin{aligned}
 h_{00} &= \omega^2 e^{-i\omega t} F(\mathbf{k} \cdot \mathbf{r}) r_m r_n \sum_{A=+, \times} h^A e_{mn}^A(\hat{\mathbf{k}}), & F(\xi) &= (e^{i\xi} - 1 - i\xi)/\xi^2 \\
 h_{0i} &= \frac{1}{2} \omega^2 e^{-i\omega t} [F(\mathbf{k} \cdot \mathbf{r}) - iF'(\mathbf{k} \cdot \mathbf{r})] [\hat{\mathbf{k}} \cdot \mathbf{r} r_m \delta_{ni} - r_m r_n \hat{k}_i] \sum_{A=+, \times} h^A e_{mn}^A(\hat{\mathbf{k}}), \\
 h_{ij} &= -i\omega^2 e^{-i\omega t} F'(\mathbf{k} \cdot \mathbf{r}) [|\mathbf{r}|^2 \delta_{im} \delta_{jn} + r_m r_n \delta_{ij} - r_n r_j \delta_{im} - r_m r_i \delta_{jn}] \sum_{A=+, \times} h^A e_{mn}^A(\hat{\mathbf{k}})
 \end{aligned}$$

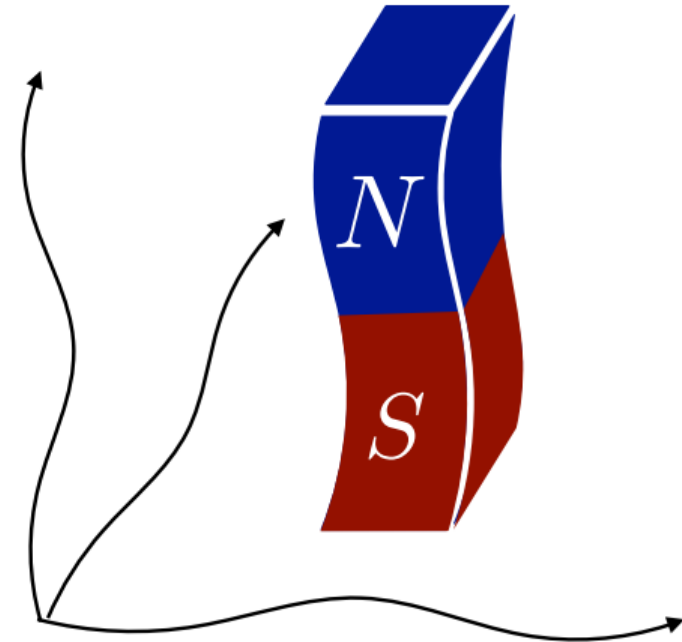
- Description of the experimental setup and observables is straightforward

Note on Frame

Proper detector frame



TT frame



Example: Solenoidal Geometry

- For single pickup loop [Domcke, Garcia-Cely, SML, Rodd '23]

$$\Phi_{\text{GW}} = \frac{e^{-i\omega t}}{144} \omega^2 B_z l r (30R^2 - 13r^2) \sin \theta_h (h^+ \cos \theta_h \sin \phi_L + h^\times \cos \phi_L) + \mathcal{O}[(\omega L)^3]$$

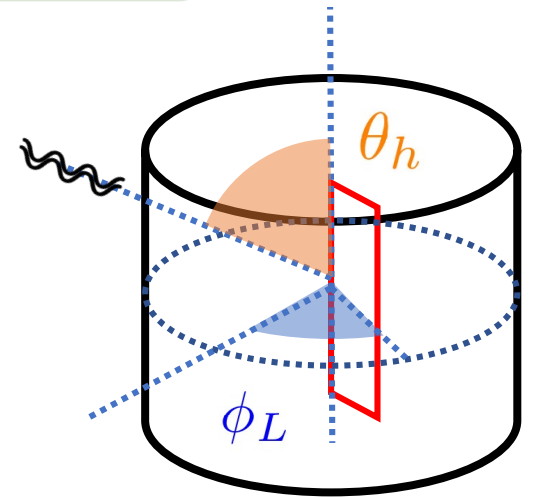
leading order

$$O(\omega^2)$$

volume effect

angular dependence

- Toroidal loop (ϕ_L integration): $\Phi_{\text{GW,Sol}} = O(\omega^3)$



This cancellation *always* happens for *cylindrically symmetric* axion detectors

Example: Toroidal Geometry

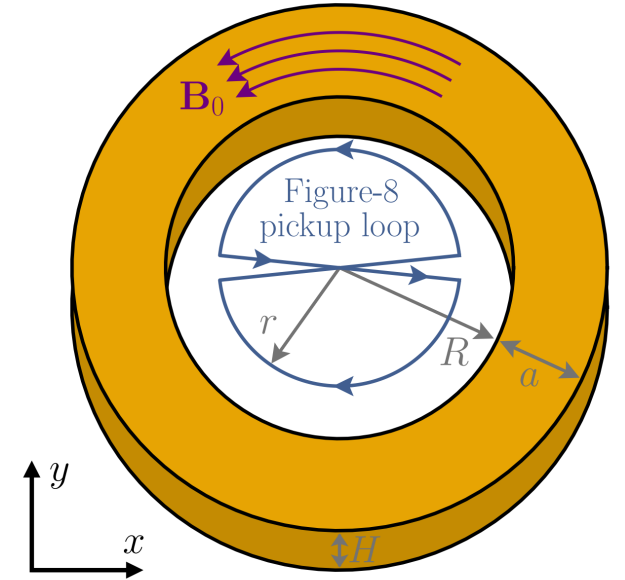
- Normal Loop

$$\Phi_{\text{GW}} = \frac{ie^{-i\omega t}}{48} \omega^3 B_{\text{max}} \pi r^2 R a (a + 2R) h^\times \sin^2 \theta_h$$

- Figure-8 Loop

$$\Phi_{\text{GW, Fig-8}} = \frac{e^{-i\omega t}}{3} \omega^2 B_{\text{max}} r^3 R \ln \left(1 + \frac{a}{R} \right) s_{\theta_h} (h^\times s_{\phi_h} - h^+ c_{\theta_h} c_{\phi_h})$$

- Kills axion sensitivity



[2202.00695]

Electromagnetism with Axion

- **Axion-Photon Coupling**

$$\mathcal{L}_{\text{int}} = -\frac{1}{4} g_{a\gamma\gamma} a F_{\mu\nu} \tilde{F}^{\mu\nu}$$

- **Effective Current**

$$\partial_\nu F^{\mu\nu} = \partial_\nu \left(g_{a\gamma\gamma} a \tilde{F}^{\nu\mu} \right) = g_{a\gamma\gamma} (\partial_\nu a) \tilde{F}^{\nu\mu} \equiv j_{\text{eff}}^\mu$$

- Axion is mainly sensitive to the magnetic field

$$\mathbf{j}_{\text{eff}} = g_{a\gamma\gamma} \sqrt{2\rho_{\text{DM}}} \cos(m_a t) \mathbf{B} \quad \rho_{\text{DM}} \simeq 0.3 \text{ GeV}/\text{cm}^3$$